

# Quick Modern Physics: A Self-Teaching Guide

Adapted from Physics Text

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# 1 Relativity I

## Frame 1

At the end of the 19th century, many scientists believed they had a good understanding of the physical world. Newton's laws of motion and his theory of gravitation, Maxwell's theory of electromagnetism, and the laws of thermodynamics seemed to explain a vast range of phenomena. However, a major revolution was brewing. In 1900, Max Planck introduced ideas that led to quantum theory, and in 1905, Albert Einstein formulated his special theory of relativity. These theories profoundly changed our understanding of nature.

This chapter focuses on Einstein's Special Theory of Relativity. What event in 1905 is particularly highlighted as a starting point for this theory? \_\_\_\_\_

Go to Frame 2.

## Frame 2

In 1905, Albert Einstein formulated his special theory of relativity. This theory was developed to address serious inconsistencies and contradictions that arose when Newtonian mechanics was applied to objects moving at very high speeds, or when considering the nature of light.

What is the accepted speed of light in a vacuum (free space), often denoted by the letter  $c$ ? (See page 2 of the original text, section 1.1) [a]  $3.00 \times 10^6$  m/s [b]  $3.00 \times 10^8$  m/s [c]  $3.00 \times 10^{10}$  m/s

Go to Frame 3.

## Frame 3

Your answer for the speed of light was [ a — b — c ].

The correct answer is [b]  $3.00 \times 10^8$  m/s. Light waves and other forms of electromagnetic radiation travel through free space at this incredibly high speed. This speed,  $c$ , sets an upper limit for the speeds of particles, waves, and the transmission of information.

Newtonian mechanics worked well for everyday experiences but failed when applied to phenomena involving speeds approaching the speed of light. For instance, experiments showed that the speed of an electron, even when accelerated by enormous voltages, never exceeds  $c$ . Newtonian mechanics, in contrast, would predict unlimited speeds.

What was Einstein's specific work called that addressed these issues? (Hint: It was introduced in 1905). \_\_\_\_\_

Go to Frame 4.

## Frame 4

In 1905, Einstein published his **special theory of relativity**. (Often just called "Special Relativity"). This theory arose from deep contradictions in the physics of the time and aimed to create a new framework consistent with experimental observations, especially concerning light and high-speed motion.

Einstein's special theory of relativity is based on two fundamental postulates. Can you name them (or at least describe their essence) \*before\* we look at them in detail? 1. \_\_\_\_\_ 2. \_\_\_\_\_

If you're unsure, that's okay! We'll cover them next. Go to Frame 5.

## Frame 5

The special theory of relativity is built upon two basic postulates:

1. **The Principle of Relativity:** The laws of physics are the same in all inertial reference frames. 2. **The Constancy of the Speed of Light:** The speed of light in a vacuum has the same value,  $c = 3.00 \times 10^8$  m/s, in all inertial frames, regardless of the motion of the light source or the observer.

Let's first discuss what an "inertial reference frame" is. Go to Frame 6.

### Frame 6

To describe a physical event, we need a frame of reference. Your studies in mechanics likely introduced Newton's laws, which are valid in **inertial frames of reference**.

What defines an inertial frame of reference? (See page 3, section 1.2). [a] A frame that is accelerating. [b] A frame in which an object subjected to no net force moves with constant velocity (or remains at rest). [c] Any frame attached to the Earth.

Go to Frame 7.

### Frame 7

Your answer for defining an inertial frame was [ a — b — c ].

The correct answer is [b]. An inertial frame is one in which Newton's first law (the law of inertia) holds true: an object with no net force acting on it will maintain a constant velocity (which could be zero). Any frame moving with constant velocity relative to an inertial frame is also an inertial frame.

Is there a "preferred" or "absolute" inertial frame in the universe according to the principle of relativity? [ Yes — No ]

Go to Frame 8.

### Frame 8

According to the **Principle of Relativity** (Einstein's first postulate), the answer is [No]. The laws of physics must take the same mathematical form for all observers in all inertial frames. This means no single inertial frame is special or preferred; all are equivalent for describing physical laws. For example, if you perform an experiment in a truck moving at a constant velocity, the laws of physics governing the experiment will be identical to those in a stationary laboratory.

Go to Frame 9.

### Frame 9

The second postulate, the **Constancy of the Speed of Light**, is quite radical. It states that the speed of light in a vacuum ( $c$ ) is always measured to be the same, regardless of the speed of the source emitting the light or the speed of the observer measuring it.

Imagine a spaceship A moving away from spaceship B at a speed of  $0.5c$ . Spaceship A fires a laser beam in its direction of motion. According to Newtonian ideas (Galilean relativity), what speed would spaceship B measure for the laser light? According to Einstein's second postulate, what speed would spaceship B measure for the laser light?

1. Newtonian prediction: \_\_\_\_\_ 2. Einstein's postulate prediction: \_\_\_\_\_

Go to Frame 10.

### Frame 10

1. Newtonian prediction: An observer on spaceship B would expect to measure the speed of light as  $0.5c(\text{speed of A}) + c(\text{speed of light relative to A}) = 1.5c$ . 2. Einstein's postulate prediction: An observer on spaceship B would measure the speed of light as simply  $c$ .

This constancy of  $c$  has profound implications. Let's consider how events are described in different inertial frames.

Suppose we have two inertial frames, S and S'. Frame S' moves with a constant velocity  $v$  along the x-axis relative to frame S. Their origins coincide at  $t = t' = 0$ . An event has coordinates  $(x, y, z, t)$  in S and  $(x', y', z', t')$  in S'. What set of equations, known as the **Galilean transformation of coordinates**, relates these coordinates in classical (Newtonian) physics? (See Figure 1.2 and surrounding text on page 4).  $x' = ?$   $y' = ?$   $z' = ?$   $t' = ?$

Go to Frame 11.

### Frame 11

The Galilean transformation equations are:  $x' = x - vt$   $y' = y$   $z' = z$   $t' = t$

A key assumption in the Galilean transformation is that time is absolute, i.e.,  $t = t'$ . This means all observers measure the same time interval between two events.

If we differentiate  $x' = x - vt$  with respect to time (assuming  $t = t'$ ), we get the Galilean addition law for velocities:  $u'_x = \frac{dx'}{dt'} = \frac{dx}{dt} - v = u_x - v$  where  $u_x$  is the velocity of an object as measured in S, and  $u'_x$  is its velocity in S'.

Does the Galilean velocity addition law  $u'_x = u_x - v$  (or  $u_x = u'_x + v$ ) correctly predict the speed of light according to Einstein's second postulate? [ Yes — No ] Explain briefly. \_\_\_\_\_

Go to Frame 12.

### Frame 12

The answer is [No]. If  $u_x = c$  (speed of light in S), then the Galilean transformation would predict  $u'_x = c - v$  in S'. This violates Einstein's second postulate, which states that the speed of light should be  $c$  in all inertial frames. This discrepancy highlights that the Galilean transformations are not consistent with special relativity, especially at high speeds.

Go to Frame 13.

### Frame 13

The **Michelson-Morley experiment** (1887) was a crucial experiment that attempted to detect the motion of the Earth through a hypothetical medium called the "ether," which was thought to be the medium through which light waves propagated. If the ether existed and the Earth moved through it, then the measured speed of light should depend on the direction of its travel relative to the Earth's motion.

What was the expected outcome if the ether theory was correct and light behaved like other waves in a moving medium (e.g., sound in wind)? (See page 7 discussion). \_\_\_\_\_

Go to Frame 14.

### Frame 14

If the ether theory was correct, the speed of light measured on Earth would vary depending on its direction relative to the Earth's motion through the ether.

- Light traveling "downwind" (in the direction of Earth's ether velocity  $v$ ) would have a speed  $c + v$ .
- Light traveling "upwind" (against Earth's ether velocity  $v$ ) would have a speed  $c - v$ .
- Light traveling "across the wind" would have a speed  $\sqrt{c^2 - v^2}$ .

The Michelson-Morley experiment used an interferometer (Figure 1.4) designed to detect these minute differences in the time it took for light to travel along two perpendicular paths as the apparatus was rotated.

What was the actual result of the Michelson-Morley experiment? [a] It confirmed the existence of the ether. [b] It found no significant difference in the speed of light, regardless of direction (a null result). [c] The results were inconclusive.

Go to Frame 15.

### Frame 15

Your answer was [ a — b — c ].

The correct answer is [b]. The Michelson-Morley experiment consistently yielded a **null result**. No fringe shift (which would indicate a difference in travel times) was observed that corresponded to the Earth's motion through a supposed ether.

This null result was deeply puzzling for classical physics but is perfectly consistent with Einstein's second postulate: the speed of light is constant in all inertial frames. Special relativity does not require an ether.

The details of the Michelson-Morley calculation (pages 8-9) show how a time difference  $\Delta t = t_1 - t_2$  between the two arms of the interferometer was expected, leading to a path difference  $\Delta d = c(2\Delta t)$  and a shift in interference fringes. The predicted shift was  $\Delta d/\lambda \approx 0.40$  fringes, but the observed shift was much smaller, consistent with zero.

Go to Frame 16.

### Frame 16

Let's re-state Einstein's postulates for Special Relativity (from page 10, section 1.4):

1. **The Principle of Relativity:** The laws of physics are the same in all inertial frames of reference. There is no preferred inertial frame. 2. **The Constancy of the Speed of Light:** The speed of light in a vacuum has the same value,  $c \approx 3.00 \times 10^8$  m/s, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

These two postulates form the bedrock of Special Relativity and lead to remarkable consequences that challenge our everyday intuitions about space and time. We will explore some of these consequences in the next part of this chapter (which would correspond to section 1.5 onwards in the original text).

This concludes the introduction to the postulates. Review these frames if needed.

End of Section.

## 2 Consequences of Special Relativity

### Frame 1

Einstein's postulates of special relativity lead to some startling predictions that challenge our everyday intuitions about space and time. We will find that concepts like length, time, and simultaneity, which we usually consider absolute, are actually relative and depend on the observer's frame of reference.

Before we dive into specific consequences, what is a fundamental requirement for describing an event in an inertial reference frame, especially when considering events at different locations? (Hint: Think about clocks, as shown in Figure 1.8 on page 13). \_\_\_\_\_

Go to Frame 2.

## Frame 2

To describe an event, we need a coordinate system and a set of synchronized clocks. An event is specified by its three space coordinates and one time coordinate. Clocks in an inertial frame must be synchronized. One way to do this is to imagine an observer at the origin sending out a light pulse at time  $t = 0$ . When the light pulse reaches a clock at a distance  $r$ , that clock is set to  $t = r/c$ . This procedure ensures that the speed of light is measured to be  $c$  in all directions.

Go to Frame 3.

## Frame 3

### Simultaneity and the Relativity of Time

A basic premise of Newtonian mechanics is that a universal time scale exists. Newton wrote that "Absolute, true, and mathematical time, of itself, and from its own nature flows equably without relation to anything external." This implies that events that are simultaneous for one observer are simultaneous for all observers.

Einstein devised a thought experiment to challenge this. Imagine a boxcar moving with uniform velocity, and two lightning bolts striking the ends of the boxcar (points A and B) simultaneously as observed by an observer O at rest on the ground, midway between A and B. (See Figure 1.9a on page 14).

Since O is midway and sees the light signals from A and B arrive at the same time, O concludes the bolts struck simultaneously.

Now consider an observer O' moving with the boxcar, also midway between A' and B' (the ends of the boxcar). When will O' see the light from the front (B') and the light from the back (A') of the boxcar? [a] O' sees both light signals arrive at the same time. [b] O' sees the light from B' (front) arrive before the light from A' (back). [c] O' sees the light from A' (back) arrive before the light from B' (front).

(Hint: O' is moving towards the light from B' and away from the light from A'). Go to Frame 4.

## Frame 4

Your answer was [ a — b — c ].

The correct answer is [b]. Observer O' is moving towards the point where the light from B' is coming from, and away from the point where the light from A' is coming from. Since the speed of light is constant for O', the light signal from the front (B') will reach O' before the light signal from the back (A').

Since O' is midway between A' and B' and sees the light from B' arrive first, what does O' conclude about when the lightning bolts struck A' and B'? [a] They struck simultaneously. [b] The bolt at B' (front) struck before the bolt at A' (back). [c] The bolt at A' (back) struck before the bolt at B' (front).

Go to Frame 5.

## Frame 5

Your answer was [ a — b — c ].

The correct answer is [b]. Since O' is midway and receives the signal from B' first, O' concludes that the lightning must have struck the front of the boxcar (B') \*before\* it struck the back (A').

This leads to a crucial conclusion: Two events that are simultaneous in one frame of reference (for observer O) are, in general, **not simultaneous** in a second frame moving with respect to the



first (for observer O'). Simultaneity is not an absolute concept but depends on the state of motion of the observer.

Go to Frame 6.

## Frame 6

### Time Dilation

Observers in different inertial frames always measure different time intervals between a pair of events if those frames are moving relative to each other.

Consider a vehicle moving with speed  $v$  (Figure 1.10a on page 15). An observer O' at rest in this vehicle has a light clock. A light pulse is emitted from a source, travels to a mirror at distance  $d$  directly above, and reflects back to the source. The time interval for this round trip as measured by O' is  $\Delta t'$ .

What is the distance the light pulse travels for observer O'? What is  $\Delta t'$  in terms of  $d$  and  $c$ ?

1. Distance: \_\_\_\_\_ 2.  $\Delta t' =$  \_\_\_\_\_

Go to Frame 7.

## Frame 7

1. Distance (for O'): The light pulse travels up a distance  $d$  and down a distance  $d$ , so the total distance is  $2d$ . 2. Time interval (for O'):  $\Delta t' = \frac{\text{distance}}{\text{speed of light}} = \frac{2d}{c}$ .

This time interval  $\Delta t'$  is called the **proper time**. The proper time interval is the time interval between two events measured by an observer who sees the events occur at the same point in space. For O', the light pulse starts and ends at the same location (the light source/detector).

Go to Frame 8.

## Frame 8

Now consider an observer O on Earth who sees the vehicle (and O's clock) moving past with speed  $v$  (Figure 1.10b on page 15). Observer O measures the time interval between the emission of the light pulse and its return as  $\Delta t$ .

During this time  $\Delta t$ , the light pulse travels a longer, diagonal path for observer O. The light still travels at speed  $c$ . The horizontal distance the vehicle moves during  $\Delta t$  is  $v\Delta t$ . Using the Pythagorean theorem on the right triangle formed by the light's path (see Figure 1.10c), we can relate  $d$ ,  $v\Delta t/2$ , and  $c\Delta t/2$ :

$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{v\Delta t}{2}\right)^2 + d^2$$

Solve this equation for  $\Delta t$ . (You'll need to substitute  $d = c\Delta t'/2$  from Frame 7). \_\_\_\_\_

Go to Frame 9 for the solution steps.

## Frame 9

Starting with:

$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{v\Delta t}{2}\right)^2 + d^2$$

Multiply by 4:

$$\begin{aligned} (c\Delta t)^2 &= (v\Delta t)^2 + (2d)^2 \\ c^2(\Delta t)^2 &= v^2(\Delta t)^2 + (2d)^2 \end{aligned}$$

Substitute  $2d = c\Delta t'$  (from  $\Delta t' = 2d/c$ ):

$$c^2(\Delta t)^2 = v^2(\Delta t)^2 + (c\Delta t')^2$$

Rearrange to solve for  $\Delta t$ :

$$\begin{aligned} (\Delta t)^2(c^2 - v^2) &= (c\Delta t')^2 \\ (\Delta t)^2 &= \frac{c^2(\Delta t')^2}{c^2 - v^2} = \frac{(\Delta t')^2}{1 - v^2/c^2} \end{aligned}$$

Taking the square root:

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

This is the famous **time dilation** formula. It is often written as  $\Delta t = \gamma\Delta t'$ , where  $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$  is the Lorentz factor.

Since  $v < c$ , is  $\gamma$  greater than, less than, or equal to 1? What does this imply about the relationship between  $\Delta t$  and  $\Delta t'$ ? 1.  $\gamma$  is [  $>$  —  $=$  ] 1. 2.  $\Delta t$  is [ longer than — shorter than — the same as ]  $\Delta t'$ .

Go to Frame 10.

### Frame 10

1. Since  $v < c$ ,  $v^2/c^2 < 1$ . Therefore,  $1 - v^2/c^2 < 1$  (but positive). The square root is also  $< 1$ . So,  $\gamma = 1/(\text{a number less than } 1)$  is **greater than 1**. (If  $v = 0$ ,  $\gamma = 1$ ). 2. Since  $\gamma > 1$ ,  $\Delta t = \gamma\Delta t'$  means that  $\Delta t$  (the time interval measured by the observer O who sees the clock moving) is **longer than  $\Delta t'$**  (the proper time interval measured by observer O' for whom the clock is at rest).

This means that a moving clock is measured to run *slower* than an identical clock at rest with respect to the observer. "A moving clock runs slow." This phenomenon is called **time dilation**.

The lifetime of unstable particles like muons provides experimental evidence for time dilation. Muons created in the upper atmosphere have a short proper lifetime (about  $2.2 \mu\text{s}$ ). If time dilation didn't occur, most would decay before reaching Earth's surface. However, because they travel at speeds close to  $c$ , their lifetime as measured by an observer on Earth is significantly dilated, allowing many more to reach the surface than classically expected (Figure 1.11 on page 17).

Go to Frame 11.

### Frame 11

#### Length Contraction

We have seen that time intervals are not absolute. What about lengths? The **proper length**  $L_p$  of an object is the length measured by an observer at rest relative to the object.

Consider a spaceship traveling between two stars. An observer on Earth measures the distance between the stars as  $L_p$  (Earth is at rest relative to the stars). The spaceship travels at speed  $v$ , and the time it takes, as measured by Earth clocks, is  $\Delta t = L_p/v$ . An observer on the spaceship measures the time for the trip as  $\Delta t'$ . Due to time dilation,  $\Delta t' = \Delta t/\gamma = L_p/(\gamma v)$ .

The spaceship observer sees the stars approaching and receding at speed  $v$ . What distance  $L$  does the spaceship observer measure between the stars, given they measure the travel time as  $\Delta t'$  and see the relative speed as  $v$ ?  $L = v\Delta t'$  Substitute  $\Delta t' = L_p/(\gamma v)$ . What is  $L$  in terms of  $L_p$  and  $\gamma$ ?  $L = \underline{\hspace{2cm}}$

Go to Frame 12.

### Frame 12

$L = v\Delta t' = v \left( \frac{L_p}{\gamma v} \right) = \frac{L_p}{\gamma}$  Since  $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ , this can also be written as:

$$L = L_p \sqrt{1 - v^2/c^2}$$

This is the **length contraction** formula.

Since  $\gamma \geq 1$  (or  $\sqrt{1 - v^2/c^2} \leq 1$ ), is the length  $L$  measured by the moving observer (spaceship) longer than, shorter than, or the same as the proper length  $L_p$ ?  $L$  is [ longer than — shorter than — the same as ]  $L_p$ .

Go to Frame 13.

### Frame 13

Since  $\gamma \geq 1$ ,  $L = L_p/\gamma$  means that  $L$  is **shorter than**  $L_p$ . The length of an object measured by an observer moving relative to the object is always less than the proper length. This contraction occurs only along the direction of motion. Lengths perpendicular to the direction of motion are unaffected. (See Figure 1.13 on page 19).

So, if an object has a proper length  $L_p$  when it is measured by an observer at rest with respect to it, its length  $L$  is measured to be shorter,  $L = L_p/\gamma$ , when it moves with speed  $v$  in a direction parallel to its length.

Go to Frame 14.

### Frame 14

#### The Twins Paradox (Optional Section from text)

This is a famous thought experiment in relativity. One twin (Speedo) takes a high-speed round trip to a distant star, while the other twin (Goslo) stays on Earth. When Speedo returns, who is younger? [a] Speedo [b] Goslo [c] They are the same age.

(The text provides a detailed explanation on pages 21-22. The key is that Speedo undergoes accelerations to turn around, making Speedo's frame non-inertial for part of the trip. The twin who remains in a single inertial frame (or effectively so, like Goslo on Earth) will age more.)

Go to Frame 15 for the brief answer.

### Frame 15

The answer is [a] Speedo. Speedo, the traveling twin, will be younger upon return.

While it seems paradoxical because motion is relative, the situation is not symmetric. Speedo must accelerate to start the trip, turn around, and stop. Goslo does not. The effects of time dilation mean that less time passes for Speedo. The text explains (page 22) that even from Speedo's perspective (where Earth and Goslo are moving), the flashes from Earth's clock will be observed to run slower during the outbound and inbound constant-velocity segments due to the relativistic Doppler shift and how distances change, ultimately confirming Speedo ages less.

Go to Frame 16.

### Frame 16

#### The Relativistic Doppler Shift (Optional Section from text)

Another important consequence of relativity is the change in observed frequency (and wavelength) of light due to relative motion between source and observer, known as the relativistic Doppler shift. This differs from the classical Doppler effect for sound. If a light source is moving

away from an observer, the observed frequency is [ lower — higher ] than the source frequency. If a light source is moving towards an observer, the observed frequency is [ lower — higher ] than the source frequency. (This is similar to the classical Doppler effect for sound).

Go to Frame 17 for the answers.

### Frame 17

If a light source is moving away from an observer, the observed frequency is **lower** (redshift). If a light source is moving towards an observer, the observed frequency is **higher** (blueshift).

The relativistic formula is more complex than the classical one, accounting for time dilation. This effect is crucial in astronomy for determining the velocities of distant galaxies.

This covers the main consequences of Special Relativity discussed in this part of the chapter. The next steps would involve Lorentz Transformations, Relativistic Momentum and Energy.

End of Section.

## 3 The Lorentz Transformation and Spacetime

### Frame 1

We've seen that the Galilean transformation is not valid when speeds approach the speed of light ( $c$ ) because it's inconsistent with Einstein's postulates. We need a new set of transformation equations that correctly relate coordinates and time between inertial frames, valid for all speeds  $0 \leq v < c$ . This new transformation is known as the **Lorentz transformation**.

What was the primary issue with the Galilean transformation that the Lorentz transformation seeks to correct? [a] It didn't account for gravity. [b] It assumed time was absolute and led to incorrect velocity additions for light. [c] It only worked for accelerating frames.

Go to Frame 2.

### Frame 2

Your answer was [ a — b — c ].

The correct answer is [b]. The Galilean transformation assumed  $t = t'$  (absolute time) and its velocity addition rule ( $u'_x = u_x - v$ ) violated the constancy of the speed of light. The Lorentz transformation must preserve the speed of light as  $c$  in all inertial frames.

Let's consider two inertial frames, S and S', where S' moves with a constant velocity  $v$  relative to S along their common x-x' axis. Their origins coincide at  $t = t' = 0$ . The Lorentz transformation equations relate  $(x, y, z, t)$  in S to  $(x', y', z', t')$  in S'. A key assumption in deriving them is linearity, and they must reduce to the Galilean transformation when  $v/c$  is very small.

The derivation (pages 26-27 of the original text) is a bit involved, but it starts by assuming  $x' = G(x - vt)$ , where  $G$  is a factor that might depend on  $v$  but not on  $x$  or  $t$ . Using the principle of relativity and the constancy of  $c$ , the factor  $G$  is found to be  $\gamma = 1/\sqrt{1 - v^2/c^2}$ .

The Lorentz transformation equations are (Eqs. 1.23-1.26 on page 27):  $x' = \gamma(x - vt)$   $y' = y$   $z' = z$   $t' = \gamma(t - \frac{vx}{c^2})$

What is the factor  $\gamma$  called, and what is its formula? 1. Name: \_\_\_\_\_ 2. Formula:  $\gamma =$  \_\_\_\_\_

Go to Frame 3.

### Frame 3

1. Name:  $\gamma$  is called the **Lorentz factor**. 2. Formula:  $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ .

Notice a crucial difference from the Galilean transformation:  $t' \neq t$ . Time is also transformed. The term  $vx/c^2$  in the equation for  $t'$  shows that the time measured in S' depends not only on the time  $t$  in S but also on the position  $x$  in S. This is directly related to the relativity of simultaneity.

If you want to transform coordinates from S' back to S (inverse Lorentz transformation), what simple changes do you make to the equations above? (Hint: Consider the relative velocity of S with respect to S'). \_\_\_\_\_

Go to Frame 4.

#### Frame 4

To get the inverse Lorentz transformation (from S' to S), we replace  $v$  with  $-v$  and interchange the primed and unprimed coordinates (Eq. 1.27 on page 28):  $x = \gamma(x' + vt')$   $y = y'$   $z = z'$   $t = \gamma(t' + \frac{vx'}{c^2})$

As a check, what happens to the Lorentz transformation equations if the relative speed  $v$  is much, much smaller than  $c$  ( $v \ll c$ )? \_\_\_\_\_

Go to Frame 5.

#### Frame 5

If  $v \ll c$ , then  $v^2/c^2 \approx 0$ , so  $\gamma = 1/\sqrt{1 - v^2/c^2} \approx 1/\sqrt{1 - 0} = 1$ . Also, the term  $vx/c^2$  in the time transformation becomes very small. So, the Lorentz transformations reduce to:  $x' \approx x - vt$   $y' = y$   $z' = z$   $t' \approx t$  These are the Galilean transformation equations, as required.

The text (Example 1.7, page 28) shows how time dilation is contained within the Lorentz transformation for time:  $t'_2 - t'_1 = \gamma(t_2 - t_1)$  if the events occur at the same position in S ( $x_1 = x_2$ ). If  $t_2 - t_1 = \Delta t_p$  (proper time in S), then  $\Delta t' = \gamma \Delta t_p$ .

Go to Frame 6.

#### Frame 6

##### Lorentz Velocity Transformation

Just as coordinates transform, velocities also transform differently under Lorentz transformations compared to Galilean ones. If an object has velocity components  $u_x = dx/dt$ ,  $u_y = dy/dt$ ,  $u_z = dz/dt$  in frame S, its velocity components  $u'_x = dx'/dt'$ ,  $u'_y = dy'/dt'$ ,  $u'_z = dz'/dt'$  in frame S' (moving at  $v$  along x-axis relative to S) are given by (see Eqs. 1.28, 1.29 on page 29):

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \quad u'_y = \frac{u_y}{\gamma(1 - \frac{u_x v}{c^2})} \quad u'_z = \frac{u_z}{\gamma(1 - \frac{u_x v}{c^2})}$$

Consider the case where an object (e.g., a photon) moves at speed  $u_x = c$  in frame S. What is its speed  $u'_x$  in frame S' according to the Lorentz velocity transformation?  $u'_x = \underline{\hspace{2cm}}$  (Substitute  $u_x = c$  into the first equation).

Go to Frame 7.

#### Frame 7

Substituting  $u_x = c$  into  $u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$ :

$$u'_x = \frac{c - v}{1 - \frac{cv}{c^2}} = \frac{c - v}{1 - \frac{v}{c}} = \frac{c - v}{\frac{c - v}{c}} = c \left( \frac{c - v}{c - v} \right) = c$$

So,  $u'_x = c$ . This confirms that the Lorentz velocity transformation correctly upholds Einstein's second postulate: if something moves at speed  $c$  in one inertial frame, it moves at speed  $c$  in all other inertial frames.

What happens to the formula for  $u'_x$  if  $u_x$  and  $v$  are both much smaller than  $c$ ? \_\_\_\_\_  
Go to Frame 8.

### Frame 8

If  $u_x \ll c$  and  $v \ll c$ , then the term  $u_x v / c^2$  in the denominator of  $u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$  becomes very close to zero. So,  $u'_x \approx \frac{u_x - v}{1 - 0} \approx u_x - v$ . This is the Galilean velocity addition formula, as expected for low speeds.

The text provides examples (pages 30-31) of applying these velocity transformations, such as "Relativistic Spaceships" and "The Speeding Motorcycle." These illustrate how relative velocities combine in ways that ensure no object's speed can exceed  $c$ .

Go to Frame 9.

### Frame 9

#### Spacetime and Causality (Section 1.7, page 31)

Special relativity fundamentally changes our view of space and time. They are no longer independent entities but are interwoven into a four-dimensional continuum called **spacetime**. An event is a point in spacetime, specified by its three spatial coordinates and one time coordinate.

The Lorentz transformation shows that space and time coordinates are not absolute but transform into each other depending on the relative motion of observers. This mixing of space and time is a core feature of relativity.

What remains invariant (unchanged) when transforming between inertial frames S and S' is not distance or time separately, but a specific combination. While not explicitly derived in this section of the text, this invariant quantity is the "spacetime interval"  $\Delta s^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$ .

Does the order of events (causality) get mixed up by relativity? Can an effect precede its cause in some reference frame? [ Yes — No ]

Go to Frame 10.

### Frame 10

The answer is generally [No], provided the cause and effect are "timelike" or "lightlike" separated, meaning a signal at or below the speed of light can connect them.

If two events are causally connected (one event can cause the other), then all observers will agree on the time order of these events, even if they disagree on the exact time interval or spatial separation. The speed of light acts as a limiting speed for any causal influence. No signal or object carrying energy or information can travel faster than  $c$ . This preserves the principle of causality.

If two events are "spacelike" separated (meaning not even light has enough time to travel between them given their spatial separation), then their time order can be different for different observers. However, since they cannot causally affect each other, this reversal of time order does not violate causality.

This concludes our look at the Lorentz Transformation and basic concepts of spacetime. The transformations are fundamental to all further study of relativistic physics.

End of Section.

## 4 Relativity II: Relativistic Momentum and Energy

### Frame 1

In the previous section, we explored the postulates of special relativity and some of its immediate consequences like time dilation, length contraction, and the Lorentz transformation. Now, we will extend the theory of special relativity to classical mechanics, specifically focusing on how concepts like momentum and energy need to be redefined to be consistent with relativity.

What is the classical (Newtonian) definition of momentum for a particle of mass  $m$  moving with velocity  $\mathbf{v}$ ?  $\mathbf{p} =$  \_\_\_\_\_

Go to Frame 2.

### Frame 2

The classical definition of momentum is  $\mathbf{p} = m\mathbf{v}$ .

The law of conservation of linear momentum states that when two bodies collide in an isolated system, the total momentum remains constant. This law is fundamental in classical mechanics. However, if we apply the classical definition of momentum  $\mathbf{p} = m\mathbf{v}$  and use the Lorentz velocity transformation for the velocities of the colliding bodies as seen from different inertial frames, we find that momentum (as defined classically) is **not** conserved in all inertial frames. (See Figure 2.1a and 2.1b on page 42 for an illustration of this problem).

This implies that to preserve the principle of relativity (laws of physics are the same in all inertial frames), we need to modify the definition of momentum.

Go to Frame 3.

### Frame 3

#### Relativistic Momentum

To ensure that momentum conservation holds in all inertial frames under a Lorentz transformation, Einstein showed that the definition of momentum must be modified. The relativistic momentum  $\mathbf{p}$  of a particle of mass  $m$  moving with velocity  $\mathbf{v}$  is given by (Eq. 2.1 on page 43):

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} = \gamma m\mathbf{v}$$

where  $\gamma = 1/\sqrt{1 - v^2/c^2}$  is the Lorentz factor, and  $v$  is the magnitude of  $\mathbf{v}$ .

What happens to this definition of relativistic momentum when the speed  $v$  is much, much smaller than  $c$  ( $v \ll c$ )? [a] It becomes infinite. [b] It reduces to the classical momentum  $m\mathbf{v}$ . [c] It becomes zero.

Go to Frame 4.

### Frame 4

Your answer was [ a — b — c ].

The correct answer is [b]. If  $v \ll c$ , then  $v^2/c^2 \approx 0$ , so  $\gamma \approx 1/\sqrt{1-0} = 1$ . Therefore,  $\mathbf{p} = \gamma m\mathbf{v} \approx 1 \cdot m\mathbf{v} = m\mathbf{v}$ . The relativistic definition of momentum correctly reduces to the classical definition at low speeds.

Notice that as  $v$  approaches  $c$ , what happens to the factor  $\gamma$ ? And consequently, what happens to the relativistic momentum  $p = \gamma mv$ ? 1. As  $v \rightarrow c$ ,  $\gamma \rightarrow$  \_\_\_\_\_ 2. As  $v \rightarrow c$ ,  $p \rightarrow$  \_\_\_\_\_

Go to Frame 5.

## Frame 5

1. As  $v \rightarrow c$ ,  $v^2/c^2 \rightarrow 1$ . Then  $1 - v^2/c^2 \rightarrow 0$ . So,  $\gamma = 1/\sqrt{1 - v^2/c^2} \rightarrow 1/0 \rightarrow \infty$ . 2. As  $v \rightarrow c$ ,  $p = \gamma mv \rightarrow \infty \cdot mv \rightarrow \infty$ .

This implies that an infinite amount of momentum (and as we'll see, infinite energy) would be required to accelerate a particle with mass to the speed of light. This is why no object with mass can reach the speed of light.

Go to Frame 6.

## Frame 6

### The Relativistic Form of Newton's Second Law

Newton's second law is classically stated as  $\mathbf{F} = m\mathbf{a} = m \frac{d\mathbf{v}}{dt}$ . A more general form, which is also valid when mass can change, is  $\mathbf{F} = \frac{d\mathbf{p}}{dt}$  (force equals the rate of change of momentum). This latter form is preserved in relativity, provided we use the relativistic momentum. So, the relativistic force  $\mathbf{F}$  acting on a particle is (Eq. 2.2 on page 43):

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(\gamma m \mathbf{v})$$

This equation is more complex than  $\mathbf{F} = m\mathbf{a}$  because  $\gamma$  depends on  $v$ , and  $v$  can change with time. If we apply a constant force to an object, will its acceleration be constant according to this relativistic form? (Hint: As  $v$  increases,  $\gamma$  increases). [ Yes — No ]

Go to Frame 7.

## Frame 7

The answer is [No]. As the speed  $v$  of the particle increases due to the force, the Lorentz factor  $\gamma$  also increases. This means that for a constant force  $\mathbf{F}$ , the resulting acceleration  $\mathbf{a}$  (rate of change of velocity) must decrease as  $v$  approaches  $c$ . The particle becomes increasingly "resistant" to further acceleration. In other words, the "effective mass" or inertia of the particle increases.

The text mentions (page 43) that if the force is perpendicular to the velocity (like in circular motion in a magnetic field), the equation can simplify, but if the force causes a change in speed, the full derivative of  $(\gamma m \mathbf{v})$  is needed.

Go to Frame 8.

## Frame 8

### Relativistic Energy (Section 2.2, page 44)

To find the relativistic expression for kinetic energy, we use the work-energy theorem: the work done by a force  $\mathbf{F}$  in displacing a particle equals the change in its kinetic energy  $K$ .  $K = W = \int \mathbf{F} \cdot d\mathbf{s}$ . Using  $\mathbf{F} = d\mathbf{p}/dt$  and some calculus (details omitted here, but involve integration by parts after substituting  $\mathbf{p} = \gamma m \mathbf{v}$ ), one arrives at the relativistic kinetic energy:

$$K = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$

This is the energy associated with the motion of the particle.

What is the classical kinetic energy  $K_{\text{classical}}$ ?  $K_{\text{classical}} = \frac{1}{2}mv^2$ . Does the relativistic kinetic energy reduce to this classical form when  $v \ll c$ ? (This requires using the binomial expansion for  $\gamma$  when  $v \ll c$ :  $\gamma = (1 - v^2/c^2)^{-1/2} \approx 1 + \frac{1}{2} \frac{v^2}{c^2} + \dots$ )

Go to Frame 9.



### Frame 9

The classical kinetic energy is  $K_{\text{classical}} = \frac{1}{2}mv^2$ .

Now, let's check if  $K = (\gamma - 1)mc^2$  reduces to this at low speeds. Using the approximation  $\gamma \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$  for  $v \ll c$ :

$$K \approx \left( \left( 1 + \frac{1}{2}\frac{v^2}{c^2} \right) - 1 \right) mc^2$$

$$K \approx \left( \frac{1}{2}\frac{v^2}{c^2} \right) mc^2 = \frac{1}{2}mv^2$$

Yes, the relativistic kinetic energy correctly reduces to the classical expression at low speeds.

Go to Frame 10.

### Frame 10

The expression for relativistic kinetic energy  $K = \gamma mc^2 - mc^2$  leads to one of the most famous insights of relativity. The term  $mc^2$  is called the **rest energy**  $E_R$  of the particle – the energy it has even when it is at rest ( $v = 0$ , so  $\gamma = 1$  and  $K = 0$ ).

$$E_R = mc^2$$

The term  $\gamma mc^2$  is interpreted as the **total energy**  $E$  of the particle.

$$E = \gamma mc^2$$

So, the total energy can be written as:

$$E = K + mc^2 = \text{Kinetic Energy} + \text{Rest Energy}$$

This equation  $E = mc^2$  (specifically  $E_R = mc^2$ ) implies that mass itself is a form of energy. A small amount of mass can be converted into a tremendous amount of energy, and vice-versa, as seen in nuclear reactions.

Go to Frame 11.

### Frame 11

#### Conservation of Relativistic Energy and Momentum (Sections 2.3, 2.4)

In relativistic mechanics, the total relativistic energy  $E = \gamma mc^2$  and the relativistic momentum  $\mathbf{p} = \gamma m\mathbf{v}$  are conserved in isolated systems (e.g., during collisions or decays). This means that in any closed system, the sum of the total energies of all particles before an interaction equals the sum of the total energies after, and similarly for momentum.

A useful relationship between total energy  $E$ , momentum  $p$ , and rest mass  $m$  can be derived: Starting from  $E = \gamma mc^2$  and  $p = \gamma mv$ :  $E^2 = (\gamma mc^2)^2 = \gamma^2 m^2 c^4$   $p^2 = (\gamma mv)^2 = \gamma^2 m^2 v^2 \implies p^2 c^2 = \gamma^2 m^2 v^2 c^2$  Subtracting  $p^2 c^2$  from  $E^2$ :  $E^2 - p^2 c^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 v^2 c^2 = \gamma^2 m^2 c^2 (c^2 - v^2)$  Since  $\gamma^2 = 1/(1 - v^2/c^2) = c^2/(c^2 - v^2)$ , we have  $\gamma^2 (c^2 - v^2) = c^2$ . So,  $E^2 - p^2 c^2 = m^2 c^2 (c^2) = (mc^2)^2$ . This gives the **energy-momentum relation**:

$$E^2 = p^2 c^2 + (mc^2)^2$$

What is the total energy  $E$  of a particle that has zero rest mass (like a photon)? (Set  $m = 0$  in the energy-momentum relation).  $E =$  \_\_\_\_\_

Go to Frame 12.

## Frame 12

If  $m = 0$  (like for a photon), the energy-momentum relation  $E^2 = p^2c^2 + (mc^2)^2$  becomes:  $E^2 = p^2c^2 + 0$   $E = pc$  So, for a massless particle like a photon, its total energy is  $E = pc$ . Since  $E = hf$  for a photon, its momentum is  $p = hf/c = h/\lambda$ . This means massless particles can still carry momentum and energy.

This completes our brief overview of relativistic momentum and energy. These concepts are essential for understanding particle physics and phenomena involving high speeds or energy-mass conversions.

End of Section.

## 5 Mass, Energy, Conservation, and General Relativity

### Frame 1

We previously defined total relativistic energy as  $E = \gamma mc^2$  and rest energy as  $E_R = mc^2$ . The equation  $E = mc^2$  (in the context of rest energy) implies that mass is a form of energy. This concept, mass-energy equivalence, has profound implications.

What does the term  $mc^2$  represent in the equation  $E = \gamma mc^2 = K + mc^2$ ? [a] Kinetic energy [b] Rest energy [c] Potential energy

Go to Frame 2.

### Frame 2

Your answer was [ a — b — c ].

The correct answer is [b] Rest energy.  $mc^2$  is the energy an object possesses by virtue of its mass alone, even when it is at rest.

The equation  $E_R = mc^2$  is Einstein's famous mass-energy equivalence equation. It shows that mass is a measure of the total energy in all forms. Any change in the energy of a system (e.g., thermal, chemical, nuclear) is accompanied by a change in its mass. Because  $c^2$  is a very large number, even a small amount of mass corresponds to a huge amount of energy.

Go to Frame 3.

### Frame 3

#### Mass as a Measure of Energy (Section 2.3, page 48)

The idea that  $E_R = mc^2$  means that if the energy of a system changes by  $\Delta E_R$ , its mass changes by  $\Delta m = \Delta E_R/c^2$ . Consider an inelastic collision where two lumps of clay stick together (Figure 2.1a, page 42). Classically, momentum is conserved, but kinetic energy is not (it's converted into internal energy, like heat). In relativity, total relativistic energy  $E = \gamma mc^2$  is always conserved.

If two identical lumps, each with rest mass  $m_0$ , collide and stick together, forming a single lump M at rest, what is the rest mass M of the combined lump if the initial kinetic energy is converted into rest mass of the combined object? (Hint: Conserve total energy. Before collision, each has  $E_i = \gamma_i m_0 c^2$ . After,  $E_f = M c^2$ ). \_\_\_\_ (This is a simplified view; the text page 48 shows the more precise  $M = 2\gamma m_0$  if the final lump has the same structure as the initial lumps but is just hotter).

Let's consider the implication from the text (page 48): the conservation of mass and the conservation of energy are no longer separate principles but are unified. The **conservation of mass-energy** states that the sum of the mass-energy of a system of particles before interaction must

equal the sum of the mass-energy of the system after interaction where the mass-energy of the  $i$ th particle is defined as its total relativistic energy  $E_i/c^2$ .

Go to Frame 4.

#### Frame 4

The text on page 48 analyzes an inelastic collision where two identical masses  $m$  (moving with velocities such that their initial total momentum is zero) collide and stick together forming a composite object of mass  $M$ . Initially, the total energy is  $E_{before} = \gamma mc^2 + \gamma mc^2 = 2\gamma mc^2$ . After they stick and are at rest (in the center of mass frame), the total energy is  $E_{after} = Mc^2$ . Conservation of total energy  $E_{before} = E_{after}$  implies  $2\gamma mc^2 = Mc^2$ , so  $M = 2\gamma m$ . Since  $\gamma > 1$ , the rest mass of the composite object  $M$  is [ greater than — less than — equal to ]  $2m$  (the sum of the initial rest masses).

Go to Frame 5.

#### Frame 5

Your answer was [ greater than — less than — equal to ].

The correct answer is [greater than].  $M = 2\gamma m > 2m$  because  $\gamma > 1$ . The "extra" mass  $\Delta M = M - 2m = 2(\gamma - 1)m$  comes from the conversion of the initial kinetic energy of the two particles into rest mass (internal energy) of the composite object.  $\Delta M = 2K/c^2$ , where  $K = (\gamma - 1)mc^2$  is the initial kinetic energy of one particle. This clearly shows an example of the conversion of kinetic energy to mass.

The text gives an example of a fission reaction (Example 2.7, page 50), where a heavy nucleus (like Uranium) splits into lighter nuclei and releases energy.  ${}^{236}_{92}\text{U} \rightarrow {}^{95}_{38}\text{Sr} + {}^{139}_{54}\text{Xe} + 2{}^1_0\text{n}$  In this process, the sum of the rest masses of the products is [ greater than — less than ] the rest mass of the original Uranium nucleus. This "missing mass"  $\Delta m$  is converted into kinetic energy of the products,  $Q = \Delta mc^2$ .

Go to Frame 6.

#### Frame 6

Your answer was [ greater than — less than ].

The correct answer is [less than]. In fission (and fusion), the sum of the rest masses of the products is less than the sum of the rest masses of the reactants. This "mass defect" is converted into released energy (kinetic energy of the products).

The energy required to break a bound system (like a nucleus or even a molecule) apart into its constituent particles at rest is called its **binding energy**, BE. This binding energy contributes negatively to the mass of the bound system. If  $M_B$  is the bound system mass and  $\sum m_i c^2$  is the sum of the rest energies of its free constituents, then (Eq. 2.17, page 51):  $M_B c^2 + \text{BE} = \sum m_i c^2$ . This means the mass of a stable bound system is always less than the sum of the masses of its individual components when free.

Go to Frame 7.

#### Frame 7

##### Conservation of Relativistic Momentum and Energy (Section 2.4, page 52)

We have seen that mass and energy are interconvertible. The most general conservation law is the conservation of total relativistic energy. In isolated systems, total momentum is also conserved. These principles are fundamental in analyzing particle interactions, decays, and creations.

Consider the decay of a pion ( $\pi^+$  meson) at rest into a muon ( $\mu^+$ ) and a neutrino ( $\nu_\mu$ ) (Example 2.9, page 52).  $\pi^+ \rightarrow \mu^+ + \nu_\mu$  The pion is initially at rest. What is the total initial momentum of the system? — What must be the total final momentum of the muon and neutrino combined? —  
Go to Frame 8.

### Frame 8

Initial momentum: Since the pion is at rest, its initial momentum is 0. Final momentum: By conservation of momentum, the total final momentum of the muon and neutrino must also be 0. This means the muon and neutrino must fly off in opposite directions with equal magnitudes of momentum.

Energy conservation is also used. Let  $m_\pi c^2$  be the rest energy of the pion. Let  $E_\mu$  and  $E_\nu$  be the total energies of the muon and neutrino, and  $p_\mu$  and  $p_\nu$  be their momenta. Conservation of energy:  $m_\pi c^2 = E_\mu + E_\nu$  Using  $E^2 = p^2 c^2 + (mc^2)^2$  for each particle, and knowing  $p_\mu = p_\nu$  (in magnitude), these equations can be solved to find the kinetic energies of the decay products.

Go to Frame 9.

### Frame 9

**General Relativity** (Section 2.5, page 53)

So far, we've dealt with Special Relativity, which applies to inertial (non-accelerating) frames of reference. Einstein sought to generalize his theory to include accelerating frames and gravity. This led to the **General Theory of Relativity** (published around 1916).

A key idea in General Relativity is the **Principle of Equivalence**, which states that there is no local experiment that can distinguish between the effects of a uniform gravitational field and the effects of being in a uniformly accelerating frame of reference.

Mass has two aspects: 1. **Inertial mass** ( $m_i$ ): A measure of an object's resistance to acceleration ( $\mathbf{F} = m_i \mathbf{a}$ ). 2. **Gravitational mass** ( $m_g$ ): A measure of the gravitational force experienced by an object (e.g.,  $F_g = G \frac{Mm_g}{r^2}$ ).

Experiments have shown with extremely high precision that inertial mass and gravitational mass are equivalent ( $m_i = m_g$ ). Einstein used this equivalence as a cornerstone of General Relativity.

What is the central concept that General Relativity uses to describe gravity? [a] A force acting at a distance between masses. [b] The curvature of spacetime caused by mass and energy. [c] An electromagnetic interaction.

Go to Frame 10.

### Frame 10

Your answer was [ a — b — c ].

The correct answer is [b]. General Relativity describes gravity not as a force in the Newtonian sense, but as a manifestation of the **curvature of spacetime**. Mass and energy tell spacetime how to curve, and the curvature of spacetime tells mass and energy how to move. Objects (and light) follow paths called "geodesics" in this curved spacetime. In regions where spacetime is nearly flat (weak gravity), these geodesics approximate straight lines or Newtonian orbits.

General Relativity is a mathematically complex theory but has passed numerous experimental tests, such as the bending of starlight by the Sun, the anomalous precession of Mercury's orbit, and the existence of gravitational waves.

This concludes our very brief introduction to some further concepts in relativity. The theory of relativity, both special and general, represents one of the most profound intellectual achievements in physics.

End of Section.

## 6 The Quantum Theory of Light

### Frame 1

At the end of the 19th century, classical physics (based on Newton's laws, Maxwell's electromagnetism, and thermodynamics) seemed to explain most known phenomena. However, a few persistent problems hinted at the need for a new way of thinking. One such problem was understanding the radiation emitted by heated objects, known as **blackbody radiation**.

What was the main challenge regarding blackbody radiation that classical physics couldn't solve? (Hint: It relates to predicting the intensity of radiation at different wavelengths for a given temperature).

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Go to Frame 2.

### Frame 2

The main challenge was that classical physics (specifically, Maxwell's theory of light emission) failed to accurately predict the observed distribution of energy (intensity vs. wavelength) emitted by a "blackbody" (an ideal absorber and emitter of radiation) at a specific temperature. Classical theories led to the "ultraviolet catastrophe," predicting infinite energy at short wavelengths, which was clearly wrong.

What is a "blackbody"? (See page 68, first paragraph under section 3.2). [a] An object that only emits black light. [b] An ideal object that absorbs all electromagnetic radiation falling on it, and also emits radiation based on its temperature. [c] An object that reflects all radiation.

Go to Frame 3.

### Frame 3

Your answer for defining a blackbody was [ a — b — c ].

The correct answer is [b]. A blackbody is an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. It is also a perfect emitter of thermal radiation, with an emission spectrum that depends only on its temperature. A small opening in a heated cavity is a good practical approximation of a blackbody (Figure 3.4, page 69).

Experimental observations showed that the total power emitted per unit area by a blackbody,  $e_{\text{total}}$ , is proportional to the fourth power of its absolute temperature  $T$ . This is known as **Stefan's Law** (or Stefan-Boltzmann Law, Eq. 3.3, page 69):

$$e_{\text{total}} = \int_0^{\infty} e_f df = \sigma T^4$$

where  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$  is the Stefan-Boltzmann constant. For real objects (not ideal blackbodies), the law is  $e_{\text{total}} = \epsilon \sigma T^4$ , where  $\epsilon$  is the emissivity ( $0 < \epsilon \leq 1$ ).

What happens to the wavelength at which the maximum power is emitted ( $\lambda_{\text{max}}$ ) as the temperature  $T$  of a blackbody increases? (See Figure 3.3 on page 70 and Wien's displacement law). [a]  $\lambda_{\text{max}}$  increases. [b]  $\lambda_{\text{max}}$  decreases. [c]  $\lambda_{\text{max}}$  stays the same.

Go to Frame 4.

### Frame 4

Your answer was [ a — b — c ].

The correct answer is [b]  $\lambda_{\max}$  decreases as temperature  $T$  increases. This is described by **Wien's displacement law** (Eq. 3.6, page 70):

$$\lambda_{\max}T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$

This means hotter objects peak at shorter wavelengths (e.g., a heated iron bar goes from red to orange to white as it gets hotter).

Classical attempts to derive the full blackbody spectrum, like the Rayleigh-Jeans law, worked well at long wavelengths but failed dramatically at short wavelengths (the "ultraviolet catastrophe"). This discrepancy highlighted a fundamental flaw in classical physics.

Go to Frame 5.

## Frame 5

### Enter Planck

In October 1900, Max Planck found a formula that perfectly matched the experimental blackbody radiation curve. His formula for the spectral energy density  $u(f, T)$  (energy per unit volume per unit frequency) was (Eq. 3.9, page 73):

$$u(f, T) = \frac{8\pi hf^3}{c^3} \frac{1}{e^{hf/k_B T} - 1}$$

where  $h$  is a new constant (Planck's constant,  $h \approx 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$ ) and  $k_B$  is Boltzmann's constant.

To derive this formula theoretically, Planck had to make a radical assumption about the energy of the oscillators (resonators) in the walls of the blackbody cavity that emit and absorb radiation. What was this revolutionary assumption? (See page 74). \_\_\_\_\_

Go to Frame 6.

## Frame 6

Planck's revolutionary assumption was that the energy of a resonator (oscillator) of frequency  $f$  could not have any arbitrary value, but could only be an integer multiple of a fundamental unit of energy,  $hf$ . That is, the allowed energy levels of a resonator are quantized:

$$E_{\text{resonator}} = nhf \quad \text{where } n = 0, 1, 2, 3, \dots$$

Furthermore, when a resonator emits or absorbs radiation, its energy changes by discrete amounts,  $\Delta E = hf$ . Energy is emitted or absorbed in discrete "packets" or "quanta."

This idea of energy quantization was a radical departure from classical physics, where energy was assumed to be continuous.

Is the energy quantization significant for macroscopic oscillators (like a pendulum)? Why or why not? (Hint: Compare  $hf$  to typical energies, or  $\Delta E/E$ ). \_\_\_\_\_

Go to Frame 7.

## Frame 7

For macroscopic oscillators, energy quantization is **not significant**. The energy quantum  $hf$  is extremely small compared to the total energy of a macroscopic system. The fractional change in energy  $\Delta E/E$  is unobservably small (as shown in the text Example 3.2, page 75, for a pendulum). Thus, energy appears continuous for macroscopic systems, consistent with classical

physics. However, for atomic-scale oscillators,  $hf$  can be comparable to their total energy, making quantization effects crucial.

Planck's work, while initially a mathematical trick to fit the data, laid the foundation for quantum theory.

Go to Frame 8.

## Frame 8

### The Photoelectric Effect (Section 3.4, page 80)

Another phenomenon that classical physics could not explain was the photoelectric effect, where electrons are emitted from a metal surface when light shines on it. Albert Einstein, in 1905 (one of his "miracle year" papers), provided the explanation, for which he later received the Nobel Prize.

Classical wave theory of light predicted several things about the photoelectric effect that were contradicted by experiments: 1. Classical: Kinetic energy (KE) of emitted electrons should increase with light intensity. Experimental: KE is independent of intensity. 2. Classical: Electrons should be emitted for any light frequency, if the intensity is high enough. Experimental: No electrons are emitted if the light frequency is below a certain **cutoff frequency**  $f_0$  (or threshold frequency), regardless of intensity. 3. Classical: There should be a time lag between light striking the surface and electron emission, especially for low intensity light. Experimental: Electrons are emitted almost instantaneously, even at very low intensities.

How did Einstein explain these observations? What fundamental idea about light did he propose? \_\_\_\_\_

Go to Frame 9.

## Frame 9

Einstein proposed that light itself is quantized. He suggested that light energy is not spread out continuously in a wave but is concentrated in discrete packets, or **quanta**, called **photons**. Each photon has an energy:

$$E_{\text{photon}} = hf$$

where  $h$  is Planck's constant and  $f$  is the frequency of the light.

In the photoelectric effect, one photon is absorbed by one electron in the metal. To escape the metal, the electron needs a certain minimum energy called the **work function**,  $\phi$  (phi), which is characteristic of the metal.

If a photon of energy  $hf$  is absorbed by an electron, what is the maximum kinetic energy ( $K_{\text{max}}$ ) of the emitted electron according to Einstein?  $K_{\text{max}} = \text{_____}$  (in terms of  $hf$  and  $\phi$ )

Go to Frame 10.

## Frame 10

According to Einstein's photoelectric equation (Eq. 3.23, page 84):

$$K_{\text{max}} = hf - \phi$$

This equation elegantly explains the experimental observations:

1.  $K_{\text{max}}$  depends only on the frequency  $f$  of the light and the work function  $\phi$ , *not* on the light intensity. Increasing intensity means more photons, thus more electrons emitted, but not with higher individual kinetic energy. 2. For electrons to be emitted,  $K_{\text{max}}$  must be greater than 0. So,  $hf > \phi$ . The minimum (cutoff) frequency  $f_0$  for emission is when  $hf_0 = \phi$ , or  $f_0 = \phi/h$ . If  $f < f_0$ ,

no electrons are emitted, no matter how intense the light. 3. Since energy is transferred in discrete photon-electron interactions, emission is practically instantaneous if  $f \geq f_0$ .

This photon model of light was a revolutionary concept, extending Planck's idea of energy quantization from oscillators to light itself, and establishing the particle-like nature of light.

Go to Frame 11.

### Frame 11

The photoelectric equation  $K_{\max} = hf - \phi$  can also be written in terms of the stopping potential  $V_s$ . The stopping potential is the retarding voltage applied that is just sufficient to stop the most energetic photoelectrons. The work done by this potential on an electron of charge  $e$  is  $eV_s$ , which must equal  $K_{\max}$ :  $K_{\max} = eV_s$ . So,  $eV_s = hf - \phi$ . A plot of  $K_{\max}$  (or  $eV_s$ ) versus  $f$  yields a straight line. What is the slope of this line? \_\_\_\_ What is the y-intercept (when  $f = 0$  theoretically, or the x-intercept related to  $f_0$ )? \_\_\_\_\_

(See Figure 3.17 on page 84).

Go to Frame 12.

### Frame 12

From  $eV_s = hf - \phi$  (or  $K_{\max} = hf - \phi$ ):

- The slope of  $K_{\max}$  vs.  $f$  is Planck's constant,  $h$ .
- The y-intercept (when  $f = 0$ ) is  $-\phi$ .
- The x-intercept (when  $K_{\max} = 0$ ) is  $f_0 = \phi/h$ , the cutoff frequency.

Experiments by Millikan in 1916 confirmed Einstein's predictions with high accuracy and provided an independent measurement of Planck's constant  $h$ .

The successful explanation of blackbody radiation by Planck (quantized energy of oscillators) and the photoelectric effect by Einstein (quantized energy of light itself - photons) marked the beginning of quantum physics, a theory that revolutionized our understanding of the atomic and subatomic world.

End of Section.

## 7 The Compton Effect, X-Rays, and Duality

### Frame 1

In 1905, Einstein proposed that light consists of quanta of energy  $E = hf$ . He later (1906) concluded that these light quanta (photons) also carry momentum. What is the momentum  $p$  of a photon with energy  $E = hf$ ?  $p =$  \_\_\_\_\_

Go to Frame 2.

### Frame 2

The momentum of a photon is  $p = E/c = hf/c$ . This momentum is directed along the photon's line of motion.

The theoretical treatment of photon-particle collisions, which firmly established this momentum aspect of light, was developed independently by Peter Debye and Arthur Holly Compton in 1923. They explained the scattering of X-ray photons from electrons by treating photons as pointlike



particles with energy  $hf$  and momentum  $hf/c$ , and by conserving relativistic energy and momentum in the collision. This completed the particle picture of light.

Before diving into the Compton effect, let's briefly review X-rays.

Go to Frame 3.

### Frame 3

#### X-Rays (Page 86)

X-rays were discovered in 1895 by Wilhelm Roentgen. How are X-rays typically produced? (See Figure 3.18 on page 87). [a] By heating a metal target until it glows. [b] By passing a high-speed beam of electrons through a gas. [c] By bombarding a metal target with a beam of high-speed electrons.

Go to Frame 4.

### Frame 4

Your answer was [ a — b — c ].

The correct answer is [c]. X-rays are produced when a beam of high-speed electrons (typically with energies of 50-100 keV) strikes a metal target. X-rays are electromagnetic waves similar to light but with much shorter wavelengths (around  $10^{-10}$  m, comparable to atomic spacing in crystals) and thus greater penetrating power.

How can the crystal structure of a material be determined using X-rays? (Hint: Think about diffraction and the regular arrangement of atoms in a crystal). \_\_\_\_\_

Go to Frame 5.

### Frame 5

The regular, periodic arrangement of atoms in a crystal can act as a three-dimensional diffraction grating for X-rays. W. L. Bragg proposed a simple method for analyzing X-ray scattering (diffraction) from parallel crystal planes. Constructive interference of X-rays scattered from successive planes occurs when the path length difference is an integer multiple of the wavelength. This leads to **Bragg's Law** (Eq. 3.25a, page 87):

$$n\lambda = 2d \sin \theta$$

where:  $n$  = an integer (order of the maximum)  $\lambda$  = X-ray wavelength  $d$  = spacing between crystal planes  $\theta$  = angle of incidence/reflection measured from the plane.

By measuring the angles  $\theta$  at which strong reflections (intensity maxima) occur for a known  $\lambda$ , the plane spacing  $d$  can be determined, revealing the crystal structure. (See Figure 3.20 for the geometry).

Go to Frame 6.

### Frame 6

The X-ray emission spectrum from a metal target (Figure 3.21b, page 88) has two main features:

1. A broad, continuous spectrum called *bremssstrahlung* (braking radiation), resulting from electrons decelerating as they interact with the target atoms.
2. Sharp, intense lines characteristic of the target material.

What is the origin of the sharp characteristic X-ray lines? (Hint: It relates to discrete atomic energy levels, though this is detailed more in Chapter 9 of the original text).

What is the significance of the minimum wavelength ( $\lambda_{\min}$ ) observed in the continuous bremsstrahlung spectrum? How is it explained? \_\_\_\_\_

Go to Frame 7.

### Frame 7

Origin of sharp lines: The sharp lines are due to transitions between discrete, inner-shell electron energy levels in the target atoms. When an incoming high-energy electron knocks out an inner-shell electron, an electron from a higher energy level drops down to fill the vacancy, emitting an X-ray photon of a specific energy (and thus wavelength) characteristic of that transition.

Significance of  $\lambda_{\min}$ : The minimum wavelength (or maximum frequency/energy) in the continuous spectrum occurs when an incident electron loses *all* of its kinetic energy in a single collision, converting it entirely into a single X-ray photon. If the accelerating voltage in the X-ray tube is  $V$ , the electron's kinetic energy is  $eV$ . So,  $hf_{\max} = hc/\lambda_{\min} = eV$ . Thus,  $\lambda_{\min} = hc/(eV)$ . (Eq. 3.26, page 89). This  $\lambda_{\min}$  is independent of the target material and depends only on the accelerating voltage  $V$ . This provides further evidence for the photon theory of light.

Go to Frame 8.

### Frame 8

#### The Compton Effect (Page 89)

Classical wave theory failed to explain the scattering of X-rays from free (or loosely bound) electrons. Classical theory predicted:

- Incident radiation of frequency  $f_0$  should cause the electron to oscillate and re-radiate at the same frequency  $f \approx f_0$ .
- The scattered wavelength should depend on the exposure time and intensity of the incident radiation.

Compton's experiments (1922) showed different results. What did Compton observe about the wavelength of scattered X-rays compared to the incident X-rays? [a] The scattered wavelength was always shorter. [b] The scattered wavelength was always the same. [c] The scattered wavelength was longer, and the shift depended on the scattering angle.

Go to Frame 9.

### Frame 9

Your answer was [ a — b — c ].

The correct answer is [c]. Compton observed that the scattered X-rays had a wavelength  $\lambda'$  that was longer than the incident wavelength  $\lambda_0$ . This increase in wavelength ( $\lambda' - \lambda_0$ ), known as the Compton shift, depended only on the scattering angle  $\theta$  and was independent of the incident X-ray intensity and the material of the target (for loosely bound electrons).

How did Compton explain this shift using the quantum (particle) model of light? (See Figure 3.22b, page 90). \_\_\_\_\_

Go to Frame 10.

### Frame 10

Compton treated the scattering as a collision between an X-ray photon (particle with energy  $E = hf$  and momentum  $p = h/\lambda$ ) and a free electron initially at rest. In this collision (see Figure 3.24, page 92 for the collision diagram):

- The incident photon transfers some of its energy and momentum to the electron, causing the electron to recoil.
- The scattered photon therefore has less energy ( $E' < E$ ) and thus a longer wavelength ( $\lambda' > \lambda_0$ ) since  $E = hc/\lambda$ .

By applying conservation of relativistic energy and relativistic momentum to this two-body collision, Compton derived an expression for the wavelength shift  $\Delta\lambda = \lambda' - \lambda_0$ .

What is the Compton shift formula? (Eq. 3.27, page 90 or 93).  $\Delta\lambda =$  \_\_\_\_\_  
Go to Frame 11.

### Frame 11

The Compton shift formula is:

$$\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta)$$

where:  $h$  = Planck's constant  $m_e$  = rest mass of the electron  $c$  = speed of light  $\theta$  = scattering angle of the photon.

The term  $h/(m_e c)$  is called the **Compton wavelength** of the electron and has a value of about 0.0243 Å or 0.00243 nm. Compton's experimental results (Figure 3.23b, page 91) perfectly matched this formula, providing strong evidence for the particle nature of photons and the validity of energy/momentum conservation in photon-electron interactions.

Why is the Compton effect typically observed with X-rays or gamma rays, and not easily with visible light? (Hint: Consider the magnitude of  $\Delta\lambda$  relative to  $\lambda_0$  for different types of radiation, as in Example 3.8 page 93). \_\_\_\_\_

Go to Frame 12.

### Frame 12

The Compton wavelength  $h/(m_e c) \approx 0.0243$  Å is very small.

- For X-rays (e.g.,  $\lambda_0 \approx 0.7$  Å), the shift  $\Delta\lambda$  can be a significant fraction of  $\lambda_0$  (e.g., about 3.4% at 90°), making it measurable.
- For visible light (e.g.,  $\lambda_0 \approx 5000$  Å), the shift  $\Delta\lambda$  is an extremely tiny fraction of  $\lambda_0$  (e.g., about  $4 \times 10^{-6}$  or 0.0004%), making it practically unobservable.

Thus, the Compton effect is prominent when the photon wavelength is comparable to or smaller than the Compton wavelength of the electron.

In Compton's experimental data (Figure 3.23b), an unshifted peak at  $\lambda_0$  was also observed alongside the shifted peak  $\lambda'$ . What causes this unshifted peak? \_\_\_\_\_

Go to Frame 13.

### Frame 13

The unshifted peak at  $\lambda_0$  is caused by X-rays scattering from electrons that are tightly bound to the atoms in the target. When a photon scatters off a tightly bound electron, it effectively scatters off the entire atom. Since the mass of an atom is much larger than the mass of an electron (e.g., carbon atom is 23,000 times  $m_e$ ), the  $m_e$  in the Compton formula should be replaced by  $M_{\text{atom}}$ . This makes the Compton wavelength  $h/(M_{\text{atom}} c)$  extremely small, and thus the shift  $\Delta\lambda$  becomes negligible.

Go to Frame 14.

#### Frame 14

##### **Particle-Wave Complementarity** (Section 3.6, page 94)

Phenomena like the photoelectric effect and the Compton effect demonstrate that light behaves as if it were composed of particles (photons) with energy  $hf$  and momentum  $h/\lambda$ . However, phenomena like interference and diffraction (which are used in X-ray crystallography with Bragg's law) clearly show that light behaves as a wave.

Is light a wave or a particle? [a] It is only a wave. [b] It is only a particle. [c] It exhibits both wave-like and particle-like properties depending on the experiment.

Go to Frame 15.

#### Frame 15

Your answer was [ a — b — c ].

The correct answer is [c]. Light exhibits **particle-wave duality**. It behaves like a wave in some experiments (e.g., diffraction, interference) and like a particle in others (e.g., photoelectric effect, Compton effect). The physicist Max Born emphasized that our common language, developed from everyday experience with macroscopic objects (which are clearly either waves or particles), is inadequate to fully describe quantum phenomena.

The wave and particle descriptions are **complementary** ways of viewing the same underlying reality of light. Neither model alone can completely describe all of light's behavior. Quantum theory provides a more comprehensive framework that unifies these aspects.

This dual nature is not unique to light; as we will see in later chapters, matter (like electrons) also exhibits wave-particle duality.

End of Section.

## 8 The Particle Nature of Matter and Atomic Models

#### Frame 1

In Chapter 3, we reviewed evidence that electromagnetic radiation (like light and X-rays) behaves like particles (photons) in certain experiments, such as the photoelectric effect and Compton scattering. This was a departure from the classical wave theory of light. Now, in this chapter, we turn our attention to matter itself. Is matter continuous, or is it composed of discrete particles?

What is the common modern understanding of the fundamental structure of matter? [a] Matter is a continuous fluid-like substance. [b] Matter is composed of indivisible spheres. [c] Matter is composed of atoms, which themselves have internal structure.

Go to Frame 2.

#### Frame 2

Your answer was [ a — b — c ].

The correct answer is [c]. Today, it's commonplace to say that the world is made up of atoms, and these atoms are composed of even smaller particles (electrons, protons, neutrons). However, establishing this "graininess" of matter was a long and fascinating scientific journey, involving indirect measurements and clever inferences due to the incredibly small size of atoms.

The text (page 107) mentions several key figures who contributed to the early atomic theory. Which of these scientists is credited with the law of multiple proportions, a key piece of evidence for atomicity? \_\_\_\_\_

Go to Frame 3.

### Frame 3

John Dalton perceived the atomicity of nature in the law of multiple proportions of compounds. Other key figures include:

- Democritus and Leucippus (ancient Greeks): Speculated about atoms in motion.
- Lavoisier: Established conservation of mass in chemical reactions.
- Avogadro: Postulated that equal volumes of gases (at same T, P) contain equal numbers of molecules.
- Maxwell: Developed the kinetic theory of gases, relating macroscopic properties to molecular motion.
- Perrin and Einstein: Work on Brownian motion provided further confirmation of the atomic-molecular hypothesis.

Go to Frame 4.

### Frame 4

**The Composition of Atoms** (Section 4.2, page 108)

If matter is made of atoms, what are atoms made of? Several key discoveries shed light on this:

1. **Electrolysis (Faraday, 1833)**: Showed that the mass of an element liberated at an electrode is proportional to the charge transferred. This suggested that charge is quantized. 2. **Identification of the Electron (J.J. Thomson, 1897)**: Thomson showed that cathode rays were streams of negatively charged particles (electrons) and measured their charge-to-mass ratio ( $e/m_e$ ). He found  $e/m_e$  was the same regardless of the cathode material, concluding the electron is a universal constituent of matter. 3. **Measurement of Electronic Charge (Millikan, 1909)**: Millikan precisely measured the elementary charge  $e$ . Combining this with Thomson's  $e/m_e$ , he determined the mass of the electron, showing it was about 1000 times less massive than a hydrogen atom. 4. **The Nuclear Model (Rutherford, Geiger, Marsden, 1913)**: Scattering experiments showed atoms consist of a tiny, dense, positively charged nucleus surrounded by orbiting electrons.

Let's look at Faraday's law of electrolysis. It states  $m = \frac{(q)(\text{molar mass})}{(96500 \text{ C})(\text{valence})}$ . What does the quantity 96,500 C represent? \_\_\_\_\_

Go to Frame 5.

### Frame 5

96,500 C is approximately one **faraday** of charge, which is the magnitude of the charge of one mole of electrons ( $N_A \times e$ , where  $N_A$  is Avogadro's number and  $e$  is the elementary charge). Faraday's work implied that:

- Matter consists of atoms/molecules.
- Electric charge is quantized (transferred in discrete units).

- Atoms contain positive and negative charges.

Go to Frame 6.

## Frame 6

### J.J. Thomson's $e/m_e$ Experiment

J.J. Thomson's experiments with cathode ray tubes (Figures 4.4, 4.5, pages 110-111) were crucial. Electrons were accelerated and then passed through a region with perpendicular electric (E) and magnetic (B) fields.

