Maximizing Network Connectivity for UAV Communications via Reconfigurable Intelligent Surfaces

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Abstract—**It is anticipated that integrating unmanned aerial vehicles (UAVs) with reconfigurable intelligent surfaces (RISs), resulting in RIS-assisted UAV networks, will offer improved network connectivity against node failures for the beyond 5G networks. In this context, we utilize a RIS to provide path diversity and alternative connectivity options for information flow from user equipment (UE) to UAVs by adding more links to the network, thereby maximizing its connectivity. This paper employs the algebraic connectivity metric, which is adjusted by the reflected links of the RIS, to formulate the problem of maximizing the network connectivity in two cases. First, we consider formulating the problem for one UE, which is solved optimally using a linear search. Then, we consider the problem of a more general case of multiple UEs, which has high computational complexity. To tackle this problem, we formulate the problem of maximizing the network connectivity as a semi-definite programming (SDP) optimization problem that can be solved efficiently in polynomial time. In both cases, our proposed solutions find the best combination between UE(s) and UAVs through the RIS. As a result, the phase shifts of the RIS are adjusted to direct the signals of the UEs to the appropriate UAVs, thus maximizing the network connectivity. Simulation results are conducted to assess the performance of the proposed solutions compared to the existing solutions.**

***Index Terms*—Network connectivity, algebraic connectivity, RIS-assisted UAV communications, graph theory.**

I. Introduction

UAVs are expected to have a remarkable impact on the economy by 2026 with a global market value of US$59.2 billion, making the incorporation of UAVs critical in beyond 5G networks [?]. One of the unique features of UAV-assisted communication is improved network connectivity by establishing line-of-sight (LoS) connections with [illegible] [?]. Meanwhile, RIS is a promising technique that can be integrated with UAVs to further improve network connectivity [?], particularly in deep fade [illegible]. In this context, RISs can be leveraged to provide path diversity and alternative connectivity solutions for information flow from UEs to UAVs in RIS-assisted UAV networks.

The prime concern of UAV communications is that UAV nodes are prone to failure due to several reasons, such as limited energy, hardware failures, or targeted attacks in the battlefield [illegible]. Such UAV failures cause network disintegration, and consequently, information flow from UEs to the UAVs can be severely impacted. Hence, it is critical to always keep the network connected, which was addressed in the literature by adding more backhaul links to the network, e.g., [?]. [illegible]

[illegible] the Laplacian matrix [illegible] the Fiedler [illegible] the second smallest eigenvalue of the Laplacian matrix representing a graph [illegible] that measures how well a graph is connected. [illegible] backhaul network [?] [illegible]

[illegible] RIS-assisted UAV networks is maximized. To the best of our knowledge, [illegible] then we consider two problem cases. First, we [illegible]

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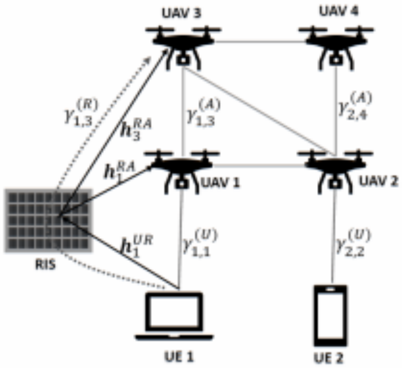

Fig. 1: A typical RIS-assisted UAV network with one RIS, 2 UEs, and 4 UAVs.

... the RIS by tuning its phase shifts to direct the UEs' signals to the appropriate UAVs, thus maximizing the network connectivity. Simulation results are conducted to assess the performance of the proposed solutions compared to the existing solutions.

II. System Model and Network Connectivity

A. System Model

We consider a RIS-assisted UAV network with a set of UAVs, one RIS, and multiple UEs that represent ground users, sensors, etc. An example of the considered network is shown in Fig. 1. The sets of UAVs and UEs are denoted as $\mathcal{A} = \{1, 2, \ldots, A\}$ and $\mathcal{U} = \{1, 2, \ldots, U\}$, respectively, where A and U are the cardinality of the sets $\mathcal{A}$ and $\mathcal{U}$. All UEs and UAVs are equipped with a single antenna. ...

... As such, the network becomes more resilient against UAV node failures by providing path diversity. However, UEs may occasionally experience deep fade. To overcome this problem and further improve network connectivity, we propose to utilize a RIS for this purpose. ... The RIS is equipped with a controller and $M = M_r \times M_c$ passive reflecting units (PRUs) to form a uniform passive array (UPA). Each column of the UPA has M_r PRUs with an equal spacing of d_r meters (m) and each row of the UPA consists of M_c PRUs with an equal spacing of d_c m. ...

Since this paper focuses on the network connectivity from data link-layer viewpoint, we abstract the physical layer factors and consider a model that relies only on the distance between the nodes. Therefore, we model only the large scale fading and ignore the small scale fading. To quantify the UEs transmission to the UAVs and the RIS, we use the signal-to-noise ratio (SNR). For the u-th UE, SNR is defined as follows [?]:

$$\gamma_{u,a}^{(U)} = \frac{p\, d_{u,a}^{-\alpha}}{N_0}, \qquad (1)$$

where $d_{u,a}$ is the distance between the u-th UE and the a-th UAV, p is the transmit power of the u-th UE, which is maintained fixed for all the UEs, N_0 is the additive white Gaussian noise (AWGN) variance, and α is the path loss exponent that depends on the transmission environment.

