# *A Theory of Crime and Vigilance*

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**Abstract**

This paper develops a novel equilibrium theory of crime and vigilance, in which (i) potential victims elect how much vigilance to exert to guard their property; (ii) potential criminals choose whether to attempt a crime; (iii) random encounters of criminals and victims produce attempted crimes; (iv) the success of an attempted crime is probabilistic; and (v) the crime rate reflects the number of successful attempted crimes.

The model permits a supply and demand interpretation, that yields a rich and tractable framework for performing equilibrium, welfare, and policy analyses. It uncovers a *vigilance force* that invariably limits the efficacy of policies aimed at discouraging crime. When vigilance expenses are greater than expected property losses, an increase in punishment leads to more crime — namely, a *criminal Laffer curve* emerges. This curve peaks earlier and rises faster when victims face higher vigilance costs. Thus, the government may wish to subsidize or mandate vigilance rather than increase legal penalties or policing. In fact, an increase in penalties may shift the levels of vigilance even further away from their socially optimal ones. Finally, this vigilance force makes the crime rate and the attempted crime rate first rise and then fall in the value of the property at stake, which seems to be consistent with the empirical evidence.

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# 1 Introduction

Previous research on the economics of crime emphasized the role of penalties and policing as key drivers of criminal behavior; however, private self-protection by individuals — or *vigilance* — is also an important determinant that has received significantly less attention in the literature. Indeed, recent evidence indicates that vigilance expenditures against crime are sizeable and even greater than public expenditures. Over and above property losses, Anderson (2012) estimates that \$480 billion is spent annually on vigilance efforts in the US, in addition to public expenditures.[1] Also, as we move towards a more digitalized and globally connected economy, cyber crime has become increasingly costly and it presents a pressing issue for corporations, which spend major resources on security (Makridis and Dean, 2018).[2]

Understanding vigilance efforts is essential for improving policy interventions. Consider, e.g., an increase in legal penalties or policing. On one hand, the intended effect of this legal policy is clear: to reduce criminal activity via greater deterrence (Becker, 1968). On the other hand, this policy blunts individuals' incentives to be vigilant, and thereby raises the expected gains of a criminal offense. Altogether, the final effect is ambiguous, for there is a *vigilance force* that will inevitably undermine the efficacy of this policy, potentially crowding out the government's goal. Thus, to correctly evaluate the effect of any policy aimed at alleviating crime, it is fundamental to know (1) how both market and technological forces influence potential victims' incentives to self-protect; (2) how those incentives impact criminal behavior; and (3) the conditions under which policies lead to undesirable outcomes.

This paper develops a novel model of crime centered on the competition between optimizing criminals and potential victims. By showing precisely how the different competing forces interact, this model provides a new vehicle for analyzing policy and its impact on crime. In this model, criminals choose whether to attempt a crime while potential victims decide how vigilant to be. While formally specified as a game, the model admits a natural *supply and demand* interpretation that yields a rich and tractable framework for performing equilibrium, welfare, and policy analyses. The theory predicts a wide range of equilibrium variables such as the crime rate, the attempted crime rate, the failure rate of attempted crimes, and potential victims' vigilance expenses.

Randomness is a crucial aspect of crime. First, few of us are crime victims in any period. According to the Bureau of Justice Statistics, in 2017, only 10.8% of 123 million households experienced one or more property victimizations in the US. The stochastic nature of crime is

[1] Altogether, Anderson (2012) estimates the cost of crime to be around \$1 trillion in the US.

[2] According to estimates of the Center for Strategic and International Studies, cybercrime costs the global economy up to \$600 billion a year. This represents a \$155 billion increase since 2014; see Lewis (2018).

captured by assuming random encounters of criminals and victims. These encounters give rise to the *attempted crime rate*, which influences potential victims' vigilant behavior.[3] Second, randomness plays another key role because not all attempted crimes succeed (e.g., one often fails to break into a house or steal a car).[4] The *failure rate* of an attempted crime determines the expected gains from crime and is positively affected by potential victims' vigilance.

As in Becker (1968), potential criminals have an extensive margin: whether to engage in crime. Stricter punishments or increased policing make crime less appealing, and so formally raise the expected legal costs of crime.[5] I assume that the incentives to attempt a crime vary across potential criminals by considering heterogeneous *opportunity costs*. On the other side, potential victims elect vigilance levels to secure, perhaps imperfectly, their property. Given vigilance, the efficacy to frustrate an attempted crime is heterogeneous: potential victims vary by their self-protection *technologies*. Finally, because criminals are often surprised by the vigilance they encounter, I assume that vigilance efforts are unobserved by criminals. This yields a rich and tractable benchmark model through which one can unveil economic forces likely to be present even if vigilance is observed, as shown in section §9.

All told, I explore the equilibrium of a model in which a continuum of heterogeneous potential criminals each chooses whether to engage in crime, while a continuum of heterogeneous potential victims each elects how vigilant to be. The number of criminals then fixes the attempted crime rate, while vigilance determines the failure rate. Potential victims are hurt by a greater attempted crime rate, while greater vigilance impedes criminal gains. In *equilibrium*, all agents optimize, and the crime rate reflects the number of successful attempted crimes. This model naturally applies to property crime where property can be defined in a broad sense ranging from physical goods (e.g., cars) to digital ones (e.g., data).

I re-interpret the equilibrium as a competitive equilibrium, depicted in a metaphorical, yet microfounded, supply and demand framework. Due to the natural asymmetry of the market, the average failure rate plays the role of a price for potential criminals, while the attempted crime rate plays the same role for potential victims. Optimal criminal behavior induces a *downward-sloping supply of crime*, as fewer crimes are committed when failing is more likely. Similarly, optimal vigilant behavior induces an *upward-sloping demand for vigilance*, because a greater victimization chance elicits a more vigilant response. The supply-demand crossing yields a unique *market equilibrium*, rendering unambiguous all analyzed comparative statics.

[3] While law enforcement may also respond to changes in the attempted crime rate, individuals can clearly vary their own vigilance significantly quicker than law enforcement.

[4] A recent test verified that TSA security prevents only 5% of criminal attempts; see www.vox.com/2016/5/17/11687014/tsa-against-airport-security.

[5] In order to isolate the missing and important interaction between criminals and their target victims, law enforcement is assumed to exogenously impact the expected legal costs of crime for criminals.

Contrary to standard supply and demand analyses, here "trade" is not win-win; rather, criminal offenses constitute an economic good for criminals, but a bad for potential victims. This distinction is fundamental when performing welfare and policy analyses.

Overview of the results. The model delivers new testable predictions. To see this, consider the most well-known claim that aggregate and individual criminal activity should fall when legal penalties or policing increase or, more generally, when the expected punishment rises (Becker, 1968). By contrast, in this paper a higher expected punishment deters criminal entry, shifting the supply curve down, while leaving the demand curve unaffected. So greater expected punishment discourages vigilance,[6] as documented by Vollaard and Koning (2009), raising the marginal profitability of every offense. On balance, there are fewer criminals but each is less likely to fail to compensate for the greater punishment. When the demand curve is sufficiently elastic, the effect on the crime rate is non-monotone. Precisely, Proposition 1 shows that the crime rate is a *hump-shaped* function of the expected punishment, resembling a *criminal Laffer curve*. Thus, an increase in punishment can actually lead to more crime.[7]

Next, Proposition 2 provides a simple sufficient condition to determine *when* the criminal Laffer curve is upward sloping: An increase in expected punishment raises the crime rate if total vigilance expenses are no less than total property losses. Proposition 3 then studies the micro-effects of raising the expected punishment. While the attempted crime rate is common to all potential victims, the ultimate property loss faced by them depends on their own vigilant behavior. I find that the *Lafferian force* is stronger for potential victims with superior technologies who will face more crime after an increase in punishment. Altogether, these results may help to explain why the empirical literature has found very small or no effects of punishment severity on crime rates; see Durlauf and Nagin (2011) for a discussion.

Because the endogenous vigilance efforts by potential victims lies at the heart of this paper, I explore how vigilance costs, or technologies, impact equilibrium outcomes. First, unlike punishment, when vigilance costs rise, or individuals with inferior technologies are more abundant, the demand falls but the supply is unaffected. Thus, there are more criminals, each with lower chance of failing, such that the crime rate rises. The criminal Laffer curve shifts up and peaks earlier, and becomes steeper as it rises (Proposition 4). So, increasing punishment can rapidly raise the levels of crime, attenuating its efficacy even further.

[6] Becker (1968) argued (on p. 188) that improved law enforcement has an ambiguous effect on total offenses. He conjectured that more policing would be partially offset by a drop in private vigilance. Pushing this logic further, this paper shows when such vigilance displacement will lead to an undesirable outcome. See Nagin (2013) and Chalfin and McCrary (2017) for excellent surveys of policing and crime.

[7] "Lafferian effects" have been found in other contexts. For instance, in the tax compliance literature (Graetz et al., 1986; Andreoni et al., 1998), an increase in taxes can lead to an increase in the tax compliance rate. In the industrial organization literature, it has been argued that consumer protection policies (e.g., price cap) can actually harm consumers (Armstrong et al., 2009).

I then explore a simultaneous variation in both the material losses for victims and gains for criminals, or in the *stakes of crime*. This naturally applies, e.g., to a fixed class of goods, as the value of the good varies for owner and criminal alike. Likewise, this change applies to understanding policies that improve the recovery rate of stolen goods, as they clearly lower the stakes of crime. In general, an increase in the stakes of crime impacts both sides of the market: facing a greater loss, potential victims grow more vigilant, while criminals respond to the increased gains with more offenses. The demand and supply curves increase, and so does the failure rate. However, Proposition 5 shows that both the crime rate and the attempted crime rate are non-monotone and highest for mid property values. These findings may help to explain, e.g., why most stolen cars are typically in the mid-range,[8] and at a more aggregated level, why as a country's wealth rises crime initially rise and then, eventually, fall. This non-monotone pattern has been observed in many countries; in particular, in the US from 1960 until now (Farrell et al., 2014).[9] From a policy perspective, improving the recovery rate of stolen goods seems to be effective only for low value property.

When vigilance is unobservable, potential victims benefit from one another's vigilance efforts, as everyone faces the same attempted crime rate independent of their choices. These positive externalities lead to an under-provision of total vigilance (Clotfelter, 1978; Shavell, 1991). Nonetheless, it is important to tie this insight to policy: if legal punishment were to rise, should we expect more or less under-provision of vigilance? Proposition 6 examines how the optimal failure rate and the degree of under-provision are affected by the expected punishment. Interestingly, raising punishment can shift the equilibrium levels of vigilance further away from their socially optimal ones. By contrast, policies targeted to aiding the demand side, such as subsidizing or mandating vigilance (Vollaard and van Ours, 2011), can reduce or even eliminate the under-provision gap.

Some forms of vigilance are noticeable, such as installing a fence or an outside camera on one's property. When vigilance is observed, criminals can effectively target their offenses. But, because criminals are ultimately affected by the success rate of their attempts — which depends upon both the chosen vigilance and the victim's *private* cost-technology — observing low vigilance is not necessarily good news for criminals. In equilibrium, potential criminals use the information contained in vigilance choices to make failure rate inferences and target their offenses, whereas potential victims understand this when choosing their optimal vigilance intensity. This novel *signaling* force is embedded into the model, which

[8] According to the National Insurance Crime Bureau, from 2006–2017, the Honda Civic and Accord have consistently been the most stolen vehicles in the U.S.; see `www.nicb.org/news/reports-statistics`.

[9] Among many competing hypotheses, these authors conclude that vigilance (security, in their paper) is the most likely explanation for why crime has declined in many countries.

yields a tractable framework to perform equilibrium analysis. Proposition 7 gives an equilibrium characterization, uncovering new forces and qualitative differences between observable and unobservable vigilance. In the former case, e.g., the attempted crime rate varies across victims but the failure rate is constant, whereas the opposite holds in the latter case. This divergence yields novel implications for crime victimization patterns. Finally, since the observed vigilance case also admits a supply and demand interpretation, the crowding out of vigilance owed to an increase in punishment is prone to emerge, by common logic.

Outline. Section §3 sets up the model, and §4 performs equilibrium analysis. §5 characterizes the criminal Laffer curve and explores its determinants. Section §6 examines the effects of vigilance costs and material values. §7 performs welfare analysis, whereas §8 studies the under-provision of vigilance and the role of policy. Finally, §9 discusses the observable vigilance case. §10 concludes. All omitted proofs and analyses are in the Appendix.

## 2 Literature Review

The groundbreaking paper by Becker (1968) gave the first economic analysis of crime. Accounting for the optimal criminal response to changes in punishment and the probability of capture, he explored the socially optimal level of law enforcement expenditures. Becker's focus is on the supply side: in his analysis, the levels of crime are determined by the criminal optimization. The demand side is missing in his analysis. Ehrlich (1981) later posited a reduced-form downward demand for crime, where the net payoff per offense acts as the price, and argued that the number of offenses is, indeed, an equilibrium variable.[10] Yet, the optimization of potential victims is absent in his paper.[11] In my analysis, the induced supply and demand curves explicitly arise from both the criminals' and victims' optimization problems, respectively. In doing so, I exploit the randomness of crime by modeling both probabilistic encounters and stochastic vigilance efficacy. Having micro founded supply and demand curves is fundamental if the goal is to unveil forces that shape policy interventions.

There is a small literature that examines how equilibrium vigilance levels are affected by whether vigilance is observed by criminals and how these levels compare to benchmarks. In a model with homogeneous victims and a fixed pool of homogeneous criminals, Shavell

[10] Conceptually, our demand curves are different: In Ehrlich, "the demand schedule for offenses represents the average potential payoff per offense at alternative frequencies of offenses"; see Ehrlich (2010) for a survey. His verbal story also suggests that victims' vigilance directly reduce criminal material payoffs. By contrast, here the key effect of vigilance, like burglar systems, is to raise the failure chance of an attempted crime and so reduce the *expected* criminal gain. See also Clotfelter (1977) and Cook (1986) for early work on the topic.

[11] Hotte et al. (2003) develop a market for offenses. While their supply follows from criminal entry, their demand reflects the criminal optimization. So their market equilibrium takes vigilance as given.

(1991) finds that, relative to the levels that minimize the victims' losses, vigilance is over provided when it is observable by criminals and under provided when unobservable. More recently, building on Lacroix and Narceau (1995), Baumann and Friehe (2013) find that vigilance may be under provided even if it is observable when there is asymmetric information between victims and criminals. Relatedly, this literature also focuses on how the observability of vigilance may divert crime to less protected victims (Koo and Png, 1994; Hotte and Van Ypersele, 2008).[12] My analysis differs from these papers along three dimensions. First, their focus is on the demand: they consider a *fixed* number of criminals, and so a constant attempted crime rate. In my analysis, the attempted crime rate is an endogenous variable that responds to variations in vigilance and is key in explaining crime rates. Second, this literature considers an exogenous theft chance function, whereas in my paper it is shaped by free criminal entry. Finally, these papers do not pursue welfare or policy analysis.

Following the normative approach to crime (see Polinsky and Shavell (2000) for a survey), there is a small literature that focuses on optimal law enforcement and rational victim behavior (Ben-Shahar and Harel, 1995; Hylton, 1996; Garoupa, 2003; Helsley and Strange, 2005). There, the government is another player who may choose the level of policing and penalties in order to optimize a welfare function. Instead, my analysis considers an exogenous government and explores how the jointly determined levels of crime and vigilance respond to different intensities of policing and penalties. This allows me to provide a full characterization of how and when vigilance will crowd out the government's policy objective.

Finally, somewhat related, Knowles et al. (2001) focus on the interaction between police and heterogeneous potential criminals and seek to identify whether police stops exhibit racial or statistical discrimination. Their criminals decide whether to carry contraband, and the probability of a police search influences their choices. The models differ in most respects but share the feature that the criminal extensive margin and police search decision (like vigilance) are jointly determined. Persico (2002) gives an equilibrium analysis of police fairness and crime. Burdett et al. (2003) develop an equilibrium theory that jointly determines wages, unemployment, and crime, assuming exogenous private vigilance efforts. Quercioli and Smith (2015) develop a counterfeiting model, focusing on a money passing game, assuming homogeneous "bad" and "good" guys. They study how the counterfeiting rate depends on the notes' denomination, without pursuing welfare analysis.

[12] For empirical studies, see Ayres and Levitt (1998); Vollaard and van Ours (2011); Cook and MacDonald (2011); Gonzalez-Navarro (2013); Zimmerman (2014); van Ours and Vollaard (2015); Priks (2015).

# 3 The Model

**Players and Matching.** Consider an economy in which the ownership of a single good is dispersed among a large population of risk-neutral agents. I call those endowed with the good *potential victims*, and those not so endowed *potential criminals*. Thus, the focus is naturally on property crime, such as theft, burglary, cyber-crime, and not on so-called "victimless" crimes such as speeding, contraband, and drug offenses. I consider a static environment, with the usual understanding that it can be seen as the steady-state of a dynamic model.

Following Becker (1968), potential criminals simply have an extensive margin — they choose whether to enter and attempt a crime. If so, they shall be labeled as *criminals*. Potential criminals are heterogeneous and differ by their outside option, or opportunity *cost* $c \geq 0$. This cost subsumes, e.g., the expected foregone income from the legal sector. The *cost mass distribution* $F$ has a density $f(c) \equiv F'(c) > 0$ on $[0, \infty)$. Since there is entry of new criminals, $F$ need not be a probability distribution with unit mass. Criminals have *higher costs* with $F_H$ than $F_L$ if $F_H(c) < F_L(c)$ all $c > 0$.[13] As in Becker (1968), criminals are apprehended with *capture probability* $p \in [0, 1]$ and, if so, face a *penalty* $x \geq 0$, where the latter is the monetary equivalent of the punishment. Altogether, criminals treat $c + \ell$ as the fixed cost of attempting a crime, where $\ell \equiv px \geq 0$ is the expected legal *punishment*.

Stealing a good may entail a deadweight loss, because owners usually value their property more than others do. If criminals must further launder their spoils, such losses are amplified, because laundering is costly. I ignore complementarities among crimes, and assume criminals focus on the theft of a specific good. A good is described by a pair $(m, M)$, where $M > 0$ is the potential victim's *property loss*, and $m > 0$ the *criminal gain*. For instance, auto- and identity- theft have a large *markdown* $(1 - m/M \gg 0)$, whereas money theft has a smaller one $(m/M \approx 1)$.[14] Finally, the criminal gain is greater than the expected punishment, $m \geq \ell$.

Randomness plays two roles in this model. First, criminals randomly meet potential victims as part of a decentralized pairwise random matching model. All potential victims are criminal targets facing the same endogenous *attempted crime rate* $\alpha \geq 0$, namely, the expected number of attempted crimes per capita. Second, if a potential victim faces an attempted crime, she can randomly deter it by, e.g., placing bars on windows, installing home alarms, or by avoidance and mental alertness.[15] All such actions are called *vigilance*, and subsumed by a scalar $v \geq 0$. Because the model is stationary, vigilance is a flow cost, or the annuity value of initial one-shot costs, and so it is a direct subtraction from utility.

[13] This extends first order stochastic dominance, applying it to distributions with different masses.

[14] Despite the property crime focus, one could think that a violent crime has a high markdown $(m/M \approx 0)$.

[15] Each adult spends one minute and 50 seconds locking and unlocking doors each day (Anderson, 2012).

Any attempted crime against a potential victim can either fail or succeed. A potential victim is able to impact the failure rate of an attempted crime by being vigilant. Potential victims vary by their private cost-technology indexed by $\xi \in [0,1]$. *Types* $\xi$ have a probability distribution $G$ and density $g(\xi) \equiv G'(\xi) > 0$ on $[0,1]$. A potential victim with technology $\xi$ can stop an attempted crime with probability $\phi \in [0,1]$ if vigilance $v = V(\phi|\xi) \geq 0$.[16] That is, $V(\phi|\xi)$ is the *vigilance cost* of securing an *individual failure rate* $\phi$ for type $\xi$. I assume a monotone and strictly convex cost $V_\phi(\phi|\xi), V_{\phi\phi}(\phi|\xi) > 0$ for $\phi > 0$, with $V(0|\xi) = V_\phi(0|\xi) = 0$ and $V_\phi(1|\xi) < \infty$. While marginal vigilance costs rise, I now posit that its elasticity $(\phi V_{\phi\phi}/V_\phi)$ falls in $\phi$. This ensures that marginal costs are log-concave, $[\log(V_\phi)]_{\phi\phi} \leq 0$, and so never jump up; also, it forces a well-defined limit $\lim_{\phi\downarrow 0} V_{\phi\phi}/V_\phi > 1$.

