`kuramoto_1d.m` - Kuramoto-Sivashinsky Solvers

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Summary

This software (and the similar `kuramoto_1da.m`) solves the Kuramoto-Sivashinsky equation using C^1 Hermite cubic elements in a periodic domain.

The Equations

There are two popular forms of the Kuramoto-Sivashinsky equations. The first mimics the form originally derived by Kuramoto [**?**].

$$d \tag{1}$$

The second has the form of the derivative of equation (1), namely

$$w_t + w_{xx} + w_{xxxx} + 2ww_x = 0.$$

Note that if we scale the spatial variable so that the computational domain occurs over the unit interval: $\bar{x} = x/L$, and define $\epsilon = 1/L^2$, then we have the following version of the Kuramoto-Sivashinsky equation

$$w_t + \epsilon w_{\bar{x}\bar{x}} + \epsilon^2 w_{\bar{x}\bar{x}\bar{x}\bar{x}} + 2\epsilon w w_{\bar{x}} = 0 \tag{2}$$

with $t > 0$, $\bar{x} \in (0,1)$, $w(0,t) = w(1,t)$ and $w_{\bar{x}}(0,t) = w_{\bar{x}}(1,t)$. The initial condition is scaled so that $w(\bar{x},0) = w_0(\bar{x})$.

A nice property of solutions to equation (2) is that the integral

$$\int_0^1 w(\bar{x},t)\ d\bar{x} \tag{3}$$

is independent of time. In other words, the average of the solutions is determined from the average of the initial conditions. This imposes a great deal of structure on the solutions as will be seen in the numerical tests.

length L	ϵ	solution behavior
12.5664	0.00633257	bifurcation
12.8767	0.00603102	heteroclinic bifurcation
13.1403	0.00579148	hopf bifurcation
402.2590	0.00000618	"chaos"

Parameters

Input parameters

- `epsilon` -
  This parameter has the effect of changing the length of the periodic domain. See the scaling of the equations above.

Internal parameters

- `n_gauss` -
  The number of Gauss point used in element integration
- `n_nodes` -
  The number of nodes (including the shared endpoints) used in the spatial discretization. The number of Hermite elements is equal to `n_nodes-1`.
- `t_initial`, `t_step`, `t_save`, and `t_final` -
  Time integration parameters. The solution is saved at multiples of `t_save`, so `t_save`/`t_step` should be a positive integer.
- `resid_tol` -
  Residual tolerance for Newton solve at each timestep.
- `step_tol` -
  Step tolerance for Newton solve at each timestep.
- `max_iterations` -
  Number of Newton iterations attempted to achieve one of the convergence tolerances in each timestep.

Dynamical Systems Analysis