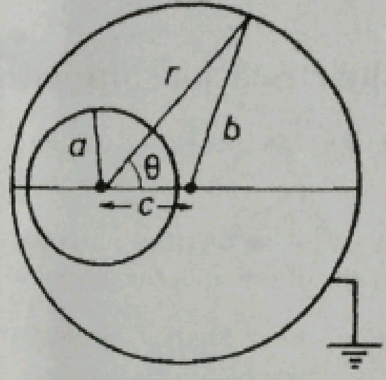
PH 601

Due September 22, 2021



Problem 1

As can be seen in the figure above, the inner conducting sphere of radius a carries charge Q, and the outer sphere of radius b is grounded. The distance between their centers is c, which is a small quantity.

(a) Show that to first order in c, the equation describing the outer sphere, using the center of the inner sphere as origin, is

$$r(\theta) = b + c\cos\theta. \tag{1}$$

(b) If the potential between the two spheres contains only $\ell = 0$ and $\ell = 1$ angular components, determine it to first order in c.

Problem 2

Take a very long cylinder of radius r made of insulating material. Spray a cloud of electrons on the surface. They are free to move about the surface so that, at first, they spread out evenly with a charge per unit area σ_0. Then put the cylinder in a uniform applied electric field perpendicular to the axis of the cylinder. You are asked to think about the charge density on the surface of the cylinder, $\sigma(\theta)$, as a function of the applied electric field E_a. In doing this you may neglect the electric polarizability of the insulating cylinder.

(a) Solve for the potentials inside and outside the cylinder – call these φ_{I} and φ_{II}

(b) In what way is this problem different from a standard electrostatic problem in which we have a charged conducting cylinder? When are the solutions to the two problems the same? (Answer in words)

(c) Calculate the solution for $\sigma(\theta)$ in the case of a conducting cylinder and state the range of value of E_a for which this solution is applicable to the case described here.

Problem 3

(a) Find the vector potential inside and outside a solenoid that generates a magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{z}}$ inside an infinite cylinder of radius R. Work in the Coulomb gauge, i.e. with $\boldsymbol{\nabla}\cdot\boldsymbol{A} = 0$.

(b) The Aharonov-Bohm effect occurs because the magnetic flux $\Phi_B = \int \boldsymbol{dl}\cdot\boldsymbol{A}$ is non-zero when the integration circuit is, say, the rim of a disk of radius $\rho > R$ which lies perpendicular to the solenoid axis. Show that $\boldsymbol{A}' = \boldsymbol{A} + \boldsymbol{\nabla}\chi$ with $\chi = -\Phi\phi/2\pi$ (ϕ is the angle in cylindrical coordinates) leads to identically zero vector potential outside the solenoid.

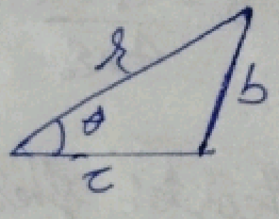
Q1(a) Applying the cosine theorem to the triangle, we have

$$b^2 = c^2 + r^2 - 2cr\cos\theta \simeq r^2 - 2cr\cos\theta.$$

or

$$r \simeq \frac{1}{2}\left(2c\cos\theta + \sqrt{4c^2\cos^2\theta + 4b^2}\right) \simeq b + c\cos\theta.$$

or, $\underline{r(\theta) = b + c\cos\theta}$



(b) Using Laplace's equation, $\nabla^2\phi = 0$

and axial symmetry, we can express the potential at a point b/w the two spheres as

$$\phi = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

If we retain only the $l=0$, and $l=1$ angular components, we have,

$$\phi = A_0 + \frac{B_0}{r} + \left(A_1 r + \frac{B_1}{r^2}\right)\cos\theta \quad \text{———} \quad (\star)$$

As the surface of the inner conductor is an equipotential surface, ϕ for $\boxed{r = a \text{ should not depend on } \theta.}$

Hence, $A_1 a + \dfrac{B_1}{a^2} = 0$ ——— (1)

