Q1. Logic

(a) Prove, or find a counterexample to, each of the following assertions:

(i) If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \land \beta) \models \gamma$

True, because of monotonicity.

(ii) If $(\alpha \land \beta) \models \gamma$ then $\alpha \models \gamma$ or $\beta \models \gamma$ (or both).

False, $\alpha \equiv A$, $\beta \equiv B$, $\gamma \equiv (A \land B)$

(iii) If $\alpha \models (\beta \lor \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

False, $\beta \equiv A$, $\gamma \equiv \neg A$

(b) Decide whether each of the following sentences is valid, unsatisfiable, or neither.

(i) $Smoke \implies Smoke$

Valid

(ii) $Smoke \implies Fire$

Neither

(iii) $(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$

Neither

(iv) $Smoke \lor Fire \lor \neg Fire$

Valid

(v) $((Smoke \land Heat) \implies Fire) \Longleftrightarrow ((Smoke \implies Fire) \lor (Heat \implies Fire))$

Valid

(vi) $(Smoke \implies Fire) \implies ((Smoke \land Heat) \implies Fire)$

Valid

(vii) $Big \lor Dumb \lor (Big \implies Dumb)$

Valid

(c) Suppose an agent inhabits a world with two states, S and $\neg S$, and can do exactly one of two actions, a and b. Action a does nothing and action b flips from one state to the other. Let S^t be the proposition that the agent is in state S at time t, and let a^t be the proposition that the agent does action a at time t (similarly for b^t).

(i) Write a successor-state axiom for S^{t+1}.

$S^{t+1} \Longleftrightarrow [(S^t \land a^t) \lor (\neg S^t \land b^t)$

(ii) Convert the sentence in the previous part into CNF.

Agent can do only one action, we know that $b^t = \neg a^t$ so we can replace b^t and obtain:

1. $(\neg s^{t+1} \vee s^t \vee \neg a^t)$
2. $(\neg s^{t+1} \vee \neg s^t \vee a^t)$
3. $(s^{t+1} \vee s^t \vee \neg a^t)$
4. $(s^{t+1} \vee s^t \vee a^t)$

Q2. First Order Logic

Consider a vocabulary with the following symbols:

- $Occuption(p, o)$: Predicate. Person p has occuption o.
- $Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
- $Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
- $Doctor, Surgeon, Lawyer, Actor$: Constants denoting occupations.
- $Emily, Joe$: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

(iii) Emily is either a surgeon or a lawyer.

$$O(E,S) \vee O(E,L)$$

(iv) Joe is an actor, but he also holds another job.

$$O(J,A) \wedge \exists o,\ o \neq A \wedge O(J,o)$$

(v) All surgeons are doctors.

$$\forall p\ O(p,S) \Rightarrow O(p,D)$$

(vi) Joe does not have a lawyer (i.e., is not a customer of any lawyer).

$$\neg E p\ C(J,p) \wedge O(p,L)$$

(vii) Emily has a boss who is a lawyer.

$$\exists p\ B(p,E) \wedge O(p,L)$$

(viii) There exists a lawyer all of whose customers are doctors.

$$\exists p\ O(p,L) \wedge \forall q\ C(q,p) \Rightarrow O(q,D)$$

(ix) Every surgeon has a lawyer.

$$\forall p\ O(p,S) \Rightarrow \exists q\ O(q,L) \wedge C(p,q)$$

Q3. [Optional] Local Search

(a) Hill Climbing

(i) Hill-climbing is complete. ☐ True ☒ False

(ii) Hill-climbing is optimal. ☐ True ☒ False

(b) Simulated Annealing

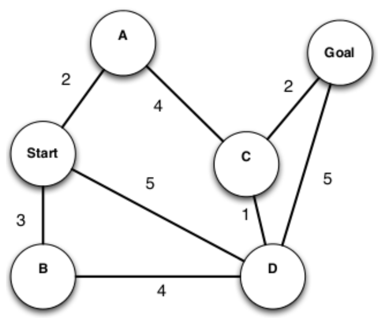
(i) The higher the temperature T is, the more likely the randomly chosen state will be expanded. ☒ True ☐ False

(ii) In one round of simulated annealing, the temperature is 2 and the current state S has energy 1. It has 3 successors: A with energy 2; B with energy 1; C with energy 1-ln 4. If we assume the temperature does not change, What's the probability that these states will be chosen to expand after S eventually?

(iii) On a undirected graph, If T decreases slowly enough, simulated annealing is guaranteed to converge to the optimal state. ☒ True ☐ False

(c) Local Beam Search

The following state graph is being explored with 2-beam graph search. A state's score is its accumulated distance to the start state and lower scores are considered better. Which of the following statements are true?



☒ States A and B will be expanded before C and D.

☐ States A and D will be expanded before B and C.

☐ States B and D will be expanded before A and C.

☐ None of above.

(d) Genetic Algorithm

(i) In genetic algorithm, cross-over combine the genetic information of two parents to generate new offspring.

☒ True ☐ False

(ii) In genetic algorithm, mutation involves a probability that some arbitrary bits in a genetic sequence will be flipped from its original state.

☒ True ☐ False

(e) Gradient Descent

(i) Gradient descent is optimal. ☐ True ☒ False

(ii) For a function $f(x)$ with derivative $f'(x)$, write down the gradient descent update to go from x_t to x_{t+1}. Learning rate is α.