Discipline of Astronomy, Astrophysics and Space Engineering (DAASE)

INDIAN INSTITUTE OF TECHNOLOGY INDORE

Spring Semester	**Academic Year 2020-21**
Mid Semester Exam	**Course Code : AA 608**
Date: **30 January 2020 (FN)**	Max. duration : **2 hr**
Max. Marks : **30**	Number of pages: 1

Instructions:

- All questions are compulsory.
- Please, answer in the most complete and clear way. Do not forget the units!
- When you define a quantity try to give a complete definition: words + formula + units.
- Provide schematic diagrams where necessary to support your answers.
- Numbers in the parentheses at the end of each question represent the allocated marks.

1. (a) What is probability? Explain it from the Bayesian and the Frequentists point of view. (**3**)
   (b) Write down the Bayes theorem. What are likelihood, prior, evidence and posterior in this context? (**3**)

2. Consider the proverbial bad penny, for which prior information has indicated that there is a probability of 0.99 that it is unbiased ("OK"); or a probability of 0.01 that it is double-headed ("DH"). What is the (Bayesian) posterior probability, given this information, of obtaining seven heads in a row? In such a circumstance, how might we consider the fairness of the coin? Or of the experimenter who provided us with the prior information? What are the odds on the penny being fair? (**6**)

3. Suppose a medical test for an allergy gives a positive result with probability 0.8 in patients with the allergy. It has a false positive probability (i.e. a positive result in patients without the allergy) of 0.1. The probability of having the allergy in the population is 0.01. If you take the test, and it is positive. What is the probability that you have the allergy? Solve this problem using Bayesian approach. (**6**)

4. A lighthouse is situated at unknown co-ordinates x_0, y_0 with respect to a straight coastline $y = 0$. It sends a series of N flashes in random directions, and these are recorded on the coastline at positions x_i. Using Bayesian approach, find the posterior distribution of x_0, y_0, given the positions x_i. (**6**)

5. In 1994 the American football player O. J. Simpson was charged with murder. Simpson was known to be violent. The defence argued that only 1/2500 of violent people commit murder, so the information that he is known to be violent is irrelevant. This statement was not challenged. Simpson was acquitted. A key piece of information has been conveniently ignored by the defence, that, there is a dead body! Given that the probability of being murdered in the USA is 0.00005, what is the probability that Simpson was the murderer given this evidence alone? Solve this problem using Bayesian approach.(**6**)