

AA 474N/674N: Radio Astronomy Assignment 4

- For a pair of conjugate variables x and s , the Fourier Transform is given by:

$$F(s) \equiv \int_{-\infty}^{+\infty} f(x) e^{2\pi i s x} dx$$

This can be compactly written as $\mathbf{f}(\mathbf{x}) \leftrightarrow \mathbf{F}(\mathbf{s})$

- The convolution of two functions $f(x)$ and $g(x)$ is given by: $f(x)*g(x) = \int_0^x f(x')g(x-x')dx'$
1. Derive the similarity theorem $f(ax) \leftrightarrow \frac{F(s/a)}{|a|}$, where a is the scaling factor.
 2. If a function $f(x)$ is shifted along the x -axis to become $f(x-a)$, show that $f(x-a) \leftrightarrow e^{-2\pi i s a} F(s)$.
 3. Find the Fourier transforms of the following:
 - (a) $f(x) = \begin{cases} 0 & x < 0 \\ e^{-x} & x > 0 \end{cases}$
 - (b) $f(x) = e^{-a|x|}$ ($a > 0$)
 - (c) the unit step function
 - (d) $f(x) = \cos \omega_0 x$
 - (e) $f(x) = \delta(x)$
 - (f) Gaussian function of the form $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$
 4. Show that $f(x)\cos(2\pi\nu x) \leftrightarrow (1/2)[F(s-\nu)+F(s+\nu)]$. What is this property called?
 5. Find the convolution of $f(x) = e^{-x}$ and $g(x) = \sin(x)$.
 6. Show that $(f(x)*\delta(x)) = f(x)$.
 7. Consider the rect function $f(x) = \begin{cases} 1 & |x| < 1/2 \\ e^{-x} & |x| \geq 1/2 \end{cases}$
Find its Fourier transform and sketch a graph of the same.
 8. Sketch the graph of the function given by $f(x) = \begin{cases} 1-|x| & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$.
What is its Fourier Transform?
 9. Show that $\int_{-\infty}^x f(x')dx' = \frac{1}{is}F(s)+\pi F(0)\delta(s)$
 10. If we replace x by $(-x)$ in the definition of Fourier transform given in the beginning, what will be the corresponding Fourier transform in terms of $F(s)$?