First, how did Thomson determine the velocity  $v_x$  of the electrons? (Hint: He adjusted E and B fields). \_\_\_\_\_

Second, once  $v_x$  was known, how did he find  $e/m_e$  by observing the deflection caused by only the electric field? (Give the conceptual steps or the resulting formula). \_\_\_\_\_

Go to Frame 7.

## Frame 7

1. **Determining  $v_x$  (Velocity Selector):** Thomson adjusted the E and B fields so that the electric force ( $qE = eE$ ) and the magnetic force ( $qv_xB = ev_xB$ ) on the electrons were equal and opposite, resulting in an undeflected beam.  $eE = ev_xB \implies v_x = E/B$ .

2. **Determining  $e/m_e$ :** With the B field off, the E field alone caused a deflection. The electron experiences an upward acceleration  $a_y = F_y/m_e = eE/m_e$ . If it spends time  $t = l/v_x$  in the E field of length  $l$ , its upward velocity is  $v_y = a_y t = (eE/m_e)(l/v_x)$ . The deflection angle  $\theta$  (for small deflections) is  $\tan \theta \approx \theta \approx v_y/v_x$ . Substituting for  $v_y$  and  $v_x = E/B$  (and noting  $E = V/d$  for parallel plates with voltage  $V$  and separation  $d$ ), he derived a formula for  $e/m_e$  (Eq. 4.7, page 112):

$$\frac{e}{m_e} = \frac{V\theta}{B^2ld}$$

(The text uses slightly different notation:  $V$  is deflecting voltage,  $l$  is length of plates,  $d$  is separation of plates,  $\theta$  is deflection angle). Thomson's key finding was that  $e/m_e$  was about 1000 times larger than  $q/m$  for hydrogen ions, indicating a very small mass for the electron.

Go to Frame 8.

## Frame 8

### Millikan's Oil Drop Experiment

Robert Millikan's experiment (1909) precisely measured the elementary electric charge  $e$ . He observed tiny charged oil droplets suspended or moving in an electric field (Figure 4.8, page 115).

What were the key forces acting on an oil droplet in his experiment? (See Figure 4.9, page 116). 1. Downward: 2. Upward (when falling at terminal velocity, E-field off): 3. Upward (when E-field is on, opposing gravity): \_\_\_\_\_

Go to Frame 9.

## Frame 9

The key forces were: 1. Downward: Gravity ( $mg$ ) 2. Upward (field off, terminal velocity  $v$ ): Viscous drag force ( $D = Cv$ , where  $C$  depends on droplet radius and air viscosity, often using Stokes's law  $D = 6\pi\eta av$ ) 3. Upward (field on, terminal velocity  $v'$ ): Electric force ( $qE$ ) plus viscous drag force ( $Cv'$ ) (if moving up) or minus drag force (if still falling but slower).

By balancing these forces ( $mg = Cv$  with field off, and  $q_1E = mg + Cv'_1$  when moving up with speed  $v'_1$  with charge  $q_1$ ), Millikan could determine the charge  $q$  on a droplet. He observed that the charge  $q$  on any droplet was always an integer multiple of a fundamental unit of charge,  $e$ .  $q = ne$ , where  $n$  is an integer. His value for  $e$  was  $1.60 \times 10^{-19}$  C (very close to the modern value). This provided direct proof of charge quantization.

Go to Frame 10.

### Frame 10

#### Rutherford's Model of the Atom (Page 119)

Before Rutherford, the prevailing model of the atom was J.J. Thomson's "plum pudding" model: a sphere of uniformly distributed positive charge with electrons embedded in it. Rutherford, Geiger, and Marsden conducted experiments scattering alpha ( $\alpha$ ) particles (helium nuclei) from thin metal foils (Figure 4.10, page 120).

What was the most surprising result of their scattering experiments, and how did it contradict the plum pudding model? \_\_\_\_\_

Go to Frame 11.

### Frame 11

The most surprising result was that a small fraction of alpha particles were scattered through very large angles, some even nearly straight back (backscattering). The plum pudding model, with its diffuse positive charge, could not explain such large-angle scattering. It predicted only small deflections due to multiple, weak interactions. Rutherford concluded that the large-angle scattering must be due to a single, strong collision.

What did Rutherford propose as the structure of the atom to explain these results? \_\_\_\_\_

Go to Frame 12.

### Frame 12

Rutherford proposed the **nuclear model** of the atom:

- The atom has a tiny, dense, positively charged **nucleus** at its center, containing most of the atom's mass. (Diameter  $\sim 10^{-14}$  m).
- Electrons orbit this nucleus, like planets around the sun. (Electron cloud diameter  $\sim 10^{-10}$  m).

The strong electrostatic repulsion between the positive alpha particle and the concentrated positive charge of the nucleus could cause the observed large-angle scattering. Rutherford derived a scattering formula based on Coulomb's law and this model, which quantitatively agreed with the experimental data (Figure 4.12, page 123). He could even estimate the size of the nucleus by finding the energy at which alpha particles deviate from this scattering formula (implying they penetrate the nucleus, Example 4.5 page 124).

Go to Frame 13.

### Frame 13

Rutherford's nuclear model was a great success, but it raised new questions: 1. If the nucleus contains  $Z$  protons, what accounts for the rest of the nuclear mass (since atomic mass is typically  $\gg Z$  times proton mass)? 2. What holds the positive protons together in the tiny nucleus against their strong electrostatic repulsion? 3. How do electrons orbit the nucleus without radiating energy

and spiraling in, as predicted by classical electromagnetism? (Classical theory says an accelerating charge radiates energy). 4. How does this model explain the discrete line spectra emitted by atoms?

The first question was later answered by the discovery of the neutron. The second by the discovery of the strong nuclear force. The third and fourth questions were addressed by Niels Bohr.

Go to Frame 14.

## Frame 14

### The Bohr Atom (Section 4.3, page 125)

Before Bohr, the study of **spectral series** provided key data. When a low-pressure gas is excited (e.g., by an electric discharge), it emits light at specific, discrete wavelengths, forming a line spectrum characteristic of the element (Figure 4.15, page 127). For hydrogen, Johann Balmer (1885) found an empirical formula for the wavelengths of its visible lines (Balmer series, Figure 4.20 page 129):

$$\lambda(\text{cm}) = C_2 \frac{n^2}{n^2 - 2^2}, \quad n = 3, 4, 5, \dots$$

Later, this and other series (Lyman, Paschen, etc.) were unified by the Rydberg formula (Eq. 4.21, page 130):

$$\frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where  $R$  is the Rydberg constant, and  $n_f, n_i$  are integers with  $n_i > n_f$ .

What was the fundamental problem with a classical "planetary" model of the atom, where electrons orbit a nucleus like planets orbit a sun? (See Figure 4.21, page 131). \_\_\_\_\_

Go to Frame 15.

## Frame 15

The fundamental problem with a classical planetary model was that, according to Maxwell's theory of electromagnetism, an accelerating charged particle (like an orbiting electron) should continuously radiate electromagnetic energy. This loss of energy would cause the electron to spiral into the nucleus, leading to atomic collapse and a continuous emission spectrum, neither of which is observed. Atoms are stable and emit discrete line spectra.

Niels Bohr (1913) proposed a new model for the hydrogen atom that incorporated quantum ideas from Planck and Einstein to address these issues.

Go to Frame 16.

## Frame 16

### Bohr's Quantum Model of the Atom

Bohr's model for hydrogen was based on four main assumptions (page 132): 1. The electron moves in circular orbits around the proton under the influence of the Coulomb force. 2. **Only certain orbits are stable** (called **stationary states**). In these orbits, the electron does not radiate energy, despite being accelerated. Classical mechanics applies to motion in these orbits. 3. Radiation is emitted (or absorbed) when the electron "jumps" from one stationary state (initial energy  $E_i$ ) to another (final energy  $E_f$ ). The frequency  $f$  of the emitted/absorbed photon is given by the Planck-Einstein relation:

$$hf = |E_i - E_f|$$



4. The size of the allowed electron orbits (and thus their energies) is determined by a quantum condition imposed on the electron's orbital angular momentum ( $L = m_e v r$ ). What was this condition? (Eq. 4.24).  $m_e v r =$  \_\_\_\_\_

Go to Frame 17.

### Frame 17

Bohr's quantization condition for angular momentum was:

$$m_e v r = n \hbar \quad \text{where } n = 1, 2, 3, \dots$$

and  $\hbar = h/(2\pi)$  ("h-bar"). This means angular momentum is quantized, taking only discrete values.

Using these assumptions, Bohr derived expressions for the allowed orbit radii and energy levels for the hydrogen atom. The radius of the  $n$ -th orbit is (Eq. 4.28, page 133):

$$r_n = n^2 \frac{\hbar^2}{m_e k e^2} = n^2 a_0$$

where  $a_0 \approx 0.0529$  nm is the **Bohr radius** (radius of the  $n = 1$  orbit, the ground state). The energy of the  $n$ -th level is (Eq. 4.30, 4.31):

$$E_n = -\frac{k e^2}{2 a_0} \frac{1}{n^2} = -\frac{13.6 \text{ eV}}{n^2}$$

The negative sign indicates the electron is bound to the proton.  $E = 0$  corresponds to ionization (electron infinitely far and at rest).

What is the energy required to ionize a hydrogen atom from its ground state ( $n = 1$ )? \_\_\_\_\_

Go to Frame 18.

### Frame 18

The ground state energy ( $n = 1$ ) is  $E_1 = -13.6 \text{ eV}/1^2 = -13.6 \text{ eV}$ . To ionize the atom, the electron must be raised to the  $E = 0$  level. So, the ionization energy is  $0 - (-13.6 \text{ eV}) = 13.6 \text{ eV}$ . This value was in excellent agreement with experimental measurements.

When an electron jumps from an initial state  $n_i$  to a final state  $n_f$  ( $n_i > n_f$ ), a photon is emitted. Its energy is  $hf = E_i - E_f$ . Using  $E_n = -(ke^2/2a_0)(1/n^2)$  and  $f = c/\lambda$ :

$$\frac{hc}{\lambda} = -\frac{ke^2}{2a_0 n_i^2} - \left( -\frac{ke^2}{2a_0 n_f^2} \right) = \frac{ke^2}{2a_0} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\frac{1}{\lambda} = \frac{ke^2}{2a_0 hc} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

This is exactly the form of the Rydberg formula! Bohr's theory predicted the Rydberg constant  $R = ke^2/(2a_0 hc)$  in terms of fundamental constants, and the calculated value agreed remarkably well with the experimental value. This was a major triumph for Bohr's model. (See Figure 4.24, page 134 for an energy level diagram showing transitions for different series).

Bohr's model, while successful for hydrogen and hydrogen-like ions (one electron atoms, by replacing  $ke^2$  with  $kZe^2$ ), had limitations. It could not explain the spectra of multi-electron atoms, the intensities of spectral lines, or the splitting of lines in magnetic fields (Zeeman effect). It also mixed classical and quantum ideas somewhat arbitrarily. Nevertheless, it was a crucial stepping stone towards modern quantum mechanics.

End of Section.

## 9 Matter Waves and Their Properties

### Frame 1

In previous chapters, we saw that light, traditionally considered a wave, exhibits particle-like properties (photons). In 1923, Louis de Broglie proposed a revolutionary idea: if waves can act like particles, perhaps particles can act like waves. He postulated that all forms of matter have wave as well as particle properties. This was a radical idea with no direct experimental confirmation at the time.

What did de Broglie suggest accompanies every moving particle, like an electron? [a] An electromagnetic wave [b] A "pilot wave" or matter wave that guides it [c] A gravitational wave

Go to Frame 2.

### Frame 2

Your answer was [ a — b — c ].

The correct answer is [b]. De Broglie proposed that a **matter wave** (or "pilot wave") accompanies every electron (and indeed any particle), guiding its motion. This wave is not electromagnetic. He reasoned that since the quantum theory of light (photons) involves frequency ( $E = hf$ ), and a purely corpuscular theory has no concept of frequency, both wave and particle aspects must be introduced simultaneously for light. He suggested the same should apply to electrons and other matter.

What are the de Broglie relations for the wavelength ( $\lambda$ ) and frequency ( $f$ ) associated with a particle having relativistic momentum  $p$  and total relativistic energy  $E$ ? (Eqs. 5.1, 5.2, page 152).

1.  $\lambda =$  \_\_\_\_\_ 2.  $f =$  \_\_\_\_\_

Go to Frame 3.

### Frame 3

The de Broglie relations are: 1. **De Broglie wavelength:**  $\lambda = \frac{h}{p}$  2. **De Broglie frequency:**  $f = \frac{E}{h}$  where  $h$  is Planck's constant,  $p$  is the relativistic momentum ( $p = \gamma mv$ ), and  $E$  is the total relativistic energy ( $E = \gamma mc^2$ ).

How did de Broglie use his matter wave hypothesis to provide a physical explanation for Bohr's quantization of angular momentum in the hydrogen atom ( $m_e v r = n\hbar$ )? (See page 153 and Figure 5.2). \_\_\_\_\_

Go to Frame 4.

### Frame 4

De Broglie proposed that allowed Bohr orbits arise because the electron's matter wave forms a **standing wave** around the nucleus. For a standing wave to exist in a circular orbit of radius  $r$ , an integral number of wavelengths must fit into the circumference of the orbit:

$$n\lambda = 2\pi r \quad (n = 1, 2, 3, \dots)$$

Substituting the de Broglie wavelength  $\lambda = h/(m_e v)$ :

$$n \frac{h}{m_e v} = 2\pi r$$

Rearranging this gives:

$$m_e v r = n \frac{h}{2\pi} = n\hbar$$

This is precisely Bohr's quantization condition for angular momentum. De Broglie's idea provided a more intuitive physical picture for why only certain orbits (and thus energies) were allowed.

Go to Frame 5.

### Frame 5

Let's calculate some de Broglie wavelengths. (a) What is the de Broglie wavelength of a 140 g baseball traveling at 27 m/s (about 60 mi/h)? (Example 5.1, page 154). (Use  $h = 6.63 \times 10^{-34}$  J·s).  $\lambda_{\text{baseball}} = h/p = h/(mv) =$  \_\_\_\_\_

(b) If a particle of charge  $q$  and mass  $m$  is accelerated from rest through a potential difference  $V$ , its kinetic energy is  $qV$ . If non-relativistic ( $K = p^2/(2m)$ ), then  $p = \sqrt{2mqV}$ . What is its de Broglie wavelength  $\lambda$  in terms of  $h, m, q, V$ ? (Example 5.2, page 154).  $\lambda_{\text{particle}} =$  \_\_\_\_\_

Go to Frame 6.

### Frame 6

(a) Baseball:

$$\lambda_{\text{baseball}} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.140 \text{ kg})(27 \text{ m/s})} \approx 1.75 \times 10^{-34} \text{ m}$$

This wavelength is incredibly small, much smaller than any physical object, which is why we don't observe wave properties for macroscopic objects.

(b) Particle accelerated through  $V$ :

$$\lambda_{\text{particle}} = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

For an electron ( $m_e, q = e$ ) accelerated through 50 V, the text calculates  $\lambda \approx 1.7 \text{ \AA}$  (0.17 nm). This wavelength is on the order of atomic dimensions, suggesting that electrons might exhibit diffraction with crystals.

Go to Frame 7.

### Frame 7

**The Davisson-Germer Experiment** (Section 5.2, page 154)

Direct experimental proof of the wave nature of electrons ( $\lambda = h/p$ ) came from the diffraction experiments of Clinton Davisson and Lester Germer in 1927 at Bell Labs. They were initially studying the scattering of low-speed electrons from a polycrystalline nickel target.

What accidental event changed their experiment and led to the observation of electron diffraction? (See page 155). \_\_\_\_\_

Go to Frame 8.

### Frame 8

A flask of liquid air accidentally broke, rupturing their vacuum system and oxidizing the hot nickel target. To remove the oxide, they heated the sample in hydrogen. This prolonged heating **annealed** the nickel, causing large single-crystal regions to form.

When they resumed the experiment with this annealed target, they observed strong variations in the intensity of scattered electrons with angle, which were characteristic of diffraction from a crystal lattice. (See Figure 5.5, page 156, for a polar plot of scattered intensity).

For a specific case (electron energy 54 eV, scattering angle  $\phi = 50^\circ$  for the first-order maximum), Davisson and Germer calculated the experimental wavelength of the electrons using a diffraction

formula similar to Bragg's law for surface reflection (Figure 5.6, page 157):  $d \sin \phi = n\lambda$  (here  $n = 1$ ). With  $d = 2.15 \text{ \AA}$  (from X-ray diffraction data for nickel), they found  $\lambda_{\text{exp}} = (2.15 \text{ \AA}) \sin 50^\circ \approx 1.65 \text{ \AA}$ .

What was the theoretical de Broglie wavelength for these 54 eV electrons? (Use  $\lambda = h/\sqrt{2m_e eV}$  from Frame 6b, with  $V = 54 \text{ V}$ ).  $\lambda_{\text{deBroglie}} =$  \_\_\_\_\_

Go to Frame 9.

## Frame 9

The theoretical de Broglie wavelength for 54 eV electrons is:

$$\lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-19} \text{ C})(54 \text{ V})}} \approx 1.67 \text{ \AA}$$

The excellent agreement between  $\lambda_{\text{exp}} \approx 1.65 \text{ \AA}$  and  $\lambda_{\text{deBroglie}} \approx 1.67 \text{ \AA}$  provided strong confirmation of de Broglie's hypothesis. Diffraction experiments were later performed with other particles like helium atoms, hydrogen atoms, and neutrons, all confirming the universality of matter waves.

The controllable wavelength of electrons, which can be much shorter than visible light, led to the development of the **electron microscope**, offering much higher resolution than light microscopes (pages 159-163).

Go to Frame 10.

## Frame 10

### Wave Groups and Dispersion (Section 5.3, page 164)

A single sinusoidal matter wave ( $\lambda = h/p$ ,  $f = E/h$ ) is infinitely extended in space and time. This cannot properly represent a localized particle. To represent a particle, which is localized, we need a **wave group** or **wave packet** – a superposition of many waves with slightly different wavelengths, interfering constructively in a small region and destructively elsewhere (Figure 5.17b).

A wave group has two important velocities associated with it: 1. **Phase velocity** ( $v_p$ ): The speed of a point of constant phase on an individual component wave (e.g., a crest). For a single wave  $y = A \cos(kx - \omega t)$ ,  $v_p = \omega/k$ . 2. **Group velocity** ( $v_g$ ): The speed of the overall envelope of the wave group.

What is the general relationship between group velocity  $v_g$ , phase velocity  $v_p$ , wavenumber  $k$ , and angular frequency  $\omega$ ? (Eqs. 5.20, 5.21, pages 167-168).  $v_g = d\omega/dk$ . How can this be expressed in terms of  $v_p$  and  $k$ ?  $v_g =$  \_\_\_\_\_

Go to Frame 11.

## Frame 11

Since  $\omega = kv_p$ , the group velocity can be written as:

$$v_g = \frac{d\omega}{dk} = \frac{d(kv_p)}{dk} = v_p + k \frac{dv_p}{dk}$$

(This is from Eq. 5.21 by evaluating the derivative at  $k_0$ , the central wavenumber).

If  $dv_p/dk = 0$  (phase velocity  $v_p$  is independent of  $k$  or  $\lambda$ ), the medium is **nondispersive**, and  $v_g = v_p$ . All component waves travel at the same speed, so the wave packet maintains its shape. If  $dv_p/dk \neq 0$  (phase velocity  $v_p$  depends on  $k$  or  $\lambda$ ), the medium is **dispersive**. Component waves travel at different speeds, and the wave packet spreads out (disperses) over time. (See Figure 5.20, page 168, for an example with a laser pulse in an optical fiber).

Go to Frame 12.

### Frame 12

#### Matter Wave Packets (Page 169)

For a de Broglie matter wave, the phase velocity is  $v_p = f\lambda = (E/h)(h/p) = E/p$ . Using relativistic energy  $E = \sqrt{p^2c^2 + m^2c^4}$ :

$$v_p = \frac{\sqrt{p^2c^2 + m^2c^4}}{p} = c\sqrt{1 + \left(\frac{mc}{p}\right)^2}$$

Alternatively, using  $E = \gamma mc^2$  and  $p = \gamma mv$ :

$$v_p = \frac{\gamma mc^2}{\gamma mv} = \frac{c^2}{v}$$

where  $v$  is the particle's speed.

Since  $v < c$  (for a particle with mass), is the phase velocity  $v_p = c^2/v$  of a de Broglie wave greater than, less than, or equal to  $c$ ?  $v_p$  is [  $>$  —  $=$  ]  $c$ .

Go to Frame 13.

### Frame 13

Since  $v < c$  (for  $m > 0$ ),  $c/v > 1$ . Therefore,  $v_p = c(c/v)$  is **greater than**  $c$ . The individual de Broglie waves (phase waves) travel faster than light! This seems problematic, but these individual waves do not carry energy or information; the wave packet (group) does.

What is the group velocity  $v_g$  for a de Broglie matter wave packet? (The text shows on page 169 that it equals the particle's classical speed  $v$ ).  $v_g =$  \_\_\_\_\_

(The derivation involves  $v_g = dE/dp$  for relativistic particles, or using  $v_g = v_p + k \frac{dv_p}{dk}$  with  $v_p = c\sqrt{1 + (mc/\hbar k)^2}$ ).

Go to Frame 14.

### Frame 14

The group velocity of a de Broglie matter wave packet is:

$$v_g = v$$

where  $v$  is the classical speed of the particle. This is a satisfying result: the wave packet representing the particle moves at the same speed as the particle itself. Even though individual phase waves travel faster than  $c$ , the group (which carries the particle's energy and probability of location) travels at the particle's speed  $v < c$ . Matter waves are dispersive even in empty space because  $v_p = c^2/v$  depends on  $v$  (and thus on  $p$  or  $k$ ).

This completes our introduction to matter waves and their group properties. The next key concept arising from wave packets is the Heisenberg Uncertainty Principle.

End of Section.

## 10 The Heisenberg Uncertainty Principle and Wave-Particle Duality

### Frame 1

We've seen that to represent a localized particle (like an electron), we need to superimpose many matter waves to form a wave packet. This wave packet has a limited spatial extent, say  $\Delta x$ . To construct such a localized wave packet, what is required regarding the range of wavenumbers ( $\Delta k$ ) of the component waves? (Recall the relationship  $\Delta x \Delta k \approx 1$  from the previous chapter's discussion of wave groups, Eq. 5.16, page 167). [a] A very small range of wavenumbers ( $\Delta k \approx 0$ ). [b] A large range of wavenumbers. [c] The range of wavenumbers is unrelated to  $\Delta x$ .

Go to Frame 2.

### Frame 2

Your answer was [ a — b — c ].

The correct answer is [b] A large range of wavenumbers. To make a wave packet that is well-localized in space (small  $\Delta x$ ), we need to superimpose waves with a wide spread of wavenumbers (large  $\Delta k$ ). Conversely, if we use a narrow range of wavenumbers (small  $\Delta k$ ), the resulting wave packet will be spread out in space (large  $\Delta x$ ). The more precisely we try to define the "position" (by making  $\Delta x$  small), the less precisely the "wavelength" (and thus wavenumber  $k = 2\pi/\lambda$ ) is defined, and vice versa. The relationship is roughly  $\Delta x \Delta k \gtrsim 1/2$  (the exact number depends on definitions, with  $1/2$  being the minimum for Gaussian wave packets).

Go to Frame 3.

### Frame 3

**The Heisenberg Uncertainty Principle** (Section 5.5, page 173)

Werner Heisenberg, in 1927, formulated a fundamental principle of quantum mechanics stemming from this wave nature of particles. He connected the uncertainty in wavenumber  $\Delta k$  to the uncertainty in momentum  $\Delta p_x$  using de Broglie's relation  $p_x = \hbar k_x$  (so  $\Delta p_x = \hbar \Delta k_x$ ).

If  $\Delta x \Delta k_x \gtrsim 1/2$ , what does this imply for the product of uncertainties in position ( $\Delta x$ ) and momentum ( $\Delta p_x$ )? Substitute  $\Delta k_x = \Delta p_x / \hbar$ .  $\Delta x \Delta p_x \gtrsim$  \_\_\_\_\_

Go to Frame 4.

### Frame 4

Substituting  $\Delta k_x = \Delta p_x / \hbar$  into  $\Delta x \Delta k_x \gtrsim 1/2$ :

$$\Delta x \left( \frac{\Delta p_x}{\hbar} \right) \gtrsim \frac{1}{2}$$

$$\Delta x \Delta p_x \gtrsim \frac{\hbar}{2}$$

This is the **Heisenberg Uncertainty Principle** for position and momentum (Eq. 5.31, page 174). It states that it is impossible to determine simultaneously with unlimited precision both the position and the momentum of a particle along a given direction. The product of their uncertainties can never be smaller than  $\hbar/2$ .

Does this uncertainty arise from imperfections in measuring instruments? [ Yes — No ] Explain briefly. \_\_\_\_\_

Go to Frame 5.

### Frame 5

The answer is [No]. The Heisenberg Uncertainty Principle is not due to imperfections in measuring instruments. It is a fundamental, inescapable consequence of the wave nature of matter. To localize a particle (wave packet) in a small region  $\Delta x$  inherently requires a superposition of waves with a broad range of momenta  $\Delta p_x$ .

A similar uncertainty relation exists between energy and time. Given  $\Delta\omega\Delta t \gtrsim 1/2$  (where  $\Delta\omega$  is the spread in angular frequencies and  $\Delta t$  is the time duration or measurement time), and using de Broglie's relation  $E = \hbar\omega$  (so  $\Delta E = \hbar\Delta\omega$ ), what is the energy-time uncertainty principle?  $\Delta E\Delta t \gtrsim$  \_\_\_\_\_ (Eq. 5.34, page 175).

Go to Frame 6.

### Frame 6

The energy-time uncertainty principle is:

$$\Delta E\Delta t \gtrsim \frac{\hbar}{2}$$

This means that the precision with which we can know the energy of a system ( $\Delta E$ ) is limited by the time interval ( $\Delta t$ ) available for measuring that energy. For example, if a particle exists for a very short lifetime  $\Delta t$ , its energy (or mass, via  $E = mc^2$ ) will have a correspondingly large uncertainty  $\Delta E$ . This is used to estimate lifetimes of very short-lived subatomic particles from the measurable spread in their mass/energy.

Heisenberg also illustrated the uncertainty principle using a "thought experiment" involving trying to measure an electron's position with a microscope (Figure 5.27, page 176). To "see" the electron, at least one photon must scatter off it and enter the microscope. What limits the precision ( $\Delta x$ ) with which the electron's position can be determined by the microscope? (Hint: Think about diffraction of light by the microscope's lens). What effect does the scattering photon have on the electron? \_\_\_\_\_

Go to Frame 7.

### Frame 7

1. **Precision of position ( $\Delta x$ ):** The ability to resolve the electron's position is limited by the diffraction of the light used. The resolving power of a microscope gives an uncertainty  $\Delta x \approx \lambda/(2 \sin \alpha)$ , where  $\lambda$  is the wavelength of light and  $2\alpha$  is the angle subtended by the microscope lens. To get a small  $\Delta x$  (good position measurement), we need short wavelength  $\lambda$  light (e.g., gamma rays).

2. **Effect on electron:** The scattering photon imparts momentum to the electron (Compton scattering). A photon with short wavelength  $\lambda$  has high momentum ( $p = h/\lambda$ ). This means a precise position measurement (using short  $\lambda$ ) results in a large and uncertain transfer of momentum to the electron, increasing the uncertainty in the electron's momentum ( $\Delta p_x$ ). Heisenberg showed that the product  $\Delta x\Delta p_x$  from this thought experiment is consistent with  $\gtrsim \hbar$ .

Go to Frame 8.

### Frame 8

**If Electrons Are Waves, What's Waving?** (Section 5.6, page 178) **The Wave-Particle Duality** (Section 5.7, page 179)

We've established that matter, like light, exhibits wave-particle duality. Electrons can act like particles (e.g., in Millikan's oil drop experiment, leaving tracks in cloud chambers) and like waves (e.g., Davisson-Germer experiment, electron microscope).

What is the "stuff" that is waving for a matter wave? The quantity that describes this wave is the **wavefunction**, denoted by  $\Psi(x, y, z, t)$  (often just  $\Psi$ ). Is  $\Psi$  itself a directly measurable physical quantity like an electric field? [ Yes — No ] If not, what physical meaning is attached to it? (Hint: Max Born's interpretation).

---

Go to Frame 9.

### Frame 9

$\Psi$  itself is [No], not a directly measurable physical quantity in the same way an electric field is. It can be a complex number. Max Born provided the accepted statistical interpretation: The square of the absolute value of the wavefunction,  $|\Psi|^2 = \Psi^*\Psi$ , represents the **probability density** of finding the particle at a particular point in space at a particular time. The probability of finding the particle in a small volume  $dV$  is  $|\Psi|^2 dV$ .

Go to Frame 10.

### Frame 10

The double-slit experiment for electrons (Figure 5.28, page 180) beautifully illustrates wave-particle duality:

- Electrons are always detected as individual, localized particles ("lumps") at the screen.
- However, if both slits are open, the accumulated pattern of these particle arrivals over time shows an interference pattern, characteristic of waves. The probability of an electron arriving at a certain spot is determined by the interference of matter waves passing through both slits.

If we try to determine which slit an electron goes through, what happens to the interference pattern? (Refer to the thought experiment in Figure 5.33, page 184). [a] The interference pattern becomes sharper. [b] The interference pattern is destroyed. [c] The interference pattern is unchanged.

Go to Frame 11.

### Frame 11

Your answer was [ a — b — c ].

The correct answer is [b]. If we design an experiment to determine which slit the electron passes through (an act of measuring its particle-like path), the very act of measurement inevitably disturbs the electron's momentum enough (due to the uncertainty principle) to destroy the delicate phase relationship needed for interference. The pattern then becomes simply the sum of two single-slit patterns (Figure 5.31, page 183).

This is a core aspect of Bohr's principle of **complementarity**: wave and particle aspects of matter (or light) are complementary. An experiment designed to observe the particle nature will necessarily obscure the wave nature, and vice versa. It is impossible to simultaneously observe both aspects with full clarity.

The wavefunction  $\Psi$  for an electron passing through two slits (when not observed) is a superposition of the wavefunctions for passing through slit 1 ( $\Psi_1$ ) and slit 2 ( $\Psi_2$ ):  $\Psi = \Psi_1 + \Psi_2$ . The probability density is then  $|\Psi|^2 = |\Psi_1 + \Psi_2|^2 = |\Psi_1|^2 + |\Psi_2|^2 + \text{interference terms}$ . If we know which slit it went through, the state is either  $\Psi_1$  or  $\Psi_2$ , and the probability is just  $|\Psi_1|^2$  or  $|\Psi_2|^2$ , and the sum gives no interference.



Go to Frame 12.

## Frame 12

**A Final Note** (Section 5.8, page 186)

Classical physics viewed the world as made of distinct parts interacting deterministically. Quantum mechanics reveals a different picture at the atomic level. An electron (or any particle) is not a fixed, unchanging entity but an "amorphous entity" possessing the potential to exhibit either wave or particle behavior. The type of experiment performed determines which aspect is observed.

The wavefunction  $\Psi$  provides the mathematical description of matter waves. In the next chapter of the original text, the Schrödinger wave equation would be introduced, which is the fundamental equation governing how  $\Psi$  evolves in space and time for a given physical system.

This concludes our exploration of matter waves and the fundamental principles of uncertainty and complementarity.

End of Section.

# 11 Wavefunctions and the Schrödinger Equation

## Frame 1

We've established that particles like electrons exhibit wave-like properties, described by a wavefunction  $\Psi(x, y, z, t)$ . For free particles (no forces acting), we saw that a wave packet can be formed by superimposing plane waves.

What happens to the localization of a matter wave packet for an atomic electron over time if it's not confined by any forces? (See Example 6.4, page 197, "Dispersion of Matter Waves"). [a] It remains localized. [b] It disperses (spreads out) very quickly. [c] It becomes more localized.

Go to Frame 2.

## Frame 2

Your answer was [ a — b — c ].

The correct answer is [b]. An unconfined electron wave packet disperses very quickly. For an electron initially localized to atomic size (0.10 nm), its localization is destroyed by dispersion in about  $1.7 \times 10^{-15}$  s, a time comparable to the period of an electron in a Bohr orbit. In contrast, a macroscopic object like a marble, even if hypothetically localized to 0.10 mm, would take vastly longer than the age of the universe to show quantum dispersion effects. This again shows why quantum effects are not apparent for macroscopic objects.

Go to Frame 3.

## Frame 3

**Wavefunctions in the Presence of Forces** (Section 6.3, page 197)

When a particle is acted on by a force  $F$ , its wavefunction  $\Psi(x, t)$  (considering one dimension for simplicity) must be found from **Schrödinger's equation**. This is the fundamental equation of non-relativistic quantum mechanics.

The time-dependent Schrödinger equation is (Eq. 6.10, page 197):

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + U(x) \Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t}$$

Where:  $\hbar = h/(2\pi)$  (reduced Planck's constant)  $m$  = mass of the particle  $U(x)$  = potential energy function corresponding to the force  $F$  (where  $F = -dU/dx$ )  $i = \sqrt{-1}$  (imaginary unit)  $\Psi(x, t)$  = the time-dependent wavefunction

Is Schrödinger's equation derivable from more basic principles? [ Yes — No ] (See page 198, end of first paragraph).

Go to Frame 4.

### Frame 4

The answer is [No]. Schrödinger’s equation is a fundamental law of quantum physics, analogous to Newton’s laws in classical mechanics or Maxwell’s equations in electromagnetism. Its validity is judged by its ability to make predictions that agree with experiments.

To solve this partial differential equation, a common technique is **separation of variables**. We look for solutions of the form  $\Psi(x, t) = \psi(x)\phi(t)$ , where  $\psi(x)$  is a function of position only, and  $\phi(t)$  is a function of time only. Substituting this into the Schrödinger equation and rearranging leads to two separate ordinary differential equations (details on page 199).

The equation for the time part  $\phi(t)$  is:  $i\hbar \frac{d\phi(t)}{dt} = E\phi(t)$ . What is the solution for  $\phi(t)$ ? (Hint: It's an exponential, and  $E$  is a separation constant which turns out to be the energy).  $\phi(t) = \underline{\hspace{2cm}}$

Go to Frame 5.

### Frame 5

The solution for the time part is (page 200):

$$\phi(t) = e^{-iEt/\hbar}$$

(This can also be written as  $e^{-i\omega t}$  where  $\omega = E/\hbar$ , which is the same time dependence as for free particle plane waves). The separation constant  $E$  is identified as the total energy of the particle.

The equation for the spatial part  $\psi(x)$  is called the **time-independent Schrödinger equation** (Eq. 6.13, page 200):

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

Solutions  $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$  are called **stationary states**. Why are they called stationary states? (Hint: Consider the probability density  $|\Psi(x,t)|^2$ ). \_\_\_\_\_

Go to Frame 6.

## Frame 6

They are called stationary states because the probability density  $|\Psi(x,t)|^2$  is independent of time:

$$|\Psi(x, t)|^2 = |\psi(x)e^{-iEt/\hbar}|^2 = |\psi(x)|^2 |e^{-iEt/\hbar}|^2$$

Since  $|e^{-i\theta}|^2 = (\cos \theta - i \sin \theta)(\cos \theta + i \sin \theta) = \cos^2 \theta + \sin^2 \theta = 1$ , we have:

$$|\Psi(x, t)|^2 = |\psi(x)|^2$$

So, for stationary states, all probabilities are static (unchanging in time) and can be calculated from the time-independent wavefunction  $\psi(x)$ . The wavefunction  $\psi(x)$  must be finite, single-valued, and continuous. Its slope  $d\psi/dx$  must also be continuous wherever  $U(x)$  is finite.

Go to Frame 7.

### Frame 7

#### The Particle in a Box (Section 6.4, page 200)

The simplest problem involving forces is particle confinement. Consider a particle of mass  $m$  confined to move along the  $x$ -axis between  $x = 0$  and  $x = L$  (an "infinite square well" or "box"). Inside the box ( $0 < x < L$ ): Potential energy  $U(x) = 0$  (particle is free). Outside the box ( $x \leq 0$  or  $x \geq L$ ): Potential energy  $U(x) = \infty$  (impenetrable walls).

Since the particle cannot be found outside the box, what must be true about the wavefunction  $\psi(x)$  for  $x \leq 0$  and  $x \geq L$ ?  $\psi(x) =$  \_\_\_\_\_

Inside the box ( $0 < x < L$ ),  $U(x) = 0$ . What does the time-independent Schrödinger equation become? \_\_\_\_\_

Go to Frame 8.

### Frame 8

1. Outside the box:  $\psi(x) = 0$  for  $x \leq 0$  and  $x \geq L$ . 2. Inside the box ( $U(x) = 0$ ): The time-independent Schrödinger equation becomes:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

or

$$\frac{d^2\psi(x)}{dx^2} = -k^2\psi(x) \quad \text{where } k^2 = \frac{2mE}{\hbar^2}$$

The general solution to  $d^2\psi/dx^2 = -k^2\psi$  is  $\psi(x) = A \sin(kx) + B \cos(kx)$ . We need to apply boundary conditions:  $\psi(x)$  must be continuous. So,  $\psi(0) = 0$  and  $\psi(L) = 0$ . Applying  $\psi(0) = 0$ :  $A \sin(0) + B \cos(0) = 0 \implies 0 + B(1) = 0 \implies B = 0$ . So,  $\psi(x) = A \sin(kx)$ . Now apply  $\psi(L) = 0$ :  $A \sin(kL) = 0$ . Since  $A \neq 0$  (otherwise  $\psi(x) = 0$  everywhere, no particle), we must have  $\sin(kL) = 0$ .

What values of  $kL$  satisfy  $\sin(kL) = 0$ ?  $kL =$  \_\_\_\_\_

Go to Frame 9.

### Frame 9

$\sin(kL) = 0$  implies  $kL = n\pi$ , where  $n = 1, 2, 3, \dots$  ( $n = 0$  would mean  $k = 0$ , so  $\psi(x) = 0$ , which is not allowed. Negative  $n$  gives the same states as positive  $n$  apart from an overall sign, which doesn't change  $|\psi|^2$ ). So, the allowed wavenumbers are  $k_n = n\pi/L$ .

Since  $k^2 = 2mE/\hbar^2$ , the allowed energies  $E_n$  are quantized:

$$\left(\frac{n\pi}{L}\right)^2 = \frac{2mE_n}{\hbar^2}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = n^2 \left( \frac{\hbar^2}{8mL^2} \right) \quad (n = 1, 2, 3, \dots)$$

(using  $\hbar = h/2\pi$ ). This is Eq. 6.17, page 202.

What is the lowest possible energy for the particle in a box (ground state energy)? Is it zero?  
[ Yes — No ] This lowest energy is called the \_\_\_\_\_.

Go to Frame 10.

### Frame 10

The lowest energy (ground state) occurs for  $n = 1$ :  $E_1 = h^2/(8mL^2)$ . This is [No], not zero. This minimum energy  $E_1$  is called the **zero-point energy**. The particle can never be at rest in the box ( $E = 0$  is not allowed). This is a purely quantum mechanical result, consistent with the uncertainty principle (if a particle is confined to  $\Delta x \approx L$ , it cannot have  $\Delta p_x = 0$ , so  $p_x$  cannot be exactly zero).

The wavefunctions for the particle in a box are  $\psi_n(x) = A \sin(n\pi x/L)$ . The constant  $A$  is found by normalization:  $\int_0^L |\psi_n(x)|^2 dx = 1$ . This gives  $A = \sqrt{2/L}$  (Eq. 6.19, page 205). So,  $\psi_n(x) = \sqrt{2/L} \sin(n\pi x/L)$ .

Figure 6.9 (page 204) shows  $\psi_n(x)$  and probability densities  $|\psi_n(x)|^2$  for  $n = 1, 2, 3$ . For  $n = 1$ , where is the particle most likely to be found? — For  $n = 2$ , where is the particle most likely to be found? Where is it \*least\* likely (probability = 0)? \_\_\_\_\_

Go to Frame 11.

### Frame 11

For  $n = 1$  (ground state): Most likely at  $x = L/2$  (center of the box). For  $n = 2$  (first excited state): Most likely at  $x = L/4$  and  $x = 3L/4$ . Least likely (probability=0) at  $x = L/2$  (center).

How does a particle get from one side of  $L/2$  to the other if it can never be found at  $L/2$  (for  $n = 2$ )? This question highlights that we shouldn't think of quantum objects as classical particles following definite paths. The wavefunction describes the probability of finding it.

The text (pages 205-209) discusses Charge-Coupled Devices (CCDs) as an application of potential wells, and the Finite Square Well as a more realistic (but more complex) potential. In a finite well, the wavefunction can penetrate slightly into the classically forbidden regions outside the well.

This concludes our introduction to the Schrödinger equation and the particle in an infinite box.  
End of Section.

## 12 The Quantum Oscillator

### Frame 1

We now examine another important potential for which exact quantum mechanical results can be obtained: a particle subject to a linear restoring force  $F = -Kx$ . This is the force exerted by an ideal spring, and the system is called a harmonic oscillator.

What is the potential energy  $U(x)$  corresponding to the force  $F = -Kx$ , if we take  $U(0) = 0$ ? (Hint:  $F = -dU/dx$ ).  $U(x) =$  \_\_\_\_\_

Go to Frame 2.

### Frame 2

The potential energy for a linear restoring force  $F = -Kx$  is:

$$U(x) = \frac{1}{2} Kx^2$$

This parabolic potential energy describes not only a mass on a spring but also approximates the potential energy near any point of stable equilibrium for a more general potential (Figure 6.17, page 212). The constant  $K$  is related to the curvature of the potential at the equilibrium point:  $K = d^2U/dx^2|_{\text{equilibrium}}$ .

Classically, a particle with total energy  $E$  in this potential oscillates between  $x = -A$  and  $x = +A$ , where  $A$  is the amplitude, and  $E = \frac{1}{2}KA^2$ . The classical particle can have any non-negative energy.

Go to Frame 3.

### Frame 3

To find the quantum mechanical behavior, we substitute  $U(x) = \frac{1}{2}Kx^2 = \frac{1}{2}m\omega^2x^2$  (where  $\omega = \sqrt{K/m}$  is the classical angular frequency) into the time-independent Schrödinger equation:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + \frac{1}{2}m\omega^2x^2\psi(x) = E\psi(x)$$

Rearranging gives (Eq. 6.25, page 213):

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2} \left( E - \frac{1}{2}m\omega^2x^2 \right) \psi(x)$$

Solving this differential equation is complex. However, we can try to guess the form of the ground state wavefunction  $\psi_0(x)$ .

What properties should the ground state wavefunction  $\psi_0(x)$  for the harmonic oscillator have? (Hint: Think about symmetry and behavior at large  $|x|$ , similar to the particle in a box ground state). 1. Symmetry: — 2. Behavior for large  $|x|$ : — 3. Number of nodes (excluding infinity): —

Go to Frame 4.

### Frame 4

Properties expected for the ground state wavefunction  $\psi_0(x)$ : 1. Symmetric about  $x = 0$  (since the potential is symmetric). 2. Should approach zero for large  $|x|$  (particle is bound). 3. Should have no nodes (be nodeless), similar to the ground state of the particle in a box.

A simple mathematical function satisfying these is the Gaussian form (Eq. 6.26, page 213):

$$\psi_0(x) = C_0 e^{-\alpha x^2}$$

where  $C_0$  and  $\alpha$  are constants to be determined.  $C_0$  is for normalization, and  $\alpha$  will be determined by substituting this guess into the Schrödinger equation.

Go to Frame 5.

### Frame 5

If we substitute  $\psi_0(x) = C_0 e^{-\alpha x^2}$  into the Schrödinger equation for the harmonic oscillator, and compare terms, we find that it is indeed a solution provided (page 214): 1.  $2\alpha = \frac{m\omega}{\hbar}$  (so  $\alpha = \frac{m\omega}{2\hbar}$ ) 2. The energy  $E_0$  of this ground state is  $E_0 = \alpha \frac{\hbar^2}{m} = \left(\frac{m\omega}{2\hbar}\right) \frac{\hbar^2}{m} = \frac{1}{2}\hbar\omega$ .

So the ground state energy of the quantum harmonic oscillator is  $E_0 = \frac{1}{2}\hbar\omega$ . This is another example of **zero-point energy**. Classically, an oscillator can have zero energy (at rest at  $x = 0$ ), but quantum mechanically, the lowest possible energy is  $\frac{1}{2}\hbar\omega$ .

The ground state wavefunction is  $\psi_0(x) = C_0 e^{-m\omega x^2/(2\hbar)}$ . Figure 6.18 (page 214) shows  $\psi_0(x)$  and  $|\psi_0(x)|^2$ . What is a striking feature of  $|\psi_0(x)|^2$  compared to the classical turning points  $x = \pm A$  (where  $A = \sqrt{2E_0/(m\omega^2)} = \sqrt{\hbar/(m\omega)}$  for the ground state)? \_\_\_\_\_

Go to Frame 6.

### Frame 6

The higher energy states (excited states) of the quantum oscillator can be found using more advanced techniques (like the power series method mentioned on page 215). The allowed energies are found to be (Eq. 6.29):

Notice  $n = 0$  gives the ground state energy  $E_0 = \frac{1}{2}\hbar\omega$ .

### Frame 7

This result provides the quantum justification for Planck’s original hypothesis about his cavity resonators in the theory of blackbody radiation. Planck assumed oscillators could only have energies that were multiples of  $hf = \hbar\omega$ . The quantum oscillator indeed emits or absorbs energy in bundles of  $\Delta E = \hbar\omega$ . (The  $\frac{1}{2}\hbar\omega$  zero-point energy doesn’t affect the energy differences).

## Frame 8

Describe two key differences between the quantum and classical probability densities for low  $n$  values (e.g.,  $n = 0, 1$ ). 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 9.

Agreement for large  $n$ : As  $n$  becomes large, the quantum probability distribution increasingly resembles the classical one. The quantum probability also becomes large near the classical turning

points and has many oscillations. This is an example of **Bohr's correspondence principle**: quantum mechanical predictions must agree with classical physics in the limit of large quantum numbers (where classical physics is known to be valid).

This concludes our look at the quantum harmonic oscillator, a fundamental system in quantum mechanics with wide applications.

End of Section.

## 13 Expectation Values, Observables, and Operators

### Frame 1

#### Expectation Values (Section 6.7, page 217)

For a particle described by a wavefunction  $\Psi(x, t)$ , some quantities like the energy  $E$  of a stationary state are "sharp" – every measurement yields the same value. Other quantities, like position  $x$ , are "fuzzy" – measurements yield a range of values, and the wavefunction only provides probabilities.

If we make many repeated measurements of the position  $x$  of a particle, we get a distribution of values (like in Table 6.1). How do we calculate the average position, also known as the **expectation value** of  $x$ , denoted by  $\langle x \rangle$ ? (Eq. 6.31, page 218).  $\langle x \rangle = \text{_____}$  (in terms of  $\Psi(x, t)$  or  $\psi(x)$  for stationary states).

Go to Frame 2.

### Frame 2

The expectation value (average value) of position  $x$  is given by:

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi(x, t)|^2 dx$$

For stationary states,  $|\Psi(x, t)|^2 = |\psi(x)|^2$ , so:

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\psi(x)|^2 dx$$

The term  $|\Psi(x, t)|^2$  is the probability density. We are essentially calculating a weighted average of  $x$ , where the weight is the probability of finding the particle at  $x$ .

Similarly, the expectation value of any function of  $x$ , say  $f(x)$ , is (Eq. 6.32):

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x) |\Psi(x, t)|^2 dx$$

For example, to find the average potential energy  $\langle U(x) \rangle$ , we would use  $f(x) = U(x)$ .

Go to Frame 3.

### Frame 3

The "fuzziness" or spread of measurements around the average value  $\langle x \rangle$  is quantified by the standard deviation  $\sigma_x$ , often called the **uncertainty** in position,  $\Delta x$ . It is defined as (Eq. 6.34, page 219):

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

To calculate this, we need the expectation value of  $x$ ,  $\langle x \rangle$ , and the expectation value of  $x^2$ ,  $\langle x^2 \rangle$ . How would you calculate  $\langle x^2 \rangle$ ?  $\langle x^2 \rangle =$  \_\_\_\_\_

Go to Frame 4.

#### Frame 4

Using the rule from Frame 2 for  $\langle f(x) \rangle$  with  $f(x) = x^2$ :

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\Psi(x, t)|^2 dx$$

If  $\Delta x = 0$ , the position is "sharp." If  $\Delta x > 0$ , the position is "fuzzy."

The text (Example 6.15, page 219) calculates  $\langle x \rangle$  and  $\Delta x$  for a particle in the ground state of an infinite box of length  $L$ . The wavefunction is  $\psi_1(x) = \sqrt{2/L} \sin(\pi x/L)$ . What is the expectation value of position  $\langle x \rangle$  for the ground state? (Think about symmetry, or see page 219).  $\langle x \rangle =$  \_\_\_\_\_

What is the uncertainty  $\Delta x$  for this state? (The calculation is on page 220).  $\Delta x = L\sqrt{1/3 - 1/(2\pi^2)} \approx 0.181L$ . This shows the position is quite fuzzy, about 1/5th the size of the box.

Go to Frame 5.

#### Frame 5

For the ground state of a particle in a box ( $n = 1$ ):  $\langle x \rangle = L/2$ . This is expected due to the symmetry of  $|\psi_1(x)|^2$  about the center of the box.

Now, what about the average momentum  $\langle p_x \rangle$ ? Classically,  $p_x(x) = mv_x(x)$ . But in quantum mechanics, there's no definite path  $x(t)$ , so we can't define  $p_x$  as a simple function of  $x$  in the same way. One approach (Eq. 6.35, page 220) is to relate average momentum to the time derivative of average position:

$$\langle p_x \rangle = m \frac{d\langle x \rangle}{dt}$$

For a stationary state, is  $\langle x \rangle$  time-dependent? [ Yes — No ] What does this imply for  $\langle p_x \rangle$  in a stationary state? 1. Time dependence of  $\langle x \rangle$ : \_\_\_\_\_ 2.  $\langle p_x \rangle$  for stationary state: \_\_\_\_\_

Go to Frame 6.

#### Frame 6

1. For a stationary state,  $|\Psi(x, t)|^2 = |\psi(x)|^2$ , which is independent of time. Therefore,  $\langle x \rangle = \int x |\psi(x)|^2 dx$  is also independent of time. 2. Since  $\langle x \rangle$  is constant for a stationary state,  $d\langle x \rangle/dt = 0$ . Thus,  $\langle p_x \rangle = 0$  for any stationary state. This makes sense for bound states like the particle in a box, where the particle is bouncing back and forth, so its average momentum is zero.

A more general (and fundamental) way to calculate expectation values involves quantum mechanical **operators**.

Go to Frame 7.

#### Frame 7

##### Observables and Operators (Section 6.8, page 221)

In quantum mechanics, every measurable physical quantity (an **observable** like position, momentum, energy) is associated with an **operator**. An operator is a mathematical instruction for an operation to be performed on the function that follows it (the operand, which is usually the wavefunction  $\Psi$ ).



The expectation value of an observable  $Q$  is written in the general form (Eq. 6.37):

$$\langle Q \rangle = \int_{-\infty}^{\infty} \Psi^*(x, t) [\hat{Q}] \Psi(x, t) dx$$

where  $\Psi^*$  is the complex conjugate of  $\Psi$ , and  $[\hat{Q}]$  (often just  $\hat{Q}$  or  $Q_{op}$ ) is the operator associated with the observable  $Q$ . The operator acts on  $\Psi(x, t)$  to its right.

What are the operators for: 1. Position  $x$ :  $[\hat{x}] = \underline{\hspace{1cm}}$  2. Potential Energy  $U(x)$ :  $[\widehat{U(x)}] = \underline{\hspace{1cm}}$  3. Momentum  $p_x$ :  $[\hat{p}_x] = \underline{\hspace{1cm}}$  (This one comes from comparing Eq. 6.36 with the general form).

Go to Frame 8.

### Frame 8

The operators are: 1. Position:  $[\hat{x}] = x$  (multiplication by  $x$ ) 2. Potential Energy:  $[\widehat{U(x)}] = U(x)$  (multiplication by  $U(x)$ ) 3. Momentum:  $[\hat{p}_x] = \frac{\hbar}{i} \frac{\partial}{\partial x} = -i\hbar \frac{\partial}{\partial x}$  (taking the partial derivative with respect to  $x$  and multiplying by  $-i\hbar$ ).

Using these, we can construct operators for other observables. For example, kinetic energy  $K = p_x^2/(2m)$ . The operator for kinetic energy  $[\hat{K}]$  is:  $[\hat{K}] = \frac{[\hat{p}_x]^2}{2m} = \frac{1}{2m} (-i\hbar \frac{\partial}{\partial x}) (-i\hbar \frac{\partial}{\partial x}) = \frac{(-i\hbar)^2}{2m} \frac{\partial^2}{\partial x^2} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$ .

The total energy operator is called the **Hamiltonian operator**  $[\hat{H}]$ :  $[\hat{H}] = [\hat{K}] + [\widehat{U(x)}] = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x)$ . (This is for one dimension. For stationary states, use  $d/dx$  instead of  $\partial/\partial x$ ).

What is the operator for total energy  $E$  that arises from the time-dependent part of the Schrödinger equation? (Hint: Look at the RHS of Schrödinger's eq:  $i\hbar \partial \Psi / \partial t$ , and the general form  $\langle E \rangle = \int \Psi^* [\hat{E}] \Psi dx$ ).  $[\hat{E}] = \underline{\hspace{1cm}}$

Go to Frame 9.

### Frame 9

The operator for total energy derived from the time-dependent Schrödinger equation is (Eq. 6.40, page 222):

$$[\hat{E}] = i\hbar \frac{\partial}{\partial t}$$

So, we have two operators that, when applied to a solution  $\Psi$  of the Schrödinger equation, should correspond to the energy:  $[\hat{H}] = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x)$  (acts on spatial part)  $[\hat{E}] = i\hbar \frac{\partial}{\partial t}$  (acts on time part) Indeed, the Schrödinger equation itself ( $[\hat{H}]\Psi = [\hat{E}]\Psi$ ) states that the action of these two operators on  $\Psi$  yields the same result.

Table 6.2 (page 222) summarizes common observables and their associated operators.

The quantum uncertainty  $\Delta Q$  for any observable  $Q$  is given by (Eq. 6.41):

$$\Delta Q = \sqrt{\langle Q^2 \rangle - \langle Q \rangle^2}$$

If  $\Delta Q = 0$ , the observable  $Q$  is said to be "sharp," and all measurements yield the same value. If  $\Delta Q > 0$ ,  $Q$  is "fuzzy."

This concludes our introduction to expectation values and quantum mechanical operators. These are central tools for calculating measurable properties of quantum systems.

End of Section.

## 14 Tunneling Phenomena: The Square Barrier

### Frame 1

In this chapter, we apply wave mechanics to particles encountering a potential **barrier**. Unlike potential wells that trap particles, barriers repel them. Our focus will be on whether a particle incident on a barrier is reflected or transmitted. A key phenomenon we'll encounter is **tunneling**, a purely wave-mechanical effect essential for many modern devices.

What is the fundamental difference in how a potential barrier interacts with a particle compared to a potential well? [a] Barriers attract particles, wells repel them. [b] Barriers repel particles, wells attract and can trap them. [c] Both attract particles.

Go to Frame 2.

### Frame 2

Your answer was [ a — b — c ].

The correct answer is [b]. Barriers repel particles, while potential wells attract and can trap particles (leading to bound states). Barriers generally do not have bound states.

We'll start with the **Square Barrier** (Section 7.1, page 231). Imagine a potential energy  $U(x)$  that is:

- $U(x) = 0$  for  $x < 0$  (Region I)
- $U(x) = U_0$  (a constant positive value) for  $0 \leq x \leq L$  (Region II, the barrier region)
- $U(x) = 0$  for  $x > L$  (Region III)

(See Figure 7.1b, page 232, for a sketch). This potential can be created, for example, by charged hollow cylinders.

Go to Frame 3.

### Frame 3

Consider a classical particle with total energy  $E$  incident on this square barrier from the left.

1. If  $E > U_0$ : What happens to the particle classically? Does it cross the barrier? Does its speed change while inside the barrier? —
2. If  $E < U_0$ : What happens to the particle classically? Does it cross the barrier? —

---

Go to Frame 4.

### Frame 4

Classically: 1. If  $E > U_0$ : The particle **crosses** the barrier. Its kinetic energy inside the barrier is  $K' = E - U_0$ , so its speed is reduced. After passing  $x = L$ , its speed returns to its initial value. 2. If  $E < U_0$ : The particle is **reflected** by the barrier at  $x = 0$ . It cannot enter or cross the barrier region because its kinetic energy would have to be negative, which is impossible classically. The region  $0 \leq x \leq L$  is classically forbidden.

Quantum mechanically, the situation is different due to the wave nature of the particle. Is there any region inaccessible to a quantum particle, regardless of its energy  $E$  relative to  $U_0$ ? [ Yes — No ] (Hint: Think about the wavefunction  $\Psi(x, t)$ ).

Go to Frame 5.

### Frame 5

The answer is [No]. Quantum mechanically, the wavefunction  $\Psi(x, t)$  associated with the particle can be non-zero everywhere, even in classically forbidden regions. This means there is a probability of finding the particle inside the barrier and even beyond it, even if  $E < U_0$ . The process of a particle passing through a barrier that it classically shouldn't be able to overcome is called **tunneling**. (See Figure 7.2a, page 233, for a typical wavefunction).

Go to Frame 6.

### Frame 6

Let's analyze the stationary state wavefunction  $\psi(x)$  for a particle with energy  $E$  incident on the square barrier from the left. We divide space into three regions:

- Region I ( $x < 0$ ,  $U(x) = 0$ ):  $\psi_I(x) = Ae^{ikx} + Be^{-ikx}$  Here,  $Ae^{ikx}$  represents the **incident wave** (traveling right) and  $Be^{-ikx}$  represents the **reflected wave** (traveling left). The wavenumber  $k = \sqrt{2mE}/\hbar$ .
- Region III ( $x > L$ ,  $U(x) = 0$ ):  $\psi_{III}(x) = Fe^{ikx} + Ge^{-ikx}$   $Fe^{ikx}$  is the **transmitted wave** (traveling right). What does  $Ge^{-ikx}$  represent, and why do we usually set  $G = 0$  for a particle incident from the left? \_\_\_\_\_

Go to Frame 7.

### Frame 7

In Region III ( $x > L$ ),  $Ge^{-ikx}$  would represent a wave incident from the right (traveling left). Since we are considering a particle incident from the left only, there is no wave coming from  $+\infty$ , so we set  $G = 0$ . Thus,  $\psi_{III}(x) = Fe^{ikx}$ .

Now consider Region II ( $0 \leq x \leq L$ ,  $U(x) = U_0$ ). Case 1:  $E > U_0$  (particle energy greater than barrier height). The time-independent Schrödinger equation is  $-\frac{\hbar^2}{2m}\psi'' + U_0\psi = E\psi$ , or  $\psi'' = -\frac{2m(E-U_0)}{\hbar^2}\psi = -k'^2\psi$ . The solution is  $\psi_{II}(x) = Ce^{ik'x} + De^{-ik'x}$ , where  $k' = \sqrt{2m(E-U_0)}/\hbar$ . This represents waves traveling in both directions inside the barrier due to reflections at  $x = L$ .

Case 2:  $E < U_0$  (particle energy less than barrier height - the tunneling case). The Schrödinger equation becomes  $\psi'' = \frac{2m(U_0-E)}{\hbar^2}\psi = \alpha^2\psi$ , where  $\alpha = \sqrt{2m(U_0-E)}/\hbar$  is real and positive (Eq. 7.6, page 234). What is the general form of the solution  $\psi_{II}(x)$  in this  $E < U_0$  case? (Hint: Solutions to  $\psi'' = \alpha^2\psi$  are real exponentials).  $\psi_{II}(x) =$  \_\_\_\_\_

Go to Frame 8.

### Frame 8

For  $E < U_0$  in Region II (the barrier), the wavefunction is:

$$\psi_{II}(x) = Ce^{\alpha x} + De^{-\alpha x}$$

(This is from Eq. 7.7, page 234, for the spatial part, where  $C$  and  $D$  are different constants from the  $E > U_0$  case). This represents an exponentially decaying and an exponentially growing solution within the barrier. Typically, for a wide barrier where particles are incident from the left, the  $Ce^{\alpha x}$  term (growing) might be discarded or be very small if the barrier is thick enough such that reflections from the  $x = L$  edge are minimal for the first part of the barrier, but in general both terms are needed to satisfy boundary conditions. The text uses both  $C$  and  $D$ .

The coefficients  $A, B, C, D, F$  are determined by applying **boundary conditions** at  $x = 0$  and  $x = L$ : Both  $\psi(x)$  and its derivative  $d\psi/dx$  must be continuous at these boundaries. This ensures a "smooth joining" of the wavefunctions (Eq. 7.8, page 234).

Go to Frame 9.

### Frame 9

We define the **reflection coefficient**  $R$  and the **transmission coefficient**  $T$ :  $R = \frac{\text{reflected probability density}}{\text{incident probability density}} = \frac{|B|^2}{|A|^2}$  (Eq. 7.2)  $T = \frac{\text{transmitted probability density}}{\text{incident probability density}} = \frac{|F|^2}{|A|^2}$  (Eq. 7.4) These represent the probability that the particle is reflected or transmitted, respectively. What must be true about the sum  $R + T$ ? (Eq. 7.5).  $R + T =$  \_\_\_\_\_  
Go to Frame 10.

### Frame 10

Since the particle must be either reflected or transmitted, their probabilities must sum to unity:

$$R + T = 1$$

The exact expression for  $T$  when  $E < U_0$  (tunneling) is given by Eq. 7.9 (page 235):

$$T(E) = \left[ 1 + \frac{U_0^2}{4E(U_0 - E)} \sinh^2(\alpha L) \right]^{-1}$$

where  $\alpha = \sqrt{2m(U_0 - E)}/\hbar$  and  $\sinh x = (e^x - e^{-x})/2$ . (For  $E > U_0$ ,  $\alpha$  becomes imaginary, and  $\sinh(i\theta) = i \sin \theta$ , leading to oscillatory behavior for  $T(E)$  as shown in Figure 7.3, page 235. Transmission resonances,  $T(E) = 1$ , can occur).

For  $E < U_0$ , and a "wide" or "tall" barrier (meaning  $\alpha L \gg 1$ ),  $\sinh(\alpha L) \approx e^{\alpha L}/2$ . In this approximation, how does the transmission coefficient  $T$  depend on the barrier width  $L$  and the factor  $\alpha$ ?  $T \approx$  \_\_\_\_\_ (Hint:  $T \approx 1/(\text{something large} \times (e^{\alpha L}/2)^2)$ ).

Go to Frame 11.

### Frame 11

For  $\alpha L \gg 1$ ,  $T(E) \approx \frac{16E(U_0 - E)}{U_0^2} e^{-2\alpha L}$ . The most important feature is the exponential dependence:  $T \propto e^{-2\alpha L}$ . This shows that the tunneling probability decreases exponentially with:

- Increasing barrier width  $L$ .
- Increasing  $\alpha = \sqrt{2m(U_0 - E)}/\hbar$ . This means it decreases with:
  - Increasing particle mass  $m$ .
  - Increasing barrier height  $U_0$  relative to particle energy  $E$ .

Example 7.1 (page 235) shows that for 7.00 eV electrons incident on a 10.0 eV barrier:

- If  $L = 5.0$  nm (50 Å),  $T \approx 0.96 \times 10^{-38}$  (extremely small).
- If  $L = 1.0$  nm (10 Å),  $T \approx 0.66 \times 10^{-7}$  (still small, but vastly larger).

This extreme sensitivity to barrier width is characteristic of tunneling.

Go to Frame 12.

### Frame 12

Applications of Tunneling mentioned in the chapter outline include:

- **Field Emission:** Electrons tunneling out of a metal surface under the influence of a strong external electric field, which effectively lowers and thins the potential barrier at the surface.
- **$\alpha$  Decay:** Alpha particles tunneling out of the nucleus, despite being bound by a potential barrier created by the strong nuclear force and Coulomb repulsion.
- **Ammonia Inversion:** The nitrogen atom in an  $\text{NH}_3$  molecule can tunnel through the potential barrier presented by the plane of the three hydrogen atoms, causing the molecule to invert its configuration. This forms the basis of ammonia masers.
- **Scanning Tunneling Microscope (STM):** Uses electron tunneling between a sharp conducting tip and a sample surface to map the surface topography with atomic resolution (discussed in the essay following the chapter).

Tunneling is a purely quantum phenomenon with no classical analog and is crucial in many areas of modern physics and technology.

This concludes our discussion of the square barrier and the basics of tunneling.

End of Section.

## 15 Essay: The Scanning Tunneling Microscope

### Frame 1

The basic idea of quantum mechanics, that particles can exhibit wave properties and vice versa, is one of the strangest concepts in science. One of the most direct consequences of this wave nature of particles is tunneling. What is tunneling in quantum mechanics? [a] Particles creating physical holes through barriers. [b] Particles passing through a potential energy barrier even if their kinetic energy is less than the barrier height. [c] Particles always reflecting off potential barriers.

Go to Frame 2.

### Frame 2

Your answer was [ a — b — c ].

The correct answer is [b]. Tunneling is the quantum mechanical phenomenon where a particle can pass through a potential energy barrier even if its total energy is less than the height of the barrier, something impossible in classical mechanics. This phenomenon is at the heart of a very practical and powerful device: the **Scanning Tunneling Microscope (STM)**.

What is the primary capability of an STM? (See page 253, first paragraph). \_\_\_\_\_

Go to Frame 3.

### Frame 3

The STM enables physicists to make highly detailed images of surfaces with resolution comparable to the size of a single atom (around  $2 \text{ \AA}$ , or  $2 \times 10^{-10} \text{ m}$ ). This allows for the study of structures and processes at the atomic scale. (Figure 1 in the essay shows an STM image of a gold surface with atomic-scale terraces and steps).

For comparison, what limits the resolution of an ordinary optical microscope, and what is its typical best resolution? 1. Limiting factor: \_\_\_\_\_ 2. Best resolution: \_\_\_\_\_

Go to Frame 4.

#### Frame 4

For an ordinary optical microscope: 1. Limiting factor: The wavelength of the light used. 2. Best resolution: About 2000 Å (200 nm), which is roughly half the wavelength of visible light. This is far too large to see individual atoms.

Electron microscopes can achieve much better resolution (e.g., 2 Å) by using electron waves with very short de Broglie wavelengths. However, high-energy electrons typically penetrate the material, giving less information about the very top surface layer of atoms. The STM excels at surface imaging.

Who invented the STM, and when did they receive the Nobel Prize for it? (See page 254). —  
Go to Frame 5.

#### Frame 5

The STM was invented by Gerd Binnig and Heinrich Rohrer at IBM Research Laboratory in Zurich. They shared the 1986 Nobel Prize in Physics for their work, remarkably quickly after their first experiments in the early 1980s.

The basic operation of an STM (Figure 3 in the essay, page 255) involves a conducting probe with a very sharp tip brought extremely close to the conducting surface to be studied. Classically, if an electron in the surface (or tip) does not have enough energy to overcome the work function (the energy needed to escape the material, which forms a potential barrier), can it cross the empty space (vacuum gap) to the tip (or surface)? [ Yes — No ]

Quantum mechanically, can it? [ Yes — No ]

Go to Frame 6.

#### Frame 6

Classically: [No], electrons cannot escape if their energy is less than the work function. Quantum mechanically: [Yes], electrons can **tunnel** across the vacuum gap (potential barrier) between the surface and the tip.

The probability of an electron tunneling across the gap (and thus the tunneling current) is extremely sensitive to the width of the gap,  $L$  (the distance between tip and surface). The characteristic length scale for tunneling,  $\delta$ , is related to the work function  $\Phi = U - E$  (barrier height an electron needs to overcome) by (page 255):

$$\delta = \frac{\hbar}{\sqrt{2m_e\Phi}}$$

For a typical work function of 4.0 eV,  $\delta \approx 1$  Å. The tunneling current density  $j$  is proportional to  $e^{-2L/\delta}$ .

If  $L$  increases by just 0.01 Å (when  $\delta \approx 1$  Å), by what factor does the current change approximately? [a] Decreases by about 2% [b] Decreases by about 20% [c] Decreases by a factor of  $e^2$

Go to Frame 7.

#### Frame 7

Your answer was [ a — b — c ].

The correct answer is [a] Decreases by about 2%. The current is multiplied by a factor  $e^{-2(\Delta L)/\delta} = e^{-2(0.01 \text{ Å})/(1.0 \text{ Å})} = e^{-0.02} \approx 1 - 0.02 = 0.98$ . So the current decreases by 2%. This extreme sensitivity to the gap distance  $L$  is the basis for the STM's operation.

The STM can be operated in two main modes (see Figure 4 in the essay, page 256):

1. **Constant Current Mode:** A feedback network adjusts the vertical position ( $z$ ) of the tip to keep the tunneling current  $I$  constant as the tip is scanned laterally ( $x, y$ ) across the surface. The image is a plot of  $z$  versus  $(x, y)$ , which maps the surface topography. Is this mode better for atomically flat surfaces or surfaces with significant height variations? \_\_\_\_\_

2. **Constant Height Mode:** The tip is scanned at a nearly constant height above the surface, and the variations in tunneling current  $I$  are measured. The image is a plot of  $I$  versus  $(x, y)$ . Is this mode faster or slower than constant current mode? \_\_\_\_\_

Go to Frame 8.

### Frame 8

1. **Constant Current Mode** is better for surfaces that are **not atomically flat**, as the tip can retract or approach to follow the contours while maintaining a constant current. However, the feedback mechanism makes this mode relatively slow.

2. **Constant Height Mode** is much **faster** because the tip doesn't need to make large vertical adjustments. It's best suited for atomically flat surfaces where current variations will primarily reflect atomic-scale features rather than large height changes. This speed allows for making "STM movies" of surface processes in real time.

Figure 7 in the essay (page 258) shows an STM image of a graphite surface. Some carbon atoms in the hexagonal rings appear higher than others. Why is this, even though all atoms in that surface layer are at the same physical level? \_\_\_\_\_

Go to Frame 9.

