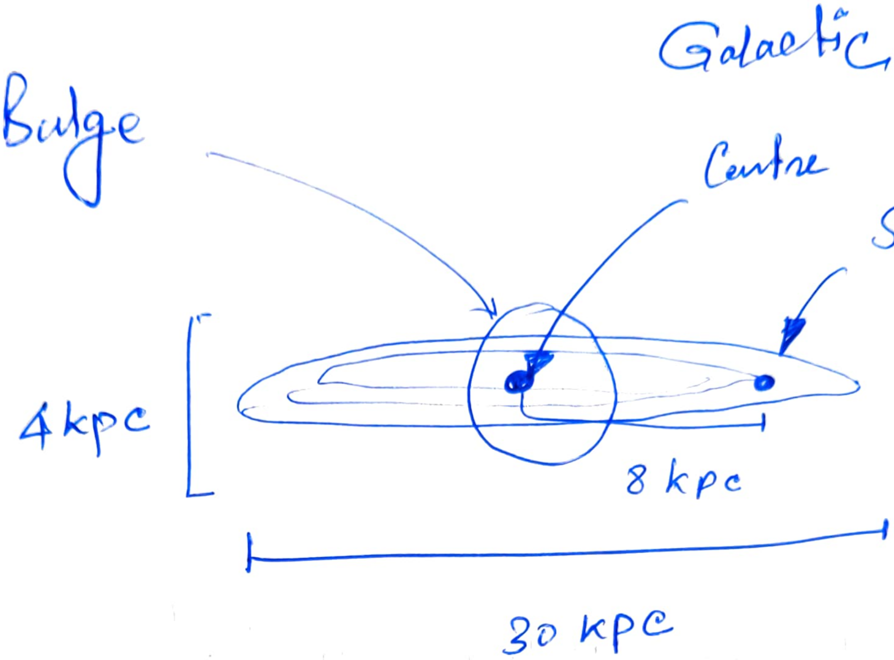
31/12/2021

AA 472/672N GAEG1

~~Galactic~~ Components Galaxy :—

Galactic Bulge

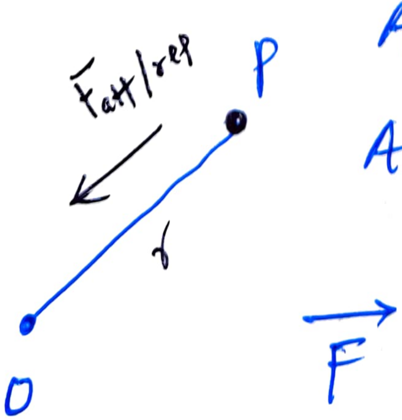


* Gas & Dust

> Oh ~~Boy~~ Be A fine Girl; Kiss Me
>
> O B A F G K M

Central Force :

A coordinate system
A origin (O)
A point of ~~infects~~ interest (P)



