Near-Field Communications: A Degree-of-Freedom Perspective

Chongjun Ouyang, Yuanwei Liu, Xingqi Zhang, and Lajos Hanzo

***Abstract*—Multiple-antenna technologies are advancing towards large-scale aperture sizes and extremely high frequencies, leading to the emergence of near-field communications (NFC) in future wireless systems. To this context, we investigate the degree of freedom (DoF) in near-field multiple-input multiple-output (MIMO) systems. We consider both spatially discrete (SPD) antennas and continuous aperture (CAP) antennas. Additionally, we explore three important DoF-related performance metrics and examine their relationships with the classic DoF. Numerical results demonstrate the benefits of NFC over far-field communications (FFC) in terms of providing increased spatial DoFs. We also identify promising research directions for NFC from a DoF perspective.**

I. Introduction

The electromagnetic (EM) radiation field emitted by antennas is divided into two regions: the far-field and the radiation near-field. The Rayleigh distance, determined by the product of the array aperture's square and the carrier frequency, serves as the boundary between these regions [?]. In the far-field region, beyond the Rayleigh distance, EM waves exhibit different propagation characteristics compared to the near-field region within it. Planar waves effectively approximate the far-field EM field, while the near-field EM field requires precise modeling using spherical waves [?].

Limited by the size of antenna arrays and the operating frequency bands, the Rayleigh distance in current cellular systems typically spans only a few meters, making the near-field effects negligible. Thus, existing cellular communications predominantly rely on theories and techniques from far-field communications (FFC). However, with the rapid advances of wireless technology, next-generation wireless communications rely on extremely large-scale antenna arrays and higher frequencies to cater for the ever-increasing thirst for communication services [?]. In these advanced scenarios, near-field communications (NFC) can extend over longer distances, surpassing the conventional proximity range. The deployment of massive antenna arrays and the utilization of high-frequency bands allow NFC to be effective at distances of hundreds of meters, thereby opening up novel opportunities for the development of NFC theories and techniques [?], [?].

In the realm of wireless communications, the degree of freedom (DoF) concept has emerged as a crucial framework for understanding the capabilities and potential of different communication systems [?]. Briefly, the DoF provides insights into the number of independent signal dimensions that can be exploited for conveying information in a wireless channel. While traditional FFC have been extensively studied within this context, the unique physical properties of NFC exhibit distinct characteristics that necessitate a fresh exploration of DoF.

The adoption of a DoF perspective in NFC is motivated by several factors. Firstly, NFC offers increased DoFs, which represents a significant advantage over FFC. By understanding the DoF characteristics of NFC systems, we can unveil the superior data capacity and transmission capabilities of NFC compared to FFC. Secondly, characterizing the DoF in NFC assists in optimizing the system parameters, such as the antenna configurations and transmission strategies, leading to improved overall performance. Thirdly, adopting a DoF perspective facilitates the development of communication protocols and algorithms specifically tailored for NFC environments, resulting in enhanced reliability, coverage, and throughput. Although there are some studies analyzing NFC's DoF [?], this field is still in its infancy.

Hence, we aim for the critical appraisal of NFC and its DoF. Our focus is on point-to-point multiple-input multiple-output (MIMO) channels under line-of-sight (LoS) propagation, as illustrated in Fig. ??. This emphasis arises from the anticipation that future NFC will operate at high frequencies, leading to a prevalence of LoS communication associated with limited multi-path effects. We commence by exploring the DoFs achieved in near-field MIMO by spatially discrete antennas (SPD-MIMO). Subsequently, we extend our analysis to the near-field MIMO supported by continuous aperture antennas (CAP-MIMO). Utilizing numerical simulations, we demonstrate the superiority of NFC over FFC concerning its DoF and establish connections between the DoF and effective DoF (EDoF). Finally, future research ideas are discussed.

II. DoFs Achieved in SPD-MIMO

In practical implementations of NFC, a viable approach is to equip the transceiver with an extensive antenna array comprising a large number of SPD patch antennas. In this section, we will delve into a comprehensive analysis of the achievable DoFs in near-field SPD-MIMO.

C. Ouyang is with the School of Information and Communication Engineering, Beijing University of Posts and Telecommunications, Beijing, 100876, China (e-mail: DragonAim@bupt.edu.cn).

