ASSIGNMENT 2 Sohini Dutta

AA 474 2003121011

Q.1. (a) Given that $m_v = 22$

We know that apparent magnitude and flux are related in the following fashion:

$$m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right) \quad \text{——— (1)}$$

We know that the star Vega is considered to be the reference point for apparent magnitude, so, $m_2 = 0$. Let $m_1 = m_v = 22$.

Flux of Vega in the V band is:

$$F_\nu = 3.636 \times 10^{-20} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$$
$$F_\lambda = 362.1 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Å}^{-1}$$

So, from (1), we can write:

$$22 = -2.5 \log_{10}\left(\frac{F_v}{3.636 \times 10^{-20}}\right)$$

$$\Rightarrow \frac{22}{-2.5} = \log_{10}\left(\frac{F_\nu}{3.636 \times 10^{-20}}\right)$$

$$\Rightarrow 10^{-8.8} = \frac{F_\nu}{3.636 \times 10^{-20}}$$

$\Rightarrow F_\nu = 5.763 \times 10^{-29}$ erg cm^{-2} s^{-1} Hz^{-1} $= 5.763 \times 10^{-6}$ Jy for the given star.

Ily, $$22 = -2.5 \log\left(\frac{F_\lambda}{363.1 \times 10^{-11}}\right)$$

$$\Rightarrow 10^{-8.8} = \frac{F_\lambda}{363.1 \times 10^{-11}}$$

$$\Rightarrow F_\lambda = 5.755 \times 10^{-18} \text{ erg cm}^{-2} \text{s}^{-1} \text{Å}^{-1}$$

We know that V-band corresponds to a wavelength range between 500 nm to 700 nm. So, assuming mean value of 600 nm, energy possessed by each photon

$$E = \frac{hc}{\lambda} = 3.315 \times 10^{-19} \text{ J} = 3.315 \times 10^{-12} \text{ ergs.}$$

$\therefore$ from F_λ, number of photons striking cm^{-2} s^{-1} is:

$$\frac{\lambda F_\lambda}{E} = \frac{5.755 \times 10^{-18} \times 6000}{3.315 \times 10^{-12}} = 1.04 \times 10^{-2} \text{ photons cm}^{-2} \text{s}^{-1}$$

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Given that the diameter of the telescope is 200 inch

$\Rightarrow$ Radius $= 100$ inch $= 254$ cm

$\Rightarrow$ Area $= 2.03 \times 10^5$ cm^2

$\Rightarrow$ Total number of photons striking the telescope per second from the source is:

$$1.04 \times 10^{-4} \times 2.03 \times 10^5 \simeq 211.4 \text{ photons s}^{-1}$$

(k) From the magnitude flux relation, we can write:

$$m_1 - m_0 = -2.5 \log_{10}\left(\frac{f_1}{f_0}\right)$$

where m_0 and f_0 are apparent magnitude and flux of Vega respectively.

now, given that for 1 arcsec^{-2} area, the magnitude is 20.4

$\therefore$ $m_1 = 20.4$

$$\Rightarrow \quad 20.4 - m_2 = -2.5 \log_{10}\left(\frac{f_1}{f_2}\right) \quad \text{—— (1)}$$

where m_2 is the apparent magnitude of the sky through the spectrograph and f_2 is the corresponding flux.

We know, that m_1 and f_1 are given for 1 arcsec^2

So, for the aperture of area $2 \times 2 = 4\,\text{arcsec}^2$,
$F_2 = 4F_1$

Using this, we have from ①,

$$20.4 - m_2 = -2.5 \log_{10}\left(\frac{\cancel{F_1}}{4\cancel{F_1}}\right)$$

$\Rightarrow 20.4 - m_2 = -2.5 \times \log 0.25 = 1.505$

$\Rightarrow 20.4 - m_2 = -1.5 \quad \Rightarrow m_2 = 18.895$

This will be the apparent magnitude of the patch of sky observable through the spectrograph

(c) The key is the narrowness of the band, which is 1/100 the width of the entire V-band (for one resolution element).
Thus, the number of photons in each resolution element from the galaxy is 0.33 photons/s, (accounting for the 10% efficiency), giving 1188 photons.
The sky is 3.1 mag brighter and thus is $10^{0.4 \times 3.1} = 17.4$ times brighter, or about 20700 photons.

