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ASSIGNMENT #1

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Que1 → If we consider the probability of spotting quasars in a particular area in sky to be independent, then P(1,2)

P(Quasar 1 in given solid angle Ω, Quasar 2 within the angular area) = P(Quasar 1 in given solid angle) × P(Quasar 2 within the angular area) = P(1) P(2)

If S_1 and S_2 are the surface density of quasar 1 and 2 respectively, then

$$P(1) = S_1 \Omega$$

and $$P(2) = S_2 \pi r^2$$

[r is the radius of the angular area]

then, $$P(1,2) = S_1 S_2 \Omega \pi r^2$$

But the probability of observing 2 quasars close together is conditional on having noticed this fact in the first place.

Thus, the probability of full event $P(1|1) = 1$.

Que2 → The probability of drawing a red ball at a single turn.

$$P(R) = \frac{N}{M+N} = \frac{N}{10}$$

Similarly, probability of getting a white ball,

$$P(W) = \frac{M}{N+M} = \frac{10-N}{10}$$

∴ Using Binomial distribution, the probability of picking R red balls in 3 tries is:

$$P(R) = {}^3C_R \left(\frac{N}{10}\right)^R \left(\frac{10-N}{10}\right)^{3-R}$$

for $R = 2$, ATQ

$$P(R) = {}^3C_2 \left(\frac{N}{10}\right)^2 \left(1-\frac{N}{10}\right) = 3\left(\frac{N}{10}\right)^2\left(1-\frac{N}{10}\right)$$

Probabilities for different values of N,

$N=1$,	$P(R) = 0.027$
$N=2$,	$P(R) = 0.096$
$N=3$,	$P(R) = 0.189$
$N=4$,	$P(R) = 0.288$
$N=5$,	$P(R) = 0.375$
$N=6$,	$P(R) = 0.432$
$N=7$,	$P(R) = 0.441$
$N=8$,	$P(R) = 0.384$
$N=9$,	$P(R) = 0.242$

∴ The ~~propo~~ probability $P(R)$ is maximum at $N=7$
Number of red balls in the urn = 7.

Que 3 → Consider the same probability for X-ray emmision technique as the general technique

The probability of getting a cluster with a dominant central galaxy $= \frac{10}{100} = 0.1$

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Then, the probability of getting n dominant central galaxies out of the 30 cluster.

$$P(n) = {}^{30}C_n (0.1)^n (0.9)^{30-n}$$

$$P(n) = \frac{30!}{n!(30-n)!} (0.1)^n (0.9)^{30-n}$$

The probability is maximum for $n=3$, $P(n) = \cancel{0.24}$ 0.24

$\therefore$ We can expect 3 centrally dominant galaxies.

Que 34 →

(i) Signal is the mean,

$$\mu = \lambda t$$

As the photons detected are solely from the objects, the noise ~~Sd~~ = Scatter, $\sigma = \sqrt{\mu} = (\lambda t)^{1/2}$

$$\therefore \frac{S}{N} = \frac{\lambda t}{(\lambda t)^{1/2}} \Rightarrow SNR \propto t^{1/2}$$

(ii) In this case, the background sky noise dominates, then noise $= \sigma_{sky} = (\lambda_{sky} t)^{1/2}$

$$\therefore SNR = \frac{\lambda t}{(\lambda_{sky} t)^{1/2}} = \frac{\lambda t}{\sigma_{sky}} \propto t^{1/2}$$

(iii). In this case, the ~~Readout~~ readout noise of the device might dominate.

$$\therefore SNR = \frac{\lambda t}{\sigma_{readout}} \propto t$$

(iv). In this case, ~~the~~ the

$$SNR = \frac{S}{\sigma_{thermal}} \sqrt{t}$$

where S is the signal from ~~Souts~~ Source and $\sigma_{thermal}$ is the thermal noise of the device.

Que 5 → The sample ~~popov~~ population or data can be estimated from the given probability distribution using Bayes Theorem. This is known as ~~By~~ Bayseain influence

Que 7 → Dividing the set of nights in equal parts.
1. the training set of Ist 5 nights
2. Targets set of 2^{nd} 5 ~~ningt~~ nights

Now, the probability that the best night will be in 2^{nd} half $= \frac{1}{2}$

If the ~~x~~ 2^{nd} best night is in ~~the~~ the training sample, then ~~pot~~ probability of this's $= \frac{5}{10} = \frac{1}{2}$

$\therefore$ The probability of picking the best ~~ning~~ night is at least $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \Rightarrow 25\%$

But the events are not independent, as actually the joint probability is more than 25%.