Y. Liu is with the School of Electronic Engineering and Computer Science, Queen Mary University of London, London, E1 4NS, U.K. (e-mail: yuanwei.liu@qmul.ac.uk).

Q. Zhang is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, T6G 2H5, Canada (e-mail: xingqi.zhang@ualberta.ca).

L. Hanzo is with the School of Electronics and Computer Science, University of Southampton, Southampton, SO17 1BJ, U.K. (e-mail: lh@ecs.soton.ac.uk).

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Fig. 1: Illustration of near-field communications.

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I. Introduction

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In the realm of wireless communications, the degree of freedom (DoF) concept has emerged as a crucial framework for understanding the capabilities and potential of different communication systems [?]. Briefly, the DoF provides insights into the number of independent signal dimensions that can be exploited for conveying information in a wireless channel. While traditional FFC have been extensively studied within this context, the unique physical properties of NFC exhibit distinct characteristics that necessitate a fresh exploration of DoF.

The adoption of a DoF perspective in NFC is motivated by several factors. Firstly, NFC offers increased DoFs, which represents a significant advantage over FFC. By understanding the DoF characteristics of NFC systems, we can unveil the superior data capacity and transmission capabilities of NFC compared to FFC. Secondly, characterizing the DoF in NFC assists in optimizing the system parameters, such as the antenna configurations and transmission strategies, leading to improved overall performance. Thirdly, adopting a DoF perspective facilitates the development of communication protocols and algorithms specifically tailored for NFC environments, resulting in enhanced reliability, coverage, and throughput. Although there are some studies analyzing NFC's DoF [?], this field is still in its infancy.

Hence, we aim for the critical appraisal of NFC and its DoF. Our focus is on point-to-point multiple-input multiple-output (MIMO) channels under line-of-sight (LoS) propagation, as illustrated in Fig. ??. This emphasis arises from the anticipation that future NFC will operate at high frequencies, leading to a prevalence of LoS communication associated with limited multi-path effects. We commence by exploring the DoFs achieved in near-field MIMO by spatially discrete antennas (SPD-MIMO). Subsequently, we extend our analysis to the near-field MIMO supported by continuous aperture antennas (CAP-MIMO). Utilizing numerical simulations, we demonstrate the superiority of NFC over FFC concerning its DoF and establish connections between the DoF and effective DoF (EDoF). Finally, future research ideas are discussed.

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A. Calculation of the DoF

1) DoF: In the context of SPD-MIMO, the overall channel response can be represented as a matrix $\mathbf{H}$ having dimensions of $N_r \times N_t$, where N_r denotes the number of receive antennas and N_t represents the number of transmit antennas. By applying the singular value decomposition (SVD) to this channel matrix, the SPD-MIMO channel can be effectively decomposed into multiple independent single-input single-output (SISO) sub-channels that operate in parallel without mutual interference. Mathematically, the number of positive singular values or the rank of the correlation matrix $\mathbf{H}\mathbf{H}^H$ corresponds to the number of sub-channels having a non-zero signal-to-noise ratio (SNR). Each of these sub-channels accommodates an independent communication mode within the MIMO channel. The total number of communication modes is referred to as the *spatial DoF* of the channel, denoted as DoF. On the other hand, for a MIMO Gaussian channel, the capacity growth rate can be shown to be $\mathrm{DoF} \cdot \log_2(\mathrm{SNR})$ at high SNR. Therefore, the DoF is also termed as the *high-SNR slope* or maximum *multiplexing gain* (relative to a SISO channel) [?].

Given a channel matrix $\mathbf{H}$, the spatial DoFs are inherently limited and cannot exceed the minimum value between N_r and N_t. In a far-field MIMO LoS channel, only a single incident angle is available due to the almost parallel planar-wave propagation. Consequently, the channel matrix is of rank-1, resulting in a very limited DoF, namely 1. By contrast, within the near-field region, the spherical waves exhibit different phase shifts and power levels for each link. This diversity leads to a higher rank for the MIMO channel matrix and subsequently a higher DoF compared to the far-field scenario. Notably, if the SPD antennas are well separated, the achievable DoFs for the near-field MIMO LoS channel can approach the minimum value between N_r and N_t. *This signifies that spatial multiplexing can be supported even in the absence of a rich scattering environment, which is a significant advantage of NFC.*