### Frame 9

In the graphite image, the atoms that appear higher are those that are **not** directly bonded to carbon atoms in the layer immediately beneath them. For these atoms, some of their electron density (which would have been involved in bonding to a subsurface atom) extends further into the space above the surface. The STM maps the topography of the electron distribution (specifically, states near the Fermi level available for tunneling). Since these atoms have more electron density protruding, the tip senses them as "higher" to maintain a constant tunneling current, or draws more current if in constant height mode.

Can STMs operate in environments other than high vacuum? [ Yes — No ] Give an example.

Go to Frame 10.

### Frame 10

The answer is [Yes]. Electron tunneling can occur not just through vacuum but also through insulating gases and liquids. For example, the STM image in Figure 5 of the essay (tantalum disulfide) was obtained with the surface and tip immersed in liquid nitrogen. STMs can even operate in air or water. This versatility is a significant advantage.

The STM's operation relies on a quantum mechanical phenomenon (tunneling) that was understood theoretically in the 1920s, yet the device itself wasn't built until the 1980s. This highlights that practical applications of fundamental science can sometimes take a long time to emerge.

This concludes our look at the Scanning Tunneling Microscope.

End of Section.

## 16 Central Forces and Angular Momentum

### Frame 1

In our previous discussions of quantum mechanics, we often used Cartesian coordinates ( $x$ ,  $y$ ,  $z$ ). While this is a natural way to extend from one to three dimensions, it's not always the most convenient for every physical situation.

Consider an atomic electron. It is attracted to the nucleus by the Coulomb force. This force is an example of a **central force**.

What does it mean for a force to be "central"? \_\_\_\_\_

Go to Frame 2 for the answer.

### Frame 2

A central force is one that is always directed towards a fixed point. In the case of an atomic electron, the nucleus is this fixed point, acting as the center of force.

Given that the force is directed towards a central point (the nucleus), what coordinate system would be most natural to describe the electron's position relative to this nucleus? (Hint: Think about symmetry. See Figure 8.5 in the original text.) [a] Cartesian coordinates ( $x$ ,  $y$ ,  $z$ ) [b] Cylindrical coordinates ( $\rho$ ,  $\phi$ ,  $z$ ) [c] Spherical coordinates ( $r$ ,  $\theta$ ,  $\phi$ )

Go to Frame 3 for the answer.

### Frame 3

Your answer was [ a — b — c ].

The most natural coordinates are [c] **spherical coordinates** ( $r$ ,  $\theta$ ,  $\phi$ ), centered on the nucleus. Here, ' $r$ ' is the distance from the nucleus, ' $\theta$ ' is the polar angle (angle from the  $z$ -axis), and ' $\phi$ ' is the azimuthal angle (angle in the  $x$ - $y$  plane from the  $x$ -axis).

If a central force is also *conservative* (meaning the work done by it is independent of the path, and it can be derived from a potential energy function), what quantity related to the particle remains constant? \_\_\_\_\_

Go to Frame 4.

### Frame 4

If the central force is conservative, the particle's total **energy** (kinetic plus potential) stays constant. This energy,  $E$ , becomes a candidate for quantization, meaning it can only take on specific, discrete values. Stationary quantum states would then be represented by wavefunctions of the form  $\Psi(r, \theta, \phi)e^{-i\omega t}$ , where  $\omega = E/\hbar$ .

For central forces, another quantity is also conserved classically. Since a central force exerts no torque about the force center, what other physical quantity of the orbiting particle is constant? —

Go to Frame 5.

### Frame 5

For central forces, **angular momentum ( $\mathbf{L}$ )** about the force center is also constant (classically).

One might naively expect that if angular momentum  $\mathbf{L}$  is constant, then all three of its components ( $L_x$ ,  $L_y$ ,  $L_z$ ) could have sharp, well-defined values simultaneously in quantum mechanics. Is this expectation correct? [ Yes — No ]

Go to Frame 6 for the answer.



### Frame 6

The answer is [No].

It is impossible to find a wavefunction for which all three components of angular momentum ( $L_x, L_y, L_z$ ) take sharp values simultaneously. This is a fundamental aspect of quantum mechanics, similar to the uncertainty principle for position and momentum.

The text states: "it is impossible to specify simultaneously any two components of angular momentum." What does this imply if one component of L, say  $L_z$ , is known precisely (has a "sharp" value)? \_\_\_\_\_

Go to Frame 7.

### Frame 7

If one component of L (e.g.,  $L_z$ ) is sharp, then the other two components ( $L_x$  and  $L_y$ ) must be "fuzzy" or uncertain.

This situation is analogous to knowing the direction of L precisely (Figure 8.6). If the direction of L were known, the particle would be confined to its orbital plane. This would mean its coordinate perpendicular to this plane is precisely known (and unchanging), and thus its momentum component perpendicular to the plane would also be precisely known (as zero). This simultaneous precise knowledge of position and momentum in that direction would violate the uncertainty principle.

So, for central forces, which quantities can be simultaneously sharp (quantized)? We know Energy (E) can be. From the discussion above, only \*one\* component of L can be sharp, typically chosen as  $L_z$ . What about the \*magnitude\* of the angular momentum vector, often denoted as L (or  $|\mathbf{L}|$ )? Can it also be sharp at the same time as E and  $L_z$ ? [ Yes — No ]

Go to Frame 8.

### Frame 8

The answer is [Yes].

For central force problems, the sharp observables that can be simultaneously quantized are:

1. Energy (E)
2. The magnitude of angular momentum (L)
3. One component of angular momentum (conventionally  $L_z$ )

To find wavefunctions where L and  $L_z$  are both sharp, we separate variables in Schrödinger's time-independent equation when expressed in spherical coordinates. We assume the wavefunction  $\psi(r, \theta, \phi)$  can be written as a product of three functions, each depending on only one variable:

$$\psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$$

Here,  $R(r)$  is the radial part, and  $\Theta(\theta)\Phi(\phi)$  together form the angular part.

Go to Frame 9.

### Frame 9

When Schrödinger's time-independent equation for a central potential  $U(r)$  is written in spherical coordinates and the product solution  $\psi = R(r)\Theta(\theta)\Phi(\phi)$  is substituted, a process of separation of variables can be performed.

The text (Eq. 8.12) shows an intermediate step after some rearrangement:

$$\frac{1}{\Phi} \frac{d^2 \Phi}{d\phi^2} = -\sin^2 \theta \left[ \frac{1}{R} \frac{d^2 R}{dr^2} + \frac{2}{rR} \frac{dR}{dr} + \frac{2mr^2}{\hbar^2} (E - U(r)) \right] - \frac{1}{\Theta \sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta}{d\theta} \right)$$

(This is a slightly re-expressed form from Eq 8.12 for clarity of separation)

The left side depends *only* on  $\phi$ . The right side depends *only* on  $r$  and  $\theta$ . For these two sides to be equal for all values of  $r, \theta, \phi$ , what must be true about each side? \_\_\_\_\_

Go to Frame 10.

### Frame 10

Each side must be equal to a **constant**. This is the fundamental principle of separation of variables.

Let's call this first separation constant  $-m_l^2$ . (The negative sign and square are conventional for reasons that become clear later). So,

$$\frac{1}{\Phi} \frac{d^2 \Phi}{d\phi^2} = -m_l^2$$

This gives the differential equation for  $\Phi(\phi)$ :

$$\frac{d^2 \Phi(\phi)}{d\phi^2} = -m_l^2 \Phi(\phi)$$

What is a general solution to this equation? And what condition must  $m_l$  satisfy for  $\Phi(\phi)$  to be physically meaningful (i.e., single-valued, meaning  $\Phi(\phi) = \Phi(\phi + 2\pi)$  since  $\phi$  and  $\phi + 2\pi$  represent the same physical angle)? \_\_\_\_\_

Go to Frame 11.

### Frame 11

A general solution for  $\frac{d^2 \Phi}{d\phi^2} = -m_l^2 \Phi$  is  $\Phi(\phi) = Ae^{im_l \phi} + Be^{-im_l \phi}$ , or simply  $Ce^{im_l \phi}$  (where C can be complex).

For  $\Phi(\phi)$  to be periodic with period  $2\pi$  (i.e.,  $\Phi(\phi + 2\pi) = \Phi(\phi)$ ),  $m_l$  must be an **integer** ( $0, \pm 1, \pm 2, \dots$ ).  $m_l$  is called the **magnetic quantum number**.

Now, equating the right side of the equation in Frame 9 to  $-m_l^2$  and rearranging (see Eq. 8.14 in the original text) leads to another separation. One side will depend only on  $r$ , and the other only on  $\theta$ . Again, both sides must equal a constant. This second separation constant is conventionally written as  $l(l+1)$ . This leads to the equation for  $\Theta(\theta)$  (see Eq. 8.15):

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta}{d\theta} \right) + \left( l(l+1) - \frac{m_l^2}{\sin^2 \theta} \right) \Theta = 0$$

For  $\Theta(\theta)$  to be a physically acceptable solution (bounded, single-valued), what are the allowed values for the new quantum number  $l$ , and what is its relationship to  $m_l$ ? (See page 269).

1. Allowed values for  $l$ : \_\_\_\_\_ 2. Relationship between  $l$  and  $m_l$ : \_\_\_\_\_

Go to Frame 12.

### Frame 12

For physically acceptable solutions  $\Theta(\theta)$ : 1.  $l$  must be a non-negative integer:  $l = 0, 1, 2, \dots$   $l$  is called the **orbital quantum number**. 2. The magnetic quantum number  $m_l$  is restricted by  $l$ :  $m_l = 0, \pm 1, \pm 2, \dots, \pm l$ . That is, for a given  $l$ , there are  $2l + 1$  possible values for  $m_l$ .

The solutions  $\Theta(\theta)$  are related to *Associated Legendre Polynomials*,  $P_l^{|m_l|}(\cos \theta)$ . The combined angular functions  $\Theta(\theta)\Phi(\phi)$  are called **spherical harmonics**, denoted  $Y_l^{m_l}(\theta, \phi)$ . (See Tables 8.2 and 8.3 in the original text for examples).

Go to Frame 13.

### Frame 13

The separation constants  $l(l + 1)$  and  $m_l$  are directly related to the quantized values of angular momentum.

The magnitude of the angular momentum  $L$  is quantized according to:

$$L = \sqrt{l(l + 1)}\hbar$$

where  $l = 0, 1, 2, \dots$

The z-component of angular momentum  $L_z$  is quantized according to:

$$L_z = m_l\hbar$$

where  $m_l = -l, -l + 1, \dots, 0, \dots, l - 1, l$ .

Notice that  $L_z$  can never be greater than  $L$  (since  $|m_l| \leq l$ ,  $\sqrt{l(l + 1)} > l$  for  $l > 0$ ). The fact that  $m_l$  is an integer implies that  $L_z$  can only take on discrete values.

What does this tell us about the possible orientations of the angular momentum vector  $L$  in space? \_\_\_\_\_

Go to Frame 14.

### Frame 14

The quantization of  $L_z$  means that the angular momentum vector  $L$  can only have certain allowed orientations in space relative to the z-axis. This is sometimes called "space quantization." The vector  $L$  can be thought of as precessing around the z-axis, such that its projection  $L_z$  remains constant.

The energy  $E$  of the particle depends on  $l$  but is independent of  $m_l$ . For a given value of  $l$ , there are  $2l + 1$  possible values of  $m_l$ . What does this independence of  $E$  on  $m_l$  imply about the energy levels? (Hint: think about "degeneracy"). \_\_\_\_\_

Go to Frame 15.

### Frame 15

The independence of energy  $E$  on the magnetic quantum number  $m_l$  means that for a given orbital quantum number  $l$ , all  $2l + 1$  states corresponding to the different possible values of  $m_l$  have the **same energy**. Such an energy level is said to be  $(2l + 1)$ -fold **degenerate**. This degeneracy arises from the spherical symmetry of any central potential.

The final step in solving the problem for a specific central potential  $U(r)$  is to solve the **radial wave equation** for  $R(r)$  (see Eq. 8.17):

$$-\frac{\hbar^2}{2m} \left( \frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} \right) + \frac{l(l + 1)\hbar^2}{2mr^2} R(r) + U(r)R(r) = ER(r)$$

Solving this equation gives the radial part of the wavefunction and the allowed energy levels  $E$ . The term involving  $l(l+1)$  acts like an effective potential energy, often called the "centrifugal barrier."

This completes the general setup for central force problems. Specific solutions depend on the form of  $U(r)$ .

End of Section.

## 17 Space Quantization and Angular Momentum

### Frame 1

We've seen that for wavefunctions satisfying the separated angular equations (from the previous section, related to Equations 8.13 and 8.15 in the original text), the orbital angular momentum magnitude  $L$  and its  $z$ -component  $L_z$  are both sharp (well-defined) and quantized. The specific values are determined by the orbital quantum number  $l$  and the magnetic quantum number  $m_l$ .

The fact that the direction of the angular momentum vector  $L$  is quantized with respect to an arbitrarily chosen  $z$ -axis is known as **space quantization**.

Let's explore the possible orientations of  $L$  for a given  $l$ . If  $l = 0$ , what are the possible values for  $m_l$ ? What are  $L$  and  $L_z$ ? 1.  $m_l$ : \_\_\_\_\_ 2.  $L$ : \_\_\_\_\_ 3.  $L_z$ : \_\_\_\_\_

Go to Frame 2 for the answers.

### Frame 2

For  $l = 0$ : 1.  $m_l = 0$  (since  $m_l$  ranges from  $-l$  to  $+l$ ). 2.  $L = \sqrt{l(l+1)}\hbar = \sqrt{0(0+1)}\hbar = 0$ . 3.  $L_z = m_l\hbar = 0\hbar = 0$ . In this case, all components of  $L$  are zero.

Now, if  $l = 1$ : What are the possible values for  $m_l$ ? What are the corresponding values for  $L_z$ ? What is the magnitude  $L$ ? 1.  $m_l$ : \_\_\_\_\_ 2.  $L_z$  values: \_\_\_\_\_ 3.  $L$ : \_\_\_\_\_

Go to Frame 3.

### Frame 3

For  $l = 1$ : 1.  $m_l$  can be  $-1, 0, +1$ . 2.  $L_z$  can be  $-\hbar, 0, +\hbar$ . 3.  $L = \sqrt{1(1+1)}\hbar = \sqrt{2}\hbar$ .

Notice that the maximum value of  $L_z$  (which is  $l\hbar = \hbar$ ) is less than the magnitude  $L = \sqrt{2}\hbar$ . Can the angular momentum vector  $L$  ever be perfectly aligned with the  $z$ -axis (i.e., can  $L_z = L$ )? [ Yes — No ] (Assume  $l > 0$ ).

Go to Frame 4.

### Frame 4

The answer is [No] (for  $l > 0$ ). Since  $L_z = m_l\hbar$  and  $L = \sqrt{l(l+1)}\hbar$ , for  $L$  to be aligned with the  $z$ -axis, we would need  $m_l\hbar = \sqrt{l(l+1)}\hbar$ , or  $m_l = \sqrt{l(l+1)}$ . Since  $m_l$  must be an integer and its maximum value is  $l$ , and  $\sqrt{l(l+1)}$  is always greater than  $l$  (for  $l > 0$ ), perfect alignment is impossible.

From a three-dimensional perspective,  $L$  must lie on the surface of a cone that makes an angle  $\theta$  with the  $z$ -axis (see Figure 8.7b). The angle  $\theta$  is also quantized. Given  $L_z = L \cos \theta$ , what is the formula for  $\cos \theta$  in terms of  $m_l$  and  $l$ ? (See Eq. 8.18). \_\_\_\_\_

Go to Frame 5.

### Frame 5

The formula for the quantized angle  $\theta$  is:

$$\cos \theta = \frac{L_z}{L} = \frac{m_l \hbar}{\sqrt{l(l+1)}\hbar} = \frac{m_l}{\sqrt{l(l+1)}}$$

Classically,  $\theta$  could take any value, meaning  $L$  could point in any direction. Quantum mechanically, only specific orientations are allowed. This is space quantization.

The text mentions that Figure 8.7 is somewhat misleading. If  $L_z$  is sharp, what about  $L_x$  and  $L_y$ ? Are they also sharply defined by the cone in Figure 8.7b? [ Yes — No ] \_\_\_\_\_

Go to Frame 6.

## Frame 6

The answer is [No]. If  $L_z$  is sharp,  $L_x$  and  $L_y$  must be "fuzzy" or uncertain. A better visualization is to imagine the vector  $L$  **precessing** around the z-axis, tracing out the cone. This way,  $L_x$  and  $L_y$  are continually changing, while  $L_z$  remains fixed at  $m_l \hbar$ .

It's important to note that these special orientations are not due to external forces, but arise from the fundamental structure of space itself in quantum mechanics.

(The section on "EXAMPLE 8.5 The Bohr Atom Revisited" shows an incompatibility between Bohr's quantization of  $L$  as  $n\hbar$  and the quantum mechanical result  $L = \sqrt{l(l+1)}\hbar$ . This highlights a limitation of the older Bohr model.)

Go to Frame 7.

## Frame 7

### OPTIONAL Section: 8.4 Quantization of Angular Momentum and Energy

This section (marked optional in the original text) delves deeper into how angular momentum properties facilitate solving Schrödinger's equation for central forces, using operator methods (introduced in Section 6.8 of the original text).

The eigenvalue condition  $[Q]\Psi = q\Psi$  is used as a tool. Here,  $[Q]$  is the operator for an observable  $Q$ ,  $q$  is its sharp measured value, and  $\Psi$  is the wavefunction. If a quantity is a constant of the classical motion, its operator will generally commute with the Hamiltonian, and it can be a sharp observable.

For energy, the eigenvalue equation is  $[E]\Psi = E\Psi$ . Since  $[E] = i\hbar \frac{\partial}{\partial t}$ , this is satisfied by stationary states  $\psi(r)e^{-i\omega t}$  with  $E = \hbar\omega$ .

For a central force, angular momentum  $L$  is a constant of motion. The operators for its components are derived from position and momentum operators. For example (Eq. 8.20):  $[L_z] = [x][p_y] - [y][p_x] = -i\hbar \left( x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$ . In spherical coordinates, these operators take simpler forms that do not involve 'r' (Eq. 8.23), for instance:  $[L_z] = -i\hbar \frac{\partial}{\partial \phi}$ .

If  $L_z$  is to be sharp with eigenvalue  $L_z$  (the measured value), then  $\Psi$  must be an eigenfunction of  $[L_z]$ :  $-i\hbar \frac{\partial \Psi}{\partial \phi} = L_z \Psi$ . What is the functional dependence of  $\Psi$  on  $\phi$  that satisfies this?  $\Psi \propto e^{i(L_z/\hbar)\phi}$

For  $\Psi$  to be single-valued (i.e.,  $\Psi(\phi + 2\pi) = \Psi(\phi)$ ), what must be true about  $L_z/\hbar$ ? \_\_\_\_\_

Go to Frame 8.

## Frame 8

For  $\Psi$  to be single-valued,  $L_z/\hbar$  must be an integer. We call this integer  $m_l$ , the magnetic quantum number. So,  $L_z = m_l \hbar$ , where  $m_l = 0, \pm 1, \pm 2, \dots$ . This means the wavefunction has a factor  $e^{im_l \phi}$ .

Can  $L_x$  (or  $L_y$ ) be simultaneously sharp with  $L_z$  if  $L_z \neq 0$ ? (Hint: Refer back to Frame 6 or page 275 of original text). [ Yes — No ] \_\_\_\_\_

Go to Frame 9.

### Frame 9

The answer is [No]. As established, if  $L_z$  is sharp,  $L_x$  and  $L_y$  are fuzzy, unless the angular momentum is exactly zero. So, we cannot generally find a wavefunction that is simultaneously an eigenfunction of  $[L_z]$  and  $[L_x]$  (or  $[L_y]$ ).

However, the magnitude squared of angular momentum,  $L^2 = L_x^2 + L_y^2 + L_z^2$ , \*is\* a constant of motion and \*can\* be made sharp simultaneously with  $L_z$ . The operator  $[L^2]$  in spherical coordinates is (Eq. 8.26):  $[L^2] = -\hbar^2 \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right]$ .

If  $L^2$  is to be sharp with eigenvalue  $L^2$  (the measured value), we require  $[L^2]\Psi = L^2\Psi$ . Substituting the  $\phi$  dependence  $e^{im_l\phi}$  (so  $\frac{\partial^2 \Psi}{\partial \phi^2} = -m_l^2 \Psi$ ), the equation for the  $\theta$  part of the wavefunction becomes (related to Eq. 8.27):  $-\hbar^2 \left[ \frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta}{d\theta} \right) - \frac{m_l^2}{\sin^2 \theta} \Theta \right] = L^2 \Theta$  (assuming  $\Psi = R(r)\Theta(\theta)e^{im_l\phi}$ ). For physically acceptable solutions  $\Theta(\theta)$ , it turns out that the eigenvalue  $L^2$  must be of the form  $l(l+1)\hbar^2$ . What are the allowed values for  $l$  and its relation to  $m_l$ ? (This is a recap). 1.  $l$ : \_\_\_\_\_ 2.  $m_l$  relation to  $l$ : \_\_\_\_\_

Go to Frame 10.

### Frame 10

As before: 1.  $l = 0, 1, 2, \dots$  (orbital quantum number). 2.  $|m_l| \leq l$ . So,  $L = \sqrt{l(l+1)}\hbar$ . The solutions  $\Theta(\theta)$  are Associated Legendre Polynomials, and  $\Theta(\theta)e^{im_l\phi}$  are the spherical harmonics  $Y_l^{m_l}(\theta, \phi)$ .

Finally, for Energy  $E$  to be sharp,  $\Psi$  must be an eigenfunction of the Hamiltonian operator  $[H] = [K] + [U]$ .  $[H]\Psi = E\Psi$ . The kinetic energy operator  $[K]$  can be split into a radial part  $[K_{rad}]$  and an orbital part  $[K_{orb}]$ , where  $[K_{orb}] = \frac{1}{2mr^2}[L^2]$  (Eq. 8.29). Since  $\Psi$  is an eigenfunction of  $[L^2]$  with eigenvalue  $l(l+1)\hbar^2$ , the Schrödinger equation becomes (Eq. 8.31):  $[K_{rad}]\Psi + \frac{l(l+1)\hbar^2}{2mr^2}\Psi + U(r)\Psi = E\Psi$ . Since  $Y_l^{m_l}(\theta, \phi)$  is an eigenfunction of the angular part, we can divide it out, leaving the radial wave equation for  $R(r)$  (as seen in Frame 15 of the previous section).

This operator approach confirms the separation of variables and the origin of the quantum numbers  $l$  and  $m_l$  from the properties of angular momentum.

End of Section.

## 18 Introduction to Atomic Structure

### Frame 1

Much of what we've learned about the hydrogen atom, with its single electron, can be directly applied to describe other single-electron ions like  $\text{He}^+$  and  $\text{Li}^{2+}$ . These are called "hydrogen-like" ions.

However, atoms with multiple electrons (multielectron atoms), such as neutral helium or lithium, introduce new complexities. What is the primary source of these extra complications in multielectron atoms? \_\_\_\_\_

Go to Frame 2 for the answer.

### Frame 2

The extra complications in multielectron atoms stem primarily from the **interactions among the atomic electrons** themselves.

Electrons, being of like charge and confined to a small space within the atom, repel each other strongly via the Coulomb force. Additionally, we will discover that atomic electrons also behave like tiny bar magnets. What kind of interactions will these magnetic properties lead to? List two.

1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 3.

### Frame 3

These magnetic properties lead to electrons: 1. Interacting magnetically with **one another**. 2. Interacting magnetically with any **external magnetic field** applied to the atom.

A key concept responsible for these magnetic properties, which we will explore, is **electron spin**.

Another crucial new physical idea presented in this chapter is the **exclusion principle**. How important is this principle for understanding multielectron atoms and the periodic table? [a] Moderately important [b] Extremely important [c] Of minor importance

Go to Frame 4.

### Frame 4

The correct answer is [b] Extremely important. The exclusion principle is fundamental to understanding the properties of multielectron atoms and the structure of the periodic table. Its implications are described as "almost as far-reaching as those of the Schrödinger equation itself."

Let's begin by examining the magnetic effects arising from an electron orbiting the nucleus.

Go to Frame 5.

## 19 Orbital Magnetism and the Normal Zeeman Effect

### Frame 5

An electron orbiting the nucleus of an atom generates magnetic effects, similar to an electric current flowing in a wire loop. This motion of charge creates a magnetic field within the atom. These magnetic interactions can be described using a single property of the atom: its **magnetic dipole moment**, denoted by  $\vec{\mu}$ .

Consider a current-carrying loop of wire with current  $i$  and area  $A$ . What is the magnitude of its magnetic moment  $\mu$ ?  $\mu =$  \_\_\_\_\_

What determines the direction of  $\vec{\mu}$ ? \_\_\_\_\_

Go to Frame 6.

### Frame 6

For a current loop: Magnitude:  $\mu = iA$ . Direction:  $\vec{\mu}$  is perpendicular to the plane of the loop, with its sense given by the right-hand rule (if fingers curl in the direction of current, the thumb points in the direction of  $\vec{\mu}$ ). (See Figure 9.1a). This current loop behaves magnetically like a tiny bar magnet (Figure 9.1b).

Now, consider an electron with charge  $q$  circulating in an orbit. If  $T$  is its orbital period, what is the equivalent current  $i$ ?  $i =$  \_\_\_\_\_

Go to Frame 7.

### Frame 7

The equivalent current for a circulating charge  $q$  with period  $T$  is  $i = q/T$ .

The text states that for orbital motion, the quantity  $A/T$  (area swept out per unit time) is related to the magnitude of the angular momentum  $L$  of the orbiting charge and its mass  $m$  by  $A/T = L/(2m)$ . Combining  $\mu = iA = (q/T)A = q(A/T)$ , substitute the expression for  $A/T$ . What is the resulting formula for the magnetic moment  $\vec{\mu}$  in terms of  $q, m$ , and  $\vec{L}$ ? (See Eq. 9.1. Pay attention to vector nature for electrons).  $\vec{\mu} =$  \_\_\_\_\_

Go to Frame 8.

### Frame 8

The magnetic moment of an orbiting charge  $q$  is:

$$\vec{\mu} = \frac{q}{2m} \vec{L}$$

For an electron,  $q = -e$  (where  $e$  is the elementary positive charge). So, for an orbiting electron:

$$\vec{\mu}_L = -\frac{e}{2m_e} \vec{L}$$

The subscript  $L$  on  $\mu$  denotes it's the \*orbital\* magnetic moment. The negative sign indicates that for an electron,  $\vec{\mu}_L$  and  $\vec{L}$  point in **opposite directions** (Figure 9.1c). The proportionality constant  $q/(2m)$  is called the **gyromagnetic ratio**.

Go to Frame 9.

### Frame 9

The fundamental unit of angular momentum in atomic systems is  $\hbar$ . This implies a natural unit for atomic magnetic moments. This unit is called the **Bohr magneton**,  $\mu_B$ . What is its definition in terms of  $e, \hbar$ , and  $m_e$ ? (See Eq. 9.2).  $\mu_B =$  \_\_\_\_\_

What is its approximate value in SI units (Joules/Tesla)?  $\mu_B \approx$  \_\_\_\_\_

Go to Frame 10.

### Frame 10

The Bohr magneton is defined as:

$$\mu_B = \frac{e\hbar}{2m_e}$$

Its value is approximately  $9.274 \times 10^{-24}$  J/T.

Since the orbital magnetic moment  $\vec{\mu}_L$  is proportional to the angular momentum  $\vec{L}$ , and we know  $\vec{L}$  is subject to space quantization,  $\vec{\mu}_L$  must also be subject to space quantization. The  $z$ -component of the orbital magnetic moment,  $\mu_{Lz}$ , is related to the magnetic quantum number  $m_l$  by (see Eq. 9.3):  $\mu_{Lz} = -\frac{e}{2m_e} L_z = -\frac{e}{2m_e} (m_l \hbar) = -m_l \left( \frac{e\hbar}{2m_e} \right)$ . Therefore,  $\mu_{Lz} =$  \_\_\_\_\_ (in terms of  $m_l$  and  $\mu_B$ ).

Go to Frame 11.

### Frame 11



The z-component of the orbital magnetic moment is:

$$\mu_{Lz} = -m_l \mu_B$$

(See Figure 9.2 for a visualization with  $l = 2$ ). Just like  $\vec{L}$ , the magnetic moment vector  $\vec{\mu}_L$  can be visualized as precessing about the z-axis.

When an atom with a magnetic moment  $\vec{\mu}$  is placed in an external magnetic field  $\vec{B}$ , it experiences a torque  $\vec{\tau}$ . What is the formula for this torque? (See Eq. 9.4).  $\vec{\tau} =$  \_\_\_\_\_

Go to Frame 12.

### Frame 12

The torque on a magnetic moment  $\vec{\mu}$  in a field  $\vec{B}$  is:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

This torque tends to align  $\vec{\mu}$  with  $\vec{B}$ . However, because  $\vec{\mu}_L$  is proportional to  $\vec{L}$ , the moment doesn't simply align. Instead, what kind of motion does  $\vec{\mu}_L$  (and  $\vec{L}$ ) undergo? \_\_\_\_\_

Go to Frame 13.

### Frame 13

Instead of aligning, the magnetic moment vector  $\vec{\mu}_L$  (and thus  $\vec{L}$ ) undergoes **Larmor precession** around the direction of the magnetic field  $\vec{B}$ . (See Figure 9.3). The Larmor frequency  $\omega_L$  for an electron is given by (Eq. 9.5):  $\omega_L =$  \_\_\_\_\_ (in terms of  $e, B, m_e$ ).

Go to Frame 14.

### Frame 14

The Larmor frequency for an electron is:

$$\omega_L = \frac{e}{2m_e} B$$

The work done to orient a magnetic dipole in a field  $\vec{B}$  results in a magnetic potential energy  $U$ . What is the formula for  $U$ ? (See Eq. 9.6).  $U =$  \_\_\_\_\_

Go to Frame 15.

### Frame 15

The magnetic potential energy is:

$$U = -\vec{\mu} \cdot \vec{B}$$

The energy is minimal (most stable) when  $\vec{\mu}$  and  $\vec{B}$  are aligned. Since the orientations of  $\vec{\mu}_L$  are quantized, the magnetic energy  $U$  is also quantized. If we take the z-axis to be along  $\vec{B}$  (so  $\vec{B} = B\hat{k}$ ), then  $U = -\mu_{Lz}B$ . Using  $\mu_{Lz} = -m_l \mu_B$ , the quantized magnetic energy is (Eq. 9.7):  $U = (-m_l \mu_B)(-B)$  is incorrect. It should be  $U = -(-m_l \mu_B)B$ . So,  $U =$  \_\_\_\_\_ (in terms of  $m_l, \mu_B, B$ ).

What is the relationship between  $\hbar\omega_L$  and  $\mu_B B$ ? \_\_\_\_\_

Go to Frame 16.

### Frame 16

The quantized magnetic energy is:

$$U = m_l \mu_B B$$

(Note: The text Eq. 9.7 directly states  $U = \frac{e}{2m_e} \mathbf{L} \cdot \mathbf{B} = \frac{eB}{2m_e} L_z = \hbar\omega_L m_l$ . Since  $\hbar\omega_L = \hbar \frac{eB}{2m_e} = \mu_B B$ , then  $U = m_l(\mu_B B)$ .)

The quantity  $\mu_B B$  is the quantum of magnetic energy, often called the **Zeeman energy**. The total energy of an electron in a magnetic field is its energy in the absence of the field ( $E_0$ ) plus this magnetic energy:  $E = E_0 + U = E_0 + m_l \mu_B B$ .

For hydrogen,  $E_0$  depends only on the principal quantum number  $n$ . Consider an electron in the  $n = 2$  state of hydrogen placed in a magnetic field  $B$ . For  $n = 2$ ,  $l$  can be 0 or 1. If  $l = 0$ , what is  $m_l$ ? What is the magnetic energy  $U$ ?  $m_l(\text{for } l = 0) = \underline{\hspace{2cm}}$   $U(\text{for } l = 0) = \underline{\hspace{2cm}}$

If  $l = 1$ , what are the possible  $m_l$  values? What are the corresponding magnetic energies  $U$ ?  $m_l(\text{for } l = 1) = \underline{\hspace{2cm}}$   $U(\text{for } l = 1) = \underline{\hspace{2cm}}$

Go to Frame 17.

### Frame 17

For  $n = 2$ : If  $l = 0$ :  $m_l = 0$ .  $U = 0 \cdot \mu_B B = 0$ . The magnetic field has no effect on the energy of the  $l = 0$  (s-state) level.

If  $l = 1$ :  $m_l = -1, 0, +1$ . The corresponding magnetic energies are  $U = -\mu_B B, 0, +\mu_B B$ . So, the original  $n = 2, l = 1$  energy level (which was degenerate) is split into **three** distinct energy levels by the magnetic field. (See Figure 9.4).

This splitting of spectral lines due to an external magnetic field is known as the **Zeeman effect**. When an atom transitions from an excited state to a lower state, it emits a photon. If the excited state is split by the Zeeman effect, what will happen to the emitted spectral line compared to the case with no magnetic field?  $\underline{\hspace{2cm}}$

Go to Frame 18.

### Frame 18

The single spectral line (seen with no magnetic field) will be split into multiple lines. In the simple case described (normal Zeeman effect, involving only orbital angular momentum), an original line splits into **three** lines:

- One at the original frequency  $\omega_0$  (corresponding to transitions where  $m_l$  doesn't change, or from/to states not split).
- Two satellite lines at frequencies  $\omega_0 \pm \omega_L$  (where  $\omega_L = \mu_B B / \hbar$  is the Larmor frequency). These correspond to transitions where  $m_l$  changes by  $\pm 1$ .

(See Figure 9.4 for transitions from  $n = 2$  to  $n = 1$ ).

The text mentions selection rules for optical transitions (Figure 9.5). For transitions involving a change in  $l$  by  $\pm 1$  (like  $p \rightarrow s$  or  $d \rightarrow p$ ), by how much can  $m_l$  change?  $\Delta m_l = \underline{\hspace{2cm}}$

Go to Frame 19.

### Frame 19

The selection rule for  $m_l$  in optical transitions (where  $\Delta l = \pm 1$ ) is:

$$\Delta m_l = 0, \pm 1$$

This rule limits the number of observed Zeeman lines. Even for more complex initial splittings (e.g., from  $n = 3$ , Figure 9.5), only lines corresponding to these  $\Delta m_l$  values are typically seen, resulting in satellites only at the Larmor frequency  $\omega_L$  (or multiples if higher order effects were dominant, but they are not here).

The simple splitting into three lines is called the **normal Zeeman effect**. However, often more complex splittings (more lines, unequally spaced) are observed. This is called the **anomalous Zeeman effect**. What new property of the electron is responsible for the anomalous Zeeman effect? \_\_\_\_\_

Go to Frame 20.

### Frame 20

The anomalous Zeeman effect (and other phenomena like fine structure in spectral lines) is attributed to a new magnetic moment arising from the **electron spinning on its axis**. This is the concept of **electron spin**.

Just as orbital motion of charge creates an orbital magnetic moment  $\vec{\mu}_L$ , the spinning of the charged electron creates a **spin magnetic moment**  $\vec{\mu}_s$ .

A rotating body of charge can be viewed as a collection of charge elements  $\Delta q$  each with mass  $\Delta m$ , rotating in circular orbits about the spin axis S (Figure 9.6). Each element  $\Delta q_i$  has an orbital angular momentum  $L_i$  relative to the spin axis, and thus a magnetic moment  $\mu_i$ . The total spin magnetic moment  $\vec{\mu}_s$  is the sum of these.

End of Section 9.1. Go to Frame 21 to begin Section 9.2.

## 20 The Spinning Electron

### Frame 21

We've established that the anomalous Zeeman effect and fine structure suggest the electron possesses an intrinsic spin, leading to a spin magnetic moment  $\vec{\mu}_s$ .

This concept will be explored further.

This concludes the self-teaching adaptation for the provided text sections.

End of Section.

## 21 The Spinning Electron (Continued)

### Frame 21

We've established that phenomena like the anomalous Zeeman effect and fine structure in spectral lines (an observed doubling of many lines) point to the existence of a new magnetic moment for the electron. This moment arises from the electron effectively **spinning on its axis** and is called the **spin magnetic moment**,  $\vec{\mu}_s$ .

To find the spin moment, the text (page 302, Figure 9.6) suggests viewing a rotating body of charge  $q$  and mass  $m_e$  as a collection of tiny charge elements  $\Delta q_i$  with mass  $\Delta m_i$ , each rotating in a circular orbit about a fixed axis (the spin axis). Each element  $\Delta q_i$  has an orbital angular momentum  $\vec{L}_i$  relative to this spin axis and thus an associated magnetic moment  $\vec{\mu}_i = (\Delta q_i / 2\Delta m_i) \vec{L}_i$ .

If the charge-to-mass ratio ( $\Delta q_i / \Delta m_i$ ) is uniform throughout the body, what is the relationship between the total spin magnetic moment  $\vec{\mu}_s$  and the total spin angular momentum  $\vec{S} = \sum \vec{L}_i$ ? (See Eq. 9.9).  $\vec{\mu}_s =$  \_\_\_\_\_

Go to Frame 22.

### Frame 22

If the charge-to-mass ratio is uniform, the total spin magnetic moment is:

$$\vec{\mu}_s = \frac{q}{2m_e} \vec{S}$$

where  $q$  is the total charge ( $-e$  for an electron) and  $m_e$  is its total mass, and  $\vec{S}$  is the total spin angular momentum. The spin angular momentum  $\vec{S}$  points along the axis of rotation (right-hand rule).

If the charge-to-mass ratio is \*not\* uniform, the gyromagnetic ratio ( $q/2m_e$ ) needs to be multiplied by a dimensionless constant called the **g-factor**.

What experiment first demonstrated the existence of a spin magnetic moment for the electron?

Go to Frame 23.

### Frame 23

The existence of electron spin magnetic moment was first demonstrated in 1921 by the **Stern-Gerlach experiment**. This experiment originally aimed to show space quantization associated with \*orbital\* angular momentum.

In the experiment (Figure 9.7a):

- A beam of silver atoms was passed through a **nonuniform** magnetic field.
- The beam was detected on a collector plate.

Why is a \*nonuniform\* magnetic field necessary to exert a net force on a magnetic dipole? \_\_\_\_\_

Go to Frame 24.

### Frame 24

A nonuniform magnetic field is necessary because if the field were uniform, the forces on the opposite "poles" of the tiny atomic magnet would be equal and opposite, resulting in a net torque but no net translational force. A nonuniform field creates a gradient, leading to a net force that deflects the atom. The amount of deflection depends on the orientation of the atom's magnetic moment relative to the field gradient (z-axis in Fig 9.7b).

If atomic magnetic moments could have any orientation (classical expectation), what would the pattern on the collector plate look like? (Fig 9.7c, "Classical pattern"). \_\_\_\_\_

If orientations are space-quantized due to orbital angular momentum  $l$ , how many discrete components (lines) would be expected for a given  $l$ ? \_\_\_\_\_

Go to Frame 25.

### Frame 25

Classical expectation: A **continuous fanning** of the atomic beam. Quantum expectation (based on orbital angular momentum  $l$ ): The beam should split into  $2l + 1$  discrete components.

What was the staggering result of the Stern-Gerlach experiment with silver atoms? \_\_\_\_\_

Why was this particularly surprising given the expected orbital angular momentum of silver's outermost electron? \_\_\_\_\_

Go to Frame 26.

### Frame 26

The staggering result: The silver atomic beam split into only **TWO** components. This was surprising because:

- If it were due to orbital angular momentum,  $2l + 1$  should be 2. This implies  $2l = 1$ , or  $l = 1/2$ , which is not an allowed integer value for orbital angular momentum.
- Moreover, silver atoms in their ground state have their outermost electron in an s-state, which means  $l = 0$ . If  $l = 0$ , then  $m_l = 0$ , and  $2l + 1 = 1$ , so no splitting (or only one undeflected beam component) would be expected from orbital effects.

The experiment was repeated with hydrogen atoms (eliminating complexities of silver) and yielded the same two-component splitting.

What did Goudsmit and Uhlenbeck propose in 1925 to explain this? \_\_\_\_\_  
Go to Frame 27.

### Frame 27

Goudsmit and Uhlenbeck proposed that the observed magnetic moment and its splitting originated from the **spinning motion of the atomic electrons** themselves. This spin angular momentum  $\vec{S}$  was proposed to obey the same quantization rules as orbital angular momentum  $\vec{L}$ .

The observed two-component splitting ( $2s + 1 = 2$ ) in the Stern-Gerlach experiment indicates that the spin quantum number  $s$  for the electron must be:  $s =$  \_\_\_\_\_  
Go to Frame 28.

### Frame 28

The spin quantum number for the electron is  $s = 1/2$ . This value is dictated by the observation of two components, since  $2s + 1 = 2$  implies  $2s = 1$ , so  $s = 1/2$ .

This leads to the semiclassical model of electron spin:

- The spin quantum number  $s$  for the electron is  $1/2$ .
- This implies there are  $2s + 1 = 2(1/2) + 1 = 2$  possible orientations for the spin axis. These are described as "spin-up" and "spin-down".
- The z-component of spin angular momentum  $S_z$  is quantized as  $S_z = m_s \hbar$ . What are the allowed values for the spin magnetic quantum number  $m_s$  for an electron? (See Eq. 9.10).  
 $m_s =$  \_\_\_\_\_

Go to Frame 29.

### Frame 29

The allowed values for the spin magnetic quantum number  $m_s$  for an electron ( $s = 1/2$ ) are:

$$m_s = +1/2 \quad (\text{spin-up}, \uparrow)$$

$$m_s = -1/2 \quad (\text{spin-down}, \downarrow)$$

The fact that  $s$  is half-integral suggests spin is a distinct quantum phenomenon, not just another form of orbital motion.

What is the magnitude of the spin angular momentum  $|\vec{S}|$  for an electron? (Use the general formula  $|\vec{J}| = \sqrt{j(j+1)}\hbar$ , replacing  $j$  with  $s$ . See Eq. 9.11).  $|\vec{S}| =$  \_\_\_\_\_  
Go to Frame 30.

### Frame 30

The magnitude of the spin angular momentum for an electron ( $s = 1/2$ ) is:

$$|\vec{S}| = \sqrt{s(s+1)}\hbar = \sqrt{\frac{1}{2}\left(\frac{1}{2} + 1\right)}\hbar = \sqrt{\frac{1}{2} \cdot \frac{3}{2}}\hbar = \frac{\sqrt{3}}{2}\hbar$$

An important characteristic: This magnitude  $|\vec{S}|$  **never changes**. It's an intrinsic property of the electron, like its mass or charge.

The spin magnetic moment  $\vec{\mu}_s$  is related to  $\vec{S}$  by  $\vec{\mu}_s = g_s \frac{q}{2m_e} \vec{S}$ . For an electron,  $q = -e$ . What is the experimentally determined g-factor for electron spin,  $g_s$ ?  $g_s \approx$  \_\_\_\_\_

Go to Frame 31.

### Frame 31

The g-factor for electron spin is  $g_s \approx 2$ . (More precisely,  $g_s \approx 2.00232$ ). This means the spin magnetic moment is approximately twice as large as would be expected if the charge-to-mass distribution were uniform (where  $g_s$  would be 1). So, for an electron:

$$\vec{\mu}_s \approx -2 \frac{e}{2m_e} \vec{S} = -\frac{e}{m_e} \vec{S} = -2 \frac{\mu_B}{\hbar} \vec{S}$$

The recognition of electron spin means an electron's state in an atom must be described by \*four\* quantum numbers. What are they? 1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_ 4. \_\_\_\_\_

Go to Frame 32.

### Frame 32

The state of an electron in hydrogen (and other atoms) is described by four quantum numbers:  
1. Principal quantum number,  $n$  2. Orbital angular momentum quantum number,  $l$  3. Magnetic quantum number (for orbital),  $m_l$  4. Spin magnetic quantum number,  $m_s$

The total magnetic moment  $\vec{\mu}$  of an electron now has contributions from both its orbital motion and its spin (Eq. 9.12):  $\vec{\mu} = \vec{\mu}_L + \vec{\mu}_s = -\frac{e}{2m_e}(\vec{L} + g_s \vec{S}) \approx -\frac{e}{2m_e}(\vec{L} + 2\vec{S})$ . Because  $g_s \neq 1$ , the total magnetic moment  $\vec{\mu}$  is generally **not** collinear with the total angular momentum  $\vec{J} = \vec{L} + \vec{S}$ .

Go to Frame 33.

## 22 The Spin-Orbit Interaction and Other Magnetic Effects

### Frame 33

The existence of both orbital ( $\vec{\mu}_L$ ) and spin ( $\vec{\mu}_s$ ) magnetic moments for an electron leads to their mutual interaction. This is called the **spin-orbit interaction**.

How can we understand this interaction from the electron's perspective? (Hint: think about relative motion, see Figure 9.10). \_\_\_\_\_

Go to Frame 34.

### Frame 34

From the electron's perspective as it orbits, the nucleus appears to be circling it (Figure 9.10a, right). This apparent orbital motion of the charged nucleus generates a magnetic field  $\vec{B}_{internal}$  at the electron's location (Figure 9.10b). The electron's spin magnetic moment  $\vec{\mu}_s$  then interacts with

this internal magnetic field, leading to a potential energy  $U_{SO} = -\vec{\mu}_s \cdot \vec{B}_{internal}$ . This is an internal Zeeman effect.

The electron has higher energy when its spin is "up" (aligned with  $\vec{B}_{internal}$ ) and lower energy when its spin is "down" (anti-aligned). This energy difference between the two spin orientations is responsible for what observed phenomenon in atomic spectra? \_\_\_\_\_

Go to Frame 35.

### Frame 35

The energy difference due to spin-orbit interaction is responsible for the **fine structure doubling** of many atomic spectral lines. For example, the  $2p \rightarrow 1s$  transition in hydrogen is split into two closely spaced lines because the  $2p$  level itself is a "spin doublet" (Figure 9.11). The  $1s$  level ( $l = 0$ ) is not split by spin-orbit interaction because there is no orbital motion to create an internal magnetic field.

The coupling of spin and orbital moments implies that neither  $\vec{L}$  nor  $\vec{S}$  are conserved separately if this interaction is present. What quantity \*is\* conserved (in the absence of external torques)? \_\_\_\_\_

Go to Frame 36.

### Frame 36

The **total angular momentum**,  $\vec{J} = \vec{L} + \vec{S}$ , is conserved. This means quantum states can exist for which the magnitude of  $\vec{J}$  (denoted by quantum number  $j$ ) and its z-component  $J_z$  (denoted by quantum number  $m_j$ ) are sharp. The quantization rules are (Eq. 9.13):  $|\vec{J}| = \sqrt{j(j+1)}\hbar$   $J_z = m_j\hbar$ , where  $m_j = -j, -j+1, \dots, j-1, j$ .

How are the possible values of the total angular momentum quantum number  $j$  determined from the orbital quantum number  $l$  and the spin quantum number  $s$  (which is  $1/2$  for an electron)? (See Eq. 9.14). For a single electron,  $j =$  \_\_\_\_\_

Go to Frame 37.

### Frame 37

For a single electron with orbital quantum number  $l$  and spin  $s = 1/2$ :

- If  $l = 0$ , then  $j = s = 1/2$  only.
- If  $l > 0$ , then  $j = l + s = l + 1/2$  AND  $j = |l - s| = l - 1/2$ .

This means that for any  $l > 0$ , the energy level is split into two sub-levels (a doublet) corresponding to these two different  $j$  values due to the spin-orbit interaction.

Consider the  $n = 2$  states of hydrogen (Example 8.7 was about this, now with spin). For the  $2s$  state:  $n = 2, l = 0$ . What is  $j$ ? For the  $2p$  state:  $n = 2, l = 1$ . What are the possible  $j$  values? \_\_\_\_\_

Go to Frame 38.

### Frame 38

For the  $2s$  state ( $n = 2, l = 0, s = 1/2$ ):  $j = 0 + 1/2 = 1/2$ . The spectroscopic notation is  $2S_{1/2}$ .

For the  $2p$  state ( $n = 2, l = 1, s = 1/2$ ):  $j = 1 + 1/2 = 3/2$   $j = 1 - 1/2 = 1/2$  The spectroscopic notations are  $2P_{3/2}$  and  $2P_{1/2}$ . The spin-orbit interaction splits the  $2P$  level into these two sub-levels.

The number of possible  $m_j$  values for a given  $j$  is  $2j + 1$ . This is always an \*even\* number for a single electron system (where  $j$  is always a half-integer like  $1/2, 3/2, 5/2, \dots$ ). This leads to an even number of orientations for  $\vec{J}$  in space (Figure 9.12b), unlike the odd number for  $\vec{L}$  alone.

This concludes the main points of sections 9.2 and 9.3. The remainder of the text discusses examples like the sodium doublet and further details on transitions.

End of Section.

## 23 Exchange Symmetry and the Exclusion Principle

### Frame 1

As we've discussed, the existence of electron spin means that the state of an atomic electron must be specified by **four** quantum numbers. If we ignore spin-orbit interaction for a moment, these are  $n, l, m_l$ , and  $m_s$ . If we include spin-orbit interaction,  $m_l$  and  $m_s$  are replaced by  $j$  and  $m_j$ . In either case, four quantum numbers are needed.

This leads to an important question for systems with two or more electrons: "How many electrons in an atom can have the same four quantum numbers (i.e., be in the same quantum state)?" What is the name of the principle, formulated by Wolfgang Pauli in 1925, that answers this question? \_\_\_\_\_

Go to Frame 2.

### Frame 2

The principle is called the **Pauli exclusion principle**.

State the Pauli exclusion principle. \_\_\_\_\_

Go to Frame 3.

### Frame 3

The Pauli exclusion principle states that: **No two electrons in an atom can have the same set of four quantum numbers.**

If this principle were not valid, what would be the consequence for the structure of atoms and the chemical behavior of elements? (Hint: Where would all electrons tend to go?) \_\_\_\_\_

Go to Frame 4.

### Frame 4

If the exclusion principle were not valid, every electron in an atom would tend to occupy the lowest energy state, which is the 1s atomic state. This would mean:

- The chemical behavior of elements would be drastically different.
- Nature as we know it would not exist.

The exclusion principle is fundamental to understanding the diversity of elements and their properties.

The exclusion principle itself stems from a deeper concept regarding identical particles in quantum mechanics. What is this concept? \_\_\_\_\_

Go to Frame 5.

### Frame 5

The exclusion principle follows from our belief that electrons are **identical particles** – it is impossible to distinguish one electron from another.



Consider a classical collision between two electrons (Figure 9.14a and 9.14b). Even if they are identical, can we distinguish them after the collision if we could follow their paths? [ Yes — No ] .

Now, consider the quantum mechanical view. Are the paths of electrons sharply defined during a collision? [ Yes — No ] (Hint: think about wave properties and the uncertainty principle). —

Go to Frame 6.

### Frame 6

Classical collision: [Yes], we could distinguish them by following their distinct paths. Quantum mechanical view: [No], the paths are blurred by wave properties (inherent fuzziness, de Broglie wavelengths). During the collision, their wavefunctions overlap, and they become indistinguishable (Figure 9.14c).

This inherent **indistinguishability** of identical particles in quantum mechanics is key. It means that if we formally interchange two identical particles in our mathematical description, there should be no observable physical consequences.

Specifically, if  $\psi(\mathbf{r}_1, \mathbf{r}_2)$  is the wavefunction for a system of two electrons (where  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are their coordinates), the probability density  $|\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$  must be unchanged if we swap the electrons. So,  $|\psi(\mathbf{r}_1, \mathbf{r}_2)|^2 = |\psi(\mathbf{r}_2, \mathbf{r}_1)|^2$ . This implies a property for the wavefunction  $\psi$  itself when particles are interchanged. What are the two possibilities for how  $\psi(\mathbf{r}_1, \mathbf{r}_2)$  relates to  $\psi(\mathbf{r}_2, \mathbf{r}_1)$ ? 1. — 2. —

Go to Frame 7.

### Frame 7

The condition  $|\psi(\mathbf{r}_1, \mathbf{r}_2)|^2 = |\psi(\mathbf{r}_2, \mathbf{r}_1)|^2$  implies two possibilities for the wavefunction under exchange of particles 1 and 2: 1.  $\psi(\mathbf{r}_1, \mathbf{r}_2) = +\psi(\mathbf{r}_2, \mathbf{r}_1)$  (Symmetric or even wavefunction) 2.  $\psi(\mathbf{r}_1, \mathbf{r}_2) = -\psi(\mathbf{r}_2, \mathbf{r}_1)$  (Antisymmetric or odd wavefunction)

This property is called **exchange symmetry**.

Particles described by symmetric wavefunctions are called **bosons**. Give an example of a boson.

Particles described by antisymmetric wavefunctions are called **fermions**. Give an example of a fermion. \_\_\_\_\_

Go to Frame 8.

### Frame 8

Examples:

- **Bosons** (symmetric  $\psi$ ): Photons, pions.
- **Fermions** (antisymmetric  $\psi$ ): Electrons, protons, neutrons.

(It's an experimental fact, with theoretical backing from relativistic quantum theory, that integer spin particles are bosons, and half-integer spin particles are fermions. Electrons have spin 1/2, so they are fermions.)

Since electrons are fermions, the total wavefunction for a system of two electrons, say in a helium atom, must be **antisymmetric** upon exchange of the two electrons:

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = -\psi(\mathbf{r}_2, \mathbf{r}_1)$$

Consider a simple model for a two-electron atom (like Helium, but ignoring electron-electron repulsion for now). Each electron "sees" only the nucleus. The wavefunction for electron 1 in atomic state 'a' (a collective label for its four quantum numbers) is  $\psi_a(\mathbf{r}_1)$  with energy  $E_a$ . Similarly,

for electron 2 in state 'b', it's  $\psi_b(\mathbf{r}_2)$  with energy  $E_b$ . A simple product wavefunction would be  $\Psi_{product} = \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)$ . The total energy is  $E_a + E_b$ . Is this simple product wavefunction antisymmetric as required for fermions? [ Yes — No ]

Go to Frame 9.

### Frame 9

The answer is [No]. If we exchange electrons 1 and 2 in  $\Psi_{product} = \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)$ , we get  $\psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)$ . This is generally not equal to  $-\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)$ .

However, if  $\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)$  is a solution to the (simplified) two-electron Schrödinger equation with energy  $E_a + E_b$ , then so is the "exchanged" wavefunction  $\psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)$  with the same energy. How can we combine these two product wavefunctions to create a total wavefunction that *is* antisymmetric? (See Eq. 9.19).  $\Psi_{antisymmetric}(\mathbf{r}_1, \mathbf{r}_2) =$  \_\_\_\_\_

Go to Frame 10.

### Frame 10

The correct antisymmetric combination is:

$$\Psi_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} [\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) - \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)]$$

(The  $1/\sqrt{2}$  is a normalization factor often included). You can verify that if you swap  $\mathbf{r}_1$  and  $\mathbf{r}_2$ , then  $\Psi_{ab}(\mathbf{r}_2, \mathbf{r}_1) = -\Psi_{ab}(\mathbf{r}_1, \mathbf{r}_2)$ . This wavefunction describes a state where it's impossible to tell which electron is in state 'a' and which is in state 'b'.

Now, what happens to this antisymmetric wavefunction  $\Psi_{ab}(\mathbf{r}_1, \mathbf{r}_2)$  if the two electrons are supposed to be in the *same* quantum state, i.e., if state 'a' is identical to state 'b' (so  $a = b$ )? \_\_\_\_\_

Go to Frame 11.

### Frame 11

If state 'a' is identical to state 'b' (meaning all four quantum numbers are the same for both states, so  $\psi_a = \psi_b$ ), then the antisymmetric wavefunction becomes:

$$\Psi_{aa}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} [\psi_a(\mathbf{r}_1)\psi_a(\mathbf{r}_2) - \psi_a(\mathbf{r}_2)\psi_a(\mathbf{r}_1)] = 0$$

The wavefunction is identically zero!

What does a wavefunction of zero imply about the probability of finding the system in such a state? \_\_\_\_\_

What principle does this directly lead to? \_\_\_\_\_

Go to Frame 12.

### Frame 12

A wavefunction of zero means the probability density  $|\Psi|^2$  is zero everywhere. This implies that such a state **cannot exist** or has zero probability of occurring.

This directly leads to the **Pauli exclusion principle**: Since a state where two electrons have all four quantum numbers the same (i.e.,  $a = b$ ) results in a zero (non-existent) wavefunction for fermions, it means no two electrons in an atom can occupy the exact same quantum state.

The requirement for an antisymmetric total wavefunction for identical fermions is the deeper quantum mechanical reason for the Pauli exclusion principle.

End of Section 9.4.

## 24 Atomic Structure and Interactions

### Frame 1

In this section, we'll explore some aspects of atomic structure beyond simple hydrogen-like atoms. We'll start with the helium atom, then discuss how electron interactions and screening effects are handled, and finally look at the basis for the periodic table and X-ray spectra.

Let's begin with the helium atom, which has two electrons.

Go to Frame 2.

### Frame 2

#### Example 9.5: Ground State of the Helium Atom

We want to construct the two-electron ground-state wavefunction for helium using the independent particle approximation. This means we initially ignore the repulsion between the two electrons. Each electron "sees" only the doubly charged helium nucleus ( $Z = 2$ ).

The ground state for a hydrogen-like atom has quantum numbers  $n = 1, l = 0, m_l = 0$ . The spatial wavefunction  $\psi_{100}(r)$  for such an electron around a nucleus with charge  $Ze$  is given (from previous chapters, e.g., Eq 8.42) as:

$$\psi_{100}(r) = (\text{normalization constant}) \times e^{-Zr/a_0}$$

For helium,  $Z = 2$ . So, for one electron:

$$\psi_{100}(r) = \frac{1}{\sqrt{\pi}} \left( \frac{Z}{a_0} \right)^{3/2} e^{-Zr/a_0} = \frac{1}{\sqrt{\pi}} \left( \frac{2}{a_0} \right)^{3/2} e^{-2r/a_0}$$

What must we also consider for each electron, besides its spatial wavefunction? (Hint: it's an intrinsic property of electrons). \_\_\_\_\_

Go to Frame 3 for the answer.

### Frame 3

We must consider the electron's **spin**. Each electron can have spin up ( $\uparrow$  or  $+\frac{1}{2}$ ) or spin down ( $\downarrow$  or  $-\frac{1}{2}$ ).

For the ground state of helium, the two electrons are in the lowest energy spatial state ( $n = 1$ ). According to the Pauli Exclusion Principle, can both electrons have the same spin state? [ Yes — No ]

Go to Frame 4 for the answer.

### Frame 4

The answer is [No]. The Pauli Exclusion Principle states that no two electrons in an atom can have the same set of all four quantum numbers ( $n, l, m_l, m_s$ ). Since  $n, l, m_l$  are the same for both electrons in the ground state (1, 0, 0), their spin quantum numbers ( $m_s$ ) must be different. Thus, one must be spin up and the other spin down.

The overall wavefunction for two electrons must be *antisymmetric* upon exchange of the two electrons. If the spatial part of the wavefunction is symmetric (as it is when both electrons are in the same  $\psi_{100}$  orbital), the spin part must be antisymmetric to make the total wavefunction antisymmetric.

The antisymmetric spin combination for two electrons is  $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$ . The total two-electron wavefunction (ignoring electron-electron repulsion for now) combines the spatial parts and the spin part. If  $\psi_a(1)$  is electron 1 in state  $a$  and  $\psi_b(2)$  is electron 2 in state  $b$ , and  $a$  includes  $\psi_{100}$  and spin up, and  $b$  includes  $\psi_{100}$  and spin down:

$$\Psi(r_1, r_2) = \frac{1}{\sqrt{2}}[\psi_{100}(r_1)\alpha(1)\psi_{100}(r_2)\beta(2) - \psi_{100}(r_1)\beta(1)\psi_{100}(r_2)\alpha(2)]$$

where  $\alpha$  is spin up and  $\beta$  is spin down. This can be written as:

$$\Psi(r_1, r_2) = \psi_{100}(r_1)\psi_{100}(r_2) \times \frac{1}{\sqrt{2}}(\alpha(1)\beta(2) - \beta(1)\alpha(2))$$

The first term of the antisymmetric wavefunction,  $\psi_{100}(r_1)\alpha(1)\psi_{100}(r_2)\beta(2)$ , describes electron 1 as spin up and electron 2 as spin down. What does the second term,  $-\psi_{100}(r_1)\beta(1)\psi_{100}(r_2)\alpha(2)$ , describe? \_\_\_\_\_

Go to Frame 5.

### Frame 5

The second term describes electron 1 as having spin down and electron 2 as having spin up. The total spatial part (ignoring constants) is  $e^{-2r_1/a_0}e^{-2r_2/a_0} = e^{-2(r_1+r_2)/a_0}$ . The notation  $\{\uparrow\downarrow - \downarrow\uparrow\}$  implies that the spin of one electron is just as likely to be up as down, but the other electron always has the opposite spin. This spin-spin correlation is a consequence of the exclusion principle.

**Energy of Helium Ground State (Independent Particle Model):** In this model, the energy of each electron is given by the hydrogen-like formula  $E_n = -(Z^2/n^2)(13.6 \text{ eV})$ . For helium,  $Z = 2$  and for the ground state  $n = 1$ . What is the energy of one electron ( $E_a$  or  $E_b$ )?  $E_a = E_b =$  eV.

What is the total electronic energy  $E = E_a + E_b$ ?  $E =$  \_\_\_\_\_ eV.

Go to Frame 6 for the answers.

### Frame 6

Energy of one electron:  $E_a = E_b = -(2^2/1^2)(13.6 \text{ eV}) = -4 \times 13.6 \text{ eV} = -54.4 \text{ eV}$ . Total electronic energy:  $E = -54.4 \text{ eV} - 54.4 \text{ eV} = -108.8 \text{ eV}$ .

This value ( $-108.8 \text{ eV}$ ) represents the energy required to remove \*both\* electrons from helium in this simplified model. The experimentally measured value is about  $-79.0 \text{ eV}$ .

Why is there a discrepancy between the calculated ( $-108.8 \text{ eV}$ ) and measured ( $-79.0 \text{ eV}$ ) total energies? \_\_\_\_\_

Go to Frame 7 for the explanation.

### Frame 7

The discrepancy arises because our independent particle model **ignored the mutual electrostatic repulsion between the two electrons**. This repulsion energy is positive, meaning it makes the electrons less tightly bound. Therefore, the actual energy required to remove them is less ( $-79.0 \text{ eV}$ ) than the simple model predicts ( $-108.8 \text{ eV}$ ).

Specifically, it takes about  $24.6 \text{ eV}$  to remove the \*first\* electron. The remaining  $\text{He}^+$  ion has an energy of  $-54.4 \text{ eV}$  (as it's hydrogen-like with  $Z = 2, n = 1$ ). So,  $-79.0 \text{ eV} \approx -24.6 \text{ eV} - 54.4 \text{ eV}$ . The reason it only takes  $24.6 \text{ eV}$  (not  $54.4 \text{ eV}$ ) to remove the first electron is that the second electron "screens" the nuclear charge, making it appear less positive to the first electron.

This leads us to the topic of electron interactions and screening.  
Go to Frame 8.

## Frame 8

### 9.5 Electron Interactions and Screening Effects

When we have systems with two or more electrons, we must consider the electron-electron repulsion. To ignore it entirely is too crude. To include it exactly is very difficult. So, we need approximations. A common approach is the idea of an **effective field** or **effective potential energy**  $U_{\text{eff}}(r)$ .

An electron in an atom experiences: 1. Attraction to the nucleus. 2. Repulsion from all other electrons. These largely cancel, leaving a net effective field.

How is the attraction of an outer (valence) electron to the nucleus affected by the inner (core) electrons? [a] It's increased. [b] It's decreased (shielded/screened). [c] It's unaffected.

Go to Frame 9.

## Frame 9

The answer is [b]. Inner electrons **shield** or **screen** the nuclear charge, so an outer electron experiences a weaker attraction than it would to a bare nucleus. This can be modeled by an **effective atomic number**  $Z_{\text{eff}}$ , which is less than the actual atomic number  $Z$ . The effective potential energy for an electron might be written as:

$$U_{\text{eff}}(r) = -\frac{k(Z_{\text{eff}}e)(e)}{r}$$

If there were no screening,  $Z_{\text{eff}} = Z$ . If there were perfect screening by  $Z - 1$  other electrons, what would  $Z_{\text{eff}}$  be?  $Z_{\text{eff}} =$  \_\_\_\_\_

Go to Frame 10.

## Frame 10

If there were perfect screening by  $Z - 1$  other electrons,  $Z_{\text{eff}} = Z - (Z - 1) = 1$ . The actual  $Z_{\text{eff}}$  is usually not an integer and can be found from experimental data like ionization energies.  $Z_{\text{eff}}$  also depends on the electron's shell and subshell, as electrons that penetrate closer to the nucleus experience less screening. For example, a 4s electron is generally screened more effectively (has a lower  $Z_{\text{eff}}$ ) than a 3s electron.

**Example 9.6:  $Z_{\text{eff}}$  for the 3s Electron in Sodium** The ionization energy for the 3s electron in sodium (Na) is 5.14 eV. This means the energy of this electron in the atom is  $E = -5.14$  eV. For a hydrogen-like atom,  $E_n = -(Z_{\text{eff}}^2/n^2)(13.6 \text{ eV})$ . For the 3s electron in sodium,  $n = 3$ . Calculate  $Z_{\text{eff}}$  for this electron.  $-5.14 \text{ eV} = -\frac{Z_{\text{eff}}^2}{3^2}(13.6 \text{ eV})$   $Z_{\text{eff}} =$  \_\_\_\_\_

Go to Frame 11.

## Frame 11

Calculation for  $Z_{\text{eff}}$ :

$$\begin{aligned} 5.14 &= \frac{Z_{\text{eff}}^2}{9}(13.6) \\ Z_{\text{eff}}^2 &= \frac{5.14 \times 9}{13.6} \approx \frac{46.26}{13.6} \approx 3.40 \\ Z_{\text{eff}} &= \sqrt{3.40} \approx 1.84 \end{aligned}$$

Sodium has  $Z = 11$ . The 3s electron is screened by the 10 inner electrons, but not perfectly. It "sees" an effective charge of about  $+1.84e$ .

More sophisticated models allow  $Z_{\text{eff}}$  to vary with distance  $r$ . One such model is **Thomas-Fermi screening**:  $Z_{\text{eff}}(r) = Ze^{-r/a_{\text{TF}}}$ , where  $a_{\text{TF}}$  is the Thomas-Fermi screening length. Here,  $Z_{\text{eff}} \approx Z$  close to the nucleus ( $r \approx 0$ ) and drops off quickly for  $r \gg a_{\text{TF}}$ .

Another approach is the **quantum-defect method**, particularly for alkali metals. The energy levels are given by:

$$E_{n,l} = -\frac{ke^2}{2a_0} \frac{1}{(n - D(l))^2}$$

What does  $D(l)$  represent in this formula? \_\_\_\_\_

Go to Frame 12.

## Frame 12

$D(l)$  is the **quantum defect**. It measures how much the energy levels depart from the simple hydrogen-atom structure due to screening.  $D(l)$  depends on the orbital angular momentum quantum number  $l$  (s, p, d electrons will have different quantum defects) but is nearly independent of the principal quantum number  $n$  for a given  $l$ . Table 9.1 shows some experimental quantum defects for sodium.

More advanced is the **Hartree theory**, which treats the electron cloud as a classical charge distribution  $\rho(r)$ . The potential energy of one electron is:

$$U_{\text{eff}}(r') = -\frac{kZe^2}{r'} + ke \int \frac{\rho(r)}{|r' - r|} dV$$

The first term is nuclear attraction, the second is repulsion from other electrons. This must be solved *self-consistently*: the wavefunctions determine  $\rho(r)$ , which in turn determines  $U_{\text{eff}}$ , which is then used to find new wavefunctions, and this is iterated until consistency. **Hartree-Fock methods** are even more refined and commonly used for accurate atomic calculations.

Go to Frame 13.

## Frame 13

### 9.6 The Periodic Table

Wave mechanics, in principle, allows us to predict properties of all elements. However, for multielectron atoms, approximations are essential. The electronic structure can be viewed as a succession of filled energy levels, with outermost electrons determining chemical properties.

In the central field approximation (where each electron moves in an average field from the nucleus and other electrons), levels are labeled by  $n$  (principal quantum number) and  $l$  (orbital angular momentum quantum number). What rule dictates the maximum number of electrons in a subshell defined by  $n$  and  $l$ ? And what is that maximum number? Rule: —Maximum number: —

Go to Frame 14.

## Frame 14

Rule: The **Pauli Exclusion Principle**. Maximum number of electrons in a subshell ( $n, l$ ) is  $2(2l + 1)$ . (This accounts for the  $2l + 1$  possible values of  $m_l$  and 2 spin states for each).

The energy of an electron depends mainly on  $n$  and, to a lesser extent, on  $l$ . Electrons fill subshells according to the **minimum energy principle**: once a subshell is filled, the next electron goes into the vacant subshell that is lowest in energy.

What is the electron configuration of an atom? \_\_\_\_\_  
Go to Frame 15.

### Frame 15

The **electron configuration** is the specification of  $n$  and  $l$  for each atomic electron. (Often written like  $1s^2 2s^2 2p^6 \dots$ ).

Let's list a few:

- Hydrogen (1 electron):  $1s^1$
- Helium (2 electrons):  $1s^2$  (1s subshell is closed, Helium is inert)
- Lithium (3 electrons):  $1s^2 2s^1$  (third electron goes to 2s, which is slightly lower in energy than 2p)
- Beryllium (4 electrons):  $1s^2 2s^2$  (2s subshell is closed)
- Boron (5 electrons):  $1s^2 2s^2 2p^1$

For Carbon (6 electrons), it has two 2p electrons ( $1s^2 2s^2 2p^2$ ). **Hund's rule** states that electrons usually fill different orbitals within a subshell with unpaired (parallel) spins, rather than pairing up in the same orbital. This is because electrons in the same orbital are closer and their mutual repulsion increases energy. (See Fig 9.15 for filling of 2p orbitals from B to Ne). Neon (10 electrons) has configuration  $1s^2 2s^2 2p^6$ . The 2p subshell is closed. Neon is exceptionally stable and chemically inert.

