we will get from

$A = 15$ km/s kpc^{-1}

$B = -13$ km/s kpc^{-1}

(a) Keplerian profile

$$\Omega = \sqrt{\frac{GM}{R^3}}$$

$$v_c \sim R^{-1/2} \Rightarrow V = V_0 \left(\frac{R}{R_0}\right)^{-1/2}$$

$$\left.\frac{dV}{dR}\right|_{R_0} = -\frac{1}{2} \frac{V_0}{\sqrt{R_0}} (R_0)^{-3/2}$$

$$\frac{dv}{dr} = -\frac{1}{2} \frac{V_0}{R_0}$$

$$A = \frac{1}{2}\left(\frac{V_0}{R_0} + \frac{1}{2}\frac{V_0}{R_0}\right) = \frac{3}{4}\frac{V_0}{R_0}$$

$$B = -\frac{1}{2}\left(\frac{V_0}{R_0} - \frac{1}{2}\frac{V_0}{R_0}\right) = \frac{1}{4}\frac{V_0}{R_0}$$

(b) $V_c \propto R \Rightarrow V_c = \frac{V_0 R}{R_0}$

$$\frac{dV_c}{dR} = \frac{V_0}{R_0}$$

$$A = \frac{1}{2}\left(\frac{V_0}{R_0} - \frac{V_0}{R_0}\right)$$

$$B = -\frac{1}{2}\left(\frac{V_0}{R_0} + \frac{V_0}{R_0}\right)$$

$$A = 0$$

$$B = -\frac{V_0}{R_0}$$

(c) $V = V_0 = 220\ km/s$

$R_0 = 8\ kpc$

then

$$A = +\frac{1}{2}\frac{V_0}{R_0}$$

$$B = -\frac{1}{2}\frac{V_0}{R_0}$$

$M_r (R < R_0)$

$$\frac{V_0}{R_0} = \omega$$

Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi R_0}{V_0}$

$$= \frac{2\pi \times 8\ kpc}{220\ km/s}$$

1 kpc

$$= \frac{2\pi \times 8 \times 3.08 \times 10^{16}}{220}\ s$$

1 Myr $= 3.15 \times 10^{13}$ sec

$$= \frac{7.03 \times 10^{15}\ sec}{3.15 \times 10^{13}} = 223\ Myr$$

$$\frac{kg\, m\, s^{-2}}{} \, m^{-2} kg \qquad M \approx 8 \times 10^{8} M_{\odot}$$

Q. 1 $v^2 = \frac{GM}{R}$

$$M = \frac{v^2 R}{G}$$

$$= \frac{220^2\, km^2 s^{-2} \times 8\, kpc}{6.67 \times 10^{-11}\, N m^{2}\, kg^{-2}}$$

$$= \frac{220^2 \times 10^6 \times 3.08 \times 10^{19} \times 8}{6.67 \times 10^{-11}}\, kg$$

=

u

(i) Wedge plot of 20 galaxies

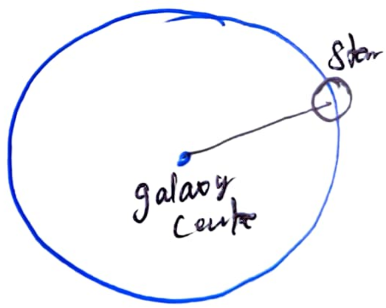
Master wedge plot of all 218 galaxies

(a) Answer of given question in Manual.

AA 472/672 <u>Extragalactic</u>

(i) LSR – local standard of rest

a fictious star which is rotate in exact circular orbit ~~around~~



$$R_{LSR} = R_0 = 8.1 \text{ pc}$$

(ii) Peculiar Velocity

$$V_{LSR} = \overset{0}{\overset{\shortparallel}{u}},\ \underset{\underset{V_0}{\shortparallel}}{v},\ \overset{0}{\overset{\shortparallel}{w}} = 0, V_0, W$$

