

Experimental design and Statistical Parametric Mapping - Karl Friston

1. Intro

- Characterizing a regionally specific effect rests on estimation and inference
- Functional specialization and integration serve as motivation for most analyses of neuroimaging data
- These two have to be combined for full understanding of brain mapping results
- Statistical parametric mapping is generally used to identify functionally specialized brain responses
- Characterizes functional anatomy and disease-related changes
- voxel-based, classical inference, comments on regionally specific responses to experimental factors

2. Functional specialization and integration

- Brain has two fundamental principles of functional organization
- Functional integration
- Functional specialization

2a. Functional specialization and segregation

- Functional role of a brain component is defined by its cortical connections
- Functional segregation demands that cells with common functional properties are grouped together
- The analysis of functional neuroimaging data is divided into:
 - Spatial processing
 - Estimating parameters of a statistical model
 - Inference on parameter estimates with appropriate statistics

3. Spatial Realignment and Normalization (Section I: Computational Neuroanatomy)

- Analysis of neuroimaging data starts with series of spatial transformations
- reduce unwanted variance components in voxel time-series induced by movement or shape differences among series of scans
- voxel-based analyses assume data are derived "locally" (enabling reporting regionally-specific effects)
- First step is the realign data
- then transform using linear or nonlinear warps into a standard anatomical space
- finally, data are spatially smoothed

(Chapter 2: Rigid body registration)

3a. Realignment

- Changes in signal intensity over time arise from head motion, disrupting fMRI study results
- Realignment involves:
 - estimating the 6 parameters of an affine 'rigid-body' transformation that minimizes the differences (LSA)
 - first-order approximation of the Taylor expansion of the effect of movement on signal intensity using spatial derivatives
 - allows for a simple iterative least squares solution corresponding to a Gauss-Newton search
 - applying the transformation by re-sampling the data using tri-linear, sinc or spline interpolation.

3b. Adjusting for movement related effects in fMRI

- as much as 90% of variance in fMRI time-series can be effects of movement after realignment
- caused by effects that cannot be modeled using a linear affine model
- nonlinear effects include:
 - subject movement between slice acquisition
 - interpolation artifacts
 - nonlinear distortion due to magnetic field inhomogeneities
 - spin-excitation history effects
- these effects create movement-related signal "y", a nonlinear function of displacement "x" in current and previous scans
- $y_n = f(x_n, x_{n-1}, \dots)$
- this estimated signal is then subtracted from original data
- adjustment can be carried out pre-processing step or embodied in model estimation during the analysis
- this considers spatial realignment, not temporal realignment
- temporal realignment: using sinc interpolation over time and only when:
 - temporal dynamics of evoked responses are important
 - TR (time repetitions) sufficiently small to permit interpolation
 - timing effects are usually unimportant
- provided that effects of latency differences are modelled, this renders temporal realignment unnecessary usually

(Chapter 3: Spatial Normalization using basis functions)

3c. Spatial normalization

- After realignment, a mean image of the series is used to estimate some warping parameters that map it into a template that conforms to a standard anatomical space
- Estimation can use a variety of models for the mapping:
 - 12-parameter affine transformation
 - parameters constitute a spatial transformation matrix
 - low-frequency basis spatial functions
 - discrete cosine set or polynomials
 - parameters are coefficients of basis functions
 - vector field specifying the mapping for each control point (eg voxel)
 - parameters are vast and vector field is bigger than image
- Estimation of parameters in any case can be done through Bayesian framework, finding deformation parameters that have maximum posterior probability $p(\theta|y)$ given data "y"
- $p(\theta|y)p(y) = p(y|\theta)p(\theta)$
- ie, finding deformation (most likely) given the data
- deformation can be found by maximizing probability of getting the data, assuming current estimate of deformation is true, times probability that estimate is true
- deformation is updated iteratively using Gauss-Newton scheme to maximize maximum posterior probability $p(\theta|y)$
 - involves jointly minimizing the likelihood and prior potentials
 - likelihood potential is sum of squared differences between template and deformed image
 - reflects probability of actually getting that image if the transformation was correct