I further assume that potential victims with higher types $\xi$ face greater costs and a faster increase in marginal costs: $V_\xi(\phi|\xi) > 0$ and $V_{\phi\phi\xi}(\phi|\xi) > 0$ for $\phi > 0$. The latter condition implies that higher types $\xi$ have greater marginal costs $V_{\phi\xi}(\phi|\xi) > 0$ for $\phi > 0$.[17] Finally, I assume that the marginal cost elasticity $(\phi V_{\phi\phi}/V_\phi)$ increases in $\xi$. All told, vigilance cost $V(\phi|\xi)$ rises in $\xi$, and its marginal $V_\phi$ is supermodular in $(\phi, \xi)$ and log-supermodular in $(\phi, \xi)$.[18] These assumptions not only simplify the analysis, but will discipline the comparative statics. Any convex geometric function $V(\phi|\xi) = (1+\xi)\phi^\gamma, \gamma \geq 1$, meets all these properties.

Criminals cannot observe potential victims' types $\xi$ and vigilance $v$,[19] and so each criminal conjectures that a type $\xi$ potential victim elects vigilance to ensure an individual failure rate $\phi(\xi)$. Because $\xi$ varies across potential victims, criminals expect that their attempted crimes will fail with probability $\varphi \equiv \int_0^1 \phi(\xi) dG(\xi) \in [0,1]$, the average or expected *failure rate*. Naturally, the average criminal *success rate* is $1 - \varphi$ and the *success odds* is $(1-\varphi)/\varphi$. The actual *crime rate* $\kappa \equiv \alpha(1-\varphi) \in [0,1]$ denotes the mass of successful attempted crimes.

**Optimizations and Equilibrium.** Because losses realize when a crime succeeds, a potential victim with individual failure $\phi$ has expected property losses $\alpha(1-\phi)M$. Potential victims seek to minimize their expected total property losses of crime plus vigilance $v$. But since $v = V(\phi|\xi)$ is increasing for all $\xi$, one can simply think of the victim as choosing the

[16] $\xi$ may index the alertness or attention level of an individual, or the opportunity cost of being vigilant.

[17] Since $V_\phi(0|\xi) = 0$ for all $\xi$, it follows that $V_{\phi\xi}(\phi|\xi) = \int_0^\phi V_{\phi\phi\xi}(t|\xi)dt > 0$ for $\phi > 0$.

[18] A real valued function $x \mapsto h$ on a lattice $X \subseteq \mathbb{R}^n$ is *supermodular (submodular)* if $h(\max\{x, x'\}) + h(\min\{x, x'\}) \geq (\leq) h(x) + h(x')$. When $h$ is twice differentiable, then $h$ is supermodular (submodular) iff $h_{x_i x_j}(x) \geq (\leq) 0$ for all $i \neq j$, by Topkis (1998). These definitions are strict if the inequalities are strict. A positive function $h > 0$ is *log-supermodular (log-submodular)* if $\log(h)$ is supermodular (submodular).

[19] In reality, vigilance also subsumes observable actions. For instance, installing a fence or bars on windows are observable to criminals. Yet, these observable actions are likely to be similar across potential victims that have similar characteristics (e.g., in a residential neighborhood). Thus, given a "submarket," the encounters between criminals and victims can be seen as random, and the variation of actual crimes across victims can be seen as a result of unobservable and idiosyncratic vigilance actions (e.g., installing deadbolt locks on house doors, or following "best avoidance practices"). Section §9 explores the observable vigilance case.

failure rate $\phi \in [0,1]$ and express *losses from crime* for a type $\xi$ potential victim as:

$$\mathcal{L}(\phi, \alpha, \xi) \equiv \alpha(1-\phi)M + V(\phi|\xi). \tag{1}$$

Reflecting an underlying competitive assumption that no potential victim impacts the attempted crime rate $\alpha$, potential victims choose $\phi$ to minimize (1), taking $\alpha$ *as given*. In this respect, vigilance (or private defense) is different from law enforcement (or public defense), which surely has aggregate effects; see, e.g., Polinsky and Shavell (2000). To isolate the impact of vigilance on crime, policing behavior is unmodelled and simply embedded into the legal punishment $\ell$ with the understanding that policing is positively associated with $\ell$.

A potential criminal with total cost $c + \ell$ cannot observe the failure rates produced by potential victims, and so he takes the average failure rate $\varphi$ as given, and attempts a crime with gain $m$ if and only if his expected *net criminal profits* $\Pi(\varphi|c) \geq 0$, where:

$$\Pi(\varphi|c) \equiv (1-\varphi)m - \ell - c. \tag{2}$$

The *marginal potential criminal* $\bar{c}(\varphi) \equiv \max\{(1-\varphi)m - \ell, 0\}$ earns non-positive criminal profits. Thus, naturally, potential criminals attempt a crime provided $c \leq \bar{c}$.

Notice the embedded causation. The potential victims' losses reflect the aggregated criminal behavior, via the attempted crime rate, and the criminal profits are only impacted by the average failure rate. Also, the only spillovers of potential victims onto each other are indirectly channeled by the impact on criminal behavior, and conversely, criminals only affect other criminals indirectly via the potential victims' vigilance. Likewise, law enforcement indirectly impacts potential victims to the extent that it affects criminal participation.

An *equilibrium* is a pair $(\phi^*(\cdot), \bar{c}^*)$ such that $(i)$ each potential victim $\xi \in [0,1]$ expends vigilance $v^*(\xi) = V(\phi^*(\xi)|\xi)$, where the individual failure rate $\phi^*(\xi)$ minimizes expected losses $\mathcal{L}(\phi, \alpha^*, \xi)$ in (1), given the attempted crime rate $\alpha^* = F(\bar{c}^*)$, and $(ii)$ a potential criminal attempts a crime if and only if his cost is $c \leq \bar{c}^*$, where profits $\Pi(\varphi^*|\bar{c}^*) = 0$ in (2), for an expected failure rate $\varphi^* = \int_0^1 \phi^*(\xi)dG(\xi)$.

# 4 A Supply and Demand Equilibrium Analysis

**The Demand Curve.** Consider the potential victims' optimization problem (1). Given an attempted crime rate $\alpha \geq 0$, the best response for a potential victim $\xi$ is an *optimal failure rate* $\hat{\phi}(\alpha|\xi) \in [0,1]$ that minimizes losses $\mathcal{L}(\phi, \alpha|\xi)$. Clearly, if there is no attempted crime, then individuals elect no vigilance, or $\hat{\phi}(0|\xi) = 0$. If the optimal failure rate is interior, i.e.,

$\hat{\phi}(\alpha|\xi) \in (0, 1)$, then it obeys the first-order condition for (1):

$$\mathcal{L}_\phi(\hat{\phi}, \alpha|\xi) = -\alpha M + V_\phi(\hat{\phi}|\xi) = 0. \tag{3}$$

The second order condition holds by strict convexity of $V$. Notice that the marginal losses from crime are negative when the attempted crime rate $\alpha > V_\phi(1|\xi)/M$, and so the optimal failure rate is interior when $0 < \alpha < V_\phi(1|\xi)/M$, and obeys (3). Also, the optimal failure rate $\hat{\phi}(\alpha|\xi)$ rises from zero as $\alpha$ rises. Now, potential victims with higher types $\xi$ have higher vigilance cost $V(\cdot|\xi)$ and marginal cost $V_\phi(\cdot|\xi)$, and thus choose a lower failure rate, i.e., $\hat{\phi}_\xi(\alpha|\xi) < 0$, and face greater losses from crime (by the Envelope theorem applied to (1)). Finally, in terms of vigilance, the individual level of vigilance expenditures $V(\hat{\phi}(\alpha|\xi)|\xi)$ is hump-shaped in $\xi$, provided the attempted crime rate $\alpha$ is neither too high nor too low; see Claim A.1. The total level of vigilance expenses is defined as $\bar{v} \equiv \int_0^1 V(\hat{\phi}(\alpha|\xi)|\xi)dG(\xi)$.

The optimal failure rate $\hat{\phi}(\alpha|\xi)$ can naturally be interpreted as an induced *individual demand* for vigilance. At the aggregate, the market *demand* for vigilance, given the attempted crime rate $\alpha$, is captured by:

$$\mathcal{D}(\alpha) \equiv \int_0^1 \hat{\phi}(\alpha|\xi)dG(\xi). \tag{4}$$

The demand vanishes with no attempted crime, $\mathcal{D}(0) = 0$, and is continuous and upward sloping (see Figure 1), because crime is an economic bad. It hits perfect security $\mathcal{D}(\alpha) = 1$ once every single potential victim chooses $\hat{\phi}(\alpha|\xi) = 1$, namely, when $\alpha \geq \bar{\alpha} \equiv V_\phi(1|1)/M$. I, therefore, call the threshold $\bar{\alpha}$ the *attempted crime ceiling*. As potential victims take more precautions when they risk losing a more valuable good, the demand $\mathcal{D}$ increases in the property loss $M$, rising towards perfect security as $M \uparrow \infty$.

When is the market demand $\mathcal{D}$ more or less elastic? Consider two type-distributions for potential victims, namely, $G_L$ and $G_H$ with respective densities $g_L$ and $g_H$. Appendix A.2 shows that, *demand $\mathcal{D}$ is less elastic with $G_H$ than $G_L$ if $g_H(\xi)/g_L(\xi)$ is increasing*. Recall that when the likelihood ratio $g_H(\xi)/g_L(\xi)$ is monotone, then $G_H$ dominates $G_L$ in the sense of first-order and second-order stochastic dominance (see, e.g., Athey, 2002). Thus, the market demand falls when potential victims with higher vigilance costs are more abundant, because the individual demand $\hat{\phi}(\alpha|\xi)$ is decreasing in $\xi$. That is, the demand is lower with $G_H$ than $G_L$. Likewise, by standard results, the demand falls when vigilance costs are less dispersed in the population, provided the individual demand $\hat{\phi}(\alpha|\xi)$ is convex in type $\xi$ (this is, e.g., the case when vigilance costs $V(\phi|\xi)$ are geometric in $\phi$ for all $\xi$).

**The Supply Curve.** Given an expected failure rate $\varphi$, the marginal potential criminal is $\bar{c} = (1 - \varphi)m - \ell$ if and only if $\varphi \leq \bar{\varphi} \equiv 1 - \ell/m$, the *failure rate ceiling*. No crimes are

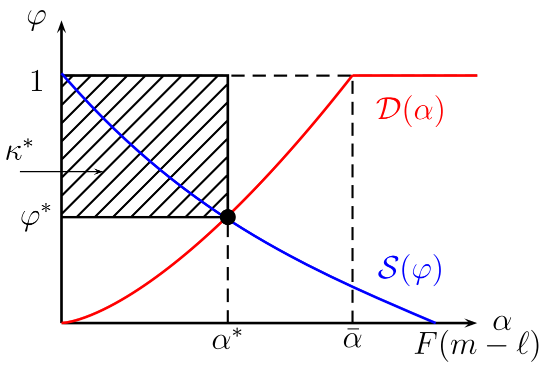


Figure 1: **Equilibrium Existence and Uniqueness.** For additional clarity, the figure assumes that the failure rate ceiling $\bar{\varphi} \approx 1$. The demand slopes upward, while the supply curve slopes down. The equilibrium crime rate $\kappa^*$ is the dashed area $(1-\varphi^*)\alpha^*$ of successful attempted crimes.

attempted when the expected failure rate is too high, $\varphi > \bar{\varphi}$. Hence, the *supply* of crime is

$$\mathcal{S}(\varphi) \equiv F((1-\varphi)m - \ell), \tag{5}$$

for $\varphi \leq \bar{\varphi}$, and $\mathcal{S}(\varphi) \equiv 0$ for $\varphi > \bar{\varphi}$. The supply curve simply reflects the (almost everywhere) differentiable and increasing map from the marginal criminal $\bar{c}$ to the attempted crime rate $\alpha = F(\bar{c})$. As noted after (3), if the failure rate $\varphi$ is less than $\bar{\varphi}$, the attempted crime rate is at most $V_\phi(1|1)/M$, and thus is boundedly finite, as seen in Figure 1.

The supply curve is downward sloping, falling in $\varphi$ (see Figure 1) and vanishing as $\varphi \uparrow \bar{\varphi}$. It rises in the criminal gain $m$, but falls in punishment $\ell$. Also, *the supply elasticity falls when the cost distribution $F$ improves in the monotone likelihood ratio sense* (Appendix A.3). Therefore, the supply is lower when more potential criminals have higher opportunity costs.

Notice that, for potential victims, the failure rate $\varphi$ plays the role of a demand quantity, while the attempted crime rate $\alpha$ serves as a price. But conversely, for potential criminals, the failure rate $\varphi$ acts as a supply price, while the attempted crime rate $\alpha$ as a supply quantity. Thus, while a more elastic supply curve in Figure 1 is more horizontal per usual, a more elastic demand curve in Figure 1 is more vertical.

The falling supply and rising demand curves imply that *there exists a unique equilibrium* $(\phi^*(\cdot), \bar{c}^*)$, balancing market forces (see Appendix A.4). Figure 1 depicts this unique equilibrium. In such equilibrium, the crime rate $\kappa^*$ is the rectangular shaded area $(1-\varphi^*)\alpha^*$.

# 5 The Criminal Laffer Curve

Understanding the deterrence effects of different policies aimed at alleviating crime is a challenge that lies at the heart of the economics of crime literature (Becker, 1968). Policies not only differ in their effects on crime, but also in their implementation costs. For instance, it

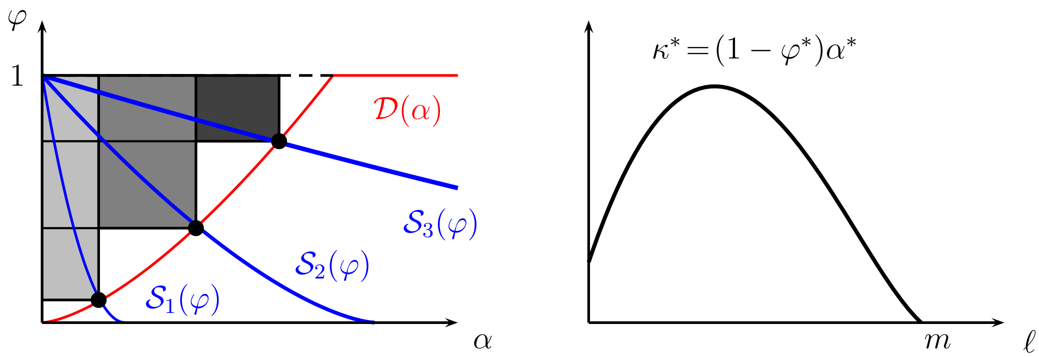


Figure 2: **More Punishment and the Criminal Laffer Curve.** Left: When expected punishment $\ell$ rise, supply shifts left, and the failure rate $\varphi^*$ and attempted crime rate $\alpha^*$ fall. Right: The crime rate $\kappa^*$ is hump-shaped and so highest for mid punishment levels.

is arguably the case that deterring individuals from criminal participation is more economical than incapacitating them in prisons. Chalfin and McCrary (2017) argue that, "offenders who are deterred from committing crime in the first place do not have to be identified, captured, prosecuted, sentenced, or incarcerated..." (pp. 5). While there are several ways of making crime less attractive, perhaps the most direct one is increasing either legal penalties ($x$) or policing ($p$), and thereby the expected legal punishment of crime ($\ell$).

So motivated, in this section, I study the equilibrium effects of raising an offense's expected punishment, $\ell = px$. I explore how potential victims' vigilance impacts the determination of crime rates and the efficacy of public policy, as the expected punishment rises.

It is well-known in the economics of crime literature that, at least theoretically, an increase in the expected punishment must unambiguously reduce crime (Becker, 1968). This insight owes to the decision theory nature of most crime models, and also to the uniform treatment of attempted and actual crimes. Indeed, when attempted and actual crimes are carefully disentangled,[20] an increase in punishment unambiguously lowers only the attempted crime rate $\alpha$. Meanwhile, potential victims respond to variations in $\alpha$, reducing their vigilance as the attempted crime rate $\alpha$ falls. As a result, *the crime rate $\kappa$ need not be monotone.* Thus, legal policies that appear effective ex-ante may actually be ineffective ex-post.

Proposition 1 characterizes how increases in expected punishment impact individual behavior and market outcomes. Before stating the result, say that the demand $\mathcal{D}$ is *sufficiently elastic* if its elasticity is greater than the criminal success odds, namely, $\mathcal{E}_\alpha(\mathcal{D}) \geq (1-\mathcal{D})/\mathcal{D}$.

**Proposition 1.** *If expected legal punishment $\ell$ rises, then vigilance expenditures $v^*$, the failure rate $\varphi^*$, and the attempted crime rate $\alpha^*$ all fall. The crime rate $\kappa^*$ is a hump-shaped function of legal punishment $\ell \in [0, m]$ if the demand $\mathcal{D}$ is sufficiently elastic for low enough punishments; otherwise, the crime rate is monotone decreasing in $\ell$.*

[20]See, e.g., Farrell (2016) and Tseloni et al. (2017) for empirical research that accounts for this difference.

The logic is the following. Assume that punishment $\ell$ rises. In Figure 2, as $\ell$ rises through $\ell_3 < \ell_2 < \ell_1$, the supply curve (5) shifts left from $\mathcal{S}_3$ to $\mathcal{S}_2$ to $\mathcal{S}_1$, while the demand curve $\mathcal{D}$ holds constant. The attempted crime rate $\alpha^*$ and failure rate $\varphi^*$ unambiguously fall: fewer crimes are attempted, but each succeeds more often, as potential victims grow less vigilant.

Next, to understand why the crime rate is hump-shaped, it is useful to first step back and explore how the crime rate varies along the demand curve. Since a change in punishment shifts the supply curve $\mathcal{S}$ *along* the demand curve $\mathcal{D}$, the elasticity of the demand determines whether the crime rate ultimately rises or falls. Indeed, *the crime rate falls along the demand iff the demand is sufficiently elastic.* This means that a 1% increase in the attempted crime lowers the average criminal success rate more than 1%, resulting in a drop in crime.[21] Lemma A.6.1 shows that *along the demand curve, the crime rate* $\alpha(1-\mathcal{D}(\alpha))$ *is hump-shaped*, as the demand is sufficiently elastic when the attempted crime rate is high enough.

Finally, since punishment $\ell$ shifts the supply along the demand, the crime rate can be written as $\kappa^*(\ell) \equiv \alpha^*(\ell)(1 - \mathcal{D}(\alpha^*(\ell)))$. Observe that when penalties $\ell = m$, no criminal obtains revenues; thus, there are no attempted crimes $\alpha^*(m) = 0$, and so $\kappa^*(m) = 0$. But when legal penalties $\ell = 0$, the attempted crime rate is clearly positive and below the attempted crime rate ceiling, i.e., $\alpha^*(0) \in (0, \bar{\alpha})$, whereas the failure rate is imperfect $\varphi^*(0) < 1$. Thus, since $\alpha^*(\ell) \in [0, \alpha^*(0)]$ is monotone decreasing, and the map $\alpha \mapsto \alpha(1 - \mathcal{D}(\alpha))$ is quasi-concave (by Lemma A.6.1), it follows that $\kappa^*(\ell)$ is quasi-concave in $\ell \in [0, m]$. Finally, if the demand is sufficiently elastic at $\alpha = \alpha^*(0)$, then the map $\alpha \mapsto \alpha(1-\mathcal{D}(\alpha))$ is decreasing around $\alpha = \alpha^*(0)$, and thus the equilibrium crime rate $\kappa^*(\ell)$ is increasing around $\alpha = \alpha^*(0)$ because $\kappa^*(\cdot)$ is the composition of two decreasing functions.

For an intuition, consider two extreme cases. In the "police state," massive legal penalties hold the attempted crime rate $\alpha^*$ very low, and so decreases in legal penalties lead to proportionately large increases in $\alpha^*$, and thus the crime $\kappa^* = (1 - \varphi^*)\alpha^*$ rises. In other words, if potential victims are over-protected by police, their behavior is not responsive enough to variations in penalties. At the opposite extreme, in the "Wild West," policing is very slight, and crime is held at bay by individual vigilance — the quick draw. In this case, the criminal success rate $1 - \varphi^*$ is extremely low, and so decreases in legal penalties lead to proportionately large increases $1 - \varphi^*$, and thus the crime rate $\kappa^* = (1 - \varphi^*)\alpha^*$ falls. In less extreme cases, the impact of increasing criminal costs on the crime rate $\kappa^* = (1 - \varphi^*)\alpha^*$ is unclear, a priori. Proposition 1 clarifies this showing that the equilibrium crime rate is a hump-shaped function of the expected punishment, resembling a *criminal Laffer curve.*[22]

[21] To see this, notice that along the demand curve, the crime rate obeys $\alpha(1 - \mathcal{D}(\alpha))$. Thus, its derivative is negative, i.e., $1 - \mathcal{D}(\alpha) - \mathcal{D}'(\alpha)\alpha < 0$, iff $\mathcal{E}_\alpha(\mathcal{D}) > (1 - \mathcal{D})/\mathcal{D}$, or $\mathcal{E}_\alpha(1 - \mathcal{D}) < 1$.

[22] See Ayres (2016) for a related and intuitive discussion of the criminal Laffer curve.

