

Near-Field Communications: A Degree-of-Freedom Perspective

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Abstract—Multiple-antenna technologies are advancing towards large-scale aperture sizes and extremely high frequencies, leading to the emergence of near-field communications (NFC) in future wireless systems. To this context, we investigate the degree of freedom (DoF) in near-field multiple-input multiple-output (MIMO) systems. We consider both spatially discrete (SPD) antennas and continuous aperture (CAP) antennas. Additionally, we explore the relationship between DoF and performance metrics and examine their relationships with the DoF. Numerical results demonstrate the benefits of NFC over far-field communication (FFC) in terms of providing increased spatial DoFs. We also identify promising research directions for NFC from a DoF perspective.

I. Introduction

The electromagnetic (EM) radiation field emitted by antennas is divided into two regions: the far-field and the radiation near-field. The Rayleigh distance, determined by the product of the array aperture's square and the carrier frequency, serves as the boundary between these regions [?]. In the far-field region, beyond the Rayleigh distance, EM waves exhibit different propagation characteristics compared to the near-field region within it. Planar waves effectively approximate the far-field EM field, while the near-field EM field requires precise modeling using spherical waves [?].

Limited by the size of antenna arrays and the operating frequency bands, the Rayleigh distance in current cellular systems typically spans only a few meters, making the near-field effects negligible. Thus, existing cellular communications predominantly rely on theories and techniques from far-field communications (FFC). However, with the rapid advances of wireless technology, next-generation wireless communications rely on extremely large-scale antenna arrays and higher frequencies to cater for the ever-increasing thirst for communication services [?]. In these advanced scenarios, near-field communications (NFC) can extend over longer distances, surpassing the conventional proximity range. The deployment of massive

antenna arrays and the utilization of high-frequency bands allow NFC to be effective at distances of hundreds of meters, thereby opening up novel opportunities for the development of NFC theories and techniques [?], [?].

In the realm of wireless communications, the degree of freedom (DoF) concept has emerged as a crucial framework for understanding the capabilities and potential of different communication systems [?]. Briefly, the DoF provides insights into the number of independent signal dimensions that can be exploited for conveying information in a wireless channel. While traditional FFC have been extensively studied within this context, the unique physical properties of NFC exhibit distinct characteristics that necessitate a fresh exploration of DoF.

The adoption of a DoF perspective in NFC is motivated by several factors. Firstly, NFC offers increased DoFs, which represents a significant advantage over FFC. By understanding the DoF characteristics of NFC systems, we can unveil the superior data capacity and transmission capabilities of NFC compared to FFC. Secondly, characterizing the DoF in NFC assists in optimizing the system parameters, such as the antenna configurations and transmission strategies, leading to improved overall performance. Thirdly, adopting a DoF perspective facilitates the development of communication protocols and algorithms specifically tailored for NFC environments, resulting in enhanced reliability, coverage, and throughput. Although there are some studies analyzing NFC's DoF [?], this field is still in its infancy.

Hence, we aim for the critical appraisal of NFC and its DoF. Our focus is on point-to-point multiple-input multiple-output (MIMO) channels under line-of-sight (LoS) propagation, as illustrated in Fig. ???. This emphasis arises from the anticipation that future NFC will operate at high frequencies, leading to a prevalence of LoS communication associated with limited multi-path effects. We commence by exploring the DoFs achieved in near-field MIMO by spatially discrete antennas (SPD-MIMO). Subsequently, we extend our analysis to the near-field MIMO supported by continuous aperture antennas (CAP-MIMO). Utilizing numerical simulations, we demonstrate the superiority of NFC over FFC concerning its DoF and establish connections between the DoF and effective DoF (EDoF). Finally, future research ideas are discussed.

II. DoFs Achieved in SPD-MIMO

In practical implementations of NFC, a viable approach is to equip the transceiver with an extensive antenna array

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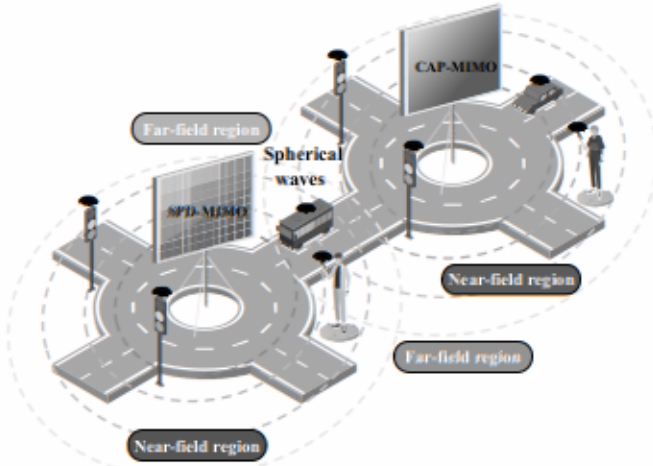


