CENTRE OF ASTRONOMY

INDIAN INSTITUTE OF TECHNOLOGY INDORE

Spring Semester		**Academic Year 2017-18**
End Semester Exam		**Course Code : AA 472/672**
Date: **26 April 2018**	Max. Time Duration : **3 Hr**	Max Marks : **50**
Number of Questions : 5		Number of pages in the Question Paper: 2

Instructions:

- Possession of Mobile Phone during the Exam is **strictly prohibited** and will be treated as *Use of Unfair Means*
- Exchange of question paper and examination related materials and talking to other students during the examination will also be treated as *Use of Unfair Means*
- All Questions are compulsory. The sub-questions within each question should be answered in order, however, the ordering of answering 5 main questions is not important.

Table 1: **Physical Constants in CGS units**

Light Speed	3.0×10^{10} cm s^{-1}	Mass of Sun	2.0×10^{33} g
Gravitational Constant	6.67×10^{-8}cm^3/(g.s^2)	Luminosity of Sun	3.83×10^{33} ergs s^{-1}
Boltzmann Constant	1.38×10^{-16} erg/K	Planck Constant	6.63×10^{-27} ergs s
Mass of electron	9.12×10^{-28} g	Charge of Electron	4.8×10^{-10} esu

1. *Active Galactic Nuclei* – **Total : 10 marks**

   (a) How do Active galaxies differ from normal galaxies ? What are the two main sub-classes of Active galaxies? State differences between these two sub-classes. **[3 marks]**

   (b) High resolution images of jets from AGNs often show *hotspots* or *knots* moving in time. Quantifying the speed of such blobs results in values much higher than speed of light. Derive the expression of such an apparent superluminal transverse motion of *knots*? **[4 marks]**

   (c) Find the angle at which this transverse apparent velocity will be maximum and estimate the minimum value of $\beta = v/c$ of the jet required to observe superluminal motion. **[3 marks]**

2. *Jeans Equations and Virial Theorem* – **Total : 10 marks**

   (a) Write down the *Jeans Equations* and explain in details the physical importance the term containing the velocity dispersion σ_{ij}^2 **[2 marks]**.

   (b) Starting from the set of momentum equations given below, derive the general expression for tensor virial theorem.

$$\frac{\partial(\rho < v_j >)}{\partial t} + \frac{\partial(\rho < v_i v_j >)}{\partial x_i} + \rho \frac{\partial \Phi}{\partial x_j} = 0$$

   Further, express the above derived expression for Virial theorem (a) if the system is in steady state and (b) in its scalar form. **[5 marks]**.

   (c) Using the scalar form of Virial theorem for a spherically symmetric molecular cloud show that the critical mass (*Jeans mass*) needed to initiate a collapse and form stars within it is given by

$$M_J = \left(\frac{5 k_B T}{G \mu m_H}\right)^{\frac{3}{2}} \left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

   [3 marks].

3. *Synchrotron Emission* – **Total : 10 marks**

(a) What are the main differences between thermal and non-thermal emission? Justify why synchrotron emission is considered to be a type of non-thermal emission. **[3 marks]**

(b) There exists a typical frequency,ν_s, associated to the synchrotron process. This is related to the inverse of a typical time. Show that for a relativistic electron, $\nu_s = \gamma^3 \nu_B$, where $\nu_B = \frac{v}{2\pi r_L}$, with v being velocity of that electron and r_L is the Larmor radius. **[3 marks]**

(c) An ultra-relativistic electron emits synchrotron radiation. Show that its energy decreases with time according to:

$$\gamma(t) = \frac{\gamma_0}{1 + A\gamma_0 t}, \quad \text{where} \quad A = \frac{2e^4 B_\perp^2}{3m^3c^5}$$

Here γ_0 is the initial value of Lorentz factor γ and $B_\perp = B\sin\alpha$, with α being the angle between the velocity vector and magnetic fields. Show that the time for the electron to lose half its energy is

$$t_{1/2} = \frac{5.1 \times 10^8}{\gamma_0 B_\perp^2} \quad \text{(in sec)}.$$

[4 marks].