What is the general order of filling subshells by energy, according to Fig 9.15 and Table 9.2? (List the first few).  $1s < 2s < \dots$  \_\_\_\_\_

Go to Frame 16.

### Frame 16

The approximate order of filling subshells by energy is:  $1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d \dots$  (and so on, as listed on page 321). Notice that the 4s subshell fills \*before\* the 3d subshell, and 5s before 4d. This is because energy separating subshells ( $l$  values) within a given shell ( $n$ ) becomes comparable to or larger than energy separating consecutive shells for higher  $n$ .

What are **transition series** elements characterized by? (e.g., Scandium  $Z=21$  to Zinc  $Z=30$ ).

Go to Frame 17.

### Frame 17

Transition series elements are characterized by the progressive filling of an inner  $d$  subshell (e.g., 3d) while the outer electron configuration (e.g.,  $4s^2$ ) remains largely unchanged. This results in similar chemical properties across the series. The **lanthanide series** ( $Z=57$  to  $Z=70$ ) shows similar behavior with the filling of the inner  $4f$  subshell while the valence configuration is often  $6s^2$ . These are also known as rare earths.

Table 9.2 and Figure 9.16 show ionization energies and atomic volumes. Generally, ionization energy tends to: [a] Increase within a shell (as  $Z$  increases) [b] Decrease within a shell And then it: [c] Drops sharply when a new shell begins [d] Increases sharply when a new shell begins

Go to Frame 18.

### Frame 18

Ionization energy tends to [a] **increase within a shell** as  $Z$  increases (due to stronger nuclear attraction) and then [c] **drops sharply when a new shell begins** (as the outermost electron is further away and better screened). Atomic volume shows an inverse trend: decreasing within a shell and then increasing sharply for a new shell. These recurring patterns give the periodic table its name.

Go to Frame 19.

## Frame 19

### 9.7 X-Ray Spectra and Moseley's Law

Electronic transitions within the \*inner shells\* of heavy atoms involve large energy changes. If this energy is released as a photon, X-rays are emitted. Consider Molybdenum (Mo,  $Z = 42$ ). The innermost K-shell electrons ( $n = 1$ ) are tightly bound because they experience an almost unscreened nuclear charge. The energy for a K-shell electron (using  $Z_{eff} \approx Z - 1$  or similar for a simple estimate, or just  $Z$  from Eq. 8.38 as a first approximation given in the text):  $E_1 \approx -(13.6 \text{ eV})(Z^2/n^2)$ . For Mo with  $Z = 42, n = 1$ :  $E_1 \approx -(13.6 \text{ eV})(42)^2 \approx -23990.4 \text{ eV}$ .

If an electron is knocked out of the K-shell (e.g., by energetic electron bombardment), a vacancy is created. How can this vacancy be filled, and what is emitted in the process shown in Figure 9.17?

---

Go to Frame 20.

## Frame 20

The vacancy in an inner shell (e.g., K-shell) can be filled by an electron from a higher-lying shell (L, M, N, etc.) making a downward transition. The energy difference is emitted as an **X-ray photon**. Transitions ending in the:

- K-shell ( $n = 1$ ) form the **K series** ( $K_\alpha, K_\beta$ , etc.)
- L-shell ( $n = 2$ ) form the **L series** ( $L_\alpha, L_\beta$ , etc.)
- M-shell ( $n = 3$ ) form the **M series**

(This explanation refers to Figure 9.17). These X-ray energies are characteristic of the emitting atom, a fact used by Moseley to order elements by atomic number.

Go to Frame 21.

## Frame 21

### Chapter 11: Molecular Structure (Introduction)

Except for inert gases, elements generally combine to form chemical compounds, whose basic unit is the **molecule** – an aggregate of atoms joined by chemical bonds. Properties of molecules derive from:

- Their constituent atoms
- Arrangement of atoms
- How they interact (bonds)
- Individual electronic structures



Like atoms, molecules have allowed energy levels and can emit or absorb photons, giving characteristic spectra.

Besides electronic transitions, what other ways can molecules store or release energy that are not found in individual atoms? (List two from page 372). 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 22.

## Frame 22

Molecules can also store/release energy through: 1. **Rotation:** Kinetic energy of rotation of the molecule about its center of mass. 2. **Vibration:** Energy due to the vibration of the molecule's constituent atoms.

Both rotational and vibrational energies are quantized and lead to their own unique spectra, making molecular spectra more complex than atomic spectra. The total energy of a molecule in the gas phase can be categorized as:  $E = E_{\text{el}} + E_{\text{trans}} + E_{\text{rot}} + E_{\text{vib}}$  (electronic, translational, rotational, vibrational). Translational energy is usually not important for interpreting molecular spectra related to internal structure.

Go to Frame 23.

## Frame 23

### 11.1 Bonding Mechanisms: A Survey

Two atoms combine to form a molecule because of a net *attractive force* between them. The total energy of the bound molecule is *less* than the total energy of the separated atoms.

What is the fundamental nature of the forces that cause bonding in a molecule? [a] Gravitational forces [b] Nuclear forces [c] Electrostatic forces

Go to Frame 24.

## Frame 24

The answer is [c] **Electrostatic forces**.

When two atoms are very far apart, the force between them is zero. As they are brought closer:

- At large separations: dominant forces are attractive.
- At small separations: repulsive forces (between like charges of nuclei, and between electron clouds due to Pauli exclusion) begin to dominate.

The total potential energy  $U(r)$  of a two-atom system is often approximated by (see Figure 11.1):

$$U(r) = -\frac{A}{r^n} + \frac{B}{r^m}$$

The first term represents attraction, the second repulsion. At the equilibrium separation,  $r_{\text{eq}}$ , the potential energy is at a minimum, and the net force is zero.

What is the **dissociation energy** of a molecule? \_\_\_\_\_

Go to Frame 25.

## Frame 25

The **dissociation energy** is the energy that must be supplied to break apart the molecule into its constituent neutral atoms. It's the magnitude of the potential energy minimum relative to separated atoms.

We will now briefly survey different types of bonds, in order of decreasing strength.

Go to Frame 26.

### Frame 26

**Ionic Bonds** (Page 374) An ionic bond forms when one or more electrons are *transferred* from one atom to another, creating oppositely charged ions that attract each other via Coulomb forces. Example: Sodium Chloride (NaCl).

- Na ( $1s^2 2s^2 2p^6 3s^1$ ) gives up its 3s electron  $\rightarrow \text{Na}^+$ . Energy required = Ionization energy of Na = 5.1 eV.
- Cl ( $1s^2 2s^2 2p^5$ ) accepts an electron  $\rightarrow \text{Cl}^-$ . Energy released = Electron affinity of Cl = 3.7 eV.

What is the net energy cost (activation energy) to form  $\text{Na}^+$  and  $\text{Cl}^-$  ions from isolated neutral atoms? Energy cost = \_\_\_\_\_ eV.

Go to Frame 27.

### Frame 27

Net energy cost (activation energy) = 5.1 eV (to ionize Na) – 3.7 eV (gained by Cl) = 1.4 eV. As the  $\text{Na}^+$  and  $\text{Cl}^-$  ions are brought closer, their mutual electrostatic attraction lowers the system's energy. At the equilibrium separation (0.24 nm for NaCl), the total energy has a minimum value of –4.2 eV relative to the neutral separated atoms. So, the dissociation energy for NaCl is 4.2 eV. If ions are brought closer than 0.24 nm, what happens? \_\_\_\_\_

Go to Frame 28.

### Frame 28

If the ions are brought closer than the equilibrium separation, electrons in the closed shells of  $\text{Na}^+$  and  $\text{Cl}^-$  begin to overlap. This leads to a strong repulsive force, partly electrostatic and partly due to the Pauli Exclusion Principle (forcing electrons into higher energy states).

Go to Frame 29.

### Frame 29

**Covalent Bonds** (Page 374-375) A covalent bond is one in which electrons are *shared* by two atoms. Example:  $\text{H}_2$  molecule. The two electrons are shared, forming a "molecular orbital" where they are more likely to be found between the two nuclei (Figure 11.3). This concentration of negative charge between the positive nuclei leads to attraction. Due to the exclusion principle, the two electrons in the  $\text{H}_2$  ground state molecular orbital must have *antiparallel* spins.

Can an  $\text{H}_3$  molecule form stably in a similar way? Why or why not? \_\_\_\_\_

Go to Frame 30.

### Frame 30

An  $\text{H}_3$  molecule does not form stably. If a third H atom approaches  $\text{H}_2$ , its electron would have to occupy a higher-energy quantum state due to the exclusion principle (as the lowest energy molecular orbital is already filled with two electrons of opposite spin). This higher energy state makes  $\text{H}_3$  energetically unfavorable.

More complex molecules like  $\text{H}_2\text{O}$ ,  $\text{CO}_2$ ,  $\text{CH}_4$  also feature covalent bonds. In  $\text{CH}_4$  (methane, Figure 11.4), carbon forms four covalent bonds with four hydrogen atoms, arranged tetrahedrally.

Go to Frame 31.

### Frame 31

**van der Waals Bonds** (Page 375-376) These are weaker bonds that can occur *between* neutral molecules or atoms that don't form ionic or covalent bonds (like inert gases). They are due to weak electrostatic forces. There are three types: 1. **Dipole-dipole force**: Interaction between two molecules that *\*each\** have a permanent electric dipole moment (e.g., HCl, H<sub>2</sub>O). 2. **Dipole-induced force**: A polar molecule (with a permanent dipole) induces a temporary dipole in a nearby nonpolar molecule, leading to attraction. 3. **Dispersion force (London force)**: Occurs even between two *\*nonpolar\** molecules. Temporary, fluctuating charge distributions in one molecule can induce a correlated temporary dipole in a nearby molecule, leading to a net attractive force.

How does the strength of van der Waals forces generally compare to ionic or covalent bonds?  
[a] Stronger [b] Weaker [c] Similar strength

Go to Frame 32.

### Frame 32

The answer is [b] **Weaker**. Van der Waals forces are much weaker than ionic or covalent bonds. They typically fall off with separation distance  $r$  as  $1/r^7$ . These forces are responsible for the condensation of inert gases into liquids and solids at very low temperatures.

Go to Frame 33.

### Frame 33

**The Hydrogen Bond** (Page 377) A hydrogen atom, having only one electron, typically forms one covalent bond. However, it can also form a second, weaker bond called a hydrogen bond. Example: (HF<sub>2</sub>)<sup>-</sup> ion (Figure 11.5). Two negative F<sup>-</sup> ions are bound by a positively charged proton (H<sup>+</sup>) between them. In H<sub>2</sub>O, the H atoms are covalently bonded to O. The electrons are pulled towards the oxygen, leaving the hydrogen atoms as somewhat bare protons. This positive end can be attracted to the negative (oxygen) end of another H<sub>2</sub>O molecule.

What is a critical role of hydrogen bonds mentioned in the text, despite their relative weakness (binding energy 0.1 eV)? \_\_\_\_\_

Go to Frame 34.

### Frame 34

Hydrogen bonds are critical for **linking biological molecules and polymers**, such as in the double-helix structure of DNA, where they link the turns of the helix. They are also responsible for the structure of ice.

This concludes our brief survey of bonding mechanisms and introduction to molecular structure. The following frames would delve into molecular rotation and vibration if we were to continue this chapter.

End of Section.

## 25 Molecular Structure (Adapted Sections)

### Frame 1

In addition to the van der Waals forces that cause attraction between molecules, another important type of bond can form, especially involving hydrogen. This is called a **hydrogen bond**.

A hydrogen atom, having only one electron, typically forms a covalent bond with one other atom in a molecule. However, it can also form a second, weaker bond *between* molecules.

Consider the  $(\text{HF}_2)^-$  ion shown in Figure 11.5. What holds the two negative fluorine ions ( $\text{F}^-$ ) together? \_\_\_\_\_

Go to Frame 2 for the answer.

## Frame 2

In the  $(\text{HF}_2)^-$  ion, the two negative fluorine ions are bound by the positively charged proton ( $\text{H}^+$ ) located between them. This is an example of a hydrogen bond.

Water ( $\text{H}_2\text{O}$ ) is another prime example. In a water molecule, the covalent bonds between oxygen and hydrogen are polar. The electrons are more likely to be found near the oxygen atom.

What does this leave at the positions of the hydrogen atoms? [a] A net negative charge [b] Essentially bare protons [c] No net charge

Go to Frame 3 for the answer.

## Frame 3

Your answer was [ a — b — c ].

The correct answer is [b]. The polarity of the O-H bonds in water leaves essentially bare, unshielded protons at the hydrogen atom positions.

This unshielded positive charge on a hydrogen atom in one water molecule can be attracted to the negative end (the oxygen atom) of another water molecule. This attraction forms a hydrogen bond.

Are hydrogen bonds strong or weak compared to typical covalent bonds? \_\_\_\_ (Hint: The text mentions a binding energy around 0.1 eV, and that ice melts at  $0^\circ\text{C}$  because of this.)

Go to Frame 4 for the answer.

## Frame 4

Hydrogen bonds are relatively **weak** compared to covalent bonds (which typically have energies of several eV). They can be broken with about 0.1 eV of energy.

Despite their weakness, hydrogen bonds are crucial. List one example mentioned in the text of their importance in biological systems. \_\_\_\_\_

Go to Frame 5.

## Frame 5

Hydrogen bonds are critical for linking biological molecules and polymers. For example, in the DNA double-helix structure, hydrogen bonds formed by sharing a proton between two atoms create linkages between the turns of the helix.

Now we will move on to discussing how molecules rotate and vibrate.

Go to Frame 6.

## Frame 6

## Molecular Rotation and Vibration

Just like with atoms, we can study the structure and properties of molecules by examining the radiation (light) they emit or absorb. First, let's understand the different ways a molecule can possess energy.

The total energy of an individual molecule in a gas can be divided into four categories. Can you list them? (One is electronic energy). 1. Electronic energy 2. \_\_\_\_\_ 3. \_\_\_\_\_ 4. \_\_\_\_\_  
Go to Frame 7 for the answers.

### Frame 7

The energy of a molecule can be divided into:

1. **Electronic energy** ( $E_{el}$ ): Due to interactions between the molecule's electrons and nuclei.
2. **Translational energy** ( $E_{trans}$ ): Due to the motion of the molecule's center of mass through space.
3. **Rotational energy** ( $E_{rot}$ ): Due to the rotation of the molecule about its center of mass.
4. **Vibrational energy** ( $E_{vib}$ ): Due to the vibration of the molecule's constituent atoms.

So, the total energy is approximately  $E = E_{el} + E_{trans} + E_{rot} + E_{vib}$ .

Which of these energy components is considered unimportant for interpreting molecular spectra related to internal structure? \_\_\_\_\_

Go to Frame 8 for the answer.

### Frame 8

**Translational energy** ( $E_{trans}$ ) is unimportant in interpreting molecular spectra because it's related to the motion of the molecule as a whole through space, not its internal structure.

Electronic energies are complex and will be discussed later (Section 11.4 in the original text). We will now focus on rotational and vibrational energy states, which give rise to spectral lines primarily in the infrared region.

Go to Frame 9.

### Frame 9

#### Molecular Rotation

Let's consider a diatomic molecule rotating about its center of mass. (Figure 11.6a). A diatomic molecule has how many rotational degrees of freedom if we consider rotations perpendicular to the molecular axis? [a] 1 [b] 2 [c] 3

(Note: Rotation \*about\* the molecular axis itself has very high excitation energy and is usually not observed for diatomic molecules because most of the mass is concentrated along this axis, giving a tiny moment of inertia for that rotation.)

Go to Frame 10 for the answer.

### Frame 10

Your answer was [ a — b — c ].

The correct answer is [b] 2. A diatomic molecule has 2 rotational degrees of freedom, corresponding to rotations about the two axes perpendicular to the bond (e.g., y and z axes if the molecule lies along x).

The rotational energy of a rigid rotating molecule is purely kinetic. If  $L$  is the angular momentum and  $I$  is the moment of inertia, the classical rotational energy is:

$$E_{rot} = \frac{L^2}{2I}$$

This is analogous to the translational kinetic energy  $p^2/2m$ . What does the moment of inertia  $I$  measure in rotation, analogous to what mass  $m$  measures in translation? \_\_\_\_\_

Go to Frame 11.

### Frame 11

The moment of inertia  $I$  measures the molecule's **resistance to changes in its rotational motion**, just as mass  $m$  measures a particle's resistance to changes in its translational motion.

For a diatomic molecule with atomic masses  $m_1$  and  $m_2$  and atomic separation  $R_0$ , the moment of inertia about an axis through the center of mass ( $I_{CM}$ ) is given by:

$$I_{CM} = \mu R_0^2$$

where  $\mu$  is the **reduced mass** of the molecule, defined as:

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Go to Frame 12.

### Frame 12

In quantum mechanics, angular momentum is quantized. The square of the angular momentum  $L^2$  is given by:

$$L^2 = \ell(\ell + 1)\hbar^2$$

where  $\ell = 0, 1, 2, \dots$  is the **rotational quantum number** and  $\hbar = h/2\pi$ .

Substituting this into the energy equation  $E_{rot} = L^2/(2I_{CM})$ , what is the formula for the allowed rotational energies of a diatomic molecule? (See Eq. 11.5) \_\_\_\_\_

Go to Frame 13.

### Frame 13

The allowed rotational energies for a diatomic molecule are:

$$E_{rot} = \frac{\hbar^2}{2I_{CM}} \ell(\ell + 1)$$

where  $\ell = 0, 1, 2, \dots$  (Figure 11.6b shows a sketch of these energy levels).

Notice that the energy levels are not equally spaced. The spacing between adjacent rotational levels increases as  $\ell$  increases. If a molecule transitions from a higher rotational state  $\ell$  to an adjacent lower state  $\ell - 1$ , what is the energy difference  $\Delta E = E_\ell - E_{\ell-1}$ ? \_\_\_\_\_

Go to Frame 14.

### Frame 14

Let's calculate the energy difference  $\Delta E = E_\ell - E_{\ell-1}$ :

$$E_\ell = \frac{\hbar^2}{2I_{CM}} \ell(\ell + 1)$$

$$E_{\ell-1} = \frac{\hbar^2}{2I_{CM}}(\ell-1)(\ell-1+1) = \frac{\hbar^2}{2I_{CM}}(\ell-1)\ell$$

So,

$$\Delta E = \frac{\hbar^2}{2I_{CM}}[\ell(\ell+1) - \ell(\ell-1)]$$

$$\Delta E = \frac{\hbar^2}{2I_{CM}}[\ell^2 + \ell - \ell^2 + \ell]$$

$$\Delta E = \frac{\hbar^2}{2I_{CM}}(2\ell) = \frac{\hbar^2}{I_{CM}}\ell$$

This energy is typically released or absorbed as a photon with frequency  $f = \Delta E/h$ , or angular frequency  $\omega = \Delta E/\hbar = (\hbar/I_{CM})\ell$ . These transitions give rise to spectra in the microwave range. (See Table 11.1 for CO molecule data).

Go to Frame 15.

## Frame 15

### Molecular Vibration

Molecules are not rigid; their atoms can vibrate about their equilibrium positions, as if connected by "effective springs." (Figure 11.7). For small displacements from the equilibrium separation  $R_0$ , the potential energy  $U(r)$  can be approximated by that of a simple harmonic oscillator:

$$U = \frac{1}{2}K\xi^2$$

where  $K$  is the effective spring (force) constant and  $\xi = (r - R_0)$  is the displacement from equilibrium. (Actually, for a diatomic molecule,  $\xi = \xi_1 - \xi_2$  where  $\xi_1, \xi_2$  are displacements of individual atoms).

The kinetic energy of vibration, in the center-of-mass frame, can be written as  $p_1^2/(2\mu)$ , where  $p_1$  is the momentum of one atom relative to the center of mass, and  $\mu$  is the reduced mass. The Schrödinger equation for this 1D oscillator (with coordinate  $\xi$ ) is (Eq. 11.7):

$$-\frac{\hbar^2}{2\mu} \frac{d^2\psi}{d\xi^2} + \frac{1}{2}K\xi^2\psi(\xi) = E_{vib}\psi(\xi)$$

What are the allowed vibrational energy levels  $E_{vib}$  for this quantum harmonic oscillator? (Hint: Recall from Chapter 6 or see Eq. 11.8).  $v$  is the vibrational quantum number. \_\_\_\_\_

Go to Frame 16.

## Frame 16

The allowed vibrational energies are:

$$E_{vib} = \left(v + \frac{1}{2}\right) \hbar\omega$$

where  $v = 0, 1, 2, \dots$  is the **vibrational quantum number**. The classical angular frequency of vibration  $\omega$  is related to the force constant  $K$  and reduced mass  $\mu$  by:

$$\omega = \sqrt{\frac{K}{\mu}}$$

(See Figure 11.8 for a diagram of these levels).

What is the energy of the lowest vibrational state (the zero-point energy)? What is the energy difference between any two successive vibrational levels (e.g.,  $E_{v+1} - E_v$ )? \_\_\_\_\_

Go to Frame 17.

### Frame 17

For the lowest vibrational state,  $v = 0$ , so the zero-point energy is:

$$E_{vib}(v = 0) = \frac{1}{2}\hbar\omega$$

The energy difference between successive levels is:

$$\Delta E_{vib} = E_{v+1} - E_v = \left(v + 1 + \frac{1}{2}\right)\hbar\omega - \left(v + \frac{1}{2}\right)\hbar\omega = \hbar\omega$$

This means vibrational energy levels are **equally spaced** by  $\hbar\omega$ . Transitions between these levels typically involve absorption or emission of photons in the infrared region of the spectrum.

Go to Frame 18.

### Frame 18

The simple harmonic oscillator model for molecular vibration is an approximation. Real interatomic forces are not perfectly harmonic, especially for larger vibrational amplitudes (higher energies). A more realistic potential is the **Morse potential** (Figure 11.9a). How do the energy levels of a Morse oscillator differ from those of a harmonic oscillator, especially at higher energies? (See Figure 11.9b and surrounding text). \_\_\_\_\_

Go to Frame 19.

### Frame 19

For a Morse oscillator, unlike the harmonic oscillator, the spacing between successive vibrational energy levels **decreases** as the vibrational energy (and quantum number  $v$ ) increases. This is due to the **anharmonic effects** of the true interatomic potential.

The Morse potential also accounts for thermal expansion: as vibrational energy increases (e.g., with temperature), the average internuclear separation  $\bar{r}$  increases because the potential well is asymmetric.

Go to Frame 20.

### Frame 20

## Molecular Spectra

In general, a molecule rotates and vibrates simultaneously. To a first approximation, these motions are independent. The total rotational-vibrational energy is (Eq. 11.10):

$$E_{rot-vib} = \frac{\hbar^2}{2I_{CM}}\ell(\ell + 1) + \left(v + \frac{1}{2}\right)\hbar\omega$$

Each vibrational level (indexed by  $v$ ) has a whole set of rotational levels (indexed by  $\ell$ ) associated with it (Figure 11.10).



Optical transitions between these molecular levels involve absorption or emission of photons. What are the two main "selection rules" that govern these transitions for diatomic molecules when photons are involved? (See Eqs. 11.12 and 11.13). 1. For rotational quantum number  $\ell$ :  $\Delta\ell = \pm 1$ . For vibrational quantum number  $v$ :  $\Delta v = \pm 1$ .

Go to Frame 21.

### Frame 21

The selection rules for optical transitions in diatomic molecules (in the harmonic oscillator, rigid rotor approximation) are: 1.  $\Delta\ell = \pm 1$  (The molecule gains or loses one unit of rotational angular momentum, carried by the photon which is a spin-1 particle). 2.  $\Delta v = \pm 1$  (This reflects the harmonic character of the interatomic force for lower vibrational levels).

Can a molecule undergo a pure rotational transition ( $\Delta v = 0$ ) by absorbing or emitting a photon? Can it undergo a pure vibrational transition ( $\Delta\ell = 0$ )? [ Yes/Yes — Yes/No — No/Yes — No/No ] Based on the selection rules above.

Go to Frame 22.

### Frame 22

Your answer was [ ... ].

According to the selection rules  $\Delta\ell = \pm 1$  and  $\Delta v = \pm 1$ , neither a pure rotational transition ( $\Delta v = 0$ ) nor a pure vibrational transition ( $\Delta\ell = 0$ ) is allowed by photon absorption/emission. Both quantum numbers must change. (Note: The original text mentions these are strict rules for ideal cases. Real molecules have anharmonicity and non-rigid rotation, which can lead to violations or additional "forbidden" transitions, though often weaker).

This means absorption of a photon (typically from the  $v = 0$  state at normal temperatures) leads to  $v = 1$  and either  $\ell \rightarrow \ell + 1$  or  $\ell \rightarrow \ell - 1$ . This results in two branches in the absorption spectrum (Figure 11.11). The energies of absorbed photons are (Eq. 11.14):

- $\Delta E = \hbar\omega + \frac{\hbar^2}{I_{CM}}(\ell + 1)$  for  $\Delta\ell = +1$  (where  $\ell = 0, 1, 2, \dots$  is initial  $\ell$ )
- $\Delta E = \hbar\omega - \frac{\hbar^2}{I_{CM}}\ell$  for  $\Delta\ell = -1$  (where  $\ell = 1, 2, 3, \dots$  is initial  $\ell$ )

Notice the central frequency  $\omega$  (from  $\hbar\omega$ ) is "missing" because  $\Delta\ell = 0$  is forbidden.

Go to Frame 23.

### Frame 23

Molecules can also *scatter* radiation without first absorbing and then re-emitting it.

- **Rayleigh scattering:** Photon energy is unchanged. Stronger for shorter wavelengths (explains blue sky).
- **Raman scattering:** Photon energy \*changes\* (inelastic scattering). The energy lost or gained by the photon corresponds to a change in the rotational and/or vibrational state of the molecule.

For Raman scattering involving rotational states, what is the selection rule for  $\Delta\ell$ ? (See page 388).  $\Delta\ell = \pm 1$

Go to Frame 24.

### Frame 24

The selection rule for rotational Raman scattering is  $\Delta\ell = \pm 2$ .

If an incident photon of energy  $E$  scatters and excites the molecule to a higher rotational state (e.g., from  $\ell$  to  $\ell+2$ ), the scattered photon emerges with a reduced energy  $E'$ . The energy difference,  $E - E' = \Delta E_{\text{molecule}}$ , is called the **Raman shift**. (Figure 11.13). Measurements of Raman shifts provide information about the molecule's moment of inertia.

Go to Frame 25.

## Frame 25

### Fluorescence and Phosphorescence (Figure 11.14)

These processes involve changes in the \*electronic state\* of the molecule, as well as vibrational/rotational states. **Fluorescence** is a three-step process:

1. Photon absorption excites the molecule to a higher electronic-vibrational state.
2. The molecule loses some vibrational energy (non-radiatively, e.g., through collisions).
3. The molecule de-excites back to the original electronic state by emitting a photon.

Is the emitted photon in fluorescence typically of higher, lower, or the same energy as the absorbed photon? Why? \_\_\_\_\_

Go to Frame 26.

## Frame 26

The emitted photon in fluorescence is typically of **lower energy** (longer wavelength) than the absorbed photon. This is because some energy is lost as vibrational energy in step 2 before the photon is emitted in step 3. The energy difference between absorbed and emitted photons is called the **Stokes shift**.

**Phosphorescence** is similar, but in step 2 the molecule transitions to a metastable electronic state. The subsequent emission (step 3) is from this metastable state and is often a "forbidden" transition (violates selection rules), making it much slower. This results in an afterglow that can last seconds, minutes, or even hours.

Go to Frame 27.

## Frame 27

### Electron Sharing and the Covalent Bond (Introduction)

We now briefly touch upon how electrons behave in molecules to form bonds, focusing on the simplest cases.

Consider the **hydrogen molecular ion  $\text{H}_2^+$** . It consists of one electron in the field of two protons. (Figure 11.15). What forces act on the electron? \_\_\_\_\_

Go to Frame 28.

## Frame 28

In  $\text{H}_2^+$ , the single electron is attracted to *both* protons by the Coulomb force. This sharing of the electron between the two protons is what can lead to a stable bond.

The detailed quantum mechanical treatment involves solving the Schrödinger equation for the electron in the potential created by the two protons. This leads to bonding and antibonding molecular orbitals.

This is a very brief introduction to the chemical bond. The full section in the original text would delve deeper into molecular orbitals.

End of Adapted Section.

## 26 Electron Sharing and the Covalent Bond

### Frame 29

We will now explore how electrons are shared between atoms to form covalent bonds. The simplest case to consider is the **hydrogen molecular ion,  $\text{H}_2^+$** .

What are the constituent particles of  $\text{H}_2^+$ ? [a] Two electrons and one proton [b] One electron and two protons [c] Two electrons and two protons

Go to Frame 30 for the answer.

### Frame 30

Your answer was [ a — b — c ].

The correct answer is [b]. The  $\text{H}_2^+$  ion consists of **one electron** in the field of **two protons**. (See Figure 11.15 in the original text).

The potential energy  $U(r)$  of the electron in the field of these two protons (one at the origin  $r = 0$ , the other at position  $R$ ) is given by (Eq. 11.16):

$$U(r) = -\frac{ke^2}{|\mathbf{r}|} - \frac{ke^2}{|\mathbf{r} - \mathbf{R}|}$$

The equilibrium separation  $R$  of the protons in  $\text{H}_2^+$  is about 0.1 nm.

Go to Frame 31.

### Frame 31

To understand the behavior of the electron in  $\text{H}_2^+$ , it's helpful to consider two limiting cases for the proton separation  $R$ :

1. **United Atom Limit ( $R \rightarrow 0$ ):** The two protons coalesce. What atomic nucleus does this effectively form? \_\_\_\_\_
2. **Separated Atom Limit ( $R \rightarrow \infty$ ):** The electron essentially sees only one proton. What atomic system does this resemble? \_\_\_\_\_

Go to Frame 32 for the answers.

### Frame 32

1. **United Atom Limit ( $R \rightarrow 0$ ):** The two protons form a nucleus of Helium (specifically, a  $\text{He}^+$  ion, since there's only one electron). The energies are those of  $\text{He}^+$  ( $Z=2$ ). The ground state energy is -54.4 eV.
2. **Separated Atom Limit ( $R \rightarrow \infty$ ):** The electron is effectively bound to a single proton, forming a Hydrogen atom. The ground state energy is -13.6 eV.

(See Figure 11.16 for a correlation diagram showing energy levels versus  $R$ ).

In the separated atom limit, the ground state is doubly degenerate. Why is there this extra degeneracy? \_\_\_\_\_

Go to Frame 33.

### Frame 33

In the separated atom limit ( $R \rightarrow \infty$ ), the ground state is doubly degenerate because the electron can be in a hydrogen-like orbital around *either* the first proton *or* the second proton. These two possibilities have the same energy.

Now, what happens at intermediate values of  $R$ ? If  $R$  is large but finite, the electron, initially around one proton, can tunnel through the potential barrier to the other proton. This is similar to ammonia inversion.

Due to this tunneling and the symmetry of  $H_2^+$ , the stationary state wavefunctions are not localized on one proton. Instead, they are combinations of atomic-like wavefunctions centered on each proton. What are the two simplest combinations? (Hint: symmetric and antisymmetric, see Eq. 11.17 and 11.18). 1.  $\psi_+(r) = \psi_a(r) + \psi_a(r - R)$  2.  $\psi_-(r) = \psi_a(r) - \psi_a(r - R)$  (Where  $\psi_a(r)$  is an atomic orbital centered at the origin, and  $\psi_a(r - R)$  is the same orbital centered at  $R$ ).

Go to Frame 34.

### Frame 34

The two simplest combinations of atomic orbitals are:

1. **Symmetric combination:**  $\psi_+(\mathbf{r}) = \psi_a(\mathbf{r}) + \psi_a(\mathbf{r} - \mathbf{R})$

2. **Antisymmetric combination:**  $\psi_-(\mathbf{r}) = \psi_a(\mathbf{r}) - \psi_a(\mathbf{r} - \mathbf{R})$

(See Figure 11.17 for sketches of these wavefunctions and their probability densities).

Which of these two states,  $\psi_+$  or  $\psi_-$ , do you expect to have lower energy? Why? \_\_\_\_\_

Go to Frame 35.

### Frame 35

The symmetric state  $\psi_+$  has **lower energy**. This is because in the  $\psi_+$  state, the electron has a higher probability of being found in the region *between* the two protons. In this region, it is strongly attracted to both protons simultaneously, lowering its potential energy. This state is called the **bonding orbital**.

The antisymmetric state  $\psi_-$  has a node (zero probability) exactly midway between the protons. The electron is more confined to the vicinity of one or the other nucleus, leading to a higher kinetic energy and thus a higher total energy. This state is called the **antibonding orbital**.

As  $R \rightarrow 0$ , the bonding orbital energy approaches the ground state of  $He^+$  (-54.4 eV), and the antibonding orbital energy approaches the first excited state of  $He^+$  (-13.6 eV).

Go to Frame 36.

### Frame 36

The energy of the bonding orbital  $E_+$  for  $H_2^+$  (when  $\psi_a$  is the hydrogen ground state) can be approximated as (Eq. 11.20):

$$E_+ = -1 - \frac{2}{1 + \Delta} \left[ \frac{1}{R} - \frac{1}{R}(1 + R)e^{-2R} + (1 + R)e^{-R} \right] \quad (\text{in Rydbergs})$$

where  $\Delta$  is an overlap integral.

This energy  $E_+$  is for the electron only. To get the total molecular energy, we must also include the Coulomb repulsion between the two protons, which is  $ke^2/R$  (or  $2/R$  in Rydbergs). The molecule is stable if the total energy has a minimum at some separation  $R_0 > 0$ .

The model predicts a bond length  $R_0 \approx 0.132$  nm and a total molecular energy of  $-1.13$  Ry. The energy needed to separate  $H_2^+$  into  $H + H^+$  (the bond energy) is then  $1.13 \text{ Ry} - 1 \text{ Ry} = 0.13 \text{ Ry} \approx 1.77 \text{ eV}$ . (The experimental value is about  $2.65 \text{ eV}$ ).

What is the energy behavior of the antibonding orbital  $E_-$  as  $R$  decreases from infinity? Does it lead to a stable molecule when proton repulsion is included? [Yes/No] \_\_\_\_\_

Go to Frame 37.

### Frame 37

The energy of the antibonding orbital  $E_-$  (Eq. 11.21) *steadily increases* as  $R$  decreases from the separated atom limit ( $-1 \text{ Ry}$ ). It always lies above the energy of a separated H atom and  $H^+$  ion. Therefore, the antibonding state does **not** lead to a stable  $H_2^+$  molecule; it is repulsive at all separations when nuclear repulsion is included.

Go to Frame 38.

### Frame 38

#### The Hydrogen Molecule $H_2$

Now we add a second electron to  $H_2^+$  to form the neutral hydrogen molecule,  $H_2$ . How does the bond energy of  $H_2$  ( $4.5 \text{ eV}$ ) compare to that of  $H_2^+$  ( $2.65 \text{ eV}$ )? Is the bond stronger or weaker? How does the bond length of  $H_2$  ( $0.074 \text{ nm}$ ) compare to that of  $H_2^+$  ( $0.106 \text{ nm}$ )? Is it shorter or longer?

Go to Frame 39.

### Frame 39

The bond energy of  $H_2$  ( $4.5 \text{ eV}$ ) is **greater** than that of  $H_2^+$  ( $2.65 \text{ eV}$ ), so the  $H_2$  bond is **stronger**. The bond length of  $H_2$  ( $0.074 \text{ nm}$ ) is **shorter** than that of  $H_2^+$  ( $0.106 \text{ nm}$ ).

The second electron adds more "glue" for covalent bonding.

If both electrons in  $H_2$  occupy the bonding molecular orbital (derived from  $\psi_+$ ), what does the Pauli exclusion principle require regarding their spins? [a] Their spins must be parallel. [b] Their spins must be opposite (antiparallel). [c] There is no restriction on their spins.

Go to Frame 40.

### Frame 40

Your answer was [ a — b — c ].

The correct answer is [b]. If both electrons occupy the same spatial bonding orbital, the Pauli exclusion principle requires that their spins must be **opposite** (one spin up, one spin down).

What would happen if their spins were parallel? \_\_\_\_\_

Go to Frame 41.

### Frame 41

If the two electrons in  $H_2$  had parallel spins, the Pauli exclusion principle would force one of them into the higher-energy **antibonding orbital**. Figure 11.19 shows that this configuration (one electron in bonding, one in antibonding, with parallel spins) does *not* lead to a stable  $H_2$  molecule; the total energy is higher than that of two separated H atoms.

Go to Frame 42.

## Frame 42

### Bonding in Complex Molecules

(Optional Section as per original text)

The bonds in  $\text{H}_2^+$  and  $\text{H}_2$  are called **sigma ( $\sigma$ ) bonds**. What atomic orbitals overlap to form these sigma bonds, and what is a key characteristic of their electron density distribution? 1. Overlapping orbitals: \_\_\_\_\_ 2. Electron density characteristic: \_\_\_\_\_

Go to Frame 43.

## Frame 43

Sigma ( $\sigma$ ) bonds, like those in  $\text{H}_2$ , arise from the overlap of atomic **s states** (or other orbitals like  $p_z$  that point along the bond axis). Their electron density is **axially symmetric** about the line joining the two atoms (the bond axis).

Bonding between atoms with more electrons often involves the overlap of  $p$  orbitals, leading to different types of bonds. Consider the nitrogen molecule,  $\text{N}_2$ . The electron configuration of N is  $1s^2 2s^2 2p^3$ . The three  $2p$  valence electrons are primarily responsible for bonding.

Atomic  $p$  orbitals ( $p_x$ ,  $p_y$ ,  $p_z$ ) are lobed and have directional characteristics. When two N atoms approach, one axis (say,  $z$ ) becomes the molecular axis.

- The overlap of the two  $p_z$  orbitals (pointing along the molecular axis) forms what type of bond? [ $\sigma$  —  $\pi$ ]
- The overlap of  $p_x$  orbitals (perpendicular to the molecular axis) forms what type of bond? [ $\sigma$  —  $\pi$ ]

Go to Frame 44.

## Frame 44

- The overlap of the two  $p_z$  orbitals forms a  **$\sigma$  bond** because the electron density is axially symmetric along the bond axis.
- The side-to-side overlap of  $p_x$  orbitals (and similarly  $p_y$  orbitals) forms a **pi ( $\pi$ ) bond**. A  $\pi$  bond has a nodal plane containing the molecular axis. (See Figure 11.20).

In  $\text{N}_2$ , there is one  $\sigma$  bond and two  $\pi$  bonds, resulting in a very stable triple bond with a high bond energy (9.8 eV).

Pi bonds are generally [ stronger — weaker ] than sigma bonds. Why? \_\_\_\_\_  
Go to Frame 45.

## Frame 45

Pi ( $\pi$ ) bonds are generally **weaker** than sigma ( $\sigma$ ) bonds. This is because the side-to-side overlap of  $p$  orbitals in a  $\pi$  bond is less effective (involves less electron overlap between the nuclei) than the end-to-end overlap in a  $\sigma$  bond.

This concludes our adaptation of the provided text on molecular structure and bonding.  
End of Section.

## 27 Statistical Physics

### Frame 1

Thermodynamics deals with macroscopic properties like temperature and pressure. **Statistical physics** (or statistical mechanics) aims to explain these bulk properties based on the motion and interactions of individual atoms and molecules.

While we could, in principle, use Newton's laws or the Schrödinger equation for each particle, the sheer number of particles (e.g.,  $\sim 10^{22}$  atoms/cm<sup>3</sup>) makes this impractical. Thus, we rely on a statistical approach.

In this chapter, we'll explore how a fixed amount of energy is distributed among a large number of particles in thermal equilibrium at a temperature  $T$ . This involves finding the average number of particles with a specific energy, or the probability that a single particle has a certain energy.

Go to Frame 2.

### Frame 2

#### The Maxwell–Boltzmann Distribution

The connection between thermodynamics and atomic properties was established in the late 1800s by physicists like James Clerk Maxwell, Ludwig Boltzmann, and Josiah Willard Gibbs.

Maxwell, building on Clausius's work, derived the functional form of the equilibrium speed distribution for gas molecules. What did Clausius reason about the speeds of gas molecules that contradicted the idea of a single high speed for all molecules? \_\_\_\_\_

Go to Frame 3 for the answer.

### Frame 3

Clausius reasoned that gas molecules do not all travel at a single high speed. Instead, there is a **well-defined distribution of molecular speeds** in a gas, which depends on the gas temperature. Furthermore, molecules collide, leading to zigzag paths rather than straight-line travel over long distances.

Boltzmann extended Maxwell's work. What two main things did he want to achieve regarding the distribution of molecular speeds? 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 4 for the answers.

### Frame 4

Boltzmann aimed to: 1. Establish the properties of the **equilibrium (or most probable) distribution** of molecular speeds. 2. Describe the **evolution in time of a gas towards this Maxwellian distribution** (the approach-to-equilibrium problem).

Gibbs, working more quietly, established statistical mechanics on a rigorous mathematical basis.

Go to Frame 5.

### Frame 5

Let's examine the basic assumptions underlying the Maxwell–Boltzmann (MB) distribution for a system of particles:

1. Particles are identical in physical properties but **distinguishable** (e.g., by position or path). This implies particle size is small compared to average inter-particle distance.

2. The equilibrium distribution is the **most probable** way of distributing particles among allowed energy states, given a fixed total number of particles and fixed total energy.
3. There's no theoretical limit on how many particles can be in a given energy state, but due to low density and high temperature, it's unlikely for **more than one particle** to occupy the same specific state at the same time.

Which of these assumptions primarily relates to the classical nature of the particles? [a] Assumption 1 [b] Assumption 2 [c] Assumption 3

Go to Frame 6 for the answer.

### Frame 6

Your answer was [ a — b — c ].

The correct answer is [a]. Assumption 1, that particles are distinguishable, is a key feature of classical particles. Quantum particles of the same type are inherently indistinguishable.

To make these ideas concrete, consider distributing a total energy of  $8E$  (where  $E$  is an indivisible unit) among six distinguishable particles. Figure 10.1a shows 20 possible ways (arrangements or macrostates) if the particles were indistinguishable. For *distinguishable* particles, each of these 20 arrangements can be formed in multiple ways, called **microstates**.

The number of microstates ( $N_{MB}$ ) for a given arrangement where  $n_1$  particles are in energy state 1,  $n_2$  in state 2, etc., for a total of  $N$  particles, is given by (Eq. 10.1):

$$N_{MB} = \frac{N!}{n_1!n_2!n_3!\dots}$$

For example, if  $N = 6$  particles, and we have an arrangement with 3 particles in state  $0E$  (let's call this  $n_0 = 3$ ), 2 particles in state  $1E$  ( $n_1 = 2$ ), and 1 particle in state  $6E$  ( $n_6 = 1$ ), and 0 particles in other states. How many microstates does this arrangement correspond to?  $N_{MB} = \underline{\hspace{2cm}}$

Go to Frame 7 for the calculation and answer.

### Frame 7

For  $N = 6$ ,  $n_0 = 3$ ,  $n_1 = 2$ ,  $n_6 = 1$  (and all other  $n_i = 0$ ):

$$N_{MB} = \frac{6!}{3!2!1!(0!\dots)} = \frac{720}{(6)(2)(1)(1\dots)} = \frac{720}{12} = 60$$

This matches the number in parentheses for the corresponding diagram in Figure 10.1a (fourth diagram from left, top row).

The basic postulate of statistical mechanics is that **any individual microstate is as likely as any other individual microstate**. If there are a total of  $M_{total}$  microstates for a given total energy and particle number, the probability of any specific microstate is  $1/M_{total}$ . The probability of a particular *arrangement* (macrostate) is then  $N_{MB}/M_{total}$ .

Go to Frame 8.

### Frame 8

To find the average number of particles  $\bar{n}_j$  with a particular energy  $E_j$ , we sum the number of particles with that energy in each arrangement, weighted by the probability of that arrangement (Eq. 10.2):

$$\bar{n}_j = n_{j1}p_1 + n_{j2}p_2 + \dots$$



where  $n_{j1}$  is the number of particles with energy  $E_j$  in arrangement 1, and  $p_1$  is the probability of arrangement 1.

For the system of 6 particles with total energy  $8E$  (Figure 10.1a), the text states there are a total of 1287 microstates. The calculation for  $\bar{n}_0$  (average number of particles with energy  $0E$ ) yields  $\bar{n}_0 = 2.307$ .

The probability  $p(E_j)$  of finding a randomly chosen particle with energy  $E_j$  is  $\bar{n}_j/N$ . So,  $p(0E) = \bar{n}_0/6 = 2.307/6 = 0.385$ . The text provides a list of probabilities for other energies (Figure 10.2). What is the general trend of  $p(E)$  as  $E$  increases? [a] Increases [b] Decreases [c] Stays roughly constant

Go to Frame 9.

### Frame 9

Your answer was [ a — b — c ].

The correct answer is [b]. The probability  $p(E)$  of finding a particle with energy  $E$  (and thus the average number of particles  $\bar{n}_E$ ) **decreases** approximately exponentially as energy  $E$  increases. This means higher energy states are less likely to be occupied.

For a large number of particles in thermal equilibrium at temperature  $T$ , this distribution takes a specific exponential form, the Maxwell-Boltzmann probability  $f_{MB}$ :

$$f_{MB}(E_i) = Ae^{-E_i/k_B T}$$

where  $E_i$  is the energy of a state,  $k_B$  is Boltzmann's constant, and  $A$  is a normalization constant.

If  $g_i$  is the number of states with the same energy  $E_i$  (called degeneracy or statistical weight), then the number of particles  $n_i$  with energy  $E_i$  is:

$$n_i = g_i f_{MB}(E_i) = g_i A e^{-E_i/k_B T}$$

Go to Frame 10.

### Frame 10

When energy states are very numerous and closely spaced, we can use continuous functions. The density of states  $g(E)$  is the number of energy states per unit volume in the energy interval  $dE$ . The number of particles per unit volume  $n(E)dE$  with energies between  $E$  and  $E + dE$  is:

$$n(E)dE = g(E)f_{MB}(E)dE = g(E)Ae^{-E/k_B T}dE$$

The normalization constant  $A$  is determined by requiring that the total number of particles  $N$  (or total number per unit volume  $N/V$ ) is obtained by summing (or integrating)  $n_i$  (or  $n(E)dE$ ) over all possible energies.

$$\frac{N}{V} = \int_0^\infty n(E)dE = \int_0^\infty g(E)Ae^{-E/k_B T}dE$$

Go to Frame 11.

### Frame 11

#### The Maxwell Speed Distribution for Gas Molecules in Thermal Equilibrium at Temperature T

Maxwell derived a famous formula for  $n(v)dv$ , the number of gas molecules per unit volume with speeds between  $v$  and  $v + dv$ . For an ideal gas of point particles, the energy is purely translational kinetic energy:  $E = \frac{1}{2}mv^2$ .

To find  $g(E)dE$ , we first find the number of states  $f(v)dv$  with speeds between  $v$  and  $v+dv$ . This is proportional to the volume of a spherical shell in "velocity space" (a 3D space with axes  $v_x, v_y, v_z$ ). The volume of a spherical shell of radius  $v$  and thickness  $dv$  is  $4\pi v^2 dv$ . So,  $f(v)dv = C \cdot 4\pi v^2 dv$ , where  $C$  is a constant. Since  $E = \frac{1}{2}mv^2$ , states with speeds between  $v$  and  $v+dv$  correspond to energies between  $E$  and  $E+dE$ . Thus,  $g(E)dE = f(v)dv$ .

Substituting into  $n(E)dE = g(E)f_{MB}(E)dE$  and converting variables from  $E$  to  $v$  (since  $dE = mv dv$ ), we get:

$$n(v)dv = A' \cdot 4\pi v^2 e^{-mv^2/2k_B T} dv$$

where  $A'$  is a new normalization constant. After normalization, Maxwell's speed distribution is (Eq. 10.8):

$$n(v)dv = \frac{N}{V} \left( \frac{m}{2\pi k_B T} \right)^{3/2} 4\pi v^2 e^{-mv^2/2k_B T} dv$$

(See Figure 10.4 for a sketch).

What does the  $v^2$  term cause the distribution  $n(v)$  to do as  $v \rightarrow 0$ ? What does the  $e^{-mv^2/2k_B T}$  term cause  $n(v)$  to do as  $v \rightarrow \infty$ ? \_\_\_\_\_

Go to Frame 12.

### Frame 12

As  $v \rightarrow 0$ , the  $v^2$  term causes  $n(v) \rightarrow 0$ . (Few molecules have very low speeds). As  $v \rightarrow \infty$ , the exponential term  $e^{-mv^2/2k_B T}$  causes  $n(v) \rightarrow 0$  very rapidly. (Few molecules have very high speeds). This results in a distribution peaked at some "most probable speed"  $v_{mp}$ .

The average speed  $\bar{v}$  is found by:

$$\bar{v} = \frac{\int_0^\infty v \cdot n(v)dv}{\int_0^\infty n(v)dv} = \frac{1}{N/V} \int_0^\infty v \cdot n(v)dv$$

The result is (Eq. 10.12):

$$\bar{v} = \sqrt{\frac{8k_B T}{\pi m}}$$

How does the average speed  $\bar{v}$  depend on temperature  $T$  and molecular mass  $m$ ?  $\bar{v}$  is proportional to \_\_\_\_\_ and inversely proportional to \_\_\_\_\_.

Go to Frame 13.

### Frame 13

The average speed  $\bar{v}$  is proportional to  $\sqrt{T}$  (increases with temperature) and inversely proportional to  $\sqrt{m}$  (lighter molecules move faster on average at the same T).

The root-mean-square speed  $v_{rms}$  is defined as  $v_{rms} = \sqrt{\bar{v}^2}$ . It is found to be (Eq. 10.14):

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

$v_{rms}$  is slightly larger than  $\bar{v}$ .

Go to Frame 14.

### Frame 14

#### The Equipartition of Energy

The average translational kinetic energy of a molecule is  $\overline{K} = \frac{1}{2}m\overline{v^2} = \frac{1}{2}mv_{rms}^2$ . Using  $v_{rms} = \sqrt{3k_B T/m}$ , what is  $\overline{K}$ ?  $\overline{K} =$  \_\_\_\_\_  
 Go to Frame 15.

### Frame 15

$$\overline{K} = \frac{1}{2}m \left( \frac{3k_B T}{m} \right) = \frac{3}{2}k_B T$$

This result is consistent with the **equipartition theorem** (or equipartition of energy). It states: A classical molecule in thermal equilibrium at temperature  $T$  has an average energy of  $\frac{1}{2}k_B T$  for each independent **degree of freedom** (mode of motion).

For translational motion in 3D, there are 3 degrees of freedom (motion along x, y, z axes). So, the average kinetic energy for motion along the x-axis is  $\frac{1}{2}m\overline{v_x^2} = \frac{1}{2}k_B T$ . Similarly for y and z. The total average translational kinetic energy is  $3 \times (\frac{1}{2}k_B T) = \frac{3}{2}k_B T$ .

Degrees of freedom are associated with each variable that appears *squared* in the energy formula. A 1D harmonic oscillator has energy  $E = \frac{1}{2}mv_x^2 + \frac{1}{2}Kx^2$ . How many degrees of freedom does it have? \_\_\_\_\_ What is its average total energy at temperature  $T$ ? \_\_\_\_\_

Go to Frame 16.

### Frame 16

A 1D harmonic oscillator has **2 degrees of freedom**:

1. One for kinetic energy ( $\frac{1}{2}mv_x^2$ , associated with  $v_x^2$ )
2. One for potential energy ( $\frac{1}{2}Kx^2$ , associated with  $x^2$ )

Each degree of freedom contributes  $\frac{1}{2}k_B T$  to the average energy. So, the average total energy of a classical 1D harmonic oscillator at temperature  $T$  is  $\frac{1}{2}k_B T + \frac{1}{2}k_B T = k_B T$ .

This concludes the section on the Maxwell-Boltzmann distribution.

End of Section.

## 28 Statistical Physics: Maxwell-Boltzmann Statistics

### Frame 1

This section delves into the world of statistical physics, focusing on the conditions under which the classical Maxwell-Boltzmann (MB) distribution is applicable. We'll also touch upon quantum statistics.

Before we dive into the main topic, the original text presents two exercises. We won't work through them in detail here, but they relate to:

- Exercise 1: Deriving the energy distribution  $n(E)dE$  for an ideal gas and showing its relation to the equipartition theorem for total energy per unit volume.
- Exercise 2: Confirming Maxwell's result about the "spread-outness" of speed distribution, specifically by showing the standard deviation  $\sigma_v$  is proportional to  $\sqrt{T}$ .

These exercises build upon foundational concepts in statistical mechanics.

Our main goal now is to understand when classical statistics (Maxwell-Boltzmann) can be used and when we need to consider quantum effects.

Go to Frame 2.

## Frame 2

The Maxwell-Boltzmann (MB) distribution for classical particles was derived with certain assumptions. One key assumption is that particles are **distinguishable**.

However, quantum mechanics tells us that particles, especially when close together, exhibit wave-like behavior. What happens to the distinguishability of particles when their wavefunctions overlap? [a] They become more distinguishable. [b] Their distinguishability is unaffected. [c] They become indistinguishable.

Choose an answer and go to Frame 3.

## Frame 3

Your answer was [ a — b — c ].

The correct answer is [c]. When particles are close enough for their wavefunctions to overlap, they become **indistinguishable**. You can't tell one identical particle from another in this situation. (This relates to concepts like Exchange Symmetry and the Exclusion Principle, see Section 9.4 of the original text).

If swapping particle A for particle B no longer counts as a new configuration (because they are indistinguishable), what does this imply for the classical MB distribution? [a] The MB distribution remains perfectly valid. [b] The MB distribution might need to be replaced by a quantum distribution. [c] The MB distribution becomes even more accurate.

Go to Frame 4.

## Frame 4

Your answer was [ a — b — c ].

The correct answer is [b]. If particles are indistinguishable, the number of ways a given energy distribution can be realized changes. This means the equilibrium (most probable) distribution also changes. Thus, the classical Maxwell-Boltzmann distribution must be replaced by a **quantum distribution** when wavefunction overlap is significant or particle concentration is high.

However, the MB distribution is often a good approximation. Under what common conditions is the MB distribution still a valid approximation to the correct quantum distribution? (Hint: Think about typical gases). \_\_\_\_\_

Go to Frame 5.

## Frame 5

The MB distribution is a valid approximation in the common case of **gases at ordinary conditions** (ordinary temperatures and pressures). In these situations, the particle concentration is relatively low, and wavefunction overlap is minimal.

When are quantum statistics absolutely required? List two examples given in the text (page 344). 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 6.

## Frame 6

Quantum statistics are required for cases involving: 1. High particle concentrations, such as **electrons in a metal**. (The density of conduction electrons in a metal is thousands of times higher than gas molecules at STP). 2. **Photons in a blackbody cavity**.

Go to Frame 7.

### Frame 7

To determine more precisely when the classical MB distribution is valid, we need a criterion. We can say MB distribution is valid when the **average distance between particles**,  $d$ , is large compared with the **quantum uncertainty in a particle's position**,  $\Delta x$ .

Mathematically, this condition is:

$$\Delta x \ll d \quad (10.15)$$

How can we estimate  $\Delta x$ ? Which fundamental principle of quantum mechanics relates uncertainty in position to uncertainty in momentum? \_\_\_\_\_

Go to Frame 8.

### Frame 8

We use the **Heisenberg Uncertainty Principle** to find  $\Delta x$ . The principle states (in one dimension)  $\Delta x \Delta p_x \gtrsim \hbar/2$ , where  $\hbar$  is the reduced Planck constant. For our estimation, we can use  $\Delta x \Delta p_x \approx \hbar$ .

For a particle of mass  $m$  in thermal equilibrium at temperature  $T$ , the average kinetic energy associated with motion in the x-direction is given by the equipartition theorem as  $\langle E_{kx} \rangle = \frac{1}{2} k_B T$ . Since  $E_{kx} = p_x^2/(2m)$ , we can say that on average,  $p_x^2/(2m) \approx k_B T/2$ . Thus, a typical magnitude for  $p_x$  is  $p_x \approx \sqrt{mk_B T}$ . We can use this as an estimate for the uncertainty  $\Delta p_x$ .

So,  $\Delta p_x \approx \sqrt{mk_B T}$ . (Equation 10.16)

Using  $\Delta x \approx \hbar/\Delta p_x$ , substitute the expression for  $\Delta p_x$  to find an estimate for  $\Delta x$ .  $\Delta x \approx$  \_\_\_\_\_

Go to Frame 9.

### Frame 9

Substituting  $\Delta p_x \approx \sqrt{mk_B T}$  into  $\Delta x \approx \hbar/\Delta p_x$ , we get:

$$\Delta x \approx \frac{\hbar}{\sqrt{mk_B T}} \quad (10.17)$$

(The text uses  $h$  instead of  $\hbar$ , so it would be  $\Delta x \approx h/(2\sqrt{mk_B T})$  if using  $h$ , or  $\Delta x \approx h/\sqrt{mk_B T}$  if ignoring factors of 2 and  $\pi$  in the estimation). Let's stick with the text's approach on page 345, which leads to  $\Delta x \approx \frac{h}{2\sqrt{mk_B T}}$ .

The average distance  $d$  between particles can be estimated if we know the number of particles  $N$  in a volume  $V$ . Each particle occupies an average volume of  $V/N$ . If we imagine this volume as a cube of side  $d$ , then  $d^3 = V/N$ , so  $d = (V/N)^{1/3}$ .

Now, substitute these expressions for  $\Delta x$  and  $d$  into the criterion  $\Delta x \ll d$ . \_\_\_\_\_  $\ll$  \_\_\_\_\_

Go to Frame 10.

### Frame 10

Substituting into  $\Delta x \ll d$ :

$$\frac{h}{2\sqrt{mk_B T}} \ll \left( \frac{V}{N} \right)^{1/3}$$

To simplify this, let's isolate the particle concentration  $N/V$ . Rearrange the inequality. (Hint: you might want to cube both sides at some point). The result is shown in Equation 10.18. Try to derive it.

Go to Frame 11.

### Frame 11

Rearranging the inequality:

$$\frac{h}{2\sqrt{mk_B T}} \ll \left(\frac{V}{N}\right)^{1/3}$$

Cube both sides:

$$\left(\frac{h}{2\sqrt{mk_B T}}\right)^3 \ll \frac{V}{N}$$

$$\frac{h^3}{8(mk_B T)^{3/2}} \ll \frac{V}{N}$$

Now, invert and rearrange to get  $N/V$  on one side:

$$\frac{N}{V} \ll \frac{8(mk_B T)^{3/2}}{h^3}$$

Or, often written as:

$$\frac{N}{V} \frac{h^3}{8(mk_B T)^{3/2}} \ll 1 \quad \text{or simply} \quad \frac{N}{V} \left(\frac{h^2}{2\pi mk_B T}\right)^{3/2} \ll 1 \quad (\text{using } \hbar)$$

The text's Equation 10.18 is:

$$\left(\frac{N}{V}\right) \frac{h^3}{(8(mk_B T))^{3/2}} \ll 1$$

This equation tells us that the Maxwell-Boltzmann distribution holds for:

- **Low** particle concentration ( $N/V$ )
- **High** particle mass ( $m$ )
- **High** temperature ( $T$ )

Do these conditions make intuitive sense for when classical behavior (distinguishable particles, minimal wave effects) should dominate? [ Yes — No ] Explain briefly. \_\_\_\_\_

Go to Frame 12.

### Frame 12

Yes, these conditions make intuitive sense.

- **Low concentration** ( $N/V$ ): Particles are far apart, so their wavefunctions are less likely to overlap, making them more distinguishable.
- **High mass** ( $m$ ): Heavier particles have smaller de Broglie wavelengths ( $\lambda = h/p$ ). Smaller wavelengths mean less "fuzziness" or wave-like spread, reducing overlap.
- **High temperature** ( $T$ ): Higher temperature means higher average momentum ( $p \approx \sqrt{mk_B T}$ ). Again, this leads to smaller de Broglie wavelengths, reducing quantum overlap effects.

The text then provides Example 10.2 to apply this criterion.  
Go to Frame 13.

### Frame 13

#### Example 10.2: When Can We Use Maxwell-Boltzmann Statistics?

(a) Are MB statistics valid for hydrogen gas ( $H_2$ ) at Standard Temperature and Pressure (STP: 273 K, 1 atm)?

Given:

- 1 mole of  $H_2$  ( $6.02 \times 10^{23}$  molecules) occupies  $V = 22.4 \times 10^{-3} \text{ m}^3$ .
- Mass of  $H_2$  molecule  $m_{H_2} \approx 3.34 \times 10^{-27} \text{ kg}$ .
- $k_B T$  at 273 K is  $3.77 \times 10^{-21} \text{ J}$ .
- $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$  (Planck's constant).

Calculate the value of the left side of Equation 10.18:  $\left(\frac{N}{V}\right) \frac{h^3}{(8(mk_B T))^{3/2}}$ . Is this value  $\ll 1$ ? \_\_\_\_\_

Go to Frame 14 for the solution to part (a).

### Frame 14

For  $H_2$  at STP:

$$\frac{N}{V} = \frac{6.02 \times 10^{23} \text{ molecules}}{22.4 \times 10^{-3} \text{ m}^3} \approx 2.69 \times 10^{25} \text{ m}^{-3}$$

The denominator term:  $8(mk_B T) = 8 \times (3.34 \times 10^{-27} \text{ kg}) \times (3.77 \times 10^{-21} \text{ J}) \approx 1.008 \times 10^{-46} \text{ J}\cdot\text{kg}$   
 $(8(mk_B T))^{3/2} \approx (1.008 \times 10^{-46})^{3/2} \approx 1.012 \times 10^{-69} (\text{J}\cdot\text{kg})^{3/2}$   
 $h^3 = (6.626 \times 10^{-34})^3 \approx 2.90 \times 10^{-100} (\text{J}\cdot\text{s})^3$

So, the expression is:

$$(2.69 \times 10^{25}) \frac{2.90 \times 10^{-100}}{1.012 \times 10^{-69}} \approx (2.69 \times 10^{25}) \times (2.866 \times 10^{-31}) \approx 7.71 \times 10^{-6}$$

(The text calculates this as  $8.83 \times 10^{-8}$  using  $1.055 \times 10^{-34} \text{ J}\cdot\text{s}$  for  $h/(2\pi)$  which appears to be a mix-up in application, as  $h^3$  is needed. Using  $h \approx 6.626 \times 10^{-34}$  and  $(mk_B T)^{3/2}$  and  $8^{3/2}$  as separate factors in the denominator leads to results closer to the text's if their  $h^3/(8(mk_B T)^{3/2})$  simplifies differently. Let's follow the text's final numerical answer for its expression format.)

The text finds  $N/V \times (\text{term}) \approx (6.02 \times 10^{23}/22.4 \times 10^{-3}) \times \frac{(1.055 \times 10^{-34})^3}{[(3.34 \times 10^{-27})(3.77 \times 10^{-21})]^{3/2}}$  (this appears to be using  $\hbar^3$  and missing the factor of 8 in the denominator of Eq 10.18 or combining it) leading to their answer of  $8.83 \times 10^{-8}$ .

Regardless of the exact numerical path, the result is a very small number (e.g.,  $\approx 10^{-6}$  or  $10^{-8}$ ). Is this value  $\ll 1$ ? [ Yes — No ] What does this imply about MB statistics for  $H_2$  at STP? \_\_\_\_\_

Go to Frame 15.

### Frame 15

Yes,  $7.71 \times 10^{-6}$  (or the text's  $8.83 \times 10^{-8}$ ) is much less than 1. This implies that Maxwell-Boltzmann statistics are **valid** for hydrogen gas at STP, even though it's the lightest gas.

(b) Are MB statistics valid for conduction electrons in silver at 300 K?

Given:

- Silver density:  $10.5 \text{ g/cm}^3$
- Molar weight of silver:  $107.9 \text{ g/mol}$
- Assume one free electron per silver atom.
- Avogadro's number:  $6.02 \times 10^{23} \text{ atoms/mol}$  (or electrons/mol here)
- Electron mass  $m_e = 9.109 \times 10^{-31} \text{ kg}$ .
- $k_B T$  at 300 K is  $4.14 \times 10^{-21} \text{ J}$ .

First, calculate the number density of free electrons ( $N/V$ ) in silver.  $N/V = \text{_____}$  electrons/ $\text{m}^3$ .  
Go to Frame 16.

### Frame 16

Calculating  $N/V$  for electrons in silver: Density =  $10.5 \text{ g/cm}^3 = 10.5 \times 10^6 \text{ g/m}^3$ . Number of moles per  $\text{m}^3 = (10.5 \times 10^6 \text{ g/m}^3)/(107.9 \text{ g/mol}) \approx 9.73 \times 10^4 \text{ mol/m}^3$ . Number of electrons per  $\text{m}^3$  ( $N/V$ ) =  $(9.73 \times 10^4 \text{ mol/m}^3) \times (6.02 \times 10^{23} \text{ electrons/mol})$

$$\frac{N}{V} \approx 5.86 \times 10^{28} \text{ electrons/m}^3$$

Now, calculate the criterion term  $\left(\frac{N}{V}\right) \frac{h^3}{(8(m_e k_B T))^{3/2}}$  for these electrons. \_\_\_\_\_  
Go to Frame 17 for the result.

### Frame 17

Using  $m_e = 9.109 \times 10^{-31} \text{ kg}$  and  $k_B T = 4.14 \times 10^{-21} \text{ J}$ :  $8(m_e k_B T) = 8 \times (9.109 \times 10^{-31}) \times (4.14 \times 10^{-21}) \approx 3.01 \times 10^{-50}$   $(8(m_e k_B T))^{3/2} \approx (3.01 \times 10^{-50})^{3/2} \approx 1.65 \times 10^{-75}$  The expression is:

$$\begin{aligned} (5.86 \times 10^{28}) \frac{(6.626 \times 10^{-34})^3}{(1.65 \times 10^{-75})} &\approx (5.86 \times 10^{28}) \frac{2.90 \times 10^{-100}}{1.65 \times 10^{-75}} \\ &\approx (5.86 \times 10^{28}) \times (1.758 \times 10^{-25}) \approx 10.3 \times 10^3 = 1.03 \times 10^4 \end{aligned}$$

The text calculates this value as 4.64. The discrepancy again might be due to using  $h$  vs  $\hbar$  and how factors of  $2\pi$  or 8 are incorporated in the simplified form. The critical point is the order of magnitude and whether it's  $\ll 1$ .

Is the value 4.64 (or  $1.03 \times 10^4$ )  $\ll 1$ ? [ Yes — No ] What does this imply about MB statistics for electrons in silver at 300 K? What kind of statistics should be used instead? \_\_\_\_\_  
Go to Frame 18.

### Frame 18

No, 4.64 (or  $1.03 \times 10^4$ ) is **not** much less than 1. It is greater than 1. This implies that the Maxwell-Boltzmann distribution **does not hold** for conduction electrons in silver at 300 K. This is due to the small mass of the electron and the very high free electron density.

The text states that the correct quantum distribution for electrons (which are fermions) is the **Fermi-Dirac distribution**.

This example highlights that the classical approximation breaks down when quantum effects (indistinguishability, wave nature) become dominant, which is common for electrons in metals due to their high density and small mass.



The chapter then moves on to discuss Quantum Statistics (Section 10.3), which we will briefly touch upon.

Go to Frame 19.

## Frame 19

### 10.3 Quantum Statistics

We've seen that if particle wavefunctions overlap, particles become indistinguishable. This forces the system wavefunction to have a specific symmetry under particle exchange. What are the two types of symmetry mentioned for system wavefunctions when particles are exchanged? (See page 346, under "Wavefunctions and the Bose-Einstein Condensation...") 1. \_ (for bosons) 2. \_ (for fermions)

Go to Frame 20.

## Frame 20

When individual particle wavefunctions overlap, the system wavefunction must be: 1. **Symmetric** (even) under particle exchange for **Bosons**. 2. **Antisymmetric** (odd) under particle exchange for **Fermions**.

Consider a simple system of two independent particles (1 and 2 at positions  $\mathbf{r}_1, \mathbf{r}_2$ ) and two quantum states ( $a, b$ ). If the particles were distinguishable, two possible system wavefunctions are:  $\Psi_A = \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)$  (Particle 1 in state  $a$ , Particle 2 in state  $b$ )  $\Psi_B = \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)$  (Particle 1 in state  $b$ , Particle 2 in state  $a$ )

How is the wavefunction for **Bosons** ( $\Psi_B$ ) constructed from  $\Psi_A$  and  $\Psi_B$ ? (It must be symmetric).  $\Psi_B = \frac{1}{\sqrt{2}}[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) + \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)]$  How is the wavefunction for **Fermions** ( $\Psi_F$ ) constructed? (It must be antisymmetric).  $\Psi_F = \frac{1}{\sqrt{2}}[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) - \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)]$  (Hint: page 346 provides these, including normalization constants).

Go to Frame 21.

## Frame 21

For indistinguishable particles: The symmetric wavefunction for Bosons is:

$$\Psi_B = \frac{1}{\sqrt{2}}[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) + \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)]$$

(Adding them makes it symmetric upon exchange of particle labels 1 and 2).

The antisymmetric wavefunction for Fermions is:

$$\Psi_F = \frac{1}{\sqrt{2}}[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) - \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)]$$

(Subtracting them makes it antisymmetric:  $\Psi_F(\mathbf{r}_2, \mathbf{r}_1) = -\Psi_F(\mathbf{r}_1, \mathbf{r}_2)$ ). The  $1/\sqrt{2}$  is a normalization constant.

Now, what happens if both particles try to occupy the \*same\* state, say state  $a$  (so  $b = a$ )? What does  $\Psi_B$  become? What does  $\Psi_F$  become? \_\_\_\_\_

Go to Frame 22.

## Frame 22

If both particles are in state  $a$  (so we replace  $b$  with  $a$ ): For Bosons:

$$\Psi_B = \frac{1}{\sqrt{2}}[\psi_a(\mathbf{r}_1)\psi_a(\mathbf{r}_2) + \psi_a(\mathbf{r}_2)\psi_a(\mathbf{r}_1)] = \frac{2}{\sqrt{2}}\psi_a(\mathbf{r}_1)\psi_a(\mathbf{r}_2) = \sqrt{2}\psi_a(\mathbf{r}_1)\psi_a(\mathbf{r}_2)$$

The probability density  $|\Psi_B|^2 = 2|\psi_a(\mathbf{r}_1)|^2|\psi_a(\mathbf{r}_2)|^2$ . The text (page 346) notes that this is twice the probability compared to distinguishable particles occupying the same state ( $|\Psi_{MB}|^2$  where  $\Psi_{MB} = \psi_a(\mathbf{r}_1)\psi_a(\mathbf{r}_2)$ ). This enhanced probability for bosons to occupy the same state is a purely quantum effect, as if there's an attractive "force" (though there isn't a real physical one). This leads to **Bose-Einstein Condensation (BEC)** at low temperatures, where many bosons can occupy the single ground state.