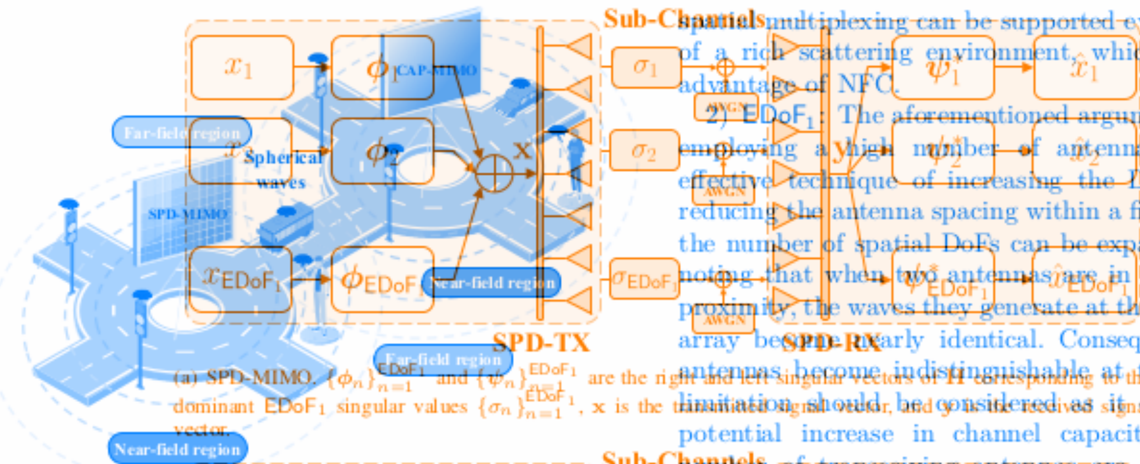
2) EDoF_1: The aforementioned arguments suggest that employing a high number of antennas constitutes an effective technique of increasing the DoFs in NFC. By reducing the antenna spacing within a fixed aperture size, the number of spatial DoFs can be expanded. It is worth noting that when two antennas are in each other's close proximity, the waves they generate at the receiver antenna array become nearly identical. Consequently, these two antennas become indistinguishable at the receiver. This limitation should be considered as it could restrict the potential increase in channel capacity, when a large number of transceiving antennas are incorporated into a fixed aperture. This limitation has been theoretically demonstrated in [?], [?].

[illegible] singular values of matrix $\mathbf{H}$ [illegible] fall off slowly until a critical threshold, beyond which they decay rapidly. [illegible] *effective degree of freedom (EDoF)*, [illegible] EDoF_1 [illegible] Furthermore, for a large number of antennas, Miller [?] concludes that the upper limit of EDoF_1 is proportional to the product of transmitter and receiver areas and it is inversely proportional to the link distance. [illegible] using the uniform spherical wave (USW) model described in [?, Eq. (35)]. [illegible] near-field region, where the communication distance exceeds the uniform-power distance, exhibiting uniform channel gains and [illegible] the link distance becomes comparable to the transceiver sizes [illegible] USW model [illegible] EDoF_1 derived in [?] [illegible] To address this, Dardari introduced a more general formula for EDoF_1 based on the [illegible] uniform spherical wave (NUSW) model of [?]. Although this formula may present tractability challenges, again, it reveals that the upper limit of EDoF_1 is proportional to the product of the transmitter and receiver areas, while it is inversely proportional to the link distance. [illegible] our understanding of EDoF in NFC systems.

In summary, the conclusions drawn from [?] and [?] suggest that the number of dominant communication modes and channel capacity can be enhanced in two primary means: increasing the aperture size and reducing the communication distance. Remarkably, these strategies align with the commonly employed techniques for supporting NFC, emphasizing the superior spatial EDoF capabilities of NFC systems.

