ASSIGNMENT 3
AA 474/674

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Q1. (a) We first note that the conversion between f_λ and f_ν is the same as between I_λ and I_ν, which is given by the relation:

$$I_\lambda = \frac{c}{\lambda^2} I_\nu$$

The flux per Hz at 5500 Å is:

$$f_\nu = \frac{\lambda^2}{c} f_\lambda = \frac{(4250 \times 10^{-10}\ m)}{3 \times 10^8\ m s^{-1}} 2.3 \times 10^{-8} W m^{-2} Å^{0-1} (10^{10} Å m^{-1})$$

$$= 1.38 \times 10^{-29}\ W\ m^{-2}\ Hz^{-1}$$

$$= 1.38 \times 10^{-3}\ Jy$$

This is 4 orders of magnitude smaller than the radio-frequency flux density.

Now converting the radio frequency flux density measurement to match that in the visible. We get:

$$f_\lambda = \frac{c}{\lambda^2} f_\nu = \frac{\nu^2}{c} f_\nu$$

$$= \frac{2.22\times10^{10}\ Hz}{3\times10^{8}\ m/s}\ 4.2\times10^{-25}\ Wm^{-2}\ Hz^{-1}\left(10^{-10}\ m\,\mathring{A}^{-1}\right)$$

$$= 6.9\times10^{-23}\ W\,m^{-2}\ \mathring{A}^{-1}$$

This is 15 orders of magnitude smaller than the visible wavelength flux density.

(b) The detected fluxes are given by multiplying the measured flux densities by the bandwidths.
For the radio, this is:

$$F = 4.2\times10^{-26}\ W\,m^{-2}\ Hz^{-1}\times50\times10^{6}\ Hz$$
$$= 2.1\times10^{-18}\ W\,m^{-2}$$

while for the visible, we find

$$F = 2.3\times10^{-8}\ W\,m^{-2}\ \mathring{A}^{-1}\times50\ \mathring{A}$$
$$= 1.15\times10^{-6}\ W\,m^{-2}$$

$\therefore$ the total detected flux at visible wavelengths is much larger

Q.2. (a) $L = 5\times10^{6}\ J/100\ sec = 5\times10^{4}\ W = 50\ kW$

(b) Here we can use the equivalence of intensity and surface brightness. The intensity is the power emitted per area of the emitting surface per solid angle of the transmitted beam per bandwidth, i.e.:

$$I_\nu = \frac{5 \times 10^4 \text{ W}}{(\text{A of transmitting antenna}) \times (\Omega \text{ of beam}) \times \Delta\nu}$$

We are given that the transmitting area, beam solid angle, and bandwidth are:

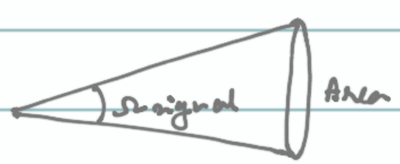
$$A = \pi(50.0\,\text{m})^2 = 7.85 \times 10^3\,\text{m}^2, \quad \Omega = 9.3 \times 10^{-6}\,\text{sr}$$

and $\Delta\nu = 1.00 \times 10^6$ Hz

$$\text{So, } I_\nu = \frac{5 \times 10^4}{(7.85 \times 10^3\,\text{m}^2) \times (9.3 \times 10^{-6}\,\text{sr}) \times 1 \times 10^6\,\text{Hz}}$$

$$= 0.685 \text{ W Hz}^{-1}\,\text{m}^{-2}\,\text{sr}^{-1}$$

(c) i. By beaming the radiation, the alien civilization directs and confines the radiation to a cone of small angle Ω_{signal}. Now, as the radiation travels outward, the area of the end of the cone is given by $\Omega_{signal}\, d^2$, as depicted below:



The flux, then, is given:

$$f = \frac{L}{\Omega_{signal}\, d^2}$$

To measure the flux density at the distance of the Earth is:

$$f_\nu = \frac{L}{\Omega_{emitted}\, d^2\, \Delta\nu_{emitted}}$$

$$= \frac{5 \times 10^4 \text{ W}}{9.3 \times 10^{-6} \text{ sr} [7 \text{ ly} \times 3.09 \times 10^{16} \text{ m/pc}]^2 \times 10^6 \text{ Hz}}$$

Then, the flux density is:

$$f_\nu = 1.15 \times 10^{-31} \text{ W Hz}^{-1} \text{ m}^{-2} = 1.15 \times 10^{-5} \text{ Jy}$$

This is very small and would be challenging to detect.

ii. Detected power

= (detected flux density) × (area of telescope) × (bandwidth)

$= 1.15 \times 10^{-31} \text{ W Hz}^{-1} \text{ m}^{-2} \times \pi \times 3.5^2 \times 5 \times 10^5$

$= 2.21 \times 10^{-24}$ W

Q.3.(a) As given by the equation

$$I_\nu = \frac{f_\nu}{\Omega}$$

the average intensity is the flux density divided by the solid angle of the source, and since the source is found to be a uniform circle, the intensity equals the average intensity. We first need to find the solid angle, since we

are only given the angular diameter.
Using the equation

$$\theta(\text{radians}) \simeq \frac{l}{r} \quad (\text{for small angles})$$

the source subtends an angle of:

$$\Omega = \frac{\pi}{4}\left(30'' \frac{1^\circ}{3600''} \times \frac{\pi}{180}\right)^2$$

$$= 1.66 \times 10^{-8}\ \text{sr}$$

The intensity is then:

$$I_\nu = \frac{20\ \text{Jy}}{1.66 \times 10^{-8}\ \text{sr}} = 1.2 \times 10^{-14}\ \text{W m}^{-2}\ \text{Hz}^{-1}\ \text{sr}^{-1}$$

(b) The total detected power equals the flux density multiplied by the bandwidth and the collecting area of the telescope,

$$P = F_\nu A_{\text{telescope}} \Delta\nu$$

So,

$$P = 20\ \text{Jy}\ (10^{-26}\ \text{W m}^{-2}\ \text{Hz}^{-1}/\text{Jy})\ (300\ \text{m}^2)(250 \times 10^3\ \text{Hz})$$
$$= 1.5 \times 10^{-17}\ \text{W}$$

(c) The luminosity of the source, over the observed spectral range, is given by the observed flux density times the bandwidth, and then using the equation

$$f = \frac{L}{4\pi d^2}$$

So,

$$L = 20\ \mathrm{Jy}\left(10^{-26}\ \mathrm{W\ cm^{-2}\ Hz^{-1}/Jy}\right) \times 250 \times 10^{4}\ \mathrm{Hz}\left(4\pi\left(20\times10^{6}\ \mathrm{pc} \times 3.09\times10^{16}\ \mathrm{m/pc}\right)^{2}\right)$$
$$= 2.4 \times 10^{29}\ \mathrm{W}$$

Q. 4. (a) The stone bronze is solid and opaque and so the Planck function is a good approximation to the intensity emitted. Therefore, we can use

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/KT)-1}$$

to solve for the temperature.

$$1.46\times10^{-28}\ \mathrm{W\ Hz^{-1}\ m^{-2}\ sr^{-1}} = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{KT}\right)-1}$$

$$= \frac{2(6.626\times10^{-34}\ \mathrm{Js})(3.33\times10^{14}\ \mathrm{Hz})^3}{3\times10^{8}\ \mathrm{m/s}}\frac{1}{\exp\left(\frac{(6.626\times10^{-34}\ \mathrm{Js})(3.33\times10^{14}\ \mathrm{Hz})}{(1.38\times10^{-23}\ \mathrm{J/K})\ T}\right)-1}$$

This gives : $\exp\left(\frac{1.6\times10^{4}K}{T}\right)-1 = \frac{5.44\times10^{-7}}{1.46\times10^{-28}} = 3.73\times10^{-21}$