For Fermions:

$$\Psi_F = \frac{1}{\sqrt{2}}[\psi_a(\mathbf{r}_1)\psi_a(\mathbf{r}_2) - \psi_a(\mathbf{r}_2)\psi_a(\mathbf{r}_1)] = 0$$

What fundamental principle does this result for fermions demonstrate? \_\_\_\_\_

Go to Frame 23.

### Frame 23

The result  $\Psi_F = 0$  when two fermions try to occupy the same quantum state demonstrates the **Pauli Exclusion Principle**: No two identical fermions can occupy the same quantum state simultaneously.

These different behaviors lead to two distinct quantum statistical distributions:

- **Bose-Einstein distribution** for bosons (particles with integral spin, e.g., photons, alpha particles). No limit on particles per state.
- **Fermi-Dirac distribution** for fermions (particles with half-integral spin, e.g., electrons, protons, neutrons). Only one particle per quantum state.

The text then gives an example (page 347) of calculating the average number of particles in the zero-energy level for a simple system using Bose-Einstein statistics, showing it to be higher than what MB statistics would predict if particles were distinguishable but could still pile up.

This concludes our adapted overview of this section. The original text provides more depth, especially regarding the specific distribution formulas and their consequences.

End of Section.

## 29 Quantum Statistics and Applications

### Frame 1

In the previous section, we established that classical Maxwell-Boltzmann statistics, which assume distinguishable particles, break down when quantum effects become significant. This typically occurs at high particle densities or low temperatures/masses, where particle wavefunctions overlap, making them indistinguishable.

This indistinguishability leads to different statistical behaviors depending on the type of particle. What are the two main categories of quantum particles based on their spin and the symmetry of their system wavefunctions? 1. Particles with integral spin are called \_\_\_\_\_. Their system wavefunction is \_\_\_\_\_. 2. Particles with half-integral spin are called \_\_\_\_\_. Their system wavefunction is \_\_\_\_\_.

Go to Frame 2 for the answers.

### Frame 2

The two main categories are: 1. Particles with integral spin are called **Bosons**. Their system wavefunction is **symmetric** under particle exchange. 2. Particles with half-integral spin are called **Fermions**. Their system wavefunction is **antisymmetric** under particle exchange.

We saw that this leads to:

- Bosons having an increased probability of occupying the same quantum state (leading to Bose-Einstein Condensation or BEC at low temperatures).
- Fermions having zero probability of occupying the same quantum state (Pauli Exclusion Principle).

These distinct behaviors give rise to two fundamental quantum distribution functions. What are they called? \_\_\_\_\_ and \_\_\_\_\_.

Go to Frame 3.

### Frame 3

The two fundamental quantum distribution functions are: 1. The **Bose-Einstein (BE) distribution** (for bosons). 2. The **Fermi-Dirac (FD) distribution** (for fermions).

To obtain the Bose-Einstein distribution, what assumption from Maxwell-Boltzmann statistics regarding the number of particles per state is retained? (See page 347, "Bose-Einstein and Fermi-Dirac Distributions"). \_\_\_\_\_

To obtain the Fermi-Dirac distribution, what key stipulation is made about particle occupancy in a given quantum state? \_\_\_\_\_

Go to Frame 4.

### Frame 4

For the Bose-Einstein distribution: The MB assumption of **no theoretical limit on the number of particles per state** is retained. Examples of bosons include alpha particles (spin 0), photons (spin 1), and deuterons (spin 1).

For the Fermi-Dirac distribution: It is stipulated that **only one particle can occupy a given quantum state**. (If states can have two spin orientations, like for electrons, then two electrons can occupy that energy level, one spin up, one spin down, but these are distinct quantum states). Examples of fermions include electrons, protons, and neutrons (all spin 1/2).

The text (page 347-348) then revisits a simple system of six indistinguishable particles with total energy 8E to illustrate how these constraints affect the average number of particles in energy levels, comparing BE and FD statistics to the previous MB case.

- For BE statistics (Fig 10.6), the distribution tends to have more particles in the lowest energy levels compared to MB.
- For FD statistics (Fig 10.7), with a constraint (e.g., no more than two particles per energy state), the distribution shows a distinct "leveling off" at low energies.

Go to Frame 5.

### Frame 5

When dealing with a large number of quantum particles, continuous distribution functions can be derived. The probability of finding a particle in a particular state of energy  $E$  at temperature  $T$  is given by  $f(E)$ .

For Bose-Einstein statistics, the distribution function is (Eq. 10.19):

$$f_{BE}(E) = \frac{1}{Be^{E/k_BT} - 1}$$

For Fermi-Dirac statistics, the distribution function is (Eq. 10.20):

$$f_{FD}(E) = \frac{1}{He^{E/k_BT} + 1}$$

The parameters  $B$  and  $H$  depend on system temperature and particle density.

The number of particles per unit volume with energy between  $E$  and  $E + dE$  is  $n(E)dE = g(E)f(E)dE$ , where  $g(E)$  is the density of states (number of available quantum states per unit energy per unit volume).

For a system of bosons whose number is \*not\* fixed with temperature (like photons in a black-body cavity or phonons in a solid), what does the parameter  $B$  in  $f_{BE}(E)$  become? (See page 349).

$B =$  \_\_\_\_\_

Go to Frame 6.

### Frame 6

For bosons whose number is not fixed with temperature (e.g., photons, phonons), the parameter  $B = 1$ . So, for photons or phonons:

$$f(E) = \frac{1}{e^{E/k_BT} - 1}$$

For the Fermi-Dirac distribution, the parameter  $H$  is often written in an explicitly temperature-dependent form as  $H = e^{-E_F/k_BT}$ , where  $E_F$  is a crucial energy level. What is  $E_F$  called?  $E_F =$  \_\_\_\_\_

Go to Frame 7.

### Frame 7

$E_F$  is called the **Fermi energy**.

With this substitution, the Fermi-Dirac distribution function becomes (Eq. 10.25):

$$f_{FD}(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1}$$

This is a very important formula.

Consider the probability of finding an electron with an energy exactly equal to the Fermi energy ( $E = E_F$ ). What is the value of  $f_{FD}(E_F)$  at any temperature  $T$ ? (Substitute  $E = E_F$  into the equation above).  $f_{FD}(E_F) =$  \_\_\_\_\_

Go to Frame 8.

### Frame 8

If  $E = E_F$ , then  $E - E_F = 0$ . So,  $e^{(E-E_F)/k_BT} = e^0 = 1$ . Therefore,

$$f_{FD}(E_F) = \frac{1}{1+1} = \frac{1}{2}$$

This gives a key interpretation of the Fermi energy: The Fermi energy  $E_F$  is the energy level at which the probability of occupation by an electron is exactly 1/2, at any temperature.

Figure 10.8 (page 350) shows a comparison of MB, BE, and FD distributions at 5000 K. At  $T = 0$  K, what is the value of  $f_{FD}(E)$  for  $E < E_F$ ? And for  $E > E_F$ ? (See Figure 10.11a). For  $E < E_F$ ,  $f_{FD}(E) =$  \_\_\_\_\_ For  $E > E_F$ ,  $f_{FD}(E) =$  \_\_\_\_\_

Go to Frame 9.

### Frame 9

At  $T = 0$  K: For  $E < E_F$ ,  $f_{FD}(E) = 1$  (all states below  $E_F$  are completely filled). For  $E > E_F$ ,  $f_{FD}(E) = 0$  (all states above  $E_F$  are completely empty).

This means that at absolute zero, electrons fill up all available energy states from the lowest energy up to the Fermi energy  $E_F$ . Electrons at the Fermi level possess significant kinetic energy and speed ( $v_F$ , the Fermi speed, related by  $\frac{1}{2}m_e v_F^2 = E_F$ ), even at  $T = 0$  K. This is a purely quantum mechanical effect due to the Pauli Exclusion Principle.

As temperature  $T$  increases above 0 K (Figure 10.11b), what happens to the  $f_{FD}(E)$  distribution around  $E_F$ ? \_\_\_\_\_

Go to Frame 10.

### Frame 10

As  $T$  increases, the Fermi-Dirac distribution "rounds off" slightly near  $E_F$ . States just below  $E_F$  (within about  $k_B T$  of  $E_F$ ) lose some population (electrons are excited to higher energies). States just above  $E_F$  (within about  $k_B T$  of  $E_F$ ) gain some population. For energies far from  $E_F$  (i.e.,  $|E - E_F| \gg k_B T$ ), the distribution remains close to 1 (for  $E \ll E_F$ ) or 0 (for  $E \gg E_F$ ).

The text now discusses applications. **10.4 Applications of Bose-Einstein Statistics** One key application is **Blackbody Radiation**. Here, electromagnetic radiation in an enclosure is treated as a gas of photons. Are photons bosons or fermions? \_\_\_\_\_ What is their spin? \_\_\_\_\_

Go to Frame 11.

### Frame 11

Photons are **bosons** and have spin 1.

The number of photons per unit volume with energy between  $E$  and  $E + dE$  is  $n(E)dE = g(E)f_{BE}(E)dE$ . The energy density (energy per unit volume) in this range is  $u(E)dE = E \cdot n(E)dE$ . Since photons are bosons whose number is not fixed,  $f_{BE}(E) = 1/(e^{E/k_B T} - 1)$ . So,

$$u(E)dE = \frac{g(E)E dE}{e^{E/k_B T} - 1} \quad (10.26)$$

The text mentions that the density of states  $g(E)$  for photons in an enclosure is (Eq. 10.27):

$$g(E) = \frac{8\pi E^2}{(hc)^3}$$

(This is derived from considering the number of standing wave modes in a cavity). Substituting this  $g(E)$  into  $u(E)dE$  would yield Planck's radiation law.

Go to Frame 12.

### Frame 12

Another application of BE statistics (not covered in detail in this excerpt, but Example 10.4 touches on it) is related to **phonons** in a solid (quantized lattice vibrations), which also behave as bosons. This is key to understanding the specific heat of solids at low temperatures.

**10.5 An Application of Fermi-Dirac Statistics: The Free Electron Gas Theory of Metals** In metals, the outer conduction electrons are weakly bound and can be treated as a gas of fermions trapped within the metal. The number of electrons per unit volume with energy between  $E$  and  $E + dE$  is:

$$n(E)dE = g(E)f_{FD}(E)dE \quad (10.22)$$

where  $f_{FD}(E) = 1/(e^{(E-E_F)/k_B T} + 1)$ .

The density of states  $g(k)dk$  for electrons (matter waves) confined to a cavity, accounting for two spin states, is given by (Eq. 10.36):

$$g(k)dk = \frac{k^2 dk}{\pi^2}$$

(This is twice the density of states for electromagnetic waves because electrons have two spin states per momentum state). To get  $g(E)dE$ , we need to relate  $k$  (wavenumber) to  $E$  (energy) for non-relativistic free electrons. What is this relationship?  $E = \underline{\hspace{2cm}}$  (in terms of  $k$ ,  $m_e$ , and  $\hbar$ )

Go to Frame 13.

### Frame 13

For non-relativistic free electrons, the energy is purely kinetic:

$$E = \frac{p^2}{2m_e}$$

Since  $p = \hbar k$  (de Broglie relation),

$$E = \frac{(\hbar k)^2}{2m_e} = \frac{\hbar^2 k^2}{2m_e}$$

From this, we can find  $k(E)$  and  $dk(E)$  to substitute into  $g(k)dk$  to get  $g(E)dE$ . The result for the density of states for free electrons is (Eq. 10.39):

$$g(E)dE = DE^{1/2}dE$$

where  $D = \frac{8\sqrt{2}\pi m_e^{3/2}}{h^3}$  (Eq. 10.40).

Thus, the number of electrons per unit volume with energy between  $E$  and  $E + dE$  is:

$$n(E)dE = \frac{DE^{1/2}dE}{e^{(E-E_F)/k_B T} + 1} \quad (10.41)$$

Figure 10.12 shows a plot of this  $n(E)$ .

The total number of electrons per unit volume  $N/V$  is found by integrating  $n(E)dE$  over all energies:

$$\frac{N}{V} = \int_0^\infty n(E)dE$$

At  $T = 0K$ ,  $f_{FD}(E) = 1$  for  $E < E_F$  and 0 for  $E > E_F$ . So the integral simplifies to:

$$\frac{N}{V} = \int_0^{E_F} DE^{1/2}dE = D \left[ \frac{2}{3} E^{3/2} \right]_0^{E_F} = \frac{2}{3} DE_F^{3/2} \quad (10.43)$$

This allows calculation of the Fermi energy at 0K,  $E_F(0)$ , if  $N/V$  is known (Eq. 10.44). Table 10.1 lists  $E_F(0)$  for various metals.

Go to Frame 14.

### Frame 14

One interesting consequence of Fermi-Dirac statistics for electrons in metals is their contribution to the **heat capacity**. Classically,  $N$  electrons would contribute  $(3/2)N_A k_B T = (3/2)RT$  to the internal energy per mole, leading to a molar heat capacity  $C_{el} = (3/2)R$ .

However, experimentally, the electronic contribution to heat capacity is much smaller. Why is this, according to FD statistics and Figure 10.12? (Hint: Which electrons can actually absorb thermal energy  $k_B T$  when the metal is heated from  $T = 0\text{K}$ ?) \_\_\_\_\_

Go to Frame 15.

### Frame 15

According to FD statistics, when a metal is heated from  $T \approx 0\text{K}$ , only electrons within an energy range of about  $k_B T$  below the Fermi energy  $E_F$  can be thermally excited to states just above  $E_F$ . The vast majority of electrons deep within the Fermi sea (i.e., with  $E \ll E_F - k_B T$ ) cannot absorb  $k_B T$  of energy because the states  $k_B T$  above them are already occupied (Pauli Exclusion Principle).

The fraction  $f$  of electrons that can be excited is approximately  $k_B T / E_F = T / T_F$ , where  $T_F = E_F / k_B$  is the Fermi Temperature (typically very high,  $\sim 10^4 - 10^5$  K, see Table 10.1). So, only a small fraction  $f$  of electrons actually contributes to the heat capacity. The total thermal energy gained per mole is roughly  $U \approx (N_A f) k_B T = N_A (T / T_F) k_B T = RT^2 / T_F$ . This leads to an electronic heat capacity  $C_{el} = dU / dT \approx R(T / T_F)$ , which is much smaller than  $(3/2)R$  at ordinary temperatures (where  $T \ll T_F$ ). This resolved a long-standing puzzle in solid-state physics.

The chapter concludes with Example 10.5 calculating  $E_F$ ,  $v_F$ , and  $T_F$  for gold.

End of Section.

## 30 Applications of Quantum Statistics

### Frame 1

In the previous section (Frame 11 of "Statistical Physics: Maxwell-Boltzmann Statistics"), we set up the application of Bose-Einstein (BE) statistics to Blackbody Radiation. We established:

- Photons are bosons (spin 1).
- The energy density of photons in an enclosure with energy between  $E$  and  $E + dE$  is  $u(E)dE = E \cdot n(E)dE = E \cdot g(E)f_{BE}(E)dE$ .
- For photons,  $f_{BE}(E) = \frac{1}{e^{E/k_B T} - 1}$ .
- The density of states for photons is  $g(E) = \frac{8\pi E^2}{(hc)^3}$ .

Combining these, what is the expression for  $u(E)dE$ , the energy density of photons in the range  $E$  to  $E + dE$ ? (This is Equation 10.28 in the text).  $u(E)dE =$  \_\_\_\_\_

Go to Frame 2.

### Frame 2

Substituting  $g(E)$  and  $f_{BE}(E)$  into  $u(E)dE = E g(E) f_{BE}(E) dE$ :

$$u(E)dE = E \left( \frac{8\pi E^2}{(hc)^3} \right) \left( \frac{1}{e^{E/k_B T} - 1} \right) dE$$

$$u(E)dE = \frac{8\pi}{(hc)^3} \frac{E^3 dE}{e^{E/k_B T} - 1} \quad (10.28)$$

This is the energy density distribution in terms of energy  $E$ .

To connect this back to Planck's blackbody formula (which is usually in terms of frequency  $f$ ), we use the relation  $E = hf$ . If we convert Equation 10.28 from energy  $E$  to frequency  $f$ , what famous formula do we retrieve? (See Eq. 3.9 in the text). 

---

Go to Frame 3.

### Frame 3

Converting from photon energy  $E$  to frequency  $f$  using  $E = hf$  (and  $dE = hdf$ ) in Equation 10.28, we immediately retrieve the **Planck blackbody formula**:

$$u(f, T) = \frac{8\pi h}{c^3} \frac{f^3}{e^{hf/k_B T} - 1} \quad (3.9)$$

This shows that Planck's formula for a blackbody follows directly and simply from Bose-Einstein statistics applied to a gas of photons.

Go to Frame 4.

### Frame 4

#### Example 10.3: Photons in a Box (page 352)

(a) The number of photons per unit volume with energies between  $E$  and  $E + dE$  in a cavity at temperature  $T$  is  $n(E)dE = g(E)f_{BE}(E)dE$ . Using  $g(E) = \frac{8\pi E^2}{(hc)^3}$  and  $f_{BE}(E) = \frac{1}{e^{E/k_B T} - 1}$ , write out  $n(E)dE$ :  $n(E)dE =$  

---

(b) To find the total number of photons per unit volume ( $N/V$ ) for all energies, we need to integrate  $n(E)dE$  from  $E = 0$  to  $E = \infty$ .

$$\frac{N}{V} = \int_0^\infty n(E)dE$$

The text shows this integral can be transformed by letting  $z = E/k_B T$ :

$$\frac{N}{V} = 8\pi \left( \frac{k_B T}{hc} \right)^3 \int_0^\infty \frac{z^2 dz}{e^z - 1}$$

The definite integral  $\int_0^\infty \frac{z^2 dz}{e^z - 1}$  is a standard integral equal to approximately 2.40.

Go to Frame 5 to see the result for (a) and proceed with (c).

### Frame 5

For part (a), the expression is:

$$n(E)dE = \frac{8\pi E^2}{(hc)^3 (e^{E/k_B T} - 1)} dE$$

(c) Calculate the number of photons/cm<sup>3</sup> inside a cavity whose walls are heated to 3000 K. Compare this with a cavity whose walls are at 3.00 K. Given: Integral value  $\approx 2.40$ .  $k_B \approx 8.62 \times 10^{-5}$  eV/K  $hc \approx 1.24 \times 10^{-4}$  eV·cm (useful conversion: 1 eV · cm = 10<sup>-2</sup> eV · m)

First, calculate  $N/V$  at  $T = 3000$  K.  $(N/V)_{3000K} \approx$  \_\_\_\_\_ photons/cm<sup>3</sup>.

Then, calculate  $N/V$  at  $T = 3.00$  K.  $(N/V)_{3.00K} \approx$  \_\_\_\_\_ photons/cm<sup>3</sup>.



How many orders of magnitude does the photon density decrease by? \_\_\_\_\_  
 Go to Frame 6 for the results.

### Frame 6

At  $T = 3000$  K:

$$\left(\frac{k_B T}{hc}\right)^3 = \left(\frac{(8.62 \times 10^{-5} \text{ eV/K})(3000 \text{ K})}{1.24 \times 10^{-4} \text{ eV} \cdot \text{cm}}\right)^3 \approx \left(\frac{0.2586}{1.24 \times 10^{-4} \text{ cm}}\right)^3 \approx (2085 \text{ cm}^{-1})^3 \approx 9.07 \times 10^9 \text{ cm}^{-3}$$

$$\left(\frac{N}{V}\right)_{3000K} = 8\pi(9.07 \times 10^9 \text{ cm}^{-3})(2.40) \approx 5.47 \times 10^{11} \text{ photons/cm}^3$$

At  $T = 3.00$  K:  $k_B T = (8.62 \times 10^{-5} \text{ eV/K})(3.00 \text{ K}) = 2.586 \times 10^{-4} \text{ eV}$ .

$$\left(\frac{k_B T}{hc}\right)^3 = \left(\frac{2.586 \times 10^{-4} \text{ eV}}{1.24 \times 10^{-4} \text{ eV} \cdot \text{cm}}\right)^3 \approx (2.085 \text{ cm}^{-1})^3 \approx 9.07 \text{ cm}^{-3}$$

$$\left(\frac{N}{V}\right)_{3.00K} = 8\pi(9.07 \text{ cm}^{-3})(2.40) \approx 5.47 \times 10^2 \text{ photons/cm}^3$$

The photon density decreases from  $\sim 5 \times 10^{11}$  to  $\sim 5 \times 10^2$ , which is a decrease by a factor of  $10^9$ , or **9 orders of magnitude**.

Go to Frame 7.

### Frame 7

The text then briefly introduces **Einstein's Theory of Specific Heat**. The molar specific heat  $C$  of a substance is defined as the ratio of the differential thermal energy  $dU$  added to a mole, divided by the resulting differential increase in temperature  $dT$ :

$$C = \frac{dU}{dT} \quad (10.29)$$

To develop a theory for  $C$ , one needs an expression for the internal thermal energy  $U$  of the solid as a function of temperature  $T$ . Differentiating  $U(T)$  then yields  $C$ .

The model treats the solid as a collection of atoms vibrating independently. (This is a simplification; a more advanced model, Debye's theory, considers collective modes or phonons, which are quantized lattice vibrations and are treated as bosons).

This particular OCR snippet doesn't go into the details of deriving  $U(T)$  for Einstein's model or for phonons using BE statistics, but these are standard applications of statistical mechanics.

Go to Frame 8.

### Frame 8

The chapter includes a guest essay on **Bose-Einstein Condensation (BEC)**. This phenomenon was predicted by Einstein for an ideal gas of bosons at sufficiently low temperatures.

What is a Bose-Einstein Condensation? (See page 370, first paragraph of essay, or page 347 of main text). \_\_\_\_\_

When was it experimentally confirmed, and by whom? \_\_\_\_\_

Go to Frame 9.

### Frame 9

A Bose-Einstein Condensation (BEC) is a state where a sizable fraction of bosons in a gas condenses into a **single quantum state**, typically the ground state. This is a macroscopic quantum phenomenon where all individual particle wavefunctions are in phase. It occurs when the de Broglie wavelength  $\lambda = h/p$  of the Bose atoms becomes comparable to the interatomic spacing.

It was experimentally confirmed in **1995** by a group at the University of Colorado led by **Eric Cornell and Carl Wieman**, using rubidium atoms cooled to less than 100 nK.

The essay describes the complex cooling and trapping techniques used:

- Magneto-optic trap (MOT)
- Optical molasses and Sisyphus cooling
- Optical pumping into a specific Zeeman state
- Loading into a modified magnetic trap (spherical quadrupole with rotating zero point)
- Final cooling via "evaporization"

These techniques increased phase space density by many orders of magnitude.

Go to Frame 10.

## Frame 10

One key challenge in observing BEC in atomic gases is to cool the atoms to extremely low temperatures while maintaining a sufficiently low density. Why is maintaining low density important? (See page 370, "Figure 5 shows...").

---

Go to Frame 11.

## Frame 11

Maintaining low density (large interatomic distance) is important to: 1. Preserve the **ideal gas character** of the cloud. 2. Minimize **interatomic forces**. 3. Prevent **liquefaction** (turning into a regular liquid or solid), which would mask the BEC.

The essay also mentions that once atoms are in a Bose condensate, their energy is defined by the localization energy  $\Delta x \Delta p \approx \hbar/2$ . Relaxing the confining trap forces can reduce  $\Delta p$ , effectively lowering the "temperature" of the condensate into the picokelvin range.

Potential applications of BEC, like atom lasers, are still being explored. The essay also mentions the remarkable slowing and stopping of light pulses in a BEC, achieved by Lene Vestergaard Hau.

This concludes the framed adaptation of the provided text on applications of Bose-Einstein statistics and the BEC essay.

End of Section.

# 31 Lasers

## Frame 1

Lasers are fascinating devices that demonstrate the basic principles of how radiation interacts with atoms. They have countless engineering applications, from CD players and barcode scanners to precision surgery and optical fiber communications. In this section, we'll explore the principles behind laser operation.

The key to understanding lasers lies in how atoms absorb and emit light. We'll look at three fundamental processes described by Albert Einstein in 1917.

Go to Frame 2.

## Frame 2

To understand lasers, we need to be familiar with three ways atoms interact with radiation (light):

1. Absorption
2. Spontaneous Emission
3. Stimulated Emission

We will look at each of these in turn.

First, let's consider an atom with at least two energy levels, a lower energy state  $E_1$  and a higher energy state  $E_2$ . What must be true about the energy of a photon ( $hf$ , where  $h$  is Planck's constant and  $f$  is the frequency) for it to be absorbed by the atom, causing a transition from  $E_1$  to  $E_2$ ?

[a]  $hf > E_2 - E_1$  [b]  $hf < E_2 - E_1$  [c]  $hf = E_2 - E_1$

Choose an answer and go to Frame 3.

## Frame 3

Your answer for the photon energy needed for absorption from  $E_1$  to  $E_2$  was [ a — b — c ].

The correct answer is [c]. For an atom to absorb a photon and transition from  $E_1$  to  $E_2$ , the photon's energy must exactly match the energy difference between the states:  $hf = E_2 - E_1$ .

Einstein proposed that the probability of absorption per unit time depends on the radiation field the atom is in. Specifically, it depends on the energy density of the radiation at the correct frequency,  $u(f, T)$  (where  $T$  is temperature, relevant in thermal equilibrium cases). He defined the probability of absorption per unit time per atom as  $B_{12}u(f, T)$ .

What does the coefficient  $B_{12}$  represent? \_\_\_\_\_

Go to Frame 4 for the answer.

## Frame 4

$B_{12}$  is Einstein's coefficient of absorption. It represents the inherent likelihood of an atom absorbing a photon of the correct frequency, given a certain radiation density.

Now, once an atom is in the excited state  $E_2$ , what can happen to it? One possibility is spontaneous emission.

Go to Frame 5.

## Frame 5

An atom in an excited state  $E_2$  doesn't stay there forever. It can return to the lower state  $E_1$  by emitting a photon, even without any external radiation influencing it. This is called **spontaneous transition** or **spontaneous emission**.

The photon emitted has energy  $hf = E_2 - E_1$ .

Does the rate of spontaneous emission depend on the external radiation density  $u(f, T)$ ? [ Yes — No ]

Go to Frame 6 for the answer.

### Frame 6

The rate of spontaneous emission does **not** depend on the external radiation density. It's an inherent property of the excited state.

The rate of spontaneous emission is characterized by Einstein's coefficient  $A_{21}$  (transitions per unit time). The inverse of this,  $t_s = 1/A_{21}$ , is the average lifetime of the atom in the excited state before it spontaneously emits a photon.

Spontaneously emitted photons are sent out in random directions. (See Figure 12.41b in the original text for a visual).

What is the third interaction process Einstein described? \_\_\_\_\_

Go to Frame 7 for the answer.

### Frame 7

The third process is **stimulated emission**.

This is crucial for lasers. If a photon with energy  $hf = E_2 - E_1$  interacts with an atom that is \*already\* in the excited state  $E_2$ , it can trigger or "stimulate" the atom to emit a \*second\* photon.

What are the properties of this second, stimulated photon compared to the first, incoming photon? List two key properties mentioned in the original text (page 447). 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 8 for the answer.

### Frame 8

The stimulated photon has the same energy (and frequency) as the incoming photon, and crucially: 1. It vibrates **in-phase** with the stimulating photon. 2. It travels in the **same direction** as the stimulating photon.

Photons that are in-phase and travel in the same direction are called **coherent**.

Einstein defined the probability of stimulated emission per unit time per atom as  $B_{21}u(f, T)$ . Notice it also depends on the radiation density  $u(f, T)$ .

(See Figure 12.41c in the original text for a visual).

Let's summarize the three processes visually (based on Figure 12.41): (a) **Absorption**: Atom in  $E_1$  absorbs  $hf$ , goes to  $E_2$ . (b) **Spontaneous Emission**: Atom in  $E_2$  emits  $hf$  randomly, goes to  $E_1$ . Lifetime  $t_s$ . (c) **Stimulated Emission**: Atom in  $E_2$  hit by  $hf$ , emits a second  $hf$  coherently, goes to  $E_1$ .

Go to Frame 9.

### Frame 9

Now, consider a collection of these atoms in thermal equilibrium with radiation at temperature  $T$ . Atoms are constantly undergoing these three processes. In equilibrium, the number of atoms moving up must balance the number moving down.

The number of atoms in state  $E_1$  is  $N_1$  and in state  $E_2$  is  $N_2$ . According to the Boltzmann relation (from statistical mechanics), how are  $N_1$  and  $N_2$  related at temperature  $T$ ? (See Eq. 12.37). \_\_\_\_\_

Go to Frame 10 for the answer.

### Frame 10

The Boltzmann relation states:

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/k_B T} = e^{-hf/k_B T}$$

where  $k_B$  is the Boltzmann constant.

Since  $hf$  is positive, and  $k_BT$  is positive, the exponent is negative. This means  $N_2 < N_1$  in thermal equilibrium. There are always fewer atoms in the higher energy state.

In equilibrium, the rate of upward transitions must equal the rate of downward transitions. Rate up (Absorption):  $N_1 \times (\text{absorption probability/time}) = N_1 B_{12} u(f, T)$  Rate down (Spontaneous + Stimulated):  $N_2 \times (\text{spontaneous rate}) + N_2 \times (\text{stimulated rate/time}) = N_2 A_{21} + N_2 B_{21} u(f, T)$

Setting Rate up = Rate down gives:

$$N_1 u(f, T) B_{12} = N_2 [A_{21} + B_{21} u(f, T)]$$

Go to Frame 11.

### Frame 11

We can rearrange the equilibrium equation from Frame 10 to solve for the energy density  $u(f, T)$ :

$$u(f, T)(N_1 B_{12} - N_2 B_{21}) = N_2 A_{21}$$

$$u(f, T) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} = \frac{A_{21}}{(N_1/N_2) B_{12} - B_{21}}$$

Now substitute the Boltzmann relation  $N_1/N_2 = e^{hf/k_BT}$  (the inverse of Eq. 12.37):

$$u(f, T) = \frac{A_{21}}{B_{12} e^{hf/k_BT} - B_{21}}$$

This expression was derived from considering the atomic transitions. But we also know the formula for energy density from Planck's blackbody radiation law (Chapter 3):

$$u(f, T) = \frac{8\pi h f^3}{c^3} \frac{1}{e^{hf/k_BT} - 1}$$

By comparing these two expressions for  $u(f, T)$ , what relationship must hold between the Einstein coefficients  $B_{12}$  and  $B_{21}$ ? And what relationship between  $A_{21}$  and  $B_{21}$  (or  $B_{12}$ )? (See Eqs. 12.40, 12.41).

1.  $B_{12}$  vs  $B_{21}$ : \_\_\_\_\_ 2.  $A_{21}$  vs  $B_{21}$ : \_\_\_\_\_

Go to Frame 12 for the answers.

### Frame 12

Comparing the two formulas for  $u(f, T)$  yields Einstein's relations: 1.

$$B_{12} = B_{21}$$

The probability coefficient for absorption is equal to the probability coefficient for stimulated emission. 2.

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h f^3}{c^3}$$

This relates the rate of spontaneous emission to stimulated emission.

The first result is particularly important. It means that if an atom is in  $E_1$ , an incoming photon is just as likely to cause absorption (upward transition) as it is to cause stimulated emission (downward transition) if the atom is in  $E_2$ .

Knowing this, and knowing that in thermal equilibrium  $N_1 > N_2$ , what is the \*net\* effect when light shines on this collection of atoms? [a] Net absorption [b] Net stimulated emission [c] No net change

Go to Frame 13.

### Frame 13

Your answer was [ a — b — c ].

The correct answer is [a] Net absorption. Because  $B_{12} = B_{21}$  but  $N_1 > N_2$ , more photons will be absorbed ( $N_1 B_{12} u$ ) than will be produced by stimulated emission ( $N_2 B_{21} u$ ). The collection of atoms absorbs energy from the light field.

To make a laser, we need the opposite: we want amplification of light, meaning more stimulated emission than absorption. What condition would allow this? \_\_\_\_\_

Go to Frame 14.

### Frame 14

To get net stimulated emission (amplification), we need the rate of stimulated emission to be greater than the rate of absorption:

$$N_2 B_{21} u(f, T) > N_1 B_{12} u(f, T)$$

Since  $B_{12} = B_{21}$ , this simplifies to:

$$N_2 > N_1$$

This condition, where there are more atoms in the higher energy state ( $E_2$ ) than in the lower energy state ( $E_1$ ), is the opposite of thermal equilibrium. It is called **population inversion**.

How can we achieve population inversion? Can it be done just by shining light (with frequency  $f = (E_2 - E_1)/h$ ) very intensely on a system with only two levels,  $E_1$  and  $E_2$ ? [ Yes — No ]

Go to Frame 15.

### Frame 15

The answer is [No].

As explained on page 450 (first bullet point under the figure), with only two levels, intense pumping (shining light) can at best make the populations equal ( $N_1 = N_2$ ). This is because absorption and stimulated emission have equal probabilities ( $B_{12} = B_{21}$ ). Once  $N_1 = N_2$ , any further pumping causes just as many stimulated emissions (down) as absorptions (up). There is no net amplification, only saturation.

Therefore, to achieve population inversion, we need a more clever approach, typically involving at least *three* energy levels.

Go to Frame 16.

### Frame 16

A **laser** (Light Amplification by Stimulated Emission of Radiation) is a device that achieves light amplification using population inversion. Most lasers share three essential features:

1. **An Energy Source (Pump):** To create the population inversion. This isn't thermal energy. Examples include: \* Electrical discharge (collisions, as in Helium-Neon lasers) \* Intense light flashes (**optical pumping**, as in Ruby lasers) \* Current injection (**injection pumping**, in semiconductor lasers)

Go to Frame 17.

### Frame 17

**2. A Lasing Medium with Appropriate Energy Levels:** Usually needs at least three levels (or four) as shown schematically in Figure 12.42: \*  $E_0$ : Ground state \*  $E_2$ : A high-energy "pump state" \*  $E_1$ : An intermediate "metastable" state

The pumping mechanism excites atoms from  $E_0$  to  $E_2$ . The atoms then quickly decay (non-radiatively or via photons we don't care about) to the metastable state  $E_1$ .

What property must the metastable state  $E_1$  have for population inversion between  $E_1$  and  $E_0$  to occur? (Hint: think about lifetimes). \_\_\_\_\_

Go to Frame 18.

### Frame 18

The metastable state  $E_1$  must have a relatively *long lifetime* ( $t_s$ ) compared to the lifetime ( $t_2$ ) of the pump state  $E_2$ .

This allows atoms to "pile up" in state  $E_1$  faster than they leave it via spontaneous emission back to  $E_0$ . If the pump is strong enough, we can achieve  $N_1 > N_0$  (where  $N_1$  is population of  $E_1$ ,  $N_0$  is population of  $E_0$ ). Laser action then occurs via stimulated emission from  $E_1$  down to  $E_0$ .

Go to Frame 19.

### Frame 19

**3. A Method for Photon Containment (Optical Resonator):** Once spontaneous emission starts the process, the initial photons need to be contained within the medium to stimulate more emission from other excited atoms, leading to a cascade (amplification).

How is this typically achieved in practice? (See page 450, third bullet point). \_\_\_\_\_

Go to Frame 20.

### Frame 20

Photon containment is usually achieved by placing **mirrors** at the ends of the lasing medium, forming an optical resonator (or cavity).

Photons bounce back and forth, passing through the medium multiple times, stimulating more emission. The waves form standing waves within the cavity (Figure 12.43). One mirror is made partially transparent to allow a fraction of the coherent, amplified light to escape as the laser beam.

These three features lead to the unique properties of laser light: high monochromaticity (single frequency), high intensity, spatial coherence (waves aligned in space), and low beam divergence (stays narrow).

Go to Frame 21.

### Frame 21

A particularly important type is the **semiconductor laser**, often used in fiber optics and CD/DVD players. These differ from gas or crystal lasers in a few ways.

Instead of discrete energy levels, semiconductors have **energy bands**. The key bands are the lower **valence band** (normally full of electrons) and the higher **conduction band** (normally empty or nearly empty), separated by an energy gap  $E_g$ . (See Figure 12.45a).

What process replaces optical pumping or electrical discharge for exciting electrons in a semiconductor laser diode? (See page 452, first paragraph). \_\_\_\_\_

Go to Frame 22.

### Frame 22

Semiconductor lasers use **injection pumping**.

A large forward current is passed through a p-n junction (a diode). This injects electrons into the conduction band on one side and "holes" (missing electrons, acting like positive charges) into the valence band on the other side.

When an electron from the conduction band recombines with a hole in the valence band, what is emitted? (See Figure 12.45b). \_\_\_\_\_

Go to Frame 23.

### Frame 23

When an electron and hole recombine, a photon is emitted with energy approximately equal to the band gap energy,  $hf \approx E_g$ .

To get laser action, we need population inversion. In a semiconductor, this means creating a situation where there are many electrons filling the bottom of the conduction band and many holes emptying the top of the valence band near the junction (Figure 12.46b).

Why does this situation lead to amplification for photons with energy  $hf = E_g$ ? (Hint: Consider absorption vs. stimulated emission, and Figure 12.46 caption). \_\_\_\_\_

Go to Frame 24.

### Frame 24

With inversion (Fig 12.46b), incoming photons with energy  $E_g$  can stimulate recombination (emission) readily. However, absorption requires *\*more\** energy than  $E_g$ . This is because an electron absorbing  $E_g$  from the valence band would need to jump into an *\*unoccupied\** state in the conduction band, which are only available *\*above\** the filled region (level Y in the figure). Since the incoming photons don't have enough energy for this absorption ( $E_a > E_g$ ), stimulated emission dominates, leading to amplification.

Semiconductor laser technology is very advanced, involving different materials (like GaAs, AlGaAs, PbSnTe) and structures (like double heterojunctions) to optimize wavelength, efficiency, and operating conditions.

This concludes our quick overview of laser principles. You may want to review the frames or consult the original text for more details, especially regarding the figures and footnotes.

End of Section.

## 32 The Solid State: Bonding

### Frame 1

Matter in the solid state has fascinated people since the beginnings of civilization, admired for beauty (gemstones) and utility (metal tools). Today, solid-state physics (or condensed-matter physics) is the largest subfield, crucial for countless industrial applications.

Solids can range from highly ordered **crystalline** structures (like salt or diamond) to disordered **amorphous** structures (like glass). Understanding how atoms bond together is key to understanding the properties of these solids.

In this section, we will explore the primary ways atoms bond to form solids.

Go to Frame 2.

### Frame 2



The bonding mechanisms in solids are similar to those for molecules (which you might have studied previously, e.g., in Chapter 11 of the source text). The main types are:

- Ionic Bonding
- Covalent Bonding
- Metallic Bonding
- Molecular Bonding (including van der Waals and Hydrogen bonds)

We will look at each of these. Let's start with ionic solids.

Go to Frame 3.

### Frame 3

Many crystals, like common table salt (NaCl), form by **ionic bonding**. What is the dominant force responsible for ionic bonding?

- [a] Electron sharing [b] Magnetic attraction [c] Coulomb (electrostatic) interaction between ions  
[d] Gravitational attraction

Choose an answer and go to Frame 4.

### Frame 4

Your answer was [ a — b — c — d ].

The correct answer is [c]. Ionic bonding is primarily due to the electrostatic attraction between positively charged ions (cations) and negatively charged ions (anions).

Consider the NaCl crystal structure (visualize Figure 12.1). Each positive sodium ion ( $\text{Na}^+$ ) is surrounded by several negative chlorine ions ( $\text{Cl}^-$ ), and vice versa. How many \*nearest-neighbor\*  $\text{Cl}^-$  ions does each  $\text{Na}^+$  ion have? (Hint: Think 3D structure).

- [a] 2 [b] 4 [c] 6 [d] 8

Go to Frame 5.

### Frame 5

Your answer was [ a — b — c — d ].

The correct answer is [c]. In the NaCl structure, each  $\text{Na}^+$  has six nearest-neighbor  $\text{Cl}^-$  ions (one above, one below, one left, one right, one front, one back), and each  $\text{Cl}^-$  also has six nearest-neighbor  $\text{Na}^+$  ions.

The net attractive potential energy between a pair of ions due to all interactions in the crystal lattice can be written as:

$$U_{\text{attractive}} = -\alpha k \frac{e^2}{r}$$

where  $k$  is the Coulomb constant,  $e$  is the elementary charge,  $r$  is the ion separation, and  $\alpha$  is a dimensionless constant.

What is the name given to the constant  $\alpha$ ? \_\_\_\_\_

Go to Frame 6.

### Frame 6

The constant  $\alpha$  is called the **Madelung constant**. Its value depends only on the specific geometric arrangement (crystal structure) of the ions. For NaCl,  $\alpha \approx 1.7476$ .

However, this attractive force isn't the whole story. When ions get very close, their electron shells start to overlap. What kind of interaction does this cause?

[a] Additional attraction [b] Repulsion [c] No significant interaction

Go to Frame 7.

### Frame 7

Your answer was [ a — b — c ].

The correct answer is [b] Repulsion. The overlap of closed electron shells leads to a strong repulsive force, partly electrostatic and partly due to the Pauli exclusion principle. This repulsive potential energy is often modeled as  $B/r^m$ , where  $B$  and  $m$  are constants ( $m$  is typically around 10).

The total potential energy per ion pair is the sum of the attractive and repulsive terms:

$$U_{\text{total}} = -\alpha k \frac{e^2}{r} + \frac{B}{r^m}$$

A plot of  $U_{\text{total}}$  versus  $r$  (like Figure 12.2) shows a minimum value at a specific separation distance. What is this distance called? \_\_\_\_\_

Go to Frame 8.

### Frame 8

The ion separation distance at which the total potential energy is minimized is called the **equilibrium separation**, denoted as  $r_0$ . (See Figure 12.2).

The minimum potential energy value itself,  $U_0$ , which occurs at  $r = r_0$ , is related to the energy needed to break the crystal apart. The magnitude  $|U_0|$  is called the **ionic cohesive energy**. It represents the energy required to separate the solid into isolated positive and negative ions.

Is this the same as the energy needed to separate the solid into neutral atoms? [ Yes — No ]

Go to Frame 9.

### Frame 9

The answer is [No].

The **ionic cohesive energy** is the energy to get isolated \*ions\*. The **atomic cohesive energy** is the energy needed to get isolated \*neutral atoms\*.

To find the atomic cohesive energy from the ionic cohesive energy, we must account for the energy involved in neutralizing the ions. For NaCl:

- Energy needed to separate NaCl solid into  $\text{Na}^+$  and  $\text{Cl}^-$  ions (Ionic cohesive energy) = 7.84 eV
- Energy *gained* by converting  $\text{Na}^+$  ion to Na atom = 5.14 eV (ionization energy of Na)
- Energy *needed* to convert  $\text{Cl}^-$  ion to Cl atom = 3.61 eV (electron affinity of Cl)

So, the Atomic cohesive energy = 7.84 eV - 5.14 eV + 3.61 eV = 6.31 eV per pair.

Go to Frame 10.

### Frame 10

Ionic crystals have several general properties resulting from this strong Coulomb bonding. Which of the following is generally TRUE for ionic solids like NaCl?

[a] They are good electrical conductors. [b] They have low melting points. [c] They are relatively hard and stable. [d] They are opaque to visible light.

Go to Frame 11.

### Frame 11

Your answer was [ a — b — c — d ].

The correct answer is [c]. Ionic solids form stable, relatively hard crystals with high melting points.

Let's consider the other options: (a) False: They are poor electrical conductors because electrons are tightly bound to the ions and there are no free electrons to carry current. (b) False: They have high melting points because significant thermal energy is needed to overcome the strong cohesive energy. (d) False: They are often transparent to visible light because visible photons don't have enough energy to excite the tightly bound electrons to higher energy shells.

Why do ionic solids absorb strongly in the \*infrared\* region? (Hint: Consider the particles involved and their mass). \_\_\_\_\_

Go to Frame 12.

### Frame 12

Ionic solids absorb strongly in the infrared because the vibrations of the relatively massive ions themselves occur at low frequencies, which correspond to the energy of infrared photons. Visible light has too high a frequency (too much energy) to resonantly excite these ionic vibrations.

Another property: Ionic solids tend to dissolve in polar liquids like water. Why? \_\_\_\_\_

Go to Frame 13.

### Frame 13

Polar molecules (like water, H<sub>2</sub>O) have a permanent electric dipole moment (a separation of positive and negative charge). These dipoles exert attractive forces on the positive and negative ions in the crystal, which can be strong enough to break the ionic bonds and pull the ions into solution.

Now let's move to the next type of bonding.

Go to Frame 14.

### Frame 14

**Covalent solids** are formed by sharing valence electrons between adjacent atoms, similar to covalent bonds in molecules. The classic example is diamond (a form of carbon).

Carbon's electron configuration is  $1s^2 2s^2 2p^2$ . How many more electrons does it need to achieve a stable, filled outer shell (like  $2p^6$ )?

[a] 2 [b] 4 [c] 6 [d] 8

Go to Frame 15.

### Frame 15

Your answer was [ a — b — c — d ].

The correct answer is [b] 4. Carbon needs four more electrons to complete its outer shell.

Therefore, in diamond, each carbon atom forms strong covalent bonds by sharing electrons with *four* other carbon atoms. This arrangement forms a very stable structure. (Visualize Figure 12.3). The geometry where one atom bonds to four others equally spaced is called a **tetrahedral**

structure. The bond angles are about 109.5 degrees. Silicon (Si) and Germanium (Ge) are other important covalent solids with this diamond structure.

Go to Frame 16.

### Frame 16

Based on the strength of covalent bonds (sharing electrons), what properties would you expect covalent solids like diamond to have compared to ionic solids?

[a] Harder, higher melting point, higher cohesive energy [b] Softer, lower melting point, lower cohesive energy [c] Similar hardness, melting point, and cohesive energy

Go to Frame 17.

### Frame 17

Your answer was [ a — b — c ].

The correct answer is [a]. Covalent bonds are very strong and directional. Covalent solids like diamond are typically:

- Extremely hard (diamond is the hardest natural substance).
- Have very high melting points (diamond melts around 4000 K).
- Have the highest atomic cohesive energies (see Table 12.2, diamond is 7.37 eV/atom).
- Good electrical insulators, because valence electrons are localized in the bonds and not free to move.
- Often transparent to visible light (for the same reason as ionic solids - not enough energy to excite bound electrons).

Go to Frame 18.

### Frame 18

The third major type is **metallic solids**, formed by **metallic bonding**. This applies to elements like copper, silver, gold, iron, sodium, etc.

In simple terms, how is the metallic structure often visualized? (Refer to Figure 12.4).

[a] Alternating positive and negative ions. [b] Atoms sharing electron pairs. [c] A lattice of positive ions surrounded by a "sea" or "gas" of mobile electrons.

Go to Frame 19.

### Frame 19

Your answer was [ a — b — c ].

The correct answer is [c]. The simple model views a metal as a regular array (lattice) of positive ions (the atom cores, consisting of the nucleus and inner shell electrons) immersed in a "gas" of valence electrons that are free to move throughout the entire crystal.

What provides the "glue" holding the positive ions together in this model? \_\_\_\_\_

Go to Frame 20.

### Frame 20

The "glue" is the attractive electrostatic force between the fixed positive ion cores and the mobile negative electron gas that permeates the lattice.

What are some characteristic properties of metals resulting from this structure? List at least two mentioned in the text. 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 21.

### Frame 21

Metals typically exhibit:

- Good electrical conductivity (due to the mobile electron gas).
- Good thermal conductivity (also due to mobile electrons).
- Shininess (opacity) - free electrons interact strongly with light, absorbing and re-emitting it near the surface.
- Ductility/Malleability - the non-directional nature of the bonds allows atoms to slide past each other without breaking the structure easily.
- Ability to form alloys (solid solutions) - different metal atoms can often substitute into the lattice easily.

Metallic bonds (and cohesive energies, typically 1-4 eV/atom, see Table 12.3) are generally weaker than ionic or covalent bonds, leading to softer materials with lower melting points than diamond, for instance.

Go to Frame 22.

### Frame 22

The fourth class is **molecular crystals**. These form from substances where the molecules themselves have stable, closed electron shells, and electrons are not readily available for ionic, covalent, or metallic bonding. Examples include solid inert gases (like Argon), organic molecules (like methane, CH<sub>4</sub>), and ice (H<sub>2</sub>O).

What kind of weak forces hold these molecules together in the solid state? (Hint: Two main types were mentioned). 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 23.

### Frame 23

The primary forces are: 1. **Van der Waals forces**: Weak attractions between temporary, fluctuation-induced dipoles. These exist between all atoms/molecules. 2. **Dipole-dipole forces**: Attractions between molecules that have permanent electric dipole moments (like water).

A special case of dipole interaction involving hydrogen is the **hydrogen bond**, which is relatively strong compared to van der Waals forces (e.g., cohesive energy of ice is 0.5 eV/molecule) but still much weaker than ionic/covalent/metallic bonds.

What properties would you expect molecular crystals to have due to these weak intermolecular forces? \_\_\_\_\_

Go to Frame 24.

### Frame 24

Due to the weak intermolecular forces, molecular crystals generally have:

- Low melting points.

- Low boiling points.
- Low cohesive energies (often around 0.1 eV/molecule or less for van der Waals).
- They are typically soft and easily deformed.
- Often poor conductors of heat and electricity.

Go to Frame 25.

### Frame 25

So far, we've mostly discussed **crystalline solids**, characterized by their regular, repeating atomic arrangement (long-range order). However, many solids lack this long-range order and are called **amorphous solids** or **glasses**. (See Figure 12.5).

How are amorphous solids typically formed from a liquid?

[a] Very slow cooling [b] Very rapid cooling [c] Applying high pressure

Go to Frame 26.

### Frame 26

Your answer was [ a — b — c ].

The correct answer is [b] Very rapid cooling. If a liquid is cooled quickly enough, the atoms or molecules don't have time to settle into the lowest-energy, ordered crystalline arrangement. They get "frozen" in a disordered state characteristic of the liquid phase.

Even metals can be made amorphous (forming **metallic glasses**) using techniques like **melt spinning** (Figure 12.6) that achieve extremely high cooling rates (e.g.,  $10^6$  K/s).

Although amorphous solids lack long-range order, do they have any structural regularity at all?

[ Yes — No ]

Go to Frame 27.

### Frame 27

The answer is [Yes].

Amorphous solids possess significant **short-range order**. This means that the distances and angles between an atom and its nearest neighbors are relatively consistent, similar to those in the crystal. The disorder accumulates over longer distances.

How can we experimentally determine the atomic arrangement in solids (both crystalline and amorphous)? Name one technique. \_\_\_\_\_

Go to Frame 28.

### Frame 28

Common techniques include:

- X-ray diffraction
- Electron diffraction
- Neutron diffraction

These methods measure the intensity of scattered waves ( $I_s(\theta)$ ) as a function of scattering angle  $\theta$  (Figure 12.7). This scattering pattern contains information about the atomic arrangement.

The scattering data can be mathematically transformed (using a Fourier transform) to obtain the **Radial Distribution Function (RDF)**,  $\rho(r)$ . What does  $\rho(r)dr$  represent? \_\_\_\_\_

Go to Frame 29.

### Frame 29

$\rho(r)dr$  represents the probability of finding a neighboring atom at a distance between  $r$  and  $r + dr$  from an atom chosen as the origin.

How would you expect the RDF to look for a perfect crystal compared to an amorphous solid (glass)? (Think about the regularity of neighbor distances). \_\_\_\_\_

Go to Frame 30.

### Frame 30

For a perfect crystal, atoms exist in distinct shells at very specific distances ( $r_1, r_2, r_3, \dots$ ). Therefore, the RDF for a crystal should show a series of **sharp peaks** at these specific distances.

For an amorphous solid (glass), the nearest-neighbor distances are somewhat consistent (short-range order), but the distances to farther neighbors become increasingly varied. Therefore, the RDF for a glass should show a few **broad peaks** at short distances, which fade out quickly at longer distances into a smooth curve representing the average density. (Compare 'Crystal' and 'Glass' in Figure 12.9).

Real RDFs (like Figure 12.10 for Germanium) confirm this difference. Note that even crystalline peaks are broadened due to thermal vibrations.

Go to Frame 31.

### Frame 31

Amorphous solids, despite their disorder, have important technological applications. Table 12.4 lists several examples. Can you name one application mentioned? \_\_\_\_\_

Go to Frame 32.

### Frame 32

Some applications include:

- Oxide glasses (like  $\text{SiO}_2\text{-Na}_2\text{O}$ ) for windows.
- Oxide glasses (like  $\text{SiO}_2\text{-GeO}_2$ ) for ultra-transparent optical fibers.
- Organic polymers (like Polystyrene) for strong, light plastics.
- Chalcogenide glasses (like Se,  $\text{As}_2\text{Se}_3$ ) in xerography.
- Amorphous silicon ( $\text{Si}_{0.9}\text{H}_{0.1}$ ) for solar cells.
- Metallic glasses (like  $\text{Fe}_{0.8}\text{B}_{0.2}$ ) for efficient transformer cores.

This concludes our look at bonding in solids and the distinction between crystalline and amorphous structures.

End of Section.

## 33 Band Theory of Solids

### Frame 1

In previous sections, we discussed metals using models (classical and quantum) that treated them as a gas of free electrons. However, we know materials like insulators behave very differently, with much higher resistivity. This difference arises from their electronic configurations and how electrons interact with the lattice of ions in the solid.

To understand the electronic properties of solids (conductors, insulators, semiconductors), we need to consider the effect of the lattice ions on the electrons. There are two main approaches:

1. The Isolated-Atom Approach: How do energy levels change as individual atoms are brought together?
2. The Periodic Potential Approach: How do electron waves behave in the periodic potential created by the lattice ions?

We will start with the isolated-atom approach because it provides a simpler initial explanation for the differences between material types.

Go to Frame 2.

### Frame 2

Imagine two identical atoms (like sodium, with its outermost electron in the 3s state) that are initially very far apart. Do their electronic energy levels interact? [ Yes — No ]

Go to Frame 3 for the answer.

### Frame 3

The answer is [No]. When atoms are very far apart, they don't interact, and their energy levels are just the distinct levels of isolated atoms.

Now, what happens as these two sodium atoms are brought closer together? Their wavefunctions begin to overlap. The original, single 3s energy level, which was the same for both atoms (degenerate), splits into two distinct energy levels. (See Figure 12.16a in the original text).

Why does this splitting occur? It relates to how the electron wavefunctions combine. Consider the individual wavefunctions  $\psi_1$  and  $\psi_2$ . Two possible combinations (approximate wavefunctions for the pair) are  $\psi_1 + \psi_2$  and  $\psi_1 - \psi_2$ . (See Figure 12.17).

In which combined state,  $\psi_1 + \psi_2$  or  $\psi_1 - \psi_2$ , is the electron more likely to be found \*between\* the two atomic cores? \_\_\_\_\_

Go to Frame 4.

### Frame 4

The electron has a substantial probability of being found midway between the cores in the state  $\psi_1 + \psi_2$ . In the state  $\psi_1 - \psi_2$ , the probability density is zero at the midpoint.

Since the electron in the  $\psi_1 + \psi_2$  state spends time between the two \*attractive\* ion cores, is its energy higher or lower than an electron in the  $\psi_1 - \psi_2$  state? [ Higher — Lower ]

Go to Frame 5.

### Frame 5



The answer is [Lower]. Because the electron in the  $\psi_1 + \psi_2$  state experiences attraction from both cores when it's in the middle, it is more tightly bound and thus has lower energy. The electron in the  $\psi_1 - \psi_2$  state avoids the middle region and is less tightly bound (higher energy).

This energy difference is the reason the original degenerate 3s level splits into two distinct levels when two atoms are brought together.

Go to Frame 6.

### Frame 6

What happens if we bring many atoms together, say  $N$  atoms (where  $N$  is huge, like  $10^{23}$  in a solid)?

The single energy level from the isolated atom (like the 3s level in sodium) will split into how many distinct levels? [a] 2 levels [b]  $N$  levels [c] An infinite number of levels

Go to Frame 7.

### Frame 7

The answer is [b]  $N$  levels. Each original atomic level splits into  $N$  closely spaced levels when  $N$  atoms form a solid.

If  $N$  is very large, these  $N$  levels are extremely close together. What do we call this collection of very closely spaced energy levels that originates from a single atomic level? (See page 426). —

Go to Frame 8.

### Frame 8

This collection of closely spaced levels is called an **energy band**. For sodium, the levels originating from the 3s atomic state form the **3s band**.

Solids have multiple energy bands, corresponding to the different energy levels of the isolated atoms (e.g., 1s band, 2s band, 2p band, 3s band, etc., as shown for sodium in Figure 12.18).

What are the regions of energy \*between\* the allowed energy bands called? \_\_\_\_\_

Go to Frame 9.

### Frame 9

The energy regions between allowed bands are called **energy gaps** or **forbidden energy regions**. Electrons in the solid cannot have energies corresponding to these gaps.

Sometimes, atomic levels broaden so much when forming a solid that the resulting bands overlap (like the 3s and 3p bands in sodium, Figure 12.18). If bands overlap, is there an energy gap between them? [ Yes — No ]

Go to Frame 10.

### Frame 10

The answer is [No]. If bands overlap, the energy gap between them disappears, and electrons can have a continuous range of energies across the merged bands.

Now let's consider how many electrons can fit into a band. An atomic level with orbital angular momentum  $l$  can hold  $2(2l + 1)$  electrons (including spin). If the solid contains  $N$  atoms, the corresponding band, which has  $N$  levels, can hold how many electrons in total? \_\_\_\_\_

Go to Frame 11.

### Frame 11

The capacity of a band arising from a level with angular momentum  $l$  in a solid with  $N$  atoms is  $2(2l + 1)N$  electrons.

Example: Sodium (Na). Atomic levels: 1s ( $l = 0$ ), 2s ( $l = 0$ ), 2p ( $l = 1$ ), 3s ( $l = 0$ ). Sodium has 11 electrons total per atom. In a solid with  $N$  sodium atoms: - 1s band capacity =  $2(2(0) + 1)N = 2N$  electrons. - 2s band capacity =  $2(2(0) + 1)N = 2N$  electrons. - 2p band capacity =  $2(2(1) + 1)N = 6N$  electrons. - 3s band capacity =  $2(2(0) + 1)N = 2N$  electrons.

How many electrons are actually \*in\* the 3s band in solid sodium? (Remember each Na atom contributes its outermost electron, which is 3s). \_\_\_\_\_

Go to Frame 12.

### Frame 12

Since each of the  $N$  sodium atoms contributes one 3s electron, the 3s band contains  $N$  electrons.

Given that the capacity of the 3s band is  $2N$  electrons, the 3s band in sodium is: [a] Completely empty [b] Half full [c] Completely full

Go to Frame 13.

### Frame 13

The answer is [b] Half full. The 1s, 2s, and 2p bands are completely full, while the 3s band is only half full. The 3p band is empty. (See Figure 12.18).

This band structure explains the differences in electrical conductivity between metals, insulators, and semiconductors.

**Metals:** Consider sodium again. Its highest occupied band (the 3s band) is only half full. If an electric field is applied, can electrons in this band easily gain a small amount of energy and move into unoccupied energy states within the \*same\* band? [ Yes — No ]

Go to Frame 14.

### Frame 14

The answer is [Yes]. In a metal like sodium, the highest occupied band (called the conduction band if it's partially filled or overlaps with an empty band) has many empty states immediately available at slightly higher energies. Electrons near the highest filled level (the Fermi energy,  $E_F$ , which lies in the middle of the band for Na at  $T=0K$ , see Figure 12.19) can easily gain energy from an electric field and move into these empty states, allowing current to flow readily. This makes sodium a good conductor.

Go to Frame 15.

### Frame 15

**Insulators:** Now consider a material where the highest energy band containing any electrons is completely full, and the next highest band is completely empty. These are called the **valence band** (filled) and the **conduction band** (empty). They are separated by a large **energy gap**,  $E_g$  (typically 10 eV for a good insulator). (See Figure 12.20).

Can electrons in the filled valence band move to slightly higher energy states \*within\* the valence band to carry current? [ Yes — No ]

Can electrons easily gain enough energy from a typical electric field or room temperature thermal energy ( $k_B T \approx 0.025$  eV) to jump across the large gap  $E_g$  into the empty conduction band? [ Yes — No ]

Go to Frame 16.

### Frame 16

The answers are [No] and [No]. Electrons in a completely filled band cannot move to slightly higher energies within that band because there are no empty states available. To conduct, they must gain enough energy to jump across the energy gap  $E_g$  to the empty conduction band.

In an insulator,  $E_g$  is very large compared to thermal energy ( $k_B T$ ) or the energy typically gained from an electric field. Therefore, very few electrons make it to the conduction band, and the material has very high resistivity. The Fermi level  $E_F$  lies in the middle of the gap.

Go to Frame 17.

### Frame 17

**Semiconductors:** Semiconductors are like insulators, but with a much smaller energy gap  $E_g$  (typically around 1 eV). (See Figure 12.21 and Table 12.8).

At  $T=0K$ , are semiconductors conductors or insulators? [ Conductors — Insulators ]

At room temperature ( $T=300K$ ), can a significant number of electrons be thermally excited from the valence band across the small gap  $E_g$  into the conduction band? [ Yes — No ]

Go to Frame 18.

### Frame 18

The answers are [Insulators] and [Yes]. At  $T=0K$ , the valence band is full and the conduction band is empty, so semiconductors act as insulators. However, at room temperature, the thermal energy ( $k_B T$ ) is large enough relative to the small gap  $E_g$  that an appreciable number of electrons are excited into the conduction band.

These electrons in the conduction band can carry current. What is left behind in the valence band when an electron is excited? \_\_\_\_\_

Go to Frame 19.

### Frame 19

When an electron leaves the valence band, it leaves behind a vacant site called a **hole**. This hole acts like a positive charge carrier because an electron from a neighboring site can move into it, causing the hole to effectively migrate in the opposite direction.

In a pure (**intrinsic**) semiconductor, the number of conduction electrons equals the number of holes. Both contribute to conductivity. (See Figure 12.22).

Does the conductivity of a semiconductor increase or decrease as temperature increases? [ Increase — Decrease ] Why? \_\_\_\_\_

Go to Frame 20.

### Frame 20

Conductivity [Increases]. Because higher temperature means more thermal energy ( $k_B T$ ), leading to more electrons being excited across the gap, creating more conduction electrons and more holes. This is opposite to metals, where conductivity generally decreases slightly with temperature due to increased scattering.

Go to Frame 21.

### Frame 21

The existence of energy bands and gaps can also be understood by considering how electron waves interact with the periodic lattice of ions in the solid. This is a different approach than the isolated-atom model.

A completely free electron has energy  $E = \hbar^2 k^2 / (2m_e)$ , where  $k$  is the wavenumber ( $k = 2\pi/\lambda$ ). This relationship is a continuous parabola (Figure 12.23).

What happens when an electron wave travels through a periodic lattice with atomic spacing  $a$ ? For most wavelengths, the wave passes through relatively unaffected. However, when the wavelength  $\lambda$  is such that waves reflected from adjacent atoms interfere constructively, the wave cannot propagate easily. What is the condition for strong reflection? (Hint: Bragg reflection, path difference. See Figure 12.24). [a]  $\lambda = a$  [b]  $\lambda = 2a$  [c]  $a = n\lambda$  ( $n$ =integer)

Go to Frame 22.

### Frame 22

The condition for strong constructive interference of reflected waves is [b]  $\lambda = 2a$  (or more generally  $n\lambda = 2a$ , see Eq 12.29).

In terms of wavenumber  $k = 2\pi/\lambda$ , this condition becomes  $k = n\pi/a$ . At these specific wavenumbers, the electron wave is strongly reflected, and simple traveling waves are no longer solutions. Instead, the solutions become standing waves.

Two types of standing waves can form at  $k = \pi/a$ :  $\psi_+$  and  $\psi_-$ . (See Eqs 12.33, 12.34). Where does the  $\psi_+$  wave concentrate its probability density (charge)? Where does  $\psi_-$  concentrate its charge? (See Figure 12.26).  $\psi_+$ : \_\_\_\_\_  $\psi_-$ : \_\_\_\_\_

Go to Frame 23.

### Frame 23

$\psi_+$  concentrates charge *between* the positive ion cores.  $\psi_-$  concentrates charge *over* the positive ion cores.

Which standing wave state,  $\psi_+$  or  $\psi_-$ , has lower potential energy? (Hint: Where is the electron relative to the attractive ions? Recall Footnote 6.) \_\_\_\_\_

Go to Frame 24.

### Frame 24

The state  $\psi_-$  has lower potential energy because it places the electron charge density directly over the attractive positive ion cores. The state  $\psi_+$  places the charge between the ions and has higher potential energy.

Since both standing wave states have the same kinetic energy (same magnitude of momentum  $k$ ), the difference in potential energy leads to a difference in total energy. The  $\psi_-$  state has slightly lower total energy than a free electron with wavenumber  $k = \pi/a$ , and the  $\psi_+$  state has slightly higher total energy.

What does this energy difference at  $k = n\pi/a$  create in the Energy vs.  $k$  diagram for the electron in the periodic lattice? (See Figure 12.25). \_\_\_\_\_

Go to Frame 25.

### Frame 25

This energy difference creates the **forbidden energy gaps** at the specific wavenumbers  $k = n\pi/a$ . Electrons cannot have energies within these gaps while propagating through the periodic lattice. The allowed energies form bands. This approach confirms the existence of energy bands and gaps derived from the isolated-atom approach.

Go to Frame 26.

### Frame 26

We can modify the conductivity of semiconductors dramatically by adding specific impurities, a process called **doping**.

If we add an impurity atom with five valence electrons (like arsenic) to a semiconductor made of Group IV elements (like silicon), four electrons form bonds, leaving one extra electron. This type of impurity is called a **donor** because it donates an electron. This electron is loosely bound and has an energy level ( $E_d$ ) just below the conduction band (Figure 12.27b). Does adding donors increase or decrease the number of conduction electrons? [ Increase — Decrease ]

Go to Frame 27.

### Frame 27

Adding donors [Increases] the number of conduction electrons, as the donated electron is easily excited into the conduction band. This creates an **n-type semiconductor** (n for negative charge carriers).

Similarly, adding impurities with three valence electrons creates holes, resulting in **p-type semiconductors**. Fabricating regions of n-type and p-type material next to each other forms the basis of **p-n junctions**, which are fundamental to transistors, diodes, and semiconductor lasers (as mentioned briefly in Figure 12.27a).

This concludes the section on Band Theory of Solids.

End of Section.

## 34 Classical Free Electron Model of Metals

### Frame 1

Following the discovery of the electron, early models tried to explain the properties of metals. Drude and Thomson proposed the **free electron theory**.

What is the basic idea of this model? How does it picture a metal? \_\_\_\_\_

Go to Frame 2 for the answer.

### Frame 2

The model treats a metal as a **classical gas of conduction electrons** moving freely through a **fixed lattice of positive ion cores**. (Think of it like a pinball machine, where the electrons are the balls and the ions are the fixed posts, though the electrons move randomly until a field is applied).

This model was developed by Thomson, Drude, and Lorentz to explain properties like electrical and thermal conductivity.

Go to Frame 3.

### Frame 3

In this classical model, how do the electrons move \*without\* an external electric field applied? (See Figure 12.11a). \_\_\_\_\_

Go to Frame 4.

### Frame 4

Without an electric field, the electrons move **randomly** in straight lines between collisions with the fixed lattice ions.

The model uses classical physics. According to the equipartition theorem from classical statistical mechanics, the average kinetic energy of a gas particle is related to temperature  $T$ . For an electron with mass  $m_e$ , the root-mean-square speed ( $v_{rms}$ ) is given by:

$$\frac{1}{2}m_e\overline{v^2} = \frac{1}{2}m_ev_{rms}^2 = \frac{3}{2}k_BT$$

Which rearranges to:

$$v_{rms} = \sqrt{\frac{3k_BT}{m_e}}$$

At room temperature (around 300 K), this speed is quite high, roughly  $10^5$  m/s.

Go to Frame 5.

### Frame 5

The straight path an electron travels between collisions is called a "free path". Two important related concepts are:

- **Mean Free Path (L):** The average distance traveled between collisions.
- **Mean Free Time ( $\tau$ ):** The average time interval between collisions.

How are  $L$ ,  $\tau$ , and the average speed (approximated by  $v_{rms}$ ) related?

$$L = \dots$$

Go to Frame 6.

### Frame 6

The relationship is simply distance = speed  $\times$  time:

$$L = v_{rms}\tau$$

In the original Drude model,  $L$  was assumed to be very small, around the distance between atoms (a few angstroms).

Go to Frame 7.

### Frame 7

Now, what happens when an external electric field  $\mathbf{E}$  is applied to the metal? An electron with charge  $-e$  experiences a force  $\mathbf{F} = -e\mathbf{E}$ . This force causes an acceleration  $\mathbf{a} = \mathbf{F}/m_e = -e\mathbf{E}/m_e$  between collisions.

This acceleration causes a small, directed motion superimposed on the large random thermal motion. What is this average, directed velocity called? (See page 415). \_\_\_\_\_

Go to Frame 8.

### Frame 8

This small average velocity caused by the electric field is called the **drift velocity**, denoted by  $\mathbf{v}_d$ . It is typically \*much\* smaller than  $v_{rms}$ . (Figure 12.11b exaggerates  $v_d$  for clarity).

We can estimate  $v_d$ . The displacement  $s$  of an electron due to the field during one mean free time  $\tau$  is approximately  $s \approx \frac{1}{2}a\tau^2 = \frac{1}{2}\left(\frac{eE}{m_e}\right)\tau^2$ . The average drift speed is this displacement divided

by the time,  $v_d = s/\tau$ . A more careful calculation considering the averaging over many collision times (Eq 12.7 to 12.10) gives:

$$v_d = \frac{eE}{m_e}\tau$$

Note: The vector velocity  $\mathbf{v}_d$  is opposite to  $\mathbf{E}$  because of the electron's negative charge.