The charge density on the surface of the inner sphere is

$$\sigma = -\varepsilon_0\left(\frac{\partial\phi}{\partial r}\right)_{r=a}$$

and we have,

$$\int_0^\pi \sigma\, 2\pi a^2 \sin\theta\, d\theta = Q.$$

$$\Rightarrow B_0 = \frac{Q}{4\pi\varepsilon_0} \qquad \text{—— (2)}$$

As the outer sphere is grounded, $\boxed{\phi = 0 \text{ for } r \simeq b + c\cos\theta.}$

Thus, we have

$$A_0 + \frac{B_0}{b+c\cos\theta} + \left[A_1(b+c\cos\theta) + \frac{B_1}{(b+c\cos\theta)^2}\right]\cos\theta = 0. \qquad \text{—— (3)}$$

To first order in c, we have the approximations

$$(b+c\cos\theta)^{-1} = b^{-1}\left(1+\frac{c}{b}\cos\theta\right)^{-1} \simeq \frac{1}{b}\left(1-\frac{c}{b}\cos\theta\right) \text{ —— (4)}$$

$$(b+c\cos\theta)^{-2} = b^{-2}\left(1+\frac{c}{b}\cos\theta\right)^{-2} \simeq \frac{1}{b^2}\left(1-\frac{2c\cos\theta}{b}\right) \text{ —— (5)}$$

Using values of (4) and (5) in (3), we have,

$$A_0 + \frac{B_0}{b}\left(1-\frac{c}{b}\cos\theta\right) + \left[A_1(b+c\cos\theta) + \frac{B_1}{b^2}\left(1-\frac{2c}{b}\cos\theta\right)\right]\cos\theta = 0$$

$$A_0 + \frac{B_0}{b}\left(1-\frac{c}{b}\cos\theta\right) + \left[A_1(b+c\cos\theta)\right]\cos\theta + \frac{B_1\left(1-\frac{2c}{b}\cos\theta\right)\cos\theta}{b^2} = 0$$

$$A_0 + \left[\frac{B_0}{b} - \frac{B_0 c\cos\theta}{b}\right] + \left(A_1 b\cos\theta + A_1 c\cos^2\theta\right)$$

$$+ \left[\frac{B_1}{b^2}\cos\theta - \frac{2B_1 c}{b^2}\cos^2\theta\right] = 0$$

Neglecting the higher order terms, we are left with

$$A_0 + \frac{B_0}{b} - \frac{B_0 c\cos\theta}{b} + A_1 b\cos\theta + \frac{B_1}{b^2}\cos\theta = 0$$

$$A_0 + \frac{B_0}{b} + \left[-\frac{B_0 c}{b} + A_1 b + \frac{B_1}{b^2}\right]\cos\theta = 0 \quad \text{—— (6)}$$

Thus, from (6), we can write

$$A_0 + \frac{B_0}{b} = 0 \quad \text{—— (7)}$$

$$-\frac{B_0 c}{b} + A_1 b + \frac{B_1}{b^2} = 0 \quad \text{—— (8)}$$

for (6) to be independent and valid for given value of θ.

From (4) and (8), and (2) and (7), we have

from (2) and (7) – $A_0 + \dfrac{Q}{b\,4\pi\varepsilon_0} = 0 \Rightarrow A_0 = \dfrac{-Q}{b\,4\pi\varepsilon_0}$

$$\Rightarrow \boxed{A_0 = \frac{-Q}{4\pi\varepsilon_0 b}, \quad B_0 = \frac{Q}{4\pi\varepsilon_0}}$$

from 5 and 8 -

$$\frac{-Q\,\tau}{4\pi\varepsilon_0 b} + A_1 b + \frac{B_1}{b^2} = 0. \quad \text{— (9)}$$