Peculiar velocity of the sun :-

$$V_\odot = U_\odot, V_\odot, W_\odot \Rightarrow V_0 = (U_0, V_0, W_0)$$

$$\cancel{\langle U_\odot \rangle = 0}$$

$$\cancel{\langle W_\odot \rangle = 0}$$

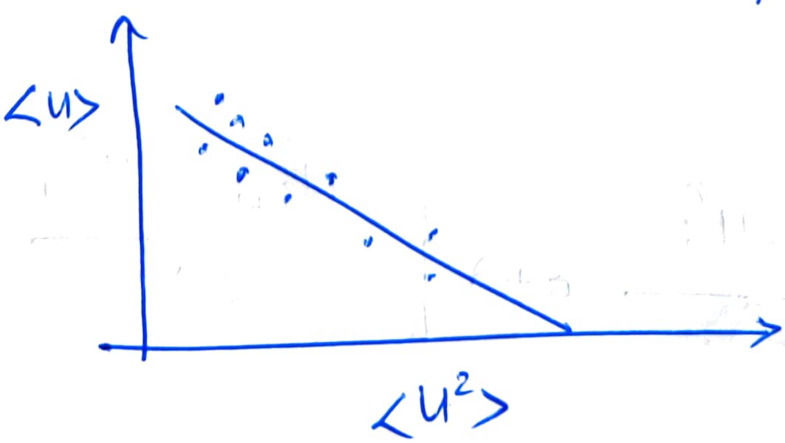
$$\cancel{\langle V_0 \rangle = c \langle U^2 \rangle}$$

Consider a velocity of star neighbourhood of Sun

$$V^* = (U^*, V^*, W^*)$$

$$\langle U^* \rangle = 0 \quad , \langle W^* \rangle = 0 \quad , \langle V^* \rangle = -c \langle U^2 \rangle$$

$\langle U^2 \rangle$ = mean dispersion of radial velocity



Snyder

a = peculiar velocity of Sun

b = " " " Star

$\cancel{V_0 (-10, 5)}$

$V_0 = (-10\ km/s,\ 5 km/s,\ 7 km/s)$

AA609

Computional Method

Advection eq^n :-

$$\frac{\partial u}{\partial t} + \lambda \frac{\partial u}{\partial x} = 0$$

Our Notations

U_i^n → n^{th} time step, i → i^{th} grid position

FTCS

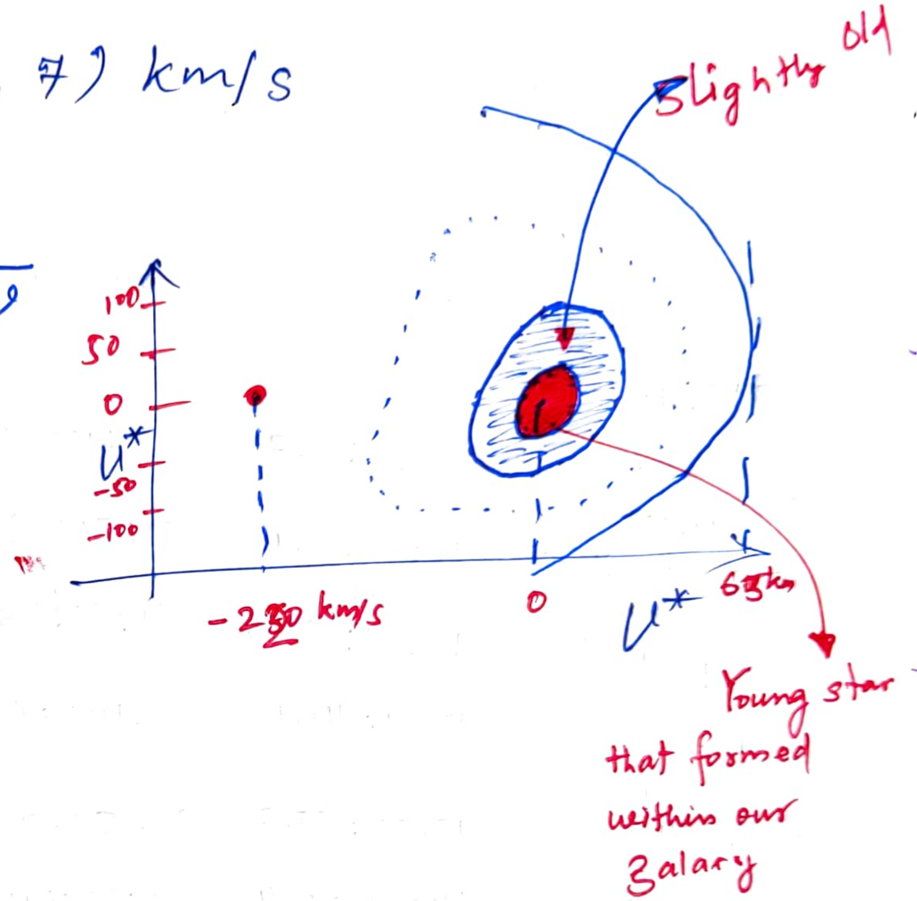
$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \lambda \left[\frac{u_{i+1}^n - u_{i-1}^n}{2 \Delta x} \right] = 0$$