UAVs hover at high altitudes, thus we reasonably assume that they maintain LoS channel between each other. The path loss between the a-th and the a'-th UAVs can be expressed as

$$\Gamma_{a,a'} = 20 \log\left(\frac{4\pi f_c d_{a,a'}}{c}\right), \qquad (2)$$

where $d_{a,a'}$ is the distance between the a-th UAV and the a'-th UAV, f_c is the carrier frequency, and c is light speed. The SNR in dB between the a-th UAV and the a'-th UAV is $\gamma_{a,a'}^{(A)} = 10 \log P - \Gamma_{a,a'} - 10 \log N_0$, where P is the transmit power of the a-th UAV, which is maintained fixed for all the UAVs. Note that the SNR of the u-th UE determines whether it has a successful connection to the corresponding UAV a. In other words, the a-th UAV is assumed to be within the transmission range of the u-th UE if $\gamma_{u,a}^{(U)} \geq \gamma_0^{\mathrm{UE}}$, where γ_0^{UE} is the minimum SNR threshold for the communication links between the UEs and the UAVs. Similarly, we assume that ...

the u-th UE if $\gamma_{u}^{(U)} \ge \gamma_0^{\rm UE}$, where $\gamma_0^{\rm UE}$ is the minimum SNR threshold for the communication links between the UEs and the UAVs. Similarly, we assume that UAV a and UAV a' have a successful connection provided that $\gamma_{a,a'}^{(A)} \ge \gamma_0^{\rm UAV}$, where $\gamma_0^{\rm UAV}$ is the minimum SNR threshold for the communication links between the UAVs.

Since the RIS is deployed in the higher altitude, the signal propagation of UE-to-RIS link is adopted to be a simple yet reasonably accurate LoS channel model [?]. The LoS channel vector between the u-th UE and the RIS is given by [?]

$$\mathbf{h}_u^{\rm UR} = \sqrt{\frac{\beta_0}{(d_u^{\rm UR})^2}}\,\bar{\mathbf{h}}_u^{\rm UR}, \tag{3}$$

where $d_u^{\rm UR}$ is the distance between the u-th UE and the RIS, β_0 denotes the path loss at the reference distance $d_{\rm ref} = 1$ m, and $\bar{\mathbf{h}}_u^{\rm UR}$ represents the array response component which can be denoted by

$$\bar{\mathbf{h}}_u^{\rm UR} = \left[1, e^{-j\frac{2\pi d_r}{\lambda}\phi_u^{\rm UR}\psi_u^{\rm UR}}, \dots, e^{-j\frac{2\pi d_r}{\lambda}(M_r-1)\phi_u^{\rm UR}\psi_u^{\rm UR}}\right]^T \otimes \left[1, e^{-j\frac{2\pi d_c}{\lambda}\varphi_u^{\rm UR}\psi_u^{\rm UR}}, \dots, e^{-j\frac{2\pi d_c}{\lambda}(M_c-1)\varphi_u^{\rm UR}\psi_u^{\rm UR}}\right]^T,$$

where $\phi_u^{\rm UR}$, $\varphi_u^{\rm UR}$, and $\psi_u^{\rm UR}$ are related to the sine and cosine terms of the vertical and horizontal angles-of-arrival (AoAs) at the RIS [?], and given by $\phi_u^{\rm UR} = \frac{y_u - y_R}{\sqrt{(x_u-x_R)^2+(y_u-y_R)^2}}$, $\varphi_u^{\rm UR} = \frac{x_u - x_R}{\sqrt{(x_u-x_R)^2+(y_u-y_R)^2}}$, $\psi_u^{\rm UR} = \frac{z_R}{d_u^{\rm UR}}$, λ is the wavelength, and T denotes transpose. On the other hand, the RIS and UAVs are deployed in the higher altitudes, thus the reflected signal propagation of the RIS-to-UAV link typically occurs in clear airspace where the obstruction or reflection effects diminish. The LoS channel vector between the RIS and the a-th UAV is given by

$$\mathbf{h}_a^{\rm RA} = \sqrt{\frac{\beta_0}{(d_a^{\rm RA})^2}}\,\bar{\mathbf{h}}_a^{\rm RA}, \tag{4}$$

where $d_a^{\rm RA}$ is the distance between the RIS and the a-th UAV, and $\bar{\mathbf{h}}_a^{\rm RA}$ represents the array response component which can be denoted by

$$\bar{\mathbf{h}}_a^{\rm RA} = \left[1, e^{-j\frac{2\pi d_r}{\lambda}\phi_a^{\rm RA}\psi_a^{\rm RA}}, \dots, e^{-j\frac{2\pi d_r}{\lambda}(M_r-1)\phi_a^{\rm RA}\psi_a^{\rm RA}}\right]^T \otimes \left[1, e^{-j\frac{2\pi d_c}{\lambda}\varphi_a^{\rm RA}\psi_a^{\rm RA}}, \dots, e^{-j\frac{2\pi d_c}{\lambda}(M_c-1)\varphi_a^{\rm RA}\psi_a^{\rm RA}}\right]^T,$$

where $\phi_a^{\rm RA}$, $\varphi_a^{\rm RA}$, and $\psi_a^{\rm RA}$ are related to the sine and cosine terms of the vertical and horizontal angles-of-departure (AoDs) from the RIS to the a-th UAV [?], and respectively given by $\phi_a^{\rm RA} = \frac{y_R - y_a}{\sqrt{(x_R-x_a)^2+(y_R-y_a)^2}}$, $\varphi_a^{\rm RA} = \frac{x_R - x_a}{\sqrt{(x_R-x_a)^2+(y_R-y_a)^2}}$, and $\psi_a^{\rm RA} = \frac{z_R - z_a}{d_a^{\rm RA}}$.