- prior potential is used to incorporate prior info about likelihood of a given warp
- can be determined empirically or motivated by constraints on the mappings
- play a more essential role as the number of parameters increases and are central to high dimensional warping schemes
- Affine or spatial basis function warps and iterative least squares are used to minimize posterior potential

3d. Co-registration of functional and anatomical data

- Can be useful
- Distortion is not an issue if functional data is spatially normalized

3e. Spatial smoothing

- Motivations for smoothing data:
 - by the matched filter theorem, the optimum smoothing kernel corresponds to the size of the anticipated effect
 - by the central limit theorem, smoothing data will render errors more normally distributed and ensure validity of inferences based on parametric tests
 - when inferring about regional effects using Gaussian random field theory, the assumption is that error terms are a reasonable lattice representation of an underlying and smooth Gaussian field
 - in context of inter-subject averaging, often necessary to smooth more to project data onto a spatial scale where homologies in functional anatomy are expressed among subjects

3f. Summary

- Products of spatial normalization are bifold:
 - a spatially normalized image and a deformation field
 - deformation field contains important info about anatomy
 - key part of computational neuroanatomy
 - tensor fields can be analyzed directly (deformation-based morphometry)
 - tensor fields can create maps of specific anatomical attributes (compressions, shears)
 - maps can be analyzed by voxel (tensor-based morphometry)
 - normalized structural images can undergo statistical analysis (voxel-based morphometry)
 - voxel-based morphometry is most common voxel-based neuroanatomical procedure

(Sections II and III: Modeling and Inference)

4. Statistical Parametric Mapping

- Statistical Parametric Mapping: the construction of spatially extended statistical processes to test hypotheses about regionally specific effects
- SPMs (maps) are image processes with voxel values that are distributed according to a known PDF, usually Student T or F
- T-maps or F-maps
- One analyzes each voxel and the resulting parameters are assembled into an image (the SPM)
- SPMs are interpreted as spatially extended processes by referring to the probabilistic behavior of Gaussian fields

- Gaussian random fields (GRF) model probabilistic characteristics of a SPM and any non-stationary spatial covariance structure
- 'Unlikely' excursions of the SPM are interpreted as regionally specific effects (sensorimotor or cognitive process)
- SPM uses the general linear model (GLM) and GRF to infer data through SPMs
- GLM estimates parameters that could explain spatially continuous data
- GRF is used to resolve multiple comparison problem that ensues when making inferences over a volume of the brain
- Reason behind SPM:
 - acknowledge Significance Probability Mapping, the use of interpolated pseudo-maps of p values used to summarize the analysis of multi-channel ERP (event-related potential) studies
 - parametric statistics that comprise the maps
- Subtle motivations despite simplicity of method:
 - mass-univariate analyses rather than multivariate analyses
 - multivariate does not support inferences about regionally specific effects
 - multivariate requires more observations than the dimension (number of voxels)
 - in dimension reduction, multivariate approach is less sensitive to focal effects
 - multivariate uses too many parameters (increasing variability of estimate of a parameter), thus inefficient
 - the minimal parameterization lends SPM added sensitivity
 - GRF theory implicitly imposes constraints on non-sphericity implied by the continuous and extended nature of data
- Bayesian alternative to classical inference with SPMs:
 - uses Posterior Probability Maps (PPMs), less common than SPMs

4a. The General Linear Model (Chapter 7)