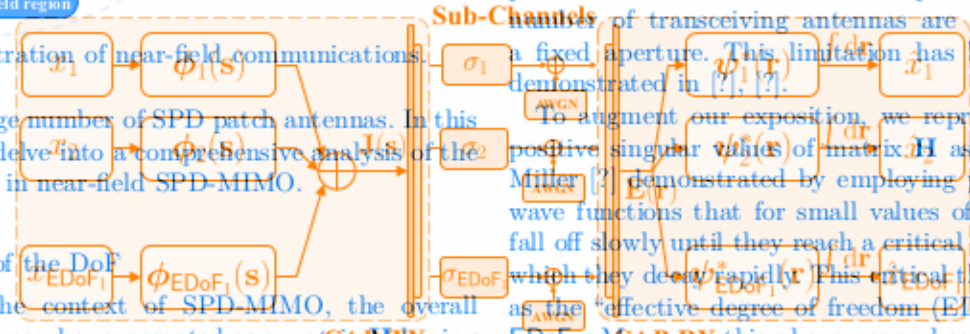
B. Exploitation of the DoF

To fully utilize the increased DoFs or EDoFs (i.e., EDoF_1) offered by near-field SPD-MIMO, it is crucial to apply SVD to the channel matrix $\mathbf{H}$. This allows for the identification of the right [illegible]



(a) SPD-MIMO. $\{\phi_n\}_{n=1}^{\mathsf{EDoF}_1}$ and $\{\psi_n\}_{n=1}^{\mathsf{EDoF}_1}$ are the right and left singular vectors of $\mathbf{H}$ corresponding to the dominant EDoF_1 singular values $\{\sigma_n\}_{n=1}^{\mathsf{EDoF}_1}$, $\mathbf{x}$ is the transmitted signal vector, and $\mathbf{y}$ is the received signal vector.

Fig. 1: Illustration of near-field communications.



(b) CAP-MIMO. $\{\phi_n(\cdot)\}_{n=1}^{\mathsf{EDoF}_1}$ and $\{\psi_n(\cdot)\}_{n=1}^{\mathsf{EDoF}_1}$ are the right and left singular functions of Green's function corresponding to the dominant EDoF_1 singular values $\{\sigma_n\}_{n=1}^{\mathsf{EDoF}_1}$, $\mathbf{E}(\mathbf{r})$ is the electric radiation field generated at the receiver, $\mathbf{s}$ is the source point within the CAP surface of the transmitter, and $\mathbf{r}$ is the field point within the CAP surface of the receiver.

Fig. 2: Communication architecture based on orthogonal parallel sub-channels for MIMO NFC. In this figure, $\{x_n\}_{n=1}^{\mathsf{EDoF}_1}$ are the transmitted symbols, $\{\hat{x}_n\}_{n=1}^{\mathsf{EDoF}_1}$ are the received symbols, $\{\sigma_n\}_{n=1}^{\mathsf{EDoF}_1}$ are the dominant singular values of the MIMO channel (channel matrix or Green's function), and AWGN stands for additive white Gaussian noise.

comprising a large number of SPD patch antennas. In this section, we will delve into a comprehensive analysis of the achievable DoFs in near-field SPD-MIMO.

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spatial multiplexing can be supported even in the absence of a rich scattering environment, which is a significant advantage of NFC.

2) EDoF_1: The aforementioned arguments suggest that employing a high number of antennas constitutes an effective technique of increasing the DoFs in NFC. By reducing the antenna spacing within a fixed aperture size, the number of spatial DoFs can be expanded. It is worth noting that when two antennas are in each other's close proximity, the waves they generate at the receiver antenna array become nearly identical. Consequently, these two antennas become indistinguishable at the receiver. This limitation should be considered as it could restrict the potential increase in channel capacity, when a large number of transceiving antennas are incorporated into a fixed aperture. This limitation has been theoretically demonstrated in [?], [?].

To augment our exposition, we represent the ordered positive singular values of matrix $\mathbf{H}$ as $\sigma_1 \geq \ldots \geq \sigma_{\mathsf{DoF}}$. Miller [?] demonstrated by employing prolate spheroidal wave functions that for small values of n, the σ_n values fall off slowly until they reach a critical threshold, beyond which they decay rapidly. This critical threshold is termed as the "effective degree of freedom (EDoF)", denoted as EDoF_1. Moreover, this phenomenon becomes more prominent as the number of transceiving antennas increases. These findings indicate that although harnessing more antennas can lead to an increased number of independent sub-channels, only the dominant EDoF_1 ones can be effectively utilized for supporting reliable communications.