Given the aforementioned channel models, the concatenated channel for the UE-RIS-UAV link between the u-th UE and the a-th UAV through the RIS is given by $h_{u,a}^{\rm URA} = (\mathbf{h}_a^{\rm RA})^H \boldsymbol{\Theta} \mathbf{h}_u^{\rm UR}$ [?]. Accordingly, the SNR of the reflected link between the u-th UE and the a-th UAV through the RIS can be written as $\gamma_{u,a}^{(R)} = \frac{p|h_{u,a}^{\rm URA}|^2}{N_0}$ [?]. For successful connection between UE u and UAV a via RIS r, $\gamma_{u,a}^{(R,r)} \ge \gamma_0^{\rm RIS}$, where $\gamma_0^{(RIS)}$ is the minimum SNR threshold for the communication links between the UEs and the UAVs via the RISs.

We model the considered RIS-assisted UAV network as an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \dots, v_V\}$ is the set of nodes (i.e., UAVs and UEs) in the network, $\mathcal{E} = \{e_1, e_2, \dots, e_E\}$ is the set of all edges, $V = |\mathcal{V}|$ and $E = |\mathcal{E}|$ are the numbers of vertices and edges in the graph, respectively. The graph $\mathcal{G}$ implies that all the links in the network are bidirectional, i.e., a node v is able to reach node v', and vice versa. The edge between any two nodes is created based on a typical SNR threshold.

B. Network Connectivity

For an edge e_k, $1 \le k \le E$, that connects two nodes $\{v_n, v_m\} \in \mathcal{V}$, let $\mathbf{a}_k$ be a vector, where the n-th and m-th elements in $\mathbf{a}_k$ are given by $a_{k,n} = 1$ and $a_{k,m} = -1$, respectively, and zero otherwise. The incidence matrix $\mathbf{A} \in \mathbb{R}^{V\times E}$ of a graph $\mathcal{G}$ is the matrix with the k-th column given by $\mathbf{a}_k$. Hence, in undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, the Laplacian matrix $\mathbf{L}$ is an V by V matrix, which is defined as follows [?]:

$$\mathbf{L} = \mathbf{A}\mathbf{A}^T = \sum_{k=1}^{E} \mathbf{a}_k \mathbf{a}_k^T, \tag{5}$$

where the entries of $\mathbf{L}$ are given as follows:

$$L(n,m) = \begin{cases} D_{v_n} & \text{if } v_n = v_m, \\ -1 & \text{if } (v_n, v_m) \in \mathcal{E}, \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where v_n, v_m are the indices of the nodes and D_{v_n} is the degree of node v_n, which represents the number of all its neighboring nodes.

In network connectivity, algebraic connectivity, also called the Fiedler metric or the second smallest eigenvalue [?], measures how well a graph $\mathcal{G}$ that has the associated Laplacian matrix $\mathbf{L}$ is connected. From its name, this metric is usually denoted as $\lambda_2(\mathbf{L})$. The motivation of $\lambda_2(\mathbf{L})$ to be used as a network connectivity metric comes from the following two main reasons [?]. First, $\lambda_2(\mathbf{L}) > 0$ if and only if $\mathcal{G}$ is connected, i.e., $\mathcal{G}$ is only one connected graph. It is worth mentioning that when $\lambda_2(\mathbf{L}) = 0$, the graph is disconnected in which at least one of its vertices is unreachable from any other vertices in the graph. Second, $\lambda_2(\mathbf{L})$ is monotone increasing in the edge set, i.e., if $\mathcal{G}_1 = (\mathcal{V}, \mathcal{E}_1)$ and $\mathcal{G}_2 = (\mathcal{V}, \mathcal{E}_2)$ and $\mathcal{E}_1 \subseteq \mathcal{E}_2$, then $\lambda_2(\mathcal{G}_2) \ge \lambda_2(\mathcal{G}_1)$. This implies that $\lambda_2(\mathbf{L})$ qualitatively represents the connectivity of a graph in the sense that

III. Problem Formulation

... Based on that, we consider $\lambda_2(\mathbf{L})$ as a quantitative measure of the network resiliency in this paper, similar to [?], [?].

III. Problem Formulation

Given a RIS-assisted UAV network represented by a graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, what are the optimum combinations between the UEs and the UAVs through the RIS in order to maximize $\lambda_2(\mathbf{L})$ of the resulting network? Essentially, adding the RIS to the network may result in connecting multiple UEs to multiple UAVs, which were not connected together. It may also result in adding new alternative options to the UEs if their scheduled UAVs have failed. In this context, we leverage the RIS to add more links to the network, and by adjusting its phase shifts, RIS can smartly beamform the signals of the UEs to suitable UAVs to maximize the network connectivity.

