Mid Semester Examination

AA 474/674

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Q.1. Luminosity of the black body: $4\pi R^2 \sigma T^4$

$$= 4 \times 3.14 \times (7 \times 10^{8})^2 \times (5.67 \times 10^{-8}) \times (10^4)^4$$

$$= 3489.5 \times 10^{24} \text{ W}$$

Then, flux density at a distance of $d = 3.3$ pc:

$$F = \frac{L}{4\pi d^2} = \frac{3489.5 \times 10^{24}}{4\pi \times 10^{34}}$$

$$= 277.83 \times 10^{-10} \text{ W m}^{-2}$$

For a black body, using Wein's law,

$$\lambda_{max} = \frac{2.9}{T} = 2.9 \times 10^{-7} \text{ m}$$

$$\Rightarrow \nu = 1.03 \times 10^{15}$$

$$\Rightarrow S_\nu = \frac{F_\nu}{\Delta \nu} = \frac{277.83 \times 10^{-10}}{1.03 \times 10^{15}}$$

$$= 269.73 \times 10^{-25} \text{ W m}^{-2} \text{ Hz}^{-1}$$

$$= 2697.3 \text{ Jy}$$

Q.2. The Rayleigh Jeans limit is given by

$$h\nu \ll KT.$$

here, $T = 2.7$ K. $\Rightarrow \nu \ll \frac{1.38 \times 10^{-23} \times 2.7}{6.63 \times 10^{-34}}$

$\Rightarrow \nu \ll 5.9 \times 10^{10}$ Hz

This is the safe limit for using the Rayleigh Jeans approximation.

Q.3. Using the equation

$$P = F_\nu A_{eff} \Delta\nu$$

we can write

$$F_\nu = \frac{P}{A_{eff} \Delta\nu}$$

$$= \frac{1.2 \times 10^{-19} \text{ W}}{1.2 \text{ m}^2 \times 2 \times 10^{6} \text{ Hz}}$$

$$= 5 \times 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} = 5 \text{ Jy}$$

Q.4. The brightness temperature is given by:

$$T_b = \frac{\lambda^2 I_\nu^2}{2K_B}$$

$$I_\nu = c\,u_\nu = \cancel{c} \cdot \frac{4\pi}{\cancel{c}} B_\nu = 4\pi \cdot \frac{2hc}{\lambda^3} \frac{1}{\exp\left(\frac{hc}{\lambda KT}\right) - 1}$$

Antenna temperature can be given by:

$$T_A = \frac{A_e S_\nu}{2K}$$

given that $S_\nu = 50\ mJy$; effective area $A_e = \eta A$

where A is geometric area

hence, $A_e = 0.5 \times 1000 = 500\ m^2$

Using these values, antenna temperature is:

$$T_A = \frac{500 \times 50 \times 10^{-3} \times 10^{-26}}{2 \times 1.38 \times 10^{-23}}$$

$$T_A = 9.06 \times 10^{-3}\ K$$

Q.5. (a) Brightness temperature is given by:

$$T_b = \frac{\lambda^2 I_\nu}{2K_B} \quad \text{— (1)}$$

For blackbody, $I_\nu = 4\pi B_\nu$ — (2)

$$B_\nu(T) \simeq \frac{2KT}{\lambda^2} = 2.944 \times 10^{-18}$$

∵ Rayleigh-Jeans approximation is valid here.

Using this in (2), we have:

$$I_\nu = 3.699 \times 10^{-17}$$

using this in (1),

$$T_b = \frac{0.15^2 \times (3.699 \times 10^{-17})}{2 \times 1.38 \times 10^{-23}}$$

$$= 3.01 \times 10^4 \text{ K}$$

(b) Flux density is given by:

$$S_\nu = \frac{P}{A_{eff} \Delta\nu} \quad \text{— (3)}$$

; $A_{eff} = \eta A_{geometric}$

$$= 0.6 \times \pi \times 10^2$$

$$= 60\pi$$

Given that $\Delta\nu = 2 \times 10^6$ Hz

and power : $P = k \Delta\nu T_{eq} = 100 \, k \Delta\nu$

Using these in ③, we have:

$$S_\nu = \frac{100 k \cancel{\Delta\nu}}{60\pi \cancel{\Delta\nu}} = 7.32 \times 10^{-24} \ W m^{-2} Hz^{-1}$$
$$= 732.1 \ Jy$$