- Statistical analysis of imaging data corresponds to:
 - modeling the data to partition observed neurophysiological responses into components of interest, confounds and errors
 - making inferences about the interesting effects in relation the error variance
- the T statistic provides a more versatile and generic way of assessing the significance of regional effects and is preferred over correlation coefficient
- GLM is aka 'analysis of covariance' or 'multiple regression analysis'
 - the matrix X that contains the explanatory variables is called the "design matrix"
 - the column of design matrix corresponds to an effect built into the experiment (explanatory variables, covariates or regressors)
 - the relative contribution of each column is assessed using standard least squares and inferences using T or F stats
- Design matrix:
 - can contain both covariates and indicator variables
 - each column has an associated unknown parameter (only some are of interest)
 - the remaining parameters pertain to confounding effects and are not interesting
 - inference about parameter estimates are made using estimated variance
 - this allows testing null hypothesis (that all estimates are zero) using F stat to give SPM{F} or that a particular linear combination is zero using SPM {T}

- the T stat is obtained by dividing a contrast/compound of the ensuing parameter estimates by its standard error
- standard error of compound is estimated using variance of the residuals about the least-squares fit
- In most analysis, the design matrix contains indicator variables or parametric variables encoding the experimental manipulations
- An important instance of GLM is the linear time invariant (LTI) model
- it explicitly treats the data-sequence as an ordered time-series and enables a signal processing perspective that is useful

[1. LTI systems and temporal basis functions]

4b. Statistical inference and Random Field theory

- Classical inferences using SPMs can be of two sorts
- Anatomically constrained hypothesis
- uncorrected p value associated with the height or extent of that region in the SPM can be used to test the hypothesis
- Anatomically open hypothesis
- The theory of random fields provides a way of adjusting the p value that takes into account the fact that neighboring voxels are not independent by continuity
- For smooth data, the GRF correction is more sensitive than a Bonferroni correction
- GRF theory deals with multiple comparisons problems in the context of continuous, spatially extended statistical field
- Difference between GF and Bonferroni corrections:
- Bonferroni correction controls expected number of false positive voxels
- GRF correction controls expected number of false positive regions
- the corrected threshold under GRF is much more sensitive consequently
- Two assumptions underlying use of GRF correction:
- the error fields are a reasonable lattice approximation to an underlying random field with multivariate Gaussian distribution
- the error fields are continuous, differentiable, invertible
- assumptions are violated only if data are not smoothed (violating reasonable lattice assumption) or model is mis-specified (errors are not normally distributed)

[1. Anatomically closed hypotheses]

- Inferences about regional effects in SPMs can be predicted, but activations may want to be considered near the location
- Two approaches:
- pre-specify a small search volume and make GRF correction
- use uncorrected p value based on spatial extent of nearest cluster
- Both procedures are based on distributional approximations from GRF theory

[2. Anatomically open hypotheses and levels of inference]

- set-level inferences are generally more powerful than cluster-level inferences (more powerful than voxel-level inferences)

- price for increased sensitivity is reduced localizing power
- voxel-level tests permit individual voxels to be identified as significant
- cluster-level only allow cluster significance
- set of clusters only allow set significance
- Typically, voxel-level inferences are used and a spatial extent threshold of zero
- reflects fact that characterizations of functional anatomy are generally more useful when specified with a high degree of anatomical precision

5. Experimental Design

- Different sorts of designs in neuroimaging studies
- Experimental designs can be single-factor or multifactorial designs
- levels of each factor can be categorical or parametric

5a. Categorical designs, cognitive subtraction and conjunctions

- cognitive subtraction: two tasks are separate cognitive or sensorimotor components, thus regionally specific differences in hemodynamic responses identify functionally specialized areas
- cognitive conjunction: extension of subtraction technique, combines a series of subtractions
- conjunction tests several hypotheses, rather than just one, to see if activations, in pairs, are jointly significant
- allows demonstration of context-invariant nature of regional responses
- important in multi-subject fMRI studies

5b. Parametric designs

- parametric design: regional physiology will vary systematically with the degree of cognitive or sensorimotor processing
- neurometric functions may be linear or nonlinear
- using polynomial regression (GLM) identify nonlinear relationships between stimulus parameters (using $SPM\{F\}$)
- clinical neuroscience studies use parametric designs by looking for neuronal correlation of clinical ratings over subjects