~~$~~ For total N no. number of nights, let the length of training sample be n.

~~If best fing night is~~ $n+1$, then probability of this is $\frac{1}{N}$.

If best night is at $n+2\left(\frac{1}{N}\right)$, then probability of 2^{nd} best night being in training sample : $\frac{n}{n+1}$

$\therefore$ then the probability of picking the best night is $\frac{1}{N}\left(\frac{n}{n+1}\right)$

Proceeding in this way, then probability of choosing bed night -

$$P(B) = \frac{1}{N} + \frac{1}{N}\left(\frac{n}{n+1}\right) + \frac{1}{N}\left(\frac{n}{n+2}\right) + \cdots$$

$$\Rightarrow \frac{1}{N}\sum_{i=0}^{N-n+1}\left(\frac{n}{n+i}\right)$$

$$P(B) = \frac{n}{N}\sum_{i=0}^{N-n+1}\frac{1}{n+i}$$

$$= \ln\left(\frac{2n+2(N-n)+1}{2n-1}\right) = \ln\left(\frac{2N+1}{2n-1}\right)$$

$$\therefore P(B) = \frac{n}{N}\ln\left(\frac{2N+1}{2n-1}\right) \quad \left(\text{For large } N \text{ and } n\right)$$

Now, for maximum probability at on n.

$$\frac{d\,P(B)}{dn} = 0$$

$$\Rightarrow \frac{d}{dn}\left(\frac{n}{N}\ln\left(\frac{2N+1}{2n-1}\right)\right) = 0$$

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$\cancel{\ln\left(\frac{2N+1}{2n-1}\right)}$ $\ln \frac{2N+1}{2n-1} - \frac{2n}{2n-1} = 0$

$\Rightarrow \ln \frac{N}{n} = 1$ $\left[\text{For large } N \text{ and } n\right]$

$\Rightarrow \frac{N}{n} = e$, $n = N/e$

$\therefore$ The optimum length of the training Sample Should be N/e.

Que 8 → We have prior information,

$P(\text{penny being fair}) = P(M_{ok}) = 0.99$

$P(\text{penny being double head}) = P(M_{dh}) - 0.01$

If we get n heads in a row i.e O_{nh} for Ist n tosses, then

$P(A) = P(O_{nh}) = P\left(\frac{O_{nh}}{M_{ok}}\right) P(M_{ok}) + P\left(\frac{O_{nh}}{M_{dn}}\right) \times P(M_{dn})$

Now, $P\left(\frac{O_{nh}}{M_{ok}}\right) = \left(\frac{1}{2}\right)^n$

and $P\left(\frac{O_{nh}}{M_{d.h}}\right) = 1$

Now, posterior probability of coin being for getting n heads in a row,

$P\left(\frac{M_o}{O_{nh}}\right) = \frac{P(O_{nh}/M_{ok}) \times P(M_{ok})}{P(O_{nh})}$

For $n = 7$

$$P(M_{OK}/O_{7n}) = \frac{\left(\frac{1}{2}\right)^7 \times (0.99)}{\left(\frac{1}{2}\right)^7 \times 0.99 + 1 \times (0.01)} = 0.436$$

$\therefore$ there is less than 50% chance for coin being fair.

The ~~\$~~inference is influenced by the given prior, more coint tosses might give a better judgement of the fairness of ~~cost~~ coin.

Que 6 → The total score after each toss of the coin either goes up or down by one. Although the average score of either player after N tosses is zero (considering the same game to be fair, the scatter in the scores will be $N^{1/2}$ (it follows poisson distribution). The score at any point is Σx_i where each x_i can be +1 or −1. The root mean square is $(\sum_i x_i^2)^{1/2}$. This means that one player can get a long way ahead, and consequently changes of lead are likely to happen near the beginning of the game. By its time symmetry, they are also likely at the end. The number of changes is likely to be small and the most common ~~no~~ number of changes ~~the~~ of lead is one.