Now, from 1, we have $A_1 = -\dfrac{B_1}{a^3}$

Using this value in 9, we get

$$\frac{-Q\tau}{4\pi\varepsilon_0 b} - \frac{B_1 b}{a^3} + \frac{B_1}{b^2} = 0$$

$$\Rightarrow \frac{B_1}{b^3} - \frac{B_1}{a^3} = \frac{Q\tau}{4\pi\varepsilon_0 b^2}$$

$$\Rightarrow \cancel{B_1 = \frac{Q\tau(a^3 - b^3)}{4\pi\varepsilon_0 b^2\, a^3 b^3}}$$

$$\Rightarrow B_1\left(\frac{a^3 - b^3}{a^3 b^3}\right) = \frac{Q\tau}{4\pi\varepsilon_0 b}$$

$$\Rightarrow B_1 = \frac{Q\tau a^3}{4\pi\varepsilon_0 (a^3 - b^3)} = \frac{-Q\tau a^3}{4\pi\varepsilon_0 (b^3 - a^3)}$$

Now, using this value, we get

$$A_1 = \frac{Q\tau}{4\pi\varepsilon_0 (b^3 - a^3)}$$

Taking $\dfrac{\Phi}{4\pi\varepsilon_0} = B_0$, for simplicity, we get

$$A_1 = \frac{B_0 c}{(b^3-a^3)} \; ; \; B_1 = \frac{-B_0 a^3 c}{(b^3-a^3)} \; ; \; A_0 = \frac{-B_0}{b}$$

Using these in ☆, we get

$$\phi = -\frac{B_0}{b} + \frac{B_0}{r} + \left(\frac{B_0 c r}{(b^3-a^3)} - \frac{B_0 a^3 c}{r^2(b^3-a^3)}\right)\cos\theta$$

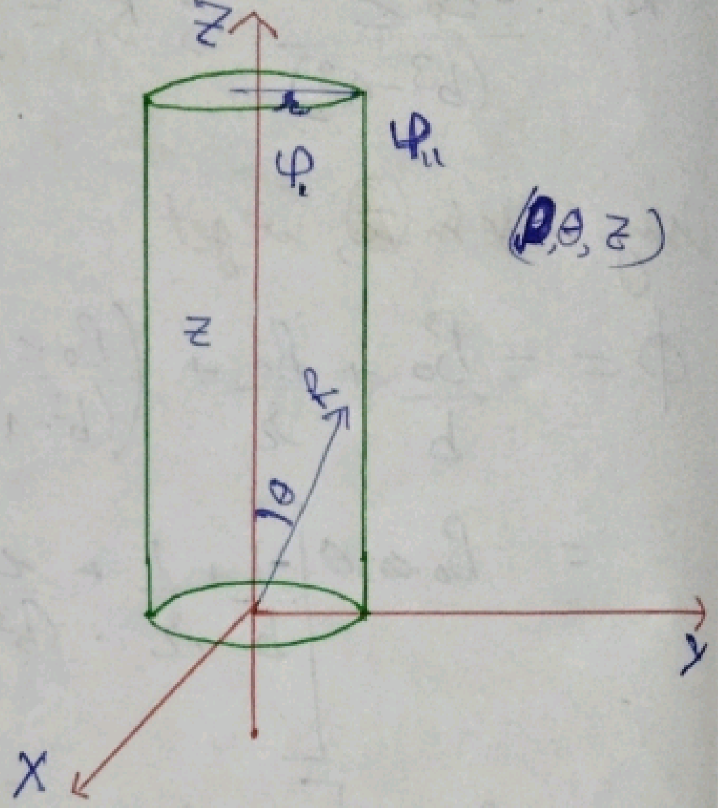
$$= B_0 \cos\theta \left[\frac{-1}{b} + \frac{1}{r} + \frac{cr}{(b^3-a^3)} - \frac{a^3 c}{r^2(b^3-a^3)}\right]$$

$$= B_0 \cos\theta \left[\frac{-1}{b} + \frac{1}{r} + \frac{cr}{(b^3-a^3)}\left\{1 - \frac{a^3}{r^3}\right\}\right]$$

$$= \frac{\Phi \cos\theta}{4\pi\varepsilon_0}\left[\frac{-1}{b} + \frac{1}{r} + \frac{cr}{(b^3-a^3)}\left(1 - \left(\frac{a}{r}\right)^3\right)\right]$$

Q 3. (a) We consider a long dielectric cylinder of radius r.

