Formatting Instructions for NIPS 2013

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Abstract

1 The model

Our data is a time-series of multielectrode recordings $\mathbf{X} \equiv (\mathbf{x}_1, \cdots, \mathbf{x}_T)$, consisting of T recordings from C channels. The set of recording times lie on regular grid with interval length Δ, while $\mathbf{x}_t \in \mathbb{R}^C$ for all t. This time-series of electrical activity is driven by an unknown number of neurons, and we let this number be infinite. A consequence is that as time passes, the total number of observed neurons (the number of observed 'species') increases. The neurons themselves emit sequences of action potentials which are superimposed to produce the recordings $\mathbf{X}$. We model the output of each neuron as a series of idealized spikes convolved with an appropriate smoothing kernel (the waveform shape). Each neuron has its own distribution over waveform shapes. We describe this process for a single channel below.

We model the spiking activity of each neuron as a homogeneous Poisson process. Let r_i be the random firing rate for neuron i, and E_i be the set of spike times, so that

$$E_i \sim \text{PoissProc}(r_i) \tag{1}$$

From the superposition property of the Poisson process the overall spiking activity $E = \cup_{i=1}^{\infty} E_i$ is a realization of a Poisson process with rate $R = \sum_{i=1}^{\infty} r_i$. The number of spikes observed in any finite interval is finite, so that the total rate R must be finite; moreover, in all applications, this will be dominated by a few neurons. Additionally, each neuron has a parameter θ associated with it that characterizes its distribution over waveform shapes.

A natural framework that captures all these modelling requirements is that of completely random measures (**?**): we map the infinite collection of pairs $\{r_i, \theta_i\}$ to an atomic measure on Θ:

$$R(\mathrm{d}\theta) = \sum_{i=1}^{\infty} r_i \delta_{\theta_i} \tag{2}$$

We model $R(\cdot)$ as a realization of a completely random measure (CRM). A completely random measure is characterized by a Levy intensity that describes the distribution of the weights r_i, while the locations θ_i are drawn i.i.d. from a base measure $H(\theta)$. We use a Gamma process; this is a CRM with Levy intensity $r^{-1} \exp(-r\alpha)$; this has the convenient property that the total mass $R \equiv R(\Theta) = \sum_{i=1}^{\infty} r_i$ is Gamma distributed (and thus conjugate to the Poisson). The Gamma distribution has shape parameter 1 and scale parameter α. Since this is finite almost surely, so too is E. The Gamma process is also closely connected with the Dirichlet process, which will prove useful later on.

We model each action potential shape as a linear combination of shared dictionary of K basis function $A \equiv (A_1, \cdots, A_K)$. For the ith neuron, the jth spike $e_{ij} \in E_i$, is associated with a random weight vector y_{ij}, and this spike is convolved with a smoothing kernel given by Ay_{ij}. We let y_{ij} be normally distributed, with $\theta_i \equiv (\mu_i, \Sigma_i)$ setting the mean and variance. The overall model is then:

$$R(\mathrm{d}\theta) \sim \Gamma\mathrm{P}(\alpha) \tag{3}$$

$$E_i \sim \mathrm{PoissProc}(r_i) \tag{4}$$

$$x_i(t) = \sum_{j=1}^{|E_i|} A y_j \delta_{(t-e_j)} \tag{5}$$

$$X = \sum_{i=1}^{\infty} x_i \tag{6}$$

Each spike of each neuron is associated with a time e and a weight vector y, and one can view the model above as a doubly stochastic Poisson process on the product space.

However, it will be more convenient to view the set of pairs $\{(e_{ij}, y_{ij})\}$ as a realization of a marked Poisson process in time. The overall process E is a rate R Poisson process, with each event assigned a label or mark indicating the neuron to which it is assigned (or equivalently, the parameter θ associated with that neuron). These marks are drawn from a probability measure $G(\mathrm{d}\theta) = \frac{1}{R}R(\mathrm{d}\theta)$. From the properties of the Gamma process, the probability measure G a Dirichlet process, moreover G is independent of R. This suggests the following model, equivalent to the one above:

$$R \sim \Gamma(1, \alpha) \tag{7}$$

$$G(\mathrm{d}\theta) \sim \mathrm{DP}(\alpha) \tag{8}$$

$$E \sim \mathrm{PoissProc}(R) \tag{9}$$

$$y_e \sim G \quad \forall e \in E \tag{10}$$

$$\mathbf{x}(t) = \sum_{e \in E} A y_e \delta_{(t-e)} \tag{11}$$

Our data is in a form that makes discrete-time modelling more natural, the Bernoulli approximation to the Poisson process suggests the following approximation: draw the random Poisson process rate R drawn from a Gamma$(1, \alpha)$ distribution. Simultaneously, draw a random probability measure G from a Dirichlet process. Assign an event to an interval independently with probability $R\Delta$, and to each event, assign a random mark drawn from the DP. Given the marks, we can evaluate the recordings at each time.