End Semester Exam

AA 674 / 474 : Radio Astronomy

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Q.1. (c)

Q.2. (a)

Q.3. (a)

Q.4. (e)

Q.5. (b)

Q.6 (b)

Q.7. (c)

Q.9. (b)

Q.9. (a)

Q.10. (a)

SECTION B:

Q.1. (a) Given that synodic period of mars $S = 780$ days.
Mars is a superior planet & we know in that case,

$$\frac{1}{S} = 1 - \frac{1}{P} \Rightarrow P = 687 \text{ days.}$$

Using Kepler's law,
$$\left(\frac{P_1}{P_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

period of body 1

orbital dist. of body 1

period of body 2

orbital dist. of body 2.

Let Earth be body 1.
Then, $P_1 = 365$ days
and $a_1 = 1$ AU.
Given that for Mars, $P_2 = 687$ days.
Using these,

$$\left(\frac{365}{687}\right)^2 = \left(\frac{1}{a_2}\right)^3$$

$$\Rightarrow a_2^3 = \frac{1}{0.531}$$

$$\Rightarrow a_2 \simeq 1.52 \text{ AU}$$

(b) Angular resolution of an interferometer type array is given by:

$$\theta = \frac{1.02\,\lambda}{b_{max}}$$

∴ the maximum resolution will be achieved by the shortest wavelength. So,

$$\theta = \frac{1.02 \times 3 \times 10^{-6}}{1000} = 3.06 \times 10^{-9} \text{ radians}$$

$$= 0.00063117''$$

The diameter of the earth is: 12,742 km

$= 1.2742 \times 10^{7}$ m.

At a distance of 10 pc, this will subtend an angle of:

$$\theta = \frac{1.2742 \times 10^{7}}{10 \times 3.086 \times 10^{16}} = 4.129 \times 10^{-11} \text{ radians}$$

$$= 8.52 \times 10^{-6} \text{ arcsec}$$

This is much, much smaller that the best resolution obtainable by TPF.

The distance between Earth & Sun is 1AU

At 10 pc, this subtends an angle of:

$$\theta = \frac{1.49 \times 10^{11}}{10 \times 3.086 \times 10^{16}} = 4.89 \times 10^{-7} \text{ radians}$$
$$= 0.10086''$$

This distance can be resolved by the TPF since the angular size is larger than the resolution.

(c) Given that the S+B count is: $S+B = n_1$

B count is: $B = n_2$

Time duration is the same for both cases.

Then, signal count can be given as:

$$S = (S+B) - B = n_1 - n_2 \quad \text{—— ①}$$

now considering the two observations to be independent,

$$\sigma_S^2 = \sigma_{S+B}^2 + \sigma_B^2$$

Both $(S+B)$ and B observations are Poissonian.

$$\therefore \sigma_{S+B} = \sqrt{S+B} = \sqrt{n_1} \quad \text{and} \quad \sigma_B = \sqrt{B} = \sqrt{n_2}$$

$$\Rightarrow \sigma_S^2 = n_1 + n_2 \quad \text{—— ②}$$

Signal to noise ratio is given by $\frac{S}{\sigma_S}$

$\therefore$ from ① and ②,

$$\boxed{\frac{S}{\sigma_S} = \frac{n_1 - n_2}{\sqrt{n_1 + n_2}}}$$

(d)
$$x_{av} = \frac{\int_0^\infty x \cancel{N_i} \exp\left(-\sigma n x\right) dx}{\int_0^\infty \cancel{N_i} \exp\left(-\sigma n x\right) dx}$$

$$= \frac{\left(\frac{\cancel{+}1}{\sigma^{\cancel{2}} n^{\cancel{2}}}\right) \cancel{\left[\exp\left(-\sigma n x\right)\right]}\Big|_0^\infty}{\left(\frac{1}{\cancel{-}\sigma n}\right) \cancel{\left[\exp\left(-\sigma n x\right)\right]}\Big|_0^\infty}$$

$$\Rightarrow x_{av} = \frac{1}{\sigma n}$$

Q.2.(a) From the given equation,

$$R = f_v \cos\left(\pi \frac{b}{\lambda} \sin(\omega_E t_{HA})\right)$$

The information we can get from this interferometer does not come from this single measurement. We need to detect the output for a range of times to detect the oscillation in fringes.

(b) The answer to this question can not be infered from this single measurement. But from the set up of the question, we know that the amplitude is 3.00 Jy. and the phase difference can be measured by
$\Delta\phi = 2\pi \frac{b}{\lambda} \sin\omega t$

(c) For the completion of the full cycle, we need to find the t for which the phase difference is 2π.
$\therefore$ we need:

$$\Delta\phi = 2\pi$$

From part (b), $\Delta\phi = 2\pi \frac{b}{\lambda} \sin\omega t$

So, equating the two, we find:

$$\cancel{2\pi} = \cancel{2\pi} \frac{b}{\lambda} \sin \omega t$$

$$\Rightarrow \quad \frac{\lambda}{b} = \sin \omega t$$

$$\Rightarrow \quad \frac{1}{\omega} \sin^{-1}\left(\frac{\lambda}{b}\right) = t$$

given that $\omega = 7.29 \times 10^{-5}$ radians/s

$$\Rightarrow \quad t = 1.372 \times 10^{4} \sin^{-1}\left(\frac{\lambda}{b}\right) \text{ seconds}$$

(d) Given that $\theta = 0.71' = 2.0653 \times 10^{-4}$ radians

$\lambda = 1\,m$

The angular resolution of an interferometer system is given by:

$$\theta = \frac{1.02\,\lambda}{b_{max}} \quad b_{max} \rightarrow \text{Baseline}$$

For my system to be able to resolve the radio galaxy, the angular resolution θ of my system $< 0.71'$.