5c. Multifactorial designs

- factorial designs enable inferences about interactions
- interactions are associated with factorial designs
- the effect of one factor on another is assessed by interaction term
- interaction effects can be interpreted as:
 - the integration of multiple cognitive processes
 - the modulation of one perceptual process by another
- in clinical studies, interactions are central
- can also embody parametric factors
- can be expressed as a difference in regression slope of regional activity on the parameter, under both levels of the other categorical factor

6. Designing fMRI Studies (Chapter 11: Analysis of fMRI time series)

- fMRI time-series as a linear admixture of signal and noise
- signal corresponds to neuronally mediated hemodynamic changes modeled as a convolution of some underlying neuronal process, responding to changes in experimental factors, by a hemodynamic response function (HRF)
- noise has neuronal and nonneuronal sources
- neuronal noise is neurogenic signals not modeled by explanatory variables with the same frequency structure as signal
- nonneuronal components are low frequency or wide-band
- superposition of all components induces temporal correlations among error terms that effect sensitivity to experimental effects
- sensitivity depends on:
 - relative amounts of signal and noise
 - efficiency of experimental design (reliability of parameter estimates, defined as inverse of variance of contrast of parameters)
- two important considerations from this perspective:
 - optimal experimental design
 - optimum convolution of the time-series to obtain most efficient parameter estimates

6a. The hemodynamic response function and optimum design

- LTI model of neuronally mediated signals in fMRI suggests that only experimentally induced signals that survive convolution with HRF can be estimated
- by convolution theorem the frequency structure of experimental variance should match the transfer function of HRF

6b. Serial correlations and filtering

- conventional signal processing approaches dictate that whitening the data engenders the most efficient parameter estimation
- filtering with a convolution matrix that is inverse of intrinsic convolution matrix
- the 'whitening' strategy renders the least square estimator equivalent to ML or Gauss-Markov estimator
- since the form of intrinsic correlations are unknown, must be estimated

6c. Spatially coherent confounds and global normalization

- implicit in use of high-pass filtering is removal of low-frequency components that are regarded as confounds
- also, signal components that are artifactual or have no regional specificity, called global confounds
- thus, global normalization is needed
- global estimator enters into statistical model as a confound
- in fMRI, instrumentation effects the scale data motivate global normalization before the data enter into the statistical model
- it is important to differentiate between global confounds and their estimators

6d. Nonlinear system identification approaches

- The above only considers LTI models and first order HRFs
- this signal processing perspective is by nonlinear system identification
- characterizing evoked hemodynamic responses in fMRI based on nonlinear system ID, particularly using Volterra series
- enables estimating Volterra kernels that describe relationship between stimulus presentation and hemodynamic responses
- essentially high order extensions of linear convolution models
- kernels represent nonlinear characterization of HRF modeling responses and interaction of stimuli
- in fMRI, kernel coefficients can be estimated by:
- using second order approximation to the Volterra series for GLM
- expanding kernels for temporal basis functions

6e. Event and epoch-related designs

- in experimental design, there is a crucial distinction between event- and epoch-related designs
- fMRI allows measure of event-related responses
- choice of inter-stimulus interval or SOA (stimulus onset asynchrony) is important
- designs can be stochastic or deterministic depending on whether there is a random element to their specification
- stochastic designs specify probabilities of an event occurring
- deterministic designs the event occurring is specified by stimulus
- an efficient design for one effect may not be optimal for another, even within the same experiment

7. Inferences about subjects and populations

Precision is the inverse of variance

- critical issue is whether inference is on effect related to "within-subject variability" or "between-subject"
- difference between "fixed" and "random" effect analysis
- random effects analysis allow inference to be generalized to population

7a. Random-effects analyses

- taking contrasts of parameter estimates from a "first-level" (fixed-effect) analysis and entering them into a "second-level" (random-effect) analysis
- second-level design matrix tests null hypothesis that contrasts are zero