Furthermore, for a large number of antennas, Miller [?] concludes that the upper limit of EDoF_1 is proportional to the product of transmitter and receiver areas and it is inversely proportional to the link distance. These findings are derived using the uniform spherical wave (USW) model described in [?, Eqn. (35)]. The USW model is applicable in the near-field region, where the communication distance exceeds the uniform-power distance, exhibiting uniform channel gains and non-linear phase-shifts. However, it is important to note that as the link distance becomes comparable to the transceiver sizes (i.e., NFC within the uniform-power distance), the accuracy of the USW model and the EDoF derived in [?] diminishes. To address this, Dardari introduced a more general formula for EDoF_1 [illegible]

channel capacity, the water-filling algorithm can be utilized for judiciously sharing the power among the EDoF_1 parallel sub-channels. Fig. ?? illustrates the detailed architecture that outlines the exploitation of DoF in NFC relying on SPD antennas.

By considering the asymptotic scenario of a large number of transceiving antennas [?], [?], [?], [?]. These elegant expressions are derived using Green's function model, which may appear impervious to newcomers, who are experts in other fields. This leads to an important question: *Can there be DoF-related metrics that evaluate NFC performance in a non-asymptotic manner in closed form?* The answer is affirmative, and the following parts provide the details of these metrics.

C. Discussion and Outlook

[illegible]

TABLE I: Summary of DoF-related metrics for MIMO NFC supported by SPD antennas.

Metric		Degree of Freedom: DoF	Effective Degree of Freedom: EDoF		
			EDoF_1	EDoF_2	EDoF_3
Definition		[?]	[?]	[?]	[?]
Values Range		$\in \mathbb{Z}^+$, $[1, N_{\min}]$	$\in \mathbb{Z}^+$, $[1, N_{\min}]$	$\in \mathbb{R}^+$, $[1, N_{\min}]$	$\in \mathbb{R}^+$, $(0, N_{\min}]$
SNR Ranges		High-SNR region	Low&Medium-SNR region	Low-SNR region	All SNR ranges
Relation with Sub-Channels		Number of sub-channels with a non-zero SNR	Number of dominant sub-channels	No direct relation with the number of sub-channels	Number of equivalent sub-channels
SPD-MIMO Fae-Field (Calculation)	LoS	1	[illegible]	1	≤ 1
	NLoS	Rank of $\mathbf{HH}^{\mathrm{H}}$, ≥ 1; Upper bound: $N_{\min}$	Obtained from SVD of $\mathbf{H}$, ≥ 1; Upper bound: $N_{\min}$	$(\mathrm{tr}(\mathbf{HH}^{\mathrm{H}})/\|\mathbf{HH}^{\mathrm{H}}\|_{\mathrm{F}})^2$ [?], ≥ 1; Upper bound: $N_{\min}$	$\frac{\mathrm{d}}{\mathrm{d}\delta}C(\mathsf{SNR}\cdot 2^{\delta})\big\|_{\delta=0}$ [?], $\leq N_{\min}$; Upper bound: $N_{\min}$
SPD-MIMO Near-Field (Calculation)	LoS	Rank of $\mathbf{HH}^{\mathrm{H}}$, ≥ 1; Upper bound: $N_{\min}$	Obtained from SVD of $\mathbf{H}$, ≥ 1; Upper limit: $\propto A_{\mathrm{t}}A_{\mathrm{r}}$, $\propto d$ [?]	$(\mathrm{tr}(\mathbf{HH}^{\mathrm{H}})/\|\mathbf{HH}^{\mathrm{H}}\|_{\mathrm{F}})^2$ [?, Eqn. (8)], ≥ 1; Upper limit: [?, Eqn. (8)], $\propto d$	$\frac{\mathrm{d}}{\mathrm{d}\delta}C(\mathsf{SNR}\cdot 2^{\delta})\big\|_{\delta=0}$, $\leq N_{\min}$; Upper bound: $N_{\min}$
	NLoS	Rank of $\mathbf{HH}^{\mathrm{H}}$, ≥ 1; Upper bound: $N_{\min}$	Obtained from SVD of $\mathbf{H}$, ≥ 1; Upper limit: $\propto A_{\mathrm{t}}A_{\mathrm{r}}$, [?], [?]	$(\mathrm{tr}(\mathbf{HH}^{\mathrm{H}})/\|\mathbf{HH}^{\mathrm{H}}\|_{\mathrm{F}})^2$, ≥ 1; Upper bound: $N_{\min}$	$\frac{\mathrm{d}}{\mathrm{d}\delta}C(\mathsf{SNR}\cdot 2^{\delta})\big\|_{\delta=0}$, $\leq N_{\min}$; Upper bound: $N_{\min}$