$$\vec{v}_p^{\,*} = \vec{v}_0 + \underbrace{\Delta\vec{v}}_{\text{velocity of star measured w.r. to sun.}}$$

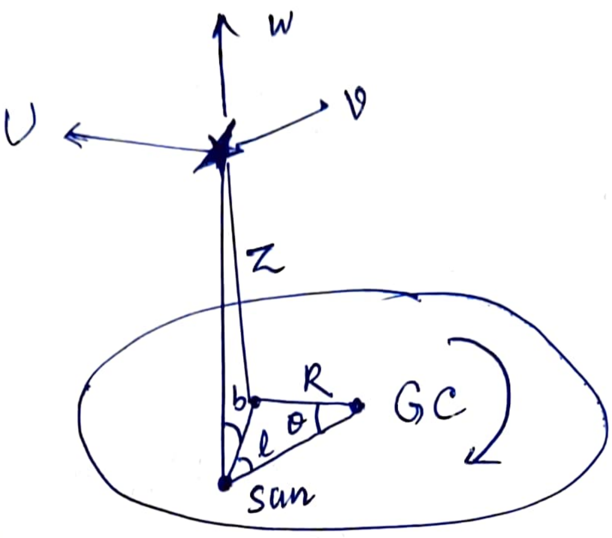
$$\vec{v}_\odot = (-10, 5, 7)\ \text{km/s}$$

$$v_p^* = \vec{v}_\odot + \Delta\vec{v}$$

$$u^*, v^*, w^*$$



AA 672 Galactic & Extra galactic



Galactic Coordinate

R, θ, Z — Local standard of rest (LSR)

(l, b) — pure "Circular velocity"

Peculer velocity :-

$$V_p^* = (U, V, W)$$

$$= (U - \overset{0}{U_{LSR}}, V - \overset{V_0}{V_{LSR}}, W - \overset{0}{W_{LSR}})$$

$$v = (U, V - V_0, W)$$

AA 472/672

Galactic and Extragalactic Astronomy

1. $$V(r) = -V_0 \exp(-\lambda^2 r^2) = -V_0\, e^{-\lambda^2/u^2}$$

angular momentum $= L$

$$\frac{d^2u}{d\theta^2} + u = \frac{1}{L^2u^2}\frac{dV}{dr}$$

$$= \frac{1}{L^2u^2}\frac{d}{dr}\left(-V_0\, e^{-\lambda^2/u^2}\right)$$

$$= -\frac{V_0}{L^2u^2}\, e^{-\lambda^2/u^2}$$

$$F(r) = \frac{dV}{dr} = -V_0 \exp(-\lambda^2 r^2) \cdot -\lambda^2 \cdot 2r$$

$$\frac{d^2u}{d\theta^2} + u = +\frac{V_0}{mL^2u^2} \exp(-\lambda^2 r^2)\, \lambda^2\, 2r$$

$$\frac{d^2u}{d\theta^2} + u = \frac{\lambda^2 V_0}{L^2 \cdot u^2} \exp\left(-\frac{\lambda^2}{u^2}\right)\frac{2}{u}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{2\lambda^2 V_0}{mL^2u^3} \exp\left(-\frac{\lambda^2}{u^2}\right)$$

