Formatting Instructions for NIPS 2013

Anonymous Author(s)
Affiliation
Address
email

Abstract

1 The model

Our data is a time-series of multielectrode recordings $\mathbf{X} \equiv (\mathbf{x}_1, \cdots, \mathbf{x}_T)$, consisting of T recordings from C channels. The set of recording times lie on regular grid with interval length Δ, while $\mathbf{x}_t \in \mathbb{R}^C$ for all t. This time-series of electrical activity is driven by an unknown number of neurons. We let this be infinite, though only a few neurons dominate. These neurons contribute the majority of the activity, though as time passes, the total number of observed neurons (the number of observed 'species') increases. The neurons themselves emit sequences of action potentials which are superimposed to produce the recordings $\mathbf{X}$. We model the output of each neuron as a series of idealized spikes convolved with an appropriate smoothing kernel (the latter determines the shape of the action potential). Each neuron has its own distribution over waveform shapes. We describe this process for a single channel below.

The spiking activity of each neuron is modelled as a homogeneous Poisson process. Let r_i be the unkown firing rate for neuron i, and E_i be the set of spike times (with the time between successive elements of E exponentially distributed with mean $1/r_i$). We write this as

$$E_i \sim \text{PoissProc}(r_i) \tag{1}$$

From the superposition property of the Poisson process [1], the overall spiking activity $E = \cup_{i=1}^{\infty} E_i$ is a realization of a Poisson process with rate $R = \sum_{i=1}^{\infty} r_i$. Since only a finite number of spikes are observed in any finite interval, the total rate R must also be finite; moreover, as we described earlier, we want this to be dominated by a few r_i. We also associate a parameter θ with each neuron; this characterizes the distribution over the shapes of action potentials that that neuron produces.

A natural framework that captures these three modelling requirements is that of completely random measures [2]. We map the infinite collection of pairs $\{r_i, \theta_i\}$ to an atomic measure on Θ:

$$R(\mathrm{d}\theta) = \sum_{i=1}^{\infty} r_i \delta_{\theta_i} \tag{2}$$

We model $R(\cdot)$ as a realization of a completely random measure (CRM); this is a random measure where for any two disjoint subsets $\mathit{\Theta}_1$ and $\mathit{\Theta}_2 \in \Theta$, the measures $R(\Theta_1)$ and $R(\Theta_2)$ are independent (here, $R(\mathit{\Theta}) = \sum_{i \ s.t.\ \theta_i \in \Theta} r_i$). A completely random measure is characterized by a Levy intensity that describes the distribution of the weights r_i, while the locations θ_i are drawn i.i.d. from a base probability measure $H(\theta)$. The CRM we choose is the Gamma process; this has Levy intensity $r^{-1}\exp(-r\alpha)$. This has the convenient property that the total mass $R \equiv R(\Theta) = \sum_{i=1}^{\infty} r_i$ is Gamma distributed (and thus conjugate to the Poisson). The Gamma process is also closely connected with the Dirichlet process, which will prove useful later on.

We model each action potential shape as a linear combination of shared dictionary of K basis function $A \equiv (A_1, \cdots, A_K)$. For the ith neuron, the jth spike $e_{ij} \in E_i$, is associated with a random

k-dimensional weight vector y_{ij}, and this spike is convolved with a smoothing kernel given by Ay_{ij}. We let y_{ij} be normally distributed, with $\theta_i \equiv (\mu_i, \Sigma_i)$ determining the mean and variance. The overall model is then:

$$R(\mathrm{d}\theta) \sim \Gamma\mathrm{P}(\alpha) \tag{3}$$

$$E_i \sim \mathrm{PoissProc}(r_i) \quad i \text{ in } 1, 2, \cdots \tag{4}$$

$$x_i(t) = \sum_{j=1}^{|E_i|} Ay_j \delta_{(t-e_j)} \tag{5}$$

$$X = \sum_{i=1}^{\infty} x_i \tag{6}$$

However, it will be more convenient to view the set of pairs $\{(e_{ij}, y_{ij})\}$ as a realization of a marked Poisson process in time. The overall process E is a rate R Poisson process, with each event assigned a label or mark indicating the neuron to which it is assigned (or equivalently, the parameter θ associated with that neuron). These marks are drawn from a probability measure $G(\mathrm{d}\theta) = \frac{1}{R}R(\mathrm{d}\theta)$. From the properties of the Gamma process, the probability measure G a Dirichlet process, moreover G is independent of R. This suggests the following model, equivalent to the one above:

$$R \sim \Gamma(1, \alpha) \tag{7}$$

$$G(\mathrm{d}\theta) \sim \mathrm{DP}(\alpha) \tag{8}$$

$$E \sim \mathrm{PoissProc}(R) \tag{9}$$

$$y_e \sim G \quad \forall e \in E \tag{10}$$

$$\mathbf{x}(t) = \sum_{e \in E} Ay_e \delta_{(t-e)} \tag{11}$$

Our data is in a form that makes discrete-time modelling more natural, the Bernoulli approximation to the Poisson process suggests the following approximation: draw the random Poisson process rate R drawn from a Gamma$(1, \alpha)$ distribution. Simultaneously, draw a random probability measure G from a Dirichlet process. Assign an event to an interval independently with probability $R\Delta$, and to each event, assign a random mark drawn from the DP. Given the marks, we can evaluate the recordings at each time.

References

[1] J. F. C. Kingman. *Poisson processes*, volume 3 of *Oxford Studies in Probability*. The Clarendon Press Oxford University Press, New York, 1993. Oxford Science Publications.

[2] J.F.C. Kingman. Completely random measures. *Pacific Journal of Mathematics*, 21(1):59–78, 1967.