* $A_{\mathrm{t/r}}$ is the effective aperture size of the transmitter/receiver

** $N_{\min}$ is the minimum value between N_{r} and N_{t}

*** d is the link distance between the transmitter and receiver.

sub-channels, as depicted in Fig. ??, and can be employed for evaluating the NFC performance [?]. However, it is crucial to note that these statements lack mathematical rigor and may lead to misinterpretations of the actual meaning and implications of EDoF_2.

The concept of EDoF_2 was originally introduced by Muharemovic *et al.* [?], who built upon Verdú's previous work [?] to approximate the MIMO channel capacity as $\mathsf{EDoF}_2 \cdot [\log_2(\frac{E_b}{N_0}) - \log_2(\frac{E_b}{N_0}_{\min})]$ in the low-SNR regime. Here, $\frac{E_b}{N_0}$ represents the bit energy over noise power spectral density, and $\frac{E_b}{N_0}_{\min}$ is the minimum value required for reliable communications. Additionally, $\frac{E_b}{N_0}$ is determined by the product of the channel capacity and the SNR [?, Eqn. (14)]. By considering the insights gleaned from [?] and [?], it becomes evident that EDoF_2 possesses a distinct physical interpretation when compared to EDoF_1 and DoF. Generally, the value of EDoF_2 is not directly associated with the number of dominant sub-channels depicted in Fig. ??. However, an exception occurs when the dominant sub-channels have nearly identical channel gains, i.e., $\sigma_1 \approx \cdots \approx \sigma_{\mathsf{EDoF}_1} \gg \sigma_{\mathsf{EDoF}_1+1} > \cdots > \sigma_{\mathsf{DoF}}$. In such cases, EDoF_1 can be approximately represented by the value of EDoF_2. Our numerical results in Section ?? suggest that this scenario can happen in certain LoS channels. Nonetheless, this approximation remains heuristic, and its generality lacks mathematical rigor.

To summarize, EDoF_2 serves as a significant performance metric for NFC in the low-SNR region, yet it cannot be simply interpreted as the equivalent number of sub-channels. Its significance and interpretation are different from those of EDoF_1 and DoF. Hence, it is important to discern its distinct role in NFC.

3) EDoF_3: To fully harness the spatial DoFs offered by NFC MIMO, it is desirable to operate the system in the high-SNR region. In such scenarios, the channel capacity should exhibit roughly linear growth vs. the DoF or EDoF_1, given a fixed transmit power. [illegible]

