AM04

Partial Differential Equations I: Elliptic and Parabolic Equations

Ecuaciones en Derivadas Parciales I: Ecuaciones Elípticas y Parabólicas

Deribatu Partzialetako Ekuazioak I: Ekuazio Eliptiko eta Parabolikoak

Organizers   **Organizadores**   **Antolatzaileak**

Iñigo Urtiaga Erneta   **Clara Torres Latorre**

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Description   **Descripción**   **Deskribapena**

Elliptic and parabolic partial differential equations are general models used to explain phenomena in fields as diverse as physics, biology, and economics. Today, these equations still pose significant mathematical challenges, including issues related to the regularity and stability of solutions, the occurrence of finite-time blow-up versus global existence, connections with other areas such as geometry and probability, and the existence of special solutions, among other topics. In our session, we aim to share the most notable recent advances in elliptic and parabolic equations made by young Spanish researchers or those based in Spain.

Las Ecuaciones en Derivadas Parciales elípticas y parabólicas son modelos generales que sirven para explicar fenómenos provenientes de campos tan variados como la física, la biología o la economía. A día de hoy, estas ecuaciones presentan aún importantes retos matemáticos por resolver, relacionados con la regularidad y estabilidad de soluciones, la explosión en tiempo finito frente a la existencia global, la relación con otras áreas como la geometría y la probabilidad, o la existencia de soluciones especiales, entre otras cuestiones. En nuestra sesión pretendemos poner en común los avances recientes más

destacados sobre ecuaciones elípticas y parabólicas llevados a cabo por la juventud investigadora española o radicada en España.

Deribatu Partzialetako Ekuazio eliptiko eta parabolikoak fisika, biologia edo ekonomia bezalako arlo ezberdinetako fenomenoak azaltzeko erabiltzen diren eredu matematiko orokorrak dira. Gaur egun, erronka matematiko esanguratsuak planteatzen dituzte ekuazio horiek horaindik, hala nola, hurrengo gaiekin lotutakoak: soluzioen erregulartasun eta egonkortasuna, leherketa denbora finituan soluzioen existentzia globala aurka, geometria eta probabilitatearekin harremana edo soluzio berezien existentzia, besteak beste. Gure sesioan, Espainako edo Espainian oinarritutato ikertzaile gazteek ekuazio eliptiko eta parabolikoei buruz lortutako azken aurrerapen nabarmenak partekatu nahi ditugu.

MSC Codes	**Códigos MSC**	**MSC Kodeak**

35J60; 35K55

(primary)

Slots	**Bloques**	**Blokeak**

2.A (Aula 0.8); 2.B (Aula 0.8); 2.C (Aula 0.8)

QR Code	**Código QR**	**QR Kodea**

Session Schedule	**Horario de la Sesión**	**Saioaren Ordutegia**

L16 | 11:00-11:20 | 0.8
Regularity of free interfaces in transmission problems arising from the jump of conductivity
María Soria-Carro (Rutgers University)

L16 | 11:30-11:50 | 0.8
Stable cones in the Alt-Phillips free boundary problem
Tomás Sanz-Perela (Universitat de Barcelona)

L16 | 12:00-12:20 | 0.8
An overdetermined Neumann problem with a nonlinearity
Jing Wu (Universidad Autónoma de Madrid)

L16 | 12:30-12:50 | 0.8
Local behaviour of high energy Laplace eigenfunctions of polygonal domains
Alba Dolores García Ruiz (ICMAT)

L16 | 16:30-16:50 | 0.8
Uniqueness of positive solutions to the nonlocal heat equation associated to certain Lévy operators
Irene Gonzálvez (Universidad Autónoma de Madrid)

L16 | 17:00-17:20 | 0.8
A quasilinear Keller-Segel model with saturated discontinuous advection
Mikel Ispizua (Euskal Herriko Unibertsitatea)

L16 | 17:30-17:50 | 0.8
Boundary regularity for nonlocal equations
Marvin Weidner (Universitat de Barcelona)

L16 | 18:00-18:20 | 0.8
Yau's conjecture for (non)local minimal surfaces
Enric Florit-Simon (ETH Zürich)

L17 | 9:00-9:20 | 0.8
Infinitely many positive solutions in a class of semilinear elliptic problems
Antonio J. Martínez Aparicio (Universidad de Almería)

L17 | 9:30-9:50 | 0.8
Symmetry breaking and rigidity results for semilinear elliptic equations
Camilla Chiara Polvara (Università degli Studi di Milano)

L17 | 10:00-10:20 | 0.8
Traveling waves for nonlinear Schrödinger equations
Laura Baldelli (IMAG & Universidad de Granada)

L17 | 10:30-10:50 | 0.8

Prescribing Gaussian and geodesic curvatures on surfaces with conical singularities and corners
Francisco J. Reyes-Sánchez (Universidad de Granada)

Regularity of free interfaces in transmission problems arising from the jump of conductivity
María Soria-Carro
(Rutgers University)

We introduce a parabolic free boundary problem motivated by the conductivity jump in composite materials undergoing a phase transition. Our goal is to establish the regularity of the free boundary at regular points following the classical strategy I.Flat implies C1,alpha II.C1,alpha implies smooth. We discuss the main ideas and techniques to achieve these results in our parabolic context, largely inspired by the seminal works of Kinderlehrer-Nirenberg-Spruck 1978 Caffarelli 1989 and DeSilva 2011

Joint work with Dennis Kriventsov.