7b. Conjunction analyses and population inferences

- motivation for conjunction analysis within multi-subject studies:
- provide inference, in fixed-effect analysis testing null hypothesis, that is more sensitive than testing average activation
- extended to make inferences about population, when conjunction of effects is established
- conjunction analysis steps:
- design matrix for explanatory variables of each experimental condition (models each subject by condition interactions)

- contrasts are specified that test for effect of interest in each subject to give series of $SPM\{T\}$
- $SPM\{T\}$ are combined at a threshold to give a $SPM\{T_{min}\}$ (ie conjunction SPM)

8. Functional Integration (Section 4)

8a. Functional and Effective connectivity (Chapter 18: Functional integration)

- functional integration is inferred on basis of correlations among measurements of neuronal activity
- functional connectivity is correlation among remote neurophysiological events
- effective connectivity is the influence that one neural system exerts over another
- effective connectivity is dynamic (activity- and time-dependent)
- it depends upon a model of interactions
- estimation procedures employed in functional neuroimaging can be classified:
 - based on linear regression models
 - based on nonlinear dynamic models
- multivariate analysis are necessary to model interactions among brain regions
- inferential or data-led (exploratory)
- based on functional connectivity or covariance patterns (exploratory)
- models of effective connectivity (inferential)

8b. Eigenimage analysis and related approaches (Chapter 19: Functional connectivity)

- most analyses of covariances among brain regions are based on singular value decomposition (SVD) of between-voxel covariances
- voxel-based PCA of neuroimaging time-series characterizes distributed brain systems implicated in sensorimotor, perceptual, or cognitive processes
- distributed systems are identified with principal components (eigenimages) corresponding to spatial modes of coherent brain activity
- simple multivariate characterization of functional neuroimaging time-series
- exploratory analysis
- PCA uses SVD to identify a set of orthogonal spatial modes for greatest variance over time
- covariance among brain regions is equal to functional connectivity
- eigenimage analysis addresses functional integration (ie connectivity)
- eigenimage analysis is limited:
 - provides only a linear decomposition of neurophysiological measurements
 - the set of eigenimages or spatial modes obtained is uniquely determined by constraints that are biologically implausible
- ICA (indep. comp. anal) uses entropy maximization to find, iteratively, spatial modes or dynamics that are approximately independent
- stronger requirement than orthogonality in PCA and involves removing high order correlations among modes
- Cluster analysis, voxels in a multidimensional scaling space are assigned probabilities to a small number of clusters
- characterizing temporal dynamics and spatial modes

8c. Characterizing nonlinear coupling among brain areas (Chapter 20: Effective connectivity)

- linear models of effective connectivity assume that multiple inputs to a brain region are linearly separable
- need for models to include interactions among inputs
- these interactions (or bilinear effects) can be put in structural equation modeling using "moderator" variables that represent the interaction between two regions causing activity in a third
- modulatory effects can be modeled with nonlinear input-output models, particularly Volterra formulation
- Volterra formulation has high face validity and biological plausibility
- its assumption is that response of a region is an analytic nonlinear function of inputs over recent past
- the influence of one region on another has two components
- direct (driving) influence of input from first (lower hierarchy) region, regardless of all other activity
- mediated by first order kernels
- activity-dependent, modulatory component that represents an interaction with inputs from the remaining (higher hierarchy) regions
- mediated by second order kernels
- context-sensitive changes in effective connectivity are most important in functional integration and have two fundamental implications for experimental design and analysis:
- experimental designs for analyses of effective connectivity are multifactorial
- because one factor is needed to evoke responses and render coupling among brain areas measurable and a second factor needs to induce change in that coupling
- models of effective connectivity embrace changes in coupling
- modeled with bilinear terms/interactions

Conclusion.

- Reviewed main components of image analysis and assessing functional integration in the brain
- key principles of functional specialization and integration were considered