With RIS deployment, a new graph $\mathcal{G}'(\mathcal{V},\mathcal{E}')$ is constructed with the same number of V nodes and a larger set of edges denoted by $\mathcal{E}'$ with $\mathcal{E}' = \mathcal{E} \cup e^R_{u,a}$, where $e^R_{u,a}$ is the new edge connecting the u-th UE to the a-th UAV through the RIS and $\mathcal{E} \subseteq \mathcal{E}'$. Note that the effect of deploying the RIS appears only in the edge set $\mathcal{E}$, and not in the node set V [?], [?], [?]. By adding those new links to the network, the gain can be realized by computing $\lambda_2(\mathbf{L}') \geq \lambda_2(\mathbf{L})$, where $\lambda_2(\mathbf{L}')$ is the resulting Laplacian matrix of a graph $\mathcal{G}'(\mathcal{V},\mathcal{E}')$.

We consider that in each time slot only one UE can transmit to the RIS, which directs the UE's signal to only one UAV. In what follows, we consider two different cases of network configurations to formulate the optimization problem of maximizing $\lambda_2(\mathbf{L}')$ in each time slot.

Case 1: One UE and One RIS

Let $\mathcal{A}_0$ be a set of reachable UAVs that have indirect ... case is formulated as follows:

$$\max_{y_a,\theta_m} \lambda_2(\mathbf{L}') \tag{7a}$$

$$\text{subject to} \sum_{a\in\mathcal{A}_0} y_a = 1, \tag{7b}$$

$$\theta_m \in [0, 2\pi) \qquad m = \{1,\ldots,M\}, \tag{7c}$$

$$y_a \in \{0,1\} \qquad \forall a \in \mathcal{A}_0, \tag{7d}$$

where constraint (??) assures that the RIS directs the signal of the UE to only one UAV. Constraint (??) is for the RIS phase shift optimization.

Case 2: Multiple UEs and One RIS

Unlike case 1 that considers one UE, case 2 adds an optimization variable that selects the u-th UE that transmits in each time slot. Let $\mathcal{A}^u_0$ be a set of reachable UAVs that have indirect communication links from the u-th UE through the RIS, i.e., $\mathcal{A}^u_0 = \{a \in \mathcal{A}\backslash\mathcal{A}_u \mid \gamma^{(R)}_{u,a} \geq \gamma^{\text{RIS}}_0\}$, where $\mathcal{A}_u$ is the set of UAVs that have direct links to the u-th UE. Let x_u be a binary variable that is equal to 1 if the u-th UE is connected to the RIS, and zero otherwise. Now, let y^u_a be a binary variable that is equal to 1 if the RIS is connected to the a-th UAV when the u-th UE is selected to transmit, and zero otherwise. The considered optimization problem in this case is formulated as follows:

$$\max_{x_u, y^u_a, \theta_m} \lambda_2(\mathbf{L}') \tag{8a}$$

$$\text{subject to} \sum_{u\in\mathcal{U}} x_u = 1, \tag{8b}$$

$$\sum_{a\in\mathcal{A}^u_0} y^u_a = 1, \tag{8c}$$

$$\theta_m \in [0, 2\pi) \qquad m = \{1,\ldots,M\}, \tag{8d}$$

$$x_u \in \{0,1\}, y^u_a \in \{0,1\} \qquad \forall a \in \mathcal{A}^u_0, \tag{8e}$$

where constraint (??) and (??) assure that only one UE can transmit to the RIS and the signal of that UE is reflected from the RIS to only one UAV. Constraint (??) is for the RIS phase shift optimization.

IV. Proposed Solutions

It is computably affordable to optimally solve (??) since it considers only one UE, however solving (??) optimally for the case of multiple UEs is computationally prohibitive. Therefore, this section proposes to solve (??) optimally using a linear search over all the possible UAV nodes. Then, the section formulates (??) as an SDP optimization problem that can be solved efficiently in polynomial time. The process of those proposed solutions are explained in subsections IV-A and IV-B, respectively.

UAV which are not connected, the graph will be ... larger set of edges denoted by $\mathcal{E}'$ with $\mathcal{E}' = \mathcal{E} \cup e^R_{u,a}$, where $e^R_{u,a}$ is the new edge connecting the u-th UE to the a-th UAV through the RIS and $\mathcal{E} \subseteq \mathcal{E}'$. Note that the effect of deploying the RIS appears only in the edge set $\mathcal{E}$, and not in the node set V [?], [?], [?]. By adding those new links to the network, the gain can be realized by computing $\lambda_2(\mathbf{L}') \geq \lambda_2(\mathbf{L})$, where $\lambda_2(\mathbf{L}')$ is the resulting Laplacian matrix of a graph $\mathcal{G}'(\mathcal{V},\mathcal{E}')$.

We consider that in each time slot only one UE can transmit to the RIS, which directs the UE's signal to only one UAV. In what follows, we consider two different cases of network configurations to formulate the optimization problem of maximizing $\lambda_2(\mathbf{L}')$ in each time slot.

