Assignment-II

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1. **Q1**

Q1) Given, pixel size = ~~1024×20~~ 1024×1024 pixels.

~~Total dimens~~ CCD dimension = 2 cm × 2 cm.

each pixel grid dimension is $= \frac{2}{1024}$ cm $\times \frac{2}{1024}$ cm

$= \frac{1}{512}$ cm $\times \frac{1}{512}$ cm.

so, one pixel size $\simeq 20\ \mu m$

plate scale $= \frac{1}{f} = \frac{206265 \times \text{Pixel size (mm)}}{\text{focal Length (mm)}}$

$= \frac{206265 \times 20 \times 10^{-3}}{1} = 4.1253$ m

2. **Q2**

Q2) plate scale $(P_s) = 14''/mm$,
Diameter $(D) = 3m$
focal length $(f) = 15.2\,m$

$$P_s = \frac{206265}{f(mm)} = \frac{206265}{15.2\times10^3} \approx 13.57''/mm.$$

$$\text{focal ratio}(R) = \frac{f}{D} = \frac{15.2}{3} = 5.067$$

Minimum energy required per pixel —

$$E_p = \frac{F\cdot A_i\cdot T}{f A_0}$$

$F \to$ flux density
$A_i \to$ Area of lens.
$A_0 \to$ Area of object on detector.
$T \to$ exposure time.
$f \to$ Number of pixels per unit ~~time~~ area.

Now, $\dfrac{\alpha_1}{S_1} = \dfrac{\alpha_2}{S_2}$ $\quad S_i \to$ size of i^{th} source.

$$\alpha_1 = 1'' = \left(\frac{1}{3600}\right)^\circ$$

$$\alpha_2 = 2' = \left(\frac{2}{60}\right)^\circ \qquad \therefore \frac{\alpha_2}{\alpha_1} = \frac{S_2}{S_1} = 120$$

$$\therefore A_0' = (120)^2 A_0$$

$$E_p' = \frac{1000F\cdot A_i\cdot t}{f(120)^2 A_0}$$

$$\therefore E_p' = E_p \Rightarrow t = \frac{(120)^2}{1000}T = 14.4\,T$$

3. **Q3**

3) Given $f_L = 7.2$, CCD size $= 24.6$ mm $\times$ 24.6 mm.

$$\text{plate scale} = \frac{206265''}{f_L\,(mm)} = \frac{206265}{7.2\times10^3}$$

$$= 28.64 \text{ arcsec/mm}.$$

$$\text{field of view} = \frac{135.3\times D}{f_L} \ (\text{arc min})$$

f_L in mm $= 7.2\times10^3$ mm.

$$\text{field of view} = \frac{135.3\times24.6\times10^{-3}}{7.2\times10^3} = 4.62\times10^{-4} \text{ arcmin}.$$

4. **Q4**

4) $\sigma_d \rightarrow$ systematic noise, t = time duration

n = photon rate.

~~Photon~~ Poission distribution of photon counter is given by,

$$P = \frac{(nt)^p e^{-nt}}{p!}$$

$\therefore$ SNR = ~~signal~~ No. of standard deviations,

$$R = \frac{nt}{\sqrt{nt + \sigma_d^2}}$$

$$\Rightarrow R^2 = \frac{(nt)^2}{nt + \sigma_d^2}$$

$$\Rightarrow R^2 nt + R^2 \sigma_d^2 = (nt)^2.$$

$$\Rightarrow \frac{R^2 t}{n} + \frac{R^2 \sigma_d^2}{n^2} = t^2.$$

$$\cancel{t = R^2} \; t^2 - \frac{R^2 t}{n} - \frac{R^2 \sigma_d^2}{n^2} = 0$$

$$\therefore \Rightarrow t = \frac{\frac{R^2}{n} \pm \sqrt{\frac{R^4}{n^2} + \frac{4 R^2 \sigma_d^2}{n^2}}}{2}$$

$$\Rightarrow t = \frac{R^2}{n} \left[\frac{1 + \sqrt{1 + 4\sigma_d^2/R^2}}{2} \right]$$

$$\Rightarrow t = \left(\frac{(SNR)^2}{Rate}\right) \left[\frac{1}{2} + \left\{ \frac{1}{4} + \left(\frac{\sigma_d}{R}\right)^2 \right\}^{1/2} \right]$$

