



## MA04

### *Dynamical Systems: Theory and Applications*

### Sistemas Dinámicos: Teoría y Aplicaciones

#### Organizers

#### Organizadores

#### Antolatzaileak

**Érika Diz Pita**

(Universidad de Santiago de Compostela)

**Lucía Pérez Pérez**

(Universidad de Oviedo)

**Salvador Borrós Cullell**

(Universitat Autònoma de Barcelona)

**Sebastián Buedo Fernández**

(Universidad De Santiago de Compostela)

#### Description

#### Descripción

#### Deskribapena

*This session will focus on the study of different aspects of dynamical systems, both in their discrete and continuous aspects, and from both a theoretical and applied point of view.*

*The topics covered will include global stability, attractors, chaos, bifurcations, classification problems and modeling of natural phenomena.*

*The session will provide a broad meeting environment for researchers in this area of mathematics, together with the related parallel session MA03 Nonlinear Dynamics and Applications.*

Esta sesión tratará sobre el estudio de diferentes aspectos de los sistemas dinámicos, tanto en su vertiente discreta como continua, y tanto desde un punto de vista teórico como aplicado.

Entre las temáticas que se tratarán se incluyen cuestiones sobre estabilidad global, atractores, caos, bifurcaciones, problemas de clasificación y modelización de fenómenos naturales.

La sesión proporcionará un entorno amplio de encuentro a investigadores/as de esta área de las matemáticas, junto con la sesión paralela afín MA03 Dinámica no lineal y aplicaciones.

MSC Codes	Códigos MSC	MSC Kodeak
	37-XX (primary)	

Slots	Bloques	Blokeak
	1.A (Aula 0.27); 1.B (Aula 0.27); 1.C (Aula 0.27)	

QR Code	Código QR	QR Kodea
		

Session Schedule	Horario de la Sesión	Saioaren Ordutegia
L13   17:30-17:50   0.27 <i>A handy Jacobian criterion for uniqueness of solution to systems of equations</i> <b>Daniel Cao Labora</b> (Universidade de Santiago de Compostela)		
L13   18:00-18:20   0.27 <i>Dynamics around a normally elliptic invariant curve in a 3D volume-preserving map</i> <b>Ainoa Murillo López</b> (Universitat de Barcelona)		
L13   18:30-18:50   0.27 <i>Study of maximal attractors of parabolic problems</i> <b>Juan García Fuentes</b> (Universidad de Sevilla)		
L13   19:00-19:20   0.27 <i>Unfolding of a unipotent fixed point</i> <b>Paula Álvarez</b> (Universidad de Oviedo)		

M14 | 15:30-15:50 | 0.27

*Nonautonomous saddle-node bifurcations in  $d$ -concave in measure equations with applications*

**Jesús Dueñas** (Universidad de Valladolid)

M14 | 16:00-16:20 | 0.27

*Conley-Zehnder index for the search of periodic solutions in planar Hamiltonian systems*

**Eduardo Muñoz Hernández** (Universidad Complutense de Madrid)

M14 | 16:30-16:50 | 0.27

*A taste of transcendental dynamics: boundaries of Fatou components*

**Anna Jové** (Universitat de Barcelona)

M14 | 17:30-17:50 | 0.27

*Dynamics of the Takagi function*

**Jesús Llorente** (Universidad Politécnica de Madrid)

M14 | 18:00-18:20 | 0.27

*Mean-Field Models of Neural Populations: The Role of Synaptic Dynamics*

**Ana Mayora-Cebollero** (Universidad de Zaragoza)

M14 | 18:30-18:50 | 0.27

*Looking into the asymptotic behaviour of non-smooth maps arising in populations models subject to combinations of constant catch and threshold harvesting strategies*

**Cristina Lois-Prados** (Universidade de Santiago de Compostela)

M14 | 19:00-19:20 | 0.27

*Chaoticity Analysis with Deep Learning: Theoretical and Real Data*

**Carmen Mayora-Cebollero** (Universidad de Zaragoza)

**Monday 13**  
**17:30-17:50**  
**[Room 0.27]**

**Lunes 13**  
**17:30-17:50**  
**[Aula 0.27]**

**Astelehena 13**  
**17:30-17:50**  
**[Gela 0.27]**

*A handy Jacobian criterion for uniqueness of solution to systems of equations*

**Daniel Cao Labora**

(Universidade de Santiago de Compostela)

We know that a real function defined on  $[a, b]$  such that  $f(a)f(b) < 0$  and whose derivative never vanishes has exactly one zero in  $[a, b]$ . Nevertheless, it is less known that similar results hold for higher dimensions. In this talk we will expose a new result in this direction, based on a combination of the Poincaré-Miranda theorem and a mix of some old and new arguments for the injectivity issue. Finally, we will apply the criterion to an example and we will discuss some applications.