Go to Frame 9.

### Frame 9

We can now relate the drift velocity to the current density  $\mathbf{J}$  (current per unit area). If there are  $n$  conduction electrons per unit volume, each with charge  $-e$ , moving with drift velocity  $\mathbf{v}_d$ , the current density is given by (see Figure 12.12 and Eq 12.11):

$$\mathbf{J} = n(-e)\mathbf{v}_d$$

Substitute the expression for  $v_d$  from Frame 8 (remembering the vector directions,  $\mathbf{v}_d$  is opposite  $\mathbf{E}$ ):

$$\mathbf{J} = n(-e) \left( \frac{-e\mathbf{E}}{m_e}\tau \right) = \frac{ne^2\tau}{m_e}\mathbf{E}$$

This equation shows that  $\mathbf{J}$  is proportional to  $\mathbf{E}$ . What is this law called? \_\_\_\_\_

Go to Frame 10.

### Frame 10

The proportionality  $\mathbf{J} \propto \mathbf{E}$  is **Ohm's Law**. We write it as  $\mathbf{J} = \sigma\mathbf{E}$ , where  $\sigma$  is the **electrical conductivity**.

From the result in Frame 9, what is the classical model's prediction for the conductivity  $\sigma$ ?

$$\sigma = \dots$$

Go to Frame 11.

### Frame 11

Comparing  $\mathbf{J} = \sigma\mathbf{E}$  with  $\mathbf{J} = \frac{ne^2\tau}{m_e}\mathbf{E}$ , the classical model predicts:

$$\sigma = \frac{ne^2\tau}{m_e}$$

We can also express this using the mean free path  $L = v_{rms}\tau$ :

$$\sigma = \frac{ne^2L}{m_e v_{rms}}$$

The reciprocal of conductivity is resistivity,  $\rho = 1/\sigma$ .

Does this classical formula accurately predict measured conductivities? (Hint: See Example 12.1c). [ Yes — No — Partially ]

Go to Frame 12.

### Frame 12

The answer is [No] (or maybe [Partially]).

Example 12.1c shows that using reasonable classical estimates for  $L$  (interatomic spacing) and  $v_{rms}$  gives a conductivity for copper that is about 10 times \*smaller\* than the measured value. Even worse, if we work backwards from the measured conductivity (as done on page 423), we find the required mean free path  $L$  is about 390 Å, which is 150 times the atomic spacing! Clearly, the classical picture of electrons bumping into every adjacent ion is incorrect.

Furthermore, the classical model predicts  $\sigma \propto 1/v_{rms} \propto 1/\sqrt{T}$ , meaning resistivity  $\rho \propto \sqrt{T}$ . However, experiments (Figure 12.13) show that at common temperatures,  $\rho \propto T$ .

Go to Frame 13.

### Frame 13

The classical free electron model was also used to explain **thermal conductivity**,  $K$ . The idea is that the same free electrons that carry charge also carry thermal energy.

A remarkable success of the classical model was predicting a relationship between thermal conductivity ( $K$ ) and electrical conductivity ( $\sigma$ ). This relationship is known as the **Wiedemann-Franz Law**. The model predicted the ratio  $K/\sigma$  should be proportional to the absolute temperature  $T$ :

$$\frac{K}{\sigma} = \mathcal{L}T$$

where  $\mathcal{L}$  is the **Lorentz number**, predicted by the classical model to be a universal constant:

$$\mathcal{L}_{classical} = \frac{3k_B^2}{2e^2} \approx 1.1 \times 10^{-8} \text{ W } \Omega \text{ K}^{-2}$$

How well does this prediction match experimental results? (See Table 12.7 and surrounding text). [a] Poorly [b] Reasonably well, but not perfectly [c] Perfectly

Go to Frame 14.

### Frame 14

Your answer was [ a — b — c ].

The best answer is [b]. The classical Wiedemann-Franz law works surprisingly well! Table 12.7 shows that the experimental Lorentz number ( $K/(\sigma T)$ ) is indeed nearly constant for different metals and temperatures, confirming the  $\propto T$  relationship. However, the measured numerical value is closer to  $2.2 - 2.5 \times 10^{-8} \text{ W } \Omega \text{ K}^{-2}$ , which is about twice the classical prediction.

So, the classical model correctly predicted the form of Ohm's Law and the Wiedemann-Franz Law, which was strong evidence that free electrons were responsible for both electrical and thermal conduction. However, it failed to predict the correct numerical values for  $\sigma$  and  $K$  individually, the correct temperature dependence of  $\rho$ , and the correct value for the Lorentz number.

What needs to be changed in the model to fix these issues? \_\_\_\_\_

Go to Frame 15.

### Frame 15

The deficiencies of the classical model are fixed by incorporating **quantum mechanics**. Two key changes are needed:

1. **Electron Statistics:** Electrons obey Fermi-Dirac statistics, not classical Maxwell-Boltzmann statistics. This means only electrons near the **Fermi energy** ( $E_F$ ) participate significantly in conduction. Consequently, the relevant speed is the **Fermi speed** ( $v_F$ ), not  $v_{rms}$ . (See Figure 12.14).
2. **Electron Mean Free Path:** Electrons are waves. They can travel through a \*perfect\* crystal lattice without scattering, meaning  $L$  would be infinite! Scattering only occurs due to **deviations**



**from periodicity**, such as: \* Vibrations of the ions (phonons) - this causes the temperature dependence. \* Impurities and defects - this causes a residual resistivity even at T=0 K (Matthiessen's Rule,  $\rho = \rho_i + \rho_L$ ).

Go to Frame 16.

### Frame 16

Replacing  $v_{rms}$  with  $v_F$  in the formulas for  $\sigma$  and  $K$ :

$$\sigma = \frac{ne^2L}{m_e v_F} \quad \text{and} \quad K = \frac{1}{3} C_v v_F L$$

Also, the electronic heat capacity  $C_v$  is much smaller than the classical prediction ( $3/2nk_B$ ) because only electrons near  $E_F$  can absorb thermal energy. The quantum model gives  $C_v \propto (k_B T/E_F)nk_B$ .

When the ratio  $K/\sigma T$  is recalculated using the quantum  $C_v$  and  $v_F$  (see page 422), the result is:

$$\mathcal{L}_{quantum} = \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \text{ W } \Omega \text{ K}^{-2}$$

This quantum Lorentz number is in excellent agreement with experimental values (Table 12.7).

This concludes our look at the classical free electron model and the quantum improvements needed. The quantum theory provides a much more accurate description of electron behavior in metals.

End of Section.

## 35 Impurity Semiconductors

### Frame 1

Most modern semiconductor devices, like diodes and transistors, rely on **impurity semiconductors**. These are created by intentionally adding small amounts of specific impurities to **intrinsic** (pure) semiconductors. This process is called **doping**.

Why are impurities deliberately added to semiconductors? \_\_\_\_\_

Go to Frame 2.

### Frame 2

Impurities are added to control and enhance the electrical conductivity of the semiconductor material. Pure semiconductors often don't conduct electricity very well under normal conditions.

Let's look at silicon (Si), a common semiconductor. Silicon is in Group IV of the periodic table. How many valence electrons does each Si atom have, and how does it bond in a crystal? (See Figure 10-27a in the original text for a visual hint). \_\_\_\_\_

Go to Frame 3.

### Frame 3

Each Silicon (Si) atom has **four** valence electrons. In a crystal, it forms four covalent bonds with its neighbors, sharing one electron with each neighbor (Figure 10-27a).

Now, suppose we dope silicon by replacing a few Si atoms with Arsenic (As) atoms. Arsenic is in Group V. How many valence electrons does Arsenic have? \_\_\_\_\_

Go to Frame 4.

#### Frame 4

Arsenic (As) has **five** valence electrons.

When an As atom replaces a Si atom in the crystal lattice (Figure 10-28a), four of its valence electrons form covalent bonds with the neighboring Si atoms, just like a Si atom would.

What happens to the fifth valence electron from the As atom? \_\_\_\_\_

Go to Frame 5.

#### Frame 5

The fifth valence electron is only weakly bound to the Arsenic atom. It doesn't fit into the regular bonding structure. This extra electron occupies an energy level that is typically just \*below\* the conduction band (Figure 10-28b).

Because this impurity atom *donates* an extra electron that can be easily excited into the conduction band to carry current, these energy levels are called **donor levels**, and the impurity (like As in Si) is called a **donor impurity**.

A semiconductor doped with donor impurities has excess negative charge carriers (electrons). What type of semiconductor is this called? [ p-type — n-type ]

Go to Frame 6.

#### Frame 6

Your answer was [ p-type — n-type ].

The correct answer is **n-type** semiconductor, because the primary charge carriers created by the doping are negative electrons.

The energy required to free this fifth electron (ionize the donor atom) is much less than the band gap energy ( $E_g$ ) needed to move an electron from the valence band to the conduction band in pure silicon. We can estimate this ionization energy.

Go to Frame 7.

#### Frame 7

The weakly bound electron orbiting the arsenic ion core ( $\text{As}^+$ ) can be modeled similarly to the electron in a hydrogen atom. However, the electron is moving within the silicon crystal, not free space. This modifies the hydrogen energy formula (Equation 4-6 from the original text, not shown here) in two main ways:

1. The electron mass  $m_e$  must be replaced by the **effective mass**  $m^*$  of an electron moving in the crystal lattice.
2. The Coulomb force is weakened because the charges are embedded in a medium with a dielectric constant  $\kappa$ . This means  $k_e = 1/(4\pi\epsilon_0)$  is replaced by  $k_e/\kappa$ .

With these changes, the energy levels are approximately (Eq. 10-43):

$$E_n = -\frac{1}{2} \left( \frac{k_e^2 m^*}{\kappa^2 m_e} \right) \frac{m_e e^4}{\hbar^2} \frac{1}{n^2} = -\frac{1}{\kappa^2} \left( \frac{m^*}{m_e} \right) \frac{13.6 \text{ eV}}{n^2}$$

(We use the known ground state energy of hydrogen, 13.6 eV). The corresponding Bohr radius is modified as (Eq. 10-44):

$$\langle r_n \rangle = a_0 \kappa \frac{m_e}{m^*} n^2$$

where  $a_0$  is the standard Bohr radius (0.0529 nm).

Go to Frame 8.

### Frame 8

For silicon,  $\kappa \approx 11.7$  and  $m^*/m_e \approx 0.2$ . Using these values in the formula from Frame 7 for the ground state ( $n = 1$ ), the ionization energy  $E_1$  for the donor electron is roughly 0.020 eV.

Compare this to the band gap energy of silicon,  $E_g \approx 1.1$  eV. Is the energy needed to free the donor electron much smaller or much larger than the band gap energy? \_\_\_\_\_

Go to Frame 9.

### Frame 9

The donor ionization energy ( 0.020 eV) is **much smaller** than the band gap energy ( 1.1 eV).

This means that even at room temperature, there is enough thermal energy ( $k_B T \approx 0.025$  eV) to easily excite these donor electrons into the conduction band, making them available to conduct electricity. The donor levels are effectively very close to the conduction band.

These easily excited electrons become the **majority carriers** in an n-type semiconductor.

Go to Frame 10.

### Frame 10

Now, let's consider doping silicon with an element from Group III, such as Gallium (Ga). How many valence electrons does Gallium have? \_\_\_\_\_

Go to Frame 11.

### Frame 11

Gallium (Ga) has **three** valence electrons.

When a Ga atom replaces a Si atom (Figure 10-29a), it can only form three covalent bonds with its neighbors. This leaves one bond incomplete, creating a vacancy or **hole** where an electron is missing.

This hole represents an energy level slightly *\*above\** the valence band (Figure 10-29b). An electron from the nearby valence band can easily gain a small amount of thermal energy to jump into this hole.

What happens in the valence band when an electron jumps into the hole associated with the Gallium atom? \_\_\_\_\_

Go to Frame 12.

### Frame 12

When an electron from the valence band moves into the hole at the impurity site, it leaves behind a **new hole** in the valence band. This new hole can then be filled by another electron from the valence band, and so on. The net effect is that the hole appears to move through the valence band, acting like a positive charge carrier.

Because the impurity atom (Ga) *accepts* an electron from the valence band, it is called an **acceptor impurity**, and the energy levels associated with the holes are called **acceptor levels**.

A semiconductor doped with acceptor impurities has excess positive charge carriers (holes). What type of semiconductor is this called? [ p-type — n-type ]

Go to Frame 13.

### Frame 13

Your answer was [ p-type — n-type ].

The correct answer is **p-type** semiconductor, because the primary (majority) charge carriers created by the doping are positive holes.

Similar to donor levels, acceptor levels lie very close to the valence band, meaning only a small amount of energy is needed for an electron to leave the valence band and fill the acceptor site, creating a mobile hole in the valence band.

In summary:

- **n-type**: Doped with donors (Group V in Si), extra electrons, majority carriers are electrons. Donor levels near conduction band.
- **p-type**: Doped with acceptors (Group III in Si), missing electrons (holes), majority carriers are holes. Acceptor levels near valence band.

Go to Frame 14.

### Frame 14

Example 10-9 in the original text shows how to use the formulas from Frame 7 (Eqs. 10-43 and 10-44) to calculate the ionization energy and orbital radius for a donor electron in Germanium (Ge), another semiconductor. The principles are the same as for Silicon, but the values for effective mass ( $m^*$ ) and dielectric constant ( $\kappa$ ) are different.

This concludes the introduction to impurity semiconductors.

End of Section.

## 36 Semiconductors

### Frame 1

We now turn our attention to semiconductors. These materials are typically insulators at very low temperatures, but their electrical conductivity increases significantly as temperature rises. This behavior is due to their specific electronic energy band structure.

Like insulators, semiconductors have a filled (or nearly filled) **valence band** and an empty (or nearly empty) **conduction band**, separated by an energy gap.

What is the key difference between a semiconductor and an insulator regarding this energy gap? [a] The gap is much larger in semiconductors. [b] The gap is much smaller in semiconductors. [c] Semiconductors have no energy gap.

Go to Frame 2.

### Frame 2

Your answer was [ a — b — c ].

The correct answer is [b]. The defining characteristic of a semiconductor is that its energy gap ( $W_g$ ) is relatively small (typically around 1 eV). For insulators, this gap is much larger (e.g., 5 eV for diamond).

Because the gap is small, at room temperature, thermal energy is sufficient to excite a noticeable number of electrons from the valence band across the gap into the conduction band. Electrons in the conduction band are free to move and conduct electricity.

The figure below shows a simplified band structure diagram. What do  $W_g$ , "Val. p.", and "Prev. p." represent? (Prev. p. is short for 'Prevodni pas', Val. p. for 'Valenčni pas').

[Figure showing Valence Band (Val. p.) below, Conduction Band (Prev. p.) above, separated by energy  $W_g$ , plotted against wave vector  $k$ ]

1.  $W_g$ : \_\_\_\_\_ 2. Val. p.: \_\_\_\_\_ 3. Prev. p.: \_\_\_\_\_  
Go to Frame 3.

### Frame 3

The labels represent: 1.  $W_g$ : The energy gap between the valence and conduction bands. 2. Val. p.: The Valence Band. 3. Prev. p.: The Conduction Band (Prevodni pas).

The horizontal axis,  $k$ , represents the wave vector, which is related to the momentum of the electrons. The diagram shows that energy ( $W$ ) depends on  $k$ . The simplest models often assume the maximum of the valence band and the minimum of the conduction band occur at  $k = 0$ , though this is not always true in real materials.

What are two common semiconductor materials mentioned in the original text, and what are their approximate band gaps? 1. Material: \_\_\_\_\_,  $W_g \approx$  \_\_\_\_\_eV 2. Material: \_\_\_\_\_,  $W_g \approx$  \_\_\_\_\_eV  
Go to Frame 4.

### Frame 4

Two common elemental semiconductors are: 1. Silicon (Si),  $W_g \approx 1.1$  eV 2. Germanium (Ge),  $W_g \approx 0.7$  eV

Compound semiconductors like Gallium Arsenide (GaAs) are also very important.

Now let's consider intrinsic semiconductors (pure, undoped materials) in more detail.

Go to Frame 5.

### Frame 5

## 14.1 Intrinsic Semiconductors

In a pure semiconductor at a temperature  $T > 0$  K, some electrons are thermally excited from the valence band to the conduction band. Let  $N_c$  be the number (or density) of electrons in the conduction band.

When an electron moves to the conduction band, it leaves behind an empty state in the valence band. This empty state behaves like a positive charge carrier and is called a **hole**. Let  $N_v$  be the number (or density) of holes in the valence band.

In an intrinsic (pure) semiconductor, how must  $N_c$  and  $N_v$  be related? [a]  $N_c > N_v$  [b]  $N_c < N_v$  [c]  $N_c = N_v$

Go to Frame 6.

### Frame 6

Your answer was [ a — b — c ].

The correct answer is [c]  $N_c = N_v$ . In a pure semiconductor, every electron that enters the conduction band leaves exactly one hole behind in the valence band. The number of electrons and holes created thermally must be equal.

Calculating  $N_c$  involves integrating the product of the density of available states in the conduction band ( $dg_p$ ) and the probability that a state is occupied (given by the Fermi-Dirac distribution function,  $f(W')$ ):

$$N_c = \int_{W_g}^{\infty} f(W') dg_p$$

where the integral is over the conduction band energies (starting from the bottom of the conduction band,  $W_g$ , assuming the bottom of the valence band is  $W = 0$ ).

The Fermi-Dirac distribution is:

$$f(W') = \frac{1}{e^{\beta(W' - W_F)} + 1}$$

Here  $\beta = 1/(k_B T)$ , and  $W_F$  is a crucial parameter. What is  $W_F$  called? \_\_\_\_\_

Go to Frame 7.

### Frame 7

$W_F$  is the **Fermi energy** or **Fermi level**. It represents the energy level at which the probability of a state being occupied is exactly 1/2. Its position relative to the bands determines the electron and hole concentrations.

Similarly, the number of holes  $N_v$  is found by integrating the probability of a state being \*empty\* ( $1 - f(W)$ ) over the valence band states ( $dg_v$ ):

$$N_v = \int_{-\infty}^0 [1 - f(W)] dg_v$$

(The exact forms for the density of states  $dg_p$  and  $dg_v$  involve effective masses  $m_e^*$  and  $m_h^*$  and depend on  $(W - W_g)^{1/2}$  and  $(-W)^{1/2}$  respectively, based on assuming parabolic bands near the band edges).

By setting the calculated  $N_c = N_v$ , one can solve for the Fermi level  $W_F$ . For an intrinsic semiconductor, where does  $W_F$  typically lie? [a] In the valence band [b] In the conduction band [c] Near the middle of the energy gap ( $W_g/2$ )

Go to Frame 8.

### Frame 8

Your answer was [ a — b — c ].

The correct answer is [c]. For an intrinsic semiconductor, the Fermi level  $W_F$  lies very close to the middle of the energy gap. If the effective masses of electrons and holes were exactly equal, it would be precisely at  $W_g/2$ . Small differences in effective mass and the exact shape of the density of states cause slight shifts, and it also has a slight temperature dependence.

Knowing that  $N_c = N_v$  and that  $W_F \approx W_g/2$  allows us to calculate the intrinsic carrier concentration.

Go to Frame 9.

### Frame 9

#### 14.1.1 Electric Current and Holes

Now, how does current flow in a semiconductor when an electric field  $E$  is applied? Both the electrons in the conduction band and the holes in the valence band contribute.

Electrons (charge -e) move opposite to the electric field  $E$ . Holes (act as charge +e) move in the same direction as the electric field  $E$ .

The total current density  $j$  is the sum of the electron current density  $j_e$  and the hole current density  $j_v$ :

$$j = j_e + j_v$$

The current densities are related to the average drift velocities  $\langle v_e \rangle$ ,  $\langle v_v \rangle$  and the carrier densities  $n_e$  (which is  $N_c$  from before) and  $n_v$  (which is  $N_v$  from before):

$$j_e = -en_e \langle v_e \rangle$$

$$\dot{j}_v = +en_v \langle v_v \rangle$$

Go to Frame 10.

## Frame 10

The average drift velocity of carriers is typically proportional to the applied electric field  $E$ . The proportionality constant is called the **mobility**, denoted by  $\beta$ .

$$\langle v_e \rangle = -\beta_e E \quad (\text{electrons move opposite to } E)$$

$$\langle v_y \rangle = +\beta_y E \quad (\text{holes move along } E)$$

where  $\beta_e$  is the electron mobility and  $\beta_v$  is the hole mobility.

Substitute these into the expressions for  $j_e$  and  $j_v$  from Frame 9. What are the resulting formulas for  $j_e$  and  $j_v$  in terms of  $n_e, n_v, \beta_e, \beta_v$ , and  $E$ ?

$$j_e = \underline{\hspace{10cm}} \quad j_v = \underline{\hspace{10cm}}$$

Go to Frame 11.

## Frame 11

Substituting the drift velocities gives:

$$j_e = -en_e(-\beta_e E) = en_e\beta_e E$$

$$j_v = +en_v(+\beta_v E) = en_v\beta_v E$$

Notice both contributions add positively to the current in the direction of  $E$ . The total current density is:

$$j = j_e + j_v = (en_e\beta_e + en_v\beta_v)E$$

This looks like Ohm's law ( $j = \sigma E$ ), where  $\sigma = (en_e\beta_e + en_v\beta_v)$  is the conductivity.

Mobility ( $\beta$ ) is a measure of how easily charge carriers move through the crystal lattice under the influence of an electric field. What units might mobility have? (Hint: velocity = mobility \* field). \_\_\_\_\_

Go to Frame 12.

## Frame 12

Velocity is (m/s). Electric field is (V/m). So, Mobility = Velocity / Field has units of (m/s) / (V/m) =  $\text{m}^2/(\text{V}\cdot\text{s})$ . The example values for Silicon (Si) given were  $\beta_e \approx 0.16 \text{ m}^2/(\text{V}\cdot\text{s})$  and  $\beta_v \approx 0.04 \text{ m}^2/(\text{V}\cdot\text{s})$  at room temperature.

This covers the basics of intrinsic semiconductors. The next step in understanding semiconductor devices involves adding impurities, known as doping.

End of Section.

## 37 Semiconductors and Doping

### Frame 1

We now turn our attention to semiconductors, materials crucial for modern electronics. Their conductivity lies between that of conductors (like metals) and insulators. A key feature is the arrangement of electron energy levels into **bands**.

For basic semiconductor behavior, we are primarily interested in two bands:

- **Valence Band:** The outermost band containing electrons involved in bonding. In an insulator or semiconductor at absolute zero temperature (0 K), this band is completely filled with electrons.
- **Conduction Band:** The next higher energy band. Electrons in this band are free to move and conduct electricity. In an insulator or semiconductor at 0 K, this band is empty.

These two bands are separated by a forbidden energy region called the **energy gap** (or band gap), denoted  $E_g$ . (See Figure 12.45a).

For Silicon (Si), a common semiconductor, the band gap  $E_g \approx 1.1$  eV (electron volts). Can electrons spontaneously jump from the valence band to the conduction band in Si at 0 K? [ Yes — No ]

Go to Frame 2.

### Frame 2

The answer is [No]. At 0 K, electrons in the valence band do not have enough energy to cross the energy gap  $E_g$  and reach the conduction band. Therefore, pure semiconductors act as insulators at absolute zero.

What happens at room temperature? Recall that the average thermal energy is about  $k_B T \approx 0.025$  eV at room temperature. Can a significant number of electrons gain enough thermal energy (1.1 eV) to jump the gap in pure Si? [ Yes — No ]

Go to Frame 3.

### Frame 3

The answer is [No]. While not strictly zero, the number of electrons with enough thermal energy to jump the 1.1 eV gap in pure silicon at room temperature is very small. Pure semiconductors (called **intrinsic semiconductors**) are poor conductors.

However, we can dramatically change the conductivity by adding specific impurities. This process is called **doping**.

Go to Frame 4.

### Frame 4

Let's consider doping silicon (which is in Group IV of the periodic table, having 4 valence electrons) with an element from Group V, such as Arsenic (As) or Phosphorus (P), which have 5 valence electrons.

When an As atom replaces a Si atom in the crystal lattice (Figure 12.27a), four of its valence electrons form covalent bonds with the neighboring Si atoms. What happens to the fifth valence electron? \_\_\_\_\_

Go to Frame 5.



### Frame 5

The fifth valence electron is left over. It is only weakly bound to its parent As atom. This extra electron requires very little energy to become free and move into the conduction band.

Atoms like As or P that contribute extra electrons are called **donor atoms**. The energy level of this fifth electron ( $E_d$ ) lies within the energy gap, but very close to the bottom of the conduction band (Figure 12.27b). Typically,  $E_c - E_d \approx 0.05$  eV, where  $E_c$  is the energy at the bottom of the conduction band.

Is the thermal energy at room temperature ( $k_B T \approx 0.025$  eV) sufficient to easily excite these donor electrons into the conduction band? [ Yes — No ]

Go to Frame 6.

### Frame 6

The answer is [Yes]. Since the energy required ( $E_c - E_d \approx 0.05$  eV) is comparable to the available thermal energy, many of these donor electrons are easily excited into the conduction band at room temperature, becoming mobile charge carriers.

Because the charge carriers contributed by donor atoms are negative electrons, a semiconductor doped with donors is called an **n-type semiconductor**.

Go to Frame 7.

### Frame 7

Now, let's consider doping silicon (Group IV) with an element from Group III, such as Indium (In), Gallium (Ga), or Aluminum (Al), which have 3 valence electrons.

When a Ga atom replaces a Si atom, its three valence electrons form covalent bonds with neighbours. However, one bond with a neighbouring Si atom is incomplete, leaving an electron deficiency (Figure 12.28a). This deficiency is called a **hole**.

What charge does a hole effectively carry? [ Positive — Negative — Neutral ]

Go to Frame 8.

### Frame 8

A hole acts as a **positive** charge carrier. An electron from the nearby valence band can easily move into the hole, effectively causing the hole to move in the opposite direction.

Atoms like Ga or Al that create these holes are called **acceptor atoms** because they can easily "accept" an electron from the valence band. The energy level associated with this hole ( $E_a$ ) lies within the band gap, but very close to the \*top\* of the valence band (Figure 12.28b). Typically,  $E_a - E_v \approx 0.05$  eV, where  $E_v$  is the energy at the top of the valence band.

At room temperature, thermal energy allows electrons from the valence band to easily jump into these acceptor levels, leaving mobile holes behind in the valence band. Because the dominant charge carriers (holes) are positive, a semiconductor doped with acceptors is called a **p-type semiconductor**.

Semiconductors doped with either donors or acceptors are called **extrinsic semiconductors** or impurity semiconductors. Typical doping densities range from  $10^{13}$  to  $10^{19}$  atoms per  $\text{cm}^3$ .

Go to Frame 9.

### Frame 9

In semiconductors, electric current can be carried by both electrons (in the conduction band) and holes (in the valence band). Let  $n_e$  be the concentration (number per unit volume, N/V) of conduction electrons and  $n_h$  (or  $p$ ) be the concentration of holes.

Under an applied electric field  $\mathbf{E}$ , electrons drift with an average velocity  $\langle \mathbf{v}_e \rangle$  and holes with  $\langle \mathbf{v}_h \rangle$  (opposite to  $\mathbf{E}$  for electrons, same direction as  $\mathbf{E}$  for holes). The relationship between drift velocity and electric field is defined by mobility,  $\beta$ :

$$\langle \mathbf{v}_e \rangle = -\beta_e \mathbf{E}$$

$$\langle \mathbf{v}_h \rangle = \beta_h \mathbf{E}$$

where  $\beta_e$  and  $\beta_h$  are the electron and hole mobilities, respectively. (The Slovene text uses  $\beta_v$  for hole mobility).

The current density  $\mathbf{j}$  (current per unit area) has contributions from both carriers. Let  $e$  be the magnitude of the electron charge. Electron current density:  $\mathbf{j}_e = (-e)n_e\langle \mathbf{v}_e \rangle = (-e)n_e(-\beta_e \mathbf{E}) = en_e\beta_e \mathbf{E}$  Hole current density:  $\mathbf{j}_h = (+e)n_h\langle \mathbf{v}_h \rangle = (+e)n_h(\beta_h \mathbf{E}) = en_h\beta_h \mathbf{E}$

The total current density is  $\mathbf{j} = \mathbf{j}_e + \mathbf{j}_h$ . What is the expression for the total current density  $\mathbf{j}$  in terms of  $n_e, n_h, \beta_e, \beta_h, e$ , and  $\mathbf{E}$ ?  $\mathbf{j} =$  \_\_\_\_\_

Go to Frame 10.

### Frame 10

The total current density is:

$$\mathbf{j} = \mathbf{j}_e + \mathbf{j}_h = (en_e\beta_e + en_h\beta_h)\mathbf{E}$$

This is a form of Ohm's law,  $\mathbf{j} = \sigma \mathbf{E}$ , where the conductivity  $\sigma = e(n_e\beta_e + n_h\beta_h)$ .

Mobility values depend on the material and temperature. For silicon at room temperature, electron mobility  $\beta_e \approx 0.16 \text{ m}^2/\text{Vs}$  and hole mobility  $\beta_h \approx 0.04 \text{ m}^2/\text{Vs}$ . (Values adapted from Slovene text,  $\beta_v \approx 0.04$ ).

Go to Frame 11.

### Frame 11

Determining the exact concentration of carriers ( $n_e, n_h$ ) involves quantum statistics, specifically the Fermi-Dirac distribution function, which gives the probability that an available energy state  $W$  is occupied by an electron:

$$f_{FD}(W) = \frac{1}{e^{(W-W_F)/k_B T} + 1}$$

Here,  $W_F$  is a crucial parameter called the **Fermi energy** or **Fermi level**. It represents the energy level at which the probability of occupation is exactly 1/2. Its position relative to the energy bands determines the material's electrical properties.

The concentration of electrons in the conduction band,  $N_e$  (or  $n_e$ ), involves integrating the product of the density of available states in the conduction band and the Fermi-Dirac probability function over the entire band. A similar integral applies for holes in the valence band. (See Eq. 14.1 and 14.2 from Slovene text for the form of these integrals, though we won't evaluate them here).

Where do you expect the Fermi level  $W_F$  to be located in an n-type semiconductor at low temperature? [a] Near the valence band [b] Near the middle of the gap [c] Near the donor levels / conduction band

Go to Frame 12.

### Frame 12

Your answer was [ a — b — c ].

The correct answer is [c]. In an n-type semiconductor at low temperature, the Fermi level  $W_F$  lies close to the donor energy levels  $E_d$  (or  $W_d$ ), just below the conduction band. This reflects the fact that the electrons available for conduction primarily come from these donor states.

Conversely, in a p-type semiconductor,  $W_F$  lies close to the acceptor levels  $E_a$ , just above the valence band. In an intrinsic (undoped) semiconductor,  $W_F$  lies near the middle of the band gap.

The position of  $W_F$  and the resulting carrier concentrations ( $n_e, n_h$ ) are temperature-dependent. As temperature increases, more electrons can be thermally excited across the main band gap ( $E_g$ ), and the material starts behaving more like an intrinsic semiconductor, shifting  $W_F$  towards the center of the gap. (This is illustrated in the graphs on Slovene page 5).

Go to Frame 13.

### Frame 13

Now, let's consider what happens when we join a piece of p-type semiconductor to a piece of n-type semiconductor. This forms a **p-n junction**, the basis for diodes, transistors, and many other devices, including semiconductor lasers.

Initially, the n-type material has many free electrons and the p-type has many free holes. What will happen immediately after joining them due to the concentration differences? (Think about diffusion). \_\_\_\_\_

Go to Frame 14.

### Frame 14

Due to the concentration gradient, electrons will diffuse from the n-side to the p-side, and holes will diffuse from the p-side to the n-side.

When electrons diffuse into the p-side, they recombine with holes. When holes diffuse into the n-side, they recombine with electrons. What is left behind in the region near the junction? (See Figure 12.29a). \_\_\_\_\_

Go to Frame 15.

### Frame 15

As the mobile carriers diffuse away and recombine near the junction, they leave behind the fixed, ionized atoms:

- Positive donor ions ( $As^+$ ) on the n-side.
- Negative acceptor ions ( $Ga^-$ ) on the p-side.

This region near the junction, which has lost its mobile carriers, is called the **depletion region** (or space-charge region).

What is the consequence of having fixed positive charges on the n-side and fixed negative charges on the p-side within the depletion region? (Think about electric fields). \_\_\_\_\_

Go to Frame 16.

### Frame 16

The fixed positive and negative ions create an internal **electric field** ( $\mathbf{E}$ ) within the depletion region, pointing from the n-side to the p-side (Figure 12.29b).

This electric field opposes further diffusion of majority carriers (electrons from n, holes from p). It creates a **potential barrier** ( $V_0$ ) across the junction (Figure 12.29c), which prevents net

current flow in equilibrium (no external voltage applied). The height of this barrier depends on the doping levels and temperature.

Go to Frame 17.

### Frame 17

The most notable feature of a p-n junction is its ability to act as a **diode**, allowing current to flow easily in one direction but not the other. This is achieved by applying an external voltage (bias).

**Forward Bias:** A positive external voltage is applied to the p-side and negative to the n-side. Does this applied voltage increase or decrease the potential barrier  $V_0$ ? [ Increase — Decrease ]

**Reverse Bias:** A positive external voltage is applied to the n-side and negative to the p-side. Does this applied voltage increase or decrease the potential barrier  $V_0$ ? [ Increase — Decrease ]

Go to Frame 18.

### Frame 18

Forward Bias: Decreases the barrier. The applied voltage opposes the internal potential  $V_0$ . With a lower barrier, majority carriers (holes from p, electrons from n) can more easily diffuse across the junction, resulting in a significant current flow. Reverse Bias: Increases the barrier. The applied voltage adds to the internal potential  $V_0$ . This makes it even harder for majority carriers to cross. Only a very small leakage current (due to minority carriers generated thermally) flows.

This directional current flow is the diode action. The current-voltage relationship for an ideal diode is given by:

$$I = I_0(e^{qV/k_B T} - 1)$$

where  $V$  is the applied voltage (positive for forward bias, negative for reverse bias),  $q$  is the electron charge magnitude,  $k_B$  is Boltzmann's constant,  $T$  is absolute temperature, and  $I_0$  is the small reverse saturation current.

Go to Frame 19.

### Frame 19

(Optional Section) Another important effect in semiconductors (and conductors) is the **Hall Effect**, which can be used to determine the type (n or p) and concentration of charge carriers.

Imagine a rectangular strip of semiconductor carrying a current  $I$  along its length (say, x-direction). A magnetic field  $\mathbf{B}$  is applied perpendicular to the current (say, z-direction). (See Figure 10-30).

The charge carriers (let's assume charge  $q$  and drift velocity  $\mathbf{v}_d$ ) moving through the magnetic field experience a magnetic force  $\mathbf{F}_m = q(\mathbf{v}_d \times \mathbf{B})$ . In which direction will this force initially push the carriers? [a] Along the current direction (x) [b] Opposite the current direction (-x) [c] Sideways across the strip (y or -y) [d] Along the magnetic field (z or -z)

Go to Frame 20.

### Frame 20

Your answer was [ a — b — c — d ].

The correct answer is [c] Sideways across the strip. According to the right-hand rule for the cross product  $\mathbf{v}_d \times \mathbf{B}$ , the magnetic force is perpendicular to both the velocity (along x) and the magnetic field (along z), so it acts along the y-direction.

This force pushes the charge carriers towards one side of the strip. What happens as charge accumulates on that side? \_\_\_\_\_

Go to Frame 21.

### Frame 21

As charge accumulates on one side, it creates a transverse electric field  $\mathbf{E}_H$  across the strip (the Hall field). This field exerts an electric force  $\mathbf{F}_e = q\mathbf{E}_H$  on the carriers, opposing the magnetic force.

Equilibrium is reached when the electric force exactly balances the magnetic force:  $F_e = F_m$ .

$$qE_H = qv_dB$$

$$E_H = v_dB$$

This electric field  $E_H$  corresponds to a measurable voltage difference  $V_H$  across the width ( $w$ ) of the strip, called the **Hall voltage**:  $V_H = E_H w = v_d B w$ .

By measuring  $V_H$ ,  $I$ ,  $B$ , and the sample dimensions, one can determine both the sign of the charge carriers  $q$  (from the polarity of  $V_H$ ) and their concentration  $n$ . (See Example 10-10 for a calculation).

This concludes the section on basic semiconductor concepts and the p-n junction.

End of Section.

## 38 Semiconductor Devices: Diodes, Transistors, and Integrated Circuits

### Frame 1

In the previous section, we explored the fundamental principles of semiconductor physics, including energy bands, electrons, and holes. Now, we'll look at how these principles are applied in essential electronic devices. We'll start with diodes and then move on to transistors and integrated circuits.

Figure 12.30 (in the original text) shows the characteristic current-voltage (I-V) curve for a real diode. Notice how the current changes dramatically depending on whether the voltage is applied in the forward or reverse bias direction.

Go to Frame 2.

### Frame 2

Recall that a p-n junction allows current to flow easily in one direction (forward bias) but restricts it strongly in the other (reverse bias). Example 12.3 in the text calculates the ratio of forward current ( $I_f$ ) to reverse current ( $I_r$ ) for a typical diode with +1.0 V (forward) and -1.0 V (reverse) applied, using the ideal diode equation (modified slightly from Eq 12.35):

$$\frac{I_{forward}}{I_{reverse}} = \frac{I_0(e^{qV/k_B T} - 1)}{I_0(e^{-qV/k_B T} - 1)}$$

At room temperature,  $k_B T \approx 0.025$  eV. With  $V = 1.0$  V and  $q = e$  (elementary charge), the term  $qV/k_B T$  becomes  $1.0/0.025 = 40$ .

Calculate the approximate ratio  $I_{forward}/I_{reverse}$ . (Note:  $e^{40}$  is enormous, and  $e^{-40}$  is extremely small, close to zero).

Ratio  $\approx$  \_\_\_\_\_

Go to Frame 3 for the calculation and result.

### Frame 3

Let's calculate the ratio:

$$\frac{I_{forward}}{I_{reverse}} = \frac{I_0(e^{40} - 1)}{I_0(e^{-40} - 1)}$$

Since  $e^{40}$  is huge,  $e^{40} - 1 \approx e^{40}$ . Since  $e^{-40}$  is tiny,  $e^{-40} - 1 \approx -1$ .

$$\frac{I_{forward}}{I_{reverse}} \approx \frac{e^{40}}{-1} = -e^{40}$$

The magnitude calculated in the text is  $2.4 \times 10^{17}$ . The negative sign just indicates the currents are in opposite directions, which we already knew. The key point is the huge \*ratio\* of the magnitudes.

This massive difference highlights the diode's primary function: acting as a one-way valve for electric current.

Go to Frame 4.

### Frame 4

Semiconductors can also interact with light, similar to how gas atoms do, but using energy bands instead of discrete levels.

Consider Figure 12.31a. An electron in the higher energy conduction band recombines with a hole in the lower energy valence band. What happens during this recombination? \_\_\_\_\_

Go to Frame 5.

### Frame 5

During recombination, the electron "falls" into the hole, and the energy difference, corresponding to the band gap ( $E_g$ ), is released, often as a **photon** of light ( $hf = E_g$ ).

Devices designed to do this, converting electrical energy into light, are called **Light-Emitting Diodes (LEDs)** or injection lasers (which we discussed previously).

Now consider the reverse process, shown in Figure 12.31b. What happens here? \_\_\_\_\_

Go to Frame 6.

### Frame 6

In Figure 12.31b, an incoming photon with energy  $hf \geq E_g$  is absorbed by an electron in the valence band. This promotes the electron to the conduction band, leaving a hole behind in the valence band. This is light absorption creating an electron-hole pair.

Devices designed to capture this light-generated energy, using a p-n junction's internal field to separate the electron and hole and collect them as current, are called **solar cells** or **photovoltaic devices**.

Go to Frame 7.

### Frame 7

The invention of the **transistor** in 1948 revolutionized electronics. It largely replaced bulky vacuum tubes. A common type is the junction transistor.

What are the two main types of junction transistors mentioned, based on the arrangement of p-type and n-type semiconductor material? 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 8.

### Frame 8

The two types are: 1. The **pnp** transistor (a narrow n-region between two p-regions) 2. The **npn** transistor (a narrow p-region between two n-regions)

Their operation is similar, so we'll focus on the pnp transistor (Figure 12.32).

What are the names given to the three regions of a junction transistor? 1. \_\_\_\_\_ (Outer region, typically heavily doped) 2. \_\_\_\_\_ (Narrow central region) 3. \_\_\_\_\_ (Other outer region)

Go to Frame 9.

### Frame 9

The three regions are: 1. **Emitter** 2. **Base** 3. **Collector**

The transistor has two p-n junctions: one between the emitter and base, and one between the base and collector.

How are these two junctions typically biased (forward or reverse) for the transistor to operate as an amplifier? (Refer to Figure 12.33). 1. Emitter-Base junction: [ Forward — Reverse ] biased  
2. Base-Collector junction: [ Forward — Reverse ] biased

Go to Frame 10.

### Frame 10

For typical amplifier operation: 1. Emitter-Base junction is **Forward** biased (e.g., by battery  $V_{eb}$ ). 2. Base-Collector junction is **Reverse** biased (e.g., by battery  $V_{ec}$ ).

In a pnp transistor, the emitter is p-type and heavily doped. When forward biased, what type of charge carrier flows predominantly from the emitter into the very narrow base? [ Electrons — Holes ]

Go to Frame 11.

### Frame 11

The charge carriers are primarily **Holes** (positive charge carriers) flowing from the heavily doped p-type emitter into the n-type base.

Because the base is very thin, most of these holes \*do not\* recombine with electrons there. Instead, what happens to them? (Hint: consider the base-collector junction bias). \_\_\_\_\_

Go to Frame 12.

### Frame 12

Most holes diffuse across the thin base and are then swept across the *reverse-biased* base-collector junction by its strong electric field, ending up in the collector. This flow of holes constitutes the collector current ( $I_c$ ).

A small number of holes \*do\* recombine in the base, or constitute charge that needs to be drawn off. This small current leaving the base terminal is the base current ( $I_b$ ).

Crucially, a small change in the base current  $I_b$  can cause a large change in the collector current  $I_c$ . This relationship is often written as:

$$I_c = \beta I_b$$

What is  $\beta$  called, and what is its typical range of values? 1. Name of  $\beta$ : \_\_\_\_\_ 2. Typical range: \_\_\_\_\_

Go to Frame 13.

### Frame 13

1.  $\beta$  is called the **current gain**. 2. Its typical range is **10 to 100** (or even higher).

This means the transistor acts as a **current amplifier**: a small input current ( $I_b$ ) controls a much larger output current ( $I_c$ ). By passing  $I_c$  through an output resistor ( $R_L$  in Figure 12.33), variations in  $I_c$  also create amplified voltage variations.

Go to Frame 14.

### Frame 14

Another major type of transistor, widely used today especially in integrated circuits, is the **Field-Effect Transistor (FET)**. A common variant is the MOSFET (Metal-Oxide-Semiconductor FET).

Look at the structure in Figure 12.34a. What are the three main terminals (connections) of a MOSFET? 1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

Go to Frame 15.

### Frame 15

The three terminals are: 1. **Source** 2. **Drain** 3. **Gate**

The source and drain are connected to n-type regions, which are joined by a narrow **n channel**. These sit on a **p-type substrate**. The gate is separated from the n channel by a thin insulating layer (typically silicon dioxide).

When a voltage ( $V_{SD}$ ) is applied between source and drain (Fig 12.34b), current flows through the n channel. How does applying a voltage to the gate ( $V_{SG}$ , Fig 12.34c) control this current? (Hint: consider the electric field from the gate). \_\_\_\_\_

Go to Frame 16.

### Frame 16

Applying a positive voltage to the gate ( $V_{SG}$ ) creates an electric field that penetrates the insulating oxide layer into the n channel below it. This field attracts more electrons into the channel (or pushes holes away from the channel boundary), effectively widening the conductive path.

If the gate voltage widens the channel, what happens to the source-drain current ( $I$ )? [ Increases — Decreases — Stays the same ]

Go to Frame 17.

### Frame 17

The source-drain current **Increases** because the resistance of the wider channel decreases.

Since a small change in the gate voltage  $V_{SG}$  can cause a large change in the source-drain current  $I$  (and thus the voltage across an output resistor), the MOSFET acts as a **voltage amplifier**.

What is another key application of MOSFETs, especially in digital circuits, due to their ability to switch the current on or off with the gate voltage? \_\_\_\_\_

Go to Frame 18.

### Frame 18

MOSFETs are widely used as electronic **switches**. By applying a suitable negative voltage to the gate (reversing  $V_{SG}$  in Fig 12.34c), the channel can be depleted of electrons and effectively



pinched off, stopping the source-drain current. This switching capability, combined with their low power consumption and ease of manufacture, is crucial for digital logic and integrated circuits.

Go to Frame 19.

### Frame 19

The invention of the **Integrated Circuit (IC)** by Jack Kilby and Robert Noyce around 1958-1959 was arguably even more revolutionary than the transistor itself.

What is an integrated circuit, in simple terms? \_\_\_\_\_

Go to Frame 20.

### Frame 20

An integrated circuit is a collection of interconnected electronic components (transistors, diodes, resistors, capacitors) all fabricated on a **single piece (chip)** of semiconductor material, usually silicon.

What were two major advantages of ICs over circuits made from discrete (individual) components like transistors? (See page 441, second paragraph). 1. \_\_\_\_\_ 2. \_\_\_\_\_ (Related to speed)

Go to Frame 21.

### Frame 21

The two main advantages mentioned were: 1. **Miniaturization** (solving the interconnection problem of wiring vast numbers of discrete components). 2. **Fast response** (signals travel much shorter distances between components packed closely on a chip).

The development of IC technology has been remarkable, following "Moore's Law" (proposed by Gordon Moore), which roughly states that the number of transistors on a chip doubles approximately every 18 months to 2 years. This exponential growth is illustrated in Figure 12.38.

This concludes our brief look at semiconductor devices.

End of Section.

## 39 Superconductivity

### Frame 1

We now turn to another fascinating phenomenon in solid-state physics: superconductivity. Some metals and compounds exhibit a remarkable property when cooled below a certain temperature.

What is the most defining characteristic of a **superconductor**? (Hint: Think about electrical resistance.) \_\_\_\_\_

Go to Frame 2.

### Frame 2

The defining characteristic is that their electrical resistance drops abruptly to *zero* when the material is cooled below a specific temperature.

This critical temperature is denoted by  $T_c$ . Above  $T_c$ , the material behaves like a normal conductor (with some resistance), but at or below  $T_c$ , its resistance vanishes completely.

Look at Table 12.9 (page 443 of original text). What is the approximate critical temperature  $T_c$  for Lead (Pb)? [a] 0.88 K [b] 4.15 K [c] 7.19 K [d] 18.0 K

Go to Frame 3.

### Frame 3

Your answer was [ a — b — c — d ].

The correct answer is [c] 7.19 K (Kelvin). Notice that early superconductors required very low temperatures, often needing liquid helium for cooling (boiling point 4 K). The discovery of materials with higher  $T_c$  (like  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  at 92 K, well above liquid nitrogen's boiling point of 77 K) was a major breakthrough.

Go to Frame 4.

### Frame 4

Zero resistance is amazing, but it's not the only unique property of superconductors. Another crucial phenomenon is the **Meissner effect**, which involves how superconductors behave in a magnetic field.

Consider a material above its critical temperature ( $T > T_c$ ) placed in an external magnetic field. The field lines will penetrate the material (Figure 12.39a). Now, imagine we cool the material down below  $T_c$  while keeping it in the magnetic field.

Based on the Meissner effect, what happens to the magnetic field lines \*inside\* the superconductor? [a] They remain unchanged. [b] They become concentrated inside. [c] They are expelled from the interior.

Go to Frame 5.

### Frame 5

Your answer was [ a — b — c ].

The correct answer is [c] They are expelled from the interior. When a material becomes superconducting, it actively pushes magnetic field lines out of its bulk (Figure 12.39b). This expulsion is the Meissner effect.

Is the Meissner effect simply a consequence of having zero resistance (being a "perfect conductor")? [ Yes — No ]

(Hint: Consider what happens if you apply the field \*after\* cooling the material below  $T_c$ . Would a perfect conductor, according to Faraday's law of induction ( $\oint \mathbf{E} \cdot d\mathbf{s} = -d\Phi_B/dt$ ), allow the internal flux  $\Phi_B$  to change once established?)

Go to Frame 6.

### Frame 6

The answer is [No]. The Meissner effect is \*more\* than just perfect conductivity.

A hypothetical "perfect conductor" cooled below  $T_c$  \*first\* and \*then\* placed in a magnetic field would trap the field inside it. Why? Because changing the magnetic flux ( $\Phi_B$ ) inside it would induce an electric field  $\mathbf{E}$  (Faraday's Law). But in a perfect conductor (zero resistance), an electric field would drive an infinite current, which is unphysical. So, the flux must remain constant. If the field was initially zero inside, it would stay zero. If it was non-zero, it would stay non-zero.

However, a true superconductor \*always\* expels the magnetic field when it enters the superconducting state, regardless of whether the field was applied before or after cooling. This active expulsion demonstrates that  $\mathbf{B} = 0$  inside a superconductor (in its bulk).

Go to Frame 7.

### Frame 7

The fact that a superconductor expels magnetic fields ( $\mathbf{B} = 0$  inside) means it behaves as a perfect **diamagnet**.

How does this explain the levitation effect shown in Figure 12.40 (page 445)? A small permanent magnet is levitating above a superconductor cooled with liquid nitrogen. (Hint: Think about induced surface currents and opposing magnetic fields). \_\_\_\_\_

Go to Frame 8.

### Frame 8

When the permanent magnet is brought near the superconductor, the changing magnetic flux induces persistent electrical currents on the \*surface\* of the superconductor (since  $\mathbf{B}$  must remain zero inside). These surface currents create a magnetic field that perfectly opposes and repels the field from the permanent magnet, causing it to levitate.

Go to Frame 9.

### Frame 9

How can we explain superconductivity at a microscopic level? A successful theory was developed in 1957 by John Bardeen, Leon N. Cooper, and J. Robert Schrieffer (the **BCS theory**).

A key concept in BCS theory is the formation of **Cooper pairs**. Under normal circumstances, electrons repel each other due to their negative charge. In certain materials below  $T_c$ , however, electrons can interact indirectly via distortions in the crystal lattice (phonons).

Does this interaction lead to a net attraction or repulsion between two electrons forming a Cooper pair? [ Attraction — Repulsion ]

Go to Frame 10.

### Frame 10

The answer is [Attraction].

An electron moving through the lattice slightly attracts the positive ion cores, creating a region of slight positive charge concentration behind it. A second electron can be attracted to this region, leading to an indirect, weak attraction between the two electrons. These two electrons form a bound state called a Cooper pair.

Cooper pairs behave differently from individual electrons. They act like **bosons** (particles with integer spin), rather than fermions (like individual electrons, which have half-integer spin). Why is this important? Bosons are not restricted by the Pauli exclusion principle in the same way fermions are. Many Cooper pairs can occupy the same quantum state.

Go to Frame 11.

### Frame 11

In the superconducting state, these Cooper pairs move collectively. An energy gap exists, meaning a minimum amount of energy is required to break a pair apart into individual electrons.

How does this collective motion of Cooper pairs explain the zero electrical resistance? (Hint: Think about scattering and energy loss). \_\_\_\_\_

Go to Frame 12.

### Frame 12

Collisions with the lattice, which normally cause resistance by scattering individual electrons and making them lose energy, are generally not energetic enough to break the Cooper pairs below

$T_c$ . The pairs move through the lattice without losing energy to such collisions, resulting in zero electrical resistance.

Go to Frame 13.

### Frame 13

For many years, superconductivity was limited to very low temperatures, requiring expensive liquid helium cooling. A major breakthrough occurred in 1986 with the discovery of materials that became superconducting at much higher temperatures.

Who led the initial discovery of these "high-temperature" superconductors? (See page 446). —  
Go to Frame 14.

### Frame 14

The initial discovery was made by J. Georg Bednorz and K. Alex Müller at the IBM Zurich Research Laboratory. They found superconductivity in an oxide of lanthanum, barium, and copper at around 30 K.

This spurred intense research, quickly leading to the discovery of materials like  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  (YBCO) with  $T_c$  around 92 K, above the boiling point of liquid nitrogen (77 K). This made potential applications much more feasible.

While BCS theory explains conventional superconductivity well, the exact mechanism behind high-temperature superconductivity (often found in complex copper-oxide ceramics) is still an active area of research, though it's believed Cooper pairs are still involved.

This concludes our brief look at superconductivity.  
End of Section.

## 40 Nuclear Structure I

### Frame 1

In 1896, the discovery of radioactivity in uranium compounds by Henri Becquerel marked the beginning of nuclear physics. Early pioneering work, particularly by Ernest Rutherford, classified the emitted radiation into three types based on their properties: alpha, beta, and gamma rays.

What fundamental property was initially used by Rutherford to classify these rays? (Hint: Think about how they behave in electric/magnetic fields). \_\_\_\_\_  
Go to Frame 2.

### Frame 2

Rutherford initially classified the rays according to the nature of the **electric charges** they possessed and their ability to penetrate matter. Later experiments showed:

- Alpha rays are helium nuclei.
- Beta rays are electrons.
- Gamma rays are high-energy photons.

Experiments by Rutherford, Geiger, and Marsden involving scattering alpha particles off thin foils revealed that an atom's positive charge and most of its mass are concentrated in a tiny central core: the **nucleus**. They also deduced the existence of a powerful short-range force, the **nuclear force**, holding the nucleus together.

Go to Frame 3.

### Frame 3

This section explores the basic properties and structure of the atomic nucleus. We'll discuss its constituents, forces, models, and the phenomenon of radioactivity.

All atomic nuclei (except for the simplest hydrogen nucleus) are composed of two types of particles. What are they? 1. \_\_\_\_\_ 2. \_\_\_\_\_

These particles are collectively called **nucleons**.

Go to Frame 4.

### Frame 4

Nuclei are composed of **protons** and **neutrons**.

To describe a nucleus, we use three key numbers:

- **Atomic Number (Z)**: The number of protons. This determines the element.
- **Neutron Number (N)**: The number of neutrons.
- **Mass Number (A)**: The total number of nucleons (protons + neutrons).

How is the mass number A related to Z and N?  $A =$  \_\_\_\_\_

Go to Frame 5.

### Frame 5

The mass number is the sum of the proton and neutron numbers:

$$A = Z + N$$

The standard notation for a nucleus is  ${}^A_ZX$ , where X is the chemical symbol for the element (determined by Z).

For example, iron (Fe) has  $Z=26$ . One common form of iron is  ${}^{56}_{26}\text{Fe}$ . For this nucleus: What is A? \_\_\_\_\_ What is Z? \_\_\_\_\_ What is N? \_\_\_\_\_

Go to Frame 6.

### Frame 6

For  ${}^{56}_{26}\text{Fe}$ :  $A = 56$  (Mass Number)  $Z = 26$  (Atomic Number = number of protons)  $N = A - Z = 56 - 26 = 30$  (Neutron Number)

Nuclei of the same element (same Z) can have different numbers of neutrons (different N, and therefore different A). What are these called? \_\_\_\_\_

Go to Frame 7.

### Frame 7

Atoms of the same element (same Z) but with different numbers of neutrons (N) are called **isotopes** of that element.

For example, Carbon ( $Z=6$ ) has isotopes like  $^{12}\text{C}$  ( $N=6$ ) and  $^{14}\text{C}$  ( $N=8$ ). Hydrogen ( $Z=1$ ) has isotopes:  $^1\text{H}$  (ordinary hydrogen,  $N=0$ ),  $^2\text{H}$  (deuterium,  $N=1$ ), and  $^3\text{H}$  (tritium,  $N=2$ ).

Go to Frame 8.

## Frame 8

### Charge and Mass

Protons carry a positive charge (+e), equal in magnitude to the electron's negative charge (-e). Neutrons have no charge.

Measuring nuclear masses accurately is crucial. Because these masses are tiny, a special unit is convenient: the **atomic mass unit (u)**.

How is the atomic mass unit (u) defined? (Hint: It's based on a specific isotope). \_\_\_\_\_

Go to Frame 9.

## Frame 9

The atomic mass unit (u) is defined such that the mass of one atom of the isotope  $^{12}\text{C}$  is *exactly* 12 u.  $1 \text{ u} = 1.660\,540 \times 10^{-27} \text{ kg}$ .

In this unit: Mass of proton  $\approx 1.007\,276 \text{ u}$  Mass of neutron  $\approx 1.008\,665 \text{ u}$  Mass of electron  $\approx 0.000\,549 \text{ u}$

Because of mass-energy equivalence ( $E = mc^2$ ), nuclear physicists often express mass in energy units, typically Mega-electron volts (MeV). The conversion uses  $c^2$  as part of the unit ( $\text{MeV}/c^2$ ).

What is the approximate energy equivalence of 1 u?  $1 \text{ u} \approx \text{_____} \text{MeV}/c^2$

Go to Frame 10.

## Frame 10

Using the precise value of  $c$  and the conversion factor between Joules and eV, we find:

$$1 \text{ u} = 931.494 \text{ MeV}/c^2$$

(Table 13.1 in the original text summarizes these masses).

Go to Frame 11.

## Frame 11

### Size and Structure of Nuclei

Rutherford's scattering experiments provided the first estimate of nuclear size. Alpha particles ( $\alpha$ , which are  $^4_2\text{He}$  nuclei) were directed at a thin metal foil (like gold). Most passed through with little deflection, but some were scattered at large angles, a few even bouncing almost straight back.

What did this scattering pattern imply about the distribution of positive charge and mass within the atom? \_\_\_\_\_

Go to Frame 12.

## Frame 12

The large-angle scattering implied that the positive charge and the majority of the atom's mass must be concentrated in a very small, dense region – the nucleus. If the positive charge were spread throughout the atom, large deflections would not occur.

By analyzing the closest approach an alpha particle makes to the nucleus before being turned back by Coulomb repulsion, Rutherford could estimate an upper limit for the nuclear size. For gold, this was found to be about  $3.2 \times 10^{-14} \text{ m}$ .

Go to Frame 13.

### Frame 13

Subsequent experiments have shown that most nuclei are roughly spherical. The nuclear radius,  $r$ , is found to depend on the mass number,  $A$ . The approximate relationship is:

$$r = r_0 A^{1/3}$$

where  $r_0$  is a constant.

What is the approximate value of  $r_0$ ? (This unit of length,  $10^{-15}$  m, is also called a femtometer or a fermi).  $r_0 \approx$  \_\_\_\_\_ fm

Go to Frame 14.

### Frame 14

The value is  $r_0 \approx 1.2 \times 10^{-15}$  m = 1.2 fm.

This formula  $r = r_0 A^{1/3}$  has an important implication for nuclear density. The volume of a sphere is  $V = \frac{4}{3}\pi r^3$ . How does the volume of the nucleus depend on the mass number  $A$ ?  $V \propto$  \_\_\_\_

Go to Frame 15.

### Frame 15

Since  $r \propto A^{1/3}$ , the volume is:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_0 A^{1/3})^3 = \left(\frac{4}{3}\pi r_0^3\right) A$$

So, the volume  $V$  is directly proportional to the mass number  $A$ .

The mass of the nucleus is also approximately proportional to  $A$  (since protons and neutrons have similar masses, and electron mass is negligible).  $\text{Mass} \approx A \times (\text{average nucleon mass})$ .

If  $\text{Volume} \propto A$  and  $\text{Mass} \propto A$ , what does this imply about the density ( $\text{Mass}/\text{Volume}$ ) of nuclear matter? \_\_\_\_\_

Go to Frame 16.

### Frame 16

It implies that the **density of nuclear matter is approximately constant**, independent of the size of the nucleus (independent of  $A$ ). Calculations show this density is incredibly high, about  $2.3 \times 10^{17}$  kg/m<sup>3</sup>.

Go to Frame 17.

### Frame 17

#### Nuclear Stability

With many positively charged protons packed into such a tiny volume, the electrostatic repulsion (Coulomb force) between them is enormous. What force must be responsible for holding the nucleus together against this repulsion? \_\_\_\_\_

This force must act between which types of nucleons? [ p-p only — n-n only — p-n only — all pairs (p-p, n-n, p-n) ]

Go to Frame 18.

### Frame 18

The nucleus is held together by the **strong nuclear force**. It is an attractive force that acts between *all* pairs of nucleons (p-p, n-n, and p-n).

Key properties of the strong nuclear force:

- It is much stronger than the Coulomb force (at short distances).
- It is very short-range, effectively dropping to zero for separations greater than a few femtometers.

Look at Figure 13.4 (plot of N vs Z for stable nuclei). For light nuclei, stable isotopes often have  $N \approx Z$ . However, for heavier nuclei, the stable isotopes lie *above* the  $N=Z$  line. Why do heavy stable nuclei require more neutrons than protons? \_\_\_\_\_

Go to Frame 19.

### Frame 19

Heavy nuclei need more neutrons than protons ( $N > Z$ ) for stability because neutrons contribute to the attractive strong nuclear force (binding all nucleons) but do *\*not\** contribute to the repulsive Coulomb force (which only acts between protons). The extra "glue" provided by the neutrons helps overcome the increasing proton-proton repulsion in large nuclei.

Nuclei far from the "line of stability" in the N-Z plot are generally unstable and undergo radioactive decay. Certain numbers of N or Z (2, 8, 20, 28, 50, 82, 126 - called "magic numbers") correspond to particularly stable nuclear configurations, analogous to electron shells in atoms.

Go to Frame 20.

### Frame 20

#### Nuclear Spin and Magnetic Moment

Protons and neutrons, like electrons, have an intrinsic quantum mechanical property called **spin**. The nucleus as a whole has a total angular momentum, called the **nuclear spin**.

Associated with spin, charged particles possess a **magnetic moment**. Even the neutral neutron has a magnetic moment (related to its internal quark structure). The magnetic properties of nuclei are exploited in technologies like Magnetic Resonance Imaging (MRI) - see Figure 13.8.

Historically, the measured values of nuclear magnetic moments provided strong evidence *\*against\** the idea that electrons could exist inside the nucleus (a model considered before the discovery of the neutron).

Go to Frame 21.

### Frame 21

#### Binding Energy

A crucial discovery was that the total mass of a nucleus is always *less* than the sum of the masses of its individual constituent protons and neutrons.

What is the significance of this "missing mass" (mass defect)? (Hint:  $E = mc^2$ ). \_\_\_\_\_

Go to Frame 22.

### Frame 22

The mass difference (mass defect), when converted to energy via  $E = mc^2$ , represents the **total binding energy** ( $E_b$ ) of the nucleus. This is the energy that must be supplied to break the nucleus apart into its separate protons and neutrons. Conversely, it's the energy released when the nucleons bind together to form the nucleus.



The formula, using atomic masses (which include electron masses that conveniently cancel out), is:

$$E_b(\text{in MeV}) = [ZM(^1\text{H}) + Nm_n - M(^A_ZX)] \times 931.494 \text{ MeV/u}$$

where  $M(^1\text{H})$  is the mass of a hydrogen atom,  $m_n$  is the neutron mass, and  $M(^A_ZX)$  is the mass of the atom with nucleus  $^A_ZX$ .

Go to Frame 23.

### Frame 23

A more useful measure for comparing stability across different nuclei is the **binding energy per nucleon**,  $E_b/A$ . A plot of this value versus mass number  $A$  (Figure 13.10) reveals important trends.

Where does the binding energy per nucleon reach its maximum? (i.e., which nuclei are the most stable?) [a] Very light nuclei ( $A \leq 20$ ) [b] Nuclei around  $A = 60$  [c] Very heavy nuclei ( $A \geq 200$ )

Go to Frame 24.

### Frame 24

Your answer was [ a — b — c ].

The correct answer is [b]. The binding energy per nucleon peaks around  $A \approx 60$ , near iron (Fe) and nickel (Ni). These are the most tightly bound (most stable) nuclei.

This curve explains why energy can be released by:

- **Nuclear Fusion:** Combining light nuclei (like hydrogen isotopes) to form heavier ones (up to  $A=60$ ) increases  $E_b/A$ , releasing energy.
- **Nuclear Fission:** Splitting very heavy nuclei (like uranium) into two mid-sized nuclei increases  $E_b/A$ , releasing energy.

Go to Frame 25.

### Frame 25

#### Saturation of Nuclear Forces

Another key feature of the binding energy curve (Figure 13.10) is that  $E_b/A$  is roughly constant for nuclei with  $A > 20$ . It doesn't keep increasing indefinitely as more nucleons are added. This implies that nucleons interact strongly only with their nearest neighbors. This property is called **saturation** of the nuclear force. If every nucleon interacted with every other one,  $E_b$  would likely increase much faster (perhaps like  $A^2$ , making  $E_b/A$  grow with  $A$ ).

Figure 13.11 shows schematic potential energy curves for the nucleon-nucleon interaction, highlighting the short-range attraction and the Coulomb repulsion for p-p.

Go to Frame 26.

### Frame 26

Let's summarize the key properties of the nuclear force discussed so far:

- **Strong:** Overcomes Coulomb repulsion.
- **Short-Range:** Acts only over a few fm.
- **Charge-Independent (Approximately):** p-p, n-n, p-n forces are similar (ignoring Coulomb for p-p).

- **Saturable:** Nucleons interact mainly with immediate neighbors.

This covers the basic static properties of nuclei. The next sections in the original text deal with nuclear models and radioactivity.

End of Section.

## 41 Nuclear Structure II

### Frame 1

Let's look at nuclear structure. Recall the concept of binding energy per nucleon (the energy required to separate a nucleus into its constituent protons and neutrons, divided by the number of nucleons). Figure 13.10 (not shown here, but described) shows that this value is highest for nuclei with a mass number  $A$  around 60 (like Iron).

Based on this trend near  $A=60$ , energy is released in two main types of nuclear reactions. Fill in the blanks: 1. When a very heavy nucleus (e.g.,  $A \gtrsim 200$ ) splits into lighter nuclei, this is called \_\_\_\_\_. 2. When very light nuclei (e.g.,  $A \lesssim 20$ ) combine to form a heavier nucleus, this is called \_\_\_\_\_.

Go to Frame 2.

### Frame 2

The processes are: 1. Fission 2. Fusion

Energy is released in both cases because the resulting nuclei (closer to  $A=60$ ) are more tightly bound (have higher binding energy per nucleon, meaning lower total energy) than the initial nucleus/nuclei.

Another key feature of the binding energy curve (Figure 13.10) is that for nuclei with  $A \gtrsim 20$ , the binding energy per nucleon is roughly constant. This observation leads to the concept of force **saturation**.

What does "saturation" imply about how a nucleon interacts within the nucleus? [a] It interacts equally strongly with all other nucleons. [b] It interacts primarily with only a limited number of nearby nucleons.

Go to Frame 3.

### Frame 3

Your answer was [ a — b ].

The correct answer is [b]. Saturation means a nucleon forms attractive bonds with only a limited number of its neighbors, much like molecules in a liquid drop. If every nucleon interacted with every other one, the binding energy per nucleon would increase significantly with  $A$ , which is not observed for  $A \gtrsim 20$ . This limited interaction is related to the short-range nature of the nuclear force.

Go to Frame 4.

### Frame 4

Experiments have revealed several key features of the force that holds nucleons (protons and neutrons) together:

1. **Stronger than Electromagnetic Force:** It overcomes the powerful Coulomb repulsion between positively charged protons within the tiny nucleus. 2. **Short-Range:** The force drops to nearly zero when nucleons are separated by more than a few femtometers (fm,  $1 \text{ fm} = 10^{-15} \text{ m}$ ).

What experimental evidence supports the short-range nature of the nuclear force? (Mention two points from the text, page 474). 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 5.

### Frame 5

Evidence for the short range comes from: 1. **Scattering experiments:** How particles deflect when interacting with nuclei reveals the force's range. The neutron-proton (n-p) potential energy plot (Figure 13.11a) shows this range clearly. 2. **Saturation of nuclear forces:** As discussed in Frame 3, the constant binding energy per nucleon (for  $A \gtrsim 20$ ) implies nucleons only interact with near neighbours.

Figure 13.11a also shows the n-p potential has a strong repulsive component at very short distances ( $\lesssim 0.4 \text{ fm}$ ), preventing nucleons from collapsing onto each other.

Go to Frame 6.

### Frame 6

A third key feature of the nuclear force:

3. **Charge Independence:** The nuclear force between two neutrons (n-n), a neutron and a proton (n-p), or two protons (p-p) is found to be approximately the same, aside from the electromagnetic effects.

Look at Figure 13.11b (the p-p potential). What causes the main difference between the p-p potential and the n-p potential (Figure 13.11a), especially at distances greater than 2 fm? \_\_\_\_\_

Go to Frame 7.

### Frame 7

The main difference is the **Coulomb repulsion** between the two positively charged protons in the p-p interaction. This adds a positive potential energy barrier that doesn't exist for the n-p interaction. At very short ranges ( $\lesssim 2 \text{ fm}$ ), where the attractive nuclear force dominates, the potentials are nearly identical.

Does the nuclear force affect electrons? [ Yes — No ]

Go to Frame 8.

### Frame 8

The answer is [No]. The charge independence (and fundamental nature) of the nuclear force means it does not directly affect electrons. This allows high-energy electrons to be used as probes to measure the charge distribution within a nucleus without being significantly affected by the nuclear forces acting between nucleons.

Go to Frame 9.

### Frame 9

What is the underlying mechanism for this strong, short-range nuclear force? One successful model is the **exchange force model**.

In this model, how do two nucleons (e.g., a proton and a neutron) attract each other? \_\_\_\_\_

Go to Frame 10.

### Frame 10

In the exchange force model, nucleons attract each other by exchanging a particle. One nucleon emits the particle, and the other absorbs it. Both nucleons are thought to be attracted to the exchanged particle, effectively creating an attraction between the nucleons.

This sounds problematic! If a proton emits a particle, shouldn't energy and momentum conservation be violated? How does the model resolve this? (Hint: uncertainty principle). \_\_\_\_\_

Go to Frame 11.

### Frame 11

The dilemma is resolved by considering the exchanged particle to be a **virtual particle**.