Rearranging, we get,

$$T = \frac{1.60\times10^{4}\ K}{\ln(3.73\times10^{21})} = 322\ K$$

(b) Since the brownie is opaque at almost all wavelengths, we can approximate the total flux emitted by the Stefan - Boltzmann law. Then, we have:

$$F = \sigma T^4 = 5.67\times10^{-8}\ W\ m^{-2}\ K^{-4}(322\ K)^4$$
$$= 609\ W/m^2$$

Q5. (a) We need to find the frequency at which B_ν peaks at this temperature, hence, we use the equation

$$\nu_{peak} = (5.879\times10^{10}\ Hz/K)T,$$

we have $\nu_{peak} = 1.82\times10^{13}\ Hz$

This corresponds to a wavelength of

$$\lambda_{peak} = \frac{c}{\nu_{peak}} = \frac{3\times10^{8}}{1.82\times10^{13}} = 16.5\ \mu m$$

which is the IR band.

(b) Now we want the peak of I_λ, which requires the equation

$$\lambda_{peak} = (2.898 \times 10^{-3} \, mK)/T$$

to get $\lambda_{peak} = 9.33 \, \mu m$ at 310 K

(c) The average energy of the photons is given by:

$$\langle E_{ph} \rangle = 2.7 \, kT = (3.73 \times 10^{-23} J/K) \, T$$
$$= (3.73 \times 10^{-16} erg \, K^{-1}) T$$

and so,

$$\langle E_{ph} \rangle = 1.16 \times 10^{-20} J \quad \text{at } 310K.$$

which corresponds to a wavelength of

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} Js)(3 \times 10^{8} m/s)}{1.16 \times 10^{-20} J}$$

$$= 17.2 \, \mu m$$

(d) If the temperature of the radiating surface increases, we will see an increase in intensity at all frequencies.

Q.6 (a) Since the flux density is measured at 6 cm, which is still in the realm where the Rayleigh Jeans approximation works well, we solve for the Temperature:

$$F_\nu = \frac{2KT}{\lambda^2}\Omega$$

So, $350\,Jy = 3.5\times10^{-24}\ W\,m^{-2}\,Hz^{-1}$

Solving for T gives us:

$$T = \frac{3.5\times10^{-34}\ W\,m^{-2}\,Hz^{-1}\,(0.06\,m)^2}{(2\times1.38\times10^{-23}\ J/K)\,7.18\times10^{-6}\,sr}$$
$$= 63.6K.$$

(b) Using the equation:

$$B_\nu(T) = \left(\frac{2h\nu^3}{c^2}\right)\frac{KT}{h\nu} = \left(\frac{2k\nu^2}{c^2}\right)T$$
$$\approx \frac{2KT}{\lambda^2} \quad \text{— (1)}$$

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{2.7\times10^{-2}} = 11.1\ GHz.$$

$$\therefore I_{11.1GHz} = \frac{2(1.38\times10^{-23}\ JK^{-1})\,63.6K}{(0.027)^2}$$
$$= 2.41\times10^{-18}\ W\,m^{-2}\,Hz^{-1}\,sr^{-1}$$

(c) As equation (1) above shows, the Rayleigh-Jeans part of the blackbody spectrum, the intensity is directly proportional to the temperature of the radiating body. So, if the second source is twice as hot as the first, then its intensity at the same wavelength will be twice as great. The intensity of the second source, then is $4.82 \times 10^{-18}\ W m^{-2} Hz^{-1} sr^{-1}$.

(d) Brightness temperature is given by:

$$T_b = \frac{\lambda^2 I_\nu^2}{2k_B}$$

from above, $I_\nu = 2.41 \times 10^{-18}\ W m^{-2} Hz^{-1} sr^{-1}$

$$T_b = \frac{(0.027)^2 \times (2.41 \times 10^{-18})}{2 \times (1.38 \times 10^{-23})} = 63.6K$$

Q.7. (a) The radiation is unpolarized only if Q, U and V are all equal to 0. Since Q and U are not zero, the radiation is polarized. Q and U are measures of linear polarization while V is a measure of circular polarization. Since V=0 and Q and U are not zero, the radiation is linearly polarized.