In this cylinder, q charge has been spread across the surface.



In the condition of equilibrium, the charge will be spread uniformly on the surface.

then, surface charge density $= \sigma_0$

As the cylinder is symmetric about the Z axis, the potential will be independent of Z.

In the equilibrium state, it can be written that $E_\theta = 0$.
i.e, the tangential component of electric field on the surface at $(R = r)$ will be Zero

Consider the electric field $\vec{E_a}$ is applied in the direction of +ive x axis., along $\theta = 0$.

Let the potential inside the sphere $= \varphi_I$

" outside " " $= \varphi_{II}$

They will be independent of z for a long cylinder, such as in this case.

As there is no charge inside and outside the cylinder, Laplace's eq^n apply as $\nabla^2\varphi_1 = \nabla^2\varphi_{11} = 0$. ——— (1)

Now, the boundary conditions are

$$\varphi_1 = \varphi_{11}\Big|_{\rho=r} \qquad \left(\frac{\partial\varphi_1}{\partial\theta}\right) = \left(\frac{\partial\varphi_{11}}{\partial\theta}\right)\Bigg|_{\rho=r} \quad \text{———} \; (\star)$$

$\varphi_1\big|_{\rho\to 0} \to 0$ should be finite, $\varphi_{11}\big|_{\rho\to\infty} \to -E_0\rho\cos\theta$

The solutions of eq^n (1) can be written as

$$\varphi_1 = A_1 + B_1\ln\rho + C_1\rho\cos\theta + \frac{D_1}{\rho}\cos\theta$$

and

$$\varphi_{11} = A_2 + B_2\ln\rho + C_2\rho\cos\theta + \frac{D_2}{\rho}\cos\theta$$

From the above boundary conditions $(\star)$, we have

$$\frac{\partial\varphi_1}{\partial\theta}\Bigg|_{\rho=r} = -C_1 r\sin\theta = 0 \Rightarrow C_1 = 0$$

$$\frac{\partial\varphi_{11}}{\partial\theta}\Bigg|_{\rho=r} = \left(E_a r - \frac{D_2}{r}\right)\sin\theta = 0 \Rightarrow D_2 = E_a r^2$$

The potentials become

$$\varphi_1 = A_1$$

$$\varphi_{11} = A_2 + B_2\ln\rho - E_a\rho\cos\theta + \frac{E_a r^2}{\rho}\cos\theta$$

Assuming $A_2 = 0$, we get

$$A_1 = B_2 \ln r - E_a r\cos\theta + E_0 r\cos\theta = B_2 \ln r.$$

Now, applying Gauss' law to unit length of cylinder, we get

$$\oint \vec{E}_{II} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

$$(\because q = \sigma_0 \, 2\pi r)$$

$$\vec{E}_{II} = -\vec{\nabla}\varphi_{II}$$

$$-\frac{B_2}{r} 2\pi r = \frac{1}{\epsilon_0}\sigma_0 \, 2\pi r$$

$$B_2 = -\frac{\sigma_0}{\epsilon_0} r$$

Therefore, $A_1 = -\dfrac{\sigma_0 r \ln r}{\epsilon_0}$

Hence $\varphi_I = -\dfrac{\sigma_0 r \ln r}{\epsilon_0}$ — (★)

$$\varphi_{II} = \frac{-\sigma_0 r \ln \rho}{\epsilon_0} - E_a \rho\cos\theta + \frac{E_0 r^2}{\rho}\cos\theta$$ — (★)

Now, electric field.

$$\underline{E_I = 0}$$

$$E_{II} = -\nabla\phi_E = \left(\frac{\sigma_0 r}{\epsilon_0 \rho} + E_a\cos\theta + \frac{E_a r^2}{\rho^2}\cos\theta\right)\hat{\rho} + \frac{E_a}{\rho}\left(\rho - \frac{r^2}{\rho}\right)\sin\theta\,\hat{\theta}$$

$$\underline{E_{II}} = \left(\frac{\sigma_0 r}{\epsilon_0 \rho} + E_a\cos\theta + \frac{E_a r^2}{\rho^2}\cos\theta\right)\hat{\rho} - E_a\left(1 - \frac{r^2}{\rho^2}\right)\sin\theta\,\hat{\theta}$$