Case 1: One UE and One RIS

Let $\mathcal{A}_0$ be a set of reachable UAVs that have indirect communication links from the UE through the RIS, i.e., $\mathcal{A}_0 = \{a \in \mathcal{A}\backslash\mathcal{A}_{UE} \mid \gamma^{(R)}_a \geq \gamma^{\text{RIS}}_0\}$, where $\mathcal{A}_{UE}$ is the set of UAVs that have direct links to the UE. Our aim is to provide an alternative link to connect the UE to a single UAV in the set $\mathcal{A}_0$. As such, the UE does not miss the communication if its scheduled UAV has failed. Now, let y_a be a binary variable that is equal to 1 if the RIS is connected to the a-th UAV ($a \in \mathcal{A}_0$), and zero otherwise. The considered optimization problem in this case is formulated as follows:

$$\max_{y_a,\theta_m} \lambda_2(\mathbf{L}') \tag{7a}$$

$$\text{subject to} \sum_{a\in\mathcal{A}_0} y_a = 1, \tag{7b}$$

$$\theta_m \in [0, 2\pi) \qquad m = \{1,\ldots,M\}, \tag{7c}$$

$$y_a \in \{0,1\} \qquad \forall a \in \mathcal{A}_0, \tag{7d}$$

where constraint (??) assures that the RIS directs the signal of the UE to only one UAV. Constraint (??) is for the RIS phase shift optimization.

Case 2: Multiple UEs and One RIS

Unlike case 1 that considers one UE, case 2 adds an ... zero otherwise. The considered optimization problem in this case is formulated as follows:

Algorithm 1 The Proposed Linear Search for Case 1

1: **Input:** One UE u, one RIS, $\mathcal{A}$ and network topology.
2: Construct $\mathcal{G}(\mathcal{V},\mathcal{E})$.
3: **for** $a = 1, 2, \ldots, |\mathcal{A}_0|$ **do**
4: Calculate θ_m of the RIS to the a-th UAV as given in (11), $\forall m \in \{1, 2, \ldots, M\}$
5: Set $\mathcal{G}' \leftarrow \mathcal{G}(\mathcal{V}, \mathcal{E} \cup \{e^R_{u,a}\})$
6: Calculate $\lambda_2(\mathbf{L}')$ of a graph $\mathcal{G}'$
7: **end for**
8: **Output:** Optimal $\lambda_2(\mathbf{L}')$.

$$\max_{x_u, y^u_a, \theta_m} \lambda_2(\mathbf{L}') \tag{8a}$$

$$\text{subject to} \sum_{u\in\mathcal{U}} x_u = 1, \tag{8b}$$

$$\sum_{a\in\mathcal{A}^u_0} y^u_a = 1, \tag{8c}$$

$$\theta_m \in [0, 2\pi) \qquad m = \{1,\ldots,M\}, \tag{8d}$$

$$x_u \in \{0,1\}, y^u_a \in \{0,1\} \qquad \forall a \in \mathcal{A}^u_0, \tag{8e}$$

where constraint (??) and (??) assure that only one UE can transmit to the RIS and the signal of that UE is reflected from the RIS to only one UAV. Constraint (??) is for the RIS phase shift optimization.

IV. Proposed Solutions

It is computably affordable to optimally solve (**??**) since it considers only one UE, however solving (**??**) optimally for the case of multiple UEs is computationally prohibitive. Therefore, this section proposes to solve (**??**) optimally using a linear search over all the possible UAV nodes. Then, the section formulates (**??**) as an SDP optimization problem that can be solved efficiently in polynomial time. The process of those proposed solutions are explained in subsections IV-A and IV-B, respectively.

A. Solution of Case 1

To optimally solve (**??**), we consider a linear search scheme that computes $\lambda_2(\mathbf{L}')$ for each possible UAV node $a \in \mathcal{A}_0$, and then calculate the corresponding phase shift of the RIS to that UAV node. In particular, the corresponding phase shift at PRU of the RIS to the a-th UAV is calculated as follows [**?**]

$$\theta_m = \pi - \frac{f_c}{c}\Big\{d_r(m_r - 1)\psi^{\text{RA}}_a \phi^{\text{RA}}_a + d_c(m_c - 1)\psi^{\text{RA}}_a \varphi^{\text{RA}}_a \ldots d_r(m_r - 1)\psi^{\text{UR}}\phi^{\text{UR}} + d_c(m_c - 1)\psi^{\text{UR}}\varphi^{\text{UR}}\Big\} \tag{9}$$

... the $|\mathcal{A}_0|$ Laplacian matrices. The steps of the proposed ... Algorithm 1.