if $\sigma_d \rightarrow 0$, then $t = \frac{(SNR)^2}{Rate} \left[\frac{1}{2} + \frac{1}{2} \right]$

$$t = \frac{(SNR)^2}{Rate}.$$

5. **Q5**

   (a) **Q5a**

5a) $r_s = r_{s+b} - r_b$

$r_{s+b} = \frac{S+B_s}{t_s}$

$r_b = \frac{B_b}{t_b}$

$\Rightarrow r_s = \frac{S+B_s}{t_s} - \frac{B_b}{t_b}$

$$\sigma_{r_s}^2 = \frac{\sigma_{S+B_s}^2}{t_s^2} + \frac{\sigma_{B_b}^2}{t_b^2} = \frac{S+B_s}{t_s^2} + \frac{B_b}{t_b^2}.$$

$$\therefore \sigma_{r_s} = \sqrt{\frac{r_{s+b}}{t_s} + \frac{r_b}{t_b}} = \sqrt{\frac{r_s + r_b}{t_s} + \frac{r_b}{t_b}}$$

$$T = t_s + t_b, \quad \sigma_{r_s} = \sqrt{\frac{r_s + r_b}{t_s} + \frac{r_b}{(T - t_s)}}$$

(b) **Q5b**

5b) Maximizing $\frac{\gamma_s}{\sigma_{\gamma_s}}$ can be done by minimizing σ_{γ_s}

$$\therefore \quad \sigma_{\gamma_s}^2 = \frac{\gamma_s+\gamma_b}{t_s} + \frac{\gamma_b}{T-t_s}$$

Differentiating w.r.t. t_s —

$$2\sigma_{\gamma_s} \cdot \frac{d\sigma_{\gamma_s}}{dt_s} = -\frac{(\gamma_s+\gamma_b)}{t_s^2} + \frac{\gamma_b}{(T-t_s)^2}$$

Now, $\frac{d\sigma_{\gamma_s}}{dt_s} = 0 \Rightarrow \frac{\gamma_s+\gamma_b}{t_s^2} = \frac{\gamma_b}{(T-t_s)^2}$

$$\Rightarrow t_s\sqrt{\gamma_b} = \sqrt{\gamma_s+\gamma_b}\,(T-t_s)$$

$$\Rightarrow t_s\left(\sqrt{\gamma_b} + \sqrt{\gamma_s+\gamma_b}\right) = \sqrt{\gamma_s+\gamma_b}\,(T)$$

$$\Rightarrow \frac{t_s}{T} = g = \frac{\sqrt{\gamma_s+\gamma_b} \cancel{- \sqrt{\gamma_b}}}{\sqrt{\gamma_b} + \sqrt{\gamma_s+\gamma_b}}$$

$$= \frac{1}{\sqrt{\frac{\gamma_b}{\gamma_s+\gamma_b}} + 1}$$

(c) **Q5c**

5c)

$r_s = r_L$, $\sqrt{\frac{r_L}{r_s+r_L}} = \sqrt{\frac{1}{2}} = 0.707$

$\frac{t_s}{T} = \frac{1}{0.707+1} \Rightarrow t_s = \frac{T}{1.707}$.

$r_s \gg r_b$ $\sqrt{\frac{r_b}{r_s+r_L}} = \sqrt{\frac{r_b}{r_s}}$.

$\therefore \frac{t_s}{T} = \frac{1}{1+\sqrt{\frac{r_b}{r_s}}} = \frac{\sqrt{r_s}}{\sqrt{r_b}}$

$r_s \ll r_b$, $\sqrt{\frac{r_b}{r_s+r_b}} = \sqrt{\frac{r_b}{r_b}} = 1$

$\therefore \frac{t_s}{T} = \frac{1}{1+1} = 0.5$

$\Rightarrow t_s = 0.5T$

6. **Q6**

Q6) x_1: Arrival of first photon,

x_2: Arrival " 2nd " .

x: arrival of a photon.

Poisson distribution, $P(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$

~~where λt = for any~~

For any arbitrary time t the no. of photons impinging $= \lambda t$.

Now, in between x_1 & $x_2 (t=T)$ no ~~th~~ photon should arrive —

$$P(x=0) = \frac{(\lambda t)^0 e^{-\lambda t}}{0!} \Big|_{t=T}$$

Probability $= e^{-\lambda T}$