Stable cones in the Alt-Phillips free boundary problem
Tomás Sanz-Perela
(Universitat de Barcelona)

In this talk I will describe a recent result, obtained in collaboration with Aram Karakhanyan, in which we obtain for the first time a stability condition for the Alt-Phillips free boundary problem. Then, I will discuss how do we use it to classify global stable axially-symmetric solutions in dimensions 3, 4, and 5.

Joint work with Aram Karakhanyan.

Thursday 16   **Jueves 16**   **Osteguna 16**
12:00-12:20   **12:00-12:20**   **12:00-12:20**
[Room 0.8]   **[Aula 0.8]**   **[Gela 0.8]**

An overdetermined Neumann problem with a nonlinearity
Jing Wu
(Universidad Autónoma de Madrid)

In this talk, we present the existence of a family of nontrivial compact subdomains in the manifold such that the overdetermined nonlinear problem with zero Neumann and constant Dirichlet boundary values admits sign-changing solutions. The proof uses a local bifurcation argument. This is a joint work with Ignace Aristide Minlend.

Thursday 16   **Jueves 16**   **Osteguna 16**
12:30-12:50   **12:30-12:50**   **12:30-12:50**
[Room 0.8]   **[Aula 0.8]**   **[Gela 0.8]**

Local behaviour of high energy Laplace eigenfunctions of polygonal domains
Alba Dolores García Ruiz
(ICMAT)

We consider the Laplace eigenvalue problem with boundary conditions. A blowup argument shows that the local behavior of a rescaled high energy eigenfunction is described by a solution to the Helmholtz equation. This is independent of the domain. A central question in spectral geometry is how the corresponding classical system (i.e. the dynamical billiard) affects this connection between eigenfunctions and monochromatic waves. We study this in the case of all polygons with integrable dynamics.

Uniqueness of positive solutions to the nonlocal heat equation associated to certain Lévy operators

Irene Gonzálvez

(Universidad Autónoma de Madrid)

We establish a representation formula for nonnegative classical solutions of a nonlocal heat equation where the diffusion operator is given by a positive symmetric Lévy kernel comparable to radial functions with mixed polynomial growths. To this aim we prove existence of initial trace, uniqueness and existence for nonnegative solutions.

A quasilinear Keller-Segel model with saturated discontinuous advection

Mikel Ispizua

(Euskal Herriko Unibertsitatea)

The Keller-Segel model is used to explain the formation and propagation of bacterial colonies, it describes how cell density moves towards a nutrient N and a chemical molecule S (chemoattractant) which is generated by the cells themselves. In this talk I will present some results for the singular limit of a chemotaxis model recently introduced in arXiv:2009.11048.

Joint work with Maria Gualdani and Nicola Zamponi.

Boundary regularity for nonlocal equations
Marvin Weidner
(Universitat de Barcelona)

Local and nonlocal problems are significantly different regarding the boundary behavior of their solutions. For instance, s-harmonic functions (fractional Laplacian nonlocal equations' solutions) are, in general, not better than C^s up to the boundary. In recent years, the huge interest in the boundary behavior of nonlocal equations' solutions has led to understanding the case of the fractional Laplacian. However, several questions remained open for more general nonlocal operators.

Joint work with Xavier Ros-Oton and Minhyun Kim.

Yau's conjecture for (non)local minimal surfaces
Enric Florit-Simon
(ETH Zürich)

In this talk we will explain a recent result on the existence of infinitely many nonlocal minimal surfaces, obtained as min-max critical points of a canonical definition of nonlocal perimeter, on any closed Riemannian manifold. We will moreover describe further directions regarding the convergence of nonlocal minimal surfaces to classical minimal surfaces, as well as a Weyl-type Law for these objects, (continues in Comments)

Joint work with Michele Caselli and Joaquim Serra.

Infinitely many positive solutions in a class of semilinear elliptic problems
Antonio J. Martínez Aparicio
(Universidad de Almería)

In this work, we study the behavior of the set of solutions of the semilinear elliptic problem

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded open subset of $\mathbb{R}^N$ and f is a nonnegative continuous real function with multiple positive zeros.

Symmetry breaking and rigidity results for semilinear elliptic equations
Camilla Chiara Polvara
(Università degli Studi di Milano)

We consider semilinear elliptic equations in spherical sectors with mixed boundary type conditions and in unbounded cones with Neumann boundary conditions. The aim of the talk is to show that a radial symmetry result of Gidas-Ni-Nirenberg type for positive solutions does not hold in general when the cone is nonconvex, while a rigidity result holds in some class of non convex cones.

Joint work with G. Ciraolo and F. Pacella.

Traveling waves for nonlinear Schrödinger equations
Laura Baldelli
(IMAG & Universidad de Granada)

Motivated by the fact that in the last years, much progress has been made to give rigorous proofs of the so-called Jones-Putterman-Roberts program, the purpose of the present talk is to analyse existence of finite energy traveling waves moving with subsonic speed for a class of nonlinear Schrödinger equations, including the Gross-Pitaevskii one, in the spirit of Berestycki-Lions, and provide a direct and simpler variational approach based on a new Sobolev-type inequality involving the momentum.

Joint work with Bartosz Bieganowski and Jaros aw Mederski.

arXiv:2406.03910

Prescribing Gaussian and geodesic curvatures on surfaces with conical singularities and corners
Francisco J. Reyes-Sánchez
(Universidad de Granada)

In this talk, we will explore the construction of conformal metrics on compact Riemannian surfaces with boundary, featuring conical singularities and corners, with prescribed Gaussian and geodesic curvatures. We will establish conditions for the existence of such metrics by studying a nonlinear elliptic PDE, taking into account the coexistence of both types of singularities.

Joint work with L. Battaglia.