Algorithm 1 The Proposed Linear Search for Case 1

1: **Input:** One UE u, one RIS, $\mathcal{A}$ and network topology
2: Construct $\mathcal{G}(\mathcal{V},\mathcal{E})$
3: **for** $a = 1, 2, \ldots, |\mathcal{A}_0^u|$ **do**
4: Calculate θ_m of the RIS to the a-th UAV as given in (11), $\forall m \in \{1, 2, \ldots, M\}$
5: Set $\mathcal{G}' \leftarrow \mathcal{G}(\mathcal{V}, \mathcal{E} \cup \{e_{a,R}^{R}\})$
6: Calculate $\lambda_2(\mathbf{L}')$ of a graph $\mathcal{G}'$
7: **end for**
8: **Output:** Optimal $\lambda_2(\mathbf{L}')$

A. Solution of Case 1

To optimally solve (??), we consider a linear search scheme that computes $\lambda_2(\mathbf{L}')$ for each possible UAV node $a \in \mathcal{A}_0$, and then calculate the corresponding phase shift of the RIS to that UAV node. In particular, the corresponding phase shift at PRU of the RIS to the a-th UAV is calculated as follows [?]

$$\theta_m = \frac{2\pi f_c}{c}\Big\{ d_r(m_r-1)\psi_a^{\rm RA}\varphi_a^{\rm RA} + d_c(m_c-1)\psi_a^{\rm RA}\varphi_a^{\rm RA} + d_r(m_r-1)\psi^{\rm UR}\varphi^{\rm UR} + d_c(m_c-1)\psi^{\rm UR}\varphi^{\rm UR}\Big\}. \quad (9)$$

We argue that the computational complexity of the proposed solution of this case is affordable since it needs to compute only $|\mathcal{A}_0|$ Laplacian matrices. The steps of the proposed method are summarized in Algorithm 1.

B. Solution of Case 2

The exhaustive search scheme to solve (??) for multiple UEs can be done by computing $\lambda_2(\mathbf{L}')$ for a total of $U\sum_{u=1}^{U}|\mathcal{A}_0^u|$ Laplacian matrices, which requires huge amount of computation for large U. For graph $\mathcal{G}'(\mathcal{V},\mathcal{E}')$, the time complexity of the exhaustive search is high for large network settings. It runs in $\mathcal{O}(4E'V^3/3)$ to compute $\lambda_2(\mathbf{L}')$ [?]. To overcome such computational intractability, we instead propose an efficient method to solve (??), which finds the feasible UE-RIS-UAV association to maximize $\lambda_2(\mathbf{L}')$ using SDP solvers [?].

We add a link connecting the u-th UE to the a-th UAV through the RIS if both x_u and y_a^u in (??) are 1. Let $\mathbf{z}$ be a vector representing the UE-RIS-UAV candidate associations, in which case $x_u = 1$ and $y_a^u = 1$, $\forall u \in \mathcal{U}, a \in \mathcal{A}$. Therefor, the problem in (??) can be seen as having a set of $|\mathbf{z}|$ UE-RIS-UAV candidate associations, and we want to select the optimum UE-RIS-UAV association among these $|\mathbf{z}|$ associations. This optimization problem can be formulated as

$$\max_{\mathbf{z}} \lambda_2(\mathbf{L}'(\mathbf{z})) \quad (10)$$
$$\text{subject to } \mathbf{1}^T\mathbf{z} = 1, \mathbf{z} \in \{0,1\}^{|\mathbf{z}|},$$

where $\mathbf{1} \in \mathbf{R}^{|\mathbf{z}|}$ is the all-ones vector and

$$\mathbf{L}'(\mathbf{z}) = \mathbf{L} + \sum_{l=1}^{|\mathbf{z}|} z_l \mathbf{a}_l \mathbf{a}_l^T, \quad (11)$$

where $\mathbf{a}_l$ is the incidence vector resulting from adding link l to the original graph $\mathcal{G}$ and $\mathbf{L}$ is the Laplacian matrix of the original graph $\mathcal{G}$. Clearly, the dimension of $\mathbf{L}$ and $\mathbf{L}'(\mathbf{z})$ is $V \times V$.

The optimization vector in (12) is the vector $\mathbf{z}$. The l-th element of $\mathbf{z}$, denoted by z_l, is either 1 or 0, which corresponds to whether this UE-RIS-UAV association should be chosen or not, respectively. The combinatorial optimization problem in (12) is NP-hard problem with high complexity. Therefore, we relax the constraint on the entries of $\mathbf{z}$ and allow them to take any value in the interval $[0,1]$. Specifically, we relax the Boolean constraint $\mathbf{z} \in \{0,1\}^{|\mathbf{z}|}$ to be a linear constraint $\mathbf{z} \in [0,1]^{|\mathbf{z}|}$, then we can represent the problem (12) as

$$\max_{\mathbf{z}} \lambda_2(\mathbf{L}'(\mathbf{z})) \quad (12)$$
$$\text{subject to } \mathbf{1}^T\mathbf{z} = 1, 0 \le \mathbf{z} \le 1.$$

We emphasize that the optimal solution of the relaxed problem in (14) is an upper bound for the optimal solution of the original problem (12) since it has a larger feasible set. In [?], it was shown that $\lambda_2(\mathbf{L}'(\mathbf{z}))$ in (14) is the point-wise infimum of a family of linear functions of $\mathbf{z}$, which is a concave function in $\mathbf{z}$. In addition, the relaxed constraints are linear in $\mathbf{z}$. Therefore, the optimization problem in (14) is a convex optimization problem [?], and it is equivalent to the following SDP optimization problem [?]

$$\max_{\mathbf{z},q} q \quad (13)$$
$$\text{subject to } q\Big(\mathbf{I} - \frac{1}{|\mathbf{z}|}\mathbf{1}\mathbf{1}^T\Big) \preceq \mathbf{L}'(\mathbf{z}), \mathbf{1}^T\mathbf{z} = 1, 0 \le \mathbf{z} \le 1,$$

where $\mathbf{I} \in \mathbf{R}^{V\times V}$ is the identity matrix and $\mathbf{F} \preceq \mathbf{L}$ denotes that $\mathbf{L} - \mathbf{F}$ is a positive semi-definite matrix.