And the surface charge density -

$$\sigma(\theta) = \epsilon_0 E_{II}^{\perp}\Big|_{\rho = r} \qquad (E_{II}^{\perp} \text{ is the normal component})$$

$$= \epsilon_0\left(\frac{\sigma_0 r}{\epsilon_0 r} + E_a\cos\theta + \frac{E_a r^2\cos\theta}{r^2}\right)$$

$$\underline{\sigma(\theta) = \sigma_0 + 2\epsilon_0 E_a\cos\theta}$$

(b) The difference between this case and the case of cylindrical conductor lies in the fact that $\sigma(\theta)$ can be positive or negative for a conductor.

In this case, $\sigma(\theta) \leq 0$. (For the current problem)

However, for $|E_a| < \left|\frac{\sigma_0}{2\epsilon_0}\right|$, the two problems have the same solution

(c) For the case of a conducting cylinder, the electrostatic field should satisfy the following:

① Inside the conductor, $E_1 = 0$ and φ_1 is constant.

② Outside the conductor,

$$\nabla^2 \varphi_{11} = 0$$

$$\left(\frac{\partial \varphi_{11}}{\partial \theta}\right)_{\rho = r} = 0 \; ; \; \varphi_{11}\Big|_{\rho\to\infty} \to -E_a \rho \cos\theta.$$

Clearly, φ_{11} has the same solution as in the case (a).

For the solution of the conductor to fit the case of an insulating cylinder, the necessary condition is $|E_0| \leq \left|\frac{\sigma_0}{2\epsilon_0}\right|$.

This ensures that the surface charge density on the cylinder is negative everywhere.

Q3. (a) By Symmetry $\vec{A} = A\hat{\phi}$

We evaluate the flux integral for circle of radius ρ

Since $\Phi = \int dS \cdot \vec{B}$

$$\Rightarrow \quad \pi B\rho^2 = 2\pi\rho A(\rho) \text{ for } \rho < R$$

$$\pi R^2 B = 2\pi\rho A(\rho) \text{ for } \rho > R$$

So, $\vec{A} = \begin{cases} \frac{B\rho}{2}\hat{\phi} & \rho \le R \\ \frac{BR^2}{2\rho}\hat{\phi} & \rho \ge R \end{cases}$ — (1)

As we have $\underline{\nabla \cdot \vec{A} = 0}$

$\Rightarrow$ We are in the Coulomb Gauge.

(b) $A' = A + \nabla\underbrace{\left(-\frac{\Phi}{2\pi}\phi\right)}_{\chi} = A - \frac{\Phi}{2\pi\rho}\hat{\phi} = A - \frac{BR^2}{2\rho}\hat{\phi}$ — (2)

Then using (1) and (2), we have

$$A' = \begin{cases} \left(\frac{B\rho}{2} - \frac{BR^2}{2\rho}\right)\hat{\phi} & \rho \le R \\ \frac{BR^2}{2\rho}\hat{\phi}(1-1) = 0 & \rho \ge R \end{cases}$$

Thus, we can say, that the vector potential is zero outside the solenoid.

(c) $B' = \nabla\times A' = B - \frac{BR^2}{2}\nabla\times\left(\frac{\hat{\Phi}}{\rho}\right)$

$$= B - \frac{\Phi}{2\pi}\nabla\times\left(\frac{\hat{\Phi}}{\rho}\right) \quad — ③ \quad (\rho \neq 0).$$

Then $\nabla\times\left(\frac{\hat{\Phi}}{\rho}\right) = \frac{\hat{z}}{\rho}\frac{\partial}{\partial\rho}\left(\rho\times\frac{1}{\rho}\right) = 0$

