Assignment 6
AA 674/474
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Q.1. $\nu = 1.4$ GHz

(a) Corresponding Wavelength:

$$c = \nu\lambda$$
$$\lambda = c/\nu$$
$$= \frac{3\times10^8}{1.4\times10^9} = 0.214 \text{ m} = 21.4 \text{ cm}$$

(b) This wavelength corresponds to radio band of electromagnetic spectrum.

(c) Diameter of radio telescope = 25 m

Resolution of radio telescope $= \frac{1.02\lambda}{D}$

$$= \frac{1.02\times0.214}{25}$$
$$= 0.0087 \text{ radians}$$
$$= 0.498^\circ \simeq 29.91'$$

(d) Average effective area of telescope $= \frac{\lambda^2}{4\pi}$

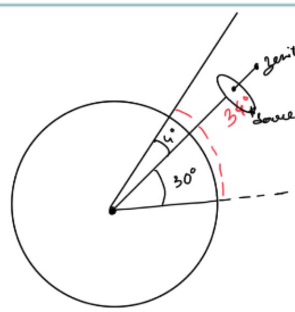
$$= \frac{0.214\times0.214}{4\pi}$$

$= 0.00364\ m^2$

$= 36.4\ cm^2$

(e) Telescope is located at $34^\circ N$ latitude

Source is located at $30^\circ N$ declination



The source will not pass through observer's zenith.

(f) The observer should look towards south as the source is 4° below the zenith.

(g) Size of source:-

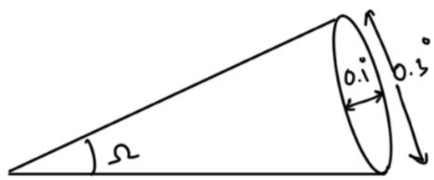
along major axis $= 0.3^\circ = \frac{0.3\pi}{180}$ radians

along minor axis $= 0.1^\circ = \frac{0.1\pi}{180}$ radians

Solid angle = Angular area subtended by source

$$(\Omega) = \pi \left[\frac{0.3}{2} \times \frac{\pi}{180}\right]\left[\frac{0.1}{2} \times \frac{\pi}{180}\right] sr$$

$$= 7.17737 \times 10^{-6}\ sr$$



Q.2. $\lambda = 6\ cm$

Solid angle $= 7.18 \times 10^{-6}\ sr$

Flux density $= F_\nu = 350\ Jy = 350 \times 10^{-26}\ W\,m^{-2}\,Hz^{-1}$

(a) Temperature of radio source can be approximated from RJ approximation.

$$I_\nu = \frac{2kT}{\lambda^2} \quad , \text{so,} \quad F_\nu = I_\nu \times \Omega$$

$$T = \frac{I_\nu \lambda^2}{2k} = \frac{F_\nu \lambda^2}{2k\Omega} = \frac{350 \times 10^{-26} \times (0.06)^2 \times 10}{2 \times 1.38 \times 10^{-23} \times 7.18 \times 10^{-6}}$$

$$= 6.358 \times 10^{21} \times 10^{-21}$$

$$= 63.58\ K$$

(b) Intensity of source at 2.7 cm

$$I_\nu = \frac{2kT}{\lambda^2} = \frac{2 \times 1.38 \times 10^{-23} \times 63.58}{(0.027)^2}$$

$$= 2.41 \times 10^{-18}\ W\,m^{-2}\,Hz^{-1}\,sr^{-1}$$

Q.3. Specifications of telescope A:

Diameter = 5 m

Detection wavelength = 2 cm

Specifications of telescope B:

Diameter = 10 cm

Detection wavelength = 6 cm

Flux density of A & B = 1 Jy at observed wavelength

$$F_\nu = \frac{\text{Power}}{\text{Area} \times \text{frequency}}$$

(a) $$\frac{\text{Power collected by A}}{\text{Power collected by B}} = \frac{P_A}{P_B} = \frac{[F_\nu \times A_{eff} \times Freq(\Delta\nu)]_A}{[F_\nu \times A_{eff} \times Freq(\Delta\nu)]_B}$$

$$= 10^{-26} \times \pi (5/2)^2$$

$$= 19625 \times 10^{-25}$$

(b) $$\frac{\text{Resolution of A}}{\text{Resolution of B}} = \frac{\theta_A}{\theta_B} = \frac{\lambda_A}{D_A} \times \frac{D_B}{\lambda_B}$$

$$= \frac{2}{6} \times \frac{10}{5}$$

$$= \frac{2}{3}$$

$\Rightarrow \theta_A : \theta_B = 2:3$

Q.4. Diameter = 2 m

Wavelength observed = 21 cm

$\Delta\nu = 1.5\ MHz = 1.5 \times 10^6\ Hz$

(a) Angular resolution with optimum edge taper

$= 1.15 \times \lambda/D$

$= 1.15 \times 0.21/2$

$= 0.121\ rad = 6.92^\circ$

(b) Maximum collecting area = $A_{geo} \times 0.82$ [due to edge taper]