By solving the SDP optimization problem in (15) efficiently using any SDP standard solver such as the CVX software package [?], the optimization variable $\mathbf{z}$ is obtained. Since the entries of the output vector $\mathbf{z}$ are continuous, we consider to round the maximum entry to 1 while others are rounded to zero. For the given association vector $\mathbf{z}$, we optimize the phase shift of the RIS as in (11) to direct the signal of the selected UE to the associated UAV.

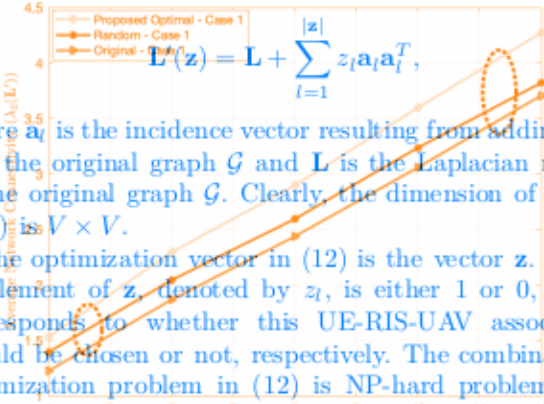

Fig. 2: The average network connectivity $\lambda_2(\mathbf{L}')$ of case 1 versus the number of UAVs A.

V. Numerical Results

For the numerical evaluations, we use the same RIS configurations and UAV communications that were used in [?] and [?], respectively. We consider a RIS-assisted UAV system in an area of 150 m × 150 m, where the RIS has a fixed location and the UEs and the UAVs are distributed randomly. The considered simulation parameters are as follows: the RIS is located at (35 m, 50 m) with an altitude of 20 m, $M = 100$, $d_r = 5$ cm, $d_c = 5$ cm, $\beta = 10^6$, $N_0 = -130$ dBm, the altitude of the UAVs is 50 m, $f_c = 3\times10^9$ Hz, $c = 3\times10^8$ m/s, $\rho = 4$, $P = 5$ watt, $\gamma_0^{(U)} = 85$ dB, $\gamma_0^{(A)} = 80$ dB. Unless specified otherwise, $A = 7$, $U = 10$ and $\gamma_0^{(RIS)} = 30$ dB.

For the sake of numerical comparison, the proposed scheme is compared to the following benchmark schemes: without RIS (Case 1), and random link association (Case 2) ... to the UAVs through the RIS. For completeness of our work, we also compare the proposed SDP scheme of case 2 with the optimal scheme that is considered as a performance upper bound ... In Fig. 2 and 3, we show the average network connectivity ... 500 ...

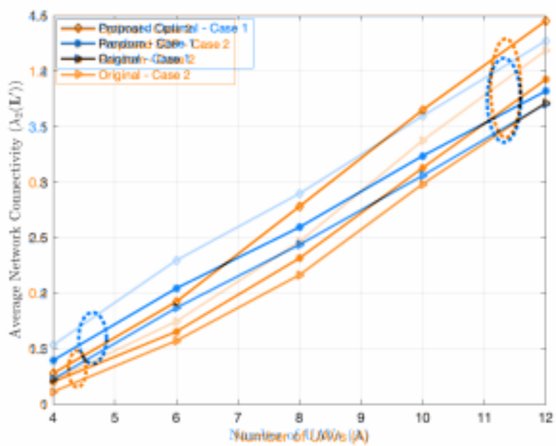

Fig. 2: The average network connectivity $\lambda_2(\mathbf{L}')$ of case 1 versus the number of UAVs A.

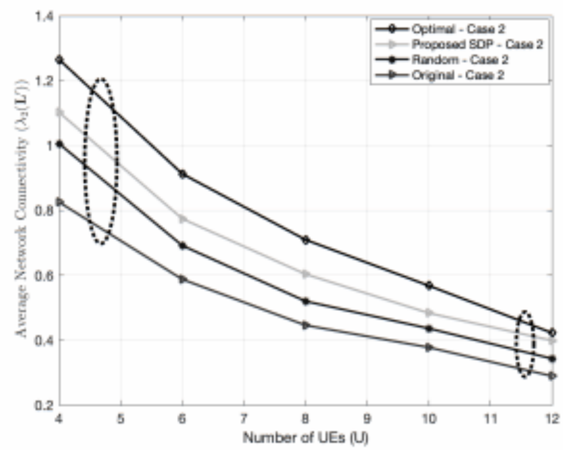

Fig. 4: The average network connectivity $\lambda_2(\mathbf{L}')$ versus the number of UEs U.

ber of UAVs in Figs. 2 and 3, the proposed optimal and SDP schemes offer a slight performance gain in terms of network connectivity compared to the original and the random schemes. This is because our proposed schemes have a few options of links, where the RIS can direct the signal of the UE to a few number of UAVs. However, when the number of UAVs increases, the proposed schemes smartly selects an effective UE-RIS-UAV link that significantly maximizes the network connectivity. It is noted that $\lambda_2(\mathbf{L}')$ of all schemes increases with the number of UAVs since adding more connected nodes to the network increases the number of edges, which increases the network connectivity. It is also noted that the values of $\lambda_2(\mathbf{L}')$ in Fig. 3 are smaller than the values of $\lambda_2(\mathbf{L}')$ in Fig. 2 for all the UAVs configurations. This is reasonably because the number of unconnected nodes that represent the UEs in Fig. 3 of case 2, i.e., $U = 10$, is larger than those of Fig. 2 of case 1, which is one UE. This makes the network of case 2 less connected (i.e., more UE nodes and no links between them), thus low network connectivity in Fig. 3. When $A > 8$ in Fig. 3, the average network connectivity of all the schemes increases significantly with A, which follows the same behaviour of Fig. 2 that is, $A > U$.

