Kishlay Singh

25/01/21

2003121005

Mid Semester Exam

Undertaking:

I, Kishlay Singh (2003121005), hereby declare that during the course of this exam, have not used any means of communication through phone, chat or any messaging ~~app~~, VoIP or social media app to discuss regarding this exam with any human or any bot.

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25/01/2021

Que 1 → (a) Distance to the Star in parsec is

$$d = \frac{1}{p}$$

$$\Rightarrow d = \left(\frac{1}{0.007}\right) pc = 142.85 \ pc$$

Radius in $R_\odot$ is given by : $\cancel{\frac{d\theta}{2}}$ $\frac{d \times \theta/2}{R_\odot}$

$$\cancel{\Rightarrow 142.85 \times 3.1 \times 10^{8}} = \frac{142.85 \times 3.08 \times 10^{16}}{6.95 \times 10^{8}}$$

$$= 6.33 \times 10^{9} \ R_\odot$$

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(b) Senstivity is a measure of the minimum signal a telescope can distinguish above random background noise. The more sensitive a telescope is, the more light it can gather from faint objects. The more light gathered, the fainter the object that can be studied photometrically or imaged

Resolution of a telescope is the smallest angle between two close objects that can be easily distinguishable by it.

Also given by $\theta = \frac{1.22\lambda}{D}$

where D is the diameter of the aperture.

No, it will not be visible to an unaided eye as apparent magnitude of 6.0 is brighter than apparent magnitude of 8.0.

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(c). Apparent magnitude :-

$$m_q - 8.0 = -2.5 \log\left(\frac{F_q}{F_*}\right)$$

According to ques. $F_q = 4F_*$

$$m_q = 8 - 2.5 \log 4 \simeq \underline{\underline{6.5}}$$

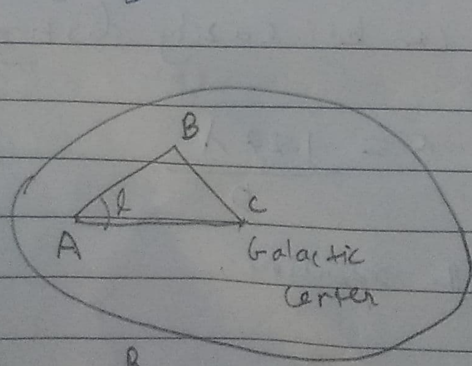
Absolute magnitude

$$6.5 - M = 5\log\left(\frac{d}{10pc}\right)$$

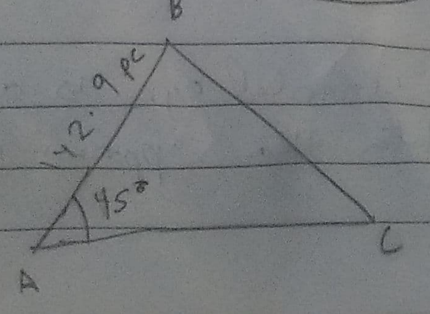
$$M = 6.5 - 5\log\left(\frac{142.9}{10}\right) = \underline{\underline{0.7}}$$

Since ~~greater~~ App. Mag of the Sytem is smaller than App. Mag. of the Single Star, the System is more brighter than a Single Star

(d).



Since $b = 0°$, the Star lies in the galactic plane



$AB = 142.9$ pc

$AC = 8K$ pc

$\angle BAC = 45°$

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By cosine rule

$$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos 45°$$

$$BC = 7899.6 \text{ pc} = 7.9 \text{ Kpc}$$

Distance to the galactic centre is 7.9 Kpc.

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Que 3 (a) $\longrightarrow$ $I(r) = I_0 \exp\left(\frac{-r}{R_d}\right)$