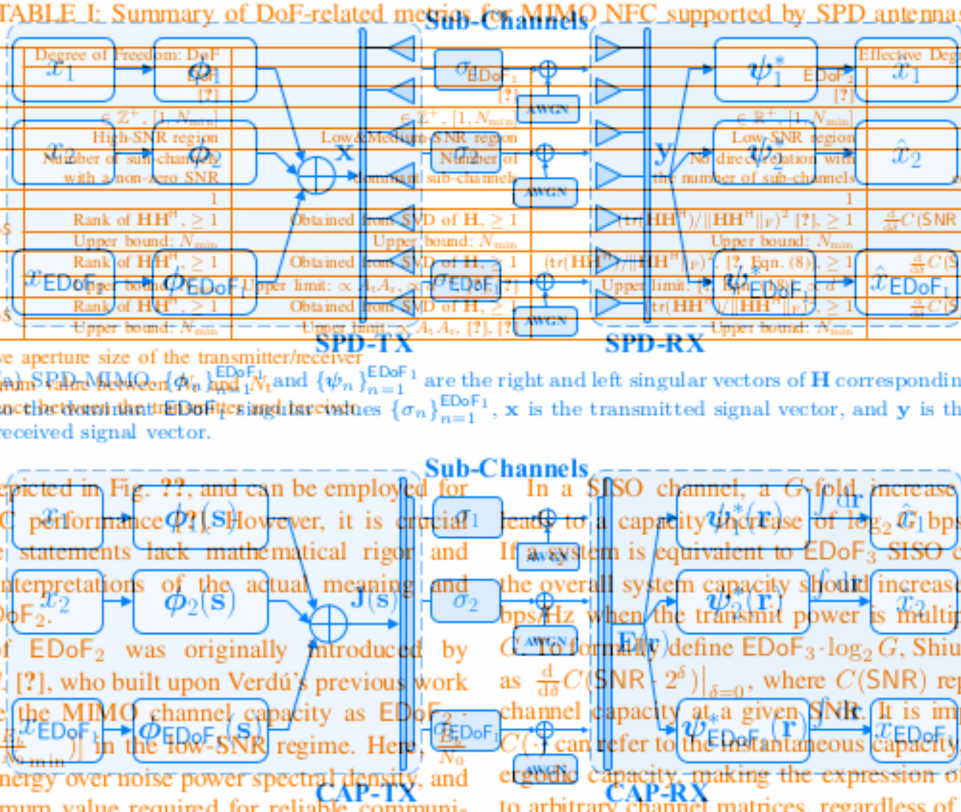
In a SISO channel, a G-fold increase in transmit power leads to a capacity increase of $\log_2 G$ bps/Hz at high SNRs. If a system is equivalent to EDoF_3 SISO channels in parallel, the overall system capacity should increase by $\mathsf{EDoF}_3 \cdot \log_2 G$ bps/Hz when the transmit power is multiplied by a factor of G. To formally define $\mathsf{EDoF}_3 \cdot \log_2 G$, Shiu *et al.* [?] express it as $\frac{\mathrm{d}}{\mathrm{d}\delta}C(\mathsf{SNR}\cdot 2^{\delta})\big|_{\delta=0}$, where $C(\mathsf{SNR})$ represents the MIMO channel capacity at a given SNR. It is important to note that $C(\cdot)$ can refer to the instantaneous capacity, outage capacity, or ergodic capacity, making the expression of EDoF_3 applicable to arbitrary channel matrices, regardless of whether the system operates in the near- or far-field regions, and under LoS or NLoS propagations. Let us consider the LoS channel as an example. In far-field LoS MIMO, the channel matrix has a rank of 1, leading to EDoF_3 being no larger than 1. Conversely, for near-field LoS MIMO, EDoF_3 could exceed 1 [?]. Observe from this comparison that the near-field effect can improve EDoF_3.

Essentially, EDoF_3 describes the number of equivalent SISO sub-channels at a given SNR, making it a valuable performance indicator for NFC in different SNR scenarios.

4) Summary and Outlook: A detailed comparison among DoF, EDoF_1, EDoF_2, and EDoF_3 is summarized in Table ??. Taken together, these four DoF-related metrics possess different physical meanings and scopes of application. As such, they should be appropriately utilized based on the practical demands of NFC. While the existing results have been primarily focused on EDoF_1, it is crucial to develop a comprehensive mathematical framework for calculating the upper limits of EDoF_2 and EDoF_3 under both LoS and NLoS scenarios. This avenue represents a potential direction for future research.

III. DoFs Achieved in CAP-MIMO

Utilizing CAP antennas presents a promising technique of improving the performance of MIMO systems having limited apertures. In contrast to SPD-MIMOs, which involve a large number of [illegible]



(a) SPD-MIMO. $\{\phi_n\}_{n=1}^{\mathsf{EDoF}_1}$ and $\{\psi_n\}_{n=1}^{\mathsf{EDoF}_1}$ are the right and left singular vectors of $\mathbf{H}$ corresponding to the dominant EDoF_1 singular values $\{\sigma_n\}_{n=1}^{\mathsf{EDoF}_1}$, $\mathbf{x}$ is the transmitted signal vector, and $\mathbf{y}$ is the received signal vector.

(b) CAP-MIMO. $\{\phi_n(\cdot)\}_{n=1}^{\mathsf{EDoF}_1}$ and $\{\psi_n(\cdot)\}_{n=1}^{\mathsf{EDoF}_1}$ are the right and left singular functions of Green's function corresponding to the dominant EDoF_1 singular values $\{\sigma_n\}_{n=1}^{\mathsf{EDoF}_1}$, $\mathbf{J}(\cdot)$ is the continuous distribution of source currents, $\mathbf{E}(\cdot)$ is the electric radiation field generated at the receiver, $\mathbf{s}$ is the source point within the CAP surface of the transmitter, and $\mathbf{r}$ is the field point within the CAP surface of the receiver.