(d) Let the expected number of source photon counts in a time duration Δt be S and the expected number of background photon counts in the same time duration Δt be B.

$\therefore$ the total on-source count should be $S+B$ and the total off source count should be B.

$\therefore$ The desired signal is given by:

$$S = (S+B) - B$$

There will be an error propagated through subtraction to S due to fluctuations in measurements of $(S+B)$ and B.

The error is given by:

$$\sigma_S^2 = \sigma_{S+B}^2 + \sigma_B^2$$

now $\sigma_{S+B} = \sqrt{S+B}$ and $\sigma_B = \sqrt{B}$

So, $\sigma_S^2 = S + B + B = S+2B$.

$$\therefore \text{Significance or SNR} = \frac{S}{\sigma_S} = \frac{S}{\sqrt{S+2B}} \quad \text{— (A)}$$

Q.2. $V = \frac{c\,\Delta\lambda}{\lambda_0}$

now, $\lambda_0 = \frac{c}{\nu} = \frac{3\times10^{8}}{110\times10^{9}} = 2.73\times10^{-3}$ m

$\therefore\ 10^{3} = \frac{3\times10^{8}\times\Delta\lambda}{2.73\times10^{-3}}$

$\Rightarrow\ \Delta\lambda = \frac{2.73}{3\times10^{8}} = 9.09\times10^{-9}$ m

$\therefore$ Resolution $= \frac{\lambda}{\Delta\lambda} = \frac{2.73\times10^{-3}}{9.09\times10^{-9}} = 3.27\times10^{5}$

Now, $\Delta\lambda = \frac{c}{\nu^2}\,\Delta\nu$

$\Rightarrow \Delta\nu = \frac{\nu^2\,\Delta\lambda}{c} = \frac{(110\times10^{9})^{2}\times9.09\times10^{-9}}{3\times10^{8}}$ Hz

$= 0.367$ MHz

Q.3. Total number of γ-rays travelling through the thickness t of the detector is:

$$N_\gamma = N_0 \exp\left(\frac{-\sigma_1 N_a \rho t}{Z}\right)$$

Now, each γ-ray emits an X-ray with a probability p.

So, total number of X-rays expected:

$$N' = p(N_0 - N_f)$$
$$N' = p N_0 \left[1 - \exp\left(-\frac{\sigma_1 N_a \rho t}{Z}\right)\right]$$

Now, the X-rays themselves have a cross section σ_2

Thus, number of X-rays coming out:

$$N_1 = N' \exp\left(-\frac{\sigma_2 N_a \rho t}{Z}\right)$$

$\therefore N_1 =$

$$p N_0 \left[\exp\left(-\frac{\sigma_2 N_a \rho t}{Z}\right) - \exp\left\{-\frac{(\sigma_1 + \sigma_2) N_a \rho t}{Z}\right\}\right]$$

Q.4. $$x_{av} = \int_0^\infty x\, N(x)\, dx \Big/ \int_0^\infty N(x)\, dx$$

$$= \frac{\int_0^\infty x \cancel{N_i}\, e^{-\sigma n x}\, dx}{\int_0^\infty \cancel{N_i}\, e^{-\sigma n x}\, dx}$$

$$= \frac{\int_0^\infty x\, e^{-\sigma n x}\, dx}{\int_0^\infty e^{-\sigma n x}\, dx}$$

Using gamma function defined as $\int_0^\infty x^n e^{-ax}\, dx = \frac{\Gamma(n+1)}{a^{n+1}}$

$$x_{av} = \frac{\frac{\Gamma 2}{(\sigma_n)^2}}{\frac{\Gamma 1}{(\sigma n)}} = \frac{1}{\sigma n}$$

Q.5. (a) Counts can be represented as:

Counts $= n \pm \sigma$

$\therefore$ Count rate can be written as:

$$\text{Count Rate} = \frac{n}{T} \pm \frac{\sqrt{n}}{T} = \frac{n}{T} \pm \frac{\sigma}{T}$$

where T is the time of observation and $\sigma = \sqrt{n}$ is the standard deviation using Poisson statistics.

