23/01/21 ASSIGNMENT #2

Kishlay Singh
2003121005

Que 1 → Let the doors be labelled a, b, c, where a is the door you chose initially and b is the door that is opened.

Let p(a) = probability that a leads to the desired prize. and B = the event that door B gets opened and leads to junk.

The aim is to calculate $P(a/b)$ i.e. the probability of opening a to find the prize given that B leads to junk.

We can use Baye's theorem for this:

$$P(a/B) = \frac{P(a, B)}{P(B)} = \frac{P(B/a)\, P(a)}{P(B)}$$

Now, clearly, $P(a) = p(b) = p(c) = 1/3$ (all are equally likely without any experiment knowledge.)

$P(B/a)$ = probability that b is opened given that a leads to prize.

Evidently, $P(B/a) = \frac{1}{2}$

Person could have opened either b or c since they both lead to junk.

and $P(B) = P(B, a) + P(B, b) + P(B, c)$
$= P(B|a)\,p(a) + P(B|b)\,p(b) + p(B|c)\,p(c)$.

We know that $p(a) = p(b) = p(c) \sim 1/3$, $P(B|a) = 1/2$
and $P(B|b) = 0$

($\because$ person will not open b since it leads to the prize in this case).

Given that you have chosen a, then if c leads to the prize, Monty Hall must open b

$\therefore$ $P(B|c) = 1$

$\therefore$ The probability that your ~~orig~~ original choice leads to the prize is :

$$P(a|B) = \frac{P(B|a)\,p(a)}{P(B|a)\,p(a) + P(B|b)\,p(b) + P(B|c)\,p(a)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{3} \times \frac{1}{2} + \left(0 \times \frac{1}{3}\right) + \left(1 \times \frac{1}{3}\right)} = \frac{1}{3}$$

So you would double your chances from $\frac{1}{3}$ to $\frac{2}{3}$ if you open the other door.

Que2 → The model probability distribution for S is :

$$p(s)\,ds = (\alpha - 1)\left(\frac{s}{s_0}\right)^{-\alpha} \frac{ds}{s_0}$$

$\alpha - 1$ and powers of s_0 arise from the normalization requirement.

$$\int_{S_0}^{\infty} ds\, p(s) = 1$$

Likelihood function L:

Dividing the flux range into [a very large number] N small bins of width ΔS so that each bin has 0 or 1 source in them. The probability of having 0 sources is $e^{-\lambda}$ and probability of having 1 source is $\lambda e^{-\lambda}$, where.

$\lambda = p(S)\,\Delta S$ is the expectation value. If $\lambda \ll 1$ (take $\Delta S \to 0$) then probabilities are $1-\lambda$ and λ respectively.

The joint probability of getting the data observed is:

$$\prod_{\text{empty cells}} (1-\lambda) \prod_{\text{Filled cells}} \lambda$$

Ignoring the first product, this becomes:

$$\prod_{\text{Filled cells}} P(S_i)\,\Delta S$$

$\therefore$ the likelihood is (the ΔS terms just affect the proportionality)

$$L(\alpha) \propto \prod_{i=1}^{n} (\alpha-1)\, S_0^{\alpha-1}\, S_i^{-\alpha}$$

Taking ~~log~~ $\log_e$ on both sides and ignoring classmate

Date ________
Page ________

a ~~cont~~ constant.

$$\ln L = \sum_{i=1}^{n} \left[\ln(\alpha - 1) + (\alpha - 1)\ln S_0 - \alpha) \ln S_i \right]$$

Maximizing $\ln L$ w.r.t. α:

$$\frac{\partial}{\partial \alpha} \ln(L) = \sum_{i=1}^{n} \left(\frac{1}{\alpha - 1} + \ln S_0 - \alpha \ln S_i \right) = 0$$

and we find the minimum for

$$\alpha = 1 + \frac{n}{\sum_{i=1}^{n} \ln\left(\frac{S_i}{S_n}\right)}$$

Suppose we observe only one Source with flux twice that of the ~~tall~~ cut-off, $S_1 = 2S_0$, then,

$$\alpha = 1 + \frac{1}{\ln 2} = 2.44$$

but with a large uncertainity.

we have two pieces of information here. ∵ So is known, as slope can be found even with one object. for example if the slope is steps, we can say that S_1 is expected to be close to S_0.