Also, if S is a surface that cuts perpendicularly through the solenoid

$$\int_S dS\cdot\nabla\times\left(\frac{\hat{\Phi}}{\rho}\right) = \oint dS\cdot\frac{\hat{\Phi}}{\rho} = \int_0^{2\pi} d\phi = 2\pi.$$

Then, $\nabla\times\left(\frac{\hat{\Phi}}{\rho}\right) = \frac{\delta(\rho)}{\rho}(\hat{z}) \quad — ④$

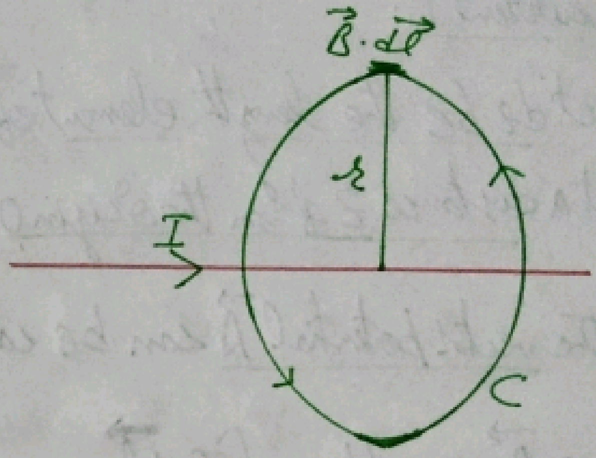
Using ③④, we can write

$$B' = B - \frac{\Phi}{2\pi}\frac{\delta(\rho)}{\rho}$$

Q4. (a) From Ampere's law we have,

$$\oint_C \vec{B}\cdot d\vec{l} = \mu I_{encl}.$$

Here, $d\vec{l}$ = line element of Amperian loop.



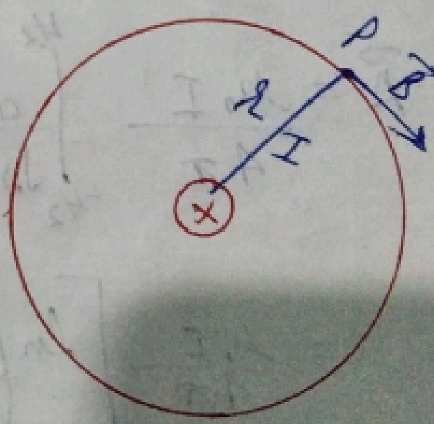
As the direction of $\vec{B}$ and $d\vec{l}$ are same

$$\int_0^{2\pi r} B\,dl = \mu_0 I$$

$$B\cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

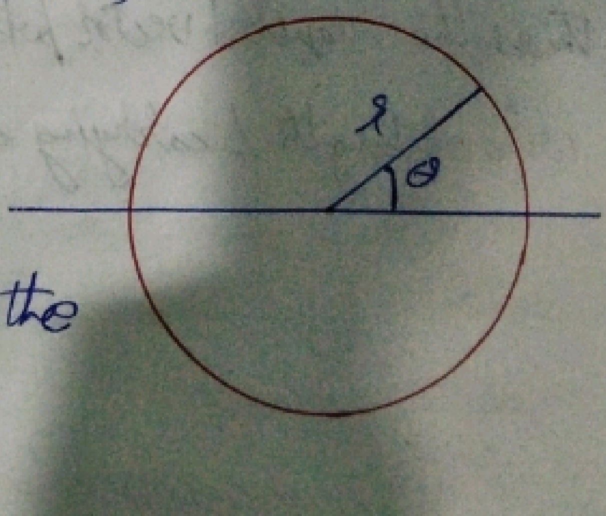
(b) Current is into the page.



From right hand curl rule, we can say that the dirn of magnetic field is in clockwise direction.

