AL07

Singularity Theory

Teoría de Singularidades

Singularitateen Teoria

Organizers   **Organizadores**   **Antolatzaileak**

Irene Macías Tarrío

(Universidad de Barcelona)

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(Basque Center for Applied Mathematics)

Description   **Descripción**   **Deskribapena**

This session is composed of nine talks concerning different aspects of the study of singularities, both in characteristic zero and positive. Concretely, we will cover topics such as singularities of plane curves, Higgs bundles, singularities of mappings, Hilbert-Kunz theory and Floer theory, among others.

Esta sesión está compuesta por nueve charlas que tratarán diversos aspectos relacionados con el estudio de las singularidades, tanto en característica cero como en característica positiva. En concreto, se abarcarán temas como singularidades de curvas planas, fibrados de Higgs, singularidades de aplicaciones, teoría de Hilbert-Kunz y teorías Floer, entre otros.

MSC Codes   **Códigos MSC**   **MSC Kodeak**

14B05

(primary)

13D40; 57R45; 32S65; 14F10; 14C17; 14E18; 14D20

(secondary)

Slots	**Bloques**	**Blokeak**

2.A (Aula 0.6); 2.B (Aula 0.6); 2.C (Aula 0.6)

QR Code	**Código QR**	**QR Kodea**

Session Schedule	**Horario de la Sesión**	**Saioaren Ordutegia**

J16 | 11:00-11:20 | 0.6
Foliations with a given set of singular points
Elvira Pérez-Callejo (Universitat Jaume I)

J16 | 11:30-11:50 | 0.6
Weighted-homogeneity and substantiality
Ignacio Breva Ribes (Universitat de València)

J16 | 12:00-12:20 | 0.6
Vanishing homology of frontal map germs
Christian Muñoz-Cabello (Universitat de València)

J16 | 12:30-12:50 | 0.6
Good real pictures of complex maps
Roberto Giménez Conejero (Mid Sweden University)

J16 | 16:30-16:50 | 0.6
Arc spaces in algebraic geometry and Floer theories in symplectic topology. A mysterious relation
Javier de la Bodega (Alfréd Rényi Institute of Mathematics)

J16 | 17:00-17:20 | 0.6
New versions of Frobenius and integral closure of ideals
Kriti Goel (BCAM)

J16 | 17:30-17:50 | 0.6
Holomorphic structures and the Chern correspondence for bundles with non-reductive structure group.
Diego Ruiz-Cases (ICMAT-UCM)

V17 | 9:30-9:50 | 0.6
Line arrangements on smooth cubic surfaces
Juan Carlos Castro Rivera (Universidad de Zaragoza)

V17 | 10:00-10:20 | 0.6
Stratification of the moduli space of plane branches with a single characteristic exponent
María de Leyva Elola-Olaso (Universitat Politècnica de Catalunya)

V17 | 10:30-10:50 | 0.6
Stratification by the poles of the complex zeta function of μ-constant plane branch deformations
Roger Gómez-López (Universitat Politècnica de Catalunya)

Foliations with a given set of singular points
Elvira Pérez-Callejo
(Universitat Jaume I)

Let $C = \{p_i\}_{i=1}^{n}$ be the set of centers of a sequence of blowups. We pose the problem of finding a foliation with the points of C as singular points. We provide an answer in which the points are also dicritical for the foliation and the foliation is algebraically integrable. We apply this result to an approach to the bounded negativity conjecture.

Joint work with Carlos Galindo and Francisco Monserrat.

Weighted-homogeneity and substantiality
Ignacio Breva Ribes
(Universitat de València)

Characterization of weighted-homogeneity is clear for functions with isolated singularity in terms of equality of Milnor and Tjurina number, thanks to Saito. The generalization to map-germs is still an open problem, Mond's conjecture. In this talk we relate the property of being weighted-homogeneous to that of being substantial, by studying the vector fields tangent to the image (or discriminant) of its stable unfolding.

Joint work with Raúl Oset Sinha.

Vanishing homology of frontal map germs
Christian Muñoz-Cabello

(Universitat de València)

A holomorphic $f : U \subseteq \mathbb{C}^n \to \mathbb{C}^{n+1}$ is frontal if its image admits a smooth field of tangent hyperplanes at every point. In this talk, we explore the vanishing homology of frontal map germs, defining an analogue of the image Milnor number and giving formulas for its computation in corank 1. We will also discuss the Mond conjecture in the context of frontal map germs, outlining the cases where it is known to be true and postulating a Mond-type conjecture.

