ASSIGNMENT 1

AA 474/674

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Q.1. (a) Given:

Observer located at $38^\circ N$ and $79^\circ W$.

Altitude of Polaris:
The altitude of Polaris is nearly equal to the altitude of the north celestial pole and this is equal to the observer's latitude, i.e. $38^\circ$.

Altitude of south celestial pole: The altitude of the south celestial pole is equal to the latitude of the observer's location, but negative, i.e. $-38^\circ$.

Altitude of celestial equator: Altitude of the celestial equator is $90^\circ -$ the observer's latitude.
$\Rightarrow$ altitude of celestial equator $= 90^\circ - 38^\circ = 52^\circ$ off the southern horizon. The longitude is irrelevant.

(b) In the Southern hemisphere, altitude of Polaris is equal to negative of the latitude of the observer $\therefore$ it is $-38^\circ$.

The altitude of the southern celestial pole is simply the latitude of the observer, i.e. $38°$. The altitude of the celestial equator is $90°$ - observer's latitude $\Rightarrow 90° - 38° = 52°$, but this time off the northern horizon. /

Once again, the longitude is irrelevant. For observers in the southern hemisphere, the equator crosses the northern sky and for the observers in the northern hemisphere, the celestial equator crosses the southern sky.

Q.2. Given that declination : $-57°$

We know that by the cosine rule, we can write

$$\cos(90° - a) = \cos(90° - \delta)\cos(90° - \phi) + \sin(90° - \delta)\sin(90° - \phi)\cos H$$

This simplifies to:

$$\sin(a) = \sin\delta \sin\phi + \cos\delta \cos\phi \cos H. \longrightarrow (1)$$

Here, $\delta$ = declination

$a$ = altitude

$\phi$ = latitude of the observer.

$H$ = hour angle

So, assuming $H = 0$, for the star to be just visible, (assuming a plain geography), the altitude has to be greater than $0°$.

⇒ From eqⁿ ①, we can write the condition for visibility of $\gamma$ Crucis:

$$0 = \sin\delta \sin\phi + \cos\delta \cos\phi$$
$$\Rightarrow \quad \sin\delta \sin\phi = -\cos\delta \cos\phi.$$
$$\Rightarrow \quad \tan\delta = -\cot\phi.$$
$$\delta = -57^\circ \quad \Rightarrow \quad \tan(57^\circ) = -\cot\phi.$$
$$\Rightarrow \quad \phi = \cot^{-1}(\tan 57^\circ)$$
$$\Rightarrow \quad \phi = 32.99^\circ.$$

⇒ $\gamma$ Crucis will always be visible South of $32.99^\circ$N
North of this latitude, its visibility depends both on the latitude and the time of the day.

The latitude at which the star will pass directly overhead, is equal to the declination of the star.
So, the latitude at which $\gamma$-Crucis will pass directly overhead is $57^\circ$ S.

Stars that never go below the horizon from a particular location, are called circumpolar stars.
Circumpolar stars are within latitude degrees of the celestial pole. They have declinations greater than

+(90 - latitude) degrees for the northern hemisphere. For the southern hemisphere, it's opposite. So,

$$\text{declination} < -(90^\circ - \text{latitude})$$

$$\Rightarrow \quad -57^\circ < -(90^\circ - \phi)$$

$$\Rightarrow \quad -57 < -90 + \phi$$

$$\Rightarrow \quad -57 + 90 < \phi.$$

$$\Rightarrow \quad \phi > 33^\circ S.$$

Hence $\gamma$- Crucis is circumpolar south of $33^\circ S$, and never sets.

**Q.3.** Consider one trip of a planet around the Sun as traversing $360^\circ$. Then the rate in degrees per year is $360^\circ / P$ where $P$ is the sidereal period of the planet in Earth years.

Let sidereal period of Earth $P_{Earth} = E$. Then, the rate, for Earth is $360^\circ / E$.

The synodic period $S$ is the number of Earth years it takes for Earth to 'lap' a planet (i.e., for outer planets, it could be from opposition to opposition while for inner planets, it could be from greatest elongation to greatest elongation).