According to the Heisenberg Uncertainty Principle relationships ( $\Delta E \Delta t \geq \hbar/2$  and  $\Delta p_x \Delta x \geq \hbar/2$ ), conservation laws can be temporarily "violated" for very short durations ( $\Delta t$ ) or within very small regions ( $\Delta x$ ).

The exchanged particle exists for such a short time  $\Delta t$  that its energy  $E = mc^2$  (where  $m$  is its mass) is "uncertain" enough ( $\Delta E \approx \hbar/\Delta t$ ) to allow its creation without violating energy conservation overall. It cannot be directly detected during this brief existence.

Go to Frame 12.

### Frame 12

The energy-time uncertainty principle gives a relationship between the mass ( $m$ ) of the exchanged particle and the maximum range ( $d_{max}$ ) of the force. The violation amount is  $\Delta E = mc^2$ . The maximum time the violation can last is  $\Delta t \approx \hbar/\Delta E = \hbar/mc^2$ . The maximum distance the particle can travel in this time (at nearly speed of light  $c$ ) is  $d_{max} \approx c\Delta t$ .

Combine these to estimate the relationship between range  $d_{max}$  and mass  $m$ . Is the range directly or inversely proportional to the mass? \_\_\_\_\_

Go to Frame 13.

### Frame 13

Substituting  $\Delta t$  into the distance equation:

$$d_{max} \approx c \left( \frac{\hbar}{mc^2} \right) = \frac{\hbar c}{mc^2} = \frac{\hbar}{mc}$$

(Note: the text uses  $\hbar$  implicitly in its derivation yielding  $d_{max} = \hbar/mc$ , although it wrote  $h$ .  $\hbar = h/2\pi$  is standard in these relations).

This shows the range ( $d_{max}$ ) is **inversely proportional** to the mass ( $m$ ) of the exchanged particle. A heavier exchange particle results in a shorter-range force.

Given the nuclear force range is about 2 fm, calculations estimate the exchange particle mass to correspond to an energy ( $mc^2$ ) of about 100-140 MeV. Particles called **pi mesons** (pions) with roughly this mass were later discovered, supporting Yukawa's exchange theory.

Go to Frame 14.

### Frame 14

While the exchange model describes the force, we need models to understand the overall structure and properties of nuclei. We'll look briefly at two important ones:

1. **Liquid-Drop Model**: Treats the nucleus like a drop of liquid. 2. **Independent-Particle Model (Shell Model)**: Treats nucleons as moving in defined orbitals, similar to electrons in atoms.

Which model is better suited for explaining the overall trend of binding energy and the process of nuclear fission? [ Liquid-Drop — Shell ]

Go to Frame 15.

### Frame 15

The **Liquid-Drop Model** is particularly useful for explaining the general trend of binding energy and provides a qualitative picture of fission (Figure 13.12 shows the drop distorting and splitting).

This model considers several effects that contribute to the total binding energy ( $E_b$ ). List the four effects mentioned on page 477. 1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_ 4. \_\_\_\_\_

Go to Frame 16.

### Frame 16

The four main contributions to binding energy in the Liquid-Drop Model are:

1. **Volume Effect:** Proportional to  $A$  ( $C_1A$ ). Arises from bulk interactions (saturation).
2. **Surface Effect:** Reduces binding, proportional to surface area ( $A^{2/3}$ ), so term is  $-C_2A^{2/3}$ . Surface nucleons have fewer neighbours.
3. **Coulomb Repulsion Effect:** Reduces binding due to proton-proton repulsion. Term is  $-C_3Z(Z-1)/A^{1/3}$ .
4. **Symmetry Effect:** Reduces binding if  $N$  (neutron number) differs significantly from  $Z$  (proton number). Term is  $-C_4(N-Z)^2/A$ . Favors  $N=Z$ .

Combining these gives the **Weizsäcker semiempirical binding energy formula**:

$$E_b = C_1A - C_2A^{2/3} - C_3\frac{Z(Z-1)}{A^{1/3}} - C_4\frac{(N-Z)^2}{A}$$

The constants  $C_1, C_2, C_3, C_4$  are adjusted to fit experimental data.

Go to Frame 17.

### Frame 17

While the liquid-drop model explains bulk properties well, it doesn't account for finer details like variations in stability for nuclei with similar  $A$ , or nuclear angular momentum. For this, we use the **Independent-Particle Model** or **Shell Model**.

What is the basic assumption of this model, analogous to the atomic shell model? \_\_\_\_\_

Go to Frame 18.

### Frame 18

The Shell Model assumes that each nucleon moves in a well-defined **orbital state** within an average field created by all the other nucleons. Nucleons exist in quantized energy states, and collisions between them are considered infrequent (unlike the liquid-drop model).

Protons and neutrons occupy separate sets of energy levels (because the potential wells are slightly different due to Coulomb forces affecting only protons).

Which fundamental principle governs how nucleons fill these available energy states? \_\_\_\_\_

Go to Frame 19.

### Frame 19

The **Pauli Exclusion Principle** applies. Since protons and neutrons are spin-1/2 particles (fermions), each orbital state can hold a maximum of two protons (one spin up, one spin down) and separately, a maximum of two neutrons (one spin up, one spin down).

How does this model explain the tendency for stable light nuclei to have  $N$  approximately equal to  $Z$ ? \_\_\_\_\_

Go to Frame 20.

### Frame 20

To achieve maximum stability (lowest total energy), nucleons fill the lowest available energy levels first. Since the proton and neutron energy levels are similar (especially for light nuclei where Coulomb effects are smaller), filling proceeds roughly in parallel for both types of nucleons.

If a nucleus had significantly more neutrons than protons ( $N \gg Z$ ), the extra neutrons would be forced into higher energy levels than if they were protons filling lower, unoccupied proton levels. This higher energy state corresponds to lower stability. Thus, configurations with  $N \approx Z$  tend to have the lowest energy and greatest stability for a given total mass number  $A$  (in light nuclei). (Example: comparing  ${}^1_5\text{B}$  and  ${}^{12}_6\text{C}$ ).

This concludes the overview of these nuclear structure topics.

End of Section.

## 42 Nuclear Structure Models

### Frame 1

We will now look at models used to describe the structure of the atomic nucleus. While the liquid-drop model (mentioned previously, see Figure 13.12) provides some insights, other models offer different perspectives. One important model is the **Independent-Particle Model**.

This model is also often called the **shell model**. It was developed independently by Maria Goeppert-Mayer and Hans Jensen, who shared the Nobel Prize in 1963 for this work.

What is the fundamental assumption of the shell model regarding the motion of individual nucleons (protons and neutrons) within the nucleus? \_\_\_\_\_

Go to Frame 2 for the answer.

### Frame 2

The fundamental assumption of the shell model is that: *Each nucleon moves in a well-defined orbital state within the nucleus in an average field produced by the other nucleons.*

This is quite different from the liquid-drop model, where nucleons are assumed to interact strongly and constantly, like molecules in a liquid drop. In the shell model, collisions between nucleons are considered relatively infrequent within the nucleus.

In what way is the nuclear shell model similar to the shell model used for electrons in atoms? And what is a key difference mentioned concerning the potential?

Similarity: \_\_\_\_\_ Difference: \_\_\_\_\_

Go to Frame 3.

### Frame 3

Similarity: Both models propose that particles (electrons in atoms, nucleons in nuclei) exist in quantized energy states or orbitals, analogous to atomic subshells.

Difference: The ordering of nuclear energy states is better modeled by a *spherical-well potential* rather than the *Coulomb potential* used for atomic electrons.

Just like electrons, nucleons (protons and neutrons) have spin  $1/2$ . What fundamental principle governs how these nucleons can occupy the available quantized energy states? \_\_\_\_\_

Go to Frame 4.

#### Frame 4

The **Pauli Exclusion Principle** applies to nucleons.

This means that each quantized orbital state can contain a maximum of two identical nucleons (two protons OR two neutrons), and these two must have opposite spins. (See Figure 13.13 in the original text).

Are the allowed energy states (levels) for protons identical to those for neutrons within the same nucleus? Why or why not? [ Yes — No ] Reason: \_\_\_\_\_

Go to Frame 5.

#### Frame 5

The allowed states for protons and neutrons are **not** identical. The answer is [No].

Reason: Protons experience both the attractive nuclear force and the repulsive Coulomb force (due to their positive charge). Neutrons experience only the nuclear force. This difference in the potential wells they move in leads to slightly different energy levels, with proton levels generally being higher in energy than neutron levels for the same quantum state, especially as the number of protons (Z) increases. (See Figure 13.13).

Go to Frame 6.

#### Frame 6

The shell model, incorporating the Pauli Exclusion Principle, can explain why stable, light nuclei often have an equal number of protons and neutrons (N=Z).

Consider building a nucleus by adding nucleons one by one. According to the model, how should nucleons fill the available energy levels to achieve maximum stability? \_\_\_\_\_

Go to Frame 7.

#### Frame 7

To achieve maximum stability for a given total number of nucleons (A), the nucleons must be packed sequentially into the *lowest available energy levels*, filling each level completely before starting the next, leaving no gaps.

Since each level holds two protons (or two neutrons), adding another nucleon beyond a filled level forces it into a higher energy state, which generally decreases stability (increases the nucleus's total energy).

Consider the isotopes Boron-12 ( ${}^{12}_5\text{B}$ , 5 protons, 7 neutrons) and Carbon-12 ( ${}^{12}_6\text{C}$ , 6 protons, 6 neutrons). According to the shell model, why is  ${}^{12}_6\text{C}$  more stable than  ${}^{12}_5\text{B}$ ? \_\_\_\_\_

Go to Frame 8.

#### Frame 8

In  ${}^{12}_6\text{C}$ , there are 6 protons and 6 neutrons. These can fill the lowest 3 proton levels and the lowest 3 neutron levels completely (2 nucleons per level). In  ${}^{12}_5\text{B}$ , there are 5 protons and 7 neutrons. The 7th neutron must occupy a higher energy level than the first 6 neutrons (which fill the lowest 3 neutron levels). This extra energy makes  ${}^{12}_5\text{B}$  less stable (higher total energy) than  ${}^{12}_6\text{C}$ .

Go to Frame 9.

#### Frame 9

To fully explain features like the "magic numbers" (nuclei with specific numbers of protons or neutrons being exceptionally stable), the basic shell model needs an addition. What interaction effect, stronger in nuclei than in atoms, must be included? \_\_\_\_\_

Go to Frame 10.

### Frame 10

Nuclear **spin-orbit effects** must be included.

This interaction is due to the nuclear force (not magnetic like the atomic spin-orbit effect) and is much stronger. It splits energy levels based on the nucleon's angular momentum. Importantly, it causes large energy gaps just above the nucleon numbers 8, 20, 28, 50, 82, and 126, explaining why nuclei with these "magic numbers" of protons or neutrons are particularly stable.

Go to Frame 11.

### Frame 11

A puzzle arises: how can nucleons move in well-defined orbitals when the nucleus is so densely packed? One might expect constant collisions. What principle helps justify the shell model's assumption of orbital motion, especially in the low-energy ground state? \_\_\_\_\_

Go to Frame 12.

### Frame 12

The **Pauli Exclusion Principle** again helps justify the model.

In the low-energy ground state, most lower energy levels are already filled. If two nucleons collide, they cannot easily change their energy state because the states they would scatter into are likely already occupied by other nucleons. Energy-changing collisions are therefore inhibited, allowing nucleons to persist in their orbital states for longer than might otherwise be expected.

Go to Frame 13.

### Frame 13

While the shell model is successful, particularly for nuclei near magic numbers, another model is needed to explain other nuclear phenomena, like collective vibrations and rotations. This is the **Collective Model**.

How does the collective model combine aspects of the liquid-drop model and the shell model?

Go to Frame 14.

### Frame 14

The collective model pictures the nucleus as having:

- A stable "core" of filled nucleon shells (like the shell model).
- Some "extra" nucleons moving in quantized orbits relative to this core (like the shell model).
- The possibility for the core itself to be deformed by the interaction with the outer nucleons, leading to collective vibrational and rotational motions of the core (like the liquid-drop model).

This hybrid approach successfully explains many nuclear properties that the other two models cannot account for individually.

This completes our overview of the Independent-Particle (Shell) and Collective models of nuclear structure.

End of Section.



## 43 Some Properties of Nuclei

### Frame 1

Let's look at some basic properties of atomic nuclei. Figure 13.4 (in the original text) shows a plot of neutron number ( $N$ ) versus proton number ( $Z$ , the atomic number) for stable nuclei. Notice that for light nuclei, stable isotopes tend to have  $N$  approximately equal to  $Z$  (the dashed line  $N=Z$ ). However, as  $Z$  increases, stable nuclei tend to have more neutrons than protons.

The shaded area in the plot represents unstable, radioactive nuclei. The band of stable nuclei is sometimes called the "line of stability."

Go to Frame 2.

### Frame 2

Interestingly, nuclei with certain specific numbers of protons or neutrons are found to be unusually stable. These numbers are called **magic numbers**:

$$Z \text{ or } N = 2, 8, 20, 28, 50, 82, 126$$

For example, Helium ( $Z=2$ ,  $N=2$ ) is very stable. This suggests a structure within the nucleus somewhat analogous to electron shells in atoms.

What model attempts to explain these magic numbers based on this observation? \_\_\_\_\_

Go to Frame 3 for the answer.

### Frame 3

The model that explains magic numbers is the **shell model** of the nucleus, which proposes that nucleons (protons and neutrons) occupy distinct energy levels or shells within the nucleus, similar to atomic electron shells.

Now, let's consider another property: nuclear spin. Just like electrons, protons and neutrons have intrinsic angular momentum (spin). The nucleus as a whole also has a total angular momentum, which comes from the combination of the orbital and spin angular momenta of all its nucleons.

The magnitude of the nuclear angular momentum is given by  $\sqrt{I(I+1)}\hbar$ , where  $\hbar$  is the reduced Planck constant. What is the quantum number  $I$  called? \_\_\_\_\_

Go to Frame 4 for the answer.

### Frame 4

The quantum number  $I$  is called the **nuclear spin**. It can be an integer (e.g., 0, 1, 2,...) or a half-integer (e.g., 1/2, 3/2, 5/2,...). The maximum projection of this angular momentum along a chosen axis (like the  $z$ -axis) is  $I\hbar$ . (See Figure 13.5 for the example  $I = 3/2$ ).

Associated with this nuclear angular momentum is a **nuclear magnetic moment**. This is analogous to the magnetic moment associated with electron spin. The magnetic moment of a nucleus is measured in units of the **nuclear magneton**,  $\mu_n$ , defined as:

$$\mu_n = \frac{e\hbar}{2m_p} \approx 5.05 \times 10^{-27} \text{ J/T}$$

where  $e$  is the elementary charge and  $m_p$  is the mass of the proton.

Recall the Bohr magneton  $\mu_B = e\hbar/(2m_e)$  is the unit for electron magnetic moments. Is the nuclear magneton  $\mu_n$  larger or smaller than the Bohr magneton  $\mu_B$ ? Why? \_\_\_\_\_  
Go to Frame 5 for the answer.

### Frame 5

The nuclear magneton  $\mu_n$  is much *smaller* than the Bohr magneton  $\mu_B$  (by about a factor of 2000). This is because the definition uses the proton mass ( $m_p$ ) in the denominator, which is much larger than the electron mass ( $m_e$ ) used for  $\mu_B$ .

Interestingly:

- The magnetic moment of a free proton is  $\approx 2.7928\mu_n$ .
- A neutron, despite having no net charge, has a magnetic moment of  $\approx -1.9135\mu_n$ .

What does the fact that the neutron has a magnetic moment suggest about its internal structure?  
Go to Frame 6 for the answer.

### Frame 6

The neutron having a magnetic moment is evidence that it must have some kind of **internal charge distribution**, even though its net charge is zero. (It's now known to be composed of charged quarks). The negative sign indicates its magnetic moment is oriented opposite to its spin angular momentum.

These nuclear magnetic moments are the basis for **Nuclear Magnetic Resonance (NMR)** and **Magnetic Resonance Imaging (MRI)**.

Go to Frame 7.

### Frame 7

When a nucleus with a magnetic moment  $\vec{\mu}$  is placed in an external magnetic field  $\vec{B}$ , it experiences a torque and precesses around the field direction (like a spinning top precessing in gravity). This precession occurs at a specific frequency called the **Larmor precessional frequency**,  $\omega_L$ , which is proportional to the magnetic field strength  $B$ .

The potential energy of the magnetic moment in the field is given by  $U = -\vec{\mu} \cdot \vec{B}$ . Due to quantum mechanics, the alignment of  $\vec{\mu}$  with  $\vec{B}$  is quantized. For a spin  $I = 1/2$  nucleus (like a proton), there are only two possible energy states. (See Figure 13.6b).

Describe these two energy states relative to the field direction. 1. Lower energy state ( $E_{min}$ ):  
2. Higher energy state ( $E_{max}$ ): \_\_\_\_\_

Go to Frame 8.

### Frame 8

For a spin  $I = 1/2$  nucleus in a magnetic field  $\vec{B}$ : 1. Lower energy state ( $E_{min}$ ): The nuclear spin (and its magnetic moment) is aligned as parallel as possible with the external field  $\vec{B}$ . 2. Higher energy state ( $E_{max}$ ): The nuclear spin (and its magnetic moment) is aligned as antiparallel as possible with the external field  $\vec{B}$ .

The energy difference between these states is  $\Delta E = E_{max} - E_{min}$ .

**Nuclear Magnetic Resonance (NMR)** is a technique used to induce and detect transitions between these nuclear spin states. How is this achieved? (Hint: What needs to be applied to the nuclei in addition to the main static field  $B$ ?). \_\_\_\_\_

Go to Frame 9.

### Frame 9

NMR involves applying a second, weak, *oscillating* magnetic field, usually perpendicular to the main static field  $\vec{B}$ .

When the frequency of this oscillating field is tuned to exactly match the Larmor precessional frequency ( $\omega_L$ ), it can cause the nuclear spins to "flip" between the  $E_{min}$  and  $E_{max}$  states. This results in a net absorption of energy from the oscillating field, which can be detected electronically. (See Figure 13.7 for a schematic).

Go to Frame 10.

### Frame 10

A widely used medical application of NMR is **Magnetic Resonance Imaging (MRI)**. The human body contains abundant hydrogen atoms (protons), which have spin  $I = 1/2$  and thus a strong NMR signal.

In MRI, the patient is placed in a strong magnetic field that is deliberately made non-uniform; it has a *gradient*. Why is this field gradient important for imaging? (Hint: How does the Larmor frequency depend on the field?). \_\_\_\_\_

Go to Frame 11.

### Frame 11

The field gradient is crucial because the Larmor frequency ( $\omega_L$ ) is proportional to the magnetic field strength ( $B$ ). Since  $B$  varies with position due to the gradient, protons in different parts of the body precess at slightly different frequencies.

By detecting the frequencies of the absorbed NMR signals, a computer can reconstruct the spatial distribution of the protons, creating an image of internal tissues.

What is a major advantage of MRI compared to X-ray or gamma-ray imaging in terms of patient safety? \_\_\_\_\_

Go to Frame 12.

### Frame 12

MRI causes minimal cellular damage. The radio frequency (RF) photons used in NMR/MRI have very low energies (typically  $\sim 10^{-7}$  eV), which is far less than the energy holding molecules together ( $\sim 1$  eV) and vastly less than the energies of X-rays ( $10^4$  to  $10^6$  eV). Therefore, RF radiation causes little disruption to biological tissues.

Now let's move to another fundamental property: the energy holding the nucleus together.

Go to Frame 13.

### Frame 13

## Binding Energy and Nuclear Forces (Section 13.2)

A key finding is that the total mass of any nucleus ( $M_A$ ) is always \*less\* than the sum of the masses of its individual constituent protons and neutrons when they are separated.

Since mass is equivalent to energy ( $E = mc^2$ ), this mass difference implies an energy difference. The total energy of the bound nucleus is less than the total energy of the separated nucleons.

What is this energy difference called, and what does it represent? \_\_\_\_\_

Go to Frame 14.

### Frame 14

This energy difference is called the **binding energy** ( $E_b$ ) of the nucleus.

It represents the energy that must be supplied to a nucleus to break it apart into its separate, constituent protons and neutrons. Equivalently, it is the energy \*released\* when separate nucleons come together to form a nucleus.

The binding energy for a nucleus  ${}^A_ZX$  (mass  $M_A$ ) composed of  $Z$  protons and  $N$  neutrons can be calculated using the mass difference:

$$E_b(\text{MeV}) = [ZM(\text{H}) + Nm_n - M_A] \times 931.494 \text{ MeV/u}$$

Here,  $M(\text{H})$  is the mass of a neutral hydrogen atom,  $m_n$  is the mass of a neutron,  $M_A$  is the mass of the neutral atom  ${}^A_ZX$ , and all masses are in atomic mass units (u). The factor 931.494 MeV/u converts mass units to energy units (MeV - Mega-electron Volts).

Why do we use the mass of the hydrogen atom  $M(\text{H})$  instead of just the proton mass  $m_p$  in this formula, given that  $M_A$  is the mass of the neutral atom? \_\_\_\_\_

Go to Frame 15.

### Frame 15

We use the mass of the neutral hydrogen atom  $M(\text{H}) = m_p + m_e$  (proton mass + electron mass) because the tabulated atomic mass  $M_A$  also includes the mass of its  $Z$  electrons.

By using  $Z \times M(\text{H})$ , we are including the mass of  $Z$  protons and  $Z$  electrons. The mass  $M_A$  includes the mass of the  $Z$  protons,  $N$  neutrons, and  $Z$  electrons. When we calculate the mass difference  $[ZM(\text{H}) + Nm_n - M_A]$ , the  $Z$  electron masses included in  $ZM(\text{H})$  conveniently cancel out the  $Z$  electron masses included in  $M_A$ . This avoids having to look up separate nuclear masses, which are less commonly tabulated than neutral atomic masses.

This concludes this section on basic nuclear properties and binding energy.

## 44 Radioactivity

### Frame 1

In 1896, Henri Becquerel discovered that certain crystals (uranyl potassium sulfate) emitted a new kind of invisible radiation. This radiation could darken a photographic plate even when light was blocked and didn't require any external stimulation like light or heat. This spontaneous emission of radiation was termed **radioactivity**.

Marie and Pierre Curie extensively investigated this phenomenon, famously isolating two new radioactive elements, polonium and radium, from tons of pitchblende ore after years of work. Their research, along with Rutherford's later experiments, showed that radioactivity is the result of the decay, or disintegration, of unstable atomic nuclei.

What are the three main types of radiation emitted by radioactive substances? \_\_\_\_\_

Go to Frame 2 for the answer.

### Frame 2

The three common types of radiation emitted are:

- **Alpha ( $\alpha$ ) radiation:** Consists of helium nuclei ( ${}^4\text{He}$ ).

- **Beta ( $\beta$ ) radiation:** Consists of electrons ( $e^-$ ) or positrons ( $e^+$ ). A positron is like an electron but with a positive charge.
- **Gamma ( $\gamma$ ) radiation:** Consists of high-energy photons (electromagnetic radiation).

How can these three types of radiation be distinguished experimentally using a magnetic field? (Think about the charge of each type). \_\_\_\_\_

Go to Frame 3 for the explanation.

### Frame 3

We can distinguish the radiation types by passing them through a magnetic field (as shown conceptually in Figure 13.14 of the original text).

- Gamma ( $\gamma$ ) rays are uncharged, so they are *not deflected* by the magnetic field.
- Alpha ( $\alpha$ ) particles are positively charged ( $\text{He}^{2+}$  nuclei), so they are deflected in one direction.
- Beta ( $\beta$ ) particles (electrons,  $e^-$ ) are negatively charged and are deflected in the *opposite* direction to alpha particles. (Positrons,  $e^+$ , if present, would deflect in the same direction as alpha particles).

Which type of radiation do you think is generally the most penetrating, and which is the least? [Most:  $\alpha - \beta - \gamma$ ] [Least:  $\alpha - \beta - \gamma$ ]

Go to Frame 4 for the answer.

### Frame 4

Your answers were: Most [ $\alpha - \beta - \gamma$ ], Least [ $\alpha - \beta - \gamma$ ].

Correct: Most penetrating is [ $\gamma$ ], least penetrating is [ $\alpha$ ]. Alpha particles can be stopped by a sheet of paper. Beta particles can penetrate a few millimeters of aluminum. Gamma rays can penetrate several centimeters of lead.

Now let's consider the rate at which radioactive decay occurs.

Go to Frame 5.

### Frame 5

The rate at which nuclei in a radioactive sample decay is found to be proportional to the number of radioactive nuclei present at that instant.

Let  $N$  be the number of radioactive (undecayed) nuclei present at time  $t$ . How can we express the rate of change of  $N$  (which is  $dN/dt$ ) in terms of  $N$ ? Remember that  $N$  is decreasing. \_\_\_\_\_

Go to Frame 6.

### Frame 6

The rate of change is proportional to  $N$ , and since  $N$  is decreasing, the rate  $dN/dt$  must be negative. We write this relationship as:

$$\frac{dN}{dt} = -\lambda N$$

Here,  $\lambda$  (lambda) is a positive constant called the **decay constant**. It represents the probability per unit time that any individual nucleus will decay. Its units are typically  $\text{s}^{-1}$  or  $\text{yr}^{-1}$ , etc.

Go to Frame 7.

### Frame 7

The equation  $\frac{dN}{dt} = -\lambda N$  is a differential equation. We can solve it to find how the number of nuclei  $N$  changes with time  $t$ . We can rearrange it as:

$$\frac{dN}{N} = -\lambda dt$$

Now, let's integrate both sides. Assume we start with  $N_0$  nuclei at time  $t = 0$ . We integrate from the initial state ( $t = 0, N = N_0$ ) to a general state ( $t, N$ ).

$$\int_{N_0}^N \frac{dN'}{N'} = \int_0^t -\lambda dt'$$

What is the result of performing these integrations? (Hint:  $\int dx/x = \ln x$ ). \_\_\_\_\_  
Go to Frame 8 for the result.

### Frame 8

Evaluating the integrals gives:

$$\begin{aligned} [\ln N']_{N_0}^N &= [-\lambda t']_0^t \\ \ln N - \ln N_0 &= -\lambda(t - 0) \\ \ln \left( \frac{N}{N_0} \right) &= -\lambda t \end{aligned}$$

To get  $N$  explicitly, we exponentiate both sides (using  $e^{\ln x} = x$ ):

$$\begin{aligned} \frac{N}{N_0} &= e^{-\lambda t} \\ N(t) &= N_0 e^{-\lambda t} \end{aligned}$$

This is the law of **exponential decay**. It shows the number of radioactive nuclei decreases exponentially with time. (See Figure 13.15).

Go to Frame 9.

### Frame 9

The **decay rate**  $R$ , also called the **activity** of the sample, is the number of decays per unit time. It's defined as the magnitude of  $dN/dt$ .

Using  $N(t) = N_0 e^{-\lambda t}$  and the original differential equation ( $dN/dt = -\lambda N$ ), find an expression for the activity  $R(t)$  in terms of the initial activity  $R_0$  (where  $R_0 = \lambda N_0$ ). \_\_\_\_\_

Go to Frame 10.

### Frame 10

The activity is:

$$R = \left| \frac{dN}{dt} \right| = |-\lambda N| = \lambda N$$

Substituting  $N = N_0 e^{-\lambda t}$ :

$$R(t) = \lambda(N_0 e^{-\lambda t}) = (\lambda N_0) e^{-\lambda t}$$

Since  $R_0 = \lambda N_0$  is the initial activity at  $t = 0$ , we have:

$$R(t) = R_0 e^{-\lambda t}$$

So, the activity also decreases exponentially with time.

Another important characteristic is the **half-life**,  $T_{1/2}$ . This is the time it takes for half of a given number of radioactive nuclei to decay. How can we relate  $T_{1/2}$  to the decay constant  $\lambda$ ? (Hint: Set  $N = N_0/2$  at  $t = T_{1/2}$  in the decay law). \_\_\_\_\_

Go to Frame 11.

### Frame 11

Setting  $N = N_0/2$  and  $t = T_{1/2}$  in  $N = N_0 e^{-\lambda t}$ :

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Taking the natural logarithm of both sides:

$$\ln\left(\frac{1}{2}\right) = -\lambda T_{1/2}$$

$$-\ln 2 = -\lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

This provides a direct relationship between the half-life and the decay constant.

Go to Frame 12.

### Frame 12

Consider a sample starting with  $N_0$  nuclei. After one half-life ( $t = T_{1/2}$ ), how many nuclei remain?  $N_0/2$ . After two half-lives ( $t = 2T_{1/2}$ ), how many nuclei remain?  $(N_0/2)/2 = N_0/4$ . After three half-lives ( $t = 3T_{1/2}$ ), how many nuclei remain?  $(N_0/4)/2 = N_0/8$ .

In general, after  $n$  half-lives, the number of nuclei remaining is  $N_0/2^n$ .

Go to Frame 13.

### Frame 13

Activity (decay rate) is measured in decays per second. The SI unit is the **becquerel (Bq)**:

$$1 \text{ Bq} = 1 \text{ decay/s}$$

A more traditional, and often larger, unit is the **curie (Ci)**:

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/s} = 3.7 \times 10^{10} \text{ Bq}$$

The curie was originally based on the approximate activity of 1 gram of radium. Smaller units like the millicurie ( $\text{mCi} = 10^{-3} \text{ Ci}$ ) and microcurie ( $\mu\text{Ci} = 10^{-6} \text{ Ci}$ ) are common.

Go to Frame 14.

### Frame 14

Let's try an example calculation. Carbon-14 ( $^{14}_6\text{C}$ ) has a half-life of 5730 years. If you start with a sample containing 1000 carbon-14 nuclei, how many will remain after 22,920 years? (See Example 13.4).

---

Go to Frame 15 for the solution.

### Frame 15

First, find how many half-lives have elapsed. Number of half-lives  $n = \frac{\text{Total time}}{\text{Half-life}} = \frac{22,920 \text{ yr}}{5730 \text{ yr}} = 4$ .

The number of nuclei remaining after  $n$  half-lives is  $N_0/2^n$ . Here  $N_0 = 1000$  and  $n = 4$ . Number remaining  $= 1000/2^4 = 1000/16 = 62.5$ .

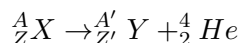
Since we can't have half a nucleus, we'd expect about 62 or 63 nuclei to remain. (Note: For such small initial numbers, statistical fluctuations would be significant. The calculation is more precise for very large  $N_0$ ).

Go to Frame 16.

### Frame 16

Now let's look more closely at the specific decay processes. As mentioned in Frame 2, the main types are alpha, beta, and gamma decay.

**Alpha Decay** In alpha decay, the nucleus emits an alpha particle ( $^4_2\text{He}$ ), which consists of 2 protons and 2 neutrons. If the parent nucleus is denoted by  $^A_ZX$  (where  $A$  is mass number,  $Z$  is atomic number), what is the daughter nucleus  $^{A'}_{Z'}Y$ ? Fill in  $A'$  and  $Z'$ .

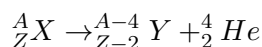


$A' =$  \_\_\_\_\_,  $Z' =$  \_\_\_\_\_

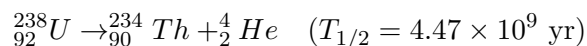
Go to Frame 17.

### Frame 17

In alpha decay: The mass number  $A$  decreases by 4. The atomic number  $Z$  decreases by 2. So,  $A' = A - 4$  and  $Z' = Z - 2$ . The decay is written:



Examples:



Go to Frame 18.

### Frame 18

**Disintegration Energy (Q value)** The energy released in a decay is called the disintegration energy,  $Q$ . It comes from the conversion of rest mass into energy. If  $M_X$  is the mass of the parent atom,  $M_Y$  is the mass of the daughter atom, and  $M_\alpha$  is the mass of the alpha particle (helium atom), then:

$$Q = (M_X - M_Y - M_\alpha)c^2$$

If masses are given in atomic mass units (u), the calculation is often done using the conversion factor 931.494 MeV/u:

$$Q = (M_X - M_Y - M_\alpha) \times 931.494 \text{ MeV/u}$$



This energy  $Q$  appears as the kinetic energy of the daughter nucleus ( $K_Y$ ) and the alpha particle ( $K_\alpha$ ).

$$Q = K_Y + K_\alpha$$

Which particle carries away most of this kinetic energy? [ Daughter nucleus Y — Alpha particle  $\alpha$  ] Why? \_\_\_\_\_

Go to Frame 19.

### Frame 19

Your answer: [ Daughter nucleus Y — Alpha particle  $\alpha$  ]. Correct: The [Alpha particle  $\alpha$ ] carries away most of the kinetic energy.

Why? Because momentum must also be conserved. Since the parent nucleus is often initially at rest, the daughter and alpha must recoil with equal and opposite momenta ( $p_Y = -p_\alpha$ ). Kinetic energy is  $K = p^2/(2m)$ . Since the alpha particle is much lighter than the heavy daughter nucleus ( $m_\alpha \ll m_Y$ ), to have the same magnitude of momentum  $p$ , the alpha particle must have a much higher velocity and thus much higher kinetic energy. (Typically  $K_\alpha \approx K_Y \times (m_Y/m_\alpha)$ ).

Go to Frame 20.

### Frame 20

**Mechanism of Alpha Decay - Tunneling** How can an alpha particle with typically  $Q \approx 5$  MeV escape the nucleus when it faces a strong Coulomb repulsion barrier that is much higher (e.g.,  $\approx 30$  MeV, see Figure 13.17)? Classically, it seems impossible.

What physical principle allows the alpha particle to escape? \_\_\_\_\_

Go to Frame 21.

### Frame 21

The alpha particle escapes via **quantum mechanical tunneling**.

In quantum mechanics, particles are described by wavefunctions ( $\psi$ ). Even if the particle's energy  $E$  is less than the potential barrier height  $U$ , the wavefunction has a small but non-zero amplitude outside the barrier (Figure 13.18). This means there is a finite probability ( $\propto |\psi|^2$ ) for the particle to be found outside the nucleus, having "tunneled" through the barrier.

The probability of tunneling depends strongly on the particle's energy and the barrier's height and width.

Go to Frame 22.

### Frame 22

**Beta Decay** In beta decay, the nucleus emits either an electron ( $e^-$ ) or a positron ( $e^+$ ). What happens to the mass number  $A$  and the atomic number  $Z$  during electron emission ( ${}^A_ZX \rightarrow {}^{A'}_{Z'}Y + e^-$ )? (Hint: An electron has negligible mass compared to nucleons and a charge of  $-1e$ . A neutron inside the nucleus effectively turns into a proton).

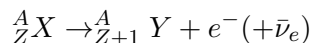
$A' =$  \_\_\_\_\_,  $Z' =$  \_\_\_\_\_

Go to Frame 23.

### Frame 23

For electron emission (often just called beta decay): The mass number  $A$  remains the same ( $A' = A$ ) because the total number of nucleons (protons + neutrons) is unchanged. The atomic

number  $Z$  increases by 1 ( $Z' = Z + 1$ ) because a neutron has changed into a proton (to conserve charge, an electron is emitted). The process is:



(Note: An antineutrino  $\bar{\nu}_e$  is also emitted, carrying away some energy and momentum, but we often omit it in simple representations).

This concludes our brief introduction to radioactivity types and the decay law.

End of Section.

## 45 Decay Processes

### Frame 1

Let's apply the quantum mechanical tunneling concepts (from Chapter 7 in the source text) to a nuclear process: alpha decay. **Example (based on 13.9):** Calculate the probability of an alpha particle escaping from a  ${}^{226}_{88}\text{Ra}$  nucleus, given a disintegration energy  $E = 5$  MeV.

The probability is given by the transmission coefficient  $T(E)$  through the Coulomb barrier. Recall the approximate formula:

$$T(E) \approx \exp\left(-\frac{2}{\hbar} \int \sqrt{2m(U(r) - E)} dr\right)$$

where the integral is over the classically forbidden region (where  $U(r) > E$ ). For alpha decay,  $U(r) = 2Zke^2/r$  for  $r > R$  (nuclear radius), and the forbidden region is from  $R$  to  $R_1 = 2Zke^2/E$ .  $Z$  is the atomic number of the \*daughter\* nucleus.

The integral to evaluate is:

$$\int_R^{R_1} \sqrt{2m\left(\frac{2Zke^2}{r} - E\right)} dr$$

This looks complicated! Let's simplify.

Go to Frame 2.

### Frame 2

The integral can be rewritten using  $E = \frac{2Zke^2}{R_1}$ :

$$\int_R^{R_1} \sqrt{2mE\left(\frac{R_1}{r} - 1\right)} dr = \sqrt{2mE} \int_R^{R_1} \sqrt{\frac{R_1}{r} - 1} dr$$

Let's make a substitution  $z = r/R_1$ . Then  $dr = R_1 dz$ . The limits become  $R/R_1$  and 1.

$$\sqrt{2mE} \int_{R/R_1}^1 \sqrt{\frac{1}{z} - 1} (R_1 dz) = \sqrt{2mER_1^2} \int_{R/R_1}^1 \sqrt{\frac{1-z}{z}} dz$$

The text notes that  $R/R_1$  is typically small (e.g.,  $R \approx 10$  fm,  $R_1 \approx 50$  fm). A first approximation is to set the lower limit to 0 and change variables again with  $z = \cos^2 \theta$ . Then  $dz = -2 \cos \theta \sin \theta d\theta$ . The limits  $z = 0$  and  $z = 1$  correspond to  $\theta = \pi/2$  and  $\theta = 0$ .

Can you show that  $\sqrt{\frac{1-z}{z}} = \sqrt{\frac{1-\cos^2\theta}{\cos^2\theta}} = \tan\theta$ ? And what does the integral  $\int_0^{\pi/2} \tan\theta(2\cos\theta\sin\theta)d\theta = 2\int_0^{\pi/2} \sin^2\theta d\theta$  evaluate to? (You might need an integral table or trigonometric identity  $\sin^2\theta = (1 - \cos 2\theta)/2$ ). \_\_\_\_\_

Go to Frame 3 for the result.

### Frame 3

The integral is:

$$\begin{aligned} 2 \int_0^{\pi/2} \sin^2\theta d\theta &= 2 \int_0^{\pi/2} \frac{1 - \cos 2\theta}{2} d\theta = \int_0^{\pi/2} (1 - \cos 2\theta) d\theta \\ &= \left[ \theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi/2} = \left( \frac{\pi}{2} - 0 \right) - (0 - 0) = \frac{\pi}{2} \end{aligned}$$

So, the approximate value of the integral part (ignoring the small lower limit  $R/R_1$ ) is  $\pi/2$ .

The text also shows a slightly better approximation by splitting the integral and noting  $\sqrt{1/z - 1} \approx \sqrt{1/z}$  for small  $z$ , leading to:

$$\int_{R/R_1}^1 \sqrt{\frac{1-z}{z}} dz \approx \frac{\pi}{2} - 2\sqrt{\frac{R}{R_1}}$$

(We won't derive this detail here).

Combining this with the constants and the definition  $R_1 = 2Zke^2/E$ , the exponent in  $T(E)$  becomes approximately (after some algebra not shown here):

$$-\frac{2}{\hbar} \sqrt{2mER_1^2} \left( \frac{\pi}{2} - 2\sqrt{\frac{R}{R_1}} \right) \approx -\frac{2\pi\sqrt{2m}}{\hbar} \sqrt{2Zke^2R_1} + \frac{8\sqrt{2m}}{\hbar} \sqrt{2Zke^2R}$$

This can be rewritten using  $E_0 = \frac{\hbar^2}{2M(r_0/2)^2}$  and  $r_0 = \frac{ke^2}{Mc^2} \times (\text{correction factor}) \approx 7.25$  fm as:

$$T(E) \approx \exp\{-4\pi Z \sqrt{E_0/E} + 8\sqrt{Z(R/r_0)}\}$$

This gives the decay probability per "attempt" or collision with the barrier.

Go to Frame 4.

### Frame 4

Now, let's plug in the numbers for  $^{226}_{88}\text{Ra}$  decaying to  $^{222}_{86}\text{Rn}$  with  $E = 5$  MeV. The daughter nucleus is Radon (Rn), so  $Z = 86$ . The radius  $R$  of the Radon nucleus is calculated using  $R = (1.2 \times 10^{-15} \text{ m})A^{1/3}$ . What is the mass number  $A$  for the Radon nucleus?  $A =$  \_\_\_\_\_. Calculate  $R$ .  $R =$  \_\_\_\_\_. fm.

Use  $E_0 = 0.0993$  MeV and  $r_0 = 7.25$  fm (values provided in the text derived from fundamental constants). Calculate the exponent:  $\text{Exponent} = -4\pi Z \sqrt{E_0/E} + 8\sqrt{Z(R/r_0)}$  Exponent = \_\_\_\_\_

Finally, calculate  $T(E) = e^{\text{Exponent}}$ .  $T(E) =$  \_\_\_\_\_

Go to Frame 5 for the results.

### Frame 5

For  $^{222}_{86}\text{Rn}$ , the mass number is  $A = 222$ .  $R = (1.2 \text{ fm})(222)^{1/3} \approx (1.2 \text{ fm})(6.05) \approx 7.27$  fm.

The exponent is:  $-4\pi(86)\sqrt{0.0993/5} + 8\sqrt{86(7.27/7.25)} \approx -1081 \times \sqrt{0.01986} + 8\sqrt{86.24} \approx -1081 \times (0.1409) + 8 \times (9.286) \approx -152.3 + 74.3 = -78.0$

The probability is  $T(E) = e^{-78.0} \approx 1.3 \times 10^{-34}$ .

This probability seems incredibly small! Yet, Radium decays. Why is the observed decay rate much higher? (Hint: How often does the alpha particle try to escape?) \_\_\_\_\_

Go to Frame 6.

### Frame 6

The observed decay rate is much higher because the alpha particle collides with the nuclear barrier extremely frequently. The collision frequency  $f$  is roughly the speed of the alpha particle inside the nucleus ( $v$ ) divided by the diameter ( $2R$ ),  $f \approx v/(2R)$ . This frequency is estimated to be around  $f \approx 10^{21}$  Hz (collisions per second).

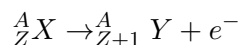
The decay rate  $\lambda$  (decays per second) is the probability per collision  $T(E)$  times the frequency of collisions  $f$ .  $\lambda = f \times T(E) \approx 10^{21} \text{ s}^{-1} \times 1.3 \times 10^{-34} \approx 1.3 \times 10^{-13} \text{ s}^{-1}$ . (The text calculation yields  $\sim 10^{-13}$  using slightly different intermediate numbers). This predicted rate is much closer to observed values (e.g., text mentions observed  $\lambda \approx 1.4 \times 10^{-11} \text{ s}^{-1}$  for  $^{226}\text{Ra}$ , suggesting our approximations are reasonable but not exact).

Go to Frame 7.

### Frame 7

Let's move on to **Beta Decay**. In this process, the number of nucleons ( $A$ ) stays the same, but the atomic number ( $Z$ ) changes by  $\pm 1$ . This means a neutron changes into a proton or vice versa within the nucleus.

The basic process where a nucleus emits an electron ( $e^-$ ) is:



(Example:  ${}^{14}_6\text{C} \rightarrow {}^{14}_7\text{N} + e^-$ )

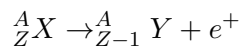
What particle is essentially an electron emitted from the nucleus? [a] Alpha particle [b] Beta particle [c] Gamma ray

Go to Frame 8.

### Frame 8

The answer is [b] Beta particle. Historically, electrons emitted from nuclei were called beta particles ( $\beta^-$ ).

There is also a process where a nucleus emits a positron ( $e^+$ ), which is the antiparticle of the electron:



(Example:  ${}^{12}_7\text{N} \rightarrow {}^{12}_6\text{C} + e^+$ )

In  $\beta^-$  decay, a neutron effectively becomes a proton ( $n \rightarrow p + e^-$ ). In  $\beta^+$  decay, a proton effectively becomes a neutron ( $p \rightarrow n + e^+$ ).

Crucially, the electron or positron is \*created\* at the moment of decay; it doesn't exist inside the nucleus beforehand.

Go to Frame 9.

### Frame 9

A major puzzle arose when measuring the kinetic energy of the emitted beta particles. If the decay is just  $X \rightarrow Y + e$ , energy conservation dictates that the electron should always have a specific kinetic energy determined by the mass difference (Q-value), assuming the heavy nucleus Y has negligible recoil energy.

However, experiments (like Figure 13.19) show that the beta particles are emitted with a \*continuous range\* of energies, from zero up to a maximum value,  $K_{max}$ .

This observation seemed to violate which fundamental conservation law? \_\_\_\_\_

Go to Frame 10.

### Frame 10

The continuous energy spectrum seemed to violate the **Law of Conservation of Energy**. It also turned out that angular momentum (spin) and linear momentum were also not conserved in the simple two-body decay  $X \rightarrow Y + e$ .

To save these fundamental laws, Wolfgang Pauli proposed in 1930 that a \*third\*, undetected particle must be emitted along with the beta particle. Enrico Fermi later named this particle the **neutrino** ( $\nu$ ).

What are the key properties proposed for the neutrino to solve the conservation problems? List three properties from page 488. 1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

Go to Frame 11.

### Frame 11

The required properties of the neutrino ( $\nu$ ) are: 1. **Zero electric charge** (to conserve charge). 2. **Very small (or zero) rest mass** (to explain why it was hard to detect and account for the maximum electron energy). Modern experiments show it's not exactly zero, but much less than the electron mass. 3. **Spin of 1/2** (to conserve angular momentum). 4. (Implied) Must carry away the "missing" energy and momentum. 5. (Implied) Interacts very weakly with matter (making it hard to detect).

Neutrinos were finally detected experimentally in 1956.

It also turns out there is an **antineutrino** ( $\bar{\nu}$ ), the antiparticle of the neutrino.

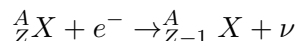
The correct beta decay equations are:  $\beta^-$  decay:  ${}_Z^AX \rightarrow {}_{Z+1}^AY + e^- + \bar{\nu}$   $\beta^+$  decay:  ${}_Z^AX \rightarrow {}_{Z-1}^AY + e^+ + \nu$

Go to Frame 12.

### Frame 12

There's a third related process called **Electron Capture (EC)**. Instead of emitting a positron, a nucleus can capture one of its own orbital electrons (usually from the innermost K-shell). This also changes a proton to a neutron.

The general reaction is:



(Example:  ${}_4^7\text{Be} + e^- \rightarrow {}_3^7\text{Li} + \nu$ )

What particle is \*emitted\* in electron capture? \_\_\_\_\_

Go to Frame 13.

### Frame 13

Only a **neutrino** is emitted in electron capture. The captured electron disappears, combining with a proton to form a neutron.

**Q-values for Beta Decay** (Energy released): The energy released,  $Q$ , determines if a decay is possible. It's calculated from the \*atomic\* masses ( $M$ , which include electron masses): For  $\beta^-$  decay:  $Q = [M({}_Z^AX) - M({}_{Z+1}^AY)]c^2$  For  $\beta^+$  decay:  $Q = [M({}_Z^AX) - M({}_{Z-1}^AY) - 2m_e]c^2$  (Need energy for  $e^+$  \*and\* an extra orbital  $e^-$  for Y) For EC:  $Q = [M({}_Z^AX) - M({}_{Z-1}^AY)]c^2$

A decay can only occur if  $Q > 0$ .

Go to Frame 14.

#### Frame 14

A very useful application of beta decay is **Radiocarbon Dating**, using the isotope Carbon-14 ( $^{14}\text{C}$ ).  $^{14}\text{C}$  is continuously produced in the upper atmosphere by cosmic rays. It mixes into the ecosystem, and the ratio  $^{14}\text{C} / ^{12}\text{C}$  in the carbon dioxide ( $\text{CO}_2$ ) of the atmosphere is roughly constant ( $\approx 1.3 \times 10^{-12}$ ).

Living organisms continuously exchange  $\text{CO}_2$  with the atmosphere, so they maintain this same  $^{14}\text{C} / ^{12}\text{C}$  ratio. What happens to this ratio when the organism dies? \_\_\_\_\_

Go to Frame 15.

#### Frame 15

When an organism dies, it stops exchanging carbon with the environment. The stable  $^{12}\text{C}$  remains, but the radioactive  $^{14}\text{C}$  decays via  $\beta^-$  emission ( $^{14}\text{C} \rightarrow ^{14}\text{N} + e^- + \bar{\nu}$ ) with a half-life ( $T_{1/2}$ ) of 5730 years. Therefore, the  $^{14}\text{C} / ^{12}\text{C}$  ratio decreases over time. By measuring this ratio (or the remaining  $^{14}\text{C}$  activity), we can determine the time since death.

**Example (based on 13.11):** An archaeologist finds 25.0 g of charcoal with a measured  $^{14}\text{C}$  activity ( $R$ ) of 250 decays/minute. How long ago did the tree die?

First, find the decay constant  $\lambda$  from the half-life  $T_{1/2} = 5730$  years. Recall the relationship:  $T_{1/2} = \ln(2)/\lambda \approx 0.693/\lambda$ . Calculate  $\lambda$  in units of  $\text{yr}^{-1}$  and  $\text{s}^{-1}$ . ( $1 \text{ yr} \approx 3.16 \times 10^7 \text{ s}$ )  $\lambda = \underline{\hspace{2cm}} \text{yr}^{-1}$   
 $\lambda = \underline{\hspace{2cm}} \text{s}^{-1}$

Go to Frame 16.

#### Frame 16

$\lambda = 0.693/T_{1/2} = 0.693/(5730 \text{ yr}) \approx 1.21 \times 10^{-4} \text{ yr}^{-1}$ .  $\lambda = (1.21 \times 10^{-4} \text{ yr}^{-1})/(3.16 \times 10^7 \text{ s/yr}) \approx 3.83 \times 10^{-12} \text{ s}^{-1}$ .

Next, we need the \*initial\* activity ( $R_0$ ) when the tree was alive. We assume the initial  $^{14}\text{C}/^{12}\text{C}$  ratio was  $1.3 \times 10^{-12}$ . Step 1: Find the number of  $^{12}\text{C}$  nuclei in 25.0 g of Carbon. (Atomic mass  $\approx 12.0 \text{ g/mol}$ , Avogadro's number  $N_A = 6.02 \times 10^{23} \text{ nuclei/mol}$ ). Number of  $^{12}\text{C}$  nuclei = \_\_\_\_\_

Step 2: Find the initial number of  $^{14}\text{C}$  nuclei ( $N_0$ ) using the ratio.  $N_0(^{14}\text{C}) = \underline{\hspace{2cm}}$

Go to Frame 17.

#### Frame 17

Number of moles of C =  $25.0 \text{ g}/12.0 \text{ g/mol} \approx 2.08 \text{ mol}$ . Number of  $^{12}\text{C}$  nuclei  $\approx (2.08 \text{ mol}) \times (6.02 \times 10^{23} \text{ nuclei/mol}) \approx 1.25 \times 10^{24}$  nuclei.

Initial number of  $^{14}\text{C}$  nuclei  $N_0 = (\text{ratio}) \times (\text{No. } ^{12}\text{C} \text{ nuclei})$   $N_0 = (1.3 \times 10^{-12}) \times (1.25 \times 10^{24}) \approx 1.63 \times 10^{12}$  nuclei.

Now, calculate the initial activity  $R_0$  using  $R_0 = \lambda N_0$ . Use  $\lambda$  in  $\text{s}^{-1}$ .  $R_0 = \underline{\hspace{2cm}} \text{decays/s}$ . Convert  $R_0$  to decays/minute (multiply by 60).  $R_0 = \underline{\hspace{2cm}} \text{decays/min}$ .

Go to Frame 18.

#### Frame 18

$R_0 = \lambda N_0 = (3.83 \times 10^{-12} \text{ s}^{-1}) \times (1.63 \times 10^{12}) \approx 6.24 \text{ decays/s}$ .  $R_0 = (6.24 \text{ decays/s}) \times (60 \text{ s/min}) \approx 374 \text{ decays/min}$ . (The text calculation gives 370 decays/min, likely due to slight rounding differences).

Finally, use the radioactive decay law for activity:  $R = R_0 e^{-\lambda t}$ . We are given  $R = 250 \text{ decays/min}$ . We found  $R_0 \approx 374 \text{ decays/min}$ . Solve for the time  $t$ .  $250 = 374 e^{-\lambda t}$   $e^{-\lambda t} = 250/374 \approx 0.668$  Take the natural logarithm of both sides:  $-\lambda t = \ln(0.668) \approx -0.403$   $t = -0.403/(-\lambda)$ . Use  $\lambda$  in  $\text{yr}^{-1}$ .  $t = \underline{\hspace{10cm}}$  years.

Go to Frame 19.

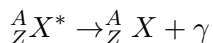
### Frame 19

$t = -0.403/(-1.21 \times 10^{-4} \text{ yr}^{-1}) \approx 3330 \text{ years}$ . So the charcoal is approximately 3300 years old. (Note: Example 13.10 asked why  $^{14}\text{C}$  is better than  $^{11}\text{C}$  (half-life 20.4 min) for dating something like the 5300-year-old Ice Man. The reason is that  $^{11}\text{C}$  decays far too quickly; essentially none would be left after 5300 years, making measurement impossible.  $^{14}\text{C}$ 's half-life is well-suited for this time scale).

Go to Frame 20.

### Frame 20

The final main type is **Gamma Decay**. Often, after an alpha or beta decay, the daughter nucleus is left in an \*excited state\* (denoted  $X^*$ ). It can then transition to a lower energy state (often the ground state  $X$ ) by emitting a high-energy photon.



where  $\gamma$  represents the gamma ray photon.

What changes for the nucleus in gamma decay? [a] Atomic number  $Z$  [b] Mass number  $A$  [c] Energy state [d] Both  $Z$  and  $A$

Go to Frame 21.

### Frame 21

The correct answer is [c] Energy state. In gamma decay, neither  $Z$  nor  $A$  changes; the nucleus simply loses excess energy by emitting a photon.

Gamma rays typically have energies from MeV to GeV. The lifetimes of excited nuclear states are usually very short, around  $10^{-10} \text{ s}$ .

Figure 13.20 shows an example where  $^{12}\text{B}$  beta decays either directly to the ground state of  $^{12}\text{C}$  or to an excited state  $^{12}\text{C}^*$ , which then gamma decays to the ground state.

Go to Frame 22.

### Frame 22

Many heavy radioactive nuclei decay through a sequence of steps, involving alpha and beta decays, until a stable nucleus is reached. These sequences are called **Radioactive Series**.

There are three main naturally occurring series, starting with long-lived isotopes: 1. Uranium Series:  $^{238}\text{U} \rightarrow \dots \rightarrow ^{206}\text{Pb}$  (stable) 2. Actinium Series:  $^{235}\text{U} \rightarrow \dots \rightarrow ^{207}\text{Pb}$  (stable) 3. Thorium Series:  $^{232}\text{Th} \rightarrow \dots \rightarrow ^{208}\text{Pb}$  (stable)

There is also an artificial series starting with  $^{237}\text{Np}$  (or  $^{241}\text{Pu}$ ) ending in  $^{209}\text{Bi}$ . Figure 13.21 shows the steps in the Thorium series. Each alpha decay decreases  $A$  by 4 and  $Z$  by 2. Each beta ( $e^-$ ) decay leaves  $A$  unchanged and increases  $Z$  by 1.

These natural series constantly replenish isotopes (like  $^{226}\text{Ra}$  from the  $^{238}\text{U}$  series) that would have long since vanished due to their shorter half-lives if they weren't being produced.

Go to Frame 23.

### Frame 23

Let's summarize the 5 main decay processes using the notation from Table 13.4. Fill in the blanks for the general forms:

Alpha decay:  ${}^A_ZX \rightarrow {}_{Z-2}^{A-4}\text{He}$  Beta decay ( $e^-$ ):  ${}^A_ZX \rightarrow {}_{Z+1}^AX + e^- + \bar{\nu}$  Beta decay ( $e^+$ ):  ${}^A_ZX \rightarrow {}_{Z-1}^AX + e^+ + \nu$  Electron capture:  ${}^A_ZX + e^- \rightarrow \text{_____} + \nu$  Gamma decay:  ${}^A_ZX^* \rightarrow \text{_____} + \gamma$   
Go to Frame 24 for the answers.

### Frame 24

Here are the completed decay equations:

Alpha decay:  ${}^A_ZX \rightarrow {}_{Z-2}^{A-4}X + {}_2^4\text{He}$  Beta decay ( $e^-$ ):  ${}^A_ZX \rightarrow {}_{Z+1}^AX + e^- + \bar{\nu}$  Beta decay ( $e^+$ ):  ${}^A_ZX \rightarrow {}_{Z-1}^AX + e^+ + \nu$  Electron capture:  ${}^A_ZX + e^- \rightarrow {}_Z^{A-1}X + \nu$  Gamma decay:  ${}^A_ZX^* \rightarrow {}^A_ZX + \gamma$   
Go to Frame 25.

### Frame 25

Two key concepts related to the \*rate\* of decay are the decay constant and half-life. If  $N_0$  is the initial number of radioactive nuclei, the number remaining after time  $t$  is  $N = N_0e^{-\lambda t}$ . The activity (decays per unit time) is  $R = \lambda N = R_0e^{-\lambda t}$ .

What does the decay constant  $\lambda$  represent physically? \_\_\_\_\_

Go to Frame 26.

### Frame 26

The decay constant  $\lambda$  represents the **probability per unit time** that a single nucleus will decay. It has units of inverse time (e.g.,  $\text{s}^{-1}$  or  $\text{yr}^{-1}$ ).

The **half-life**  $T_{1/2}$  is the time required for half of the initial nuclei to decay (or for the activity to drop by half).

What is the mathematical relationship between  $T_{1/2}$  and  $\lambda$ ?  $T_{1/2} = \text{_____}$

Go to Frame 27.

### Frame 27

The relationship is derived from setting  $N = N_0/2$  at  $t = T_{1/2}$ :

$$\frac{N_0}{2} = N_0e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Taking the natural logarithm of both sides:

$$\ln(1/2) = -\lambda T_{1/2}$$

$$-\ln(2) = -\lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln(2)}{\lambda} \approx \frac{0.693}{\lambda}$$

This concludes our review of decay processes.

End of Section.



## 46 Nuclear Reactions, Fission, and Fusion

### Frame 1

This section deals with **nuclear reactions**, where collisions between particles (like neutrons or protons) and atomic nuclei cause changes in the nuclei themselves, often producing new nuclei and particles. We will also explore the two main ways energy is released from nuclei:

- **Fission:** A heavy nucleus splits into smaller ones.
- **Fusion:** Light nuclei combine to form a heavier one.

Both processes release significant energy. Finally, we'll touch upon how radiation interacts with matter and how it's detected.

Go to Frame 2.

### Frame 2

Rutherford first observed a nuclear reaction in 1919. Today, particle accelerators allow us to study countless reactions. A general nuclear reaction can be written as:

$$a + X \rightarrow Y + b$$

where  $a$  is the incoming particle,  $X$  is the target nucleus,  $Y$  is the resulting nucleus, and  $b$  is an outgoing particle. This is sometimes written compactly as  $X(a, b)Y$ .

Example:  ${}^1_1\text{H} + {}^7_3\text{Li} \rightarrow {}^4_2\text{He} + {}^4_2\text{He}$ , or  ${}^7\text{Li}(p, \alpha){}^4\text{He}$ . (Here  $p$  denotes a proton  ${}^1_1\text{H}$ , and  $\alpha$  denotes an alpha particle  ${}^4_2\text{He}$ ).

What fundamental physical principles must be satisfied in any nuclear reaction? List at least three conservation laws mentioned on page 504. 1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

Go to Frame 3 for the answer.

### Frame 3

Nuclear reactions must obey several conservation laws: 1. **Conservation of Mass Number (A):** The total number of nucleons (protons + neutrons) must be the same before and after. In  ${}^7\text{Li}(p, \alpha){}^4\text{He}$ :  $1 + 7 = 4 + 4$ . 2. **Conservation of Charge (q or Z):** The total charge must be the same before and after. In  ${}^7\text{Li}(p, \alpha){}^4\text{He}$ :  $1 + 3 = 2 + 2$ . 3. **Conservation of Energy, Linear Momentum, and Angular Momentum.**

We can use energy conservation to calculate the **reaction energy (Q)**. This is the net kinetic energy released (if  $Q \geq 0$ ) or absorbed (if  $Q < 0$ ) in the reaction.

Assuming particle  $a$  has kinetic energy  $K_a$ , target  $X$  is at rest, and products  $Y$  and  $b$  have kinetic energies  $K_Y$  and  $K_b$ , how is  $Q$  related to these kinetic energies and the rest masses ( $M_a, M_X, M_Y, M_b$ )? (See Eq. 14.2).  $Q =$  \_\_\_\_\_

Go to Frame 4.

### Frame 4

The reaction energy  $Q$  is given by:

$$Q = (K_Y + K_b) - K_a$$

It can also be expressed in terms of the change in rest mass energy:

$$Q = (M_X + M_a - M_Y - M_b)c^2$$

where  $c$  is the speed of light.

If  $Q$  is positive, is the reaction exothermic or endothermic? — If  $Q$  is negative, is the reaction exothermic or endothermic? \_\_\_\_\_

Go to Frame 5.

### Frame 5

Correct!

- If  $Q > 0$ , rest mass is converted into kinetic energy, and the reaction is **exothermic**.
- If  $Q < 0$ , kinetic energy must be supplied to create the extra rest mass of the products, and the reaction is **endothermic**.

For an endothermic reaction ( $Q < 0$ ) to occur, the incoming particle  $a$  needs a minimum kinetic energy, called the **threshold energy** ( $K_{th}$ ). Is  $K_{th}$  simply equal to the magnitude of  $Q$  (i.e.,  $|Q|$ )? [ Yes — No ] (Why or why not?) \_\_\_\_\_

Go to Frame 6.

### Frame 6

The answer is [No].  $K_{th}$  must be greater than  $|Q|$ .

Why? Because momentum must also be conserved. The products ( $Y$  and  $b$ ) must have some kinetic energy after the reaction to carry momentum, even if the reaction just barely occurs. The incoming kinetic energy  $K_{th}$  must supply both the energy deficit  $|Q|$  \*and\* the kinetic energy needed for the products to conserve momentum.

For low-energy, non-relativistic reactions with negative  $Q$ , the threshold energy is given by (Eq. 14.3):

$$K_{th} = |Q| \left( 1 + \frac{M_a}{M_X} \right) = -Q \left( 1 + \frac{M_a}{M_X} \right)$$

Go to Frame 7.

### Frame 7

Let's apply this. Consider the reaction  ${}^1_1\text{H} + {}^7_3\text{Li} \rightarrow {}^4_2\text{He} + {}^4_2\text{He}$  (Example 14.1a). Using the formula  $Q = (M_{Li} + M_H - 2M_{He})c^2$  and atomic mass values from an appendix (or the text value), calculate the  $Q$  value. (Use the conversion factor 931.50 MeV/u, where u is the atomic mass unit).  $Q =$  \_\_\_\_\_ MeV

Go to Frame 8 for the solution.

### Frame 8

Solution for  $Q$  value (Example 14.1a): Given masses:  $M_{Li} = 7.016003$  u,  $M_H = 1.007825$  u,  $M_{He} = 4.002603$  u. Mass difference:  $\Delta M = (7.016003 + 1.007825 - 2 \times 4.002603)$  u  $\Delta M = (8.023828 - 8.005206)$  u  $= 0.018622$  u  $Q = \Delta M \times c^2 = 0.018622$  u  $\times 931.50$  MeV/u  $\approx 17.3$  MeV.

This is a highly exothermic reaction.

Now, for part (b): If the incident protons have  $K_a = 600$  keV  $= 0.6$  MeV, what is the total kinetic energy shared by the two outgoing alpha particles ( $K_{products} = K_{He1} + K_{He2}$ )?  $K_{products} =$  \_\_\_\_\_ MeV

Go to Frame 9.

### Frame 9

Solution for product kinetic energy (Example 14.1b): From the definition  $Q = K_{products} - K_{incident}$ , we have

$$K_{products} = Q + K_{incident}$$

$$K_{products} = 17.3 \text{ MeV} + 0.6 \text{ MeV} = 17.9 \text{ MeV}$$

The two alpha particles share 17.9 MeV of kinetic energy.

Go to Frame 10.

### Frame 10

When a beam of particles hits a target, not every particle causes a reaction. We need a way to quantify the probability of a reaction occurring. This is done using the concept of **cross section** ( $\sigma$ ).

Imagine throwing darts randomly at a wall with balloons (target nuclei) on it (Figure 14.1). The chance of hitting a balloon depends on the total effective area presented by the balloons. Similarly, the probability of a nuclear reaction depends on the effective area presented by each target nucleus to the incoming particle beam.

Is the cross section  $\sigma$  necessarily equal to the geometric area ( $\pi R^2$ ) of the target nucleus? [ Yes — No ] \_\_\_\_\_

Go to Frame 11.

### Frame 11

The answer is [No]. The reaction cross section  $\sigma$  can be larger than, smaller than, or equal to the geometric area. It represents an \*effective\* area for a \*specific\* reaction and often depends strongly on the energy of the incoming particle.

Consider a beam with incident rate  $R_0$  (particles/s) hitting a thin foil of area  $A$ , thickness  $x$ , containing  $n$  target nuclei per unit volume. Let  $R$  be the rate at which reaction events occur. How is the ratio  $R/R_0$  (the probability of interaction) related to  $n$ ,  $A$ ,  $x$ , and  $\sigma$ ? (See Eq. 14.4 and the preceding analogy).  $\frac{R}{R_0} =$  \_\_\_\_\_

Go to Frame 12.

### Frame 12

The probability of interaction is the ratio of the total effective target area to the total area of the foil:

$$\frac{R}{R_0} = \frac{(\text{number of nuclei}) \times (\text{area per nucleus})}{(\text{total foil area})} = \frac{(nAx)\sigma}{A}$$
$$\frac{R}{R_0} = n\sigma x$$

This shows the reaction probability is proportional to:

- The target density,  $n$
- The reaction cross section,  $\sigma$
- The target thickness,  $x$

Go to Frame 13.

### Frame 13

Now, let's consider the number of particles  $N$  that \*pass through\* the foil without reacting. If  $N_0$  particles are incident on a thin layer of thickness  $dx$ , and  $dN$  particles react, then  $dN/N = -n\sigma dx$ . The minus sign indicates particles are removed from the beam.

Integrating this from  $x = 0$  (where  $N = N_0$ ) to a final thickness  $x$  (where number remaining is  $N$ ) gives the exponential attenuation law:

$$N = N_0 e^{-n\sigma x}$$

This shows the number of transmitted particles decreases exponentially with thickness.

What is the common unit used for nuclear cross sections, and what area does it represent? Unit name: \_\_\_\_\_ Area: \_\_\_\_\_ m<sup>2</sup>

Go to Frame 14.

### Frame 14

The common unit for nuclear cross section is the **barn** (b).

$$1 \text{ barn} = 10^{-28} \text{ m}^2$$

This is roughly the geometric cross-sectional area of a medium-sized nucleus ( $R \approx 10^{-14}$  m, Area  $\approx \pi R^2 \approx 3 \times 10^{-28}$  m<sup>2</sup>).

However, actual cross sections vary enormously. Example 1:  $\bar{\nu} + p \rightarrow e^+ + n$  has  $\sigma \approx 10^{-19}$  b (very small!) Example 2:  $n + {}^{127}\text{I} \rightarrow {}^{127}\text{I}^* + n$  (inelastic scattering) has  $\sigma \approx 4$  b.

What does "inelastic scattering" mean in this context? \_\_\_\_\_

Go to Frame 15.

### Frame 15

In **inelastic scattering**, the incident particle (here, a neutron) transfers some of its kinetic energy to the target nucleus, leaving the target in an excited state ( ${}^{127}\text{I}^*$ ). The outgoing neutron has less kinetic energy than the incoming one. (In contrast, elastic scattering conserves kinetic energy).

Let's focus on neutrons. Why are neutrons particularly effective at penetrating matter and interacting with nuclei compared to charged particles like protons or electrons? \_\_\_\_\_

Go to Frame 16.

### Frame 16

Neutrons are effective because they are **electrically neutral**. They don't experience the repulsive Coulomb force from the positively charged nucleus, nor do they interact strongly with the atomic electrons. They only interact via the short-range strong nuclear force (or the weak force).

Free neutrons decay ( $\beta^-$  decay) with a mean life of about 10 minutes, but in reactors or other dense matter, they are usually absorbed by nuclei much faster than that.

Fast neutrons (energy  $> 1$  MeV) primarily lose energy by *elastic* scattering collisions with nuclei. What kind of nuclei are best for slowing down neutrons quickly? (Hint: think about colliding objects). \_\_\_\_\_

Go to Frame 17.

### Frame 17

The most energy is transferred in an elastic collision when the colliding particles have similar masses. Since neutrons have a mass similar to protons, materials rich in hydrogen (like water H<sub>2</sub>O or paraffin) are excellent **moderators** – they efficiently slow down fast neutrons. Graphite (carbon) is also used.

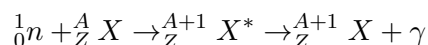
Neutrons continue to lose energy until they reach thermal equilibrium with the moderator material. Their average kinetic energy is then  $K_{av} = \frac{3}{2}k_B T$ , which is about 0.04 eV at room temperature. These slowed-down neutrons are called **thermal neutrons**.

What process becomes highly probable for thermal neutrons, especially in materials like Cadmium (Cd)? (See end of Section 14.3 and Fig 14.3). \_\_\_\_\_

Go to Frame 18.

### Frame 18

Thermal neutrons have a high probability of undergoing **neutron capture**. The process is:



The nucleus absorbs the neutron, becomes an excited isotope ( ${}^{A+1}X^*$ ), and then de-excites by emitting a gamma ray ( $\gamma$ ). The cross section for this can be very large for thermal neutrons (e.g.,  $\sim 10^4$  b for  ${}^{113}\text{Cd}$ ).

Let's work through Example 14.2: A 0.30 mm thick aluminum foil ( ${}^{27}\text{Al}$ ) is bombarded by a neutron flux  $R_0/A = 5.0 \times 10^{12}$  neutrons/cm<sup>2</sup>s. The capture cross section is  $\sigma = 2.0 \times 10^{-31}$  b =  $2.0 \times 10^{-31}$  m<sup>2</sup>. The density of Al is 2.7 g/cm<sup>3</sup>, Avogadro's number is  $N_{Av} = 6.02 \times 10^{23}$  nuclei/mol, and the molar mass is 27 g/mol.