The criminal Laffer curve systematically characterizes the trade-offs between crime rates and expected punishment, and rationalizes the empirical finding that better policing displaces private vigilance (e.g., Vollaard and Koning, 2009). It shows that the efficacy of public policy depends critically on the current level of policing and penalties for a criminal offense. It also highlights the interaction between two opposing market forces that govern the competitive nature of crime. While raising expected penalties has a positive deterrent effect *ex-ante*, since fewer crimes are attempted, increased penalties also has a negative deterrent effect *ex-post* — as more attempted crimes succeed. The relative magnitude of these positive and negative effects on any given crime is, thus, an empirical question. Notice that when the expected punishment is below a critical value, the negative effect is dominant but becomes weaker as the punishment rises. The strength of these effects depends on the demand elasticity. The next result provides a simple sufficient condition that determines *when* the negative effect dominates, or the criminal Laffer curve is upward sloping in punishment $\ell$.

**Proposition 2.** *The crime rate rises when the expected legal punishment rises, if the equilibrium property losses are less than total vigilance expenses.*

For an intuition, suppose that a decrease in punishment causes a 1% increase in the attempted crime $\alpha$. Then, potential victims raise their guard, which leads to a proportional increment in the average failure rate $\mathcal{D}$ larger than the level of vigilance expenses relative to the *average property value preserved by potential victims*, $\alpha\mathcal{D}M$. Thus, when total vigilance expenses exceed *average property losses* $\alpha(1-\mathcal{D})M$, the proportional demand increment is greater than $\alpha(1-\mathcal{D})M/(\alpha\mathcal{D}M)=(1-\mathcal{D})/\mathcal{D}$. But then, the demand curve $\mathcal{D}$ is sufficiently elastic, and so this reduction in punishment must lead to less crime, by Proposition 1.

Proposition 2 provides a condition, based on observable variables, that sheds light on when increasing expected punishment will be more than crowded out by a reduction in potential victims' vigilance. When aggregate vigilance expenses overwhelm property losses, raising penalties leads to higher crime rates.[23] But, what happens at the individual level? Notice that the criminal Laffer curve aggregates the individual responses of potential victims to variations in expected punishment. While the attempted crime rate is common to all

[23]For example, according to Ayres and Levitt (1998), the property loss of vehicles with Lojack was roughly \$1000. They estimate a theft rate in Lojack cities of 0.025, and a \$97 annuity equivalent of the \$600 Lojack installation fee. This exceeds the expected property loss of \$25, and so $\kappa M < \bar{v}$. So Proposition 2 conjectures that, if penalties for auto theft had risen, the theft rate, if anything, would have increased. For another illustration, consider the overall motor vehicle theft in the US. From the FBI, in 2017 the average property loss from motor vehicle theft was \$7,708; the theft rate was 237.4 per 100,000 inhabitants, or at least 470 per 100,000 car owners, i.e., a theft rate at least $\kappa = 0.004$. So the expected property losses are at least $\kappa M = \$30$ (conservatively assuming stolen cars are a write-off). Therefore, by Proposition 2, an increase in penalties would lead to more crime if vigilance expenses per car owner $\bar{v} \geq \kappa M = \$30$. Because a car is on average replaced every 10 years, this amounts to a one-shot investment on car security greater than \$185.

potential victims, the ultimate property loss faced by them depends on their own vigilant behavior. This means that potential victims effectively face different victimization rates. Thus, if punishment were to increase, which types of potential victims will end up facing more crime? The next result shows that, as punishment rises, the crime rate rises for potential victims with low vigilance costs and falls for those with high costs; furthermore, the proportion of potential victims who face less crime rises. To wit, the negative ex-post effect of punishment dominates for potential victims with superior self-protection technologies.

**Proposition 3.** *For any punishment $\ell \in [0, m]$, there exists a potential victim critical type $\bar{\xi} \in [0, 1]$ such that, a marginal increase in punishment $\ell$ leads to higher crime rates for all potential victims $\xi \leq \bar{\xi}$ and to lower crime rates for all potential victims $\xi > \bar{\xi}$. Moreover, the critical type $\bar{\xi}$ is decreasing in punishment $\ell$.*

**Remark 1 (Criminal Opportunity Costs and Revenues).** As is well-known, crime can be discouraged not only with sticks (e.g., penalties and policing), but also with carrots (e.g., better outside options). If the value of outside options increase, e.g., due to greater employment opportunities, the opportunity cost of attempting crime would decrease, resembling a shift of the cost distribution $F$.[24] Likewise, crime can be discouraged indirectly, e.g., by preventing the opening and functioning of black markets for stolen goods. Since criminals would face fewer opportunities to resell their stolen goods, the value of the criminal gain $m$ would decrease. In any case, an improvement in the cost distribution $F$ or a reduction in criminal gain $m$ affect *directly* the supply of crime $\mathcal{S}$, but not the demand $\mathcal{D}$. Thus, such policies would shift the supply along a constant demand, and so the same qualitative results obtained in this section would apply to these cases. That is, *policies aimed to affecting criminal opportunity costs or revenues are also susceptible to exhibit Lafferian effects.*[25]

# 6 Comparative Statics

## 6.1 The Role of Vigilance Costs

When potential victims face high vigilance costs, policymakers may wish to subsidize them, or may seek to increase, e.g., policing to counterbalance potential increases of crime. While more

[24] Indeed, Becker (1968) argued that crime should globally fall if opportunity costs increase, since "a rise in the income available in legal activities would reduce the incentives to enter illegal activities and thus would reduce the level of offenses" (p. 177).

[25] Continuously index the cost distribution with a scalar, say, $\omega \in [0, 1]$, and assume that opportunity costs are higher with greater $\omega$. By placing natural restrictions on the extreme cases ($\omega = 0, 1$), and using common logic, one can show that the crime rate is also a hump-shaped function of $\omega$. Similar reasoning establishes a hump-shape as the criminal gain $m$ rises, but the crime rate only tends to zero as $m \uparrow \infty$.

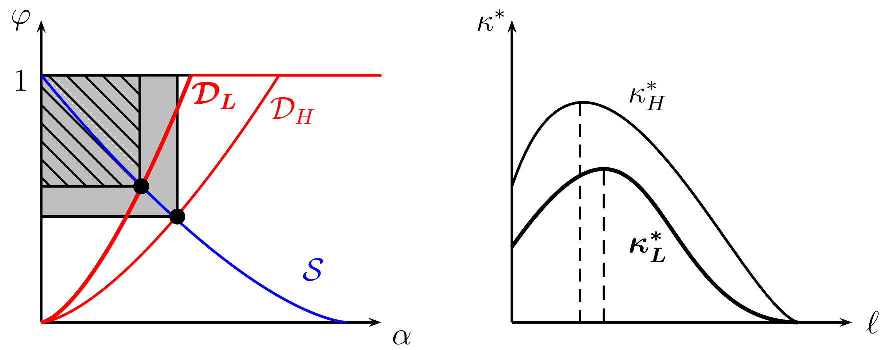


Figure 3: **Vigilance Costs and the Criminal Laffer Curve.** LEFT: When more victims face higher vigilance costs, i.e., $G$ rises from $G_L$ to $G_H$, demand shifts right from $\mathcal{D}_L$ to $\mathcal{D}_H$ along $\mathcal{S}$. The crime rate and attempted crime rate rise, while the failure rate falls. RIGHT: The criminal Laffer curve shifts up and left, and is steeper and flatter before and after it peaks, respectively.

policing can, effectively, control the attempted crime rate, it is unclear whether the number of actual crimes would decrease due to the criminal Laffer curve. Self-protection technologies impact not only the vigilance levels but also their speed of adjustment to changes in policy. These aspects should be given consideration when policy towards crime is considered.

In general, when more potential victims face higher vigilance costs, the type distribution $G$ naturally shifts. Consider two distributions for potential victims' types, $G_L$ to $G_H$. Say that $G$ *rises* if $G$ shifts from $G_L$ to $G_H$ in the monotone likelihood ratio sense. As seen in the left panel of Figure 3, *when $G$ rises the failure rate $\varphi^*$ falls, whereas the attempted crime rate $\alpha^*$ and crime rate $\kappa^*$ rise*. The reason is that demand $\mathcal{D}$ shifts down as the proportion of potential victims with high vigilance cost rises. Because supply $\mathcal{S}$ is unaffected by $G$, the success of a criminal attempt rises, and so does the crime rate as more crimes are attempted and each succeeds with a higher probability.

Thus, when more individuals have access to technologies that lower vigilance costs, the levels of crime fall. This prediction is consistent with the empirical finding that after the introduction of Lojack the auto theft rate fell substantially in the U.S. (Ayres and Levitt, 1998). It is also in line with the observed decline in auto theft rates since 2003 in the U.S., as newer cars have theft-immobilizer devices or part markings (Fujita and Maxfield, 2012).[26]

Next, I examine how vigilance costs affect the criminal Laffer curve. As seen in Figure 3, as $G$ rises the criminal Laffer curve shifts up and peaks earlier, by Lemma A.5.1. Now, to understand its responsiveness, I consider percentage changes in punishment that lead to an *equivalent* percentage change in the attempted crime rate for $G_L$ and $G_H$; see Appendix A.9 for details. Thus, under *equivalent variations in punishment* all changes in the crime rate owe to the optimizing behavior of potential victims.

[26]Likewise, Farrell et al. (2011) find that the dramatic drop in auto theft over the last 20 years in the US, Britain and Australia is negatively correlated with an increase in anti-theft devices for cars.

**Proposition 4.** *When more potential victims face higher vigilance costs: (i) the criminal Laffer curve shifts up and left; and (ii) under equivalent variations in punishment, this curve becomes steeper as it rises and flatter as it falls, provided marginal vigilance costs are concave.*

By Proposition 4, when vigilance costs are higher, the domain wherein an increase in penalties leads to more crime shrinks; yet, it is precisely in that domain where the Laffer curve becomes more responsive. Thus, *increasing punishment can drastically raise the levels of crime*. Moreover, in the domain wherein punishment is effective in reducing crime, the Laffer curve becomes less responsive, attenuating the efficacy of penalties. In light of these results, complementary policies aimed to reducing vigilance expenses could be enticing, as they would not only decrease crime, but also they would flatten and straighten up the criminal Laffer curve in the relevant regions, thereby raising the efficacy of punishment.

## 6.2 The Stakes of Crime

In this section, I explore the equilibrium effects of increasing the *stakes of crime*, or the property values $(m, M)$. As previously discussed, one could imagine that for many offenses, the value of the material loss $M$ to the victim is positively correlated with the value of the material gain $m$ to the criminal. For example, stealing a luxury vehicle would likely induce a greater loss to the owner and a greater gain to the criminal. The same logic should also hold for other offenses such as money theft and burglary. From a policy perspective, one would like to explore the effects of policies that, e.g., improve the *recovery rate of stolen goods* in the sense of lowering both the expected victims' property losses $M$ and the criminals' gain $m$.[27]

So motivated, parametrize material gains along a constant ray so that $(m, M) = ((1-\mu)t, t)$ for $t > 0$, where $\mu \in (0, 1)$ denotes a fixed criminal markdown, i.e., $\mu \equiv 1 - m/M$. Say that *the stakes of crime rise* if $t > 0$ rises. In general, moving from low to high stakes affects both the demand and supply curves, complicating the comparative statics. The next result uncovers a criminal Laffer curve for the attempted crime rate.

**Proposition 5.** *The failure rate $\varphi^*$ rises as the stake $t$ rises. If demand $\mathcal{D}$ is sufficiently elastic for high enough stakes, then the attempted crime rate $\alpha^*$ is a hump-shaped function of the stakes $t$; the crime rate $\kappa^*$ initially rises and then, eventually, falls as $t$ rises.*

First, as the stakes of crime rise, potential victims have more to lose, whereas potential criminals have more to gain. So, as seen in the left panel of Figure 4, potential victims raise

[27] Assume that a stolen good can be recovered with *recovery chance* $r \in [0, 1]$. The original model subsumes this by considering instead the effective loss $(1-r)M$ for victims and the effective gain $(1-r)m$ for criminals.

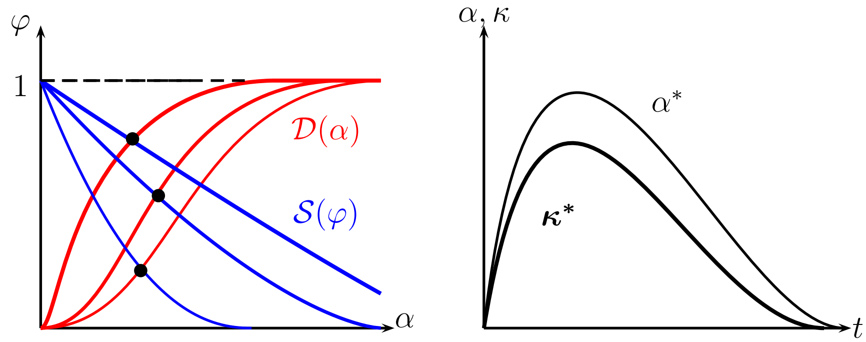


Figure 4: **Greater Criminal Gains and Property Losses.** When the stakes of crime rise demand $\mathcal{D}$ shifts left whereas supply $\mathcal{S}$ shifts right (from thin to thick lines). The failure rate $\varphi^*$ rises, while the crime rate and attempted crime rate are non-monotone (right panel).

their vigilance, shifting the demand curve $\mathcal{D}$ up. Likewise, more potential criminals attempt a crime, moving the supply curve $\mathcal{S}$ right. Thus, the failure rate $\varphi^*$ unambiguously rises. The effect on the attempted crime rate is more subtle: *The attempted crime rate $\alpha^*$ rises if and only if supply $\mathcal{S}$ shifts up more than demand $\mathcal{D}$ does.* Appendix A.10 shows that supply shifts up more than demand does iff stakes are below a critical value. The attempted crime rate is, thus, hump-shaped; see right panel of Figure 4. The crime rate $\kappa^*$ falls if stakes are high enough, for there are fewer attempts and each is more likely to fail. For low stakes, Appendix A.10 shows that $\kappa^*$ must be initially rising, as $\mathcal{S}$ vanishes as the stakes vanish.

Proposition 5 may explain, e.g., why most stolen cars and attempted car-thefts are consistently in the mid-range (in price and year) in the US, as reported by the National Insurance Crime Bureau.[28] Also, at a more aggregated level, this result suggests that, as a country's income rises (from low income) crime should initially rise and then, eventually, fall, *ceteris paribus*.[29] Finally, from a policy viewpoint, Proposition 5 argues that improving the recovery rate of stolen goods is effective only for low value property.

# 7 A Supply and Demand Welfare Analysis

Having examined the interplay between crime and vigilance, I now turn to its welfare consequences. The thrust of Becker (1968) was a treatise on the social costs of crime, focusing on the optimal public expenditures against crime. But, as argued in §1, vigilance expenses are an important source of social costs that have received much less attention among economists.

In this framework, the welfare loss of crime owes to criminals costs, vigilance costs, and

[28] See Top Vehicles Stolen at `https://www.nicb.org/news/reports-statistics`.

[29] In many countries, the property crime rate trend has been non-monotone from 1960 until now, as documented in, e.g., Figures 2–4 in Farrell et al. (2014). In particular, in the US, crime peaked in the early 1990s when the average income per capita was around \$24,000. In the case of auto theft, the drop in crime afterward has been found to be negatively correlated with an increase in car security (Farrell et al., 2011).

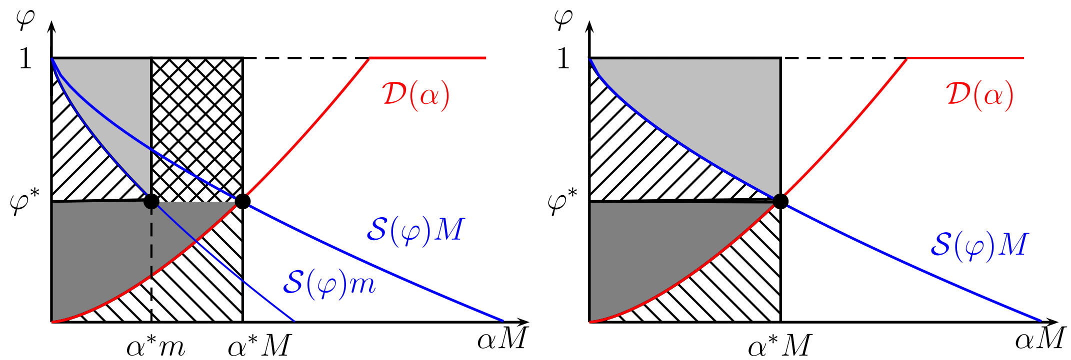


Figure 5: **The Social Costs of Crime and Rent-seeking.** LEFT: The areas are measured in dollars after I scale the units on the horizontal axis by $M$. The crosshatched region represents the transfer loss $\kappa\mu M$. The NW-dashed region is criminal profits, the light-shaded region is criminal costs, and the dark-shaded region vigilance expenses. The dashed area *below demand* $\mathcal{D}(\cdot)$ is the potential victims' surplus, i.e., vigilance benefits $\alpha^*\varphi^* M$ less total costs. RIGHT: This panel assumes $m = M$. The light shaded region is criminal costs and the dark-shaded area is the vigilance costs, which together are strictly less than the rectangle $\alpha^* M$, namely, the value of potential gains of attempted crimes. The difference is the dashed areas, owing to victim and criminal heterogeneity and vigilance cost convexity.

the *transfer loss of theft*, namely, $\kappa\mu M$, where $\mu = 1 - m/M$ is the criminal markdown. To fix ideas, let the failure rate ceiling $\bar{\varphi} \approx 1$ and consider the natural case in which the transfer loss of theft is positive, or potential victims value their property more than criminals do, i.e., $M > m$. Since the respective marginal criminal and attempted crime rate obey $\bar{c} = (1-\varphi)m - \ell$ and $\alpha = F(\bar{c})$, integrating by parts imply that total criminals costs are: $\int_0^{\bar{c}}(c+\ell)f(c)dc = \alpha(1-\varphi)m - \int_0^{\bar{c}} F(c)dc$. Next, total vigilance costs are $\int_0^1 V(\hat{\phi}(\alpha|\xi)|\xi)dG(\xi)$. Thus, given the crime rate $\kappa = \alpha(1-\varphi)$, the *social costs of crime* are:

$$\kappa\mu M + \kappa(1-\mu)M - \int_0^{\bar{c}} F(c)dc + \int_0^1 V(\hat{\phi}(\alpha|\xi)|\xi)dG(\xi). \tag{6}$$

As seen in Figure 5, the first term is the transfer loss of theft. The next two terms are gross criminal costs. For if all criminals shared a common opportunity cost, then the third term would vanish, while the second term reflects gross criminal gains, and therefore balances criminal losses, by free entry. Criminal heterogeneity is accounted for by the third term. For a criminal $c \in [0, \bar{c})$ makes profits $\bar{c} - c$, and thus total profits across all criminals amount to $\bar{c}F(\bar{c}) - \int_0^{\bar{c}} cf(c)dc = \int_0^{\bar{c}} F(c)dc$. So the social costs of crime (6) can be reinterpreted as potential victims' expected losses (summing the first two and last terms) less criminal profits.

Now let me suggestively depict the social costs in the supply and demand framework, developed in §4. In any equilibrium (variables with a star), *criminal profits* $\int_0^{\bar{c}^*} F(c)dc$ *are captured by the area above equilibrium failure rate* $\varphi^*$ *and below the value-scaled supply*

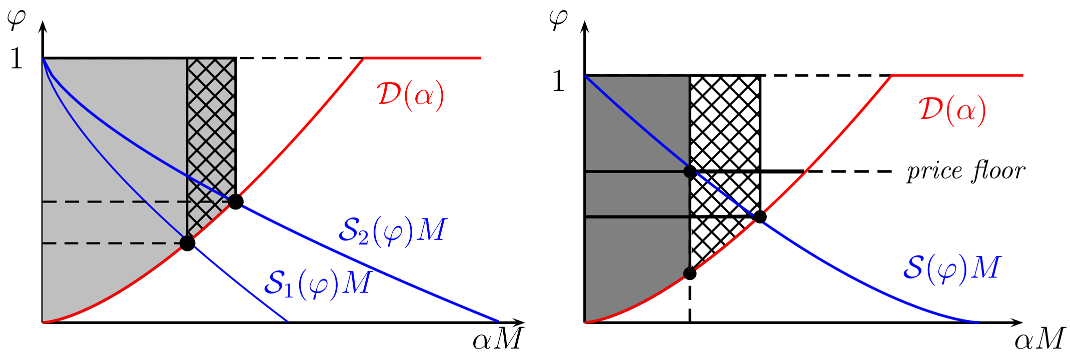


Figure 6: **Punishment and Price Floors.** To improve visual exposition, both panels assume $m = M$. LEFT: Suppose that punishment $\ell$ rises so that the supply curve shifts left from $\mathcal{S}_2$ to $\mathcal{S}_1$. While the crime rate may rise, *total* potential victims' losses unambiguously fall. In particular, the crosshatched area measures the decrease in total potential victims' losses owed to the increase in punishment. RIGHT: Mandated vigilance resembles a price floor and it lowers the crime rate and attempted crime rate when binding. The crosshatched area captures the reduction in total losses in response to less criminal activity, whereas the light-shaded area represents total potential victims' losses after the price floor is dictated.