Fig. 1: Illustration of near-field communications.

comprising a large number of SPD patch antennas. In this section, we will delve into a comprehensive analysis of the achievable DoFs in near-field SPD-MIMO.

A. Calculation of the DoF

1) **DoF**: In the context of SPD-MIMO, the overall channel response can be represented as a matrix \mathbf{H} having dimensions of $N_r \times N_t$, where N_r denotes the number of receive antennas and N_t represents the number of transmit antennas. By applying the singular value decomposition (SVD) to this channel matrix, the SPD-MIMO channel can be effectively decomposed into multiple independent single-input single-output (SISO) sub-channels that operate in parallel without mutual interference. Mathematically, the number of positive singular values or the rank of the correlation matrix $\mathbf{H}\mathbf{H}^H$ corresponds to the number of sub-channels having a non-zero signal-to-noise ratio (SNR). Each of these sub-channels accommodates an independent communication mode within the MIMO channel. The total number of communication modes is referred to as the spatial DoF of the channel, denoted as DoF. On the other hand, for a MIMO Gaussian channel, the capacity growth rate can be shown to be $\text{DoF} \cdot \log_2(\text{SNR})$ at high SNR. Therefore, the DoF is also termed as the high-SNR slope or maximum multiplexing gain (relative to a SISO channel) [?].

Given a channel matrix \mathbf{H} , the spatial DoFs are inherently limited and cannot exceed the minimum value between N_r and N_t . In a far-field MIMO LoS channel, only a single incident angle is available due to the almost parallel planar-wave propagation. Consequently, the channel matrix is of rank-1, resulting in a very limited DoF, namely 1. By contrast, within the near-field region, the spherical waves exhibit different phase-shifts and power levels for each link. This diversity leads to a higher rank for the MIMO channel matrix and subsequently a higher DoF compared to the far-field scenario. Notably, if the SPD antennas are well separated, the achievable DoFs for the near-field MIMO LoS channel can approach the minimum value between N_r and N_t . This signifies that

spatial multiplexing can be supported even in the absence of a rich scattering environment, which is a significant advantage of NFC.

2) **EDoF₁**: The aforementioned arguments suggest that employing a high number of antennas constitutes an effective technique of increasing the DoFs in NFC. By reducing the antenna spacing within a fixed aperture size, the number of spatial DoFs can be expanded. It is worth noting that when two antennas are in each other's close proximity, the waves they generate at the receiver antenna array become nearly identical. Consequently, these two antennas become indistinguishable at the receiver. This limitation should be considered as it could restrict the potential increase in channel capacity, when a large number of transceiving antennas are incorporated into a fixed aperture. This limitation has been theoretically demonstrated in [?], [?].

To augment our exposition, we represent the ordered positive singular values of matrix \mathbf{H} as $\sigma_1 \geq \dots \geq \sigma_{\text{DoF}}$. Miller [?] demonstrated by employing prolate spheroidal wave functions that for small values of n , the σ_n values fall off slowly until they reach a critical threshold, beyond which they decay rapidly. This critical threshold is termed as the “effective degree of freedom (EDoF)”, denoted as EDoF₁. Moreover, this phenomenon becomes more prominent as the number of transceiving antennas increases. These findings indicate that although harnessing more antennas can lead to an increased number of independent sub-channels, only the dominant EDoF₁ ones can be effectively utilized for supporting reliable communications.

Furthermore, for a large number of antennas, Miller [?] concludes that the upper limit of EDoF₁ is proportional to the product of transmitter and receiver areas and it is inversely proportional to the link distance. These findings are derived using the uniform spherical wave (USW) model described in [?, Eqn. (35)]. The USW model is applicable in the near-field region, where the communication distance exceeds the uniform-power distance, exhibiting uniform channel gains and non-linear phase-shifts. However, it is important to note that as the link distance becomes comparable to the transceiver sizes (i.e., NFC within the uniform-power distance), the accuracy of the USW model and the EDoF derived in [?] diminishes. To address this, Dardari introduced a more general formula for EDoF₁ based on 2D sampling theory arguments for the non-uniform spherical wave (NUSW) model of [?]. Although this formula may present tractability challenges, again, it reveals that the upper limit of EDoF₁ is proportional to the product of the transmitter and receiver areas, while it is inversely proportional to the link distance. These improvements enhance our understanding of EDoF in NFC systems.