Joint work with Sebastián Buedo Fernández.

**Monday 13**  
**18:00-18:20**  
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**Lunes 13**  
**18:00-18:20**  
**[Aula 0.27]**

**Astelehena 13**  
**18:00-18:20**  
**[Gela 0.27]**

*Dynamics around a normally elliptic invariant curve in a 3D volume-preserving map*

**Ainoa Murillo López**

(Universitat de Barcelona)

We investigate the dynamics near a normally elliptic invariant curve in a 3D volume-preserving map, reducing the map to a resonant Birkhoff normal form around the curve. This depends on the set of resonances between the tangent and normal frequencies to the curve. Single-resonances may destroy the curve, eventually leading to a chain of stability bubbles. When the elliptic curve persists, the normal dynamics become reducible, allowing the classification of 3D resonant structures around it.

Joint work with Arturo Vieiro.

**Monday 13**  
**18:30-18:50**  
**[Room 0.27]**

**Lunes 13**  
**18:30-18:50**  
**[Aula 0.27]**

**Astelehena 13**  
**18:30-18:50**  
**[Gela 0.27]**

*Study of maximal attractors of parabolic problems*

**Juan García Fuentes**

(Universidad de Sevilla)

Dissipative semigroups produce structures that are invariants and attracts every trajectory of the phase space, well known as global attractors, that are bounded. We are going to introduce the concept of an attractor that it is unbounded, the maximal attractor, and study their existence and properties, such as characterize it. Finally, we apply our result to a parabolic semilinear PDE, where the nonlinearity can be unbounded, as long as it grows linearly with a controlled growth constant.

Joint work with Matheus Bortolan, Juliana Fernandes and Piotr Kalita.

**Monday 13**  
**19:00-19:20**  
**[Room 0.27]**

**Lunes 13**  
**19:00-19:20**  
**[Aula 0.27]**

**Astelehena 13**  
**19:00-19:20**  
**[Gela 0.27]**

*Unfolding of a unipotent fixed point*

**Paula Álvarez**

(Universidad de Oviedo)

In this talk, we explore a two-parameter family of 3D diffeomorphisms related to a generic unfolding of a unipotent fixed point. We begin presenting a parameter subset with an interesting dynamical behaviour consisting of two saddle-focus fixed points with different unstable indices. After that, we examine the distances between the one-dimensional manifolds and explain how these dynamics may give rise to Tatjer's homoclinic tangencies.

**Tuesday 14**  
**15:30-15:50**  
**[Room 0.27]**

**Martes 14**  
**15:30-15:50**  
**[Aula 0.27]**

**Asteartea 14**  
**15:30-15:50**  
**[Gela 0.27]**

***Nonautonomous saddle-node bifurcations in  $d$ -concave in measure equations with applications***

**Jesús Dueñas**

(Universidad de Valladolid)

Nonautonomous saddle-node bifurcations have often been studied under the condition of concavity of the flow; in previous works, we explored them under  $d$ -concavity properties. This talk weakens that condition, identifying such bifurcations in equations with  $d$ -concavity properties in measure. The new framework allows equation coefficients to vary within large chaotic sets, in some way approaching a stochastic formulation. Applications in circuit theory and critical transitions are also presented.

Joint work with Carmen Núñez and Rafael Obaya.

[arXiv:2407.15515](https://arxiv.org/abs/2407.15515)

**Tuesday 14**  
**16:00-16:20**  
**[Room 0.27]**

**Martes 14**  
**16:00-16:20**  
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**Asteartea 14**  
**16:00-16:20**  
**[Gela 0.27]**

***Conley-Zehnder index for the search of periodic solutions in planar Hamiltonian systems***

**Eduardo Muñoz Hernández**

(Universidad Complutense de Madrid)

In this talk, we will start characterizing the Conley-Zehnder index in terms of the winding number of a linear planar periodic Hamiltonian system. This will allow us to apply the Poincaré-Birkhoff theorem in order to prove existence and multiplicity of periodic solutions in general nonlinear and nonautonomous Hamiltonian systems. Finally, some applications will be provided.