$$\vec{F} \propto f(r)$$

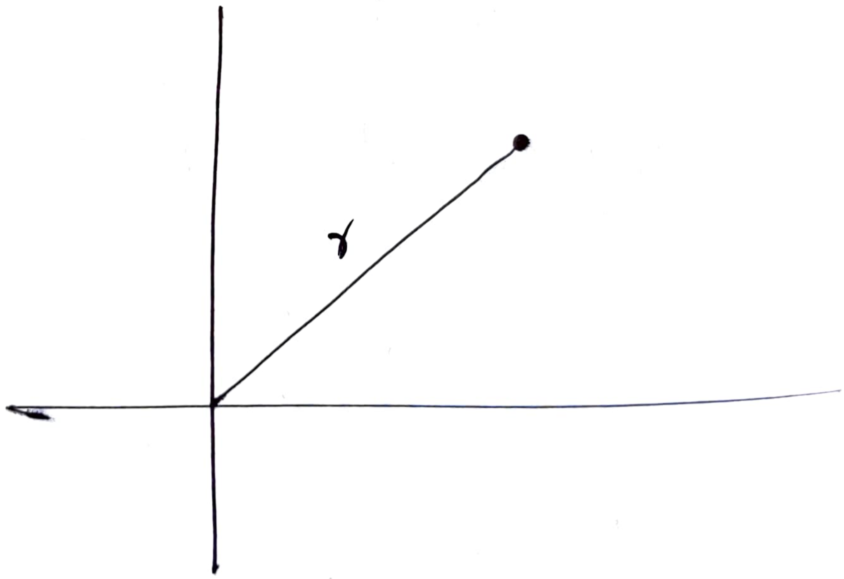
Properties of Central force

(I) Conservative force

(II) Conservative angular momentum

Magnit — dirn → Planer Motion

Central force Motion in Polar-coordinates



$$\vec{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$$

$$\frac{d\vec{r}}{dt} = \frac{\partial\vec{r}}{\partial\theta}$$

$$\hat{r} = \frac{\frac{\partial\vec{r}}{\partial r}}{\left|\frac{\partial\vec{r}}{\partial r}\right|}$$

$$= \frac{\cos\theta\,\hat{i} + \sin\theta\,\hat{j}}{\sqrt{\cos^2\theta + \sin^2\theta}} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{\theta} = \frac{\frac{\partial \vec{r}}{\partial \theta}}{\left|\frac{\partial \vec{r}}{\partial \theta}\right|} = \frac{-r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j}}{r}$$

$$= -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\frac{d\vec{r}}{dt} = \frac{\partial \vec{r}}{\partial r}\cancel{\frac{\partial r}{dt}} +$$

$$\dot{\hat{r}} = -\sin\theta\,\hat{i}\cdot\dot{\theta} + \cos\theta\,\dot{\theta}\,\hat{j} = \dot{\theta}\,\hat{\theta}$$

$$\dot{\hat{\theta}} = -\cos\theta\,\dot{\theta}\,\hat{i} - \sin\theta\,\dot{\theta}\,\hat{j} = -\dot{\theta}\,\hat{r}$$

$$\frac{d\vec{r}}{dt} = \frac{d}{dt}(r\hat{r}) = r\dot{\hat{r}} + \dot{r}\hat{r}$$

$$= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = \frac{d}{dt}(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}) = \ddot{r}\hat{r} + \dot{r}\dot{\hat{r}} + \dot{r}\dot{\theta}\hat{\theta}$$

$$+ r\ddot{\theta}\hat{\theta} + r\dot{\theta}\dot{\hat{\theta}}$$

$$= \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta}$$

$$+ r\ddot{\theta} - r\dot{\theta}^2\hat{r}$$

$$\ddot{r} - \frac{c}{r^3} = \frac{F(r)}{m} \longrightarrow \text{Non linear}$$

$$\frac{1}{r} = u$$

$$\frac{d^2u}{d\theta^2} + u = \frac{-f\left(\frac{1}{u}\right)}{m\ell^2 u^2}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

eq^n of motion $\longrightarrow$ Central force

$$\vec{F} = m\vec{a}$$

$$f(r)\hat{r} = m(\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

equating both sides

$$\boxed{f(r) = m(\ddot{r} - r\dot{\theta}^2)}$$

and

$$\boxed{r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0}$$

$$\frac{d}{dr}(r^2\dot{\theta}) = 0$$

$$\Rightarrow \boxed{r^2\dot{\theta} = \text{Const}}$$

$\hookrightarrow$ Angular Momentum is Conserve.

~~AA 672/470~~

AA 472/672 N

Galactic and Extragalactic Astronomy

Central force

- Conservative force
- Vector angular momentum $\Rightarrow$ Conserved
- Total energy is conserved.

$$E = ke + pe \Rightarrow \text{Constant}$$

$$L = mr^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{L}{mr^2}$$

$$\dot{\theta}^2 = \frac{L^2}{m^2 r^4}$$

$$\frac{mr^2\dot{\theta}^2}{2} = \frac{L^2}{2mr^2}$$

$$\boxed{U_{eff} = V(r) + \frac{L^2}{2mr^2}}$$

$$V(r) \propto -\frac{1}{r}$$

$$V(r) = -\frac{GM_{ms}}{r}$$

$$\frac{d^2u}{d\theta^2} + u = -\frac{f(\frac{1}{u})}{mc^2u^2}$$

$$F(r) = f(\tfrac{1}{u})\,\hat{r} = -\vec{\nabla} V(r)$$

$$= \frac{d}{dr} V(r)$$

$$= \frac{d}{dr} V(\tfrac{1}{u}) \quad \frac{dV}{du} V(\tfrac{1}{u}) \frac{du}{dr}$$

$$\Rightarrow \frac{d^2u}{d\theta^2} + u = \frac{1}{L^2u^2}\left[\frac{dV}{dr}(\tfrac{1}{u})\right]$$