$\therefore \quad 2.0653 \times 10^{-4} > \frac{1.02 \times 1}{b_{max}} \quad \Rightarrow b_{max} > \frac{1.02 \times 1}{2.0653 \times 10^{-4}}$

$\Rightarrow \quad b_{max} > 4938.75 \text{ m}$

$\Rightarrow \quad b_{max} > 4.94 \text{ km}$

$\Rightarrow$ The baseline of my interferometer must be larger than 4.94 km to observe Cygnus A at given wavelength.

Q.3 Given that $T_{CMB} = 2.73$ K

(a) The Planck's function is :

$$I_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\left(e^{hc/\lambda kT} - 1\right)}.$$

$$= \frac{2 \times 6.63 \times 10^{-34} \times 9 \times 10^{16}}{(10^{-3})^5 \left[\exp\left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-3} \times 1.38 \times 10^{-23} \times 2.73}\right) - 1\right]}$$

$\Rightarrow I_\nu = 6.129 \times 10^{-4} \text{ W m}^{-3} \text{ sr}^{-1}$

This is the brightness of CMB at the given wavelength.

(b) $$I_\lambda = \frac{2c}{\lambda^4} KT$$

$$T = \frac{I_\lambda \lambda^4}{2cK}$$

$$= \frac{6.129 \times 10^{-4} \times (10^{-3})^4}{2 \times 3 \times 10^8 \times 1.38 \times 10^{-23}}$$

$$= 0.074 \text{ K}.$$

This is very much less than the actual temperature of the CMB. Hence, Rayleigh Jeans approximation does not hold true in this case.

(c) At $\lambda = 90$ cm,

$$I_\lambda = \frac{2hc^2}{\lambda^5} \times \frac{1}{\left[e^{hc/\lambda KT} - 1\right]}$$

$$= \frac{2 \times 6.63 \times 10^{-34} \times 9 \times 10^{16}}{(90 \times 10^{-2})^5 \times \left[\exp\left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{90 \times 10^{-2} \times 1.38 \times 10^{-23} \times 2.73}\right) - 1\right]}$$

$$= 3.435 \times 10^{-14} \text{ W m}^{-3} \text{ sr}^{-1}$$

Using Rayleigh Jeans approximation to find

Temperature (as in pt. b) gives:

$$T = \frac{I_\lambda \lambda^4}{2cK} = \frac{3.435 \times 10^{-14} \times (90 \times 10^{-2})^4}{2 \times 3 \times 10^8 \times 1.38 \times 10^{-23}}$$

$$= 2.722 \text{ K}$$

This is much closer to the actual temperature of CMB (2.73 K). Hence Rayleigh Jeans approximation is valid in this wavelength range.

(d) $T = 100\text{K}$

$\lambda = 1\text{mm} = 10^{-3}$ m

$$I_\lambda = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{\left(e^{hc/\lambda KT} - 1\right)}$$

$$= \frac{2 \times 6.63 \times 10^{-34} \times 9 \times 10^{16}}{(10^{-3})^5 \left[\exp\left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.38 \times 10^{-23} \times 10^{-3} \times 100}\right) - 1\right]}$$

$$I_\lambda = 0.77 \text{ W m}^{-3} \text{ sr}^{-1}$$

Considering RJ approximation,

$$T = \frac{I_\lambda \lambda^4}{2cK}$$

$$= 0.77 \times (10^{-3})^4 / (2 \times 3 \times 10^8 \times 1.38 \times 10^{-23})$$

$= 92.97\,K$

$\Rightarrow$ RJ approximation holds good in this case.

Q.4. Power Transmitted $= 200\,mW = P$

Frequency $\nu = 900\,MHz$

Bandwidth $\Delta\nu = 30\,KHz$

(a) distance $= 5$ cm

$$\text{Flux} = \frac{\text{Power}}{\text{Area}} = \frac{200\,(mW)}{4\pi \times (5\,cm)^2} = 0.6366\,mW\,cm^{-2}$$

(b) Since at a distance of 5 cm, the flux is $0.63\,mW\,cm^{-2}$ which is much less than the limit of $10\,mW\,cm^{-2}$, we are not in danger from the cell phone.

(c) The flux density will be given by:

$$F_\nu = \frac{\text{Power}}{4\pi d^2 \times \Delta\nu} \qquad d = 10\,km = 10^4\,m$$

$$= \frac{200 \times 10^{-3}}{4\pi \times (10^4)^2 \times 30 \times 10^3} = 5.3 \times 10^{-15}\,W\,m^{-2}\,Hz^{-1}$$

$$= 5.3 \times 10^{11}\,Jy$$

(d) $\because$ flux density obtained above is larger than 10^9 Jy which causes compression in the receiver, the antenna is not safe.

(e) Diameter $= 45$ m.

Power received $= F_\nu \times \text{Area} \times \Delta\nu$

$$= 5.3 \times 10^{-15} \times \pi \times \left(\frac{45}{2}\right)^2 \times 30 \times 10^3$$

$$= 2.53 \times 10^{-7} \text{ W}$$

Number of photons received per second =

$$\frac{\text{Energy received in 1 second}}{\text{Energy of 1 photon}}$$

$$= \frac{2.53 \times 10^{-7}}{6.63 \times 10^{-34} \times 900 \times 10^6}$$

$$= 4.25 \times 10^{17} \text{ photons.}$$