Anubhav

MSE - AA608

Ankit meena

2003121002

(1)

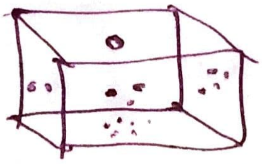
Problem: 1

(a) -

probability:-

probability is simply how likely something is to happen. whenever we're unsure about the outcome of an event we can talk about the probabilities of certain outcomes. how likely they are. The analysis of events governed by probability is called statistics.

example - There are six different outcomes



1, 2, 3, 4, 5, 6

what's the probability of rolling a one?

Soln

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total Number of favourable outcomes}}$$

$$P(1) = \frac{1}{6}$$

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Ankit Meena

2003121002

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Bayesian point of view →

probability is a 'degree-of-belief' in a proposition, allocated by an observer given the available information.

So uncertainty arising from incomplete data or noise. So This is radically different to the frequentist approach but allows us to deal with situations

Frequensitist point of view →

Probabilities are measurable frequencies assigned to objects or events. The relative frequency (probability) of an event arises from the number of times this event would occur relative to an infinite ensemble of 'identical' experiments. So This is intuitively linked to games of chance but breaks down in some obvious situation e.g.- For single events or in situation where we can not in practice measure the frequency

Problem : 1 :-> Ankit mreeny (2003121002) Ankitreey (3)

1(b)

Bayes theorem describes probabilities related to an event, given another event occurs, ->

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

where - A = an event, B = Another event

=> P(A|B) = posterior = the probability of an event occurring, given another event occurs.

=> P(B|A) = likelihood = The probability event B occurs, if event A occurs.

=> P(A) = prior = The probability an event occurs before you know if the other event occurs

=> P(B) = The normalizing const.

Arbitrary

Bayes theorem describes formula. helps us calculate. posterior probability using likelihood and. prior information together.

$$\boxed{\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{Evidence}}}$$

So ⇒ prior is the probability of the disease before having seen any test result

⇒ Evidence is also called the ~~totally~~ marginal likelihood and it ~~also call~~ it acts like a normalizing constant and is independent constant and is independent of disease status.

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Ankit Meena

20031210062

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Problem : 3 →

given in question -

p = positive result given patients with the allergy

$p(+ve \mid allergy) = 0.8$

and

p (positive result. in case of no allergy)

$= p(+ve \mid no\ allergy) = 0.1$

then

p (allergy) in the population = 0.01

from normalization - $p(allergy \mid +ve) + p(no\ allergy \mid +) = 1$

so $p(no\ allergy) = 1 - 0.01 = 0.99$

Now using Bayes' theorm

$$p(allergy \mid +) = \frac{p(+ \mid allergy)\, p(allergy)}{p(+)}$$

Ankit Meena

2003121062

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Now

$P(\text{allergy}) = 0.01$, $P(+/\text{no allergy}) = 0.1$

$P(\text{no allergy}) = 0.99$

then

$$P(\text{allergy}/+) = \frac{P(+/\text{allergy})}{P(+/\text{allergy}) + P(+/\text{no allergy}) \times \frac{P(\text{no allergy})}{P(\text{allergy})}}$$

$$P(\text{allergy}/+) = \frac{0.8}{0.8 + 0.1 \times \frac{0.99}{0.01}}$$

$$\boxed{P(\text{allergy}/+) = 0.074}$$

Anubhav

Ankit meena
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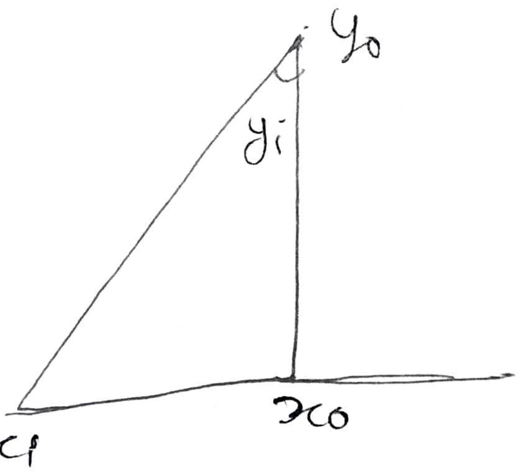
Problems4 :->



We want to prove.

$P(x_0, y_0 | \{x_i\})$

using Bayes, we write this as-

$$p(x_0, y_0 | \{x_i\}) \propto p(\{x_i\} | x_0, y_0)\, p(x_0, y_0)$$

$$\boxed{p(x_0, y_0 | \{x_i\}) \propto \prod_i p(x_i | x_0, y_0)}$$

If we assume a uniform prior for x_0, y_0 & let the angle of the direction of the flash to the coastline be φ. Then by trigonometry the position that the flash occurs at is given by-

$$\frac{x_i - x_0}{y_0} = \tan \varphi_i$$

So
$$\boxed{p(x_i | x_0, y_0) = p(\varphi_i | x_0, y_0) \left| \frac{d\varphi_i}{dx_i} \right|}$$

and for signals that are received on the shore φ is uniformly distributed in $-\pi/2 < \varphi < \pi/2$.
so $p(\varphi_i) = \frac{1}{\pi}$ in this range, independent of x_0, y_0 also.

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Ankit meena
2003121002

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$\sec^2 \varphi_i \dfrac{d\varphi_i}{dx_i} = \dfrac{1}{y_0}$

$$\left[1 + \frac{(x_i - x_0)^2}{y_0^2}\right] \frac{d\varphi_i}{dx_i} = \frac{1}{y_0}$$

and the likelihood of x_i is a cauchy distribution

$$p(x_i | x_0, y_0) = \frac{1}{\pi y_0 \left[1 + \frac{(x_i - x_0)^2}{y_0^2}\right]}$$

Hence the (unnormalised) posterior for x_0, y_0 is ⇒

$$\boxed{p(x_0, y_0 | \{x_i\}) \propto \prod_{i=1}^{N} \frac{1}{\pi y_0 \left[1 + \frac{(x_i - x_0)^2}{y_0^2}\right]}}$$

which is our desired outcome.

Assignment

Ankit Meena
2003121002

problem: 5 →

⇒ let probability of murder = $P(x)$

⇒ probability of the person is violent = $P(y)$

⇒ probability of the person murdering = $P(z)$

given in question –

$$p(z/y) = \frac{1}{2500} = 0.0004$$

and

$$p(x \mid z, y) = 1$$

then using Bayes's theorem –

$$p(z \mid x, y) = \frac{p(x \mid z, y)\, p(z \mid y)}{p(x \mid z, y)\, p(z \mid y) + p(x \mid \sim z, y)\, p(\sim z \mid y)}$$

→ (A)

Answer

Ankit meena
2003121002

Now

$p(x|z,y) = 1, \quad p(z/y) = 0.0004$

and ~~[illegible]~~

$p(\sim z|y) = 1 - p(z/y)$

$p(\sim z|y) = 1 - 0.0004$

$p(\sim z|y) = 0.9996$

put value in equation (A) →

so $p(x|\sim z,y)$ is the probability that there is a murder given the perpetrator is no a violent murder for this we take the probability of a murder is the general population

so $p(x|\sim z,y) = 0.00005$

then
$$p(z|x,y) = \frac{1 \times 0.0004}{1 \times 0.0004 + 0.00005 \times 0.9996}$$

$$\boxed{p(z|x,y) = 0.88}$$