$u = \frac{1}{r}$

$du = -\frac{dr}{r^2}$

$du = -u^2 dr$

$$\Rightarrow \cancel{\int \frac{d^2u}{d\theta^2} du + \int u\,du = \int \frac{1}{L^2u^2} \frac{dV}{dr}(\tfrac{1}{u})}\, du$$

$$\Rightarrow \int \frac{d^2u}{d\theta^2} \frac{du}{d\theta} dt + \int u \frac{du}{d\theta} dt$$

$$= \int \frac{1}{L^2u^2} \frac{dV(\frac{1}{u})}{dr} \frac{du}{d\theta} dt$$

$$\Rightarrow \frac{d^2u}{d\theta^2}\cdot\frac{du}{d\theta} + u\frac{du}{d\theta} = $$

$$\frac{d}{d\theta}\left(\frac{du}{d\theta}\right)\cdot\frac{du}{d\theta} \qquad \frac{1}{L^2u^2}\frac{dV}{dr}\left(\frac{1}{u}\right)\frac{du}{d\theta}$$

$$du = -u^2 dr \qquad \frac{dV}{dr}\left(\frac{1}{u}\right) = \frac{dV}{du}$$

$\Rightarrow$ 2

$\frac{d}{dy}(n)$

$$\frac{du}{d\theta}\frac{d^2u}{d\theta^2} + u\frac{du}{d\theta} = \frac{1}{L^2u^2}\frac{dV}{dr}\left(\frac{1}{u}\right)\frac{du}{d\theta}$$

$\frac{x^2}{2}$

$$\frac{dV}{du}\frac{du}{dr}\frac{du}{d\theta}$$

$$\frac{du}{d\theta}\frac{d}{d\theta}\left(\frac{du}{d\theta}\right) + \frac{d}{d\theta}\left(\frac{u^2}{2}\right) =$$

$$\frac{d}{d\theta}\left[\frac{1}{2}\left(\frac{du}{d\theta}\right)^2 + \frac{u^2}{2}\right] = -\frac{1}{L^2\cancel{u^2}}\cdot\cancel{u^2}\frac{dV}{\cancel{dr}}\frac{\cancel{dr}}{d\theta}$$

$$= -\frac{1}{L^2}\frac{dV}{d\theta}$$

$$\frac{d}{d\theta}\left[\left(\frac{du}{d\theta}\right)^2 + u^2 + \frac{2}{L^2}V\right] = 0$$

$$\int \frac{d}{d\theta}\left[\left(\frac{du}{d\theta}\right)^2 + u^2 + \frac{2}{L^2}V\right]d\theta = 0$$

$$\left(\frac{du}{d\theta}\right)^2 + u^2 + \frac{2}{L^2}V = \text{Constant}$$

$$\text{constant} = \frac{2E}{L^2}$$

$$\boxed{\left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{2}{L^2}(E - V)}$$

0

- Bound
- Unbound

$$\boxed{u^2 + \frac{2}{L^2}(V - E) = 0}$$

$$r_+ = \frac{1}{u_+} \qquad r_- = \frac{1}{u_-}$$

44

472) 672

Gal. & Ex.

$$\frac{d^2u}{d\theta^2} + u = -\frac{1}{L^2u^2}\frac{d\Phi}{dr}\left(\frac{1}{u}\right)$$

$$E = \frac{1}{2}\dot{r}^2 + \frac{L^2}{2r^2} + \Phi(r)$$

$$\Rightarrow \quad \frac{dr}{dt} = \pm\sqrt{2(E-\Phi) - \frac{L^2}{r^2}} \quad \text{——— } (1)$$