Fig. 3: The average network connectivity $\lambda_2(\mathbf{L}')$ of case 2 versus the number of UAVs A.

In Fig. 4, we plot the network connectivity versus the number of UEs U for case 2. From Fig. 4, we can see that the proposed SDP outperforms the original and the random schemes in terms of network connectivity. Notably, the network connectivity of all the schemes decreases as the number of UEs increases, since adding more unconnected UEs may result in a sparse graph with low network connectivity.

In Fig. 5, we show the impact of the SNR threshold $\gamma_0^{(\mathrm{RIS})}$ on the network connectivity for case 2. For small SNR threshold, all the links between the UEs and the UAVs through the RIS can satisfy this SNR threshold, [illegible] network connectivity. On the other hand, for high RIS SNR threshold, a few UE-RIS-UAV links can satisfy such high SNR threshold, [illegible] degraded, and it becomes close to the original scheme, which does not get affected by changing $\gamma_0^{(\mathrm{RIS})}$.

Fig. 5: The average network connectivity $\lambda_2(\mathbf{L}')$ versus SNR threshold $\gamma_0^{(\mathrm{RIS})}$ in dB.

It is worth remarking that while the random scheme adds a random link to the network, the original scheme does not add a link. The proposed solutions balance between the aforementioned approaches by judiciously selecting an effective link between a UE and a UAV, that maximizes the network connectivity. This utilizes the benefits of the cooperation between an appropriate scheduling algorithm design and RIS phase shift configurations. Compared to the optimal scheme, the proposed SDP has a certain degradation in network connectivity that comes as the achieved polynomial computational complexity as compared to the higher complexity of the optimal scheme that is in the order of $\mathcal{O}(2(4E'V^3/3))$ [?].

VI. Conclusion

In this paper, we propose a novel UE-UAV scheduling and RIS phase shift optimization for achieving resilient RIS-UAV networks. [illegible] RIS to add alternative links to the network by optimal UE and

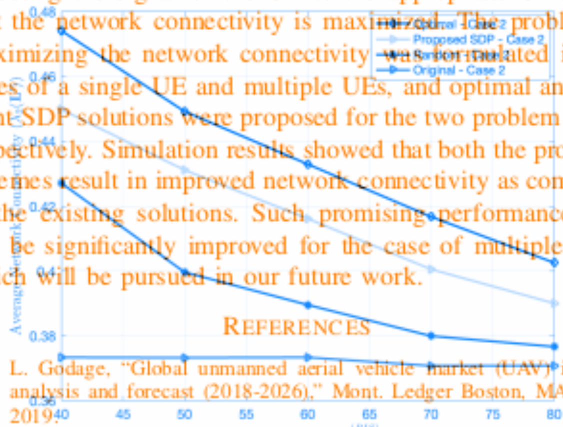

Fig. 5: The average network connectivity $\lambda_2(\mathbf{L}')$ versus SNR threshold $\gamma_0^{(RIS)}$ in dB.

the UAVs to select to maximize the network connectivity. On the other hand, for high RIS SNR threshold, a few UE-RIS-UAV links can satisfy such high SNR threshold, thus the network connectivity of all the schemes is degraded, and it becomes close to the original scheme, which does not get affected by changing $\gamma_0^{(RIS)}$.

It is worth remarking that while the random scheme adds a random link to the network, the original scheme does not add a link. The proposed solutions balance between the aforementioned aspects by judiciously selecting an effective link, between a UE and a UAV, that maximizes the network connectivity. This utilizes the benefits of the cooperation between an appropriate scheduling algorithm design and RIS phase shift configurations. Compared to the optimal scheme, our proposed SDP has a certain degradation in network connectivity that comes as the achieved polynomial computational complexity as compared to the high complexity of the optimal scheme that is in the order of $\mathcal{O}(4E'V^2/3)$ [2].

VI. Conclusion

In this paper, we proposed a novel joint UE-UAV scheduling and RIS phase shift optimization for achieving connected and resilient RIS-assisted UAV networks. We leveraged the RIS to add more links to the network by opportunistically reflecting the signal of the UE to the appropriate UAV such that the network connectivity is maximized. The problem of maximizing the network connectivity was formulated in two cases of a single UE and multiple UEs, and optimal and efficient SDP solutions were proposed for the two problem cases, respectively. Simulation results showed that both the proposed schemes result in improved network connectivity as compared to the existing solutions. Such promising performance gain can be significantly improved for the case of multiple RISs, which will be pursued in our future work.

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