*curve* $\mathcal{S}(\varphi)m$ in Figure 5. That is, $\int_0^{\bar{c}^*} F(c)dc = \int_{\varphi^*}^1 \mathcal{S}(\varphi)md\varphi$.[30] This crucially owes to the fact that crime is an economic bad for potential victims but an economic good for criminals.

Next, I depict vigilance expenses. For simplicity, consider an equilibrium in which all types $\xi$ optimally choose imperfect vigilance, i.e., $\hat{\phi}(\alpha^*|\xi) < 1$. Appendix A.11 shows that *total vigilance expenses* $v^*$ *equal the area below* $\varphi^*$ *and above the demand* $\mathcal{D}(\alpha)$.[31] In other words, $v^* = \int_0^{\alpha^* M} [\varphi^* - \mathcal{D}(\alpha)]\, d(\alpha M)$. Since the demand curve $\mathcal{D}$ depicts victims' "reservation price" at any "quantity," the area over it and below $\varphi = \varphi^*$ captures a cost, and their surplus is thus the triangular area *below* their demand. Next, I explore three applications.

**Rent-seeking.** Recall that Becker (1968) argues that in a competitive theft market, the total criminal costs should approximate the market value of the property loss (Becker, 1968, page 171, footnote 3). Along these lines, Tullock (1967) had already crystalized a more general insight that rent-seeking behavior by all agents involved might well dissipate the gains — at least absent a transfer loss of theft ($m = M$). On the other hand, his "Tullock paradox" (Tullock, 1980) later observed that rent-seeking expenditures are often swamped by the potential gains. As seen in the right panel of Figure 5, *this paradox emerges in this paper*, and it owes precisely to the strict convexity of the vigilance costs, and to potential criminal and victim heterogeneity. Indeed, it can be shown that social costs equal potential gains only in the extreme case, in which vigilance costs are linear, and criminals and victims are homogeneous, forcing demand and supply to have extreme elasticities (i.e., 0 or $\infty$).

[30] Indeed, $\int_0^{\bar{c}^*} F(c)dc = \int_1^{\varphi^*} F(\bar{c}(\varphi))\bar{c}_\varphi(\varphi)d\varphi$, where $F(\bar{c}(\varphi)) = \mathcal{S}(\varphi)$ by (5) and $\bar{c}_\varphi = -m$.

[31] If in equilibrium some types $\xi$ choose $\hat{\phi} = 1$, this area provides an underestimate of vigilance expenses.

**Expected Punishment.** What happens to potential victims' expected losses when punishment rises? Because of the criminal Laffer curve (§5), the answer is unclear a priori, as although greater penalties reduce vigilance expenses, they may also raise expected property losses (Proposition 1). However, this puzzle is easily resolved using Figure 5. Indeed, as discussed in §5, as punishment $\ell$ rises, the supply curve shifts left along the demand curve; thus, both potential victims' losses (the dark area in Figure 5's left panel plus the crime rectangle area $\alpha^*(1-\varphi^*)M$) and the net benefits of vigilance (the dashed area below $\mathcal{D}$ in the left panel of Figure 5) must unambiguously fall. That is, *greater expected punishment decreases total potential victims' losses and the net benefits of vigilance*, even if punishment raises the crime rate; see Figure 6 (left panel). The effect on criminal profits is non-monotone, since revenues are non-monotone, given the criminal Laffer curve. Finally, by Remark 1, the same insights apply if opportunity costs $F$ rise or the criminal material gain $m$ falls.

**Price Floors.** For a final application, consider the effects of regulating vigilance. van Ours and Vollaard (2015) find a large reduction of car theft in the Netherlands after the EU mandated that all new cars have electronic engine immobilizers.[32]In the same spirit, these authors also found in 2011 that burglary greatly fell in the Netherlands after a mandate for burglary-proof windows and doors in all home constructions. Using the supply and demand framework, these examples indicate that new adopted vigilance technologies intervene the demand side of the market. Moreover, these regulations can be understood as a *price floor*, since they mandate a higher vigilance level (and so a higher failure rate) than people optimally desire. As seen in the right panel of Figure 3, these regulations reduce crime rates and attempted crime rates. Also, *both regulations reduce potential victims' losses, although they increase vigilance expenses*, indicating that price floors, unlike punishment, are effective not only to reducing crime but also lowering total potential victims' losses.

# 8 Minimizing Potential Victims' Losses

Criminal activity is shaped by the unobservable vigilance efforts that potential victims make to prevent losing their property. Since all potential victims face the same attempted crime rate, independent of their vigilance choices, the more potential victims self-protect, the lower are the incentives to attempt crimes. Thus, there is a *positive externality* among potential victims as each benefits from the others' vigilance. So by standard economic logic, *total vigilance must be under-provided*, as theoretically argued by Shavell (1991) and empirically tested by Ayres and Levitt (1998). However, a missing aspect in the literature

[32]Accounting for theft diversion to older cars, van Ours and Vollaard find that the device lowered the overall rate of car theft on average by 40% during 1995–2008 in the Netherlands.

is understanding the determinants of the *degree of under-provision*. For instance, if legal punishment were to rise, should we expect more or less under-provision of vigilance?

To this end, consider a social planner who is in charge of coordinating unobservable vigilance efforts to minimize potential victims' expected losses. Unlike potential victims, who take the attempted crime rate as given, the social planner anticipates how changes in vigilance impact the attempted crime rate. Specifically, suppose that the planner desires an average failure rate $\varphi^o \in [0,1]$, and so an attempted crime rate $\alpha^o = \mathcal{S}(\varphi^o)$. The planner could implement any $\varphi$ by exploiting the *inverse demand function* $\mathcal{D}^{-1} : [0,1] \mapsto [0,\bar{\alpha}]$, where $\bar{\alpha}$ is the attempted crime rate ceiling (see §4).[33,34] Indeed, by the FOC (3), the planner can mandate individual failure rates $\hat{\phi}^o(\varphi|\xi)$ obeying, $\mathcal{D}^{-1}(\varphi)M \equiv V_\phi(\hat{\phi}^o(\varphi|\xi)|\xi)$ for each $\xi \in [0,1]$, and thus $\int \hat{\phi}^o(\varphi|\xi)dG(\xi) = \varphi$.[35] Intuitively, $\hat{\phi}^o(\varphi|\xi)$ is the failure rate optimally chosen by potential victim $\xi$ when the attempted crime rate is $\mathcal{D}^{-1}(\varphi)$. Clearly, $\hat{\phi}^o(\cdot|\xi)$ is monotone and $\hat{\phi}^o(0|\xi) = 0$ for all $\xi$. Altogether, the planner's failure rate target $\varphi^o$ solves:

$$\min_{\varphi \in [0,1]} \mathcal{S}(\varphi)(1-\varphi)M + \int_0^1 V(\hat{\phi}^o(\varphi|\xi)|\xi)dG(\xi). \tag{7}$$

To have a convex program, I assume that the density elasticity $\mathcal{E}_{1-\varphi}(f(\bar{c})) \geq -2$.[36] Appendix A.12 show that, the optimal failure rate $\varphi^o \in (0,1)$ and attempted crime rate $\alpha^o > 0$ obey:

$$\mathcal{S}(\varphi^o) - \mathcal{S}'(\varphi^o)(1-\varphi^o) = \mathcal{D}^{-1}(\varphi^o) \tag{8}$$

and $\alpha^o = \mathcal{S}(\varphi^o)$, respectively. The individual failure rates obey $\hat{\phi}^o(\varphi^o|\xi)$ for all $\xi \in [0,1]$.

As seen in Figure 7, the optimal failure rate is given by the intersection of the inverse demand curve and the *marginal crime rate supply curve* $\mathcal{MS}(\varphi) \equiv [\mathcal{S}(\varphi)(1-\varphi)]_\varphi$. The resulting attempted crime rate $\alpha^o$ is fixed by the supply curve $\mathcal{S}$. Notice the analogy with non-competitive markets, where a monopolist chooses how much to produce by equalizing marginal costs (supply) and marginal revenues, and the final price is fixed by the demand curve. The same occurs here as potential victims are the producers of vigilance, and so for them the attempted crime rate $\alpha$ resembles a price and the failure rate $\varphi$ a quantity.

Compared to the decentralized case (Figure 1), the new term $-\mathcal{S}'(\varphi^o)(1-\varphi^o) < 0$ in (8) is the *marginal deterrence effect* of vigilance (Shavell, 1991), as the planner also influences criminal entry. Since this effect is negative, *the failure rate is higher and the crime rate and the attempted crime rate are lower than in the competitive benchmark* (see the left panel of

[33] This inverse function is well-defined, since $\mathcal{D}(\cdot)$ is strictly increasing for all attempted crime rates $\alpha \leq \bar{\alpha}$.
[34] In words, $\mathcal{D}^{-1}(\varphi)$ is the attempted crime rate that induces an average failure rate $\varphi$.
[35] For simplicity, I assume that the FOC (3) holds for all potential victim types $\xi$.
[36] This condition holds when the cost distribution $F$ is either convex or not too concave.

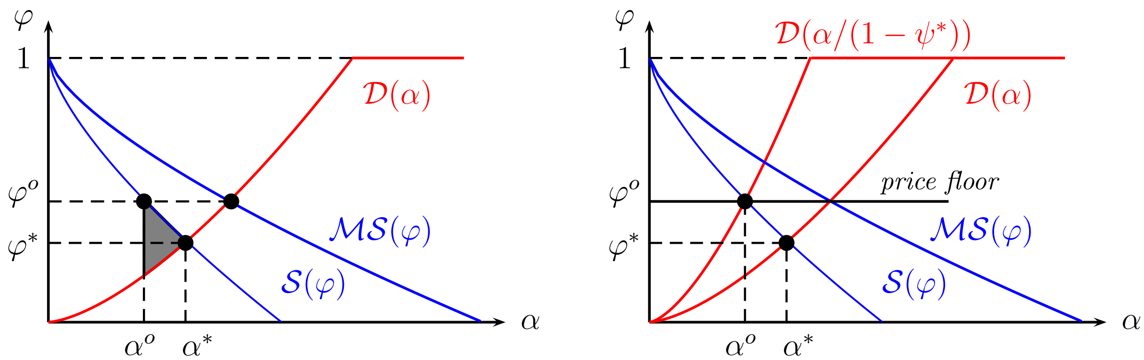


Figure 7: **Centralizing Vigilance.** LEFT: The optimal failure rate is given by the intersection of the inverse demand $\mathcal{D}^{-1}$ and marginal supply $\mathcal{MS}$. The marginal supply lies above the supply curve $\mathcal{S}$, and the respective optimal failure and attempted crime rates are greater and lower than the equilibrium ones. The shaded area captures the net benefits of centralizing vigilance. RIGHT: A subsidy $\psi^*$ on vigilance costs or a price floor implements the desired levels of crime and vigilance.

Figure 7). Also, in the same panel, the shaded triangle represents the *benefits* of coordinating vigilance efforts.[37] While total vigilance may rise compared to the benchmark, the reduction in property losses more than compensates for it. Criminal profits unambiguously fall.

The degree of vigilance under-provision is captured by the gap $\varphi^o - \varphi^*$, or by the relative excess of demand, namely, $(\mathcal{D}^{-1} - \mathcal{S})/\mathcal{S}$, as it measures the vigilance deviation from the competitive equilibrium. Furthermore, by (8), at the optimum, the degree of under-provision is equal to the elasticity of the supply curve (with respect to the failure rate) evaluated at $\varphi = \varphi^o$:

$$\frac{\mathcal{D}^{-1}(\varphi^o) - \mathcal{S}(\varphi^o)}{\mathcal{S}(\varphi^o)} = \mathcal{E}_{1-\varphi}(\mathcal{S}). \tag{9}$$

The degree of under-provision is, thus, affected by the supply elasticity $\mathcal{E}_{1-\varphi}(\mathcal{S})$, which is a function of the expected punishment $\ell$ and the criminal cost distribution $F$. Proposition 6 examines how punishment $\ell$ impact the optimal vigilance levels and the under-provision gap.

**Proposition 6.** *(i) If $(c+\ell)f(c)$ falls in $c$, the optimal failure rate $\varphi^o$ and the degree of vigilance under-provision both rise as punishment $\ell$ rises; (ii) If $(c+\ell)f(c)$ rises in $c$, the optimal failure rate $\varphi^o$ falls in punishment $\ell$, but the degree of vigilance under-provision rises in $\ell$, provided the cost distribution $F$ is log-concave and the elasticity $cf(c)/F(c)$ rises in $c$.*

The sufficient conditions in Proposition 6 do not depend on endogenous variables, and are critically related to the curvature of the cost distribution $F$ and the punishment level $\ell$. Indeed, $(c+\ell)f(c)$ is increasing in $c$ when $F$ is convex or not too concave, and it is decreasing in $c$ when $F$ is concave and the absolute rate of change of $f$ is high enough.

Proposition 6 highlights another channel through which penalties may have an undesirable effect. In the benchmark case, an increase in punishment unambiguously lowers the

[37]For a better visual exposition of welfare gains, the left panel of Figure 7 assumes equal values ($m = M$).

equilibrium failure rate $\varphi^*$ (Proposition 1). However, when vigilance is centralized, the effect of punishment on $\varphi^o$ is ambiguous: Depending on the curvature of the criminal cost distribution $F$, an increase in punishment $\ell$ may lower or raise the marginal deterrence effect of vigilance, and so it may decrease or increase the optimal failure rate $\varphi^o$. In particular, when condition (i) in Proposition 6 holds, and increase in punishment raises the optimal failure rate $\varphi^o$ and it lowers the equilibrium failure rate $\varphi^*$. Thus, as punishment rises, the equilibrium failure rate not only falls, but also becomes more distant to its desirable level.[38]

Finally, I briefly discuss how the planner could implement the optimal failure rate $\varphi^o$. This could be done by either subsidizing vigilance, or mandating it with a price floor. Indeed, suppose that the government can subsidize vigilance by paying a fraction $\psi \in (0,1)$ of vigilance costs. Potential victim $\xi$ expected losses would turn to $\alpha(1-\phi)M + (1-\psi)V(\phi|\xi)$, and individual demands would obey $\hat{\phi}(\frac{\alpha}{1-\psi}|\xi) = \arg\min_\phi \frac{\alpha}{1-\psi}(1-\phi)M + V(\phi|\xi)$ for all $\xi$. That is, potential victims would behave *as if* they faced an attempted crime rate $\frac{\alpha}{1-\psi} > \alpha$. As seen in Figure 7 (right panel), the planner can implement her desired levels of crime and vigilance with a *Pigovian subsidy* $\psi^*$ obeying $\mathcal{D}(\frac{\alpha^o}{1-\psi^*}) = \varphi^o$. Alternatively, she could mandate a minimum vigilance level, emulating a price floor (van Ours and Vollaard, 2015).

Altogether, the government need not to rely on greater punishments (as argued, their efficacy is limited), but rather its policies should be aimed to directly incentivize, or regulate, potential victims' vigilant behavior, and thereby indirectly impact criminals' incentives.

**Remark 2 (Social Cost Minimization).** One could also imagine that the social planner's goal is to minimize the social costs (6). If so, *the planner would elect a lower failure rate and more attempted crimes than those that minimize potential victims' losses* (7). For now the marginal cost of raising the average failure rate $\varphi$ not only embeds total marginal vigilance expenses but also the foregone marginal criminal profits. Also, if the criminal cost distribution $F$ is not "too concave", the failure minimizer lies between $\varphi^*$ and $\varphi^o$. However, if the cost distribution $F$ is "sufficiently" concave, the social cost minimizer entails a lower failure rate and more attempted crimes compared to the equilibrium levels. In this case, vigilance is *over-provided* in equilibrium; see Proposition B.0.1 in the appendix.[39]

# 9 The Observable Vigilance Case

In this section, I relax the assumption that vigilance is unobservable. If potential criminals can observe potential victims' vigilance actions, then a new criminal decision margin emerges:

[38] Appendix A.13 shows that if condition (ii) holds, the relative excess of demand (9) rises in punishment $\ell$.

[39] See Propositions 3A and 3B in Shavell (1991) for a related discussion.

criminals must choose, e.g., whether to burgle the gated mansion or the ungated one.

Crucially, the observability of vigilance changes the strategic nature of crime. Indeed, when vigilance is observable, a novel *signaling* force arises as potential victims' types $\xi$ are private information,. Because criminals care about the failure rate of their attempted crimes, observing, e.g., vigilance $v$ conveys information about types $\xi$, and so about the expected failure rate associated to vigilance $v$.[40] To encompass this sequentiality, the equilibrium notion, used throughout this paper, must naturally change from Nash equilibrium to perfect equilibrium. In a perfect equilibrium, potential criminals use the information contained in vigilance choices to make failure rate inferences and target their offenses, whereas potential victims understand this when choosing their vigilance intensities. In what follows, I examine *separating* equilibria in which vigilance effectively varies across potential victims.

For the sake of clarity, I now introduce some notations that are only necessary here. As discussed in §3, the failure rate of an attempted crime depends on both vigilance and the (privately known) potential victim's type. Specifically, for each type $\xi \in [0,1]$ and vigilance $v \geq 0$, the induced *failure rate function* $\tilde{\phi}(v,\xi) \in [0,1]$ obeys $v \equiv V(\tilde{\phi}(v,\xi)|\xi)$. It is increasing in vigilance $v$ (as $V_\phi > 0$) and decreasing in $\xi$ (as also $V_\xi > 0$) with partials:

$$\tilde{\phi}_v(v,\xi) = 1/V_\phi(\phi(v,\xi)|\xi) > 0 \quad \text{and} \quad \tilde{\phi}_\xi(v,\xi) = -V_\xi(\phi(v,\xi)|\xi)/V_\phi(\phi(v,\xi)|\xi) < 0. \tag{10}$$

Also, $\tilde{\phi}_{v\xi}(v,\xi) \leq 0$ (Claim C.1), so vigilance's efficacy and marginal efficacy fall as $\xi$ rises.

Now, for every observed vigilance $v \geq 0$, the *attempted crime rate function* $\tilde{\alpha}(v,\hat{\xi}) \geq 0$ is the attempted crime rate aimed to vigilance $v$ when the *criminal inference function*, $\hat{\xi}_I(\cdot)$, about the victims' type obeys $\hat{\xi}_I(v) = \hat{\xi} \in [0,1]$. I assume $\tilde{\alpha}$ is differentiable with non-vanishing partials, $\tilde{\alpha}_v \neq 0$ and $\tilde{\alpha}_{\hat{\xi}} \neq 0$. Slightly abusing notation, a potential victim's losses are: $\mathcal{L}(\xi,\hat{\xi},v|\tilde{\alpha}) \equiv \tilde{\alpha}(v,\hat{\xi})(1-\tilde{\phi}(v,\xi))M + v$. These are the losses from crime for potential victim $\xi \in [0,1]$ who chooses vigilance $v \geq 0$ when the realized criminal inference is $\hat{\xi} \in [0,1]$.

Next, consider an equilibrium candidate *vigilance function* $\chi(\cdot)$ for potential victims, where $\chi(\xi) \geq 0$ is the vigilance level exerted by potential victim $\xi \in [0,1]$. In a separating equilibrium, $\chi$ must be *one-to-one*, and so invertible. Thus, on the equilibrium path, i.e., if a potential victim chooses $v \in \chi([0,1])$ then, irrespective of her true type, potential criminals infer that the victim's type is $\hat{\xi}_I(v) \equiv \chi^{-1}(v)$ and the induced failure rate is $\tilde{\phi}(v,\chi^{-1}(v))$. Consequently, the total amount of attempted crimes obeys $\int_{\chi([0,1])} \tilde{\alpha}(v,\chi^{-1}(v))dv$. Now, off the equilibrium path, i.e., for $v \notin \chi([0,1])$, equilibrium perfection puts no restriction on the inference performed by criminals as long as $\hat{\xi}_I(v) \in [0,1]$.

[40] Baumann and Friehe (2013) study a model with observable vigilance, in which victims have private information. They consider *binary* vigilance and a *fixed* mass of criminals, which simplifies the analysis.

In equilibrium, the vigilance function $\chi(\cdot)$ must incentivize potential criminals to target their attempted crimes following $\tilde{\alpha}(\cdot, \chi^{-1}(\cdot))$ on path. Likewise, the attempted crime rate function $\tilde{\alpha}(\cdot, \hat{\xi}_I(\cdot))$ must encourage potential victims to follow the vigilance function $\chi(\cdot)$.

