AL05

Applied Nonlinear Algebra

Álgebra No Lineal Aplicada

Organizers **Organizadores** **Antolatzaileak**

Carles Checa
(University of Copenhagen)

Rodrigo Iglesias
(Universidad de la Rioja)

Pablo Mazón
(Università di Trento)

Elvira Pérez-Callejo
(Universitat Jaume I)

Description **Descripción** **Deskribapena**

Systems of polynomial equations model problems appearing in a wide range of applications. Understanding the algebraic structure and the geometry of their solution set is a widely known mathematical challenge, which can be addressed from the point of view of commutative algebra, algebraic geometry, combinatorics, tropical geometry, and other related areas. All these trends are captured under the term ``nonlinear algebra''.
The list of invited speakers includes 12 international young researchers, pre and postdoctoral, working in diverse areas lying on nonlinear algebra. We hope that this session brings positive synergies to the community, and helps researchers put in common different insights and perspectives in this field.

Los sistemas de ecuaciones polinomiales modelan problemas que aparecen en una variedad de aplicaciones. Comprender la estructura algebraica y la geometría de sus soluciones es un reto matemático ampliamente reconocido, que puede abordarse desde los puntos de vista del álgebra conmutativa, la geometría algebraica, la combinatoria, la geometría tropical y otras áreas relacionadas. Todas éstas puede enmarcarse como ``álgebra no lineal''.
La lista de ponentes invitados incluye 12 jóvenes investigadores internacionales, pre y postdoctorales, que trabajan en diversos campos dentro del álgebra no lineal. Esperamos que esta sesión aporte sinergias positivas a la comunidad y ayude a los investigadores a poner en común ideas y perspectivas.

MSC Codes **Códigos MSC** **MSC Kodeak**

13P10

(primary)

13P15; 13P20; 13P25; 14Q05; 14Q20; 14Q30; 14T20; 14T90; 68W30

(secondary)

Slots **Bloques** **Blokeak**

2.A (Aula 0.5); 2.B (Aula 0.5); 2.C (Aula 0.5)

QR Code **Código QR** **QR Kodea**

Session Schedule **Horario de la Sesión** **Saioaren Ordutegia**

J16 | 11:00-11:20 | 0.5
A BBD-engine for computations on monomial ideals
Laura Moreno Resa (Universidad de La Rioja)

J16 | 11:30-11:50 | 0.5
Complexity measures of m-ary functions
Sara Asensio Ferrero (Universidad de Valladolid)

J16 | 12:00-12:20 | 0.5
Algebraic invariants associated to graph coloring
Raquel Melgar Fernández (Université de Bordeaux)

J16 | 16:30-16:50 | 0.5
On strong Euler-homogeneity for free divisors
Abraham del Valle (Universidad de Sevilla)

J16 | 17:00-17:20 | 0.5
Hadamard products and where to find them
Dario Antolini (Università di Trento)

J16 | 18:00-18:20 | 0.5
Lower and Upper Bounds for the Riesz Energy on the Grassmannian
Pedro R. López-Gómez (Universidad de Cantabria)

V17 | 9:00-9:20 | 0.5
Computational aspects of the short resolution
Mario González-Sánchez (Universidad de Valladolid)

V17 | 9:30-9:50 | 0.5
Cox rings of blow-ups of the projective space
Luis José Santana Sánchez (Universidad de La Laguna)

V17 | 10:00-10:20 | 0.5
Algebraic geometry meets game theory: Spohn CI varieties
Javier Sendra Arranz (CUNEF Universidad)

V17 | 10:30-10:50 | 0.5
Proudfoot-Speyer degenerations of scattering equations
Barbara Betti (MPI MiS Leipzig)

Thursday 16
11:00-11:20
[Room 0.5]

Jueves 16
11:00-11:20
[Aula 0.5]

Osteguna 16
11:00-11:20
[Gela 0.5]

A BBD-engine for computations on monomial ideals

Laura Moreno Resa
(Universidad de La Rioja)

Binary Decision Diagrams (BDDs) are efficient structures for solving logic-related problems. This talk covers BDD fundamentals and explores their use in fields like commutative algebra and reliability theory. We analyze the connection between different combinatorial objects, optimizing its computational efficiency. Practical applications of BDDs using C++ libraries CoCoALib and TeDDy show improvements for monomial ideal calculations, significantly enhancing performance in real-world problems.

Joint work with Eduardo Sáenz de Cabezón.

Thursday 16
11:30-11:50
[Room 0.5]

Jueves 16
11:30-11:50
[Aula 0.5]

Osteguna 16
11:30-11:50
[Gela 0.5]

Complexity measures of m-ary functions

Sara Asensio Ferrero
(Universidad de Valladolid)

The study of the relations between different complexity measures of boolean functions led Nisan and Szegedy to state the sensitivity conjecture in 1994. This problem was solved in 2019, when Huang proved the conjecture by means of an equivalent reformulation of the problem in graph theory. We wonder if the same type of results hold for functions defined on finite alphabets of cardinality m greater than two, which we call m-ary functions, and we extend most of the results for boolean functions.

Joint work with Ignacio García-Marco and Kolja Knauer.

