Assignment 5
AA 674

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Q.1. The diffraction limited FOV is given by:

$$\frac{1.02\lambda}{D} = \frac{1.02c}{D\nu}$$

diameter of dish ← D

For VLA, $D = 25$ m. $\therefore$ $FoV = \frac{1.02c}{25\nu} = \frac{12.24}{\nu_{MHz}}$

$\therefore$ for 74 MHz:

$$FoV_{74} = \frac{12.24}{74} = 0.165 \text{ rad} = 568.6'$$

330 MHz : $FoV_{330} = \frac{12.24}{330} = 127.5'$

1400 MHz : $FoV_{1400} = \frac{12.24}{1400} = 30.06'$

5000 MHz : $FoV_{5000} = \frac{12.24}{5000} = 8.42'$

8400 MHz : $FoV_{8400} = \frac{12.24}{8400} = 5.01'$

15000 MHz : $FoV_{15000} = \frac{12.24}{15000} = 2.8'$

22000 MHz : $FoV_{22000} = \frac{12.24}{22000} = 1.91'$

43000 MHz : $FoV_{43000} = \frac{12.24}{43000} = 0.98'$

Conversion used:

$1 \text{ rad} = \left(\frac{180}{\pi} \times 60\right)$ arcminutes.

$\Rightarrow 1 \text{ rad} = 3437.75'$

Q.2. Given that aperture efficiency of EVLA at 43 GHz is 35%.

(a) RMS surface error in mm on EVLA dish:

$$\frac{A_0}{A} = \eta_{sf} \rightarrow \text{Reflector's surface efficiency}$$

$$\eta_{sf} = e^{-(4\pi\sigma/\lambda)^2} \qquad \sigma = \text{rms surface error}$$

at 43 GHz, $\lambda = \frac{3\times10^8}{43\times10^9} = 6.98\times10^{-3}$ m

$$\Rightarrow \exp\left[-\left(\frac{4\pi\sigma}{6.98\times10^{-3}}\right)^2\right] = \frac{35}{100}$$

Taking natural log on both sides,

$$-\left(\frac{4\pi\sigma}{6.98\times10^{-3}}\right)^2 = \ln 0.35 = -1.05$$

$$\Rightarrow \left(\frac{4\pi\sigma}{6.98\times10^{-3}}\right)^2 = 1.05$$

$$\Rightarrow \sigma^2 = \frac{(6.98\times10^{-3})^2 \times 1.05}{16\pi^2}$$

$$\Rightarrow \sigma = 5.69\times10^{-4}\ m = 0.569\ mm$$

(b) The indentation left is 0.5 mm. Hence, indentation would be rms surface error.

According to Ruze equation $\left(\eta_s = e^{-(4\pi\sigma/\lambda)^2}\right)$

Surface efficiency declines rapidly if the rms surface error exceeds $\frac{\lambda}{16}$

Here, $\lambda = 6.98\times10^{-3}\ m$

$\therefore$ the limit is : $\frac{6.98\times10^{-3}}{16} = 4.3625\times10^{-4}\ m$

$$\Rightarrow \frac{\lambda}{16} = 0.43625\ mm$$

As indentation of 0.5 mm > $\lambda/16$, hence, people should not be allowed to walk on the surface of the dish to maintain efficiency.

Q.3. Energy intake in a day by a cat = 300 kcal.

(a) Av power output of the day in Watts:

$$P_{av} = \frac{300 \times 1000 \times 4.185}{24 \times 60 \times 60} \quad [1\,cal = 4.185\,J]$$

$$= 14.53\ W$$

(b) To find peak of a perfect blackbody:

Using Wein's displacement law:

$$\lambda_{peak} T = 2.898 \times 10^{-3}\ mK.$$

At room temperature (300 K)

$$So,\ \lambda_{peak} = \frac{2.898 \times 10^{-3}}{300}$$

$$= 9.66 \times 10^{-3}\ m$$

$$= 0.97\ cm$$

(c) If cat is launched and appears 1" across with VLA, at room temperature ($T = 300\,K$), flux density at $\nu = 3\,GHz$ in Jy:

Flux density $= I_\nu \times \Omega$

Where $I_\nu =$ Specific Intensity

$\Omega =$ Solid angle

So, using Planck's law to evaluate I_ν:

$$I_\nu = \frac{2K\nu^2}{c^2}T \Rightarrow \frac{2\times1.38\times10^{-23}\times(3\times10^9)^2\times300}{(3\times10^8)^2}$$

$\Rightarrow I_\nu = 8.28\times10^{-19}\ \mathrm{W\,m^{-2}\,s^{-1}\,sr^{-1}\,Hz^{-1}}$

$$\Omega = \pi \times \left(\frac{1}{2}\times\frac{\pi}{180}\right)^2 \times \left(\frac{1}{60\times60}\right)^2 \text{ sr}$$

$$= 1.85\times10^{-11} \text{ sr} \qquad \left(1'' = \frac{1}{60\times60}^{\circ}\right)$$

So, flux density, $F_\nu = I_\nu \times \Omega$

$= 8.28\times10^{-19} \times 1.85\times10^{-11}$

$= 1.5318\times10^{-29}$

$= 1.5318$ mJy

Q.5. Relation of photon flux as function of frequency:
Planck's formula is given by:-

$$I_\nu = \frac{2h\nu^3}{c^2} \times \frac{1}{e^{h\nu/KT} - 1}$$

In Rayleigh-Jean part of spectrum, for small frequency i.e. in $h\nu << KT$

$$e^{h\nu/KT} - 1 = \cancel{1} + \frac{h\nu}{KT} - \cancel{1} = \frac{h\nu}{KT}$$

$\therefore$ I_ν becomes:

$$I_\nu = \frac{2h\nu^3}{c^2} \times \frac{1}{h\nu/KT} = \frac{2\cancel{h}\nu^{\cancel{3}2}}{c^2} \times \frac{KT}{\cancel{h\nu}}$$

$$\Rightarrow \quad I_\nu = \frac{2\nu^2 KT}{c^2} \quad W\,m^{-2}\,s^{-1}\,sr^{-1}\,Hz^{-1}$$

Photon flux is number of photons emitted per cm^2 per second per Hz.
Energy of 1 photon of frequency ν : $h\nu$ W

$$\Rightarrow \quad 1W = \frac{1}{h\nu} \quad \text{(Energy of 1 photon of frequency } \nu)$$

$$I_\nu = \frac{2\nu^{\cancel{2}}KT}{c^2} \times \frac{1}{h\cancel{\nu}} \quad \text{photons } m^{-2}\,s^{-1}\,sr^{-1}\,Hz^{-1}$$

$$\Rightarrow I_\nu = \frac{2\nu KT}{hc^2} \times 10^{-4} \quad \text{photons } cm^{-2}\,s^{-1}\,sr^{-1}\,Hz^{-1}$$

$\therefore$ photon flux $= 2\times10^{-4} \times \frac{2KT}{hc^2}$ photons $cm^{-2}\,s^{-1}\,Hz^{-1}$

Q.6. (a) Radiated power = 600W in freq band = 2.7–2.8 GHz

$d = 3.844 \times 10^8$ m

Bandwidth $= \Delta\nu = |2.7 - 2.8|$ GHz $= 0.1$ GHz

Flux density of source $F_\nu = \frac{P}{4\pi d^2 \Delta\nu}$

$$= \frac{600}{4\pi \times (3.844\times10^8)^2 \times 0.1\times10^9}$$

$$= 3.231 \times 10^{-24}\ W m^{-2} Hz^{-1}$$

$$= 323.128\ Jy$$

(b) In optical frequency band : 375 nm - 1000 nm

$\Rightarrow \Delta\lambda = (1000 - 375)$ nm

$= 625$ nm $= 625\times10^{-9}$ m

Flux density in optical wavelength :

$$F_\lambda = \frac{600}{4\pi \times (3.844\times10^8)^2 \times 625\times10^{-9}}\ W m^{-3}$$

$$= 5.1727 \times 10^{-10}\ W m^{-3}$$

Q.7. (a) Diameter of Green Bank Telescope : $D = 100\,m$
Bandwidth : $\Delta\nu = 200\,MHz$
Observing frequency : $\nu = 4500\,MHz$
Current flux density : $F_\nu = 1.49\,Jy$ (at ν)