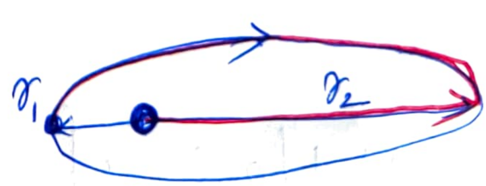
and $\quad \dfrac{d\theta}{dt} = \dfrac{L}{r^2}$

$$\therefore \quad \frac{dr}{d\theta} = \frac{\sqrt{2(E-\Phi) - \frac{L^2}{r^2}}}{\frac{L}{r^2}}$$

$$\left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{2(E-\Phi)}{L^2}$$

$$r = \frac{1}{u}$$

$$\frac{dr}{d\theta} = -\frac{1}{u^2}\frac{du}{d\theta}$$



from (1)

$$T_r = 2\int_{r_1}^{r_2} \frac{dr}{\sqrt{2(E-\Phi) - L^2/r^2}}$$

Model

$\rho(r) \rightarrow$ avg density of stars + gas + dust

$$\nabla^2 \Phi(r) = 4\pi G \rho$$

$\hookrightarrow v_c$

Potential Density Pairs

Example, Let's build a galaxy $\Rightarrow$ Assuming Spherical galaxy

$$\rho(r) = \frac{M}{4\pi} \frac{a}{r^2 (r+a)^2}$$

$M, a \rightarrow$ Constant

(i) Mass of galaxy

(ii) $\Phi(r)$

(iii) v_c

$$M_{gal} = \int_V \rho(r)\, dV$$

$$= 4\pi \int_0^\infty \frac{M}{4\pi} \frac{a}{r^2(a+r)^2} r^2\, dr$$

$$= M \int_0^\infty \frac{a}{(a+r)^2}\, dr$$

$$= M \qquad a + r = k$$
$$dr = dk$$

$$= Ma \int_a^\infty \frac{1}{k^2}\, dk$$

$$= -\frac{Ma}{a} \left[\frac{1}{k}\right]_a^\infty$$

$$\Rightarrow \boxed{M_{gal} = +M}$$

(ii) $\frac{1}{\cancel{r^2}} \frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = 4\cancel{\pi} G \frac{M}{\cancel{4\pi}} \frac{a}{\cancel{r^2}(a+r)^2}$

$$r^2 \frac{d\phi}{dr} = \int_0^r \frac{GMa}{(a+r)^2}\, dr$$

$$\cancel{r^2 \frac{d^2\phi}{dr^2} + 2r \frac{d\phi}{dr} = \frac{GMa}{(a+r)^2}}$$

$$G \int_0^r \frac{1}{(r+a)^2}\, dr$$

$r + a = k$

$$\int_{a}^{\cancel{r+a}\ r}$$

$$\left[-\frac{1}{k}\right]_0^r$$

$$-\cancel{a}\left[\frac{1}{r+a}\right]_0^r$$

$=$

$$r^2 \frac{d\phi}{dr} = GMa \int_a^{\infty} \frac{1}{k^2}\, dk$$

$$= \frac{GM\cancel{a}}{\cancel{a}} \qquad \left[\frac{1}{k}\right]_0^r$$

$$r^2 \frac{d\phi}{dr} = GM \qquad \left[\frac{1}{a+r}\right]$$

$$\phi = \int_0^{\infty} \frac{GM}{r^2}\, dr$$

$$\phi = GM\left[-\frac{1}{r}\right]$$

$$r^2 \frac{d\phi}{dr} = -GMa\left[\frac{1}{a+r}\right]_0^r$$

$$= -GMa\left(\frac{1}{a+r} - \frac{1}{a}\right)$$

$$= GMa\left(\frac{1}{a} - \frac{1}{a+r}\right)$$

$$= GM\cancel{a}\ \frac{\cancel{a}+r-\cancel{a}}{\cancel{a}(a+r)}$$

$$= \frac{GMr}{a+r}$$

$$\phi = \int \frac{GMr}{(a+r)} \cdot \frac{1}{r^2} dr$$

$$= \int_0^r \frac{GM}{r(a+r)} dr$$

$$= \frac{GM}{a} \ln\left(\frac{r}{a+r}\right) = -\frac{GM}{a} \ln\left(1+\frac{a}{r}\right)$$

Case I: when $r >> a$

$$\ln\left(1+\frac{a}{r}\right) = \frac{a}{r} - \frac{1}{2}\left(\frac{a}{r}\right)^2 + \dots$$