Joint work with Alberto Boscaggin.

[arXiv:2203.02998](https://arxiv.org/abs/2203.02998)

**Tuesday 14**  
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**Asteartea 14**  
**16:30-16:50**  
**[Gela 0.27]**

***A taste of transcendental dynamics: boundaries of Fatou components***

**Anna Jové**

(Universitat de Barcelona)

This talk is meant to be a friendly introduction to complex dynamics, starting from the iteration of  $z^2$ , and providing the definition of the basic concepts in the field: the Fatou set (stability) and the Julia set (chaos). The goal is to describe the dynamics on the boundaries of Fatou components (connected components of the Fatou set), from a measure-theoretical, symbolic and qualitative point of view, in the case when the iterated function is a transcendental entire function.

Joint work with Núria Fagella.

**Tuesday 14**  
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**Asteartea 14**  
**17:30-17:50**  
**[Gela 0.27]**

***Dynamics of the Takagi function***

**Jesús Llorente**

(Universidad Politécnica de Madrid)

The Takagi function is a classical example of a continuous nowhere differentiable function. It is defined as

$$T(x) = \sum_{n=0}^{\infty} \frac{\phi(2^n x)}{2^n}, \quad x \in [0, 1]$$

where  $\phi(x)$  denotes the distance from the point  $x$  to the nearest integer. In this talk, we will study the discrete dynamical system generated by the Takagi function, namely

$$x_{n+1} = T(x_n), \quad x_0 \in [0, 1].$$

Joint work with Zoltán Buczolich.

**Tuesday 14**  
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**Asteartea 14**  
**18:00-18:20**  
**[Gela 0.27]**

***Mean-Field Models of Neural Populations: The Role of Synaptic Dynamics***

**Ana Mayora-Cebollero**

(Universidad de Zaragoza)

The study of neural populations is of increasing interest. In the literature, there are two mean-field models representing the dynamics of heterogeneous all-to-all networks of QIF neurons with and without synaptic dynamics. In this presentation, we study the different dynamical changes observed when a parameter (linking both models and related with the synapsis) is varied, and we analyze the bifurcations underlying these changes.

Joint work with R. Barrio, J.A. Jover-Galtier, C. Mayora-Cebollero, S. Serrano and L. Pérez.

[doi:10.1103/PhysRevE.109.014301](https://doi.org/10.1103/PhysRevE.109.014301)

**Tuesday 14**  
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**[Aula 0.27]**

**Asteartea 14**  
**18:30-18:50**  
**[Gela 0.27]**

***Looking into the asymptotic behaviour of non-smooth maps arising in populations models subject to combinations of constant catch and threshold harvesting strategies***

**Cristina Lois-Prados**

(Universidade de Santiago de Compostela)

In the studies developed with E. Liz and F. M. Hilker, we considered discrete 1D population models subject to control rules that combine constant quota and threshold harvesting. These combination lead in a natural way to piecewise-smooth maps whose dynamics are challenging because multiple non-smooth bifurcations may appear. The main aim of this talk is to show how we have procced to determine the asymptotic dynamics of the models by studying the “geometry” of the associated non-smooth maps.

Joint work with E. Liz and F. M. Hilker.



**Tuesday 14****19:00-19:20****[Room 0.27]****Martes 14****19:00-19:20****[Aula 0.27]****Asteartea 14****19:00-19:20****[Gela 0.27]*****Chaoticity Analysis with Deep Learning: Theoretical and Real Data*****Carmen Mayora-Cebollero**

(Universidad de Zaragoza)

The chaoticity analysis of a dynamical system is usually performed with classical techniques as Lyapunov Exponents. Recently, Deep Learning (DL) has also been used to obtain such analysis. However, when working with real data, classical and DL techniques have drawbacks. In this presentation, we show how DL can be used to obtain the chaoticity analysis of theoretical data (3D analysis of Lorenz system), and we propose a DL chaoticity algorithm for the analysis of real data (frog heart dynamics).

Joint work with R. Barrio, F.H. Fenton, Á. Lozano, A. Mayora-Cebollero, A. Miguel, A. Ortega, S. Serrano and R. Vígara.

[doi:10.1063/5.0143876](https://doi.org/10.1063/5.0143876)