Consider the case where the Earth goes $n^\circ$ in

synodic period around the Sun. The rate of motion of the Earth is constant at $360^\circ/E$.
So, we have

$$\frac{n}{S} = \frac{360}{E} \quad \text{—— } ①$$

At the same time, the inferior planet has to go $360^\circ + n^\circ$ to make up for the Earth's motion in synodic period at the rate of $360/P$. So, we have:

$$\frac{360 + n}{S} = \frac{360}{P} \Rightarrow \frac{n}{S} = \frac{360}{P} - \frac{360}{S} \quad \text{—— } ②$$

Substituting this into equation ①, we get:

$$\frac{360}{P} - \frac{360}{S} = \frac{360}{E}$$

OR

$$\boxed{\frac{1}{S} = \frac{1}{P} - \frac{1}{E}}$$

For an Inferior planet.

Q.4. (a) Let $f(\bar{x}, \bar{p})$ be the photon distribution function in phase space, summed over the two polarization states. Then $f\,\overline{dx}\,\overline{dp}$ is the number of photons in volume $dx$ with momenta in the interval of width $dp$. The photon momentum is also expressed as

$$\bar{p} = \hbar \bar{k} = \frac{2\pi \hat{p}}{\lambda}$$

In spherical co-ordinates, we have:

$$\overline{dp} = p^2\, dp\, d\Omega$$

Here, $d\Omega$ is an element of solid angle in the direction of photon propagation; $\hat{p} = \hat{k}$.

Assuming that photons are propagating through vacuum, $p = \frac{h\nu}{c}$

Then, the density of photons in $\overline{dp}$ is:

$$f\, \overline{dp} = f\, p^2\, dp\, d\Omega = f \left(\frac{h}{c}\right)^3 \nu^2\, d\nu\, d\Omega$$

and the energy flux of these photons is:

$$f\, \overline{dp} \times h\nu \times c = f \left(\frac{h^4}{c^2}\right) \nu^3\, d\nu\, d\Omega$$

Defining specific intensity $I_\nu$ as the radiative flux per unit frequency, per unit solid angle - i.e. the energy per unit time per unit area, per unit frequency per unit solid angle. In terms of $f$, this is:

$$\text{energy flux} = I_\nu \, d\nu \, d\Omega = f\left(\frac{h^4}{c^2}\right)\nu^3 d\nu d\Omega$$

$$\Rightarrow I_\nu = \left(\frac{h^4 \nu^3}{c^2}\right) f$$

Now consider a source of luminosity $L_\nu$ measured in erg $s^{-1}$ $Hz^{-1}$. If it is at a distance $D$, the flux per Hertz is:

$$F_\nu = \frac{L_\nu}{4\pi D^2}$$

Let the projected area of the source be $A$ and let it subtend a solid angle $\Delta\Omega = A/D^2$

Then, the specific intensity is given by:

$$I_\nu = \frac{F_\nu}{\Delta\Omega} = \frac{L_\nu}{4\pi D^2} \frac{1}{A/D^2} = \frac{L_\nu}{4\pi A}$$

Which is independent of the distance $D$.
Hence proved that specific intensity is independent of distance

(b) Given:

Distance : $d$

$I_\nu$ : $I_0$ mag arcsec$^{-2}$

$I_0$ is total galaxy flux per arcsec$^{-2}$

Absolute magnitude of stars in the galaxy : $M$

The relation between apparent magnitude and flux is given by:

$$m = -2.5 \log F + \text{const.} \quad \text{—— (1)}$$

for surface brightness of a galaxy, the relation with flux is:

$$I_0 = -2.5 \log NF + \text{const} = -2.5 \log \frac{NL}{4\pi d^2} + \text{const} \quad \text{—— (2)}$$

assuming average luminosity $L$ of the stars.

