AA 474N/674N: Radio Astronomy
Assignment 5

1. Calculate the diffraction limited field-of-view (FWHM) for a single VLA antenna in arcminutes at all the frequencies that the VLA operates at (74, 330, 1400, 5000, 8400,15000, 22200, and 43000 MHz).

2. A typical aperture efficiency for the EVLA at 43 GHz is 35%.
(a) What is the rms surface error in mm on an EVLA dish?
(b) If the indentation left by someone walking on the surface of the dish can be as large as 0.5mm, is it a good idea to let people on the surface? Actually we can reduce the indentation by walking carefully on the edges of the panels, so if we get the chance, we will.

3. Consider the power output of a cat. Assume that all the energy intake in a day, 300 kcal, all goes into heat and is radiated away, and that no useful work is done (we consider here a day spent napping). Note that 1 kcal = 4185 J.
(a) Give the average power output of the day in Watts.
(b) At what wavelength (in cm) will the emission be peaked?
(c) If you launched your cat into space such that it appeared to be 1 arcsecond across with the VLA, and it kept the same room temperature you used for part (b), what would be its fluxdensity at 3 GHz in Jy?

4. Suppose that we have an ideal square antenna (constant gain across the aperture) with a diameter that can be described by a box function of width D that is unity between –D/2 and D/2 and zero everywhere else, derive the primary beam of the antenna as a function of angle (θ) from the axis of the antenna surface, and the diameter D. At what angle is the first null (where the response of the antenna goes to zero)?

5. Given a blackbody spectrum, give a relation in the Rayleigh-Jeans part of the spectrum for the photon flux (number of photons emitted per cm^2 per second per Hertz) as a function of frequency.

6. (a) What is the flux density (in Jy) of a source that radiates a power of 600 W in the radio frequency band uniformly from 2.7 to 2.8 GHz, when placed at the distance of the Moon?
(b) Repeat for an identical source if the radiation is in the optical frequency band, from 375 nm to 1000 nm. Recall 1nm = 10^{-7} cm.

7. (a) How much power (in Watts) is delivered to a receiver at the Green Bank telescope, a 100 m diameter telescope (the largest fully steerable radio telescope), when observing the bright radio source 3C286? The bandwidth is 200 MHz wide and the observing frequency is 4500 MHz. The current flux density of this calibrator at 4500 MHz is 1.49 Jy.
(b) How many photons will the GBT receive each second from 3C286?

8. A radio source has a flux density of 5.5 Jy at 1400 MHz and has an angular size of 10” x 10”. What is the temperature of the equivalent blackbody, that is what is the brightness temperature, T_b, that would give the same flux density?

9. The WMAP satellite has determined that the temperature of the cosmic microwave background (CMB) is 2.73 K.

(a) Use Planck's function to determine the brightness of the CMB at a wavelength of 1 mm.
(b) Use the value of part (a) at 1 mm and the Rayleigh-Jeans approximation to the Planck function. What temperature do you derive for the microwave background? (Basically, this shows how good/bad the RJ approximation is at this temperature and wavelength).
(c) Redo (a) and (b) at 90 cm to find the temperature you would obtain if you measured the CMB at 90 cm?
(d) Suppose you had an emitting molecular cloud at T=100 K. Redo part (b) using 1 mm wavelength to see how good the Rayleigh-Jeans approximation is at this temperature.

10. A typical cell phone transmits 200 mW of power at 850 MHz with a bandwidth of 30 kHz.

(a) What is the flux (in mW/cm^2) from the antenna at a distance of 5cm?
(b) Compare that to the "harmful" threshold set by the FCC of 10 mW/cm^2. Are you in danger from your cell phone?
(c) What is the flux density from your cell phone (in Jy) that a 25 m radio antenna 10 km away would see?
(d) If signals larger than 10^9 Jy cause gain compression in the receiver, is the antenna safe?
(e) How many photons would this 25 m antenna intercept each second?

11. A two-element interferometer operating at a wavelength of 100 cm has a separation of 100 meters. Write down an expression relating the geometric delay to the total phase delay of the wave (φ). Show that if the waves are coming from a direction 60 degrees from the horizon then an error of 0.1 m in measuring the baseline length will produce an error of 0.1π in φ_0 where $\varphi_0 = \varphi - 2\pi n$ is between 0 and 2π and is the actual phase delay to be applied. Show that a 0.1% error in the baseline length is amplified to a 5% error ($(0.1\pi/2\pi)*100$) in the phase because the separation is many wavelengths.

12. The Arecibo radio telescope in Puerto Rico is a 305 m diameter filled-aperture spherical dish, while the Very Large Array has 27 parabolic dishes each 25 m in diameter with a maximum baseline length in A configuration of 36 km.

Calculate the ratios of: (a) total collecting areas, and (b) the angular resolution for the two telescopes. (c) Which telescope might be better for detecting faint sources in a short amount of time, and which one would be better for observing structure on small angular scales?

13. Suppose you are observing a cluster of sources with the VLA in A configuration at 5 GHz and the widest separation between sources is 5 arcminutes.

(a) What is the maximum bandwidth that you can employ so as not to degrade the peak of any source due to bandwidth smearing by more than 5%?
(b) What is the maximum integration time allowable for the same limit of 5%?
(c) Is the primary beam of the VLA big enough to accommodate this observation?

14. More on preparing for observing. Lets say that your favorite compact source, 3C84, is in Perseus and you want to reach a sensitivity of 10 microJy/beam (for good polarimetry).

(a) What is the RA and Dec (J2000) to a precision of 0.1 arcseconds?
(b) What is the galactic latitude and longitude?

(c) Use the table in the VLA Status Summary or the exposure calculator to estimate how much time in total you need to ask for if you want to observe with the VLA using 2 GHz bandwidth at 5, 9 and 15 GHz to this noise limit?
(d) Add 0.3 hours for absolute flux calibration, what LST (Local Sidereal Time) range do you want to ask for?
(e) What local time range does this correspond to on March 1? Note that there are various coordinate calculators (e.g., NED) and I am not asking you to make the various transformations by hand (though you are welcome to do so).

15. You make VLA images of a point source at all 4 stokes and find I=50.0, Q=3, U=-5.2, and V=-0.1 mJy.
(a) Calculate the linearly polarized intensity, percentage of linear polarization, and give the polarization angle in degrees.
(b) Calculate the percentage of circular polarization.
(c) Note that Q, U, and V can be positive or negative. What about stokes I, can that ever be negative?