In summary, the conclusions drawn from [?] and [?] suggest that the number of dominant communication modes and channel capacity can be enhanced in two primary means: increasing the aperture size and reducing the communication distance. Remarkably, these strategies align with the commonly employed techniques for

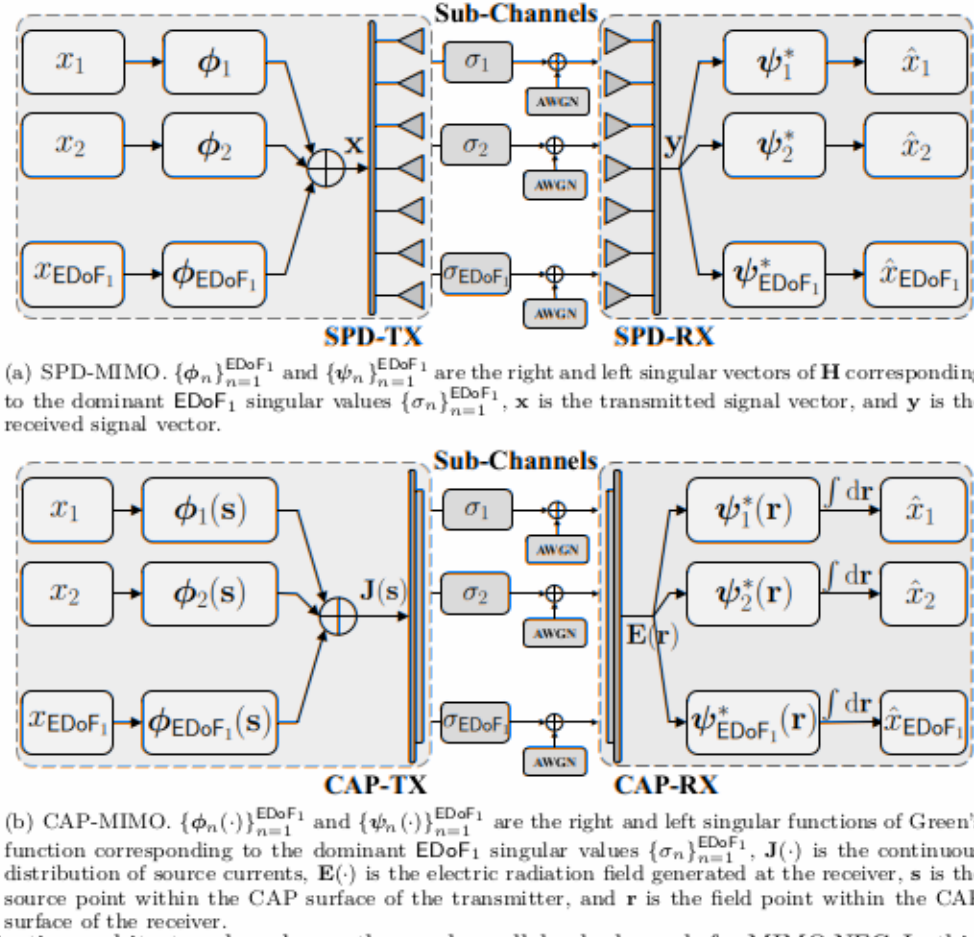


Fig. 2: Communication architecture based on orthogonal parallel sub-channels for MIMO NFC. In this figure, $\{x_n\}_{n=1}^{\text{EDoF}_1}$ are the transmitted symbols, $\{\hat{x}_n\}_{n=1}^{\text{EDoF}_1}$ are the received symbols, $\{\sigma_n\}_{n=1}^{\text{EDoF}_1}$ are the dominant singular values of the MIMO channel (channel matrix or Green's function), and AWGN stands for additive white Gaussian noise.

supporting NFC, emphasizing the superior spatial EDoF capabilities of NFC systems.

