Radio Astronomy: AA 474N/674N

Assignment II

Date: 11.01.2021 Deadline: 18.01.2021

1. (a) Compute the specific flux F_ν (in Jy, and erg/cm^2/s/Hz) and F_λ in erg cm^{-2} s^{-1} $\AA^{-1}$ from a $V = 22$ magnitude galaxy. What is the rate at which V-band photons from this object in the V band strike the mirror of the Hale 200-inch-diamter telescope from this object?

   (b) A spectrum of this galaxy is taken. The effective entrance aperture of the spectrograph is 2×2 arcsec^2 . The surface brightness of the night sky at Palomar in the V band on a dark night is 20.4 mag/arcsec^2 (i.e., 1 arcsec^2 emits a flux equivalent to that of a V = 20.4 mag object). What is the effective V magnitude of the foreground sky patch as seen by the spectrograph in its aperture?

   (c) Now assume that the overall efficiency of the telescope + spectrograph + detector is 10%. A resolution element in the spectrum is $10\AA$ wide, and we are interested in the region around $\lambda = 5500\AA$. How many counts per resolution element are detected from the galaxy spectrum alone in a 1-hour exposure? From the foreground sky?

   (d) If a blank piece of sky is measured at the same time in order to subtract the sky spectrum from the total, what is the signal-to-noise ratio per resolution element in the final, sky-subtracted galaxy spectrum? (Neglect the detector noise, and assume Poisson photon statistics.)

2. Recently, Phosphine was detected by ALMA on Venus. To get the Phosphine line profile and shape clearly we need to measure at least minimum velocity separation of 1 km/s. What is the frequency separation that can be measured by ALMA and what is the spectral resolution of ALMA which observes at 110 GHz?

3. N number of mono-energetic gamma rays of energy E are vertically incident on an absorber of thickness t (in units of cm) made of an element with atomic number Z. The predominant mode of interaction is photoelectric effect. $E >> E_k$, where E_k is the binding energy of K shell electron. Each K-shell after getting to excited states by incident photon comes back to ground state releasing K X-ray. σ_1 is the absorption cross section (in units of cm^{-1}) for the element Z at energy E and σ_2 is the cross section for the characteristics K X-rays. Assume that for each interaction, fluorescent K

X-rays are emitted with a probability p and further assume that these X-rays are emitted in the forward direction. Derive an expression for N_1, the number of K X-rays coming out of the absorber. [Hint: If N_0 is the number of photons incident on the detector and N is the number of photons surviving a distance x then $N = N_0 \exp(-\sigma N_a \rho x/Z)$ where σ is the cross-section, N_a is the Avogardo number, ρ is the density of material of detector and Z is the atomic number of the detector]

4. Find x_{av} , the average distance traversed by photons in a beam passing through a detector being attenuated as $N(x) = N_i exp(-\sigma n x)$ where n is the number of target atoms in the detector interacting with the impinging photons. Carry out the integration required by the formal definition of an average taking into account all path lengths from 0 to ∞. Hint: the probability of an interaction occurring at the distance x is proportional to the number of photons remaining in the beam at that x.

5. The expression for the significance S/σ_s of a counting measurement derived in the class is valid if the exposure times for the on-source and off-source measurements are equal. Here, let these times be different and designate them $t_s(\equiv t_{s+b})$ and t_b respectively.

   (a) Write the counting rate r_s (cts/s) of the source alone in terms of the measured rates r_{s+b} and r_b , and find the standard deviation σ_{rs} of the rate r_s in terms of these rates and the times t_s and t_b. Note that the number of background counts B_s detected "on source" may differ from the number B_b detected "off source". Be careful to distinguish total counts, $S + B_s$ and B_b from the counting rates r_{s+b} , r_b and r_s.

   (b) Use your answer to write directly the expression for the significance r_s/σ_{rs} , of the source rate. Find an expression for the time t_s that yields the maximum significance if the observing time $T = t_s + t_b$ is fixed. In other words, how should the time T be apportioned between on-source and background measurements? Hints: substitute $t_b = T - t_s$, define $g \equiv t_s/T$ and solve for the optimum value of g while noting that $r_{s+b} = r_s + r_b$.

   (c) Evaluate your expression for three cases: $r_s << r_b$; $r_s = r_b$; $r_s >> r_b$.

6. A detector has negligible dead time. However, the information is digitized and kept ready for a personal computer (PC) to collect it. The PC interrogates the system every T seconds and collects it. Essentially in the time interval T, if there is one or more event, the first event will be collected and the rest neglected. Derive a relation between the true event rate and the observed event rate.