Que 3 → The extra information in this case is that the ~~cheete~~ ~~chees~~ Cheese was stolen

Now, probability of theft of Cheese is $P(C)$.

The probability that the person is a known theif is $P(T)$.

The probability of the person starting is $P(S)$.
The probability that the person is a know theif is $P(T)$

The probability of the person starting is $P(S)$.
So, we have: $P(S/T) = 0.0004$
and ~~P~~ $P(C/S, T) = 1$

Using ~~By~~ Baye's Theorem:

$$P(S|C,T) = \frac{P(C|S,T)\ P(S/T)}{P(C/S,T)\ P(S,T) + P(C|\sim S,T)\ P(\sim S/T)}$$

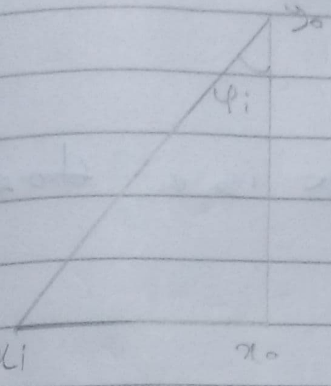
Now, $P(C|S,T) = 1$
$P(S|T) = 0.004$
and $P(\sim S/T) = 0.9996$

$P(C|\sim S,T)$ is the probability that there is a cheese theft given that the perpetual is not a theif. For this, we take the probality of a theft in the general population, i.e., $P(C|\sim S,T) = 0.00005$, so,

$$P(S|C,T) = \frac{1 \times 0.0004}{1 \times 0.0004 + 0.0005 \times 0.9996} = 0.89$$

~~Put ano~~ Putting another way, for the landlords / ladies of 100,000 ~~pretty~~ petty thieves, there will be only about 40 cheese thefts and there will only be 5 thefts by strangers.

Que 4 →



We want to know $P(x_0, y_0 | \{x_i\})$
Now given the position of the light house, ~~the~~ the probability of recording a flash is $P(x_i | x_0, y_0)$

$$\therefore P(x_0, y_0 | \{x_i\}) \propto P(\{x_i\} | x_0, y_0)\, P(x_0, y_0)$$

$$\therefore P(x_0, y_0 | \{x_i\}) \propto \prod_i P(x_i | x_0, y_0)$$

Let the angle of the direction of the flash to the normal to the coastline be φ, ~~&~~ Then by trigonometry, the position that the flash arrives at is given by

$$\tan \varphi_i = \frac{x_i - x_0}{y_0}$$

So,
$$P(x_i | x_0, y_0) = P(\varphi_i | x_0, y_0) \Big/ \left| \frac{d\varphi_i}{dx_i} \right|$$

and for signals that are ~~regu~~ required on the shore, φ_i is uniformly distributed in $-\pi/2 < \varphi < \pi/2$, So $P(\varphi_i) = 1/\pi$ in this ~~range, inde~~ ~~in~~ range, independent of x_0, y_0, Also

$$\sec^2\varphi_i \frac{d\varphi_i}{dx_i} = \frac{1}{y_0}$$

$$\Rightarrow \left[1 + \left(\frac{x_i - x_0}{y_0}\right)^2\right] \frac{d\varphi_i}{dx_i} = \frac{1}{y_0}$$

and the likelihood of x_i ~~is~~ is a ~~the~~ Cauchy distribution.

$$P(x_i | x_0, y_0) = \frac{1}{\pi y_0 \left[1 + \frac{(x_i - x_0)^2}{y_0^2}\right]}$$

Hence, the ~~the~~ unnormalized posterior for x_0, y_0 is

$$P(x_0, y_0 / \{x_i\}) \propto \prod_{i=1}^{N} \frac{1}{\pi y_0 \left[1 + \frac{(x_i - x_0)^2}{y_0^2}\right]}$$

which is our desired outcome.