B. Exploitation of the DoF

To fully utilize the increased DoFs or EDoFs (i.e., EDoF₁) offered by near-field SPD-MIMO, it is crucial to apply SVD to the channel matrix \mathbf{H} . This allows for the identification of the right and left singular vectors corresponding to the dominant EDoF₁ singular values. To further optimize the achievable channel capacity, the water-filling algorithm can be utilized for judiciously sharing the power among the EDoF₁ parallel sub-channels. Fig. ?? illustrates the detailed architecture that outlines the exploitation of DoF in NFC relying on SPD antennas.

C. Discussion and Outlook

1) MIMO NLoS Channel: The DoF of MIMO NLoS channels is influenced by the geometrical distribution of scatterers. In a rich scattering environment, the MIMO channel can achieve full rank for both the near-field and far-field regions due to the random phase shifts introduced by scatterers. As a result, the achievable DoFs in MIMO LoS channels may approach the minimum value between the numbers of receive and transmit antennas. When a

large number of transceiving antennas are employed, the authors of [?] and [?] have demonstrated by leveraging sampling theory that the upper limit of EDoF₁ is directly proportional to the effective aperture of the transceivers.

For SPD-MIMO, the exact values of DoF and EDoF₁ can be obtained from the SVD of the channel matrix \mathbf{H} for both LoS and NLoS channels. However, obtaining tractable closed-form expressions for these two performance metrics remains challenging. To address this, previous studies have investigated the upper limit of EDoF₁ under various channel conditions by considering the asymptotic scenario of a large number of transceiving antennas [?], [?], [?], [?]. These elegant expressions are derived using Green's function model, which may appear impervious to newcomers, who are experts in other fields. This leads to an important question: Can there be DoF-related metrics that evaluate NFC performance in a non-asymptotic manner in closed form? The answer is affirmative, and the following parts provide the details of these metrics.

2) EDoF₂: Recently, some researchers have introduced an alternative metric to assess NFC performance, also termed as the "effective degree of freedom (EDoF)", which is given by $(\text{tr}(\mathbf{H}\mathbf{H}^H)/\|\mathbf{H}\mathbf{H}^H\|_F)^2$ and denoted as EDoF₂.

TABLE II: Summary of DoF-related metrics for MIMO NFC supported by CAP antennas.

Metric	Degree of Freedom: DoF		Effective Degree of Freedom: EDoF	
	DoF	EDoF ₁	EDoF ₂	EDoF ₃
Definition	[?]	[?]	[?]	[?]
Values Range	$\in \mathbb{Z}^+, [1, \infty)$	$\in \mathbb{Z}^+, [1, \infty)$	$\in \mathbb{R}^+, [1, \infty)$	$\in \mathbb{R}^+, [0, \infty)$
SNR Ranges	High-SNR region	Low&Medium-SNR region	Unknown	All SNR ranges
Relation with Sub-Channels	Number of sub-channels with a non-zero SNR	Number of dominant sub-channels	No direct relation with the number of sub-channels	Number of equivalent sub-channels
CAP-MIMO Far-Field (Calculation)	LoS	1	1	≤ 1
	NLoS	Obtained from solving the eigenvalue problem, ≥ 1	Obtained from solving the eigenvalue problem, ≥ 1	The exact expression is unknown, ≥ 1
CAP-MIMO Near-Field (Calculation)	LoS	Obtained from solving the eigenvalue problem, ≥ 1	$\propto A_t A_r, \propto d^{-2}, \geq 1$, [?], [?]	$\propto d^{-1}, \geq 1$, [?], [?]
	NLoS	Obtained from solving the eigenvalue problem, ≥ 1	$\propto A_t A_r$, [?], [?]	The exact expression is unknown, ≥ 1

** $A_{t/r}$ is the effective aperture size of the transmitter/receiver, d is the link distance between the transmitter and receiver

However, it is essential to acknowledge that there are currently no studies proving that the channel capacity of CAP-MIMO satisfies $\text{EDoF}_2 \cdot [\log_2(\frac{E_b}{N_0}) - \log_2(\frac{E_b}{N_{0,\min}})]$ in the low-SNR regime. As a result, EDoF_2 remains a heuristic concept for CAP-MIMO, lacking precise physical interpretations. Further research is needed to establish a more rigorous and practical understanding of EDoF_2 in the context of CAP-MIMO.