$= \frac{\pi D^2}{2 \times 2} \times 0.82$

$= \frac{\pi (2)^2}{4} \times 0.82 = 2.576\ m^2$

(c) Power detected for 1 Jy source located at peak of a sidelobe

$P = F_\nu A_e \Delta\nu$

At sidelobe,

Power detected $(P_{sc}) = \frac{0.4}{100} \times 10^{-26} \times 2.576 \times 1.5 \times 10^6$

$= 1.545 \times 10^{-22}\ W$

Sensitivity of sidelobe is 0.4% of primary beam peak with an optical edge taper

Q.5. Separation between antenna = 30m

Observation wavelength = 60 cm

Hour angle = 1 hr

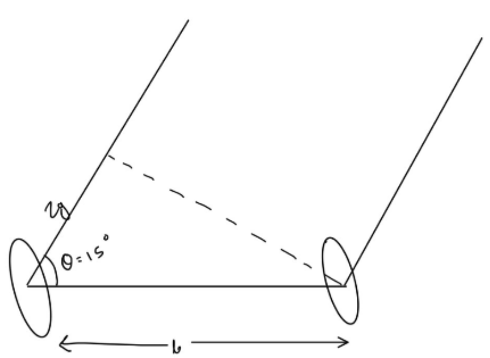
≈ 15° from meridian & zenith of baseline

(a) Time delay of wavefront

$$\tau_g = \frac{b \sin\theta}{c}$$

$$= \frac{30 \times \sin 15}{3 \times 10^8}$$

$$= 2.59 \times 10^{-8}\ s$$



(b) Phase difference between a sine wave entering 2 antenna

$$\Delta\phi = \frac{2\pi b \sin\theta}{\lambda}$$

$$= 2\pi \times \frac{30 \times \sin 15}{0.06}$$

$$= 813.1 \text{ rad}$$

$$= 258.82\pi$$

Q.6. Calibrated response of multiplicative interferometer

$$R = F_\nu \cos\left[2\pi \frac{b}{\lambda} \sin(\omega_E t_{HA}) \right]$$

$F_\nu = 3 Jy$

$\omega_E = 7.29 \times 10^{-5}$ rad/s

$\Delta\phi = 813.1$ rad $= 813.1$

Response $= 3 \cos[813.1 \text{ (rad)}]$

$= -2.522 \, Jy$

Output is measured over a range of time

(b) Amplitude of detected fringes $= 3 Jy$

Phase of fringe detected $= 813.1$ rad

$\approx 258.82\pi$ rad

Full phase value 2.58π gives how far the source is from the zenith.

Fringe phase = 0.82π

(c) One full oscillation occurs when $\Delta\phi = 2\pi$
i.e.,

$$\Delta\phi = 2\pi$$

$$2\pi \frac{b}{\lambda}\left[\sin(\omega_E t) - \sin\theta\right] = 2\pi$$

$$2\pi \times \frac{30}{0.06}\left[\sin(7.29\times10^{-5}\times t) - 258.82\pi\right] = 2\pi$$

$$t = \sin^{-1}(0.261) \times \frac{1}{7.29\times10^{-5}}$$

$$= 3619.645$$

HA = 1 h = 3600 s

So we have to wait $3619.64 - 3600 \approx 20$ s more to measure full fringe oscillation.

Q.7. Sky position of source = 14.98° to the west of zenith.

(a) Fringe phase $\phi = \frac{2\pi\, b\sin\theta}{\lambda} = \frac{2\pi \times 30\sin(14.98)}{0.06}$

$$= 812.04 \text{ rad}$$

$$= 258.48\,\pi \text{ rad}$$

Visibility amplitude would be 3 Jy

Visibility phase $= 258.82\pi - 258.48\pi$
$= 0.32\pi$

(b) Phase centre located towards point source
Visibility amplitude is unchanged but phase of fringe function = observed phase
So, visibility phase = 0

Visibility depends on the choice of source and location of phase center. So it is different in (a) and (b)

(c) Since source flux density remains the same i.e. 3Jy for baseline 10m & 40m, the visibility amplitude stays the same.
Phase center coincides with the source so visibility phase will be 0 for 2 baselines

Q.8. Two bright points of radio galaxy Cygnus A are separated by 0.71'
Wavelength of interferometer = 6 cm
baseline needed to have a resolution of 0.71' is :-

$$R = \frac{1.02\lambda}{D_{max\text{-}baseline}}$$

here $R = 0.71'$

$= 0.71 \times \frac{1}{60}^{\circ}$

$= 0.71 \times \frac{1}{60} \times \frac{\pi}{180}$ rad

$= \frac{1.02 \times 0.06}{\text{maximum baseline}}$

$\Rightarrow$ Maximum baseline $= \dfrac{1.02 \times 0.06}{0.71 \times \frac{1}{60} \times \frac{\pi}{180}}$