Flux density $F_\nu = \dfrac{\text{Power}}{\text{area} \times \Delta\nu}$

Power $P = F_\nu \times \Delta\nu \times$ area of telescope

$$= 1.49 \times 10^{-26} \times 200 \times 10^{6} \times \pi \times \left(\frac{100}{2}\right)^2$$

$$= 2.3393 \times 10^{-14}\,W$$

(b) Power $= \dfrac{\text{Energy}}{\text{time}}$

In 1s, power = energy

Energy of 1 photon $= h\nu$

$$= 6.626 \times 10^{-34} \times 4500 \times 10^{6}$$

$$= 2.9817 \times 10^{-24}\,J$$

Number of photons required $= \dfrac{\text{Energy}}{h\nu}$

$$= \frac{2.3393 \times 10^{-14}}{2.9817 \times 10^{-24}}$$

$$= 7.8455 \times 10^{9} \text{ photons}$$

Q.8. Flux density = 5.5 Jy at 1400 MHz

Angular size = 10" × 10"

Flux density = $I_\nu \times \Omega$

$$\Omega = \left(\frac{10}{3600} \times \frac{\pi}{180} \right)^2 \text{ sr}$$

$$= 2.350 \times 10^{-9} \text{ sr}$$

$$I_\nu = \frac{\text{Flux density}}{\Omega} = \frac{5.5 \times 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}}{2.350 \times 10^{-9} \text{ sr}}$$

$$= 2.34 \times 10^{-17} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$$

For a low frequency source such as a radio source, we can use Rayleigh Jeans approximation:

$$I_\nu = \frac{2\nu^2 kT}{c^2} \text{ W m}^{-2} \text{ sr}^{-1} \text{ Hz}^{-1}$$

$$\Rightarrow T_b = \frac{I_\nu c^2}{2\nu^2 k} = \frac{2.34 \times 10^{-17} \times (3 \times 10^8)^2}{2 \times (1400 \times 10^6)^2 \times 1.38 \times 10^{-23}}$$

$$\Rightarrow T_b = 3.893 \times 10^4 \text{ K}$$

Q.9. $T_{CMB} = 2.73$ K

(a) at $\lambda = 1$ mm.

The Planck's function is: $I_\nu = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$

$$= \frac{2\times 6.626\times10^{-34}\times(3\times10^{8})^{2}}{(10^{-3})^{5}\times\left[\exp\left(\frac{6.626\times10^{-34}\times10^{8}\times3}{10^{-3}\times1.38\times10^{-23}\times2.73}\right)-1\right]}$$

$$I_\nu = 6.129\times10^{-4}$$

This is the surface brightness.

(b) $I_\lambda = \frac{2c}{\lambda^4} KT$

$$T = \frac{I_\lambda \lambda^4}{2cK} = \frac{6.13\times10^{-4}\times(10^{-3})^{4}}{2\times3\times10^{8}\times1.38\times10^{-23}} = 0.074K$$

This is much less than the temperature of the CMB, $T_{CMB} = 2.73$.

So, Rayleigh Jeans approximation doesn't give good results for CMB at this frequency.

(c) at $\lambda = 90cm$

$$I_\lambda = \frac{2hc^2}{\lambda^5}\times\frac{1}{e^{hc/\lambda KT}-1}$$

$$= \frac{2\times6.626\times10^{-34}\times9\times10^{16}}{(90\times10^{-2})^{5}\times\left[\exp\left(\frac{6.626\times10^{-34}\times3\times10^{8}}{90\times10^{-2}\times1.38\times10^{-23}\times2.73}\right)-1\right]}$$

$$= 3.435\times10^{-14}\ Wm^{-3}\,sr^{-1}$$

Calculating temperature (considering RJ approximation):

$$T = \frac{I_\lambda \lambda^4}{2cK}$$

$$= \frac{3.435 \times 10^{-14} \times (90 \times 10^{-2})^4}{2 \times 3 \times 10^8 \times 1.38 \times 10^{-23}}$$

$$= 2.722 \text{ K}$$

So, RJ approximation yields good results at 90 cm wavelength since the calculated temperature is very close to the actual temperature of CMB (2.73K).