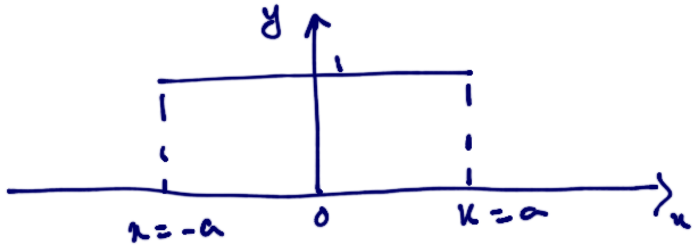
(c) Antenna temperature is given by:

$$T_A = \frac{A_e S_\nu}{2k}$$

Using values of A_e and S_ν from above,

$$T_A = \frac{60\pi \times 732.1 \times 10^{-26}}{2 \times 1.38 \times 10^{-23}} = 49.99 \simeq 50 K.$$

Q.6. $f(x) = \begin{cases} 1 & ; \; |x| < a \\ 0 & ; \; |x| > a \end{cases}$



The function is piecewise continuous and

$$\int_{-\infty}^{\infty} |f(x)|\, dx = \int_{-a}^{a} dx = 2a < \infty$$

$\therefore$ Fourier transform exists

$$\Rightarrow A(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(t) \cos(\omega t)\, dt$$

$$= \frac{1}{\pi} \int_{-a}^{a} \cos(\omega t)\, dt$$

$$= \frac{1}{\pi} \left[\frac{\sin(\omega t)}{\omega} \right]_{-a}^{a} = \frac{2\sin(\omega a)}{\pi \omega}$$

and $B(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(t) \sin(\omega t)\, dt = \frac{1}{\pi} \int_{-a}^{a} \sin(\omega t)\, dt$

$$\Rightarrow B(\omega) = 0$$

Now, fourier transform of f is defined as :

$$\hat{f}(\omega) = \sqrt{\frac{\pi}{2}} \left(A(\omega) - i\, B(\omega) \right)$$

$$\therefore \hat{f}(\omega) = \sqrt{\frac{\pi}{2}} \left(\frac{2\sin(\omega a)}{\pi \omega} - i \times 0 \right)$$

$$\Rightarrow \hat{f}(\omega) = \sqrt{\frac{2}{\pi}} \frac{\sin(\omega a)}{\omega}$$

Q.7. Given : latitude $= 22.7196°N = \phi$

LST $= 4.93$ Hrs

RA $=$ 05h 55m 10.3s $= 5.92$ Hrs

DEC $= \delta = 7°24'25'' = 7.41°$

Hour angle $H =$ LST $-$ RA $= -0.99$ Hrs $= -14.85°$

Let us denote altitude by a and azimuth by A for brevity

Now, we will use the following trigonometric formulae to calculate altitude and azimuth:

$$\sin a = \sin\delta \sin\phi + \cos\delta \cos\phi \cos H \quad \text{— (1)}$$

AND

$$\cos A = \frac{\sin\delta - \sin a \sin\phi}{\cos a \cos\phi} \quad \text{— (2)}$$

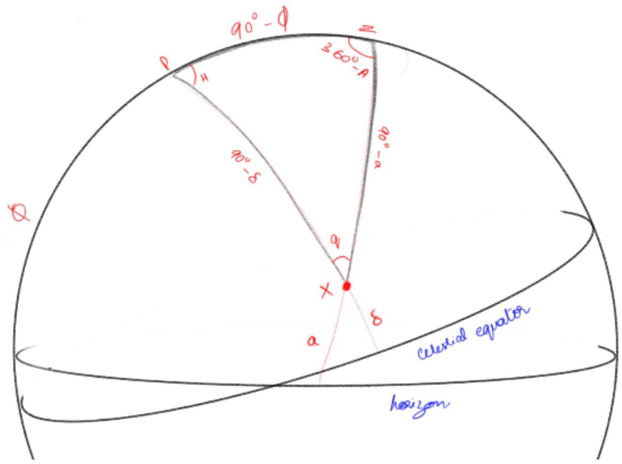
By using equation (1),

$$\sin a = \sin(7.41)\sin(22.7196) + \cos(7.41)\cos(22.7196)\cos(-14.85)$$

$$\Rightarrow \sin a = 0.934$$

$$\Rightarrow a = \sin^{-1} 0.934$$

$$\Rightarrow a = 69.07^\circ$$



Now using eq^n (2), we have:

$$\cos A = \frac{\sin(7.41) - 0.934\sin(22.4196)}{\cos(69.07)\cos(22.7196)}$$

$$\Rightarrow \cos A = -0.703$$

$$\Rightarrow A = \cos^{-1}(-0.703)$$

$$\Rightarrow A = 134.69^\circ$$

To conclude, the altitude of Betelgeuse at given time from Indore will be : 69.07°

and azimuth of Betelgeuse will be : 134.69°.

Q.8. Using trigonometric parallax, we know

$$1'' \equiv 1 pc$$

Then,

$$2'' = 0.5\ pc = 0.5 \times 3.086 \times 10^{16}\ m$$
$$= 1.54 \times 10^{16}\ m$$

Q.9. We know that scale factor is defined as:

$S = 206265''/f$

putting $f = 10$ m, we get: $S = 20.6265\ ''/mm$

now field of view of the system:

$$FOV = (90 \times 20.6265)'' \times (90 \times 20.6265)''$$
$$= 1856.385'' \times 1856.385''$$
$$= 30.94' \times 30.94'$$

Q.10. Image scale is again given by:

$$S = \frac{206265''}{f}$$ where f is the focal length.

now $\text{F-ratio} = \dfrac{\text{focal length}}{\text{Max aperture diameter}}$

given f-ratio = 8

max aperture diameter = 4

$\Rightarrow \quad 8 = \dfrac{f}{4} \quad \Rightarrow \quad f = 32\ m$

$\Rightarrow$ Image scale $S = \dfrac{206265}{32} = 6445.8\ ''/m$

Given that 1 pixel has the size of 15 μm.

$\Rightarrow$ in 1m, there are $\dfrac{1}{15 \times 10^{-6}} = 6.67 \times 10^{4}$ pixels.

$\Rightarrow$ Image scale in arcsec/pixel $= \dfrac{6445.8}{6.67 \times 10^{4}}$

$= 0.097''$ /pixel

$\therefore$ the angular dimensions of the sky observed using this set up (or FoV) is:

$(0.097 \times 2048)'' \times (0.097 \times 2048)''$

$= 198.7'' \times 198.7''$

$= 3.31' \times 3.31'$

Q.11. The detective quantum efficiency or DQE for short, is a measure of the combined effects of the signal and noise performance of an imaging system, generally expressed as a fraction of spatial frequency.

It is defined as

$$D = \frac{\left(\langle \Delta N^2 \rangle / \bar{N}^2\right)_{\text{real detector}}}{\left(\langle \Delta N^2 \rangle / \bar{N}^2\right)_{\text{ideal detector}}}$$

Given that quantum efficiency $= Q$.

Then,

$$\langle \Delta N^2 \rangle_{\text{ideal}} = \bar{N}$$

$$\langle \Delta N^2 \rangle_{\text{real}} = Q\bar{N}$$

$$\Rightarrow D = \left(\frac{\cancel{Q}\cancel{\bar{N}}}{Q^{\cancel{2}}\bar{N}^{\cancel{2}}}\right) \Big/ \left(\frac{\cancel{\bar{N}}}{\bar{N}^{\cancel{2}}}\right) = \frac{1}{Q\cancel{\bar{N}}} \times \cancel{\bar{N}}$$

$$\Rightarrow D = \frac{1}{Q}$$

When $D = Q$, $Q = \frac{1}{Q} \Rightarrow Q = 1$

$\Rightarrow$ Quantum efficiency is 1 and the detector is perfect.

$\Rightarrow$ Detector noise can be neglected.