3) EDoF_3 : The concept of EDoF_3 is also applicable to CAP-MIMO. It is evident that EDoF_3 of a far-field LoS channel cannot exceed 1, while for near-field LoS CAP-MIMO, EDoF_3 can be higher than 1. However, it is important to note that due to the lack of closed-form expressions for the channel capacity of CAP-MIMO, calculating the exact value of EDoF_3 for near-field CAP-MIMO becomes intractable [?]. Therefore, further investigations are required to address this aspect and gain a deeper understanding of EDoF_3 in the context of near-field CAP-MIMO.

4) Summary and Outlook: A detailed comparison of DoF, EDoF_1 , EDoF_2 , and EDoF_3 is summarized in Table ???. The results presented in Table ??? primarily pertain to point-to-point CAP-MIMO channels. However, investigating the spatial DoFs introduced by the near-field effect in a multiuser CAP-MIMO setup holds both theoretical and practical significance. As previously mentioned, the practical implementation of near-field CAP-MIMO is computationally intractable. Therefore, it is imperative to explore practical and scalable techniques of CAP-MIMO implementations.

IV. Numerical Results

In this section, we explore the enhanced DoFs and EDoFs offered by MIMO NFC through computer simulations in LoS channel scenarios.

A. SPD-MIMO

Fig. ?? illustrates the DoFs and EDoFs in SPD-MIMO, showcasing the increased DoFs provided by the near-field effect. Specifically, in Fig. ??, we present the singular values of the MIMO channel matrix for different link distances and numbers of antennas. Notably, the DoF of NFC is significantly higher than the value achieved by FFFC, surpassing the singular DoF threshold. As shown,

the singular values exhibit a slow decline until they reach a critical threshold, after which they decrease rapidly. The number of dominant singular values defines the EDoF_1 . From Fig. ??, we can infer that as the number of antennas increases, the singular values, and thus the channel gains of the decomposed sub-channels, experience slight improvements, with EDoF_1 converging rapidly to its upper limit (as calculated in [?]). Additionally, it is noteworthy that a shorter link distance results in a higher EDoF_1 [?], showcasing the superiority of NFC in terms of DoF enhancement.

Fig. ?? presents the plot of EDoF_2 for SPD-MIMO in the near-field. We observe that as the number of antennas increases, EDoF_2 of SPD-MIMO converges to its limit, which is equivalent to EDoF_2 of CAP-MIMO [?]. This convergence occurs more rapidly for higher link distances. Remarkably, as depicted in the graph, SPD-MIMO having half-wavelength antenna spacing can achieve nearly the same EDoF_2 as CAP-MIMO. The results in Fig. ?? indicate that the singular values of our system satisfy $\sigma_1 \approx \dots \approx \sigma_{\text{EDoF}_1} \gg \sigma_{\text{EDoF}_1+1} > \dots > \sigma_{\text{DoF}}$, where EDoF_1 can be approximated by the value of EDoF_2 . This observation aligns with the findings from Fig. ??.

In Fig. ??, we plot EDoF_3 as a function of the SNR. Observe that reducing the link distance enhances EDoF_3 , further validating the use of the near-field effect to improve channel capacity. Additionally, we note that in the high-SNR regime, EDoF_3 can exceed EDoF_1 and EDoF_2 . This phenomenon arises because the non-dominant sub-channels can also support reliable communications, when sufficient transmit power resources are available.

B. CAP-MIMO

Fig. ?? presents an analysis of the DoFs in CAP-MIMO systems. Due to the computational complexity associated with calculating the channel capacity of CAP-MIMO [?], we focus on illustrating EDoF_1 and EDoF_2 in this figure, and the numerical results for EDoF_3 are omitted.

In Fig. ?? and Fig. ??, we showcase EDoF_1 and EDoF_2 as functions of the link distance, respectively. To differentiate between the near-field and far-field regions, we mark the Rayleigh distance in both graphs. The figures demonstrate that both EDoF_1 and EDoF_2 can be enhanced by either increasing the aperture sizes of the transceivers or reducing the link distance. These strategies align with

and the computational challenges in calculating EDoF₃ for CAP-MIMO necessitate further research efforts to derive precise physical interpretations and practical implications for these metrics.

- DoF-Inspired Beamforming Design: Effective beamforming designs are crucial for fully harnessing the increased DoFs offered by NFC. However, the computational and hardware complexities, particularly in the context of CAP-MIMO implementation, pose significant challenges. Therefore, there is a pressing need to explore scalable and computation-hardware efficient beamforming techniques that can exploit the benefits of augmented DoFs in practical NFC scenarios.

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