$$L = 2\pi \int_0^1 r\, I_0 \exp\left(\frac{-r}{R_d}\right) dr$$

$$\frac{r}{R_d} = x \;,\; \Rightarrow dr = R_d\, dx$$

$$L = 2\pi \int_0^x x\, R_d\, I_0 \exp(-x)\, R_d\, dx$$

$$= 2\pi I_0 R_d^2 \int_0^x x e^{-x}\, dx$$

$$= 2\pi I_0 R_d^2 \left[-x e^{-x}\Big|_0^x - e^{-x}\Big|_0^x \right]$$

$$L = 2\pi I_0 R_d^2 \left[-x e^{-x} - e^{-x} + 1 \right]$$

when $x \longrightarrow \infty$

$$\cancel{2\pi I_0 R_d^2} \quad L = 2\pi I_0 R_d^2$$

$$L_\infty = 2\pi I_0 R_d^2$$

$$L = 2\pi I_0 R_d \left[1 - \exp(-x)(1+x) \right]$$

For half light

$$L = \frac{L_\infty}{2} = \pi I_0 R_d$$

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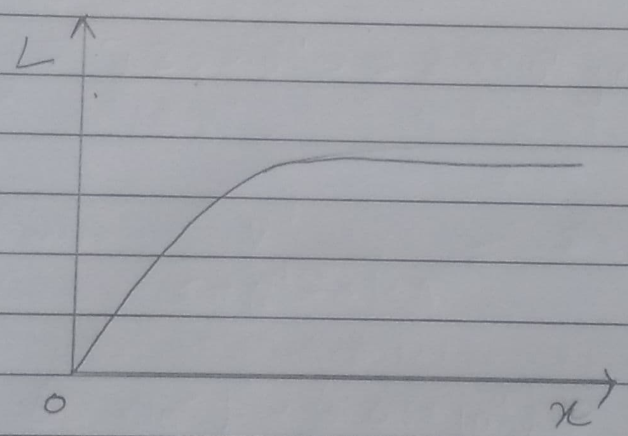
$$\pi I_0 R_d = 2\pi I_0 R_d \left[1 - e^{-x}(1+x)\right]$$

$$1 - e^{-x}(1+x) = \frac{1}{2}$$

$$\frac{1}{2} = e^{-x}(1+x)$$

~~It can be plotted as below~~

Cumulative enclosed Luminosity can be plotted as :-



Que 3. →

(b) Taking $\langle I \rangle$ as the central brightness, then we get the luminosity

$$L(x) = 2\pi x^2 \langle I \rangle$$

$$x^2 \propto \frac{L}{\langle I \rangle} \qquad - \text{①}$$

$$m(x) = \frac{V_{max}^2 \times x}{G}$$

$$x \propto \frac{m}{(V_{max})^2}$$

$$x^2 \propto \frac{m^2}{V_{max}^4}$$

$$\Rightarrow \frac{m^2}{V_{max}^2} \propto \frac{L}{\langle I \rangle} \qquad (\text{by ①})$$

$$\frac{m^2}{L^2 V_{max}^2} \propto \frac{1}{L \langle I \rangle}$$

$$\Rightarrow L \propto \left(\frac{m}{L}\right)^{-2} \times \frac{1}{\langle I \rangle} V_{max}^2$$

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~~Eqn~~ Calculating for
Radial co-ordinate r,

$$L(r) = 2\pi I_0 R_d \left(-r e^{-r/R_d} - e^{-r/R_d} + 1\right)$$

~~So~~ $\cancel{V_d^4(r) = \left(\frac{M}{L}\right)^2 \langle I \rangle 2\pi I_0 R_d}$

So,

$$V_d^4(r) = \left(\frac{M}{L}\right)^2 \langle I \rangle 2\pi I_0 R_d \left(-r e^{-r/R_d} - e^{-r/R_d} + 1\right)$$

~~Que 3(a)~~ $\rightarrow$

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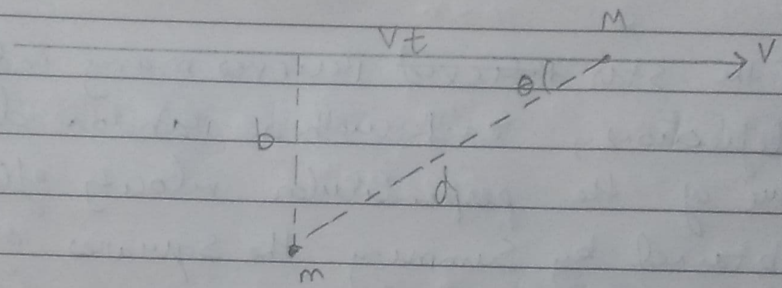
Que 4 →

(a). Stars with impact parameter $b >> r_s$ will also perturb the orbit. Path of the star will be deflected by a very small angle by one encounter.



~~b~~ ~~Let~~
Let the distance to the closest approach be b at ~~time~~ time $t=0$

Force on star M due to gravitational attraction of star m is

$$F = \frac{GMm}{d^2} = \frac{GMm}{b^2+V^2t^2}$$

$F_\perp$ (component ⊥ to the direction of star M) $= F\sin\theta$

$$\Rightarrow F \times \frac{b}{d} = \frac{GMmb}{(b^2+V^2t^2)^{3/2}}$$

Using $\quad F_\perp = M\frac{dV_\perp}{dt}$

$$\Delta V_\perp = \int_{-\infty}^{\infty} \frac{dV_\perp}{dt}\,dt = \frac{1}{M}\int_{-\infty}^{\infty} F_\perp(t)\,dt$$

$$= \frac{1}{M}\int_{-\infty}^{\infty} \frac{GMmb}{(b^2+v^2t^2)^{3/2}}\,dt = \frac{2GM}{bv}$$