In terms of absolute magnitude of the stars, we can write:

$$M = -2.5 \log \left[ \frac{L}{4\pi (10pc)^2} \right] + \text{const.} \quad \text{—— (3)}$$

Subtracting (3) from (2), we get:

$$I_0 - M = -2.5 \log_{10} \left[ \frac{N\cancel{L}}{\cancel{4\pi} d^2} \times \frac{\cancel{4\pi} (10\,pc)^2}{\cancel{L}} \right]$$

$$\Rightarrow \quad 0.4\,(M - I_0) = \log_{10}\left[N\left(\frac{10\,pc}{d}\right)^2\right]$$

$$\Rightarrow \quad 10^{0.4(M - I_0)} = N\left(\frac{10\,pc}{d}\right)^2$$

$$\Rightarrow \quad N = \left(\frac{d}{10\,pc}\right)^2 10^{0.4(M - I_0)}$$

Hence proved.

Q.5. Given :

Latitude $\phi = 22.7196^\circ N$

LST : 4.93 Hrs

RA $= 05h\ 55m\ 10.3s = 5.92$ Hrs

DEC $= \delta = 7^\circ 24' 25'' = 7.41^\circ$

Hour angle $H = LST - RA = -0.99$ Hrs $= -14.85$

Let us denote altitude by $a$ and azimuth $A$ for brevity.

Now, we will use the following trigonometric formulae to calculate altitude & azimuth:

$$\sin a = \sin\delta \sin\phi + \cos\delta \cos\phi \cos H \quad \text{—— (1)}$$

AND

$$\cos A = \frac{\sin\delta - \sin a \sin\phi}{\cos a \cos\phi} \quad \text{—— (2)}$$

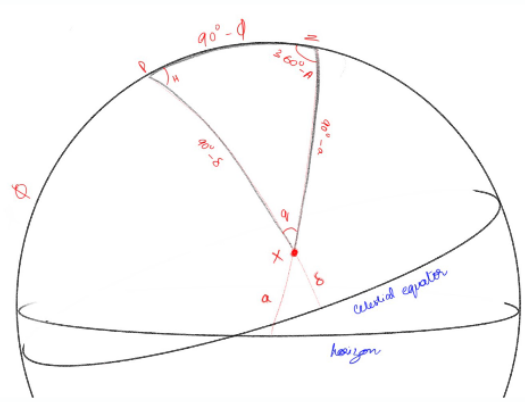
By using eq$^n$ (1),

$$\sin a = \sin(7.41)\sin(22.7196) + \cos(7.41)\cos(22.7196)\cos(-14.85)$$

$\Rightarrow \sin a = 0.934$

$\Rightarrow a = \sin^{-1} 0.934$

$\Rightarrow a = 69.07^\circ$



now using eq$^n$ (2),

$$\cos A = \frac{\sin(7.41) - 0.934 \sin(22.7196)}{\cos(69.07)\cos(22.7196)}$$

$\Rightarrow \cos A = -0.703$

$\Rightarrow A = \cos^{-1}(-0.703)$

$\Rightarrow A = 134.69^\circ$

To conclude, the altitude of Betelgeuse at given time from Indore will be: $69.07^\circ$

and azimuth of Betelgeuse will be: $134.69^\circ$

Q.6. Given that size of the image is $2\times2$ pixels and size of the object is $1''\times1''$.

$\Rightarrow$ $1\times1$ pixels corresponds to $0.5''\times0.5''$ of sky

Now CCD has $1024\times1024$ pixels.

$\therefore$ FOV $= (1024\times0.5)'' \times (1024\times0.5)''$

$= 512'' \times 512''$

Also, size of the chip, $2cm\times2cm = 20mm\times20mm$

If $s$ is the plate scale, then,

$(s\times20)'' \times (s\times20)'' = 512''\times512''$

$\Rightarrow (s \times 20)'' \times (s \times 20)'' = 512'' \times 512''$

$\Rightarrow s = 25.6$ arcsec / mm

Then, $s = \dfrac{206265}{\text{focal length}}$

$\Rightarrow$ Focal length, $f = \dfrac{206265}{25.6} = 8057.23$ mm

$= 8.057$ m

$\Rightarrow$ The focal length of the given telescope is 8.057 m

Q.7. The plate scale of the telescope is given by:

$$s = \frac{1}{f_L} \text{ (rad/m)} = \frac{206265}{f_L} \text{ (arcsec/m)}$$

For the given focal length,

$$s = \frac{206265}{15.2} \text{ (arcsec / mm)} = 13.57 \text{ (arcsec/mm)}$$

$\simeq 14''$

Hence proved.

