AA608 - END Semester Exam
(Spring Semester)

Name- Ankit Meena

Roll No - 200312002

problem : 1 :-

1(a) $a = \frac{-(1-\alpha)}{s_0^{1-\alpha}} = \frac{\alpha-1}{s_0^{1-\alpha}}$

The model probability distribution for s is

$$p(s)\, ds = (\alpha-1)\, s_0^{\alpha-1}\, s^{\alpha}\, ds$$

So where- The factor $\alpha-1$ in front of the terms arises from the normalization requirement

$$\int_{s_0}^{\infty} ds\, p(s) = 1$$

Now the Likelihood function L for n observed Sources is →

$$L = \prod_{i=1}^{n} (\alpha-1)\, s_0^{\alpha-1}\, s_i^{-\alpha}$$

$$L(\alpha/s) = \prod_{i=1}^{n} (\alpha-1) \left(\frac{s_i}{s_0}\right)^{-\alpha} \frac{1}{s_0}$$

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taking the log Both sides -

$$\ln L(\alpha) = \ln\left(\prod_{i=1}^{n} (\alpha-1)\left(\frac{S_i}{S_0}\right)^{-\alpha}\frac{1}{S_0}\right)$$

$$\ln L(\alpha) = \sum_{i=1}^{n}\left[\ln(\alpha-1) + (\alpha-1)\ln S_0 - \alpha\ln S_i\right]$$

then find maximising value of α differentiating with respect to α

$$\frac{d\ln L(\alpha)}{d\alpha} = \sum_{i=1}^{n}\left(\frac{1}{\alpha-1} + \ln S_0 - \ln S_i\right) = 0$$

$$\frac{n}{\alpha-1} = \sum_{i=1}^{n} \ln\frac{S_i}{S_0} = 0$$

$$\frac{\alpha-1}{n} = \frac{1}{\sum_{i=1}^{n}\ln\frac{S_i}{S_0}}$$

$$\boxed{\alpha = 1 + \frac{n}{\sum_{i=1}^{n}\ln\frac{S_i}{S_0}}}$$

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1(b)

Suppose we only observe one Source with flux twice the cut-off. $(n=1)$

$S_1 = 2S_0$ then,

$$\alpha = 1 + \frac{1}{\ln \frac{2S_0}{S_0}} = 1 + \frac{1}{\ln 2}$$

$$\boxed{\alpha = 2.44}$$

1(c)

To get a rough estimate of the width of the credibility interval for α, we can compute

$$\sigma_\alpha^{2} \approx -\left(\frac{\partial^2 \ln L}{\partial \alpha^2}\right)^{-1}$$

$$\sigma_\alpha^2 \approx -\left(\sum_{i=1}^{n} \frac{-1}{(\alpha-1)^2}\right)^{-1}$$

$$\sigma_\alpha^2 = -\left(\frac{-n}{(\alpha-1)^2}\right)^{-1} = -\left[\frac{(\alpha-1)^2}{-n}\right]$$

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$$\boxed{\sigma^2(\alpha) = \frac{(\alpha-1)^2}{n}}$$

from part (b) $\alpha = 2.44$ →

and $n = 1$

$$\sigma_\alpha^2 = \frac{(2.44-1)^2}{1} = (1.44)^2$$

$$\boxed{\sigma_\alpha = 1.44}$$

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Problem:2 → (a)

2(a)

The distribution of omission is

Poisson distribution → count of photos omits

in time ~~[scribbled]~~ . $\mu = \lambda t$

so probability of emitting M photons →

$$p(M) = \frac{(\mu)^M}{M!} e^{-\mu}$$

$$p(M) = \frac{(\lambda t)^M}{M!} e^{-\lambda t}$$

The detection of photon by telescope is ~~indep~~ independt event so this considering as a Binomial distribution.