4. *Thermal Bremsstrahlung* – **Total : 10 marks**

Total emissivity due to thermal bremsstrahlung for a particular frequency is given by -

$$\begin{aligned} j_{\rm ff}(\nu, T) &= \int_{v_{\rm min}}^{\infty} j_{\rm ff}(\nu, v) f(v) dv \\ &= 6.8 \times 10^{-38} T^{-0.5} \exp\left(-\frac{h\nu}{kT}\right) n_e n_p Z^2 g_{\rm ff}(\nu, T) \quad \text{ergs}^{-1}\text{cm}^{-3}\text{Hz}^{-1} \end{aligned}$$

where $f(v)$ is the Maxwell-Boltzmann distribution in velocity space and $g_{\rm ff}(\nu, T)$ is a quantum mechanical correction term (*Gaunt Factor*) which has $\mathcal{O}(1)$.

(a) What is the value of $v_{\rm min}$ and give physical reasoning behind introducing this lower limit to the above integration. **[1 mark]**

(b) Assuming Local thermodynamic equilibrium (LTE), derive the expression for the thermal free-free absorption coefficient $\alpha(\nu, T)$. Using the knowledge of solution of radiative transfer equation obtain the temperature and frequency dependence of total brightness $B(\nu, T)$ (emission + absorption). Discuss the cases of when $\tau(\nu, T) \ll 1$ and $\tau(\nu, T) \gg 1$. Finally draw the spectrum expected from total brightness. **[5 marks]**

(c) NGC1976 (Orion nebula) is one of the brightest HII regions on the sky. Its angular size is approximately 1 deg and we know that its distance is around 400 pc. This region emits thermal bremsstrahlung radiation with transition from optically thick to optically thin regime at 1 GHZ and cut-off frequency at 200 THz.

- In observations, the measure of the cutoff frequency is a way to determine the plasma temperature. Estimate the temperature of NGC1976.
- Assuming that NGC1976 has a spherical shape, estimate the number density of the region (assume $n_e = n_i$, $Z = 1$, $g_{ff} = 1$).

[4 marks]

5. *Search for Dark Matter* – **Total : 10 marks**

(a) Explain in brief how a flat rotation curve typically observed in spiral galaxies hint towards a possible existence of dark matter halos surrounding the galactic stellar disk. **[3 marks]**

(b) Estimate how long a Galaxy in the Coma cluster would take to travel from one side of the cluster to the other. Assume that the galaxy moves with a constant speed equal to the clusters radial velocity dispersion $\sigma_r = 977$ km/s. The Coma cluster has a radial size $R = 1.9 \times 10^{22}$ m. Compare this time with the Hubble time $t_H = 1./H_0$, where $H_0 = 71.0$ km/s per Mpc. What can you conclude about whether the galaxies in the Coma cluster are gravitationally bound? **[3 marks]**

(c) From the conclusion derived from the above problem, quantify the mass to light ratio assuming that Coma cluster has a spherical symmetry and contains approximately 1000 bright galaxies with an average luminosity of $8.5 \times 10^7 L_\odot$. What conclusion can you make on comparing the obtained value with a typical value of $3.0 \frac{M_\odot}{L_\odot}$. **[4 marks]**

INDIAN INSTITUTE OF TECHNOLOGY INDORE

Discipline of Astronomy, Astrophysics and Space Engineering (AASE)

Spring Semester		**Academic Year 2018-19**
End Semester Exam		**Course Code : AA 472/672**
Date: **22 April 2019**	Max. Time Duration : **3 Hr**	Max Marks : **50**
Number of Questions : 5		Number of pages in the Question Paper: 2

Instructions:

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Constants

Speed of light : $c = 3.0 \times 10^{10}$ cm/s
Boltzman Constant : $k_B = 1.38 \times 10^{-16}$ erg cm^2 g s^{-2} K^{-1}
Stefan's Constant : $\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
Proton mass : $m_p = 1.673 \times 10^{-24}$ g
Planck's Constant : $h = 6.626 \times 10^{-34}$ J s
Gravitational Constant : $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
Astronomical Unit : $1AU = 1.496 \times 10^{11}$ m
Parsec : $1pc = 3.086 \times 10^{16}$ m
Solar Radius : $R_\odot = 6.96 \times 10^5$ km
Solar Luminosity : $L_\odot = 3.83 \times 10^{26}$ W
Hubble's Constant : $H_0 = 71$ km s^{-1} Mpc^{-1}
Thomson cross-section for e$^-$: $\sigma_t = 6.65 \times 10^{-29}$ m^2

1. *Rotation Curve of Galaxy* – **Total : 10 marks** The observation of the galactic rotation curve yields a deficit of mass in the galaxy. Under the assumption of spherical symmetry of a rotating galaxy one can calculate the mass inside a sphere of a given radius from the circular velocity of the stars at its surface and compare it to an estimation from the visible stars.