(d) $T = 100K$

$\lambda = 1mm = 10^{-3} m$

$$I_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\left(e^{hc/\lambda KT} - 1\right)}$$

$$= \frac{2 \times 6.626 \times 10^{-34} \times (3 \times 10^8)^2}{(10^{-3})^5 \left[\exp\left(\frac{6.626 \times 10^{-34} \times 3 \times 10^8}{10^{-3} \times 1.38 \times 10^{-23} \times 100}\right) - 1\right]}$$

$$= 0.77 \text{ W m}^{-2} \text{ sr}^{-1}$$

Considering RJ approximation:

$$T = \frac{I_\lambda \lambda^4}{2ck} = \frac{0.77 \times (10^{-3})^4}{2 \times 3 \times 10^8 \times 1.38 \times 10^{-23}}$$

$$= 92.97 \text{ K}$$

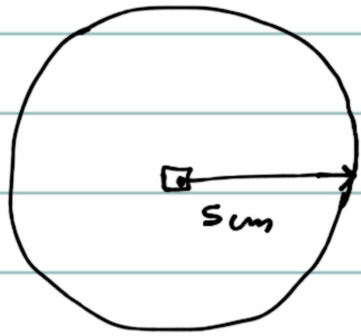
$\Rightarrow$ RJ approximation holds good.

Q.10 Power transmitted = 200 mW

at frequency ν = 850 MHz

With $\Delta\nu$ = 30 KHz

(a)



distance = 5 cm

Flux = $\frac{\text{Power}}{\text{Area}}$

$$= \frac{200 \text{ mW}}{4\pi (5 \text{ cm})^2} = 0.6366 \text{ mW cm}^{-2}$$

(b) Since flux at a distance of 5 cm $\approx 0.63 < 10 \text{ mW/cm}^2$

So, mobile phones are not really harmful

(c) Flux density at 10 km = F_ν = $\frac{\text{Power}}{4\pi d^2 \times \Delta\nu}$

$$= \frac{200 \times 10^{-3}}{4\pi \times (10 \times 10^3)^2 \times (30 \times 10^3)}$$

$$= 5.3 \times 10^{-15} \text{ W m}^{-2} \text{ Hz}^{-1}$$

$$= 5.3 \times 10^{11} \text{ Jy}$$

(d) Since flux density (5.3×10^{11} Jy) captured by 25m radio antenna 10 km away from phone $> 10^9$ Jy

This causes gain compression in the receiver

The antenna is not safe.

(e) Power received by radio antenna $= f_\nu \times \text{area} \times \Delta\nu$

$$= 5.5 \times 10^{11} \times \pi \times \left(\frac{25}{2}\right)^2 \times 30 \times 10^{3}$$

$$= 7.8 \times 10^{-8}\ W$$

Number of photons received per second:

$$n = \frac{\text{Energy received by } n \text{ photons in 1 second}}{\text{Energy of 1 photon}}$$

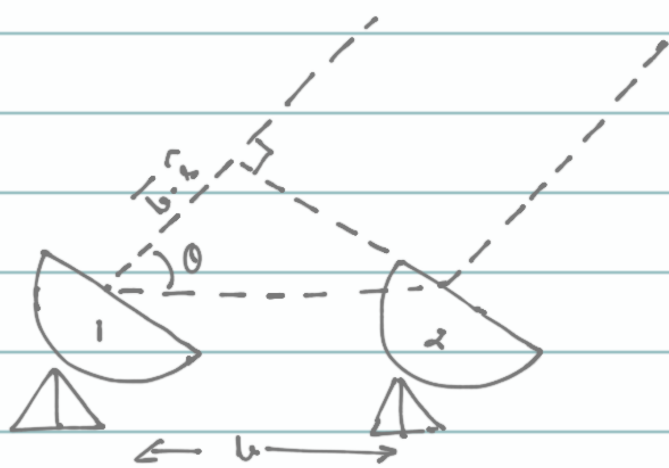
$$= \frac{7.8 \times 10^{-8}}{(6.626 \times 10^{-34} \times 850 \times 10^{6})}$$

$$= 1.387 \times 10^{17} \text{ photons}$$

Q.11. 2 element interferometer operating at wavelength of 100 cm

$d = 100m$

$\lambda = 100 cm = 1 m$



A plane wave coming from a distant point source travels an extra distance $\bar{b}.\hat{s}$ to reach antenna 1 than antenna 2. There is a geometric lag:

$$\tau_g = \frac{\bar{b}.\hat{s}}{c} = \frac{b\cos\theta}{c}$$

where $\bar{b}$ is the baseline vector

$\hat{s}$ = unit vector along the direction the antenna is pointing.