If we represent this in polar coordinates (r,θ), we write it as $-\hat{\theta}$.

i.e, $\vec{B} = -\dfrac{\mu_0 I}{2\pi r}\hat{\theta}$

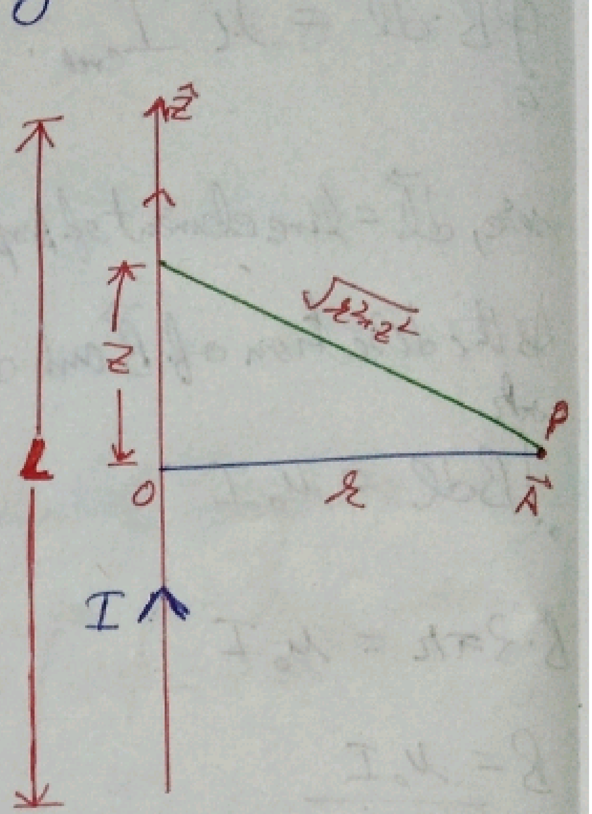


Here, $\vec{B}$ is tangential to every point on the amperian loop.

(C) We have to calculate magnetic vector potential at point P due to current carrying of wire of length L.

Consider a wire of length L and carrying current I.

Let dz be the length element of the wire at a distance z from the origin O.



The vector potential $\vec{A}$ can be calculated as

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\,\vec{dl}}{r}$$

$$\Rightarrow \vec{A} = \frac{\mu_0 I}{4\pi}\int_{-L/2}^{L/2} \frac{dz}{\sqrt{r^2+z^2}}\,\hat{z}$$

$$= \frac{\mu_0 I}{4\pi}\left[\ln\left(\frac{\frac{L}{2}+\sqrt{\frac{L^2}{4}+r^2}}{-\frac{L}{2}+\sqrt{\frac{L^2}{4}+r^2}}\right)\right]$$

This is the required vector potential at distance r from a finite wire of length L carrying current I.

(d) for an infinite wire,

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{L/2 + \sqrt{\frac{L^2}{4} + s^2}}{-L/2 + \sqrt{L^2/4 + s^2}}\right]$$

for an infinite wire, we calculate $\lim\limits_{L\to\infty} \vec{A}$

Thus, $$\vec{A} = \lim_{L\to\infty} \frac{\mu_0 I}{4\pi} \ln\left[\frac{L/2 + \sqrt{L^2/4 + s^2}}{-L/2 + \sqrt{L^2/4 + s^2}}\right]$$

$$= \lim_{L\to\infty} \frac{\mu_0 I}{4\pi} \ln\left[\frac{1/2 + \frac{1}{2}\sqrt{1 + \frac{4s^2}{L^2}}}{-1/2 + 1/2\sqrt{1+\frac{4s^2}{L^2}}}\right]$$

$$= \frac{\mu_0 I}{4\pi} \ln\left[\frac{1}{0}\right] \qquad \left(\because L\to\infty \Rightarrow \frac{1}{L^2}\to 0\right)$$

$$\vec{A} = \infty$$

$\Rightarrow$ For an infinitely long current carrying wire, the vector potential is infinity.