Formally, a *separating equilibrium* is a pair $(\chi(\cdot), \tilde{\alpha}(\cdot))$ such that: $(i)$ given the vigilance function $\chi(\cdot)$, criminal profit maximization yields attempted crime rates $\tilde{\alpha}(v, \chi^{-1}(v))$ for $v \in \chi([0,1])$, and $\tilde{\alpha}(v, \hat{\xi})$ for $v \notin \chi([0,1])$ and some $\hat{\xi}_I(v) = \hat{\xi}$; $(ii)$ given the attempted crime rate function $\tilde{\alpha}(\cdot)$, vigilance $\chi(\xi)$ minimizes expected losses for potential victim $\xi \in [0,1]$.

Given $\chi(\cdot)$ and $\hat{\xi}_I$, define $\mathcal{L}^d(\xi, \hat{\xi}_I) \equiv \inf\{\mathcal{L}(\xi, \hat{\xi}_I(v), v|\tilde{\alpha}) : v < \chi(0)\}$. The next result states the necessary conditions to sustain an equilibrium. Appendix C.2 examines when such conditions are also sufficient; see Proposition C.2.1.

**Proposition 7.** *Consider an equilibrium $(\chi(\cdot), \tilde{\alpha}(\cdot))$. Then, there exists an interior failure rate $\tilde{\varphi} \in (0,1)$ with $\chi(\xi) \equiv V(\tilde{\varphi}|\xi)$, and $\tilde{\alpha}(\chi(\xi), \xi)$ solving*

$$\left[\tilde{\alpha}_v(\chi(\xi), \xi) + \tilde{\alpha}_{\xi}(\chi(\xi), \xi)\frac{1}{\chi'(\xi)}\right] V_{\phi}(\tilde{\varphi}|\xi)(1-\tilde{\varphi})M - \tilde{\alpha}(\chi(\xi), \xi)M + V_{\phi}(\tilde{\varphi}|\xi) = 0, \tag{11}$$

*and market clearing: $\int_0^1 \tilde{\alpha}(\chi(\xi), \xi)dG(\xi) = \mathcal{S}(\tilde{\varphi})$. The vigilance function $\chi(\xi)$ is strictly increasing, whereas the attempted crime rate $\tilde{\alpha}(\chi(\xi), \xi)$ is strictly decreasing, vanishing when $\xi = 1$. The equilibrium losses $\mathcal{L}(\xi, \xi, \chi(\xi)|\tilde{\alpha}) \leq \mathcal{L}^d(\xi, \hat{\xi}_I)$ for all $\xi$ and some inference $\hat{\xi}_I$.*

Proposition 7 installs a constraint on the shape that any equilibrium can take. First, because each criminal optimally attempts a crime towards potential victims for whom the failure rate is the lowest, in equilibrium, the induced failure rates $\tilde{\phi}(\chi(\xi), \xi)$ must be constant across all types $\xi$. Consequently, from the criminals' viewpoint, all vigilance levels are equally desirable targets. The vigilance function $\chi$ is thus characterized by an *average failure rate* $\tilde{\varphi}$ obeying $\tilde{\phi}(\chi(\xi), \xi) \equiv \tilde{\varphi}$, or $\chi(\xi) \equiv V(\tilde{\varphi}|\xi)$, and so *high types are more vigilant than low types*, since $\chi'(\xi) = V_{\xi}(\tilde{\varphi}|\xi) > 0$. Second, given a common failure rate $\varphi$, the amount of attempted crimes is determined by the supply $\mathcal{S}(\varphi)$, as in §4. Since $\tilde{\alpha}(\chi(\xi), \xi)$ is the attempted crime rate faced by victim $\xi$, the failure rate $\tilde{\varphi}$ must also *clear the market*:

$$\int_0^1 \tilde{\alpha}(\chi(\xi), \xi)dG(\xi) = \mathcal{S}(\tilde{\varphi}). \tag{12}$$

Third, the differential equation (11) is a consequence of potential victim optimality and equilibrium considerations. Notice that, on path, the losses of a type $\xi$ that chooses $v$ are $\mathcal{L}(\xi, \chi^{-1}(v), v|\tilde{\alpha})$. Thus, in equilibrium, $\chi$ must be *incentive compatible*, i.e., $\chi(\xi)$ must solve,

$$\min_{v \in \chi([0,1])} \tilde{\alpha}(v, \chi^{-1}(v))(1 - \tilde{\phi}(v, \xi))M + v. \tag{13}$$

The first-order condition, evaluated at $v = \chi(\xi)$ with $\tilde{\phi}(\chi(\xi), \xi) \equiv \tilde{\varphi}$, yields (11), whereas the second-order condition implies the strict monotonicity of the attempted crime rate function $\tilde{\alpha}(\chi(\cdot), \cdot)$. In addition, $\tilde{\alpha}(\chi(1), 1)$ must vanish; for if not, victim type $\xi = 1$ could slightly raise her vigilance and lower her expected losses, for any off path inference; see Appendix C.1.

Finally, potential victims must have no incentives to elect vigilance $v \notin \chi([0,1])$. Consider an upward deviation $v > \chi(1)$. Because $\tilde{\alpha}(\chi(1), 1) = 0$, incentive compatibility (13) makes such deviations unattractive, as they lead to losses $\mathcal{L} \geq \chi(1)$. Now, take a downward deviation $v < \chi(0)$. Such vigilance would attract all criminals, as $\tilde{\phi}(v, \hat{\xi}) < \tilde{\phi}(\chi(0), \hat{\xi}) \leq \tilde{\phi}(\chi(0), 0) = \tilde{\varphi}$, for any off-path inference $\hat{\xi}_I(v) = \hat{\xi}$, yielding an attempted crime rate function $\tilde{\alpha}(v, \hat{\xi}) \equiv \mathcal{S}(\tilde{\phi}(v, \hat{\xi}))$.[41] The *lowest downward-deviation losses* are $\mathcal{L}^d(\xi, \hat{\xi}_I)$, and so for some off-path inference $\hat{\xi}_I$, the equilibrium losses must obey $\mathcal{L}(\xi, \xi, \chi(\xi)|\tilde{\alpha}) \leq \mathcal{L}^d(\xi, \hat{\xi}_I)$.

Notice the qualitative differences between observable and unobservable vigilance. In the observable case, since the attempted crime rate $\tilde{\alpha}(\chi(\xi), \xi)$ is strictly decreasing and the failure rate $\tilde{\varphi}$ is constant, the crime rate $\tilde{\alpha}(\chi(\xi), \xi)(1 - \tilde{\varphi})$ falls as $\xi$ rises. By contrast, when vigilance is unobservable, potential victims face the same attempted crime rate $\alpha^*$ but secure distinct failure rates $\hat{\phi}(\alpha^*|\xi)$; in fact, the crime rate $\alpha^*(1 - \hat{\phi}(\alpha^*|\xi))$ rises as type $\xi$ rises, since high types are less vigilant, i.e., $\hat{\phi}_\xi < 0$; see §4. This qualitative difference follows from the first-order conditions (3) and (11), for in the unobservable case the marginal cost of vigilance is *constant* across types, while it varies when vigilance is observable. Also, by Proposition 7 and Claim A.1, in the observable and unobservable cases, high-cost victims (high $\xi$) exert more and less vigilance, respectively. Still, in any case, equilibrium losses $\mathcal{L}$ are higher for high-cost victims (see Claim C.2 and §4), but each owing to different economic trade-offs.

The observability of vigilance gives rise to new forces. As in §8, the first term in (11) captures the *marginal deterrence effect of vigilance for victim* $\xi$. By Proposition 7, this effect becomes absolutely stronger as potential victims' types $\xi$ rise — namely, *high-cost victims benefit more from the observability of vigilance.*[42] Also, notice that the marginal deterrence effect is proportional to the decline in the attempted crime rate, aimed to victim $\xi = \chi^{-1}(v)$, caused by a marginal increase in vigilance $v$, $d\tilde{\alpha}/dv|_{v=\chi(\xi)}$. In particular, the effect is determined by two additive forces. Firstly, there is a *redirection effect* (proportional to $\tilde{\alpha}_v$) as criminals divert their attempted crime towards other victims in response to an increase in vigilance.[43] And secondly, there is *learning effect* (proportional to $\tilde{\alpha}_{\hat{\xi}}(1/\chi')$), owed to criminals updating their inferences and, consequently, their victim targets.

[41] Hence, the downward deviation losses for victim type $\xi$ obey $\mathcal{L}(\xi, \hat{\xi}, v|\tilde{\alpha}) = \mathcal{S}(\tilde{\phi}(v, \hat{\xi}))(1 - \tilde{\phi}(v, \xi))M + v$.

[42] Consider the FOC (11). Notice that when $\xi$ rises, the middle and right terms $-\tilde{\alpha}M$ and $V_\phi$, respectively, rise (given Proposition 7 and $V_{\phi\xi} \geq 0$). Therefore, the left term, namely, the marginal deterrence effect of vigilance, must fall in order to keep condition (11) balanced. As a result, $d\tilde{\alpha}/dv$ cannot fall too rapidly.

[43] See, e.g., Gonzalez-Navarro (2013) for an empirical analysis of this diversion effect in auto-theft.

Finally, I briefly discuss the effects of punishment $\ell$. Given a failure rate $\varphi$, the map $\varphi \mapsto \int_0^1 \tilde{\alpha} dG$ can be seen as an induced *demand for attempted crime*, as $\tilde{\alpha}$ induces a vigilance behavior governed by $\chi$, given (11). By market clearing (12), the equilibrium failure rate is fixed by the intersection of demand and supply, as in §4. So, the Lafferian effects, identified in §5, apply to the observable vigilance case, provided the demand $\int_0^1 \tilde{\alpha} dG$ rises in $\varphi$.

# 10 Concluding Remarks

Since the domestic security changes made post 9-11, it is more clear than ever that the costs of crime are not just the actual losses of individuals, but also the vigilance expenses for crimes that never happen. This paper has examined the interplay between crime and vigilance. To this end, I developed an economic framework with both heterogenous potential criminals and potential victims in which: (1) pairwise matching of criminals and potential victims produces attempted crimes; (2) not all attempted crimes succeed; and (3) the failure of an attempted crime is probabilistic, rising as the individual vigilance level rises.

From an economic perspective, the equilibrium analysis uncovered and characterized a *vigilance force* that naturally arises in these settings. This force invariably limits the efficacy of policies aimed directly at alleviating crime, such as raising an offense's legal punishment — because victims and criminals both respond when change befalls either party. This force becomes stronger when vigilance expenses are greater than property losses, unveiling a *criminal Laffer curve*, or a crime rate that is hump-shaped in legal punishment. This curve is higher and rises faster when more potential victims have high vigilance costs. In particular, potential victims with low costs are more prone to face more crime if punishment rises.

The focus on crowding out effects helps to sharpen policy intervention: if the goal is to lower the crime rate or the attempted crime rate, policies aimed at incentivizing vigilance, such as mandating or subsidizing it, could be more effective than raising punishment. In fact, raising punishment may shift vigilance even further away from its efficient level.

Finally, from an empirical perspective, these findings suggest that the relationship between crime rates and punishment can be confounded, and thus controlling for vigilance may be a fruitful exercise. More broadly, to examine the model implications, empirical designs may need to consider shifts in both the returns to crime and security investment patterns. In a cross-section of goods, e.g., the security factors inherent to a good can be embedded in the goods fixed effects (Draca et al., 2018). Thus, vigilance factors may be obtained from the differential patterns of theft across households that have similar composition of goods. This could be explored using a combination of household-level and crime victimization data.

# A Omitted Analyses and Proofs

## A.1 Optimal Vigilance Responses

**Claim A.1.** *Consider an attempted crime rate $\alpha < V_\phi(1|1)/M$. If $\alpha > V_\phi(1|0)/M$, vigilance expenditures $V(\hat{\phi}(\alpha|\xi)|\xi)$ are hump-shaped in type $\xi$; otherwise, they are decreasing in $\xi$.*

*Proof:* Fix $\alpha$, and call $\xi_h \geq 0$ the first type $\xi \in [0,1]$ for which $V_\phi(1|\xi)/M \geq \alpha$. If $\xi_h > 0$ then, by (3), all types $\xi < \xi_h$ choose $\hat{\phi}(\alpha|\xi) = 1$ (for $V_\phi(1|\xi)/M > \alpha$), whereas types $\xi \geq \xi_h$ elect $\hat{\phi}(\alpha|\xi) \in (0,1)$ obeying (3). Thus, for $\xi < \xi_h$, vigilance expenditures, $V(1|\xi)$, increase in $\xi$, since $V_\xi \geq 0$. However, for $\xi \geq \xi_h$, differentiating (3) yields $\hat{\phi}_\xi = -V_{\phi\xi}/V_{\phi\phi} \leq 0$, and:

$$\frac{dV(\hat{\phi}|\xi)}{d\xi} = V_\phi \hat{\phi}_\xi + V_\xi = V_\xi - V_\phi \frac{V_{\phi\xi}}{V_{\phi\phi}} = \frac{V_\phi V_\xi}{V_{\phi\phi}} \left( \frac{V_{\phi\phi}}{V_\phi} - \frac{V_\phi}{V} + \frac{V_\phi}{V} - \frac{V_{\phi\xi}}{V_\xi} \right) \leq 0,$$

where the inequality holds, since $V$ is log-concave in $\phi$ and log-supermodular in $(\phi,\xi)$, and also $V_\xi, V_\phi, V_{\phi\phi} \geq 0$. Altogether, vigilance $V(\hat{\phi}|\xi)$ rises for $\xi < \xi_h$ and falls for $\xi \geq \xi_h$.

Finally, notice that $\xi_h > 0$ for $\alpha > V_\phi(1|0)/M$; otherwise, $\xi_h = 0$, and so vigilance expenditures $V(\hat{\phi}|\xi)$ decrease in $\xi \in [0,1]$, as previously argued. □

## A.2 The Demand Elasticity

I now show that *demand $\mathcal{D}$ is less elastic with $G_H$ than $G_L$ if $g_H(\xi)/g_L(\xi)$ is increasing.* To this end, consider the following parametrized type-distribution $H : [0,1] \times \{0,1\} \to [0,1]$ with $(\xi,\theta) \mapsto H$ and density $h(\xi,\theta)$. Also, let $H(\xi,0) \equiv G_L(\xi)$ and $H(\xi,1) \equiv G_H(\xi)$.

Step 1: $h(\xi,\theta)$ is log-supermodular in $(\xi,\theta)$. To see this, take any $\xi' \geq \xi$, then $g_H(\xi')/g_L(\xi') \geq g_H(\xi)/g_L(\xi)$, which rearranging terms yields $g_H(\xi')g_L(\xi) \geq g_H(\xi)g_L(\xi')$, which is equivalent to $h(\xi',1)h(\xi,0) \geq h(\xi',0)h(\xi,1)$.

Step 2: $\hat{\phi}(\alpha,\xi)$ is log-submodular in $(\alpha,\xi)$. Log-differentiate the $\alpha$ derivative $M = V_{\phi\phi}\hat{\phi}_\alpha$ of the FOC (3) in $\xi$ to get:

$$0 = \frac{V_{\phi\phi\phi}}{V_{\phi\phi}}\hat{\phi}_\xi + \frac{V_{\phi\phi\xi}}{V_{\phi\phi}} + \frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} \quad \Rightarrow \quad \frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} = \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \cdot \frac{V_{\phi\xi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\phi}} = \frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} \right), \quad (14)$$

where I used $\hat{\phi}_\xi = -V_{\phi\xi}/V_{\phi\phi}$, found by differentiating (3). Since $\alpha\hat{\phi}_\alpha = V_\phi/V_{\phi\phi} > 0$ by (3),

$$\frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} - \frac{\hat{\phi}_\xi}{\hat{\phi}} \leq 0 \iff \frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} + \frac{1}{\hat{\phi}} \right) \leq 0.$$

This last inequality holds, because the parenthesized term is negative. Indeed,

$$\frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} + \frac{1}{\hat{\phi}} = \left(\frac{1}{\hat{\phi}} + \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi}}{V_{\phi}}\right) + \left(\frac{V_{\phi\phi}}{V_{\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}}\right) \leq 0,$$

because the first parenthesized expression is negative since $\phi V_{\phi\phi}/V_{\phi}$ is non-increasing in $\phi$, whereas the second parenthesized term is negative since $V_{\phi}$ is log-supermodular in $(\phi, \xi)$, by assumption. Since $V_{\phi\xi} > 0$ and $V_{\phi\phi} > 0$, we have that $\hat{\phi}$ is log-submodular in $(\alpha, \xi)$.

STEP 3: $\mathcal{D}(\alpha, \theta) = \int \hat{\phi}(\alpha, \xi) h(\xi, \theta) d\xi$ IS LOG-SUBMODULAR IN $(\alpha, \theta)$. Fix $\alpha > 0$. Since $\hat{\phi}$ is log-submodular in $(\alpha, \xi)$, we have $\hat{\phi}(\alpha|\xi)/\hat{\phi}_{\alpha}(\alpha|\xi)$ is increasing in $\xi$. By Lemma 4 in Athey (2002), since $h(\xi, \theta)$ is log-supermodular in $(\xi, \theta)$, we have that:

$$\frac{\mathcal{D}(\alpha, \theta)}{\mathcal{D}_{\alpha}(\alpha, \theta)} = \frac{\int \hat{\phi}(\alpha|\xi) h(\xi, \theta) d\xi}{\int \hat{\phi}_{\alpha}(\alpha|\xi) h(\xi, \theta) d\xi}$$

is increasing in $\theta$, which implies that $\alpha \mathcal{D}_{\alpha}/\mathcal{D}$ is decreasing in $\theta$. This means that the market demand $\mathcal{D}(\alpha, \theta)$ is log-submodular in $(\alpha, \theta)$, or $\alpha \mathcal{D}_{\alpha}(\alpha, 1)/\mathcal{D}(\alpha, 1) \leq \alpha \mathcal{D}_{\alpha}(\alpha, 0)/\mathcal{D}(\alpha, 0)$, or the demand $\mathcal{D}$ is less elastic with $G_H$ than $G_L$. □

## A.3 The Supply Elasticity

Paralleling §A.2, I now show that *supply $\mathcal{S}$ is less elastic with $F_H$ than $F_L$ if $f_H(c)/f_L(c)$ is increasing.* Consider the auxiliary and parametrized cost distribution $K : \mathbb{R}_+ \times \{0, 1\} \to [0, \infty]$ with $(c, \theta) \mapsto K$ and density $k(c, \theta)$. Also, let $K(c, 0) \equiv F_L(c)$ and $K(c, 1) \equiv F_H(c)$. Following the same logic, $k(c, \theta)$ is log-supermodular in $(c, \theta)$, since the likelihood ratio $K(c, 1)/K(c, 0)$ is monotone. Thus, the cost distribution $K(c, \theta) = \int \mathbf{1}_{[0,c]}(c') k(c', \theta) dc'$ is log-supermodular in $(c, \theta)$, since the indicator $\mathbf{1}_{[0,c]}(c')$ is log-supermodular in $(c, c')$; see Lemmas 3–4 in Athey (2002). Altogether, $k(c, \theta)/K(c, \theta)$ is increasing in $\theta$, or:

$$k(c, 1)/K(c, 1) = f_H(c)/F_H(c) \geq f_L(c)/F_L(c) = k(c, \theta)/K(c, \theta).$$

Next, using (5) with the distribution $K$, the supply elasticity is given by:

$$\mathcal{E}_{\varphi}(S) = -\frac{k(\bar{c}, \theta)}{K(\bar{c}, \theta)} \left(\frac{m - \ell - \bar{c}}{\bar{c}}\right),$$

where $\bar{c}$ is the marginal potential criminal. Since the above parenthesized term is positive, the supply is less elastic with $\theta = 1$ or $F_H$ than $F_L$. □

## A.4 Existence and Uniqueness of Equilibrium

First, since the demand $\mathcal{D}(\alpha)$ is strictly increasing in $\alpha$ for $\alpha \le \bar{\alpha}$, one can define the inverse demand map $\varphi \mapsto \mathcal{D}^{-1}(\varphi) \in [0, \bar{\alpha}]$. Next, define the *excess of supply* $\mathcal{ES}(\varphi) \equiv \mathcal{S}(\varphi) - \mathcal{D}^{-1}(\varphi)$. Clearly, when the failure rate is perfect, $\mathcal{S}(1) = 0 < \mathcal{D}^{-1}(1) = \bar{\alpha}$. Also, since the demand $\mathcal{D}(0) = 0$, its inverse obeys $\mathcal{D}^{-1}(0) = 0 < \mathcal{S}(0) = F(m - \ell)$. Altogether, the excess of supply satisfies: $\mathcal{ES}(1) < 0 < \mathcal{ES}(0)$. Thus, by the Intermediate Value Theorem (IVT), there exists $\varphi^* \in (0, 1)$ such that the excess of supply vanishes $\mathcal{ES}(\varphi^*) = 0$, or $\mathcal{S}(\varphi^*) = \mathcal{D}^{-1}(\varphi^*)$. Finally, since supply falls in the failure rate, while the inverse demand rises in it, $\varphi^*$ is unique because the excess of supply $\mathcal{ES}(\varphi)$ strictly falls in $\varphi$. □

## A.5 The Success Rate Elasticity

**Lemma A.5.1.** *Along the demand $\mathcal{D}$, the average success rate $1-\mathcal{D}$ is more elastic with $G_H$ than with $G_L$ if the likelihood ratio $g_H(\xi)/g_L(\xi)$ is increasing.*

*Proof:* As in §A.2, consider a parametrized type-distribution $H : [0, 1] \times \{0, 1\} \to [0, 1]$ with $(\xi, \theta) \mapsto H$ and density $h(\xi, \theta)$. Also, let $H(\xi, 0) \equiv G_L(\xi)$ and $H(\xi, 1) \equiv G_H(\xi)$. This implies that $k(\xi, \theta)$ is log-supermodular in $(\xi, \theta)$; see Step 1 in §A.2.