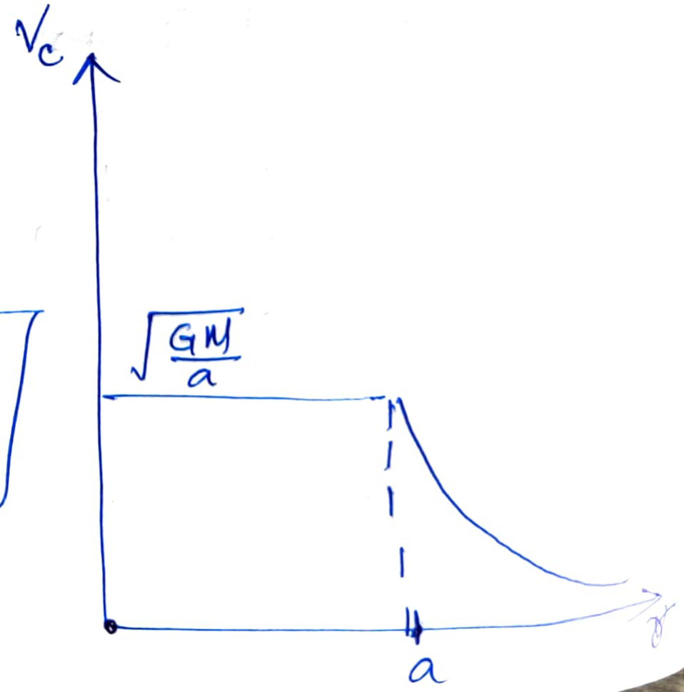
$$\phi = -\frac{GM}{a} \cdot \frac{a}{r} = -\frac{GM}{r}$$

$$\Rightarrow \boxed{\phi = -\frac{GM}{r}}$$

(iii) $$V_c^2 = r\left|\frac{d\phi}{dr}\right| = r\frac{GM}{r(a+r)}$$

$$\Rightarrow \boxed{V_c^2 = \frac{GM}{a+r}}$$

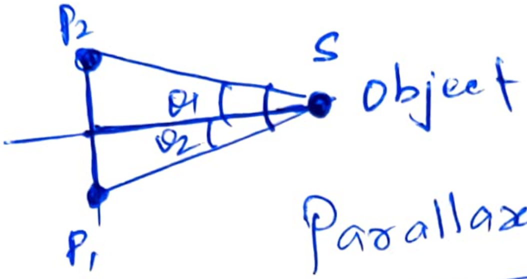
$$\Rightarrow \boxed{V_c = \sqrt{\frac{GM}{a+r}}}$$



Ex. Galactic 44-472/672N

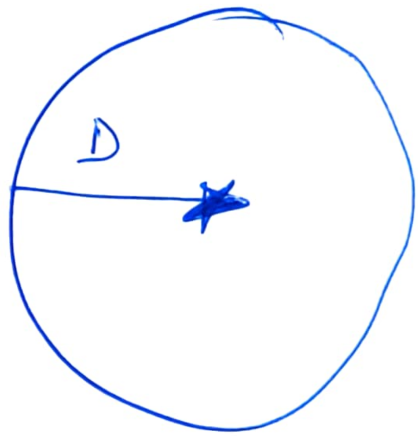
11/01/2022

— Amit Sir



Parallax Method

Luminosity Distance



flux.

$$F = \frac{L}{4\pi D^2} = \frac{\text{energy/time}}{4\pi D^2}$$

Luminosity ~~⊗~~ magnitud

$$m = -2.5 \log\left(\frac{F}{F_{vega}}\right)$$

Absolute Magnitude M = app. magnitude m at standard distance 10 pc.