$f(r)\hat{r} = \vec{F}(r) = -\vec{\nabla}V =$

<u>only $\hat{r}$</u>

$$f(r) = -\frac{\partial}{\partial r}\left(-V_0 e^{-\lambda^2 r^2}\right)$$

$$= V_0 e^{-\lambda^2 r^2}(-\lambda^2 \cdot 2r)$$

$$= -2\lambda^2 V_0 r e^{-\lambda^2 r^2}$$

<u>Now,</u>

$$\frac{d^2u}{d\theta^2} + u = -\frac{f(r)}{mL^2u^2}$$

$$\frac{d^2u}{d\theta^2} + u = +\frac{2\lambda^2 V_0 r e^{-\lambda^2 r^2}}{mL^2u^2}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{2\lambda^2 V_0 e^{-\lambda^2/u^2}}{mL^2u^3} \qquad u = \frac{1}{r}$$

$$\frac{d^2u}{d\theta^2} = \frac{2\lambda^2 V_0 e^{-\lambda^2/u^2}}{mL^2u^3} - u$$

$$\frac{d^2u}{d\theta^2} = \frac{2\lambda^2 V_0 e^{-\lambda^2/u^2} - mL^2u^4}{mL^2u^3}$$

$$\frac{1}{u^2} = t$$

$$-\frac{2}{u^3}\,du = dt$$

~~Poisson~~ Distribution:-

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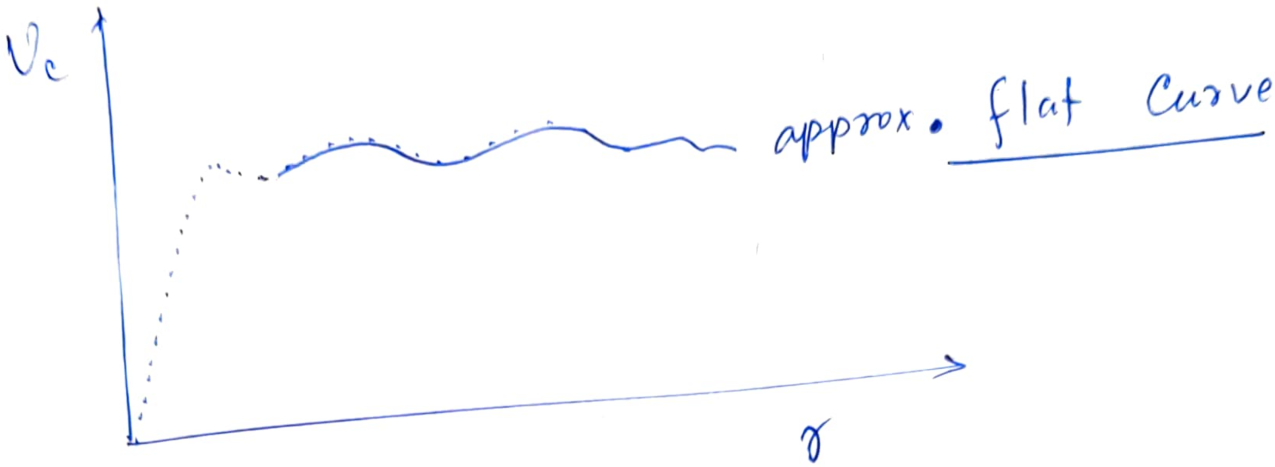