Step 1: $\hat{\phi}_\alpha(\alpha|\xi)/(1 - \hat{\phi}(\alpha|\xi))$ rises in $\xi$. Log-differentiate this expression in $\xi$ to get:

$$\frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} + \frac{\hat{\phi}_\xi}{1 - \hat{\phi}} = \frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{1}{1 - \hat{\phi}} \right),$$

where I used (14) and $\hat{\phi}_\xi = -V_{\phi\xi}/V_{\phi\phi}$, which was found in §A.2 (Step 2). Next, I show that the above (displayed) expression is negative. Indeed, adding and subtracting $V_{\phi\phi}/V_\phi$ to its right-hand side,

$$\frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{1}{1 - \hat{\phi}} \right) = \frac{V_{\phi\xi}}{V_{\phi\phi}} \left[ \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi}}{V_\phi} \right) + \left( \frac{V_{\phi\phi}}{V_\phi} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} \right) - \frac{1}{1 - \hat{\phi}} \right] < 0.$$

The inequality holds, for $\log(V_\phi)$ is concave in $\phi$ and supermodular in $(\phi, \xi)$; also, $V_{\phi\xi}, V_{\phi\phi} \ge 0$.

Step 2: $1-\mathcal{D}$ is log-supermodular in $(\xi, \theta)$. By Lemma 4 in Athey (2002), since $h(\xi, \theta)$ is log-supermodular in $(\xi, \theta)$, we have that:

$$\frac{\int (1 - \hat{\phi}(\alpha|\xi)) h(\xi, \theta) d\xi}{\int \hat{\phi}_\alpha(\alpha|\xi) h(\xi, \theta) d\xi} = \frac{1 - \mathcal{D}(\alpha, \theta)}{\mathcal{D}_\alpha(\alpha, \theta)}$$

is increasing in $\theta$. Thus, $\mathcal{E}_\alpha(1 - \mathcal{D}) = -\alpha\mathcal{D}_\alpha/(1 - \mathcal{D})$ is increasing in $\theta$. □

## A.6 The Criminal Laffer Curve: Proof of Proposition 1 Finished

**Lemma A.6.1.** *Along the demand curve, the crime rate $\alpha(1-\mathcal{D}(\alpha))$ is hump-shaped.*

I prove this result in steps. Define the *individual crime rate function* $\bar{\kappa}(\alpha,\xi) \equiv \alpha(1-\hat{\phi}(\alpha,\xi))$ for victim $\xi$, given $\alpha$. Notice that $\bar{\kappa}(0,\xi)=0$, and also $\bar{\kappa}(\alpha,\xi)=0$ for all $\alpha \geq \bar{\alpha}_\xi$, where $\bar{\alpha}_\xi \equiv V_\phi(1|\xi)/M$ is the *attempted crime rate ceiling for victim* $\xi$. Also, the *critical attempted crime rate* $\hat{\alpha}_\xi \in (0,\bar{\alpha}_\xi)$ is the one that maximizes the crime rate $\bar{\kappa}(\alpha,\xi)$, for all $\xi$.

STEP 1: THE INDIVIDUAL CRIME RATE IS QUASI-CONCAVE IN $\alpha$. Consider a specific potential victim $\xi$. As discussed in §5, think of $\bar{\kappa}(\alpha|\xi)m$ as the total criminal revenue, and $(1-\hat{\phi}(\alpha|\xi))m$ as the price. Then, the individual crime rate $\bar{\kappa}$ falls in $\alpha$ along the demand curve if $(1-\hat{\phi}(\alpha|\xi))\alpha m$ rises in $(1-\hat{\phi}(\alpha|\xi))m$. Recalling that the elasticity of a product is the sum of the elasticities, this reduces to:

$$\mathcal{E}_{(1-\hat{\phi}(\alpha|\xi))m}((1-\hat{\phi}(\alpha|\xi))\alpha m) = 1+\mathcal{E}_{(1-\hat{\phi}(\alpha|\xi))m}(\alpha) = 1-(1-\hat{\phi}(\alpha|\xi))/(\alpha\hat{\phi}_\alpha(\alpha|\xi)) > 0. \quad (15)$$

Next, I show that the left side of (15) is increasing in $\alpha$, and respectively negative and positive for low and high values of $\alpha$. For the monotonicity, $(1-\hat{\phi}(\alpha|\xi))$ is decreasing in $\alpha$, since the failure rate $\hat{\phi}(\alpha|\xi)$ is increasing in $\alpha$. Now, I claim that $\alpha\hat{\phi}_\alpha(\alpha|\xi)$ is increasing in $\alpha$. Indeed, using (3), $\alpha\hat{\phi}_\alpha(\alpha|\xi) = V_\phi(\hat{\phi}|\xi)/V_{\phi\phi}(\hat{\phi}|\xi)$. Since for each type $\xi$, marginal costs $V_\phi$ are log-concave, $V_\phi(\hat{\phi}|\xi)/V_{\phi\phi}(\hat{\phi}|\xi)$ is monotone in $\hat{\phi}$, and so in $\alpha$.

Hence, the left side of (15) rises in $\alpha$. Also, it is positive at $\alpha = V_\phi(1|\xi)/M$, since $\hat{\phi}(\alpha|\xi) = 1$; and negative as $\alpha \downarrow 0$, since $\hat{\phi}(0|\xi)=0$ and $\lim_{\phi\downarrow 0} V_\phi/V_{\phi\phi} < 1$, so that $\lim_{\alpha\downarrow 0}\alpha\hat{\phi}_\alpha(\alpha|\xi) < 1$. So, by continuity, the left side of (15) is positive for $\alpha > \hat{\alpha}_\xi$ and negative for $\alpha < \hat{\alpha}_\xi$, for some $\hat{\alpha}_\xi \in (0, V_\phi(1|\xi)/M)$. That is, $\bar{\kappa}(\alpha|\xi)$ is hump-shaped in $\alpha$, peaking at $\hat{\alpha}_\xi$. So, $\bar{\kappa}$ is quasi-concave in $\alpha$, since it is hump-shaped in $\alpha < \bar{\alpha}_\xi$ and then zero for $\alpha \geq \bar{\alpha}_\xi$. $\square$

STEP 2: THE CRITICAL ATTEMPTED CRIME RATE $\hat{\alpha}_\xi$ INCREASES IN THE INDEX $\xi$. By definition $\hat{\alpha}_\xi \equiv \arg\max_\alpha \bar{\kappa}(\alpha,\xi)$. By Topkis' Theorem (Topkis, 1998), $\hat{\alpha}_\xi$ rises in $\xi$ if $\bar{\kappa}_{\alpha\xi}(\alpha,\xi) > 0$. I argue that this is positive, i.e., that $\bar{\kappa}_{\alpha\xi} = -\hat{\phi}_\xi - \alpha\hat{\phi}_{\alpha\xi} > 0$.

Log-differentiate the $\alpha$ derivative $M = V_{\phi\phi}\hat{\phi}_\alpha$ of the FOC (3) in $\xi$ to get

$$0 = \frac{V_{\phi\phi\phi}}{V_{\phi\phi}}\hat{\phi}_\xi + \frac{V_{\phi\phi\xi}}{V_{\phi\phi}} + \frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} \quad \Rightarrow \quad \frac{-\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} = -\frac{V_{\phi\phi\phi}}{V_{\phi\phi}}\cdot\frac{V_{\phi\xi}}{V_{\phi\phi}} + \frac{V_{\phi\phi\xi}}{V_{\phi\phi}} = \frac{V_{\phi\xi}}{V_{\phi\phi}}\left(\frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}}\right),$$

where $\hat{\phi}_\xi = -V_{\phi\xi}/V_{\phi\phi}$, found by differentiating (3). Since $\alpha\hat{\phi}_\alpha = V_\phi/V_{\phi\phi}$ by (3), $\bar{\kappa}_{\alpha\xi} > 0$ iff

$$\frac{-\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} > \frac{\hat{\phi}_\xi}{\alpha\hat{\phi}_\alpha} \iff \frac{V_{\phi\xi}}{V_{\phi\phi}}\left(\frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}}\right) > \frac{-V_{\phi\xi}}{V_{\phi\phi}}\frac{V_{\phi\phi}}{V_\phi} \iff \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} > \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi}}{V_\phi}.$$

This last inequality holds, because the right side is nonpositive, as $V_\phi$ is log-concave in $\phi$; and $V_{\phi\phi\xi}, V_{\phi\xi} > 0$, as $V_\phi$ and $V$ are supermodular and log-supermodular in $(\phi, \xi)$, respectively. $\square$

Next, I show that the aggregate crime rate function $\tilde{\kappa}(\alpha) \equiv \int_0^1 \bar{\kappa}(\alpha, \xi)dG(\xi)$ is quasi-concave in $\alpha$. Building on the variation diminishing property of totally positive functions (Karlin, 1968), Choi and Smith (2017) provide a useful condition that ensures that the weighted sum of quasi-concave functions is quasi-concave.

To this end, decompose the individual crime rate function $\bar{\kappa} = \bar{\kappa}^I + \bar{\kappa}^D$ into its increasing and decreasing portions, $\bar{\kappa}^I$ and $\bar{\kappa}^D$, respectively, where $\bar{\kappa}^I$ and $-\bar{\kappa}^D$ are monotone, and $\bar{\kappa}^I$ (resp. $-\bar{\kappa}^D$) is constant right (resp. left) of the peak (argmax) of $\bar{\kappa}$. When these functions are differentiable, for any $\xi, \xi'$, say that $-\bar{\kappa}^D(\cdot, \xi)$ *grows proportionally faster* than $\bar{\kappa}^I(\cdot, \xi')$, if $\bar{\kappa}^I(\cdot, \xi')$ is more risk averse than $-\bar{\kappa}^D(\cdot, \xi)$, namely:

$$-\bar{\kappa}^I_{\alpha\alpha}(\alpha, \xi')/\bar{\kappa}^I_\alpha(\alpha, \xi') \geq -\bar{\kappa}^D_{\alpha\alpha}(\alpha, \xi)/\bar{\kappa}^D_\alpha(\alpha, \xi). \tag{16}$$

More generally, for any $\alpha_3 \geq \alpha_2 \geq \alpha_1$:

$$[\bar{\kappa}^I(\alpha_2, \xi') - \bar{\kappa}^I(\alpha_1, \xi')][\bar{\kappa}^D(\alpha_2, \xi) - \bar{\kappa}^D(\alpha_3, \xi)] \geq [\bar{\kappa}^I(\alpha_3, \xi') - \bar{\kappa}^I(\alpha_2, \xi')][\bar{\kappa}^D(\alpha_1, \xi) - \bar{\kappa}^D(\alpha_2, \xi)] \tag{17}$$

By Proposition 1 in Choi and Smith (2017), the crime rate $\tilde{\kappa}$ is quasi-concave in $\alpha$ iff $-\bar{\kappa}^D(\cdot, \xi)$ grows proportionally faster than $\bar{\kappa}^I(\cdot, \xi')$, for all $\xi', \xi$.

Step 3: $-\bar{\kappa}^D(\cdot, \xi)$ grows proportionally faster than $\bar{\kappa}^I(\cdot, \xi')$, for all $\xi, \xi'$. Whenever $\bar{\kappa}^D(\cdot, \xi)$ or $\bar{\kappa}^I(\cdot, \xi)$ is constant, (17) holds, since both sides vanish.

Assume first $\xi' \leq \xi$. Then the claim holds — for if $\alpha < \hat{\alpha}_\xi$, then $-\bar{\kappa}^D(\alpha, \xi)$ is constant, and if $\alpha \geq \hat{\alpha}_\xi$, then $\bar{\kappa}^I(\alpha, \xi')$ is constant (recalling that $\hat{\alpha}_\xi \geq \hat{\alpha}_{\xi'}$, by Step 2).

The case $\xi' > \xi$ is trickier, as both $\bar{\kappa}^I(\alpha, \xi')$ and $-\bar{\kappa}^D(\alpha, \xi')$ are increasing on an interval, namely, for $\alpha \in [\hat{\alpha}_\xi, \min\{\bar{\alpha}_\xi, \hat{\alpha}_{\xi'}\}]$. Here, $-\bar{\kappa}^D(\cdot, \xi)$ and $\bar{\kappa}^I(\cdot, \xi')$ are differentiable, and so I can use criterion (16). But on this interval, $\bar{\kappa}^I(\alpha, \xi) = \bar{\kappa}(\alpha, \xi)$ is increasing and concave, whereas $\bar{\kappa}^D(\alpha, \xi') = \bar{\kappa}(\alpha, \xi')$ is decreasing and concave, since $\bar{\kappa}(\cdot, \xi)$ and $\bar{\kappa}(\cdot, \xi')$ are concave, by Step 1. As a result, $\bar{\kappa}^I(\cdot, \xi')$ is more risk averse than $-\bar{\kappa}^D(\cdot, \xi)$:

$$-\bar{\kappa}^I_{\alpha\alpha}(\alpha, \xi')/\bar{\kappa}^I_\alpha(\alpha, \xi') > 0 > -\bar{\kappa}^D_{\alpha\alpha}(\alpha, \xi)/\bar{\kappa}^D_\alpha(\alpha, \xi). \qquad \square$$

## A.7 Higher Punishment Increases Crime: Proof of Proposition 2

Recall that the crime rate falls along the demand curve if the demand is sufficiently elastic (Proposition 1). Equivalently, $(1 - \mathcal{D}(\alpha))/(\alpha\mathcal{D}'(\alpha)) < 1$. This condition can be rewritten as $\kappa M/(\alpha^2 M\mathcal{D}'(\alpha)) < 1$, where $\kappa = (1 - \mathcal{D}(\alpha))\alpha$ is the crime rate. Now, differentiate the demand curve $\mathcal{D}(\alpha)$ in (4) to get:

$$\begin{aligned}\alpha^2 M\mathcal{D}'(\alpha) &= \alpha M\int_0^1 \frac{V_\phi(\hat\phi|\xi)}{V_{\phi\phi}(\hat\phi|\xi)}dG(\xi) \geq \int_0^1 V_\phi(\hat\phi|\xi))\frac{V_\phi(\hat\phi|\xi)}{V_{\phi\phi}(\hat\phi|\xi)}dG(\xi)\\ &= \int_0^1 V(\hat\phi|\xi)\frac{V_\phi(\hat\phi|\xi)/V(\hat\phi|\xi)}{V_{\phi\phi}(\hat\phi|\xi)/V_\phi(\hat\phi|\xi)}dG(\xi) \geq \int_0^1 V(\hat\phi|\xi)dG(\xi) = \bar v.\end{aligned}$$

To get the first equality, log-differentiate the FOC (3). To get the first inequality, use that optimal $\hat\phi$ obeys $\alpha M \geq V_\phi(\hat\phi(\alpha|\xi)|\xi)$ for all types $\xi$. Finally, use that $(V_\phi/V)/(V_{\phi\phi}/V_\phi) \geq 1$ as is well-known that if $\log V_\phi$ is concave in $\phi$, so is $\log V$ by Prékopa's Theorem (Prékopa, 1973). Altogether, if $\kappa M < \bar v$ then $\kappa M/(\alpha^2 M\mathcal{D}'(\alpha)) < 1$, and thus the crime rate falls along the demand, or the criminal Laffer curve is upward sloping. □

## A.8 Individual Crime and Punishment: Proof of Proposition 3

First, write the *individual crime rate function* for victim $\xi$ as $\bar\kappa(\alpha,\xi) \equiv \alpha(1-\hat\phi(\alpha,\xi))$. Next, let $\alpha^*(\ell)$ be the equilibrium attempted crime rate for punishment $\ell \in [0,m]$. Now differentiate the individual crime rate function in $\ell$ to get, $d\bar\kappa/d\ell = \bar\kappa_\alpha(\alpha^*(\ell),\xi)\cdot(d\alpha^*(\ell)/d\ell)$. Since $d\alpha^*(\ell)/d\ell < 0$ by Proposition 1, the individual crime rate for victim $\xi$ increases when punishment $\ell$ increases iff $\bar\kappa_\alpha(\alpha^*(\ell),\xi) < 0$. Also, by the proof of Proposition 1 (see Step 2), $\bar\kappa(\alpha,\xi)$ is supermodular in $(\alpha,\xi)$, and thus $\bar\kappa_\alpha(\alpha^*(\ell),\xi)$ is increasing in $\xi$. Thus, there exists a critical type $\bar\xi$ such that $\bar\xi$ equals one if $\bar\kappa_\alpha(\alpha^*(\ell),1) < 0$; equals zero if $\bar\kappa_\alpha(\alpha^*(\ell),0) > 0$; and solves $\bar\kappa_\alpha(\alpha^*(\ell),\bar\xi) = 0$ otherwise. Therefore, a marginal increase in punishment $\ell$ raises the crime rate for potential victim $\xi$ if and only if $\xi \leq \bar\xi$.

Finally, $\bar\xi$ falls in $\ell$, since $\bar\kappa(\alpha,\xi)$ is also quasi-concave in $\alpha$; see Appendix A.6. □

## A.9 The Criminal Laffer Curve's Slope: Proof of Proposition 4

Equivalent Variations of Legal Penalties. Does the Laffer curve become steeper or flatter as $G$ rises? To answer this question and for extra clarity, given a type-distribution $G$, respectively write the equilibrium attempted crime rate and its elasticity as $\alpha^*(\ell|G)$ and $\mathcal{E}_\ell(\alpha^*|G)$. Next, to isolate the crowding out vigilance effect associated with the Laffer curve, consider percentage changes in punishment that induce an *equivalent* percentage change in the attempted crime rate for $G_L$ and $G_H$, namely, $\mathcal{E}_\ell(\alpha^*|G_L) = \mathcal{E}_\ell(\alpha^*|G_H)$. Notice that the elasticity of the criminal Laffer curve is given by:

$$\mathcal{E}_\ell(\kappa^*|G) = \mathcal{E}_\ell(\alpha^*|G)(1+\mathcal{E}_\alpha(1-\mathcal{D}|G)). \tag{18}$$

If the respective Laffer curves (i.e., with $G_L$ and $G_H$) are increasing in punishment $\ell$ then, given (18), an equivalent variation in punishment yields $\mathcal{E}_\ell(\kappa^*|G_H) > \mathcal{E}_\ell(\kappa^*|G_L) > 0$ if and only if $1 + \mathcal{E}_\alpha(1-\mathcal{D}|G_H) < 1 + \mathcal{E}_\alpha(1-\mathcal{D}|G_L) < 0$. It is important to notice that the elasticities $\mathcal{E}_\alpha(1-\mathcal{D}|G_H)$ and $\mathcal{E}_\alpha(1-\mathcal{D}|G_L)$ are evaluated at different equilibrium attempted crime rates. Since the equilibrium $\alpha^*(\ell|G)$ rises in $G$ but falls in $\ell$, it is unclear whether the resulting $\alpha^*$ will be greater or lower with $G_L$ or $G_H$. However, under an equivalent variation of $\ell$, the effect of $G$ dominates and the equilibrium attempted crime rate is higher with $G_H$ before and after a change in punishment. This focus disciplines the comparative statics.