   (a) Give a formula which expresses the circular velocity in terms of the enclosed mass and the distance to the galactic center. Then assume the simplest case of a constant mass density ρ_0 inside a radius r_0. How does the rotation curve look like inside and outside of r_0 ? Now consider a more realistic distribution in the form of

   $$\rho(r) = \frac{\rho_0}{(1 + r/r_0)^\alpha}, \tag{1}$$

   derive the rotation curve v(r) for a value of α which gives a flat rotation curve at $r \gg r_0$ as indicated by measurements. **[5 marks]**

   (b) At $r = 10^5$ light years the measurement yields $V_{\rm meas} = 225\,$km/s and an expected velocity of $V_{\rm calc} = 15\,$km/s. Calculate the visible as well as the true galaxy mass. What is the percentage of dark matter in the galaxy? How high is the average dark matter mass density? (Use: $G = 6.67 \times 10^{11}$m^{3}kg^{-1}s^{-2}.) **[5 marks]**

2. *Search for Dark Matter and Viral Theorem* – **Total : 10 marks**

   (a) What do you mean by Virial Theorem and under what conditions one can apply this theorem? **[3 marks]**

   (b) Estimate how long a Galaxy in the Coma cluster would take to travel from one side of the cluster to the other. Assume that the galaxy moves with a constant speed equal to the clusters radial velocity dispersion $\sigma_r = 977$ km/s. The Coma cluster has a radial size $R = 1.9 \times 10^{22}$ m. Compare this time with the Hubble time $t_H = 1./H_0$, where $H_0 = 71.0$ km/s per Mpc. What can you conclude about whether the galaxies in the Coma cluster are gravitationally bound? **[3 marks]**

   (c) From the conclusion derived from the above problem, quantify the mass to light ratio assuming that Coma cluster has a spherical symmetry and contains approximately 1000 bright galaxies with an average luminosity of $8.5 \times 10^7 L_\odot$. What conclusion can you make on comparing the obtained value with a typical value of $3.0 \frac{M_\odot}{L_\odot}$. **[4 marks]**

3. *Active Galactic Nuclei: emission* – **Total : 10 marks**

   (a) What is an AGN? What distinguishes an AGN from a normal galaxy? **[2 marks]**

   (b) Can the luminosity of an AGN given a mass of the central black hole be arbitrarily large? Justify your answer. **[1 marks]**

   (c) Derive the expression for the threshold criterion for the luminosity of an AGN. **[4 marks]**

   (d) Assume the energy input from accretion is given by $\dot{M}c^2$ where $\dot{M}$ is the mass accretion rate. A fraction of the energy gets converted to the luminosity of AGN. Given the efficiency is 10% and the mass of the black hole is 10^8 $M_\odot$, calculate the critical mass accretion rate? **[3 marks]**

4. *Jets and outflows from AGN* – **Total : 10 marks**

   (a) Radio lobes from quasars are observed to traverse at velocities greater than the speed of light. How can one explain this superluminal motion? Derive an expression for apparent velocity for a radio lobe from a Quasar in terms of the true velocity and its angle with the line of sight. **[4 marks]**

   (b) For the same quasar mentioned above and for a given observed apparent velocity, derive the expression for the angle of the motion of radio lobe with respect to line of sight that corresponds to the minimum true velocity. Use this result to find the expression for the minimum true velocity. **[3 marks]**

   (c) For a quasar, the observed apparent velocity is 7.85 c (c is the velocity of light in vacuum). Find out the minimum true velocity in terms of c using the previous results that you have derived. **[3 marks]**

5. *Galaxy scaling relations* – **Total : 10 marks**

   (a) A spiral disk galaxy is observed to contain neutral hydrogen gas. The observed K-band flux is $m_K = 15.0$ and its diameter is 10 arcminutes. Estimate the distance using the empirical relation (where V_{max} is the maximum rotational velocity):

$$\frac{L_K}{3 \times 10^{10} L_{K,\odot}} = \left(\frac{V_{max}}{200\text{km/s}}\right)^{3.8} \tag{2}$$

   The observed width of the neutral hydrogen line profile is 100 km/s. Assume the galaxy to be edge-on. $M_{K,\odot} = 3.28$ **[3 marks]**

   (b) What would be the typical profile of neutral hydrogen line in the previous problem. Illustrate the profile and justify your answer. **[2 marks]**

   (c) Estimate the distance to the same galaxy given in (a) from Hubble's Law. **[2 marks]**

