

# R E Š E N J A

Klasifikacionog ispita iz Matematike za 2016. godinu:

- 1.** Uprostiti izraz  $I = \frac{x^2 - 4xy + 3y^2}{x^2 - y^2}$ .

$$\begin{aligned} I &= \frac{x^2 - 4xy + 3y^2}{x^2 - y^2} = \frac{x^2 - xy - 3xy + 3y^2}{(x-y)(x+y)} = \frac{x(x-y) - 3y(x-y)}{(x-y)(x+y)} = \\ &= \frac{(x-y)(x-3y)}{(x-y)(x+y)} = \boxed{\frac{x-3y}{x+y}}, \text{ za } x \neq \pm y. \end{aligned}$$

- 2.** Rastaviti na faktore polinom  $P(x) = x^5 - x^3 - x^2 + 1$ .

$$\begin{aligned} P(x) &= x^3(x^2 - 1) - (x^2 - 1) = (x^2 - 1)(x^3 - 1) \\ &= (x-1)(x+1)(x-1)(x^2 + x + 1) = \boxed{(x-1)^2(x+1)(x^2 + x + 1)}. \end{aligned}$$

- 3.** Uprostiti izraz  $I = \frac{1+x+(1-x)^{-1}}{1+(1-x^2)^{-1}}$ .

$$\begin{aligned} I &= \frac{1+x+(1-x)^{-1}}{1+(1-x^2)^{-1}} = \frac{1+x+\frac{1}{1-x}}{1+\frac{1}{1-x^2}} = \frac{\frac{1-x+x-x^2+1}{1-x}}{\frac{1-x^2+1}{(1-x)(1+x)}} = \\ &= \frac{(2-x^2)(1-x)(1+x)}{(2-x^2)(1-x)} = \boxed{1+x}, \text{ za } x \neq \pm 1, x \neq \pm\sqrt{2}. \end{aligned}$$

- 4.** Rešiti jednačinu  $\frac{9x-8}{7} = 7 - \frac{5x+7}{9}$ .

Ako jednačinu pomnožimo sa 63 dobijamo

$$\begin{aligned} 9(9x-8) &= 7 \cdot 63 - 7(5x+7) \iff 81x - 72 = 441 - 35x - 49 \iff \\ 116x &= 464 \iff \boxed{x = 4}. \end{aligned}$$

- 5.** Rešiti jednačinu  $\frac{x^2 - 2x}{x^2 - 4} = 0$ .

Za  $x \neq \pm 2$  je  $x^2 - 2x = 0 \iff x(x-2) = 0 \iff x = 0 \vee x-2 = 0$

$$\iff \boxed{x = 0}$$

- 6.** Rešiti sistem jednačina  $3x + 5y = 1 \wedge 3x - 2y = 8$ .

Ako prvu jednačinu pomnožimo sa  $-1$  i dodamo drugoj, dobijamo

$$\begin{aligned} -3x - 5y &= -1 \wedge 3x - 2y = 8 \iff -7y = 7 \wedge 3x - 2y = 8 \\ \iff y &= -1 \wedge 3x + 2 = 8 \iff \boxed{x = 2} \wedge \boxed{y = -1}. \end{aligned}$$

7. Rešiti jednačinu  $(x + 1)^2 - 25 = 0$ .

$$(x+1)^2 - 25 = 0 \iff (x+1)^2 - 5^2 = 0 \iff (x+1-5)(x+1+5) = 0 \iff (x-4)(x+6) = 0 \iff x-4=0 \vee x+6=0 \iff [x=4] \vee [x=-6].$$

8. Za koju vrednost parametra  $m \in \mathbb{R}$  kvadratna jednačina  $x^2 + 6x + m = 0$  ima realna rešenja?

Jednačina ima realna rešenja ako i samo ako je  $D=b^2-4ac\geq 0$ , tj. ako je  $D=6^2-4\cdot 1\cdot m\geq 0 \iff 36-4m\geq 0 \iff [m\leq 9]$ .

9. Rešiti nejednačinu  $(x - 2)^2 - 9 > 0$ .

$$\begin{aligned} (x-2)^2 - 3^2 &= (x+1)(x-5) > 0 \iff \\ [(x+1 < 0 \wedge x-5 < 0) \vee (x+1 > 0 \wedge x-5 > 0)] &\iff \\ [x < -1 \vee x > 5] &\iff [x \in (-\infty, -1) \cup (5, +\infty)]. \end{aligned}$$

10. Rešiti jednačinu  $(2016)^{x^2-5x+4} = 1$ .

$$(2016)^{x^2-5x+4} = 1 \iff (2016)^{x^2-5x+4} = (2016)^0 \iff x^2 - 5x + 4 = 0 \iff [x=1] \vee [x=4].$$

11. Rešiti jednačinu  $\log_4(3x + 4) = 3$ .

Za  $3x + 4 > 0 \iff x > -\frac{4}{3}$  je

$$\log_4(3x + 4) = 3 \iff 3x + 4 = 4^3 \iff 3x = 60 \iff [x=20].$$

12. Izračunati vrednost izraza  $I = \log_6 2 + \log_6 3$ .

$$I = \log_6 2 + \log_6 3 = \log_6(2 \cdot 3) = \log_6 6 = [1].$$

13. Rešiti nejednačinu  $\frac{x-1}{x+2} < 1$ .

$$\begin{aligned} \text{Za } x \neq -2, \text{ je } \frac{x-1}{x+2} < 1 &\iff \frac{x-1}{x+2} - 1 < 0 \iff \frac{x-1-x-2}{x+2} < 0 \iff \frac{-3}{x+2} < 0 \\ &\iff x+2 > 0 \iff x > -2 \iff [x \in (-2, +\infty)]. \end{aligned}$$

14. Rešiti jednačinu  $4^x - 5 \cdot 2^x + 4 = 0$ .

$$4^x - 5 \cdot 2^x + 4 = 0 \iff 2^{2x} - 5 \cdot 2^x + 4 = 0 \iff (2^x)^2 - 5 \cdot 2^x + 4 = 0.$$

Smenom  $2^x = t$  dobijamo:  $t^2 - 5t + 4 = 0 \iff t = 1 \vee t = 4$ , pa je:

$$4^x - 5 \cdot 2^x + 4 = 0 \iff 2^x = 1 \vee 2^x = 4 \iff [x=0] \vee [x=2].$$

15. Napisati kanonski oblik parabole  $y = x^2 - 4x + 3$ .

$$y = a \left( x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a} = 1 \cdot \left( x + \frac{-4}{2 \cdot 1} \right)^2 + \frac{4 \cdot 1 \cdot 3 - (-4)^2}{4 \cdot 1} = [(x-2)^2 - 1].$$