*Proof of (i):* The criminal Laffer curve shifts up at every punishment $\ell$ (left panel of Figure 3). Now I show that it also shifts left. First, call $\ell_{peak} \geq 0$ the punishment $\ell$ for which the crime rate $\kappa^*(\ell)$ peaks. Notice that $\ell_{peak}$ uniquely solves $\alpha^*(\ell_{peak}) = \alpha_{peak}$, where $\alpha_{peak}$ maximizes $\alpha(1-\mathcal{D}(\alpha))$. Next, recall that optimizers are invariant to monotonic transformations of the objective function. So, $\alpha_{peak} = \arg\max_\alpha \log(\alpha) + \log(1-\mathcal{D})$. But, $\partial \log(1-\mathcal{D})/\partial\alpha$ rises as $G$ rises, by Lemma A.5.1; so $\alpha_{peak}$ rises as $G$ rises, by Topkis' theorem (Topkis, 1998). Finally, since the attempted crime rate $\alpha^*$ falls in $\ell$ (Proposition 1), $\ell_{peak}$ must fall as $G$ rises. $\square$

*Proof of (ii):*

Step 1: For any $G$, $1 + \mathcal{E}_\alpha(1-\mathcal{D}) < 0 \Rightarrow 1-\mathcal{D}$ is log-concave in $\alpha$. Note that $1-\mathcal{D}$ is log-concave in $\alpha$ iff $\frac{\mathcal{D}_{\alpha\alpha}}{\mathcal{D}_\alpha} + \frac{\mathcal{D}_\alpha}{1-\mathcal{D}} \geq 0$. Also, $1 + \mathcal{E}_\alpha(1-\mathcal{D}) < 0$ implies that $1 - \frac{\alpha\mathcal{D}_\alpha}{1-\mathcal{D}} < 0$, or equivalently, $\frac{\mathcal{D}_\alpha}{1-\mathcal{D}} > \frac{1}{\alpha}$. So, $1-\mathcal{D}$ is log-concave if $\mathcal{D}_{\alpha\alpha}/\mathcal{D}_\alpha + 1/\alpha > 0$. But this latter condition is equivalent to $\alpha\mathcal{D}_{\alpha\alpha} + \mathcal{D}_\alpha > 0$ and it holds. Indeed, differentiate (3) once and twice to get:

$$\alpha\mathcal{D}_{\alpha\alpha} + \mathcal{D}_\alpha = \int (\alpha\hat{\phi}_{\alpha\alpha} + \hat{\phi}_\alpha) dG(\xi) = \int \hat{\phi}_\alpha \frac{V_{\phi\phi}}{V_\phi}\left(\frac{V_{\phi\phi}}{V_\phi} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}}\right) dG(\xi) > 0,$$

where the inequality obtains, because the integrand of this expression is positive since (1) $V$ is increasing and convex in $\phi$; (2) $V_\phi$ is log-concave in $\phi$; and (3) $\hat{\phi}$ is increasing in $\alpha$.

Step 2: If $1 + \mathcal{E}_\alpha(1-\mathcal{D}) < 0$ for $G_L$ and $G_H$, then $\mathcal{K}(\cdot|G_H)$ is steeper than $\mathcal{K}(\cdot|G_L)$ under an equivalent variation of legal punishment. As seen in Figure 3, for a fixed $\ell$, $\alpha^*(\ell|G_H) > \alpha^*(\ell|G_L)$. Thus, under an equivalent variation of legal punishment, this ranking is unaffected. Also, by Step 1, $\mathcal{E}_\alpha(1-\mathcal{D}|G)$ is decreasing in $\alpha$ for all $G$; also, by Lemma A.5.1, $\mathcal{E}_\alpha(1-\mathcal{D}|G)$ is decreasing in $G$ for all $\alpha$. These observations together yield $1 + \mathcal{E}_\alpha(1-\mathcal{D}|G_H) < 1 + \mathcal{E}_\alpha(1-\mathcal{D}|G_L)$, and so $\mathcal{K}(\cdot|G_H)$ is steeper than $\mathcal{K}(\cdot|G_L)$.

Step 3: If $1 + \mathcal{E}_\alpha(1-\mathcal{D}) > 0$ for all $G$ and $V_\phi$ is concave in $\phi$ for all $\xi$, then $1-\mathcal{D}$ is log-concave in $\alpha$ for all $G$. I'll show that $1 - \hat{\phi}(\alpha|\xi)$ is log-concave in $\alpha$ for all $\xi$, which implies $1 - \mathcal{D} = \int(1-\hat{\phi}(\alpha|\xi))dG(\xi)$ is log-concave in $\alpha$, by Prékopa's Theorem (Prékopa, 1973). Observe that $1-\hat{\phi}$ is log-concave if and only if $\hat{\phi}_{\alpha\alpha}/\hat{\phi}_\alpha + \hat{\phi}_\alpha/(1-\hat{\phi}) \geq 0$.

Using the FOC (3), and differentiating this FOC in $\alpha$ once and twice yields:

$$\frac{\hat{\phi}_{\alpha\alpha}}{\hat{\phi}_{\alpha}} + \frac{\hat{\phi}_{\alpha\alpha}}{1-\hat{\phi}} = \frac{V_{\phi}}{\alpha V_{\phi\phi}} \left( \frac{1}{1-\hat{\phi}} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \right) > 0,$$

since $V$ is increasing and convex in $\phi$, and $V_{\phi\phi\phi} \leq 0$ as $V_{\phi}$ is concave in $\phi$.

STEP 4: IF $1+\mathcal{E}_{\alpha}(1-\mathcal{D}) > 0$ FOR ALL $G$ AND $1-\mathcal{D}$ IS LOG-CONCAVE IN $\alpha$, THEN $\mathcal{K}(\cdot|G_H)$ IS FLATTER THAN $\mathcal{K}(\cdot|G_L)$. By the same logic of Step 2, notice that $0 < 1+\mathcal{E}_{\alpha}(1-\mathcal{D}|G_H) < 1+\mathcal{E}_{\alpha}(1-\mathcal{D}|G_L)$. So $\mathcal{E}_{\ell}(\mathcal{K}|G_H) > \mathcal{E}_{\ell}(\mathcal{K}|G_L)$, since $\mathcal{E}_{\ell}(\alpha^*|G) < 0$ for all $G$. □

## A.10 The Stakes of Crime: Proof of Proposition 5 Finished

Fix the markdown $\mu = 1 - m/M$ and let $M = t$ and $m = (1-\mu)t, t > 0$. Next, parametrize the demand $\mathcal{D}$ (and individual demands $\hat{\phi}$) and supply $\mathcal{S}$ curves, so that $\mathcal{D}(\alpha, t) \equiv \int \hat{\phi}(\alpha, \xi, t)dG$ and $\mathcal{S}(\varphi, t) \equiv F((1-\varphi)(1-\mu)t - \ell)$, respectively. The following observations will be useful.

(★): (i) Supply $\mathcal{S}$ vanishes, i.e. $\mathcal{S} \equiv 0$, for all failure rates $\varphi$ when stakes are low, namely, when $t \leq \ell/(1-\mu)$. (ii) Second, $t\hat{\phi}_t = V_{\phi}/V_{\phi\phi}$ by log-differentiating the FOC (3) in $t$.

**Claim A.2.** *The equilibrium failure rate $1-\varphi^*$ rises as $t$ rises.*

*Proof:* First, as $t$ rises potential victims raise their vigilance, i.e., $\hat{\phi}_t \geq 0$. On the other side, criminals attempts more crime, i.e., the marginal criminal $\bar{c}_t > 0$. As seen in Figure 4, both the demand $\mathcal{D}$ and supply $\mathcal{S}$ shift up. Thus, the failure rate $\varphi^*$ unambiguously rises. □

As discussed in the main text, the attempted crime rate $\alpha^*$ rises if and only if supply $\mathcal{S}$ shifts up more than demand $\mathcal{D}$ does. To get the respective magnitudes of the *vertical demand shift* $\varphi_t^D$ and the *vertical supply shift* $\varphi_t^S$, fix $\alpha = \alpha^*$ and, respectively, differentiate in $t$ the equilibrium conditions, $\mathcal{D}(\alpha^*, t) = \varphi^*$ and $\mathcal{S}(\varphi^*, t) = \alpha^*$. After few algebraic manipulations:

$$\varphi_t^D \equiv \int \hat{\phi}_t(\alpha^*, \xi, t)dG = \int \frac{V_{\phi}}{tV_{\phi\phi}}dG, \quad \text{and} \quad \varphi_t^S \equiv \frac{1-\varphi^*}{t} = \int \frac{1-\hat{\phi}(\alpha^*, \xi, t)}{t}dG. \tag{19}$$

**Claim A.3.** *If demand $\mathcal{D}$ is sufficiently elastic for some $t$, then the equilibrium attempted crime rate $\alpha^*$ is hump-shaped in the stakes. Otherwise, $\alpha^*$ is monotone increasing.*

*Proof.* STEP 1: THE ATTEMPTED CRIME RATE RISES FOR SMALL STAKES. Consider the *shift-difference* $\varphi_t^D - \varphi_t^S$, and small stakes $0 < t \leq \ell/(1-\mu)$. By observation (★)-(i), the supply $\mathcal{S} \equiv 0$ for all $\varphi$, and so the supply and demand curves intersect at the origin: $\varphi^* = \alpha^* = 0$. So $\varphi_t^S = 1/t$, while $\varphi_t^D < 1/t$ since $\hat{\phi}(0, \xi, t) = 0$ and $\lim_{\phi\downarrow 0} V_{\phi\phi}/V_{\phi} > 1$ (see §3).

Step 2: The attempted crime rate falls iff demand is sufficiently elastic. By (19), the shift-difference $\varphi_t^D - \varphi_t^S \geq 0$ if and only if $\int (V_\phi/V_{\phi\phi})dG \geq 1-\varphi^*$. But this latter inequality is equivalent to $\mathcal{D}$ being sufficiently elastic. Indeed, notice that, in equilibrium, $1-\mathcal{D} = 1-\varphi^*$, and $\alpha^*\mathcal{D}_\alpha = \int \alpha^* \hat{\phi}_\alpha dG = \int (V_\phi/V_{\phi\phi})dG$, by log-differentiating the FOC (3) in $\alpha$. Thus, $\varphi_t^D \geq \varphi_t^S \iff \int (V_\phi/V_{\phi\phi})dG \geq 1-\varphi^* \iff \mathcal{E}_\alpha(\mathcal{D}) \geq (1-\mathcal{D})/\mathcal{D}$.

Step 3: The shift-difference is monotone in $t$. First, assume that demand $\mathcal{D}$ is sufficiently elastic for some high stakes $t$. Then, the shift-difference is positive for high $t$ and negative for small $t$, by Steps 1 and 2. Next, I'll show that the shift-difference is monotone, ensuring a single-crossing. Differentiate the shift-difference in $t$ to get:

$$\begin{aligned}
\frac{\partial(\varphi_t^D - \varphi_t^S)}{\partial t} &= \int \left(\frac{V_\phi}{tV_{\phi\phi}}\right)\left(\frac{V_{\phi\phi}\hat{\phi}_t}{V_\phi} - \frac{1}{t} - \frac{V_{\phi\phi\phi}\hat{\phi}_t}{V_{\phi\phi}}\right) + \left(\frac{\hat{\phi}_t}{t} + \frac{1-\hat{\phi}}{t^2}\right) dG \\
&= \int \left[\left(\frac{\hat{\phi}_t}{t}\frac{V_\phi}{V_{\phi\phi}}\right)\left(\frac{V_{\phi\phi}}{V_\phi} - \frac{1}{t\hat{\phi}_t} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} + \frac{V_{\phi\phi}}{V_\phi}\right) + \frac{1-\hat{\phi}}{t^2}\right] dG \\
&= \int \left[-\left(\frac{\hat{\phi}_t}{t}\frac{V_\phi}{V_{\phi\phi}}\right)\left(\frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi}}{V_\phi}\right) + \frac{1-\hat{\phi}}{t^2}\right] dG > 0,
\end{aligned}$$

where I used observation ($\bigstar$)-(ii) for the last equality. Now, the inequality holds since $\phi_t, V_\phi, V_{\phi\phi} \geq 0$, and $V_\phi$ is log-concave in $\phi$ (and so $V_{\phi\phi\phi}/V_{\phi\phi} \leq V_{\phi\phi}/V_\phi$) and $\hat{\phi} < 1$.

Finally, because the shift-difference is negative for small stakes (Step 1), is positive for high stakes (Step 2), and the shift-difference is monotone (Step 3), there exists a unique intermediate stake such that the attempted crime rate rises if and only if stakes are below this critical vale. In other words, the attempted crime rate $\alpha^*$ is hump-shaped in $t$. □

**Claim A.4.** *If demand $\mathcal{D}$ is sufficiently elastic for some stake $t$, then the equilibrium crime rate $\kappa^*$ initially rises and eventually falls as the stake rises.*

*Proof:* First, recall that if $\mathcal{D}$ is sufficiently elastic for high $t$, then $\alpha^*$ is decreasing in $t$ by Claim A.2. Next, since the failure rate $\varphi^*$ is increasing in $t$ (by Claim A.1), the success rate $1-\varphi^*$ is decreasing in $t$. So, for high enough stakes, the crime rate $\kappa^* = \alpha^*(1-\varphi^*)$ is falls in $t$, since it is the product of two decreasing functions. Now, for small stakes, $\alpha^* = \varphi^* = 0$, and so $\kappa^* = 0$. Finally, since $\kappa^*$ is strictly positive otherwise, $\kappa^*$ must initially rise. □

## A.11 Depicting Potential Victims' Welfare Losses

Use the FOC (3) and that $\hat{\phi}(0|\xi) = 0 = V_\phi(0|\xi)$ to get:

$$\begin{aligned} v^* &= \int_0^1 V(\hat{\phi}(\alpha^*|\xi)|\xi)dG(\xi) = \int_0^1 \left( \int_0^{\alpha^*} V_\phi(\hat{\phi}(\alpha|\xi)|\xi)\hat{\phi}_\alpha(\alpha|\xi)d\alpha \right) dG(\xi) \\ &= \int_0^1 \int_0^{\alpha^*} \alpha M \hat{\phi}_\alpha(\alpha|\xi) d\alpha dG(\xi). \end{aligned}$$

Now integrate by parts to get: $\int_0^{\alpha^*} \alpha\hat{\phi}_\alpha(\alpha|\xi)d\alpha = \alpha^*\hat{\phi}(\alpha^*|\xi) - \int_0^{\alpha^*} \hat{\phi}(\alpha|\xi)d\alpha$. Therefore,

$$\begin{aligned} v^*/M &= \int_0^1 \int_0^{\alpha^*} \alpha\hat{\phi}_\alpha(\alpha|\xi)d\alpha dG(\xi) = \int_0^1 \alpha^*\hat{\phi}(\alpha^*|\xi)dG(\xi) - \int_0^1 \int_0^{\alpha^*} \hat{\phi}(\alpha|\xi)d\alpha dG(\xi) \\ &= \int_0^1 \alpha^*\hat{\phi}(\alpha^*|\xi)dG(\xi) - \int_0^{\alpha^*} \int_0^1 \hat{\phi}(\alpha|\xi)dG(\xi)d\alpha = \alpha^*\varphi^* - \int_0^{\alpha^*} \mathcal{D}(\alpha)d\alpha \\ &= \int_0^{\alpha^*} [\varphi^* - \mathcal{D}(\alpha)]d\alpha = \int_0^{\alpha^* M} [\varphi^* - \mathcal{D}(\tilde{\alpha}/M)]d\tilde{\alpha}/M, \end{aligned}$$

where I have changed variables so that $\tilde{\alpha} = \alpha M$, and used Fubini's Theorem to exchange the double integral. Altogether, $v^* = \int_0^{\alpha^* M}[\varphi^* - \mathcal{D}(\alpha)]d(\alpha M)$, namely, *vigilance expenditures* $v^*$ *equals the area below equilibrium failure rate* $\varphi^*$ *and above the demand* $\mathcal{D}(\alpha)$. □

## A.12 The Victim-Optimal Levels of Crime and Vigilance

First, notice that the optimal failure rate $\varphi^o$ must be positive, since raising $\varphi$ from zero is costless at the margin, as $V_\phi(0|\xi) = 0$ for all $\xi$ and $\mathcal{MS}(0) < 0$. Also, $\varphi^o < 1$, since a marginal drop of $\varphi$ from $\varphi = 1$ has no marginal impact on the crime rate, i.e, $\mathcal{MS}(1) = 0$. Thus, the optimal failure rate is positive and imperfect, i.e., $\varphi^o \in (0,1)$, and so it solves the FOC:

$$\mathcal{S}(\varphi^0)M - \mathcal{S}'(\varphi^o)(1-\varphi^o)M = \int_0^1 V_\phi(\hat{\phi}^o(\varphi^o|\xi)|\xi)\hat{\phi}^I_\varphi(\varphi^o|\xi)dG(\xi).$$

Next, by the definition of $\hat{\phi}^o$, $\int_0^1 \hat{\phi}^o(\varphi|\xi)dG(\xi) = \varphi$ and so $\int_0^1 \hat{\phi}^0_\varphi(\varphi|\xi)dG(\xi) = 1$. Also, $V_\phi(\hat{\phi}^o(\varphi|\xi)|\xi) = \mathcal{D}^{-1}(\varphi)M$, again by construction of $\hat{\phi}^o$. Thus, the above FOC simplifies to (8). The attempted crime rate is the mass of potential criminals that attempt a crime when $\varphi = \varphi^o$, namely, $\mathcal{S}(\varphi^o)$, and the individual failure rates obey $\hat{\phi}^o(\varphi^o|\xi)$ for all $\xi$.

Finally, since $\mathcal{E}_{1-\varphi}(f) \geq -2$, the crime rate $\mathcal{S}(\varphi)(1-\varphi)$ is convex in $\varphi$, and so is the program (7), as $\mathcal{D}^{-1}(\varphi)$ is increasing in $\varphi$. So, $(\varphi^o, \alpha^o, \hat{\phi}^o)$ solves the planner's problem. □

## A.13 The Inefficiency Gap: Proof of Proposition 6 Finished

Optimal Vigilance. Differentiate the objective function (7) in $\ell$, using (5), to get: $\mathcal{S}_\ell(\varphi)(1-\varphi)M = -f[(1-\varphi)m-\ell](1-\varphi)M$. Next, differentiate this latter expression in $\varphi$ to obtain,

$$f'[(1-\varphi)m-\ell]m(1-\varphi)M + f[(1-\varphi)m-\ell]M = f(\bar{c}(\varphi))(\mathcal{E}_{1-\varphi}(f)+1)M.$$

So the objective function is submodular in $(\varphi,\ell)$ iff $\mathcal{E}_{1-\varphi}(f(\bar{c})) \leq -1$. Thus, by Topkis (1998), the minimizer $\varphi^o$ rises as $\ell$ rises iff $\mathcal{E}_{1-\varphi}(f(\bar{c})) \leq -1$.

Next, notice that (i) if $[(c+\ell)f(c)]' \leq 0$, then $(c+\ell)f'(c)/f(c) \leq -1$ and so at $c=\bar{c}$:

$$\mathcal{E}_{1-\varphi}(f(\bar{c})) = \frac{f'[(1-\varphi)m-\ell](1-\varphi)m}{f[(1-\varphi)m]} = \frac{f'(\bar{c})(\bar{c}+\ell)}{f(\bar{c})} \leq -1,$$

where I used the definition of $\bar{c}(\varphi)$ below (2). Thus, the minimizer $\varphi^o$ rises as $\ell$ rises. Finally, following the same logic, (ii) if $[(c+\ell)f(c)]' \geq 0$, then $(c+\ell)f'(c)/f(c) \geq -1$ and so at $c=\bar{c}$:

$$\mathcal{E}_{1-\varphi}(f(\bar{c})) = \frac{f'[(1-\varphi)m-\ell](1-\varphi)m}{f[(1-\varphi)m]} = \frac{f'(\bar{c})(\bar{c}+\ell)}{f(\bar{c})} \geq -1.$$

Therefore, $\varphi^o$ falls as $\ell$ rises, by the previous arguments.

Inefficiency Gap. Now, I address the claims regarding the inefficiency gap. Part (i) is proved in the main text.

Part (ii): Differentiate the right side of (9) to get:

$$\frac{\partial \mathcal{E}_{1-\varphi}(\mathcal{S})}{\partial \ell} = \underbrace{\frac{\partial}{\partial \varphi}\left(\frac{f(1-\varphi^o)m}{F}\right)\frac{\partial \varphi^o}{\partial \ell}}_{(\clubsuit)} + \underbrace{\frac{\partial}{\partial \ell}\left(\frac{f(1-\varphi^o)m}{F}\right)}_{(\spadesuit)}.$$

Next, I note that $(\clubsuit)$ is positive. Indeed, after some algebra:

$$\frac{\partial}{\partial \varphi}\left(\frac{f(1-\varphi^o)m}{F}\right) = -\mathcal{E}_{1-\varphi}(\mathcal{S})m\left(\frac{f'}{f} - \frac{f}{F} + \frac{1}{(1-\varphi^o)m}\right) < 0,$$

where the inequality follows as $-\mathcal{E}_{1-\varphi}(\mathcal{S})m < 0$ and

$$\frac{f'}{f} - \frac{f}{F} + \frac{1}{(1-\varphi^o)m} > \frac{f'}{f} - \frac{f}{F} + \frac{1}{(1-\varphi^o)m-\ell} \geq 0.$$

The last inequality holds since $(f/F)c$ is monotone in $c$. Thus, $(\clubsuit) > 0$, since the $\partial\varphi^o/\partial\ell < 0$

as previously argued. Next, (♠) is positive, since $(f/F)$ is decreasing in $c$ and so increasing in $\ell$. Altogether, the relative excess of demand rises as punishment rises, given (9). $\square$

# B Minimizing the Social Costs of Crime

Suppose the planner's goal is to minimize the social costs of crime (6), and assume that potential victims value their property more than criminals do, i.e., $m \leq M$.[44] The planner can minimize social costs by choosing an average failure rate $\varphi^{sc} \in [0,1]$, individual demands $\hat{\phi}^o(\varphi^{sc}|\xi)$, and an attempted crime rate $\alpha^{sc} = \mathcal{S}(\varphi^{sc})$, where the average failure rate $\varphi^{sc}$ solves:

$$\min_{\varphi \in [0,1]} \mathcal{S}(\varphi)(1-\varphi)M + \int_0^1 V(\hat{\phi}^o(\varphi|\xi)|\xi)dG(\xi) - \int_0^{(1-\varphi)m-\ell} F(c)dc. \tag{20}$$

As argued in §7, the last term in (20) equals total criminal profits. To have a convex program, I assume that the density elasticity $\mathcal{E}_{1-\varphi}(f) \geq -(2 - m/M) \in (-2, -1]$ for $c = \bar{c}(\varphi)$. This condition holds if the cost distribution $F$ is either convex or not too concave.