→ probability of detected N out of M photons.

$$p(M) = \frac{M!}{M!\,(N-M)!} p^N (1-p)^{M-N}$$

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The joint probability is -

$$\boxed{p(M,N) = \frac{(\lambda t)^M}{M!} e^{-\lambda t} \cdot \frac{M!}{M!\,(N-M)!} p^N (1-p)^{M-N}}$$

$[\because P(M,N) = P(M)\,P(N)]$

2 (b)

$$P(N) = \sum_M p(M,N)$$

$\therefore$ for continuous $p(x) = \int dy\, p(x,y)$

$$p(N) = \sum_M \frac{(\lambda t)^M}{M!} e^{-\lambda t} \cdot \frac{M!}{N!\,(M-N)!} p^N \left((1-p)^{M-N}\right)$$

$$P(N) = \frac{(1-p)^{-N} p^N}{N!} \sum_M \frac{e^{-\lambda t} (1-p)^M (\lambda t)^M}{(M-N)!}$$

$$p(N) = \frac{p^N (1-p)^{-N}}{N!} e^{-\lambda t} \sum_{M \geq N}^{\infty} \frac{[(1-p)(\lambda t)]^M}{(M-N)!}$$

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$\therefore M < N$ is not defined.

$$\boxed{P(N) = \frac{p^N q^{-N} e^{-\mu}}{N!} \sum_{M=N}^{\infty} \frac{(q\mu)^M}{(M-N)!}} \quad \text{—— (A)}$$

where $\mu = \alpha t$, $q = 1-p$

2(C)

in the series →

$$\sum_{M=N}^{\infty} \frac{(\mu q)^M}{(M-N)!}$$

$$= \sum_{k=0}^{\infty} \frac{(\mu q)^{k+N}}{k!}$$

$\therefore M-N = k$

$$= (\mu q)^N \sum_{k=0}^{\infty} \frac{(\mu q)^k}{k!}$$

$$= (\mu q)^N e^{\mu q} \quad \text{—— (B)}$$

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from equation (A) & B →

$$P(N) = \frac{e^{-\mu} q^{-N} p^N}{N!} (\mu q)^N e^{\mu q}$$

$$P(N) = \frac{\left(\frac{p \cdot \mu q}{q}\right)^N e^{\mu(q-1)}}{N!} \qquad \therefore q-1 = -p$$

$$\boxed{P(N) = \frac{(p\mu)^N e^{-p\mu}}{N!}}$$

where — which is a

$p\mu$ = poision distribution

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Problem: 3 →

3 (a)

Bayes' Theorem :-

$$p(\theta/D) = \frac{p(D/\theta)\, p(\theta)}{p(D)}$$

where → $p(D/\theta)$ = likelihood probability

$p(\theta)$ = ~~[illegible]~~ prior probability

$p(\theta/D)$ = posterior probability

$p(D)$ → Evidence

The use $p(D)$ is the evidence under Bayes's theorem relates to the probability of finding evidence in relation to accured. wher Bayes's theorem concern the probability of an event and its inverse.

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3(b) If independent variables u_i are normally distributed each with mean u_i and variance σ_i^2 then the quantity χ^2 is defined by →

$$\chi^2 = \frac{(y_1 - \mu_1)^2}{\sigma_1^2} + \frac{(y_2 - \mu_2)^2}{\sigma_2^2} + \dots \frac{(y_m - \mu_m)^2}{\sigma_m^2}$$

$$\boxed{\chi^2 = \sum_{i=1}^{M} \frac{(y_i - \mu_i)^2}{\sigma_i^2}}$$

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3 (c)

The goodness of fit test is a statistical hypothesis test to see ~~which~~ how well sample data fit a distribution from a population with a normal distribution. Put differently this test shows if ~~your~~ our sample data represents the data we would expect to find in the actual population. So the flow-chart below shows how chi-square can be used to test goodness of fit

Choose a function/model

↓

Calculate expectation values for each x_i

↓

Find out m (number of data points)

↓

Calculate χ^2 using

$$\chi^2 = \sum_{i=1}^{m} \frac{(y_i - \hat{y}_i)}{\sigma_i^2}$$

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↓

Calculate χ^2/M

↓

Choose at value of significance

↓

↓

~~Choose~~

Determine corresponding values of $\chi^2_{M,\alpha}/M$

↓

Compare $\chi^2_{M,\alpha}/M$ with χ^2/M

↓ $\chi^2 > \chi^2_{M,\alpha}$ → ↓ Model can be rejected with $[(1-\alpha)\times 100]\%$ confidence

↓ $\frac{\chi^2}{M} \sim 1$ → ↓ Model is a good fit

↓ $\chi^2 < \chi^2_{M,1-\alpha}$ → ↓ Over estimated σ_i or fraudulent data

Anatomy

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Problem:6 :→

n=165	predicted NO	predicted yes
Actual NO	50 (a)	10 (b)
Actual yes	5 (c)	100 (d)

given - n=165 data points or examples

⇒ 50 = True NO (correct rejections)

⇒ 10 = False yes (false alarms)

⇒ 5 = False NO misses (lower leaked damage)

⇒ 100 = True yes hits.

we have K=2 classes here

so the accuracy is the percent of correct classifications.