Q8. (a) We know that apparent magnitudes and fluxes of two stars are related by the formula:

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right)$$

Let $F_1 = 10^6 F_2$

Then, $m_1 - m_2 = -2.5 \log_{10} 10^6$

$\Rightarrow \Delta m = -15.0$

$\Rightarrow$ The difference in magnitude of the two stars will be $\pm 15.0$.

(b) Given that magnitude of Palomar observatory, $m_p = 21$.

To get ratio of fluxes,

$$m_p - m_s = -2.5 \log_{10}\left(\frac{F_p}{F_s}\right)$$

$$\Rightarrow 21 = -2.5 \log_{10}\left(\frac{F_p}{F_s}\right)$$

$\Rightarrow$ Ratio of flux of detector plate of the observatory to flux of the star is:

$$\frac{F_p}{F_s} = 3.98 \times 10^{-9} \text{ to } 1$$

For the Hubble Space Telescope, $m_{HST} = 28$.

$\Rightarrow$

$$28 = -2.5 \log_{10}\left(\frac{f_{HST}}{f_S}\right)$$

$$\Rightarrow \quad \frac{f_{HST}}{f_S} = 6.31 \times 10^{-12} \text{ to } 1.$$

(c) $m_v = -1.44$ $\qquad d = 8.8\ LY = 2.7\ pc$

We know that apparent magnitude and absolute magnitude are related by:

$$m - M = 5 \log_{10}\left(\frac{d}{10\,pc}\right)$$ where $d$ is the distance

Let present apparent magnitude $m_0 = -1.44$

Let apparent magnitude in Andromeda be $m_1$

Distance to Andromeda: $2.5\ MLY = \dfrac{2.5 \times 10^6}{3.26}$

$$\Rightarrow \quad d_1 = 7.67 \times 10^5\ pc.$$

So, we can write, for the present case:

$$-1.44 - M = 5 \log\left(\frac{2.7}{10}\right) \qquad \text{—— (1)}$$

for Sirius in Andromeda:

$$m_1 - M = 5 \log\left(\frac{7.67\times10^5}{10}\right) \quad \text{——— ②}$$

Subtracting ② from ①, we get:

$$-1.44 - m_1 = 5 \log\left(\frac{2.7}{7.67\times10^5}\right)$$

$$\Rightarrow \quad m_1 = 25.87$$

Since apparent magnitude is smaller than $m_v$ of HST, Sirius will still be observable if it was in Andromeda.

(d) Given $M_v = 4.82$ and $d = 7.67\times10^5$ (from above)

So, $$m - 4.82 = 5 \log\left(7.67\times10^4\right)$$

$$\Rightarrow \quad m = 29.24$$

Since magnitude is higher than $m_v$ of HST, star would not be visible

Q.9. Area of the moon : $\pi (15)^2$ arcmin$^2$

$= 225\pi$ arcmin$^2$

(a) Total magnitude : $-12.7$

$\therefore$ magnitude per square arc minute : $\dfrac{-12.7}{225\pi}$

$\boxed{= -0.018}$

(b) Area of the moon in arcsec$^2$ = $225\pi \times 3600$

$= 2.54 \times 10^6$ arcsec$^2$

$\therefore$ magnitude per square arc second : $\dfrac{-12.7}{2.54 \times 10^6}$

$\boxed{= -4.99 \times 10^{-6}}$

(c) Surface brightness is given by:

$$I = m + 2.5 \log_{10}(A)$$

I = surface brightness
m = magnitude
A = area

$\therefore$ Surface brightness of the moon is:

$$I_m = -12.7 + 2.5 \log_{10}(2.54 \times 10^6)$$

$\Rightarrow I_m = 3.31$ mag arcsec$^{-2}$