Thursday 16
12:00-12:20
[Room 0.5]

Jueves 16
12:00-12:20
[Aula 0.5]

Osteguna 16
12:00-12:20
[Gela 0.5]

Algebraic invariants associated to graph coloring

Raquel Melgar Fernández
(Université de Bordeaux)

In 1995 Stanley defined $X_G(\mathbf{x})$ the chromatic symmetric function of a graph G, which specializes to $\chi_G(k)$, the polynomial counting proper colorings of G. In 2017 Shareshian, Wachs and Ellzey defined a refinement of this function for a directed graph that happens to be in the algebra of quasisymmetric functions that is of great interest in combinatorics. Our aim is to extend this work to signed graphs taking into account the hyperplane arrangement viewpoint of this problem.

Thursday 16
16:30-16:50
[Room 0.5]

Jueves 16
16:30-16:50
[Aula 0.5]

Osteguna 16
16:30-16:50
[Gela 0.5]

On strong Euler-homogeneity for free divisors

Abraham del Valle
(Universidad de Sevilla)

In 2002, it was conjectured that a free divisor satisfying the so-called Logarithmic Comparison Theorem must be strongly Euler-homogeneous and it was proved for the two-dimensional case. In 2006, it was shown that the conjecture is also true in dimension three, but, today, the answer for the general case remains unknown. In this talk I will give a new characterization of strong Euler-homogeneity that will allow us to deduce some partial answers to this problem.

Thursday 16	**Jueves 16**	**Osteguna 16**
17:00-17:20	**17:00-17:20**	**17:00-17:20**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

Hadamard products and where to find them

Dario Antolini
(Università di Trento)

The Hadamard product of two projective varieties X and Y is the Zariski closure of the set of all coordinate-wise products of an element in X and an element in Y, when this is defined. Although their definition is highly coordinate-dependent, they arise in different applications, such as the study of rigidity of plane graphs and of Restricted Boltzmann Machines. We focus on these applications and how they lead to interesting questions in Tropical Geometry and Matroid Theory.

Joint work with Nathaniel Vaduthala, Alessandro Oneto, and Guido Montúfar.

Thursday 16	**Jueves 16**	**Osteguna 16**
18:00-18:20	**18:00-18:20**	**18:00-18:20**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

Lower and Upper Bounds for the Riesz Energy on the Grassmannian

Pedro R. López-Gómez
(Universidad de Cantabria)

In recent years, the problem of minimizing certain pairwise interaction energies on compact Riemannian manifolds has attracted the interest of both the pure and applied mathematical community. In this talk, I will introduce the problem of minimizing the Riesz energy on the Grassmann manifold and I will present for the first time lower and upper bounds for the minimal Riesz energy on this space.

Joint work with Ujué Etayo.

Friday 17	**Viernes 17**	**Ostirala 17**
9:00-9:20	**9:00-9:20**	**9:00-9:20**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

Computational aspects of the short resolution

Mario González-Sánchez
(Universidad de Valladolid)

Let $I \subset R = k[x_1, \ldots, x_n]$ be a homogeneous ideal of height $n - d$, and assume that $A = k[x_{n-d+1}, \ldots, x_n]$ is a Noether normalization of R/I. The minimal graded free resolution of R/I as A-module is called its short resolution. In this talk we will show how to compute the short resolution using Gröbner bases. Moreover, we will discuss its advantages over the usual resolution when computing invariants such as the Hilbert series or the Castelnuovo-Mumford regularity of R/I.

Joint work with Ignacio García-Marco and Philippe Gimenez.

Friday 17	**Viernes 17**	**Ostirala 17**
9:30-9:50	**9:30-9:50**	**9:30-9:50**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

Cox rings of blow-ups of the projective space

Luis José Santana Sánchez
(Universidad de La Laguna)

In this talk we will discuss Cox rings of point blow-ups of the projective space. In particular, from the work of Castravet and Tevelev on the Cox ring of the projective space blown-up at points on a rational normal curve, we will see how to establish a stable base locus theorem for divisors on such a space.

Friday 17 | **Viernes 17** | **Ostirala 17**
10:00-10:20 | **10:00-10:20** | **10:00-10:20**
[Room 0.5] | **[Aula 0.5]** | **[Gela 0.5]**

Algebraic geometry meets game theory: Spohn CI varieties

Javier Sendra Arranz
(CUNEF Universidad)

In recent years, algebraic geometry has been used to study Nash and dependency equilibria, which model scenarios where players behave independently and collectively, respectively. The gap between these opposite notions is filled by the CI equilibria of a graph, where complicated dependencies among players are modeled by the graph. We analyze the geometry of CI equilibria through the algebro-geometric study of the Spohn CI variety. This work is based on a joint effort with Irem Portakal.

Joint work with Irem Portakal.

Friday 17 | **Viernes 17** | **Ostirala 17**
10:30-10:50 | **10:30-10:50** | **10:30-10:50**
[Room 0.5] | **[Aula 0.5]** | **[Gela 0.5]**

Proudfoot-Speyer degenerations of scattering equations

Barbara Betti
(MPI MiS Leipzig)

We study scattering equations of hyperplane arrangements with techniques of nonlinear algebra. We restate the problem as linear equations on a reciprocal linear space and solve it with a homotopy algorithm. This is based on the Gröbner degeneration of the coordinate ring into the Stanley-Reisner ring of the broken circuit complex. We study the regularity of the ideal defined by the equations and apply our methods to scattering equations.

Joint work with Viktoriia Borovik and Simon Telen.