Now variance for our case, considering Poisson statistics can be written as:

$$\sigma_{s+b}^2 = \sigma_s^2 + \sigma_b^2$$

where σ_{s+b}^2 is the variance of the on-source measurement over time t_s and σ_b^2 is the off-source variance over time t_b.

Hence, if $S+B$ denotes on-source count where S is the count due to source and B the count due to the background, then

$$\sigma_{s+b}^2 = \frac{S+B}{t_s^2} \quad \text{and} \quad \sigma_s^2 = \frac{S}{t_s^2}$$

$$\sigma_b^2 = \frac{B}{t_b^2}$$

$\therefore$ the standard deviation for the sample may be expressed as:

$$\sigma_s = \sqrt{\sigma_{s+b}^2 + \sigma_b^2} = \sqrt{\frac{S+B}{t_s^2} + \frac{B}{t_b^2}} \quad \text{— (1)}$$

If rates are defined as: $r_{s+B} = \dfrac{S+B}{t_s}$;

$r_B = \dfrac{B}{t_b}$;

$r_s = \dfrac{S}{t_s}$

then, $\sqrt{\dfrac{S}{t_s^2}} = \sqrt{\dfrac{S+B}{t_s^2} + \dfrac{B}{t_b^2}}$.

$\Rightarrow \sqrt{\dfrac{r_s}{t_s}} = \sqrt{\dfrac{r_{s+b}}{t_s} + \dfrac{r_b}{t_b}}$

$\Rightarrow \dfrac{r_s}{t_s} = \dfrac{r_{s+b}}{t_s} + \dfrac{r_b}{t_b}$

$\Rightarrow r_s = r_{s+b} + r_b \dfrac{t_s}{t_b} \quad \text{— (2)}$

(b) From (1), $\sigma_s = \sqrt{\dfrac{r_{s+b}}{t_s} + \dfrac{r_b}{t_b}} \quad \text{— (3)}$

$\therefore$ using (2) and (3),

$$\frac{r_s}{\sigma_s} = r_{s+b} + \frac{r_b t_s}{t_b} \Bigg/ \sqrt{\frac{r_{s+b}}{t_s} + \frac{r_b}{t_b}}$$

$$= \frac{t_s \left(\frac{r_{s+b}}{t_s} + \frac{r_b}{t_b} \right)^{1/2}}{\cancel{\sqrt{\frac{r_{s+b}}{t_s} + \frac{r_b}{t_b}}}}$$

$$\Rightarrow \frac{r_s}{\sigma_s} = t_s \sqrt{\frac{r_{s+b}}{t_s} + \frac{r_b}{t_b}} = R \quad \text{—— (4)}$$

Using $T = t_s + t_b$ and $r_{s+b} = r_s + r_b$ and solving, we have from above:

$$\frac{dR}{dt_s} = \frac{(r_s + r_b) + \frac{2 t_s r_b}{T - t_s} + \frac{t_s^2}{(T-t_s)^2} r_b}{2\sqrt{(r_s + r_b) t_s + \frac{t_s^2 r_b}{T - t_s}}} = 0$$

$$\therefore t_s = 2T \pm \sqrt{4T^2 - 4\left(\frac{r_b + r_s}{r_b}\right) T^2} \Big/ 2 \quad \text{—— (4)}$$

(c) For $r_s \gg r_w$,

from ④, in this case:

$t_s \simeq T$

Case II $r_s \simeq r_w$

from ④, $t_s \simeq T$, considering only the real part.

Case III, $r_s \ll r_w$

from ④, $t_s \simeq 0$

Q.6. Let m be true event rate and n is observed count rate.

We assume within an observation time K photons are actually detected.

For K photons, KT seconds of event registration will be missed.

As m is true event rate, so number of events missed is mKT.

So, for an observation duration of t we have

$$\frac{(mt - mKT)}{t} = \frac{K}{t}$$

or, $$mt - mKT = K$$

or, $$m = \frac{K}{t - KT} = \frac{K/t}{1 - \left(\frac{K}{t}\right)T} = \frac{n}{1 - nT}$$