First, calculate the number density  $n$  of Al nuclei (nuclei/m<sup>3</sup>).  $n = (\frac{\text{density}}{\text{molar mass}}) \times N_{Av} =$  nuclei/m<sup>3</sup>

Go to Frame 19.

### Frame 19

Calculation of number density  $n$ : Density  $\rho = 2.7$  g/cm<sup>3</sup> =  $2.7 \times 10^6$  g/m<sup>3</sup>. Molar mass  $M = 27$  g/mol.

$$n = \left( \frac{2.7 \times 10^6 \text{ g/m}^3}{27 \text{ g/mol}} \right) \times (6.02 \times 10^{23} \text{ nuclei/mol})$$

$$n = (1.0 \times 10^5 \text{ mol/m}^3) \times (6.02 \times 10^{23} \text{ nuclei/mol})$$

$$n = 6.02 \times 10^{28} \text{ nuclei/m}^3$$

Now, use the formula  $R/R_0 = n\sigma x$  to find the fraction of neutrons captured. Note  $R_0$  is the rate for the whole beam,  $R_0/A$  is the flux (rate per area). Let's calculate the capture rate per unit area,  $R/A$ . Thickness  $x = 0.30$  mm =  $0.30 \times 10^{-3}$  m. Flux  $R_0/A = 5.0 \times 10^{12}$  n/cm<sup>2</sup>s =  $5.0 \times 10^{16}$  n/m<sup>2</sup>s.  $\frac{R}{A} = \left( \frac{R_0}{A} \right) n\sigma x =$  \_\_\_\_\_ n/m<sup>2</sup>s

Go to Frame 20.

### Frame 20

Calculation of capture rate per unit area  $R/A$ :

$$\frac{R}{A} = (5.0 \times 10^{16} \text{ n/m}^2\text{s}) \times (6.02 \times 10^{28} \text{ nuclei/m}^3) \times (2.0 \times 10^{-31} \text{ m}^2) \times (0.30 \times 10^{-3} \text{ m})$$

$$\frac{R}{A} \approx 1.8 \times 10^{11} \text{ n/m}^2\text{s}$$

To compare with the incident flux per  $\text{cm}^2$ , convert this back:  $R/A \approx 1.8 \times 10^{11} \text{ n}/(10^4 \text{ cm}^2)\text{s} = 1.8 \times 10^7 \text{ n/cm}^2\text{s}$ .

This means that out of  $5.0 \times 10^{12}$  incident neutrons per  $\text{cm}^2$  per second, only  $1.8 \times 10^7$  are captured. The ratio is  $(1.8 \times 10^7)/(5.0 \times 10^{12}) \approx 3.6 \times 10^{-6}$ . Only about 4 neutrons per million are captured!

Go to Frame 21.

### Frame 21

Now we move to **Nuclear Fission**. This is the process where a very heavy nucleus (like  $^{235}\text{U}$ ) splits into two smaller fragments of comparable mass, usually initiated by capturing a thermal neutron.

What is the key observation about the total mass of the products compared to the original mass? (See page 510). \_\_\_\_\_

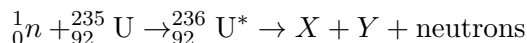
Go to Frame 22.

### Frame 22

In fission, the total mass of the product particles is *less* than the original mass. This "missing" mass is converted into a large amount of energy (typically  $\sim 200 \text{ MeV}$  per fission event) according to  $E = mc^2$ .

This energy release happens because the nucleons in the medium-mass fission fragments are, on average, more tightly bound than in the original heavy nucleus (recall the binding energy per nucleon curve, Fig 13.10).

The fission of  $^{235}\text{U}$ , discovered by Hahn, Meitner, and Strassmann, can be represented as:



$^{236}\text{U}^*$  is a short-lived excited intermediate state.  $X$  and  $Y$  are the fission fragments. What else is typically released besides the fragments  $X$  and  $Y$ ? \_\_\_\_\_

Go to Frame 23.

### Frame 23

Besides the fission fragments, several **neutrons** (typically 2 or 3, average 2.5) are also released in the fission process. This is crucial for chain reactions.

A typical fission reaction is:



The fission fragments (like Ba and Kr) tend to be neutron-rich and are therefore unstable. What type of decay do they typically undergo subsequently? \_\_\_\_\_

Go to Frame 24.

### Frame 24

The neutron-rich fission fragments are unstable and undergo a series of **beta decays** ( $\beta^-$  decay, where a neutron converts to a proton, emitting an electron and an antineutrino) to reach more stable nuclei. Gamma rays are also often emitted during this process as nuclei transition from excited states.

The liquid-drop model provides a visual analogy for fission (Figure 14.5): 1. Neutron capture excites the nucleus ( $^{236}\text{U}^*$ ). 2. Nucleus oscillates violently. 3. It distorts into a dumbbell shape; Coulomb repulsion between the ends pushes it further apart. 4. The nucleus splits (fissions) into two fragments, releasing neutrons.

Using the approximate binding energies per nucleon from Fig 13.10 (Heavy nuclei A 240: 7.6 MeV/nucleon; Medium nuclei A 120: 8.5 MeV/nucleon), estimate the energy Q released per fission.  
 $Q \approx$  \_\_\_\_\_ MeV

Go to Frame 25.

### Frame 25

Estimate of Q from binding energy difference: Energy released = (Total final BE) - (Total initial BE)  $Q \approx (240 \text{ nucleons}) \times (8.5 \text{ MeV/nucleon} - 7.6 \text{ MeV/nucleon})$   $Q \approx 240 \times 0.9 \text{ MeV} \approx 216 \text{ MeV}$ . This is consistent with the measured value of around 200 MeV. (Example 14.4 gives 208 MeV).

Now try Example 14.3: Find the number of neutrons released in these two fission modes: 1.  $^{235}\text{U} + n \rightarrow ^{140}\text{Xe} + ^{94}\text{Sr} + ?n$  2.  $^{235}\text{U} + n \rightarrow ^{132}\text{Sn} + ^{101}\text{Mo} + ?n$  Number of neutrons in (1): \_\_\_\_\_ Number of neutrons in (2): \_\_\_\_\_

Go to Frame 26.

### Frame 26

Solutions for Example 14.3: We balance mass number (A) and charge (Z). Z is conserved automatically if we use element symbols correctly (Xe=54, Sr=38; Sn=50, Mo=42; U=92). 1. A balance:  $1 + 235 = 140 + 94 + x \Rightarrow 236 = 234 + x \Rightarrow x = 2$ . Two neutrons released. 2. A balance:  $1 + 235 = 132 + 101 + y \Rightarrow 236 = 233 + y \Rightarrow y = 3$ . Three neutrons released.

Now let's discuss **Nuclear Reactors**. The key idea is using the neutrons released in one fission event to trigger further fissions, creating a **chain reaction** (Figure 14.6).

If this reaction proceeds uncontrolled, what happens? \_\_\_\_\_ What is the goal in a nuclear power reactor? \_\_\_\_\_

Go to Frame 27.

### Frame 27

An uncontrolled chain reaction leads to a nuclear **explosion** (like a bomb), releasing energy very rapidly. The goal in a nuclear reactor is to maintain a **self-sustained chain reaction** that proceeds at a *controlled*, steady rate.

Natural uranium is mostly  $^{238}\text{U}$  ( 99.3 \_\_\_\_\_)

Go to Frame 28.

### Frame 28

$^{238}\text{U}$  rarely fissions upon neutron capture; it mostly just absorbs the neutron (leading to plutonium production eventually). Therefore, reactor fuel must be artificially **enriched** to increase the percentage of  $^{235}\text{U}$  (typically to a few percent).

The average number of neutrons from a fission event that go on to cause another fission is called the **reproduction constant (K)**. What are the conditions for the reactor state in terms of K? Subcritical: K \_\_\_\_\_ 1 Critical: K \_\_\_\_\_ 1 Supercritical: K \_\_\_\_\_ 1

Go to Frame 29.

### Frame 29

Reactor states: Subcritical:  $K < 1$  (reaction dies out) Critical:  $K = 1$  (steady power output - the goal for operation) Supercritical:  $K > 1$  (reaction rate increases, potentially runaway)

Several factors affect  $K$  besides fuel enrichment: 1. **Neutron Leakage**: Neutrons escape the core. How is this minimized? \_\_\_\_ 2. **Neutron Energies**: Fission neutrons are fast (2 MeV), but  $^{235}\text{U}$  fission is most probable for thermal neutrons (0.04 eV). What component addresses this? 3. **Neutron Capture**: Non-fission capture, especially by  $^{238}\text{U}$ .

Go to Frame 30.

### Frame 30

Factors affecting  $K$ : 1. Leakage is minimized by making the reactor core sufficiently large (reducing the surface-area-to-volume ratio). 2. Fast neutrons are slowed down to thermal energies by the **moderator** (e.g., water, graphite) through elastic collisions. 3. Slowing neutrons down (moderating them) increases the chance of fission by  $^{235}\text{U}$  relative to capture by  $^{238}\text{U}$  (see Fig 14.9).

How is the power level actually controlled to keep  $K$  very close to 1 during operation? (See Fig 14.8). \_\_\_\_\_

Go to Frame 31.

### Frame 31

The power level is controlled using **control rods**. These are made of materials that strongly absorb thermal neutrons (like Cadmium or Boron). By inserting or withdrawing the rods, the number of neutrons available to cause fission is adjusted, thus controlling  $K$  and the reactor power level.

Figure 14.10 shows a common type: the **Pressurized-Water Reactor (PWR)**. Briefly describe the purpose of the primary and secondary water loops. Primary loop: \_\_\_\_\_ Secondary loop: \_\_\_\_\_

Go to Frame 32.

### Frame 32

In a PWR: **Primary loop**: Water circulates through the hot reactor core (acting as coolant and moderator), gets heated under high pressure (to prevent boiling), and transfers heat via a heat exchanger. **Secondary loop**: Water absorbs heat in the heat exchanger, turns to steam, drives a turbine (connected to an electric generator), is condensed back to water, and repeats the cycle. The loops are separate to prevent radioactive contamination of the turbine system.

Go to Frame 33.

### Frame 33

Reactor safety is crucial. Key aspects include robust design, strict operating procedures, trained personnel, and multiple containment layers (fuel cladding, reactor vessel, containment building). Safe long-term **waste disposal** of radioactive byproducts is also a major challenge, often involving sealing waste in containers and deep geological burial.

Go to Frame 34.

### Frame 34

The second method for releasing nuclear energy is **Nuclear Fusion**. This is the process that powers the sun and stars.



How does fusion fundamentally differ from fission? \_\_\_\_\_  
Go to Frame 35.

### Frame 35

Fusion is the process where two *light* nuclei combine (fuse) to form a heavier nucleus, whereas fission is the splitting of a *heavy* nucleus.

Similar to fission, the mass of the fusion product is \*less\* than the sum of the masses of the initial light nuclei. This mass difference is released as energy. This occurs because nucleons are more tightly bound in the resulting heavier nucleus (up to Iron-56) than in the very light initial nuclei (refer again to the binding energy curve).

This concludes the overview of nuclear reactions, fission, and fusion based on the provided text.  
End of Section.

## 47 Nuclear Physics Applications: Fusion and Interactions

### Frame 1

In earlier studies (e.g., Chapter 13 mentioned in the text), we learned about binding energy. Light nuclei (mass number  $\leq 20$ ) generally have \*less\* binding energy per nucleon than heavier nuclei up to iron. This suggests that energy can be released by combining light nuclei to form heavier ones.

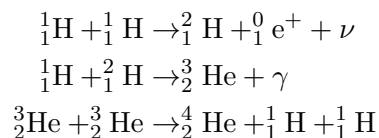
What is this process called? \_\_\_\_\_  
Go to Frame 2 for the answer.

### Frame 2

The process of combining two light nuclei to form a heavier nucleus is called **nuclear fusion**. This is the reverse process of nuclear fission.

Because the total mass of the resulting heavier nucleus is slightly less than the sum of the masses of the original light nuclei, this "missing" mass is converted into energy according to Einstein's famous equation  $E = mc^2$ .

Here are some example fusion reactions that power the Sun (part of the proton-proton cycle):



These reactions occur at extremely high temperatures (around 15 million Kelvin in the Sun's core). Why are such high temperatures necessary for fusion to occur? \_\_\_\_\_

Go to Frame 3.

### Frame 3

High temperatures are required to give the positively charged nuclei enough kinetic energy to overcome their mutual electrostatic repulsion (the **Coulomb barrier**).

Nuclei need to get extremely close (around  $10^{-14}$  m, several fermi) for the short-range strong nuclear force to take over and bind them together (See Figure 14.11). At millions of Kelvin, atoms are fully ionized, forming a state of matter called **plasma** - a collection of bare nuclei and

free electrons. Reactions driven by these high temperatures are called **thermonuclear fusion reactions**.

Which fusion reactions seem most promising for use in a terrestrial (Earth-based) fusion power reactor? (Hint: The text mentions two specific isotopes). \_\_\_\_\_

Go to Frame 4.

#### Frame 4

The most promising reactions for terrestrial fusion involve the hydrogen isotopes deuterium ( ${}^2_1\text{H}$  or D) and tritium ( ${}^3_1\text{H}$  or T):



Deuterium is abundant in water, but tritium is radioactive and must be produced artificially (often using neutrons interacting with lithium).

What is the term for the minimum temperature required for the fusion power generation rate to exceed the energy loss rate (e.g., from radiation)? \_\_\_\_\_

Go to Frame 5.

#### Frame 5

That minimum temperature is called the **critical ignition temperature**. For the D-D reaction, it's about  $4 \times 10^8 \text{ K}$  ( 35 keV). For the D-T reaction, it's lower, about  $4.5 \times 10^7 \text{ K}$  ( 4 keV).

Besides high temperature, what other two conditions are critical for achieving a net energy output from a fusion reactor? (Hint: They relate to the plasma itself and how long it's held together). 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 6.

#### Frame 6

The other two critical parameters are: 1. High **ion density** ( $n$ ): Need enough nuclei packed together to ensure frequent collisions. 2. Sufficient **confinement time** ( $\tau$ ): Need to hold the hot, dense plasma together long enough for fusion reactions to occur at a rate that generates more energy than was used to heat the plasma.

The British physicist J. D. Lawson determined that for net energy gain, the product of ion density and confinement time ( $n\tau$ ) must exceed a certain value. This is **Lawson's criterion**:

$$\begin{array}{ll} n\tau \gtrsim 10^{14} \text{ s/cm}^3 & \text{(for D-T)} \\ n\tau \gtrsim 10^{16} \text{ s/cm}^3 & \text{(for D-D)} \end{array}$$

(See Figure 14.13 for how this varies with temperature).

What are the two main techniques being investigated for confining the plasma? \_\_\_\_\_

Go to Frame 7.

#### Frame 7

The two main plasma confinement techniques are: 1. **Magnetic Field Confinement** 2. **Inertial Confinement**

Let's look at magnetic confinement first. A common device used is the **tokamak** (developed in Russia). It uses a combination of magnetic fields to hold the plasma. What are the two main field components, and what does the resulting combined field do? (See Figure 14.14a). \_\_\_\_\_  
Go to Frame 8.

### Frame 8

The tokamak uses: 1. A strong **toroidal field** (going around the donut shape, generated by external coils). 2. A weaker **poloidal field** (circling the plasma cross-section, generated by a current induced \*within\* the plasma itself).

The combination creates helical magnetic field lines that spiral around the torus. Why is this field configuration necessary? What does it prevent? \_\_\_\_\_

Go to Frame 9.

### Frame 9

The helical field confines the charged particles (ions and electrons) of the plasma, forcing them to spiral along the field lines and preventing them from hitting the walls of the vacuum chamber.

If the plasma touches the walls, two bad things happen: 1. It cools down rapidly, stopping fusion. 2. Heavy impurities are sputtered from the walls, "poisoning" the plasma and causing large energy losses via radiation.

Auxiliary heating (like injecting beams of energetic neutral particles or using radio waves) is also needed to reach ignition temperatures.

Go to Frame 10.

### Frame 10

Now let's consider the second technique: **Inertial Confinement**.

Instead of using magnetic fields, this approach aims to satisfy Lawson's criterion ( $n\tau \geq 10^{14}$ ) differently. Does it rely on a very high ion density ( $n$ ) or a very long confinement time ( $\tau$ )? [ High  $n$  — Long  $\tau$  ]

Go to Frame 11.

### Frame 11

Inertial confinement relies on achieving extremely **high ion density** ( $n \sim 5 \times 10^{25}$  particles/cm<sup>3</sup>, about 200 g/cm<sup>3</sup>) for a very **short confinement time** ( $\tau \sim 10^{-11}$  to  $10^{-9}$  s).

During this incredibly short time, the inertia of the particles prevents them from moving significantly from their compressed state, allowing fusion to occur before the plasma disperses.

What is the most common method used to achieve this rapid compression and heating? (See Figure 14.16). \_\_\_\_\_

Go to Frame 12.

### Frame 12

The most common method is **laser fusion**. Multiple, powerful, symmetrically arranged laser beams strike a tiny pellet (about 1 mm) containing Deuterium-Tritium fuel.

What happens sequence when the lasers hit the pellet? 1. Surface 2. This causes \_\_\_\_\_ on the core. 3. Core is \_\_\_\_\_ and \_\_\_\_\_. 4. Result is \_\_\_\_\_.

Go to Frame 13.

### Frame 13

The sequence is: 1. Surface *evaporates* (ablates). 2. This causes an *implosion* (inwardly moving shock wave) on the core due to reaction force. 3. Core is *compressed* (high density) and *heated* (high temperature). 4. Result is *fusion ignition* (a mini-explosion).

Go to Frame 14.

#### Frame 14

Fusion reactors, regardless of confinement type, need to handle the energy released. In the D-T reaction ( ${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n} + 17.6 \text{ MeV}$ ), how is the energy distributed between the alpha particle ( ${}^4_2\text{He}$ ) and the neutron ( ${}^1_0\text{n}$ )? \_\_\_\_\_

How is the energy from each particle typically captured in a reactor design? (See Figure 14.18).  
Alpha: \_\_\_\_\_ Neutron: \_\_\_\_\_

Go to Frame 15.

#### Frame 15

Energy distribution in D-T fusion: \* Alpha particle: 20\* Neutron: 80

Energy capture: \* Alpha: Being charged, it's trapped in the plasma and its energy contributes to keeping the plasma hot (plasma heating). \* Neutron: Being neutral, it escapes the plasma and is absorbed in a surrounding "blanket," typically containing molten lithium. The neutron's kinetic energy becomes heat in the lithium.

What are two advantages of using lithium in the blanket? 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 16.

#### Frame 16

Advantages of a lithium blanket: 1. It efficiently absorbs the high-energy neutrons, converting their kinetic energy to heat, which can then be used (via a heat exchanger) to generate steam and drive turbines for electricity. 2. Neutron capture by lithium ( ${}^1_0\text{n} + {}^6_3\text{Li} \rightarrow {}^3_1\text{H} + {}^4_2\text{He}$ ) breeds tritium ( ${}^3_1\text{H}$ ), which is essential fuel for the D-T reaction but is not naturally abundant.

List two major advantages and two potential problems of fusion power compared to fission power. Advantages: 1. \_\_\_\_\_ 2. \_\_\_\_\_ Problems: 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 17.

#### Frame 17

Advantages of Fusion: 1. Abundant, low-cost fuel (deuterium from water). 2. Less radioactive waste / inherent safety (no runaway chain reactions possible). 3. Reduced radiation hazard compared to fission products.

Potential Problems of Fusion: 1. Achieving sustained ignition / confinement (feasibility not fully proven). 2. Very high cost of proposed plants. 3. Scarcity of lithium (for D-T cycle fuel breeding). 4. Limited supply of helium (for cooling superconducting magnets, although high-temp superconductors might mitigate this). 5. Structural damage/induced radioactivity from neutron bombardment. 6. Thermal pollution.

#### Frame 18

Let's switch topics to how energetic particles interact with matter. This is important for understanding radiation shielding, particle detectors, and biological effects. We'll consider heavy charged particles, electrons, and photons.

First, **heavy charged particles** (like alpha particles, protons). When moving through a medium (solid, liquid, gas), how do they primarily lose energy? \_\_\_\_\_

They travel a well-defined distance before stopping. What is this distance called? \_\_\_\_\_  
Go to Frame 19.

### Frame 19

Heavy charged particles primarily lose energy through: \* **Excitation** and **ionization** of atoms in the medium. \* (Some energy loss via elastic collisions with nuclei).

This well-defined stopping distance is called the **range** of the particle.

Look at Figure 14.19. For a given energy (e.g., 5 MeV), which particle has a longer range in air, a proton or an alpha particle? Why? Particle: \_\_\_\_\_ Reason: \_\_\_\_\_

Go to Frame 20.

### Frame 20

The **proton** has a much longer range (about 10 times longer) than an alpha particle of the same energy.

Reason: The alpha particle has twice the charge (+2e vs +1e) and is more massive. Its higher charge leads to stronger interactions (more ionization/excitation per unit path length). Its higher mass means it travels slower for a given energy, giving it more time to interact with each atom it passes. Both factors contribute to the alpha losing energy more rapidly and having a shorter range.

The rate of energy loss per unit length,  $-dE/dx$ , is called the **stopping power**. How does it generally depend on the particle's energy? (See Figure 14.20). \_\_\_\_\_

Go to Frame 21.

### Frame 21

The stopping power ( $-dE/dx$ ) is low at very high energies, increases as the particle slows down, reaches a maximum, and then decreases again at very low energies (where elastic collisions become more important than ionization).

Now, consider **electrons**. How does their energy loss and path in matter compare to heavy charged particles? What additional energy loss mechanism is significant for electrons, especially high-energy ones? Comparison: \_\_\_\_\_ Additional mechanism: \_\_\_\_\_

Go to Frame 22.

### Frame 22

Electrons lose energy via ionization and excitation like heavy particles, but their path is \*not\* well-defined due to their small mass. They scatter much more readily and undergo large deflections. This phenomenon is called **straggling**.

The additional energy loss mechanism important for electrons (especially at high energy and in dense materials) is **bremsstrahlung** (German for "braking radiation") - the emission of electromagnetic radiation (photons) when the electron is decelerated by collisions.

Finally, let's consider **photons** (like gamma rays or X-rays). Since they are uncharged, they don't directly ionize matter in the same way charged particles do. How are they removed from a beam as it passes through matter? Name the three main processes (Figure 14.21). 1. \_\_\_\_ 2. \_\_\_\_ 3. \_\_\_\_

Go to Frame 23.

### Frame 23

The three main processes by which photons interact with matter are: 1. **Photoelectric Effect**: Photon is absorbed, ejecting an electron (Dominant at low energies,  $\leq 0.5$  MeV in lead). 2.

**Compton Scattering:** Photon scatters off an electron, transferring some energy to it (Dominant at intermediate energies). 3. **Pair Production:** Photon creates an electron-positron pair near a nucleus (Requires photon energy  $\geq 1.02$  MeV, dominant at high energies).

As a beam of photons passes through matter, its intensity decreases. This is called **attenuation**. The intensity  $I(x)$  after traveling a distance  $x$  is given by:

$$I(x) = I_0 e^{-\mu x}$$

What are  $I_0$  and  $\mu$ ?  $I_0$ : \_\_\_\_\_  $\mu$ : \_\_\_\_\_  
Go to Frame 24.

#### Frame 24

$I_0$  = Incident photon intensity (e.g., photons/m<sup>2</sup>/s).  $\mu$  = **Linear absorption coefficient** (unit: m<sup>-1</sup> or cm<sup>-1</sup>).

The coefficient  $\mu$  depends on the photon energy and the material (see Tables 14.2, 14.3). A related concept is the **half-value thickness** - the thickness of material required to reduce the beam intensity by a factor of 2.

#### Frame 25

Radiation interacting with matter can cause damage. In materials like metals in reactors, high neutron fluxes can cause **atomic displacements**, leading to fatigue. In alkali halide crystals, X-rays can create **color centers**.

What is the primary mechanism for radiation damage in biological organisms? \_\_\_\_\_  
Go to Frame 26.

#### Frame 26

The primary mechanism for biological damage is **ionization effects** within cells. This can create reactive ions and radicals (e.g., from water) that break bonds in vital molecules like proteins and DNA, or the radiation can directly ionize these vital molecules.

Damage is often categorized as: \* **Somatic damage:** Affects body cells (can lead to cancer, organ damage). \* **Genetic damage:** Affects reproductive cells (can lead to defective offspring).

To quantify radiation exposure and its effects, several units are used.

What is the definition of the **roentgen (R)**? (Now mostly historical). \_\_\_\_\_  
Go to Frame 27.

#### Frame 27

The **roentgen (R)** is the amount of ionizing radiation that produces  $3.33 \times 10^{-10}$  C of charge in 1 cm<sup>3</sup> of standard air (or deposits  $8.76 \times 10^{-3}$  J in 1 kg of air). It measures \*exposure\*.

A more common unit measures the \*energy absorbed\* per unit mass of material. What is this unit called (the traditional one)? \_\_\_\_\_

Go to Frame 28.

#### Frame 28

The unit of absorbed energy is the **rad** (Radiation Absorbed Dose). 1 rad =  $1 \times 10^{-2}$  J of energy absorbed per kg of material.

Is 1 rad of alpha particles biologically equivalent to 1 rad of X-rays? [ Yes — No ] \_\_\_\_\_  
Go to Frame 29.

### Frame 29

The answer is [No]. Different types of radiation cause different amounts of biological damage for the same absorbed energy (rad).

To account for this, the **RBE (Relative Biological Effectiveness)** factor is introduced. It compares the biological damage of a given radiation type to that of X-rays or gamma rays (for which RBE=1). (See Table 14.4).

A unit that incorporates both the absorbed dose (rad) and the RBE is used to quantify biological effective dose. What is this traditional unit called? \_\_\_\_\_

Go to Frame 30.

### Frame 30

The unit of biological effective dose is the **rem** (Radiation Equivalent in Man).

$$\text{Dose (rem)} = \text{Dose (rad)} \times \text{RBE}$$

1 rem of any radiation type is assumed to produce the same biological damage.

What are the modern SI units that have formally replaced the rad and the rem? Replaces rad: (Symbol: \_\_\_\_\_) Replaces rem: \_\_\_\_\_ (Symbol: \_\_\_\_\_)

Go to Frame 31.

### Frame 31

The SI units are: \* Replaces rad: **gray (Gy)**. 1 Gy = 1 J/kg = 100 rad. \* Replaces rem: **sievert (Sv)**. 1 Sv = (1 J/kg)  $\times$  RBE = 100 rem.

(Note: The RBE factor itself is dimensionless and used with both systems, though quality factors Q are sometimes used with SI units).

### Frame 32

Now let's briefly look at devices used to detect radiation.

The **ion chamber** uses the electron-ion pairs created by radiation passing through a gas between two charged plates to produce a current pulse. If designed to measure the energy of a particle that stops within it, it's called a **proportional counter**.

What common detector uses the same principle but involves an "avalanche" effect, making it sensitive but unable to measure energy accurately? \_\_\_\_\_

Go to Frame 33.

### Frame 33

That detector is the **Geiger counter** (or Geiger-Müller tube). It uses a high voltage between a central wire and an outer cylinder filled with low-pressure gas. Entering radiation creates initial ion pairs, which are accelerated strongly enough to cause further ionizations, leading to an avalanche and a large, easily detectable pulse. (See Figure 14.23).

What detector uses a reverse-biased p-n junction? \_\_\_\_\_

Go to Frame 34.

### Frame 34

That is the **semiconductor diode detector**. Radiation creates electron-hole pairs in the junction region, which are swept apart by the internal field, creating a current pulse.

What type of detector uses a material that emits light flashes when radiation hits it, coupled with a device to convert these light flashes into electrical pulses? Detector Type: Light-to-Electric Device: \_\_\_\_\_

Go to Frame 35.

### Frame 35

Detector Type: **Scintillation counter**. The material (solid or liquid) is the scintillator. Light-to-Electric Device: **Photomultiplier (PM) tube**.

The PM tube uses a photocathode to convert light to electrons, and then a series of electrodes (dynodes) at increasing potential to multiply the number of electrons, resulting in a large output pulse from a single incident particle. (See Figure 14.24).

What general class of detectors is needed to visualize the paths of very high-energy particles, often used in conjunction with magnetic fields to determine momentum? \_\_\_\_\_

Go to Frame 36.

### Frame 36

Those are **Track detectors**. Examples include: \* **Photographic emulsion**: Ionization leaves developable tracks. \* **Cloud chamber**: Ions seed condensation tracks in supersaturated vapor. \* **Bubble chamber**: Ions seed boiling tracks in superheated liquid. \* **Spark chamber**: Particle path creates sparks between high-voltage plates. \* **Wire chamber/Drift chamber**: Collects ionization electrons on grids of wires for electronic readout and track reconstruction.

Neutrons are harder to detect directly. How can they be detected? (Mention method for fast and slow neutrons). Fast: \_\_\_\_\_ Slow: \_\_\_\_\_

Go to Frame 37.

### Frame 37

Neutron detection methods: \* Fast neutrons: Detect recoil protons they knock out of hydrogenous material (like in an ion chamber filled with  $\text{H}_2$ ). \* Slow neutrons: Use a nuclear reaction, e.g., fill an ion chamber with  $\text{BF}_3$  gas. Slow neutrons cause Boron ( $^{10}\text{B}$ ) to capture them and emit highly ionizing alpha particles ( $^{10}\text{B} + \text{n} \rightarrow ^7\text{Li} + \alpha$ ), which are then detected.

### Frame 38

Finally, let's touch on uses of radiation. One major use is **tracing**. Radioactive isotopes (radioisotopes) are used to follow the path of elements in chemical or biological processes.

Give one medical example mentioned in the text. \_\_\_\_\_

Go to Frame 39.

### Frame 39

Medical examples include: \* Using radioactive iodine ( $^{131}\text{I}$ ) to assess thyroid function. \* Injecting radioactive sodium to check for constrictions in the circulatory system by timing its arrival elsewhere.

Tracers are also widely used in agricultural research.

This concludes this section on Nuclear Fusion and the Interaction of Radiation with Matter.



## 48 Nuclear Physics Applications and Elementary Particles

### Frame 1

Detectors like wire chambers, drift chambers, and spark chambers help visualize the paths of charged particles. Wire chambers use grids of parallel wires to detect the electrons created by an ionizing particle passing through a gas. By using grids with wires oriented perpendicularly (e.g., x and y directions), the position of the particle can be determined.

What is the purpose of stacking several such xy locating grids along a third axis (z)? \_\_\_\_\_  
Go to Frame 2 for the answer.

### Frame 2

Stacking the xy grids allows the determination of the particle's **trajectory in three dimensions**.

Detecting neutrons is more challenging than detecting charged particles. Why is this the case? [a] Neutrons are too small. [b] Neutrons decay too quickly. [c] Neutrons do not interact electrically with atoms.

Go to Frame 3.

### Frame 3

Your answer was [ a — b — c ].

The correct answer is [c]. Because neutrons have no electric charge, they don't directly ionize atoms as they pass through matter, making them harder to detect using standard ionization methods.

How are fast neutrons (high energy) typically detected? And how are slow neutrons (e.g.,  $1\text{ MeV}$ ) detected? 1. Fast Neutrons: \_\_\_\_\_ 2. Slow Neutrons: \_\_\_\_\_

Go to Frame 4.

### Frame 4

1. **Fast neutrons** can be detected indirectly. An ion chamber filled with hydrogen gas is used. The fast neutrons collide with protons (hydrogen nuclei), causing the protons to recoil at high speed. These recoiling protons then ionize the gas, which is detected. 2. **Slow neutrons** don't transfer enough energy in collisions. They are often detected using an ion chamber filled with Boron trifluoride ( $\text{BF}_3$ ) gas. Boron captures a slow neutron and undergoes nuclear disintegration, emitting highly ionizing alpha particles, which are then easily detected.

Now let's look at some uses of radiation.

Go to Frame 5.

### Frame 5

#### Uses of Radiation: Tracing

Radioactive isotopes can be used as "tracers" to follow the path of chemicals in various processes, particularly in medicine and biology.

Consider iodine, needed by the thyroid gland. The normal isotope is  $^{127}\text{I}$  (nonradioactive). An artificial radioactive isotope,  $^{131}\text{I}$ , can be produced. If a patient drinks a small amount of sodium iodide containing  $^{131}\text{I}$ , how can doctors assess the thyroid's function? \_\_\_\_\_

Go to Frame 6.

## Frame 6

Doctors assess thyroid function by measuring the **radiation intensity** from  $^{131}\text{I}$  accumulating in the thyroid gland (located in the neck area) over time. The rate and amount of uptake indicate how well the gland is working.

Another medical use involves injecting radioactive sodium ( $^{24}\text{Na}$ ) into a vein (e.g., in the leg) and tracking its arrival time elsewhere in the body with a counter. What information does the elapsed time provide? \_\_\_\_\_

Go to Frame 7.

## Frame 7

The elapsed time for the radioactive sodium to travel indicates the condition of the **circulatory system**, revealing potential constrictions or blockages.

Tracers are also used in agriculture (e.g., tracking fertilizer uptake by tagging nitrogen) and industry (e.g., measuring engine wear by making piston rings radioactive and checking the oil).

Go to Frame 8.

## Frame 8

### Neutron Activation Analysis (NAA)

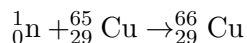
Besides chemical analysis and spectral analysis, NAA is a third technique for identifying elements in a sample. Compared to the other two methods, what are the main advantages of NAA mentioned in the text? 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 9.

## Frame 9

The main advantages of NAA are: 1. It is often **non-destructive** (or requires only a very small sample). 2. It can detect **extremely small quantities** of an element.

The basic principle is to irradiate the sample with neutrons. Nuclei in the sample capture neutrons, transforming into different isotopes, many of which are radioactive. For example:



This newly created isotope,  $^{66}\text{Cu}$ , is radioactive. How does its radioactivity allow us to detect the presence of the original copper ( $^{65}\text{Cu}$ ) in the sample? \_\_\_\_\_

Go to Frame 10.

## Frame 10

The presence of the original  $^{65}\text{Cu}$  is detected by observing the characteristic radiation emitted when the radioactive  $^{66}\text{Cu}$  decays.  $^{66}\text{Cu}$  undergoes beta decay with a specific half-life (5.1 min) and emits beta particles and gamma rays with characteristic energies (e.g., 1.04 MeV gamma ray). By identifying this unique radiation signature, even tiny amounts of the original copper can be detected.

NAA is used in checking airline luggage for explosives and was famously used to analyze hair samples from Napoleon, suggesting arsenic poisoning. What other field uses NAA to detect forgeries or hidden layers? \_\_\_\_\_

Go to Frame 11.

## Frame 11

NAA is used by **art historians**. Different pigments used throughout history react differently to neutron activation, allowing detection of forgeries or revealing hidden paintings beneath newer layers.

### **Radiation Therapy**

Radiation is particularly damaging to cells that are dividing rapidly. Why does this property make radiation useful in cancer treatment? \_\_\_\_\_

Go to Frame 12.

## **Frame 12**

Radiation therapy is useful because **cancerous tumor cells typically divide much more rapidly** than surrounding healthy cells, making them more susceptible to damage by radiation.

Radiation can be delivered via focused beams (e.g., from  $^{60}\text{Co}$  sources or X-ray machines, see Fig 14.27) or by implanting small radioactive "seeds" directly into the tumor.

Go to Frame 13.

## **Frame 13**

### **Food Preservation**

High levels of radiation can kill bacteria and mold spores. How is this used in food preservation? What steps are taken after irradiation? \_\_\_\_\_

Go to Frame 14.

## **Frame 14**

Exposure to gamma rays, electron beams, or x-rays is used to **destroy or incapacitate spoilage organisms** in food. After irradiation, the food is typically **sealed in containers** to prevent re-contamination, allowing for long-term storage without significant loss of taste or nutritional value. Major health organizations endorse the safety of this process.

Go to Frame 15.

## **Frame 15**

### **Introduction to Elementary Particles**

The word "atom" comes from the Greek \*atomos\*, meaning "indivisible." For a long time, atoms were considered the fundamental, elementary particles. Discoveries in the early 20th century showed atoms themselves were composed of constituents.

What three particles were initially thought to be the fundamental constituents of atoms, explaining the properties of elements? 1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

Go to Frame 16.

## **Frame 16**

The three constituents were **electrons, protons, and neutrons**.

However, starting around 1945, experiments with high-energy collisions began revealing hundreds of new, highly unstable particles (with very short half-lives). This complicated picture is sometimes called the "particle zoo."

A major simplification came around 1970 with the idea that most of these particles (except electrons, photons, etc.) are actually made of even smaller entities. What are these smaller entities called? \_\_\_\_\_

Go to Frame 17.

## Frame 17

These smaller entities are called **quarks**. Protons and neutrons, for instance, are now understood to be composite particles made of quarks. The theory describing quark interactions is called **quantum chromodynamics (QCD)**.

The modern "Standard Model" of particle physics divides elementary particles into quarks, leptons, and force-carrying particles. To understand their behavior, we first need to understand the fundamental forces.

List the four fundamental forces in nature, from strongest to weakest. 1. (Strongest) 2. 3. 4. (Weakest)

Go to Frame 18.

## Frame 18

The four fundamental forces are: 1. **Strong force** 2. **Electromagnetic force** 3. **Weak force** 4. **Gravitational force**

The **strong force** binds quarks together to form protons, neutrons, etc. It's extremely short-range (  $10^{-15}$  m). The force holding protons and neutrons together in the nucleus is considered a residual effect of this fundamental quark force.

The **electromagnetic force** binds electrons and nuclei to form atoms and molecules. It's long-range (inverse square law) and about 100 times weaker than the strong force.

Go to Frame 19.

## Frame 19

The **weak force** is responsible for processes like beta decay and the decay of heavier quarks and leptons. It is very short-range (  $10^{-18}$  m) and much weaker than the electromagnetic force (about  $10^{-6}$  times the strong force). (It's now seen as unified with the electromagnetic force into the "electroweak" force).

The **gravitational force** holds planets, stars, and galaxies together. It is long-range but incredibly weak at the particle level (about  $10^{-43}$  times the strong force) and is usually negligible in particle physics.

Go to Frame 20.

## Frame 20

In quantum field theory, forces are transmitted or "mediated" by the exchange of specific field particles (quanta), which are all bosons.

The electromagnetic force is mediated by the exchange of **photons**.

What particles mediate the other three forces? (Refer to Table 15.1 if needed). 1. Strong force: \_\_\_\_\_  
2. Weak force: \_\_\_\_\_ 3. Gravitational force: \_\_\_\_\_

Go to Frame 21.

## Frame 21

The mediating particles are: 1. Strong force: **Gluons** 2. Weak force: **W<sup>+</sup>** , **W<sup>-</sup>** , and **Z<sup>0</sup> bosons** 3. Gravitational force: **Gravitons** (hypothetical, not yet detected)

The lifetime of an unstable particle's decay is often a good indicator of which force is responsible, with stronger forces leading to shorter lifetimes (see Table 15.1).

Go to Frame 22.

## Frame 22

### Positrons and Other Antiparticles

In the 1920s, Paul Dirac developed a relativistic version of quantum mechanics. While successful in explaining electron spin, his theory predicted solutions with both positive and negative total energies ( $E = \pm\sqrt{p^2c^2 + m_e^2c^4}$ ).

Why was the existence of negative energy states a problem? What would be expected to happen to ordinary electrons if these states existed and were accessible? \_\_\_\_\_

Go to Frame 23.

## Frame 23

The problem was that electrons in positive energy states should rapidly **transition (fall) into these lower negative energy states**, emitting photons in the process. This contradicted observations, as electrons are stable in positive energy states.

How did Dirac resolve this difficulty? What did he postulate about these negative energy states? \_\_\_\_\_

Go to Frame 24.

## Frame 24

Dirac postulated that **all the negative energy states are normally filled** with electrons. This collection is called the "Dirac sea."

According to the Pauli exclusion principle (which states that no two electrons can occupy the same quantum state), an electron in the Dirac sea cannot transition to another filled state, nor can an external electron fall into the sea. Thus, the sea is normally unobservable.

Under what condition can an electron \*leave\* the Dirac sea? What is left behind? (See Figure 15.1). \_\_\_\_\_

Go to Frame 25.

## Frame 25

An electron can leave the Dirac sea if it is given enough energy (at least  $2m_ec^2$ , the energy gap between the sea and the positive states) to excite it into a positive energy state.

What is left behind is a **vacant state**, or a "hole," in the Dirac sea. This hole behaves like a particle identical to the electron but with a **positive charge**. This predicted particle is the **antiparticle** of the electron, called the **positron** ( $e^+$ ).

Carl Anderson experimentally discovered the positron in 1932 while studying cosmic rays using a cloud chamber in a magnetic field. What common process produces positrons? \_\_\_\_\_

Go to Frame 26.

## Frame 26

A common source of positrons is **pair production**.

In this process, a high-energy gamma-ray photon ( $\gamma$ ) interacts with a nucleus (which helps conserve momentum), and its energy is converted into mass, creating an electron-positron pair ( $e^- + e^+$ ).

What is the minimum energy the gamma ray must have for pair production to occur? (Hint: consider the rest energy of the particles created). \_\_\_\_\_

Go to Frame 27.

### Frame 27

The minimum energy required is the sum of the rest energies of the electron and the positron. Since they have the same mass ( $m_e$ ), the minimum energy is  $E_{min} = m_e c^2 + m_e c^2 = 2m_e c^2$ . Given  $m_e c^2 = 0.511$  MeV, the gamma ray needs at least 1.022 MeV.

## 49 Elementary Particles: Positrons and Other Antiparticles

### Frame 1

In the 1920s, the physicist Paul Dirac developed a version of quantum mechanics that included Einstein's theory of special relativity. This was a major advance, as it naturally explained properties of the electron like its spin and magnetic moment.

However, Dirac's theory also presented a puzzle. His equations had solutions that predicted electrons could exist not only with positive energy but also with *negative* energy.

Why was the existence of negative energy states a problem? What would we expect an ordinary electron (with positive energy) to do if these lower negative energy states were available? \_\_\_\_\_

Go to Frame 2 for the answer.

### Frame 2

The problem was that if negative energy states existed and were accessible, an electron in a positive energy state should quickly transition (fall) into one of these lower states, releasing energy (likely as a photon) in the process.

What would be the ultimate consequence if all electrons could do this? [a] They would gain infinite energy. [b] They would all end up locked in negative energy states. [c] They would cease to exist.

Go to Frame 3.

### Frame 3

Your answer was [ a — b — c ].

The correct answer is [b]. If negative energy states were empty and available, eventually all electrons in the universe would fall into them, contradicting our observation that electrons exist with positive energy.

How did Dirac resolve this apparent paradox? What did he postulate about these negative energy states? \_\_\_\_\_

Go to Frame 4.

### Frame 4

Dirac postulated that **all the negative energy states are already filled** with electrons. This collection of electrons in negative energy states is called the **Dirac sea**.

Why can't we directly observe the electrons in the Dirac sea? (Hint: Consider the Pauli exclusion principle). \_\_\_\_\_

Go to Frame 5.

### Frame 5

We can't directly observe the electrons in the Dirac sea because the Pauli exclusion principle prevents them from interacting with external forces. Since all negative energy states are full, an electron in the sea has no available state to transition to in response to a typical external force. It behaves like an isolated system.

However, there is a way to interact with the Dirac sea. What is required to excite an electron out of the Dirac sea into a positive energy state? (See Figure 15.1). \_\_\_\_\_

Go to Frame 6.

### Frame 6

To excite an electron out of the Dirac sea, we must provide it with enough energy to lift it into a positive energy state. Since the gap between the top of the Dirac sea (at approximately  $-m_e c^2$ ) and the lowest positive energy state (at  $+m_e c^2$ ) is about  $2m_e c^2$ , we need to supply at least this much energy. ( $m_e$  is the electron rest mass,  $c$  is the speed of light).

When an electron is excited out of the Dirac sea, what is left behind? \_\_\_\_\_

Go to Frame 7.

### Frame 7

When an electron is lifted out of the Dirac sea, it leaves behind a vacancy or a **hole**.

This hole is observable! It interacts with external forces similarly to an electron, but Dirac's theory predicted it would behave as if it had the opposite charge.

What is the name given to this "hole" or the antiparticle of the electron? What is its charge?

Name: \_\_\_\_\_ Charge: \_\_\_\_\_

Go to Frame 8.

### Frame 8

The antiparticle to the electron is called the **positron** ( $e^+$ ). It has the same mass as the electron but a **positive** charge ( $+e$ ).

Who experimentally discovered the positron, and when? \_\_\_\_\_

Go to Frame 9.

### Frame 9

The positron was discovered experimentally by **Carl Anderson** in **1932**.

He observed cosmic rays (high-energy particles from space) passing through a cloud chamber placed in a magnetic field. Some particles left tracks like electrons but curved the \*opposite\* way in the magnetic field, indicating they had a positive charge.

This discovery confirmed Dirac's prediction of antiparticles. It is now accepted that essentially every particle has a corresponding **antiparticle**.

What are the general relationships between a particle and its antiparticle regarding mass, spin, and charge? Mass: \_\_\_\_\_ Spin: \_\_\_\_\_ Charge: \_\_\_\_\_

Go to Frame 10.

### Frame 10

A particle and its antiparticle have: Mass: **Equal** mass Spin: **Equal** spin Charge: **Equal** magnitude but **opposite** sign charge. (They also have opposite magnetic moments and other quantum numbers like strangeness, which we might discuss later).

Are there any particles that are their own antiparticles? [ Yes — No ] Can you name one? \_\_\_\_

Go to Frame 11.

### Frame 11

Yes, some neutral particles are their own antiparticles. The most common example is the **photon** ( $\gamma$ ). Others mentioned include the neutral pion ( $\pi^0$ ) and the eta meson ( $\eta$ ).

Go to Frame 12.

### Frame 12

Positrons are commonly produced through a process called **pair production**.

What happens in pair production? What particle initiates it, and what is created? (See page 550). Initiating particle: \_\_\_\_\_ Particles created: \_\_\_\_\_

Go to Frame 13.

### Frame 13

In pair production: Initiating particle: A high-energy **photon** (gamma ray,  $\gamma$ ) Particles created: An **electron-positron pair** ( $e^-$  and  $e^+$ )

This process usually happens when the photon interacts with a nucleus, which helps conserve momentum.

What is the minimum energy the photon must have for pair production to occur? Why? Minimum Energy: \_\_\_\_\_ Reason: \_\_\_\_\_

Go to Frame 14.

### Frame 14

Minimum Energy:  $2m_e c^2 \approx 1.02$  MeV (Mega-electron Volts). Reason: This is the minimum energy required to create the rest mass energy of both the electron ( $m_e c^2$ ) and the positron ( $m_e c^2$ ), according to Einstein's  $E = mc^2$ . Any excess photon energy goes into the kinetic energy of the created pair.

Go to Frame 15.

### Frame 15

The reverse process can also happen. What occurs when an electron and a positron meet, especially at low speeds? What is produced? This process is called electron-positron **annihilation**.

Particles interacting: \_\_\_\_\_ Particles produced: \_\_\_\_\_

Go to Frame 16.

### Frame 16

Particles interacting: An electron ( $e^-$ ) and a positron ( $e^+$ ). Particles produced: Typically, **two gamma-ray photons** ( $\gamma$ ).

Why are \*two\* photons usually produced instead of just one? (Hint: Consider conservation laws, see Example 15.1). \_\_\_\_\_

Go to Frame 17.

### Frame 17

Two photons are produced to conserve **momentum**.

If the electron and positron meet at low speeds, their total initial momentum is nearly zero. A single photon always carries momentum ( $p = E/c$ ). To conserve momentum (starting near zero,



ending at zero), two photons must be created traveling in opposite directions, so their momenta cancel out. The total energy of the two photons equals the initial total energy (rest mass energy + kinetic energy) of the electron-positron pair.

Go to Frame 18.

### Frame 18

Electron-positron annihilation is used in a medical imaging technique. What is this technique called? \_\_\_\_\_

Go to Frame 19.

### Frame 19

The technique is called **Positron Emission Tomography (PET)**.

Briefly, how does PET work? 1. Patient receives: \_\_\_\_ 2. Radioactive substance decays emitting: \_\_\_\_ 3. Emitted particle interacts with body tissue producing: \_\_\_\_ 4. Detectors pinpoint: \_\_\_\_  
Go to Frame 20.

### Frame 20

How PET works: 1. Patient receives: A substance like glucose tagged with a **positron-emitting** radioactive isotope. 2. Radioactive substance decays emitting: **Positrons** ( $e^+$ ). 3. Emitted positron quickly annihilates with an electron ( $e^-$ ) in nearby tissue, producing: **Two gamma photons** traveling in opposite directions. 4. Detectors surrounding the patient detect these pairs of photons, and a computer reconstructs the location of the annihilation, pinpointing areas of high glucose uptake (like active brain regions or potentially tumors).

This concludes the section on positrons and antiparticles based on the provided text.

## 50 The Eightfold Way and Quarks

### Frame 1

Scientists often look for patterns in data to understand nature better. The periodic table is a prime example, organizing over 100 elements based on the interactions of just three particles (protons, neutrons, electrons). Similarly, hundreds of atomic nuclei can be built from just protons and neutrons.

Particle physics has also uncovered hundreds of particles and resonances. Could these also be built from a smaller set of fundamental entities? Following the success of the periodic table, physicists searched for patterns among these particles.

Let's consider a group of particles called baryons (which include protons and neutrons) that have a spin quantum number of  $1/2$ . Examples listed in Table 15.2 (from the original text) include p, n,  $\Lambda^0$ ,  $\Sigma^+$ ,  $\Sigma^0$ ,  $\Sigma^-$ ,  $\Xi^0$ , and  $\Xi^-$ .

Two important properties used for classification are electric charge (Q) and a property called "strangeness" (S). What happens if we plot Strangeness versus Charge for these 8 particles?

Go to Frame 2.

### Frame 2

Plotting Strangeness versus Charge for the spin- $1/2$  baryons reveals a fascinating pattern, often shown using a sloping coordinate system for charge (see original text Figure 15.11a).

What shape do six of these baryons form in this plot? [a] A square [b] A hexagon [c] A triangle [d] A straight line

Where are the remaining two baryons located relative to this shape? \_\_\_\_\_  
Go to Frame 3 for the answers.

### Frame 3

The correct shape is [b] A hexagon. The remaining two particles ( $\Lambda^0$  and  $\Sigma^0$ ) are located at the **center** of the hexagon.

A similar pattern emerges when plotting the nine spin-0 mesons (like  $\pi^+$ ,  $\pi^0$ ,  $\pi^-$ ,  $K^+$ ,  $K^0$ , etc.). Again, a hexagon is formed, with particles on the perimeter lying opposite their antiparticles, and the remaining three particles at the center (see original text Figure 15.11b).

These patterns were developed independently by Murray Gell-Mann and Yuval Ne'eman in 1961. Gell-Mann called this classification scheme the **eightfold way**.

Go to Frame 4.

### Frame 4

The eightfold way revealed other patterns too. For example, the family of baryons with spin-3/2 formed a pattern resembling bowling pins (a triangle, see original text Figure 15.12). When this pattern was first proposed, nine particles were known, but there was a missing spot at the bottom.

Based on the pattern, Gell-Mann predicted the properties of this missing particle. What did he call it, and what were its predicted properties (spin, charge, strangeness, approximate energy)?

1. Name: \_\_\_\_\_ 2. Spin: \_\_\_\_\_ 3. Charge: \_\_\_\_\_ 4. Strangeness: \_\_\_\_\_ 5. Approx. Rest Energy: \_\_\_\_\_

Go to Frame 5 for the answers.

### Frame 5

Gell-Mann's predicted particle: 1. Name: **Omega-minus** ( $\Omega^-$ ) 2. Spin: 3/2 3. Charge: -1 4. Strangeness: -3 5. Approx. Rest Energy: 1680 MeV

Was this particle found experimentally? [ Yes — No ]

Go to Frame 6.

### Frame 6

The answer is [Yes]. In 1964, scientists at Brookhaven National Laboratory found the  $\Omega^-$  particle in bubble chamber photographs (see original text Figure 15.13), confirming Gell-Mann's predictions based on the eightfold way pattern.

This success strongly suggested that the patterns were not accidental and hinted at a deeper underlying structure for these particles (known collectively as hadrons).

Go to Frame 7.

### Frame 7

While leptons (like electrons and neutrinos) appear to be fundamental particles with no internal structure, the sheer number and patterns of hadrons suggested they are composite.

In 1963, Gell-Mann and George Zweig independently proposed that hadrons are made of more fundamental constituents called **quarks**.

In the original model, how many types (or "flavors") of quarks were proposed to explain all the known hadrons at the time? [a] Two [b] Three [c] Four [d] Six

Go to Frame 8.

### Frame 8

The answer is [b] Three. The original quarks were named **up (u)**, **down (d)**, and **strange (s)**. (Originally "sideways", but now "strange").

These quarks needed some unusual properties to make the model work. Look at Table 15.3 in the original text. What is unusual about their electric charges? \_\_\_\_\_

What is the baryon number assigned to each quark? \_\_\_\_\_

Go to Frame 9.

### Frame 9

Quarks have **fractional** electric charges: \* u quark:  $+2/3 e$  \* d quark:  $-1/3 e$  \* s quark:  $-1/3 e$  (where  $e$  is the magnitude of the electron charge).

Each quark is assigned a **Baryon Number** of  $1/3$ . They also have spin  $1/2$ . The u and d quarks have Strangeness = 0, while the s quark has Strangeness = -1.

Every quark also has a corresponding **antiquark** (denoted  $\bar{u}, \bar{d}, \bar{s}$ ) with opposite charge, baryon number, and strangeness.

Go to Frame 10.

### Frame 10

With these properties, the composition of all known hadrons could be explained by simple rules:

1. A **meson** (like a pion  $\pi$  or kaon K) consists of one *quark* and one *antiquark* ( $q\bar{q}$ ). What is the total baryon number of a meson according to this rule? \_\_\_\_\_  
2. A **baryon** (like a proton p or neutron n) consists of *three quarks* ( $qqq$ ). What is the total baryon number of a baryon according to this rule? \_\_\_\_\_  
3. An **antibaryon** consists of three antiquarks ( $\bar{q}\bar{q}\bar{q}$ ).

(These combinations are held together by particles called gluons, which transmit the strong force).

Go to Frame 11.

### Frame 11

Answers: 1. Meson baryon number:  $(1/3) + (-1/3) = 0$ . This matches experimental observation. 2. Baryon baryon number:  $(1/3) + (1/3) + (1/3) = 1$ . This also matches experimental observation.

Table 15.4 (original text) shows the quark compositions of several hadrons. Let's check a couple.  
\* Proton (p): uud. Charge =  $(+2/3) + (+2/3) + (-1/3) = +3/3 = +1$ . Correct. \* Neutron (n): udd. Charge =  $(+2/3) + (-1/3) + (-1/3) = 0$ . Correct. \* Pi-plus meson ( $\pi^+$ ):  $u\bar{d}$ . Charge =  $(+2/3) - (-1/3) = +3/3 = +1$ . Correct. Baryon number =  $(1/3) + (-1/3) = 0$ . Correct.

Which quark is needed to build "strange" particles (like  $\Lambda^0$ ,  $K^+$ )? \_\_\_\_\_

Go to Frame 12.

### Frame 12

The **strange (s)** quark is required to build strange particles (those with non-zero strangeness). Note that protons and neutrons, the constituents of ordinary matter, only require u and d quarks. (See original text Figure 15.14 for diagrams).

The original three-quark model was very successful, but some discrepancies led physicists to propose a fourth quark flavor in 1967. What was this quark called? \_\_\_\_\_

Go to Frame 13.

### Frame 13

The fourth quark was named **charm (c)**. It was assigned a new quantum number, Charm ( $C=+1$ ), while its antiquark ( $\bar{c}$ ) has  $C=-1$ . All other quarks (u, d, s) have  $C=0$ . Charm is conserved in strong and electromagnetic interactions.

The existence of charm was confirmed in 1974 with the discovery of a heavy particle called the  $J/\psi$ . What was the quark composition of the  $J/\psi$ ? [a] ccc [b]  $c\bar{d}$  [c]  $c\bar{c}$  [d] udc

Go to Frame 14.

#### Frame 14

The answer is [c]  $c\bar{c}$ . The  $J/\psi$  was a meson made of a charm quark and its antiquark. Its large mass (around 3100 MeV/ $c^2$ ) and relatively long lifetime were strong evidence for this new quark. Other charmed mesons like  $c\bar{d}$  were found soon after.

The story continues! In 1975, evidence for a fifth lepton (the **tau**,  $\tau$ ) was found. This led physicists to predict the existence of two more quarks to maintain a symmetry often seen between leptons and quarks. What names were proposed for these fifth and sixth quarks? 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 15.

#### Frame 15

The fifth and sixth quarks were named **bottom (b)** (or beauty) and **top (t)** (or truth). They carry new quantum numbers, bottomness ( $B'$ ) and topness ( $T$ ). (Note:  $B'$  for bottomness is often used to avoid confusion with Baryon Number  $B$ ).

\* b quark:  $B'=-1$ ,  $T=0$  \* t quark:  $B'=0$ ,  $T=+1$  \* Antiquarks  $\bar{b}$  and  $\bar{t}$  have opposite quantum numbers. \* u, d, s, c quarks have  $B'=0$  and  $T=0$ .

Evidence for the bottom quark came in 1977 with the discovery of the massive Upsilon ( $\Upsilon$ ) meson. What is its likely quark composition? \_\_\_\_\_

When was the top quark finally discovered? \_\_\_\_\_

Go to Frame 16.

#### Frame 16

The Upsilon ( $\Upsilon$ ) meson is composed of  $b\bar{b}$ .

The very massive **top quark** was finally discovered in **1995** at Fermilab.

So, the current understanding (Standard Model) is that there are six flavors of quarks (u, d, c, s, t, b) and six flavors of leptons (e,  $\mu$ ,  $\tau$ , and their corresponding neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ ). These 12 particles (plus their antiparticles) are considered the fundamental fermions (spin-1/2 particles) that make up matter. (See original text Table 15.5).

However, quarks have never been observed in isolation. Why not?

Go to Frame 17.

#### Frame 17

Quarks are believed to be permanently **confined** inside hadrons. This is attributed to the nature of the strong force acting between them. Unlike electromagnetism, the force between quarks \*increases\* or stays constant as they are pulled apart. Trying to separate them requires so much energy that new quark-antiquark pairs are created from the energy, forming new hadrons, rather than isolating a quark.

This unique force requires another property for quarks. Consider the  $\Omega^-$  baryon (sss) or the  $\Delta^{++}$  (uuu). These consist of three identical quarks with parallel spins (to give total spin 3/2). What fundamental principle of quantum mechanics appears to be violated here? \_\_\_\_\_

Go to Frame 18.

### Frame 18

This composition seems to violate the **Pauli Exclusion Principle**, which states that no two identical fermions can occupy the same quantum state simultaneously. Having three identical quarks (s or u) with the same spin alignment in the same particle would violate this principle.

To resolve this, Han and Nambu proposed in 1965 that quarks possess an additional property called **color charge** (or simply "color"). This is \*not\* related to visual color.

How many different "colors" (varieties of this charge) are needed for quarks? [a] One [b] Two [c] Three [d] Six

Go to Frame 19.

### Frame 19

The answer is [c] Three. The quark colors are typically called **red**, **green**, and **blue**. Antiquarks carry corresponding "anticolors" (antired, antigreen, antiblue).

How does the concept of color resolve the Pauli principle violation in the  $\Omega^-$  (sss) baryon? —  
Go to Frame 20.

### Frame 20

The violation is resolved if the three strange quarks in the  $\Omega^-$  each have a \*different\* color: one red, one green, and one blue. Now the quarks are no longer in identical quantum states, and the Pauli principle is satisfied.

A fundamental rule is that all naturally occurring hadrons must be **colorless** (or "white"). How is colorlessness achieved for: 1. Baryons (three quarks)? — 2. Mesons (quark-antiquark)? —  
Go to Frame 21.

### Frame 21

Colorlessness is achieved by specific combinations: 1. Baryons ( $qqq$ ): Must contain one quark of each color (Red + Green + Blue = White). 2. Mesons ( $q\bar{q}$ ): Must contain a quark of one color and an antiquark of the corresponding anticolor (e.g., Red + Antired = White).

The theory describing the interactions based on color charge is called **Quantum Chromodynamics (QCD)**.

Go to Frame 22.

### Frame 22

In QCD, the strong force between quarks is called the **color force**. Like colors repel, opposite (color-anticolor) attract strongly. Differently colored quarks also attract, but less strongly.

What particle mediates the color force, analogous to the photon mediating the electromagnetic force? \_\_\_\_\_

How many types of these mediating particles are there in QCD? \_\_\_\_\_

Do these mediators carry color charge themselves? [ Yes — No ]

Go to Frame 23.

### Frame 23

The color force is mediated by **gluons (g)**. There are **eight** types of gluons. Unlike photons (which are electrically neutral), gluons **do** carry color charge (specifically, a combination of a color and an anticolor, like "blue-antired").

This self-interaction of gluons (since they carry the charge they mediate) is a key reason why the color force behaves differently from electromagnetism and leads to confinement.

Experimental evidence for quarks and gluons comes indirectly, for example, from electron scattering experiments at SLAC and the observation of **jets** of hadrons produced in high-energy collisions (see original text Figure 15.16), which are interpreted as the signatures of underlying quarks and gluons materializing into colorless particles.

Go to Frame 24.

### Frame 24

Finally, how does QCD explain the familiar nuclear force that binds protons and neutrons together in a nucleus?

While hadrons like protons and neutrons are overall colorless, the color forces between their constituent quarks don't perfectly cancel out at short distances. This **residual color force** is what we experience as the strong nuclear force.

Instead of exchanging pions (Yukawa's older model), the interaction between a proton and neutron can be described in QCD by the exchange of which particles between the quarks inside them? (See original text Figure 15.17b).

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This concludes this section on the Eightfold Way and the Quark Model.

End of Section.

## 51 Electroweak Theory and the Standard Model

### Frame 1

In this section, we explore the Electroweak Theory and the Standard Model of particle physics. First, let's recall some properties of the **weak interaction**. It's an extremely short-range force, with an interaction distance of about  $10^{-18}$  m.

What does such a short range imply about the mass of the particles that carry (mediate) this force? [a] They must be very light (low mass). [b] They must be quite massive (high mass). [c] Their mass is irrelevant to the range.

Go to Frame 2 for the answer.

### Frame 2

Your answer was [ a — b — c ]. The correct answer is [b] They must be quite massive. A short-range force is mediated by massive particles. The quantized particles that carry the weak field are the spin-1  $W^+$ ,  $W^-$ , and  $Z^0$  bosons. These are indeed quite massive, comparable to krypton atoms, despite being structureless, point-like particles (see Table 15.5 in your textbook for masses).

What are some key roles of the weak interaction in particle physics? (Mention two from page 580, first paragraph). 1. \_\_\_\_\_ 2. \_\_\_\_\_

Go to Frame 3 for the answer.

### Frame 3

The weak interaction is responsible for: 1. Neutron decay and the beta decay of other, heavier baryons. 2. The decay of heavier quarks (c, s, b, t) into lighter, more stable u and d quarks, and the decay of massive leptons ( $\mu, \tau$ ) into electrons.

Essentially, the weak interaction governs the stability of basic matter particles.

A peculiar feature of the weak interaction is its lack of symmetry, especially compared to the strong, electromagnetic, and gravitational forces. For example, it's not symmetric under mirror reflection or charge exchange. What does "not symmetric under charge exchange" mean for a particle reaction governed by the weak interaction? \_\_\_\_\_

Go to Frame 4.

#### Frame 4

"Not symmetric under charge exchange" means that if you take a reaction and convert all electric charges to their opposites (positives to negatives, vice versa), the new reaction occurs at a different rate (often less frequently) than the original direct reaction.

The example given is the decay of the  $K^0$  (neutral kaon):

$$K^0 \rightarrow \pi^- + e^+ + \nu_e$$

occurs much more frequently than

$$\bar{K}^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e$$

(Note: The text has  $K^0$  in both lines, but typically one would be the antiparticle  $\bar{K}^0$  for charge exchange symmetry discussion, or if discussing  $K^0$  decay, the second reaction would be the charge-conjugated decay of  $K^0$ , which is different. The core point is the asymmetry).

Go to Frame 5.

#### Frame 5

The distinct nature of the electromagnetic force (mediated by massless photons, infinite range) and the weak force (mediated by massive bosons, short range) was a puzzle. In 1979, Sheldon Glashow, Abdus Salam, and Steven Weinberg received the Nobel Prize for developing a theory that unifies these two.

What is this unifying theory called? \_\_\_\_\_

What does this theory postulate about the weak and electromagnetic interactions at very high particle energies? \_\_\_\_\_

Go to Frame 6 for the answers.

#### Frame 6

The unifying theory is called the **Electroweak Theory**.

It postulates that the weak and electromagnetic interactions have the **same strength** at very high particle energies. They are viewed as two different manifestations of a single, unifying electroweak interaction.

What particles play key roles in the electroweak theory? (Mention the photon and the three weak bosons). \_\_\_\_\_

Go to Frame 7.

#### Frame 7

The key particles in the electroweak theory are:

- The photon ( $\gamma$ )
- The three massive bosons ( $W^+$ ,  $W^-$ , and  $Z^0$ )

One of the most spectacular predictions of this theory was the masses of the W and Z particles. What were these predicted masses (approximately)? W bosons: \_\_\_\_\_ Z boson: \_\_\_\_\_  
Go to Frame 8.

### Frame 8

The electroweak theory predicted the masses of the W and Z particles to be: W bosons: about 80 GeV/c<sup>2</sup> Z boson: about 91 GeV/c<sup>2</sup>

Carlo Rubbia and Simon van der Meer received the 1984 Nobel Prize for their work leading to the discovery of these particles at CERN with these exact masses.

Go to Frame 9.

### Frame 9

The combination of the electroweak theory with QCD (Quantum Chromodynamics, the theory of the strong interaction) is known as the **Standard Model** of high-energy physics.

What fundamental constituents of matter and forces does the Standard Model include? (List categories of particles and forces, see page 580, bottom paragraph).

- Constituents of matter: \_\_\_\_\_
- Forces and their field particles: \_\_\_\_\_

Which fundamental force is \*not\* included in the Standard Model at this time? \_\_\_\_\_  
Go to Frame 10 for the answers.

### Frame 10

The Standard Model includes:

- Constituents of matter: Six leptons and six quarks.
- Forces and their field particles: The electromagnetic, weak, and strong forces (and their associated bosons like photons, W/Z bosons, gluons).

The fundamental force \*not\* included in the Standard Model at this time is the **gravitational force**.

Physicists hope that string theory might provide the unification of gravity with the Standard Model. Does the Standard Model answer all questions in particle physics? [ Yes — No ]

Go to Frame 11.

### Frame 11

The answer is [No]. The Standard Model does not answer all questions. For example, it requires over a dozen measured numbers (like lepton and quark masses) as input parameters but doesn't explain why these values are what they are, or the mysterious ratios of these masses. It also doesn't explain why there are only six types of leptons and six types of quarks.

A key question related to field particles is: Why does the photon have no mass while the W and Z bosons are massive? This difference leads to the distinct behaviors of electromagnetic and weak forces at low energies. At high energies, where the rest energies of W and Z are small compared to their total energies, the forces become similar (symmetric). The transition from symmetric behavior at high energies to distinct behavior at low energies is called \_\_\_\_\_.

Go to Frame 12.



## Frame 12

The transition from symmetric behavior at high energies to distinct behavior at low energies is called **symmetry breaking**.

In 1964, Peter Higgs proposed a mechanism for this electroweak symmetry breaking. What did he propose? [a] A new set of undiscovered quarks. [b] A new field that permeates all of space. [c] A modification to the theory of relativity.

Go to Frame 13.

## Frame 13

Your answer was [ a — b — c ]. The correct answer is [b] A new field that permeates all of space.

This proposed field is called the **Higgs field**. How does the Higgs field give particles their mass, roughly speaking? \_\_\_\_\_

Go to Frame 14.

## Frame 14

Roughly, the Higgs field can be viewed as causing a kind of "drag force" on particles as they interact with it. This interaction gives particles their characteristic inertia, which we perceive as mass.

Like other classical fields, the quantization of the Higgs field results in a force-carrying particle. What is this particle called? \_\_\_\_\_

Has it been definitively found (as of the text's writing in 2005)? [ Yes — No ] What is its expected mass range? \_\_\_\_\_

Go to Frame 15 for the answers.

## Frame 15

The force-carrying particle of the Higgs field is called the **Higgs boson**.

As of the text's writing (2005), the Higgs boson had \*not\* yet been found. (Note: The Higgs boson was famously discovered at CERN in 2012, after this textbook was published). Physicists (at the time of the text) thought its mass should be less than 1 TeV ( $10^{12}$  eV).

To produce and detect particles like the Higgs boson, extremely high energy collisions are needed. Why are **colliders** (colliding-beam accelerators) preferred over conventional accelerators with fixed targets for achieving these high effective collision energies? \_\_\_\_\_

Go to Frame 16.

## Frame 16

Colliders are preferred because when particles with equal masses and kinetic energies traveling in opposite directions collide head-on, their total momentum is zero. This means that \*all\* of their kinetic energy is available for the reaction (e.g., to create new, massive particles). In fixed-target collisions, a significant portion of the incoming particle's energy must go into the kinetic energy of the products to conserve momentum, reducing the energy available for particle creation.

The text mentions several colliders:

- LEP (Large Electron-Positron Collider) at CERN
- Stanford Linear Collider (SLC) in California

- Super Proton Synchrotron (SPS) at CERN (accelerates protons and antiprotons)
- Tevatron at Fermi National Laboratory (protons, almost 1 TeV)
- LHC (Large Hadron Collider) at CERN (proton-proton, designed for 14 TeV, under construction at time of text)

Go to Frame 17.

### **Frame 17**

This concludes our discussion of the Electroweak Theory and the Standard Model as presented in section 15.11. It's a framework that describes fundamental particles and their interactions (excluding gravity) with remarkable success, though it still leaves some fundamental questions unanswered and relies on experimentally determined parameters. The search for particles like the Higgs boson and a deeper understanding of symmetry breaking were (and continue to be) major drivers in experimental particle physics.

End of Section 15.11.