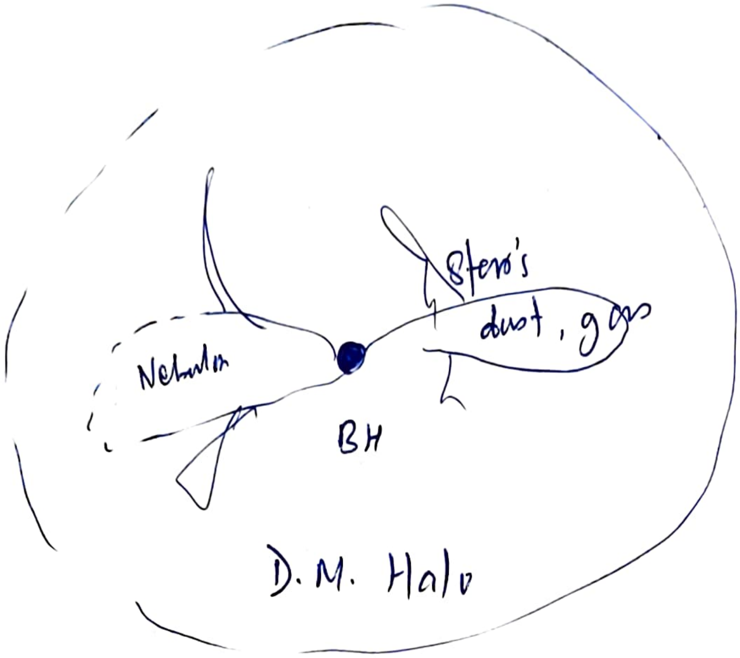
$V_c \rightarrow$ Const $\rightarrow V_0$

$$V_c^2 = \frac{G M(r)}{r} \Rightarrow M(r) \propto r$$

$$\int \rho(r)\, dV = M(r)$$

Def^n of galaxy $\Rightarrow$ ✓ Dark Matter halo
✓ Super Massive Black hole

(I) group of star
(II) Dust, gas
(III) Bounded by central force



➢ Chemical Enrichment

H, He
Buiding Block → metal

$Z_\odot$ → Metallicity → Fraction of metal / Total (Mass or Number)

$Y_\odot$ → Mass of Helium / Total Mass — $\frac{m_{He}}{M_T}$

$X_\odot$ → $\frac{m_H}{m_T}$

For Sun

$$Z_\odot, X_\odot, Y_\odot$$

$$X_\odot = \frac{\text{Mass of Hydrogen in Sun}}{\text{Total Mass of Sun}}$$

$$Y_\odot = \frac{\text{Mass of Helium in Sun}}{\text{Total Mass of Sun}}$$

$$Z_\odot = \frac{\text{Mass of all Metals in Sun}}{\text{Total mass}}$$

For galactic

$$[A|B]^G = \log\left(\frac{\text{\# of element A for galaxy}}{\text{\# of element B for galaxy}}\right)$$

Metalicity of galaxy is always a "relative measurement."

$$Z(t) = \frac{M_h(t)}{M_g(t)}$$

$$\frac{dZ}{dt} = \frac{1}{M_g(t)} \frac{dM_h}{dt} - \frac{M_h}{M_g^2} \frac{dM_g}{dt}$$

$$\Rightarrow \frac{dZ}{dt} = \frac{1}{M_g} \left[\frac{dM_h}{dt} - \frac{M_h}{M_g} \frac{dM_g}{dt} \right]$$

$$\Rightarrow \frac{dZ}{dt} = \frac{1}{M_g} \left[\frac{dM_h}{dt} - Z \frac{dM_g}{dt} \right]$$

$$\Rightarrow \frac{dZ}{dt} = \frac{1}{M_g} \left[\frac{dM_h}{dt} + Z \frac{dM_s}{dt} \right]$$

Date - 28/1/2022

Metalicity at $t=0$

$$\Rightarrow \boxed{Z(0) = 0}$$

~~at~~ @ $t=0$, there is no heavy element i.e. $M_h(0) = 0$

Galaxy → Box ⟵ our Assumption

(i) Galaxy → Premixed gas → Homogenous Components
→ equal component of gas

(ii) High Mass star enrichment is more much faster compared to star formation.

(iii) Box is closed.