   (d) Emission lines from gas near the center of M87 have been observed with the Hubble Space Telescope. The observations are best fit with orbital speeds of 1000 km s^{-1} at 0.1″ from the center. Assuming that the distance $d = 16$ Mpc, estimate the mass present inside this radius. **[3 marks]**

INDIAN INSTITUTE OF TECHNOLOGY INDORE

Discipline of Astronomy, Astrophysics and Space Engineering (AASE)

Spring Semester		**Academic Year 2018-19**
Mid Semester Exam		**Course Code : AA 472/672**
Date: **8 March 2019**	Max. Time Duration : **2 Hr**	Max Marks : **40**
Number of Questions : 4		Number of pages in the Question Paper: 2

Instructions:

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1. *Central Force and Potential Density Pair* – **Total : 10 marks**

   (a) A particle moves in a potential $V(r) = -V_0 \exp^{-\lambda^2 r^2}$, if the angular momentum is given by L find the radius of the circular orbit [*Answer can be in the implicit form*]. What is the largest value of L for which a circular orbit exists and what is the value of effective potential $V_{\rm eff}(r)$ at this critical orbit? [**3 marks**]

   (b) If the angular momentum of star close to the black hole is L, find the potential $V(r)$ such that the path taken by the star is given by a spiral $r = C\theta^k$, where C and k are constants. Plot the potential for two values of $k = \pm 2$. Take all constants as unity if required. [**3 marks**]

   (c) Obtain the density distribution and circular velocity for a Jaffe potential given by -

   $$\Phi(r) = \frac{GM}{a} \ln\left(\frac{r}{r+a}\right)$$

   where M and a are constants and show that at large distances the density corresponding the above potential goes as r^{-4}. Further plot the dependence (in the same curve) of circular velocity with radial co-ordinate for cases (i) $r \gg a$ and (ii) $a \gg r$ also compare the same with a standard Kepler profile [**4 marks**]

2. *Galaxy morphology and characteristics* – **Total : 10 marks**

   (a) Explain how the galaxies can the classified on the basis of their morphology. Comment on the difference in stellar population for the two primary broad classes of galaxies.[**2 marks**]

   (b) The disk of a galaxy can be modeled as a slab of uniform mass density ρ. Let us assume that the galaxy has a thickness $2H$ along the z direction and effectively infinite in the x and y directions. Assume that the mass density for $z > H$ is zero. Compute the effective gravitational attraction per unit mass at an arbitrary distance, z, inside and above the disk Find the speed, v_z that a star must have, starting at the middle of the disk, to get above height H, i.e., just outside of the mass distribution. Express your answer in terms of ρ and H. [**4 marks**]

   (c) The mean free path of a particle in a gas is given as $\lambda = 1/(n\sigma)$ where n and σ are respectively the number density and the cross-section of the particles undergoing collision. Let us consider the gas in a spiral galaxy which comprised of hydrogen and dust which collides with each other. Suppose the dust grain radius is a=0.1 μm, the number density of gas is $n_g = 1$ cm^{-3} and the typical velocity of hydrogen atoms is 0.5 km/s (assume the radius of hydrogen atoms and velocity of dust grains to be comparatively negligible). Calculate the timescale of collision. Now assume the stellar population of the galaxy of be gas where each star behaves like particles in a gas. Suppose the number density of stars is $n_s = 1.17 \times 10^{-56}$ cm^{-3} and typical stellar velocity is 50 km/s. Assuming solar radius ($R_\odot = 6.95 \times 10^5$ km) and applying the methods as before calculate collision timescales of stars. Compare this two timescales and comment on the implications of this result on the distribution of gas and stars in the galaxy. [**4 marks**]

3. *Spiral Density wave and Oorts Constant* – **Total : 10 marks**

(a) Explain the winding problem of spiral arms and discuss the Lin-Shu density wave theory briefly explaining the epicyclic frequency and Lindbald resonances [**3 marks**]

(b) Derive the expressions for Oort constants that characterize the local rotational properties of our galaxy. Sketch a diagram between observed velocity and Galactic longitude l and show how these constants are estimated from observations.[**5 marks**]

(c) How could one extend the local approximation to measure the rotation curve of the entire Milky way galaxy and state the possible sources of errors/limitations? [**2 marks**]

4. *Scaling relations and galaxy luminosity* – **Total : 10 marks**

(a) Starting from the radial variation of luminosity for elliptical galaxies obtain the fundamental plane relation in terms of the radius, surface brightness and velocity dispersion for these galaxies. [**3 marks**]