Globular cluster :- Stars Born at same time

$$\cancel{d = 10^{\left(\frac{m-M}{10}\right)}}$$

$$\frac{m-M}{5} = \log\left(\frac{D}{10}\right)$$

$$\log D = \cancel{10^{\frac{m-M}{5}}} \; \frac{2}{5} + 1$$

$$= 10^{2/5} \text{ pc} = 7/5$$

$$D = 10^{7/5} \text{ pc}$$

$$= 10^{1.4} \text{ pc}$$

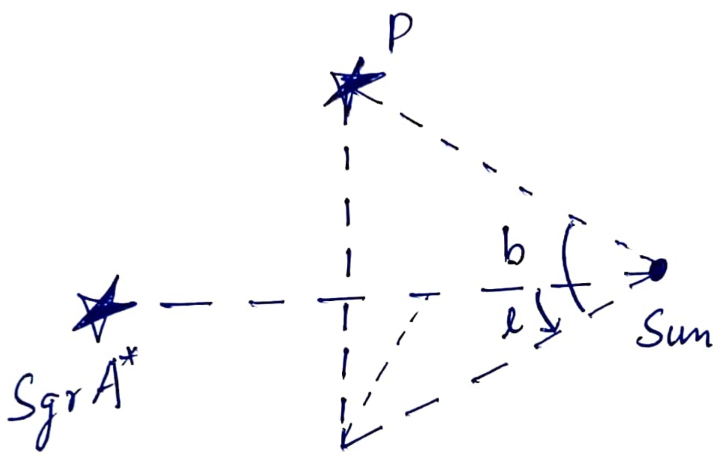
$$= 25 \text{ pc}$$

Date - 18/01/2022 AA 472/672 N

Proxima Century $\rightarrow$ Closed approaches

Galactic Coordinate $\rightarrow$ Spherical Coordinat

Our Sun $\rightarrow$ Origin



Assumption

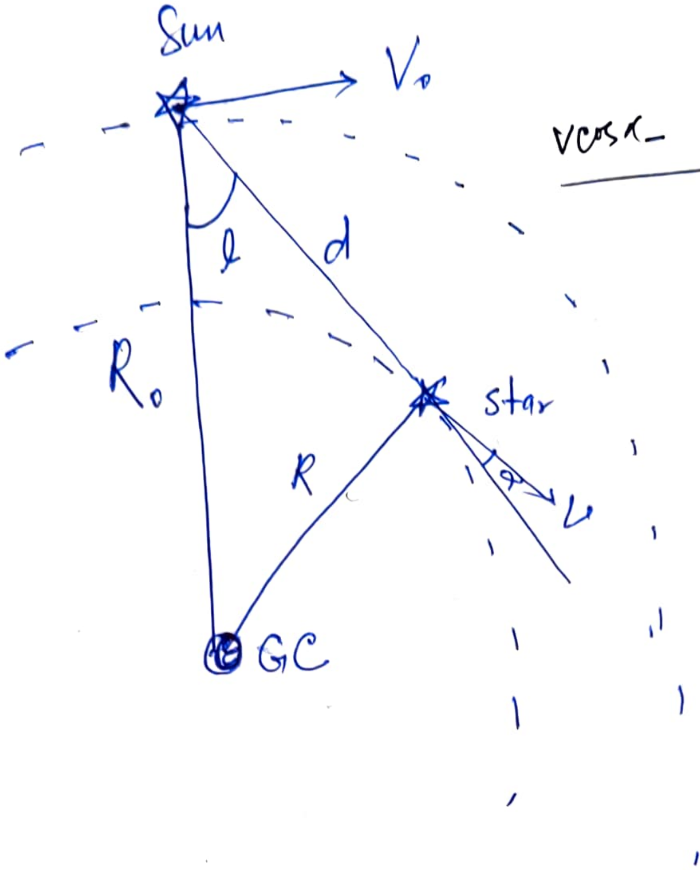
(i) All stars $\rightarrow$ moves in Circular Orbit

$\downarrow$

Ω_c

(ii) Solar Neighbourhood

Observation in Sun's frame.