$M_g(t)$ → gass mass at t

$M_s(t)$ → star mass at t

$M_h(t)$ → mass of heavy metals at t

Metalicity

$$Z(t) = \frac{M_h(t)}{M_g(t)}$$

$M_g + M_s$ → Const

$$\frac{dM_g}{dt} + \frac{dM_s}{dt} = 0$$

$$\Rightarrow \frac{dM_g}{dt} = -\frac{dM_s}{dt}$$

The heavy element is generated due to enplosion of heavy Mass stars.

$$\frac{dM_h}{dt} = p\frac{dM_s}{dt} - z\frac{dM_s}{dt}$$

P: Yield of cycle

Rate of adding heavy element to the enviroment "Rate of creation".

Rate of closing heavy elem… in remant of star.

"Rate of Destruction".

$$\frac{dz}{dt} = \frac{1}{M_g}\left[p\frac{dM_s}{dt} - z\frac{dM_s}{dt} + z\frac{dM_s}{dt}\right]$$

$$= \frac{p}{M_g}\frac{dM_s}{dt}$$

$$\frac{dz}{dt} = -\frac{p}{M_g}\frac{dM_g}{dt}$$

$$dz = -p\frac{dM_g}{M_g}$$

$$z = -p\ln M_g + c$$

$$z(t) = -p\ln M_g(t)$$

at $t=0$

$$z(0) = 0 - p\ln M_g(0) + c$$

$\cancel{p\ln M_g}$ $c = z(0) + p\ln M_g(0)$

$$\therefore\ z(t) = z(0) - p\ln\left(\frac{M_g(t)}{M_g(0)}\right)$$

$$\Rightarrow z(t) = z(0) + p\ln\left(\frac{M_g(0)}{M_g(t)}\right)$$

$$\frac{M_g(0)}{M_g(t)} > 1$$

$z(t)$ is increased with time.

$$dz = \frac{p}{M_g}\,\frac{dM_s}{\cancel{\phi}}$$

$$dM_s = \frac{M_g(t)}{p}\,dz$$

$$\cancel{M_g(t) = e^{\left(\frac{z(t)-z(0)}{p}\right)}}$$

$$p\ln\left(\frac{M_g(t)}{M_g(0)}\right) = z(0) - z(t)$$

$$M_g = M_g(0)\,e^{\left(\frac{z(0)-z(t)}{p}\right)}$$

$$\therefore\ dM_s = \frac{M_g(0)}{p}\,e^{\frac{z(0)-z(t)}{p}}\,dz$$

$$M_s = \frac{M_g(0)}{p}\int_0^z e^{\frac{z(0)-z(t)}{p}}\,dz$$

$$= \frac{M_g(0)}{p}\,e^{z_0/p}\int_0^z e^{-z/p}\,dz$$

$$= \frac{M_g(0)}{p}\,e^{z(0)/p}\left(-\frac{e^{-z}}{p}\right)_0^z$$

$$= \frac{M_g(0)}{p}\,e^{z(0)/p}\left(\frac{1}{p} - \frac{e^{-z}}{p}\right)$$

$M_s(<z(t$

M_s

$\frac{dM_s}{dz}$

Solar

$<z>$

$M_g(0$

$M_g($

$$M_s(<Z(t)) = M_s(t) = M_g(0) - M_g(t)$$

$$= M_g(0) - M_g(0)\, e^{\left(\frac{Z(0)-Z(t)}{p}\right)}$$

$$= M_g(0)\left[1 - e^{\frac{Z(0)-Z(t)}{p}}\right]$$

M_s in betn metallicity $\underline{Z \text{ \& } Z+dZ}$

$$\frac{dM_s}{dZ}$$

Solar Neighbourhood :

$$\langle Z \rangle = 0.7 Z_\odot$$

$$M_g(0) = 35\ M_\odot/pc^2$$

$$M_g(t) = 15\ M_\odot/pc^2 \qquad \cancel{0.3 Z_\odot}$$

$$p = \frac{Z(0) - Z(t)}{\ln\left(\frac{M_g(t)}{M_g(0)}\right)} = \frac{Z_0 - 0.7Z_0}{\ln\left(\frac{15}{35}\right)}$$