(b) The percentage of polarization is given by L/I where L is given by

$$L = \sqrt{Q^2 + U^2}$$

and $I = 0.5$ Jy/beam

∴ the polarization is:

$$2\sqrt{Q^2+U^2} = 2 \times (-0.003)^2 + 0.005^2 = 0.01$$

$\Rightarrow$ 1% polarized.

The polarization angle is given by:

$$\theta = \frac{1}{2} \tan^{-1}\left(\frac{U}{Q}\right)$$

∴ angle of polarization is:

$$\theta = \frac{1}{2} \tan^{-1}\left(\frac{0.004}{0.003}\right) = 63.4^\circ$$

Q.8. (a) The RA of the zenith is given by LST, so RA = 3h 00m 00s, and since the lines of declination are extensions of the lines of latitude, the Dec of an observer's zenith is the same as the observer's latitude, so the astronomers' zenith is at Dec = +35°

(b) The HA is easily calculated by subtracting the RA from the LST of the target source, so,

HA = 03h 00m 00s − 12h 30m 00s = −9h 30m 00s

This means the best time to observe the source is in another 9 hours or so.

(c) Since Dec of the source is 0°, it is on the celestial equator. Hence, it is above the horizon for exactly half the time, i.e. 12h per day. So it rises 6 hours before it transits, or when its HA = −6h. The HA of the object is more negative than −6h and so, the object is further from the meridian than the horizon is, and hence the object is below the horizon.

(d) When an object in the sky rises, it is on the horizon and so its altitude at that time is 0°. To find out its azimuth, we note that the object is on the celestial equator, and so it rises due East. This happens because the celestial equator passes through the points directly East and west for observers at all latitudes, and so its azimuth at that time is $+90^\circ$.

(e) When a body transits, it is on the meridian and so it is either at zenith or due North or South of zenith. We can find out the exact direction by comparing the declinations of the object and the zenith (latitude of the observer)

In this case, $Dec_{source} < Dec_{zenith}$, so, it will be due South of the zenith when it transits. Thus, its azimuth will be $+180°$.

$$\text{altitude} = 90° - (\text{Dec of zenith} - \text{Dec of sky position})$$
$$= 90° - (35° - 0°)$$
$$= 55°$$

Q.9. Using the expression:

$$\langle I_\nu \rangle = \frac{2K}{A_{eff}\,\Omega_{main}} T_A$$

converting antenna temperature to intensity, we get:

$$\langle I_\nu \rangle = \frac{2 \times 1.38 \times 10^{-23}}{23.1 \times 5.58 \times 10^{-6}} 0.03 = 6.42 \times 10^{-21} \ W m^{-2} Hz^{-1} sr^{-1}$$

In terms of brightness temperature, this is:

$$\langle T_B \rangle = \frac{(3 \times 10^8)^2}{2 \times 1.38 \times 10^{-23} \times (22.2 \times 10^9)^2} \times 6.42 \times 10^{-21}$$
$$= 0.425K$$

To get the answer in Jy/beam, we set $\Omega_{main} = 1$ and convert to Jy. This gives:

$$\langle I_\nu \rangle = \frac{2 \times 1.38 \times 10^{-23}}{23.1} \times 0.83 \times 10^{26} = 35.8\ Jy/beam$$

The source's minor axis is smaller than the main beam and so some of the main beam is empty. The source's average intensity, therefore, is probably larger than 35.8 Jy/beam. But, the source's major axis is somewhat larger than the main beam, and so some of the source's flux density is not detected. We can infer, then, that the source's total flux density is a little larger than 35.8 Jy, and its average intensity is larger than $6.42 \times 10^{-21}\ W\ m^{-2}\ Hz^{-1}\ sr^{-1}$