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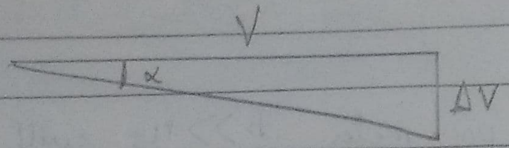
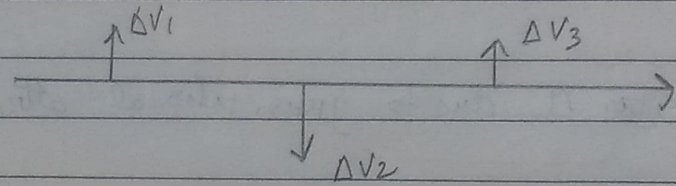


$$\alpha \simeq \tan\alpha = \frac{\Delta V_\perp}{V} = \frac{2Gm}{bV^2}$$

If the star ~~recieved~~ recieves many independent deflections, each with a random direction, expected value of the perpendicular velocity after time t is obtained by summing the squares of the individual velocity kicks.

$$\langle \Delta V_\perp^2 \rangle = \Delta V_1^2 + \Delta V_2^2 + \Delta V_3^2 + \cdots$$



Assuming there are many kicks

$$\langle \Delta V_\perp^2 \rangle = \int_{b_{min}}^{b_{max}} \left(\frac{2GM}{bV}\right)^2 dN$$

where dN is the expected number of encounters that occur in time t between impact parameter b and b + db

$$dN = n \times Vt \times 2\pi b\,db$$

Number density of perturbing stars

Distance star travels in time t

Area of annulus between impact parameter b + db

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which gives

$$\langle \Delta V_\perp^2 \rangle = \int_{b_{min}}^{b_{max}} nVt \left(\frac{2Gm}{bV}\right)^2 2\pi\, b\, db$$

$$= \frac{8\pi G^2 m^2 n t}{V} \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{8\pi G^2 m^2 n t}{V} \ln\left[\frac{b_{max}}{b_{min}}\right]$$

After a long time, the stars ~~t~~ speed will on average grow to equal its original speed which is also defined as the relaxation time.

$$V^2 = \langle \Delta V_\perp^2 \rangle = \frac{8\pi G^2 m^2 n t_{relax}}{V} \ln\left[\frac{b_{max}}{b_{min}}\right]$$

$$t_{relax} = \frac{V^3}{8\pi G^2 m^2 n \ln(b_{max}/b_{min})}$$

Recalling that the strong encounter time scale

$$t_s = \frac{V^3}{4\pi G^2 m^2 n}$$

~~t rela~~ $$t_{relax} = \frac{t_s}{2\ln(b_{max}/b_{min})} = \frac{t_s}{2\ln\Lambda}$$

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(b). For Globular Cluster

(a) $t_{cross} = \left(\frac{3}{4\pi G\rho}\right)^{1/2}$ ~~where~~ $\cancel{\rho = \frac{3M}{4\pi R^3}}$

where $\rho = m \times n$

$$= \frac{2\times10^{30} \times 10^{4}}{(3.08\times10^{16})^{3}} \ Kg\,m^{-3}$$

$$= 6.85\times10^{-16} \ Kg\,m^{-3}$$

$$t_{cross} = \left(\frac{3}{4\pi\times 6.67\times10^{-11} \times 6.85\times10^{-16}}\right)^{1/2}$$

$$= 2.29\times10^{12}\,s = 7.26\times10^{4} \text{ years}$$

(b) $t_{relax} = \frac{v^3}{8\pi G^2 m^2 n \times \ln\Lambda} = \frac{10^{12}\times(3.08\times10^{16})^{3}}{8\times20\times(6.67\times10^{-11})^{2}\times4\times10^{60}\times10^{4}}$

$$= 3.26\times10^{14}\,s = 1.03\times10^{7} \text{ years}$$

(c) $t_s = 2\times t_{relax} \times 20 = 4.13\times10^{8}$ yrs

~~For So galaxy~~

$\cancel{(a) = \left(\frac{3}{4\pi\times 6.67\times10^{-11}\times 6.84\times10^{-24}}\right)^{1/2}}$

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For So Galaxy

(a) $t_{cross} = \left(\frac{3}{4\pi G \rho}\right)^{1/2}$

$\rho = mn = \frac{2\times10^{30}\times 0.1}{(3.08\times10^{16})^3} = 6.84\times10^{-21}\,kg\,m^{-3}$

$t_{cross} = \left(\frac{3}{4\pi G\rho}\right)^{1/2} = \left(\frac{3}{4\times\pi\times6.67\times10^{-11}\times6.84\times10^{-21}}\right)^{1/2}$

$= 7.23\times10^{14}\,s = 2.3\times10^{7}$ years

(b) $t_{relax} = \frac{v^3}{8\pi G^2 m^2 n \ln\Lambda}$ =

$= \frac{(30\times10^3)^3\times(3.08\times10^{16})^3}{8\times20\times(6.67\times10^{-11})^2\times4\times10^{60}\times0.1} = 8.83\times10^{20}\,s$

$= 2.8\times10^{13}$ years

(c) = $t_s = 40\times t_{relax} = 1.12\times10^{15}$ years