$$\hbar\omega = \frac{4}{3} r^2 \hbar\omega_0$$

$$\omega = \frac{4}{3} r^2 \omega_0$$

$r = 4$ $\quad \omega = \frac{4}{3}\, 16\ 10\, GHz$

$$\omega = 21.3\ GHz$$

Audio Freq - 20 Hz - 20 kHz

Radio Freq - 20 kHz - GHz

$\langle z \rangle \sim 0.7\, Z_\odot$

$M_s(t) = 35\, M_\odot/pc^2$

$M_g(t) = 15\, M_\odot/pc^2$

$$Z(t) = Z(0) - p \ln\left(\frac{M_g}{M_s + M_g}\right)$$

$$0.7 Z_\odot = Z_\odot - p \ln\left(\frac{15}{50}\right)$$

$$p = \frac{-0.3\, Z_\odot}{\ln\left(\frac{15}{50}\right)} = 0.24\, Z_\odot$$

$$\frac{M_s(< 0.25\, Z_\odot)}{M_s(< 0.7\, Z_\odot)} = \frac{1 - \exp\left(\frac{-0.25 Z_\odot}{p}\right)}{1 - \exp\left(\frac{-0.7\, Z_\odot}{p}\right)}$$

$\simeq 52\%$

G-dwarf $\rightarrow$ 2-3%.

$$z\frac{dM_s}{dt} = -z\frac{dM_g}{dt} + z f(t)$$

M_g

$$\frac{dz}{dt} = \frac{1}{M_g}\left[\frac{dM_h}{dt} - z\frac{dM_g}{dt}\right]$$

$$= \frac{1}{M_g}\left[p\frac{dM_s}{dt} - z\frac{dM_s}{dt} + z\frac{dM_s}{dt} - z f(t)\right]$$

$$= \frac{1}{M_g}\left[p\frac{dM_s}{dt} - z f(t)\right]$$

$$= \frac{1}{M_g}\left[-p\frac{dM_g}{dt} + p f(t) - z f(t)\right]$$

$$= -\frac{p}{M_g}\frac{dM_g}{dt}$$

AA 672

Accreting Box

$\hookrightarrow$ Printine gas (~~z~~zero metallicity material) is coming to model.

$$\frac{dM_h(t)}{dt} = (p-z)\frac{dM_s}{dt}$$

Total mass

$$\frac{dM_g(t)}{dt} = -\frac{dM_s}{dt} + \underbrace{f(t)}_{\rightarrow \text{Accretion rate}}$$

Accretion ~~Accretion~~ rate

$$\frac{dz}{dt} = -\frac{1}{M_g}\left[\cancel{p\frac{dM_s}{dt} - z\frac{dM_s}{dt}} + z\frac{dM_s}{dt}\right]$$

$$\cancel{= -\frac{1}{M_g}\left[p\frac{dM_s}{dt} - z\frac{dM_s}{dt} - \right.}$$

assume @ time t

$$M_g(t) \rightarrow \text{const}$$

$$\frac{dM_g}{dt} = 0$$

$$\therefore \frac{dz}{dt} = \frac{1}{M_g}(P-z)\frac{dM_s}{dt}$$

$$z(t) = P\left[1 - \exp\left(-\frac{M_s}{M_g}\right)\right]$$

$$\underline{z(0) = 0}$$

$$M_s = -M_g \ln\left(1 - \frac{z}{P}\right)$$

$$\frac{M_s(z < 0.25)}{M_s(z < 0.7)} \sim 0.03$$

9/12/2022

Extra Galactic Keplerian Orbit

(I) $$T^2 = \frac{4\pi^2}{GM} r^3$$

M = mass
L = Luminosity
V = velocity

(II) $$V = \frac{2\pi r}{T}$$

(III) $$\frac{M}{L} = 20 \quad \text{(constant)} \Rightarrow M \propto L$$

(IV) $$F = \frac{GMm}{r^2}$$