(c) Given - $A: \Rightarrow \vec{B} = \vec{\nabla} \times \vec{A}$ —①

Now, taking curl on both sides, we have

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla} \times \vec{B} \quad \text{—②}$$

But, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}(r)$ —③

$$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{j}(r)$$

Now, we choose A in such a way that its divergence is non zero.

i.e $\vec{\nabla} \cdot \vec{A} = v(r)$ ~~(Coulomb gauge)~~

We have, $-\nabla^2 \vec{A} = \mu_0 \vec{j}(r) + v(r)$

$$\text{or} \quad \nabla^2 \vec{A} = -\mu_0 \vec{j}(r) + v(r)$$

$$\Rightarrow \boxed{A_i(r) = \frac{\mu_0}{4\pi} \int \frac{j_i(\vec{r})}{|\vec{r} - \vec{r}'|} dV' + v(r)} \quad (i = x, y, z) \quad \text{—④}$$

(b) → (g)

Here the divergence of A, $\vec{\nabla} \cdot \vec{A}$ has no physical significance.

Thus, we are completely free to choose $\vec{\nabla} \cdot \vec{A}$ to be whatever we like.

The electric scalar potential is undetermined to an arbitrary additive constant. As $\phi \to \phi + c$ Leaves the electric field invariant.

Also, the magnetic field is invariant under the transform $A \to A - \nabla \psi$

This is what we call gauge transformations. The choice of a particular function ψ or a constant c is reffered to as choice of gauge.

The usual choice of gauge for the scalar potential ϕ is such that $\phi \to 0$ at infinity.

Then, the usual gauge for $\vec{A}$ is such that

$$\vec{\nabla} \cdot \vec{A} = 0 \qquad \text{(Coulomb Gauge)}$$

Then, the equation ④ becomes

$$\boxed{A_i(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{j_i(\vec{r}')}{|\vec{r}-\vec{r}'|} dV'} \quad \text{—— (☆)} \qquad (i = x, y, z)$$

or $$A(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{j(\vec{r}')\, d^3r'}{|\vec{r}-\vec{r}'|}$$

(h) The vector potential is parallel to the direction of the current.

The equation of $\vec{A}$ is a vector equation. In cartesian coordinates, the eq^n separates into three scalar equations.

From (☆)

$$A_x(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{j_x(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3r'$$

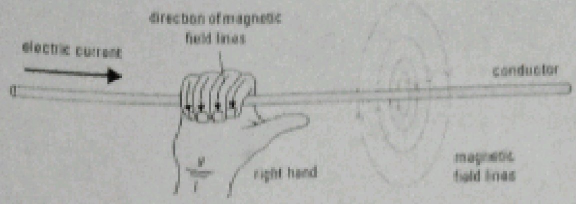
$$A_y(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{j_y(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3r'$$

$$A_z(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{j_z(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3r'$$

In this form, clearly components of $\vec{A}$ in a given direction depend only on the components of $\vec{j}$ that are in the same direction. If the current is carried in a long straight wire, $\vec{A}$ points in the same direction as the current in the wire.

(c) The result in part (b) implies that we could eliminate the Aharonov-Bohm effect by a gauge transformation. Show, however, that the new magnetic field corresponds to a different physical problem, where

$$B' = B - \frac{\Phi}{2\pi\rho}\delta(\rho)\hat{z} \tag{2}$$



Problem 4: A and B and their relationship with j, i.e. current/density

(a) For the current I in Fig. , which is flowing along the x–axis, find the magnetic field around it using Ampere's Law.

(b) If we look down the current carrying wire, so that the current is into the page, as in Fig. , and if we use the r, θ co-ordinate system, what is the direction of the magnetic field?

(c) Now, find the vector potential $\boldsymbol{A}$ for a wire of length L. [Hint: you may need to integrate an expression!]

(d) For an infinitely long wire carrying current, is the vector potential finite or infinite?

(e) Now, use $\nabla \times \boldsymbol{A} = \boldsymbol{B}$, and two Maxwell's equations: $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$ and $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$, or any other way, to find the relationship between $\boldsymbol{A}$ and $\boldsymbol{j}$, i.e. the current.

(f) Are Maxwell's equations affected by $\nabla \cdot \boldsymbol{A}$? Why or why not?

(g) Use your answer in the previous question to simplify the equation you got in Q. (e).

(h) From the previous question, or otherwise, figure out the direction of the vector potential with respect to the current.