Joint work with Juan José Nuño-Ballesteros and Raúl Oset Sinha.

arXiv:2407.16635

Good real pictures of complex maps
Roberto Giménez Conejero

(Mid Sweden University)

For a holomorphic map germ $f_{\mathbb{C}}(\mathbb{C}^n, 0) \to (\mathbb{C}^p, 0)$, the problem we study is finding a real map germ $f_{\mathbb{R}} : (\mathbb{R}^n, 0) \to (\mathbb{R}^p, 0)$ such that its complexification is equivalent to $f_{\mathbb{C}}$ and all the topological data of $f_{\mathbb{C}}$ can be found in $f_{\mathbb{R}}$. More precisely, one wants to find that the topology of the a generic perturbation of $f_{\mathbb{C}}$ (equivalent of the Milnor fiber for maps) is realised as a real object.

Joint work with Ignacio Breva Ribes.

Arc spaces in algebraic geometry and Floer theories in symplectic topology. A mysterious relation
Javier de la Bodega
(Alfréd Rényi Institute of Mathematics)

Two invariants can be attached to a hypersurface singularity: contact loci, sets of arcs with a fixed intersection multiplicity, and the Milnor fibration. In 2019, Budur, Bobadilla et al. conjectured that the cohomology of contact loci equals the Floer homology of the iterates of a symplectic monodromy. In this talk, we will give an overview of the problem and explain how de Lorenzo Poza and the speaker solved the conjecture for plane curves. Also, we will present some related on-going projects.

Joint work with Eduardo de Lorenzo Poza.

arXiv:2308.00051
arXiv:2408.01533

New versions of Frobenius and integral closure of ideals
Kriti Goel
(BCAM)

We define new versions of integral and Frobenius closures of ideals which incorporate an auxiliary ideal and a real parameter. In the case of tight closure, similar generalizations exist due to N. Hara and K.I. Yoshida, as well as A. Vraciu. We study their basic properties and give computationally effective calculations of the new tight, Frobenius, and integral closures in the case of affine semigroup rings in terms of the convex geometry of exponent sets.

Joint work with Kyle Maddox and William D. Taylor.

Thursday 16	**Jueves 16**	**Osteguna 16**
17:30-17:50	**17:30-17:50**	**17:30-17:50**
[Room 0.6]	**[Aula 0.6]**	**[Gela 0.6]**

Holomorphic structures and the Chern correspondence for bundles with non-reductive structure group.
Diego Ruiz-Cases
(ICMAT-UCM)

The aim of this talk is to report on some ongoing work to prove a Hitchin-Kobayashi correspondence for principal bundles E with non-reductive algebraic structure group. A first step (the Chern correspondence) is to identify holomorphic structures on E with unitary connections on a reduction of the structure group to a Levi subgroup plus a section of an associated bundle. This generalizes the known cases of extensions and filtrations of vector bundles.

Joint work with Oscar García-Prada.

Friday 17	**Viernes 17**	**Ostirala 17**
9:30-9:50	**9:30-9:50**	**9:30-9:50**
[Room 0.6]	**[Aula 0.6]**	**[Gela 0.6]**

Line arrangements on smooth cubic surfaces
Juan Carlos Castro Rivera
(Universidad de Zaragoza)

It is well known that any smooth cubic surface S contains exactly 27 lines L and that they can be built blowing-up 6 general points in $\mathbb{P}^2$, so S \ L is homeomorphic to the complement of a plane curve C. We study the relationship between the stratification of the moduli space of (S, L) using the number and position of Eckardt points by means of the fundamental group $\pi_1(\mathbb{P}^2 \setminus C)$.

Stratification of the moduli space of plane branches with a single characteristic exponent
María de Leyva Elola-Olaso
(Universitat Politècnica de Catalunya)

We study the moduli space of plane branches with a single characteristic exponent through a stratification using the semimodule of values of the Jacobian ideal of the branch. We provide an algorithmic procedure to describe the strata and their dimensions. This stratification refines a previously known one based on the Zariski invariant studied by Peraire in 1998.

Stratification by the poles of the complex zeta function of μ-constant plane branch deformations
Roger Gómez-López
(Universitat Politècnica de Catalunya)

The complex zeta function is an analytic family of distributions which has a meromorphic extension to $\mathbb{C}$. Its poles are related with the roots of the Bernstein-Sato polynomial. We study the stratification by the poles of the complex zeta function of any μ-constant deformation of a plane branch. The results we obtain also enable the explicit computation of stratifications without relying on Gröbner bases.

Joint work with Guillem Blanco.