Phase delay associated with it can be defined as:

$$\phi = 2\pi\nu\tau_g = 2\pi\nu\frac{b\cos\theta}{c} = \frac{2\pi b\cos\theta}{\lambda}$$

Incoming plane waves are at an angle of 60° with the horizon

also, $\frac{\Delta\phi}{\phi} = \frac{\Delta b}{b}$ $\Delta\phi$ & Δb are errors

$$\Rightarrow \Delta\phi = \phi \times \frac{\Delta b}{b} = \frac{2\pi b\cos\theta}{\lambda} \times \frac{\Delta b}{b}$$

$\theta = 60°$

$$\Rightarrow \Delta\phi = 2\pi \times 0.1 \times \cos 60 = 0.1\pi$$

An error of 0.1 m in the measurement of the baseline can lead to an error of 0.1π radians of error in the measurement of ϕ.

Corresponding delay in phase is:

$$\frac{\Delta\phi}{\phi} = \frac{0.1}{100}$$

Max value $= 2\pi \leftarrow \phi$

Error in phase delay: $\frac{\Delta\phi}{\phi} \times 100\% = \frac{0.1\cancel{\pi}}{\cancel{2}\cancel{\pi}} \times \cancel{100}^{50}$

$= 5\%$

So the given error in baseline measurement induces a 5% error in phase delay

Q.12. Collecting area of telescope $= \pi\left(\frac{D}{2}\right)$ → diameter

for Arecibo, $D = 305m$: 1 dish
for VLA $D = 25m$: 27 dishes, 36km baseline.

(a) Ratio of total collecting area is $\frac{A_A}{A_V}$ → Arecibo, → VLA

This ratio can be given by:

$$\frac{A_A}{A_V} = \frac{[\pi(D/2)^2]_A}{[\pi(D/2)^2]_V} \times \frac{\text{number of dishes at Arecibo}}{\text{number of dishes at VLA}}$$

$$= \frac{(305)^2}{(25)^2 \times 27}$$

$$= 5.51$$

(b) Angular resolution of single dish telescope $\theta = \frac{1.02\lambda}{D}$

Angular resolution of an array of telescopes : $\theta = \frac{1.02\lambda}{b_{max}}$

$$\frac{\theta_A}{\theta_V} = \frac{1.02\lambda/D}{1.02\lambda/b_{max}} = \frac{b_{max}}{D} = \frac{36\times10^3}{305} = 118.03$$

(c) A telescope having a large collecting area is needed for detecting faint source. So, Arecibo radio telescope is a better choice for such a source.

For observing structures on small angular scale, θ should be smaller. So VLA is the better option for such a source.

Q.15. For a VLA images of point sources:

$$\left.\begin{aligned} I &= 50.0 \\ Q &= 3 \\ U &= -5.2 \\ V &= -0.1 \text{ mJy} \end{aligned}\right] \text{Stoke's parameters}$$

(a) Linearly polarized intensity

$$L = \sqrt{Q^2 + U^2}$$

$$= \sqrt{3^2 + (-5.2)^2}$$

$$= 6.0 \text{ mJy}$$

% of linear polarization $= \frac{L}{I} \times 100$

$$= \frac{6}{\cancel{50}} \times \overset{2}{\cancel{100}} = 12\%$$

Polarization angle $\theta = \frac{1}{2} \tan^{-1}\left(\frac{U}{Q}\right)$

$$= \frac{1}{2} \tan^{-1}\left[\frac{-5.2}{3}\right]$$

$$= 59.99^\circ$$

(b) % of circular polarization $= \frac{V}{I} \times 100$

$$= \frac{0.1}{\cancel{50}} \times \cancel{100}^{2}$$

$$= 0.2\%$$

(c) Stokes parameter I can not be negative as it is $\sqrt{Q^2+U^2+V^2}$

hence, it is non-negative.