(b) What is the mass-to-light ratio of a stellar population with a Salpeter Initial Mass Function

$$n(M) = M^{-\alpha} dM \tag{1}$$

with $\alpha = 2.35$, of zero age (i.e. assume all stars are alive and well and on the main sequence and that $L \propto M^3$)? Assume any additional parameters you need with justification. ($M_\odot = 2 \times 10^{30}$ kg) [**3 marks**]

(c) Show that the surface brightness of a resolvable galaxy is distance independent. [**2 marks**]

(d) Obtain the relation between surface brightness in expressed magnitude per arcsec^2 and the physical surface brightness expressed in units of $L_\odot/\mathrm{pc}^2$ (1 pc $= 3.068 \times 10^{16}$ m). [**2 marks**]

Discipline of Astronomy, Astrophysics and Space Engineering (DAASE)

INDIAN INSTITUTE OF TECHNOLOGY INDORE

Spring Semester	**Academic Year 2019-20**
Mid Semester Exam	**Course Code : AA 608**
Date: **27 February 2020 (FN)**	Max. duration : **2 hr**
Max. Marks : **30**	Number of pages: 2

Instructions:

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- Exchange of question paper and examination related materials and talking to other students during the examination will also be treated as the *Use of Unfair Means*.
- All questions are compulsory.
- Please, answer in the most complete and clear way. Do not forget the units!
- When you define a quantity try to give a complete definition: words + formula + units.
- Provide schematic diagrams where necessary to support your answers.
- Numbers in the parentheses at the end of each question represent the allocated marks.

1. (a) What is probability? Explain it from the Bayesian and the Frequentists point of view. (**3**)
   (b) Write down the Bayes theorem. What are likelihood, prior, evidence and posterior in this context? (**2**)

2. We wish to estimate the mean μ of a population $\{x_i\}$, $i = 1...N$, each independent and with variance σ^2. Defining an estimator $\hat{\mu} = \sum_i w_i x_i$ for some weights w_i, show that for the estimator to be unbiased (i.e. $\langle\hat{\mu}\rangle = \mu$) we require $\sum_i w_i = 1$. By minimising the variance of $\hat{\mu}$, show that the minimum variance estimator is $\hat{\mu} = (1/N)\sum_i x_i$, and show that its distribution for repeated trials is a Gaussian centred on μ with a variance σ^2/N. (**5**)

3. A lighthouse is situated at unknown co-ordinates x_0, y_0 with respect to a straight coastline $y = 0$. It sends a series of N flashes in random directions, and these are recorded on the coastline at positions x_i. Using Bayesian approach, find the posterior distribution of x_0, y_0, given the positions x_i. (**5**)

4. A known petty thief was lodging in a house when a theft of a piece of gold jewellery takes place. The defence lawyer has argued that only 1/2500 of known thieves (T) who are in lodgings steal expensive jewellery from their hosts, so the information that he is a known thief is irrelevant and must be ignored. $p(S|T) = 0.0004$. You are the prosecution lawyer, and you are fairly sure that the thief has stolen the jewellery (this possible event we call S). How do you counter this argument? You are in possession of the knowledge that the probability of a theft of jewellery (J) from lodgings is 1/20000. (**5**)

5. Write notes on :
   (a) Random error and Systematic error (with example).
   (b) One-sided hypotheses and two-sided hypotheses.
   (c) Accuracy and Precision (with example).
   (**3**)

6. A and B both measure the speed of a moving ball. A measures it as 4.05 ± 0.35 km/s and B measures it as 3.50 ± 0.15 km/s. Are these two measurements consistent? (**2**)

7. A doctor believes that the proportions of births in this country on each day of the week are equal. A simple random sample of 700 births from a recent year is selected, and the results are given in the table below. At a significance level of $\alpha = 0.01$, is there enough evidence to support the doctor's claim?

$\chi^2 = 16.8$

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Frequency	65	103	114	116	115	112	75

Table 1:

(**3**)

8. If random samples of size ≥ 30 are drawn from a population with known population variance (σ^2), the sample means follow (choose the correct answer/answers from below):

(a) F-distribution

(b) t-distribution

(c) normal distribution

(d) χ^2 distribution

(1)

9. The null and alternative hypotheses divide the sample space into (choose the correct answer/answers from below):

(a) two sets that overlap

(b) two non-overlapping sets

(c) Both of these

(d) None of these

(1)