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(13)

for this model

$$accuracy = \frac{a+d}{a+b+c+d}$$

$$accuracy = \frac{TN + TY}{total}$$

$$= \frac{50+100}{165}$$

$$accuracy = \frac{150}{165} \simeq 0.9090...$$

$$\boxed{accuracy = 0.\overline{90}}$$

⇒ recall or True yes rate or Sensitivity

$$= \frac{TY}{actual\ yes} = \frac{d}{c+d} = \frac{100}{5+100}$$

$$= \frac{100}{105} = 0.952380952$$

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(14)

⇒ Precession or predicted yes value →

$$= \frac{TY}{\text{predicted yes}}$$

$$= \frac{d}{b+d} = \frac{100}{10+100} = \frac{100}{110} = 0.9090\ldots$$

$$= 0.\overline{90}$$

⇒ Now False alarm or false yes rate →

$$= \frac{FY}{\text{actual No}} = \frac{b}{(a+b} = \frac{10}{50+10}$$

$$= \frac{10}{60} = 0.16666 = 0.17$$

Problem: 7 →

The major advantages or disadvantages →

k-Means advantages →

① if variables are huge, then k-means most of the times computationally faster than hierarchical clustering, if we keep k small.

② k-means produce tighter clusters than hierarchical clustering, especially if the clusters are globular.

k-means Disadvantages →

① difficult to predict k-value

② with global cluster, it did not work well

③ Different initial partitions can result in different final clusters.

④ It does not work well with clusters of Different size and Different density.

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Problem: 8 →

The Naive Bayes classification probability →

$$P\left(\frac{yes}{today\ or\ feature}\right) = \frac{P\left(\frac{Feature}{yes}\right) \times P(yes)}{P\left(\frac{Feature}{yes}\right) \times P(yes) + P\left(\frac{Feature}{No}\right) \times P(No)}$$

(A)

Now this question, feature values for the given data point are outlook = sunny

Temperature = Hot
Humidity = Normal
windy = False

then since all the features are independent

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we can use Multiplication theorem of probability to get $P(\text{Feature}/\text{yes})$ and $P(\text{Feature}/\text{No})$

$$\cancel{P\left(\frac{\text{Feature}}{\text{yes}}\right) = P\left(\frac{\text{sunny}}{\text{yes}}\right) \times P\left(\frac{\text{Hot}}{\text{yes}}\right)}$$

$$P(\text{Feature}/\text{yes}) = P\left(\frac{\text{sunny}}{\text{yes}}\right) \times P\left(\frac{\text{Hot}}{\text{yes}}\right) \times P\left(\frac{\text{normal}}{\text{yes}}\right) \times P\left(\frac{\text{False}}{\text{yes}}\right)$$

$$\boxed{P(\text{Feature}/\text{yes}) = \frac{3}{9} \times \frac{2}{9} \times \frac{6}{9} \times \frac{6}{9}} \quad \text{— (B)}$$

In the same way →

$$P\left(\frac{\text{Feature}}{\text{No}}\right) = \frac{2}{5} \times \frac{2}{5} \times \frac{1}{5} \times \frac{2}{5} \quad \text{— (C)}$$

Now; $P(\text{yes}) = \frac{9}{14}$ — (D)

$P(\text{No}) = \frac{5}{14}$ — (E)

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in equat (A) putting equation (B), (C), (D), (E) :

$$P\left(\frac{yes}{(Feature)}\right) = \frac{\left(\frac{3}{9} \times \frac{2}{9} \times \frac{6}{9} \times \frac{6}{9}\right) \times \frac{9}{14}}{\left(\frac{3}{9}\right) \times \frac{2}{9} \times \frac{6}{9} \times \frac{6}{9}\Big) \times \frac{9}{14} + \left(\frac{2}{5} \times \frac{2}{5} \times \frac{1}{5} \times \frac{2}{5}\right) \times \frac{5}{14}}$$

$$= \frac{0.021164}{0.021164 + 0.004571428}$$

$$= \frac{0.021164}{0.025735428}$$

$$\boxed{P\left(\frac{yes}{feature}\right) = 0.822368293}$$