$V\cos\alpha - V_0\cos(90 - l)$

Sun's frame

$$\Omega_0 = \frac{V_0}{R_0}, \quad \Omega = \frac{V}{R}$$

$$V_{obs,r} = V_{star,r} - V_{sun,r} = V\cos\alpha - V_0\cos(90-l)$$

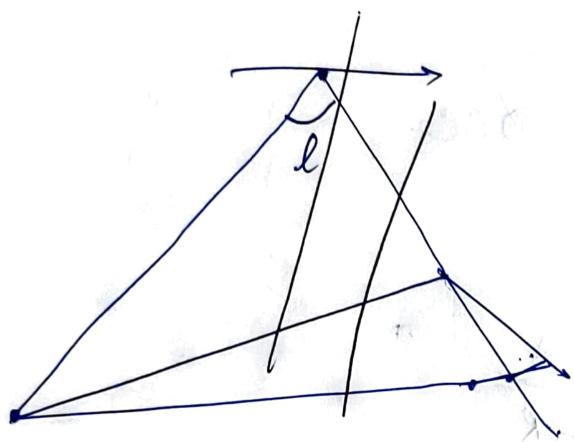
$$= V\cos\alpha - V_0 \sin l$$

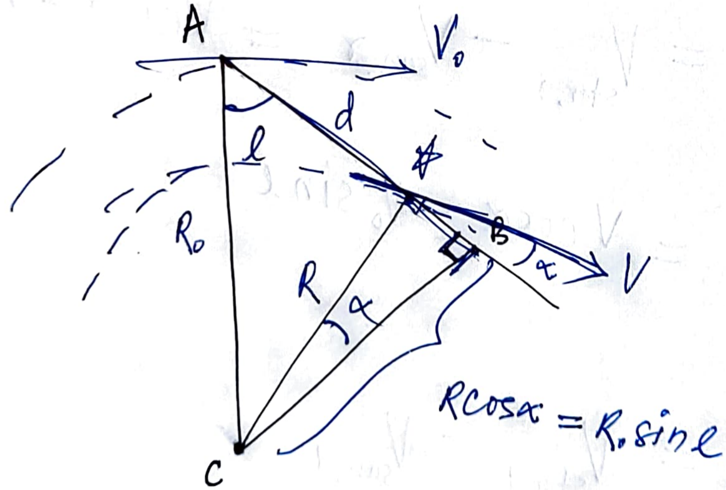
$$V_{obs,t} = V_{star,t} - V_{sun,t}$$

$$= V\sin\alpha - V_0\cos l$$

$$V_{obs,r} = \Omega R\cos\alpha - \Omega_0 R_0 \sin l$$

$$V_{obs,t} = \Omega R \sin\alpha - \Omega_0 R_0 \cos l$$





~~R cos~~ $R\cos\alpha = R_0 \sin l = BC$

$R\sin\alpha = R_0\cos l - d = B\star$

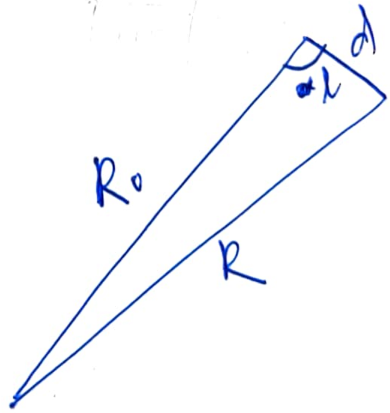
$$V_{obs,r} = (\Omega - \Omega_0) R_0 \sin l$$

$$V_{obs,t} = (\Omega - \Omega_0) R_0 \cos l - \Omega d$$

<u>Sun's neighbourhood stars</u>

$$R - R_0 \longrightarrow \text{Small}$$

$$\Omega(R) = \Omega_0 + (R - R_0) \left.\frac{d\Omega}{dR}\right|_{R_0} + \cdots$$



$$R - R_0 = -d \cos l$$

<u>Therefore,</u>

$$V_{obs,r} = -R_0 \left.\frac{d\Omega}{dR}\right|_{R_0} d \cos l \sin l$$

$$V_{obs,r} = -R_0 \left.\frac{d\Omega}{dR}\right|_{R_0} d \cos^2 l$$

$$V_{obs,r} = Ad\sin(2l)$$

$$V_{obs,t} = Ad\cos 2l + Bd$$

$$\boxed{\begin{aligned} A &= -\frac{1}{2} R_0 \left.\frac{d\Omega}{dR}\right|_{R_0} \\ B &= -\frac{1}{2} R_0 \left.\frac{d\Omega}{dR}\right|_{R_0} - \Omega \end{aligned}}$$

Oorts Constant

AA 472/672 :- Galactic and Extra galactic

$V_r = Ad \sin 2l$

$V_t = Ad \cos 2l + Bd$

$$A = \frac{1}{2}\left(\frac{V_0}{R_0} - \left.\frac{dV}{dR}\right|_{R_0}\right)$$

$$B = -\frac{1}{2}\left(\frac{V_0}{R_0} + \left.\frac{dV}{dR}\right|_{R_0}\right)$$