$= 296.32$ m

Q.9. Fringe frequency (given):

$$\frac{d\phi}{dt} = \omega_F = \pm \omega_E \frac{b}{\lambda} \left[\cos\gamma \cos\delta \cos\omega_E t_{HA}\right]$$

where $\omega_E = 7.3 \times 10^{-5}$ rad/s

(a) $\alpha = 8h\ 54m\ 48.86s$

$\delta = 20^{\circ}\ 06'\ 30''.6 = 20.108^{\circ}$

$HA = 2h = 7200s$

$\Rightarrow \lambda = 1.35$ cm

$\frac{d\phi}{dt} = -0.432$ rad/s

$\gamma = 0$

Using the relation from fringe frequency gives :-

$$\frac{d\phi}{dt} = \pm \omega_E \frac{b}{\lambda} \left[\cos\gamma \cos\delta \cos\omega_E t_{HA}\right]$$

$$-0.432 = \pm 7.3\times10^{-5} \times \frac{b}{0.0135} \left[\cos 20.108 \cos(7.3\times10^{-5}\times 7200)\right]$$

$$\Rightarrow b = \frac{-0.432 \times 0.0135}{7.3\times10^{-5} \times 1 \times \cos 20.108 \times \cos(7.3\times10^{-5}\times 7200)}$$

$$= 15.668 \text{ m}$$

(b) $\alpha = 13\text{h } 24 \text{ min } 12.09\text{ s} =$

$\delta = 40^\circ 48' 11.4'' = 40.803^\circ$

$$\frac{d\phi}{dt} = 0.288$$

$HA = ?$

$$0.288 = 7.3\times10^{-5} \times \frac{15.668}{0.0135}\left[\cos 40.803 \cos(7.3\times10^{-5}\times t_{HA})\right]$$

$$\cos(7.3\times10^{-5}\times t_{HA}) = \frac{0.288 \times 0.0135}{7.3\times10^{-5}\times 15.668 \times \cos 40.803}$$

$$t_{HA} = \frac{1}{7.3\times10^{-5}} \cos^{-1}\left[\frac{0.288\times0.0135}{7.3\times10^{-5}\times15.668\times\cos 40.803}\right]$$

$$t_{HA} = 10605.9 \text{ s}$$

$$= 2.95 \text{ h}$$

Q.10. SMA has 8 6m dishes

VLA has 27 25m dishes

Baseline of SMA $= {}^{8}C_2 = \dfrac{8!}{2! \times 6!}$

$= \dfrac{8 \times 7}{2} = 28$

Number of baseline in VLA ${}^{27}C_2$

$= \dfrac{27!}{2! \times 25!}$

$= \dfrac{27 \times 26}{2}$

$= 27 \times 13 = 351$

If one more antenna is added,

number of baselines for

SMA $= {}^{9}C_2 = \dfrac{9!}{2! \times 7!} = \dfrac{9 \times 8}{2} = 36$

% of decrease in baselines $= \dfrac{36-28}{28} \times 100$

$= 28.57\%$

Number of baselines for VLA:

${}^{28}C_2 = \dfrac{28!}{2! \times 26!} = \dfrac{28 \times 27}{2} = 378$

% of increase in baselines $= \frac{379-351}{351} \times 100$

$= 7.69\%$

Each new baseline added corresponds to new points in U-V plots, so, more number of points corresponds to better resolution.

% of increase in baseline on addition of an antenna is seen to be more for SMA than VLA. So, if needed, SMA would benefit more from this antenna addition.

Q.11. Angular size = $3'$
Central bright core has size = $5''$
$\lambda = 1.35$ cm

For a resolution of $5''$, longer baseline is needed with 3 synthesized beams across the source, corresponding baseline would be $5/3''$

Angular size of source $= 3'$

$= 3 \times \frac{1}{60} \times \frac{\pi}{180}$ rad

$= 8.73 \times 10^{-4}$ rad

Angular size of core $= 5''$

$$= \frac{5}{60\times60} \times \frac{\pi}{180} \text{ rad}$$

$$= (8.07 \times 10^{-6}) \text{ rad}$$

UV distance needed will be inverse of angular size of source

$$= \frac{1}{8.73\times10^{-4}} = 1145.4 \text{ rad}^{-1}$$

For a good map of core, UV distance will be $\frac{1}{8.07\times10^{-6}} = 123915$

At $\lambda = 1.35$ cm, baseline needed will be from (0.0135×1145.4) to (0.0135×123915) m

$= 15.4$ m to 1672.86 m

We will need baseline length minimum $= 15$ m & maxima $= 1673$ m for a good map of source.