AA 405/605

Assignment II

Date: 07.09.2021 Deadline: 16.09.2021

1. We have a digital camera with a CCD chip 1024×1024 pixels, sensitive to optical light (0.55 nm). The Chip dimension is 2 cm $\times$ 2 cm. We want to use this CCD for astronomical observations. What should be the focal length if we want to ensure that the corresponding telescope creates an image of a star (point source) as a blob of 2×2 pixels on the CCD?

2. Confirm that the Lick 3-m telescope has plate scale of $\sim 14''$/mm, given its focal length of 15.2 m. What is the focal ratio R of this telescope? A detectable image of a distant star (i.e., a point source) can be obtained on film at the focus of the telescope in an exposure of duration T if the flux density (W m^{-2}) at the telescope is F . The circular image size is $1''$ in radius due to variable atmospheric refraction. How long does it take to detect a uniformly bright nebula of angular radius $2'$ if its total flux is $1000F$?

3. Suppose a telescope has a focal length of 7.2 m. Calculate its plate scale. If the associated CCD is 24.6 mm $\times$ 24.6 mm, what is the field of view of the system?

4. What happens to the SNR of the intensity when your detector is defective and adds a systematic noise of σ_d? Prove that in this case

$$t = \left[\frac{SNR^2}{rate}\right]\left[\frac{1}{2} + \left[\frac{1}{4} + \left[\frac{\sigma_d}{SNR}\right]^2\right]^{\frac{1}{2}}\right]$$

   Assume a photon counting detector with Poissonian noise measuring photon intensity. What happens when $\sigma_d \to 0$?

5. The expression for the significance S/σ_s of a counting measurement derived in the class is valid if the exposure times for the on-source and off-source measurements are equal. Here, let these times be different and designate them $t_s (\equiv t_{s+b})$ and t_b respectively.

(a) Write the counting rate r_s (cts/s) of the source alone in terms of the measured rates r_{s+b} and r_b , and find the standard deviation σ_{rs} of the rate r_s in terms of these rates and the times t_s and t_b. Note that the number of background counts B_s detected "on source" may differ from the number B_b detected "off source". Be careful to distinguish total counts, $S + B_s$ and B_b from the counting rates r_{s+b} , r_b and r_s.

(b) Use your answer to write directly the expression for the significance r_s/σ_{rs} , of the source rate. Find an expression for the time t_s that yields the maximum significance if the observing time $T = t_s + t_b$ is fixed. In other words, how should the time T be apportioned between on-source and background measurements? Hints: substitute $t_b = T–t_s$, define $g \equiv t_s/T$ and solve for the optimum value of g while noting that $r_{s+b} = r_s + r_b$.

(c) Evaluate your expression for three cases: $r_s << r_b$; $r_s = r_b$; $r_s >> r_b$.

6. *Some fun with Poisson distribution*
Let X be the number of photons impinging in time T on a detector. The photon distribution follows a Poisson distribution. Let us assume the mean rate of photons is λ which remains constant (intrinsic). Then calculate the probability that at least a time T has passed between the arrival of two photons?