Next, notice that the average failure rate $\varphi^{sc}$ must be interior. Indeed, differentiate the objective function (20) in $\varphi$ and evaluate it at $\varphi = 0$ and $\varphi = 1$. In the former case, the marginal returns of raising $\varphi$ from $\varphi = 0$ are negative and equal to $\mathcal{S}'(0)M - \mathcal{S}(0)(M-m) < 0$, as $\mathcal{S}'(0) < 0$ and $m \leq M$. In the latter case, the absolute marginal returns of lowering $\varphi$ from $\varphi = 1$ are positive and equal to $\mathcal{D}^{-1}(1) > 0$. Thus, $\varphi^{sc} \in (0,1)$ and is characterized by the FOC:

$$\mathcal{S}(\varphi^{sc}) - \mathcal{S}'(\varphi^{sc})(1-\varphi^{sc}) = \mathcal{D}^{-1}(\varphi^{sc}) + \mathcal{S}(\varphi^{sc})m/M. \tag{21}$$

Comparing (8) and (21), one can deduce that $\varphi^{sc} < \varphi^o$, namely, *the planner elects a lower failure rate compared to the level that minimizes potential victims' losses.* Thus,the attempted crime rate $\alpha^{sc} > \alpha^o$ and individual vigilance $\hat{\phi}^o(\varphi^{sc}|\xi) < \hat{\phi}^o(\varphi^o|\xi)$ for all $\xi$.

Could $\varphi^{sc}$ be lower than its equilibrium counterpart $\varphi^*$? Since the excess of supply $\mathcal{S}(\varphi) - \mathcal{D}^{-1}(\varphi)$ vanishes when $\varphi = \varphi^*$, the social cost minimizer $\varphi^{sc} < \varphi^*$ if and only if $\varphi^{sc}$ induces excess of supply, or $\mathcal{S}(\varphi^{sc}) - \mathcal{D}^{-1}(\varphi^{sc}) > 0$. By (21),

$$\mathcal{S}(\varphi^{sc}) - \mathcal{D}^{-1}(\varphi^{sc}) = \mathcal{S}(\varphi^{sc})m/M + \mathcal{S}'(\varphi^{sc})(1-\varphi^{sc}) = \mathcal{S}(\varphi^{sc})\left(\frac{m}{M} - \mathcal{E}_{1-\varphi}(\mathcal{S})\right).$$

Thus, $\varphi^{sc}$ induces excess of supply if, at the optimum, the supply elasticity with respect to the success rate is low enough, i.e., $\mathcal{E}_{1-\varphi}(\mathcal{S}) < m/M$. Conversely, $\varphi^{sc}$ induces excess of demand if this elasticity is high enough, $\mathcal{E}_{1-\varphi}(\mathcal{S}) > m/M$.

[44] Otherwise, if $m$ is too high the planner may want zero vigilance and thus raise criminal profits.

**Proposition B.0.1.** *(i) If $F$ is sufficiently concave, i.e. $F(c)^{\varrho}/(c+\ell)$ is decreasing in $c$, where $\varrho \equiv M/m$, then the social cost minimizer $\varphi^{sc} < \varphi^*$; (ii) If $F$ is convex, or not too concave, i.e. if $F(c)^{\varrho}/(c+\ell)$ is increasing in $c$, the social cost minimizer $\varphi^{sc} > \varphi^*$.*

*Proof:* For proving (i) and (ii), I exploit the fact that $\mathcal{E}_{1-\varphi}(\mathcal{S}) = (\bar{c}+\ell)f(\bar{c})/F(\bar{c})$, where $\bar{c} = (1-\varphi^{sc})m - \ell$. Part (i): Log-differentiate $F(c)^{\varrho}/(c+\ell)$ in $c$ to get, $\frac{\varrho f(c)}{F(c)} - \frac{1}{c+\ell} \leq 0$, where the inequality follows since $F(c)^{\varrho}/(c+\ell)$ falls in $c$. Next, rearrange this inequality to obtain $(c+\ell)f(c)/F(c) \leq (1/\varrho) = m/M$. Thus, $\mathcal{E}_{1-\varphi}(\mathcal{S}) \leq (m/M)$ and so $\varphi^{sc} < \varphi^*$.

Part (ii): If $F$ is convex, the secant $F(c)/c$ is increasing, and so $(c+\ell)f(c)/F(c) \geq cf(c)/F(c) \geq 1 > m/M$, implying the desired conclusion. □

# C The Observable Vigilance Case

**Claim C.1.** *The failure rate function $\tilde{\phi}$ is submodular in $(\phi, \xi)$, i.e., $\tilde{\phi}_{v\xi} \leq 0$.*

*Proof:* Log-differentiate $\tilde{\phi}_v(v,\xi)$, using (10), to get:

$$\frac{\tilde{\phi}_{v\xi}}{\tilde{\phi}_v} = -\left(\frac{V_{\phi\phi}}{V_\phi}\tilde{\phi}_\xi + \frac{V_{\phi\xi}}{V_\phi}\right) = -\left(\frac{V_{\phi\phi}}{V_\phi}\frac{-V_\xi}{V_\phi} + \frac{V_{\phi\xi}}{V_\phi}\right) = -\frac{V_\xi}{V_\phi}\left(-\frac{V_{\phi\phi}}{V_\phi} + \frac{V_{\phi\xi}}{V_\xi}\right).$$

Now, notice that the above parenthesized term is positive,

$$-\frac{V_{\phi\phi}}{V_\phi} + \frac{V_{\phi\xi}}{V_\xi} = \left(\frac{V_\phi}{V} - \frac{V_{\phi\phi}}{V_\phi}\right) + \left(\frac{V_{\phi\xi}}{V_\xi} - \frac{V_\phi}{V}\right) \geq 0,$$

as $V$ is log-concave in $\phi$ and log-supermodular in $(\phi, \xi)$. Thus, $\tilde{\phi}_{v\xi} \leq 0$, as $\tilde{\phi}_v, V_\xi, V_\phi > 0$. □

Before proving Proposition 7, slightly abuse notation and rewrite victims' losses (1) as:

$$\mathcal{L}(\xi, \hat{\xi}, v|\tilde{\alpha}) \equiv \tilde{\alpha}(v, \hat{\xi})(1 - \tilde{\phi}(v, \xi))M + v.$$

for $v \geq 0$ and $\xi, \hat{\xi} \in [0,1]$, and $\tilde{\alpha}(\cdot) \geq 0$ with $\tilde{\alpha}_v, \tilde{\alpha}_{\hat{\xi}} \neq 0$.

## C.1 Equilibrium Characterization: Proof of Proposition 7

Step 1: The failure rate $\tilde{\phi}(\chi(\xi), \xi)$ is constant for all $\xi \in [0,1]$. First, I show that there is no interval for which $\tilde{\phi}$ is strictly monotone. By contradiction, suppose that, given $\chi$, the failure rate function $\tilde{\phi}$ is strictly monotone for types $\xi \in [\xi', \xi'']$ with $\xi'' > \xi'$. WLOG assume $\tilde{\phi}$ is strictly increasing in this set. Then, if $\xi'$ attracts no criminals, then so does $\xi \in [\xi', \xi'']$; thus, all types would want to mimic the type that chose the lowest vigilance level. Now, if $\xi'$ does attract criminals, then any type $\xi \in (\xi', \xi'']$ does not (as $\tilde{\phi}$ is strictly increasing); therefore, any type $\xi \in (\xi', \xi'']$ is better off imitating a type that attracts no criminal and elects vigilance strictly less than $\chi(\xi)$.

Now, I show that $\tilde{\phi}(\chi(\xi), \xi)$ must be continuous in $[0,1]$. Suppose that $\tilde{\phi}$ is discontinuous, and call $\xi_1 \in [0,1]$ its first discontinuity point. As previously argued, $\tilde{\phi}$ must be constant for all $\xi \neq \xi_1$ in a neighborhood of $\xi_1$. WLOG suppose that $\tilde{\phi}(\chi(\xi_1), \xi_1) > \tilde{\phi}(\chi(\xi_0), \xi_0)$ for $\xi_0 \neq \xi_1$ in this neighborhood, and $\xi_0$ arbitrarily close to $\xi_1$. Now, since $\tilde{\phi}$ is continuous, $\chi$ must be discontinuous at $\xi_1$, with $\epsilon \equiv \chi(\xi_1) - \chi(\xi_0) > 0$. Clearly, the losses of $\xi_1$ are $\chi(\xi_0) + \epsilon$, whereas the losses for $\xi_0$ are $\tilde{\alpha}(\chi(\xi_0), \xi_0)(1 - \tilde{\phi}(\chi(\xi_0), \xi_0))M + \chi(\xi_0)$. So, if $\epsilon \geq \tilde{\alpha}(\chi(\xi_0), \xi_0)(1 - \tilde{\phi}(\chi(\xi_0), \xi_0))M$, $\xi_1$ is better off imitating $\xi_0$; otherwise, $\xi_0$ is better off imitating $\xi_1$.

Altogether, there exists a failure rate $\tilde{\varphi} \in (0,1)$ with $\chi(\xi) \equiv V(\tilde{\varphi}|\xi) > 0$ with $\tilde{\phi}(\chi(\xi), \xi) = \tilde{\varphi}$ for all $\xi \in [0,1]$.[45] As a byproduct, vigilance $\chi(\cdot)$ is strictly increasing in $\xi$, as $V_\xi(\tilde{\varphi}|\cdot) > 0$.

Step 2: The attempted crime rate function $\tilde{\alpha}$ solves (12) and the ODE (11). First, the ODE. In a separating equilibrium, incentive compatibility (13) requires $\mathcal{L}(\xi, \xi, \chi(\xi)|\tilde{\alpha}) \leq \mathcal{L}(\xi, \chi^{-1}(v), v|\tilde{\alpha})$ for all types $\xi \in [0,1]$, and vigilance $v \in \chi([0,1])$. But, since $\chi$ is one-to-one, for each $v \in \chi([0,1])$ there is a unique $\hat{\xi} = \chi^{-1}(v) \in [0,1]$, and so (13) can be rewritten as:

$$\xi \in \arg\min_{\hat{\xi} \in [0,1]} \tilde{\alpha}(\chi(\hat{\xi}), \hat{\xi})(1 - \tilde{\phi}(\chi(\hat{\xi}), \xi))M + \chi(\hat{\xi}),$$

for all $\xi \in [0,1]$. Thus, the first-order condition FOC, evaluated at $\hat{\xi} = \xi$, must hold:

$$\left[\tilde{\alpha}_v(\chi(\xi), \xi) + \tilde{\alpha}_{\hat{\xi}}(\chi(\xi), \xi)\frac{1}{\chi'(\xi)}\right](1 - \phi(\chi(\xi), \xi))M - \tilde{\alpha}(\chi(\xi), \xi)\phi_v(\chi(\xi), \xi)M + 1 = 0.$$

Now, use (10) to substitute $\phi_v$, and multiply both sides of the above first-order condition by $V_\phi(\tilde{\varphi}|\xi)$ to get (11), where $\chi(\xi) = V(\tilde{\varphi}|\xi)$. Finally, in equilibrium, $\tilde{\alpha}$ and $\tilde{\varphi}$ must be consistent with criminal optimization, namely, the market clearing condition (12) must hold.

Step 3: The attempted crime rate function $\tilde{\alpha}$ is strictly decreasing. First, since $\hat{\xi} = \xi$ minimizes expected losses $\mathcal{L}(\xi, \hat{\xi}, \chi(\hat{\xi})|\tilde{\alpha})$ over $\hat{\xi} \in [0,1]$, the following second-order condition, evaluated at $\hat{\xi} = \xi$, must hold:

$$(\clubsuit) = \mathcal{L}_{\hat{\xi}\hat{\xi}}(\xi, \xi, \chi(\xi)) + 2\mathcal{L}_{v\hat{\xi}}(\xi, \xi, \chi(\xi))\chi'(\xi) + \mathcal{L}_{\hat{\xi}}(\xi, \xi, \chi(\xi))\chi''(\xi) + \mathcal{L}_{vv}(\xi, \xi, \chi(\xi))[\chi'(\xi)]^2 \geq 0$$

Also, the FOC holds for all $\xi \in [0,1]$, and so $\mathcal{L}_{\hat{\xi}}(\xi, \xi, \chi(\xi)) + \mathcal{L}_v(\xi, \xi, \chi(\xi))\chi'(\xi) \equiv 0$. Differ-

[45] If $\tilde{\varphi} = 0$ then $\chi(\xi) = 0$ for all $\xi$, and so a separating equilibrium cannot be sustain. Likewise, if $\tilde{\varphi} = 1$, all victims face no property losses; thus, each victim has incentives to deviate from $\chi(\cdot) = V(1|\cdot)$.

entiating the above identity with respect to $\xi$ yields:

$$(\clubsuit) + \mathcal{L}_{\hat{\xi}\hat{\xi}}(\xi, \xi, \chi(\xi)) + \mathcal{L}_{v\xi}(\xi, \xi, \chi(\xi))\chi'(\xi) = 0.$$

Therefore, since $(\clubsuit) \geq 0$ by the second-order conditions,

$$\begin{aligned} 0 \ \geq \ & \mathcal{L}_{\hat{\xi}\hat{\xi}}(\xi, \xi, \chi(\xi)) + \mathcal{L}_{v\xi}(\xi, \xi, \chi(\xi))\chi'(\xi) \\ & = -[\tilde{\alpha}_{\hat{\xi}}(\chi(\xi), \xi)\tilde{\phi}_{\xi}(\chi(\xi), \xi) + \tilde{\alpha}_v(\chi(\xi), \xi)\phi_{\xi}(\chi(\xi), \xi)\chi'(\xi) + \tilde{\alpha}(\chi(\xi), \xi)\phi_{v\xi}(\chi(\xi), \xi)\chi'(\xi)]. \end{aligned} \tag{22}$$

But, since $\tilde{\phi}_{v\xi} < 0 < \chi'$, the immediately above inequality implies:

$$\tilde{\alpha}_{\hat{\xi}}(\chi(\xi), \xi)\tilde{\phi}_{\xi}(\chi(\xi), \xi) + \tilde{\alpha}_v(\chi(\xi), \xi)\phi_{\xi}(\chi(\xi), \xi)\chi'(\xi) > 0.$$

Finally, because $\tilde{\phi}_{\xi} < 0$, the immediately above inequality reduces to:

$$\frac{d\tilde{\alpha}(\chi(\xi), \xi)}{d\xi} = \tilde{\alpha}_v(\chi(\xi), \xi)\chi'(\xi) + \tilde{\alpha}_{\hat{\xi}}(\chi(\xi), \xi) < 0.$$

That is, $\tilde{\alpha}(\chi(\xi), \xi)$ must be strictly decreasing in $\xi$.

Step 4: The attempted crime rate function $\tilde{\alpha}$ vanishes at $\xi = 1$. By contradiction, suppose $\tilde{\alpha}(\chi(1), 1) > 0$. If type $\xi = 1$ deviates and chooses $v^d = \chi(1) + \epsilon$, with $0 < \epsilon < \tilde{\alpha}(\chi(1), 1)(1 - \tilde{\varphi})M$, then potential criminals, after observing $v^d$, would expect an associated failure rate that is higher than the equilibrium one, since for any inference $\hat{\xi}$:

$$\tilde{\phi}(v^d, \hat{\xi}) \geq \tilde{\phi}(v^d, 1) = \tilde{\phi}(\chi(1) + \epsilon, 1) > \tilde{\phi}(\chi(1), 1) = \tilde{\varphi}.$$

Thus, no criminal would target potential victims choosing $v^d$. This deviation is also profitable, as $\mathcal{L}(1, 1, \chi(1)|\tilde{\alpha}) > \chi(1) + \epsilon$. Thus, in equilibrium, $\tilde{\alpha}(\chi(1), 1) = 0$. □

## C.2 The Envelope Formula and Sufficient Conditions

**Claim C.2.** *Consider an equilibrium $(\chi(\cdot), \tilde{\alpha}(\cdot))$. Then, potential victims' equilibrium losses $\mathcal{L}(\xi, \xi, \chi(\xi)|\tilde{\alpha})$ are strictly increasing in type $\xi$ and obey the envelope formula:*

$$\mathcal{L}(\xi, \xi, \chi(\xi)|\tilde{\alpha}) = \chi(1) + \int_{\xi}^{1} \tilde{\alpha}(\chi(t), t)\tilde{\phi}_{\xi}(\chi(t), t)M dt. \tag{23}$$

*Proof:* First, since $\tilde{\phi}(v, \cdot)$ is differentiable for all $v > 0$, it follows that $\mathcal{L}_{\xi}(\xi, \hat{\xi}, \chi(\hat{\xi})) = -\tilde{\alpha}(\chi(\hat{\xi}), \hat{\xi})\tilde{\phi}_{\xi}(\chi(\hat{\xi}), \xi)M < -\tilde{\alpha}(\chi(0), 0)\tilde{\phi}_{\xi}(\chi(1), \xi)M$, where the inequality holds as $\tilde{\alpha}$ is strictly decreasing (Proposition 7), $\tilde{\phi}_\xi(v,\xi)$ strictly decreasing in $v$ (Claim C.1), and $\chi' > 0$ (Proposition 7). Next, $\int_0^1 -\tilde{\alpha}(\chi(0),0)\tilde{\phi}_\xi(\chi(1),\xi)d\xi = \tilde{\alpha}(\chi(0),0)[\tilde{\phi}(\chi(1),0) - \tilde{\phi}(\chi(1),1)] < \infty$. Finally, the envelope formula (23) holds, by Theorem 2 in Milgrom and Segal (2002). $\square$

**Proposition C.2.1.** *Consider a pair $(\chi,\tilde{\alpha})$, satisfying the necessary conditions of Proposition 7. If, in addition, $(\chi,\tilde{\alpha})$ obeys formula (23), the candidate pair $(\chi,\tilde{\alpha})$ is an equilibrium.*

*Proof:* It is enough to show that, given $(\chi,\tilde{\alpha})$, potential victims have no incentives to deviate on the equilibrium path. Pick arbitrary $\xi,\hat{\xi} \in [0,1]$. Using formula (23):

$$\begin{aligned}
\mathcal{L}(\xi,\hat{\xi},\chi(\hat{\xi})|\tilde{\alpha}) &= \mathcal{L}(\hat{\xi},\hat{\xi},\chi(\hat{\xi})|\tilde{\alpha}) + \tilde{\alpha}(\chi(\hat{\xi}),\hat{\xi})[\tilde{\phi}(\chi(\hat{\xi}),\hat{\xi}) - \tilde{\phi}(\chi(\hat{\xi}),\xi)]M \\
&= \mathcal{L}(\xi,\xi,\chi(\xi)|\tilde{\alpha}) - \int_\xi^{\hat{\xi}} \tilde{\alpha}(\chi(t),t)\tilde{\phi}_\xi(\chi(t),t)Mdt + \tilde{\alpha}(\chi(\hat{\xi}),\hat{\xi})[\tilde{\phi}(\chi(\hat{\xi}),\hat{\xi}) - \tilde{\phi}(\chi(\hat{\xi}),\xi)]M \\
&= \mathcal{L}(\xi,\xi,\chi(\xi)|\tilde{\alpha}) \underbrace{- \int_\xi^{\hat{\xi}} \tilde{\alpha}(\chi(t),t)\tilde{\phi}_\xi(\chi(t),t)Mdt + \tilde{\alpha}(\chi(\hat{\xi}),\hat{\xi}) \int_\xi^{\hat{\xi}} \tilde{\phi}_\xi(\chi(\hat{\xi}),t)Mdt}_{(\bigstar)}
\end{aligned}$$

Notice that $(\bigstar) \geq 0$ because the map $t \mapsto \tilde{\alpha}(\chi(t),t)\tilde{\phi}_\xi(\chi(t),t')$ is increasing for all $t' \in [0,1]$, by inequality (22). Thus, $\mathcal{L}(\xi,\xi,\chi(\xi)|\tilde{\alpha}) \leq \mathcal{L}(\xi,\hat{\xi},\chi(\hat{\xi})|\tilde{\alpha})$, and so $\chi$ is incentive compatible given $\tilde{\alpha}$, i.e., condition (13) holds. $\square$

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