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B. Exploitation of the DoF

To fully utilize the increased DoFs or EDoFs (i.e., EDoF_1) offered by near-field SPD-MIMO, it is crucial to apply SVD to the channel matrix $\mathbf{H}$. This allows for the identification of the right and left singular vectors corresponding to the dominant EDoF_1 singular values. To further optimize the achievable channel capacity, the water-filling algorithm can be utilized for judiciously sharing the power among the EDoF_1 parallel sub-channels. Fig. ?? illustrates the detailed architecture that outlines the exploitation of DoF in NFC relying on SPD antennas.

C. Discussion and Outlook

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large number of transceiving antennas are employed, the authors of [?] and [?] have demonstrated by leveraging sampling theory that the upper limit of EDoF_1 is directly proportional to the effective aperture of the transceivers.

For SPD-MIMO, the exact values of DoF and EDoF_1 can be obtained from the SVD of the channel matrix $\mathbf{H}$ for both LoS and NLoS channels. However, obtaining tractable closed-form expressions for these two performance metrics remains challenging. To address this, previous studies have investigated the upper limit of EDoF_1 under various channel conditions by considering the asymptotic scenario of a large number of transceiving antennas [?], [?], [?], [?]. These elegant expressions are derived using Green's function model, which may appear impervious to newcomers, who are experts in other fields. This leads to an important question: Can there be DoF-related metrics that evaluate NFC performance in a non-asymptotic manner in closed form? The answer is affirmative, and the following parts provide the details of these metrics.

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Relation with Sub-Channels		Number of sub-channels with a non-zero SNR	Number of dominant sub-channels	No direct relation with the number of sub-channels	Number of equivalent sub-channels
SPD-MIMO Far-Field (Calculation)	LoS	1	1	1	≤ 1
	NLoS	Rank of $\mathbf{H}\mathbf{H}^H$, ≥ 1; Upper bound: $N_{\min}$	Obtained from SVD of $\mathbf{H}$, ≥ 1; Upper bound: $N_{\min}$	$(\mathrm{tr}(\mathbf{H}\mathbf{H}^H)/\|\mathbf{H}\mathbf{H}^H\|_F)^2$, [?, Eqn. (8)], ≥ 1; Upper bound: $N_{\min}$	$\frac{d}{d\delta}C(\mathsf{SNR}\cdot 2^{\delta})\|_{\delta=0}$, $\leq N_{\min}$; Upper bound: $N_{\min}$
SPD-MIMO Near-Field (Calculation)	LoS	Rank of $\mathbf{H}\mathbf{H}^H$, ≥ 1; Upper bound: $N_{\min}$	Obtained from SVD of $\mathbf{H}$, ≥ 1; Upper limit: $\propto A_t A_r \propto d^{-2}$, [?], [?]	$(\mathrm{tr}(\mathbf{H}\mathbf{H}^H)/\|\mathbf{H}\mathbf{H}^H\|_F)^2$, [?, Eqn. (8)], ≥ 1; Upper bound: $N_{\min}$	$\frac{d}{d\delta}C(\mathsf{SNR}\cdot 2^{\delta})\|_{\delta=0}$, $\leq N_{\min}$; Upper bound: $N_{\min}$
	NLoS	Rank of $\mathbf{H}\mathbf{H}^H$, ≥ 1; Upper bound: $N_{\min}$	Obtained from SVD of $\mathbf{H}$, ≥ 1; Upper limit: $\propto A_t A_r$, [?], [?]	$(\mathrm{tr}(\mathbf{H}\mathbf{H}^H)/\|\mathbf{H}\mathbf{H}^H\|_F)^2$, ≥ 1; Upper bound: $N_{\min}$	$\frac{d}{d\delta}C(\mathsf{SNR}\cdot 2^{\delta})\|_{\delta=0}$, $\leq N_{\min}$; Upper bound: $N_{\min}$

* $A_{t/r}$ is the effective aperture size of the transmitter/receiver.
** $N_{\min}$ is the minimum value between N_t and N_r.
*** d is the link distance between the transmitter and receiver.

A. Calculation of the DoF

We consider a scenario where both the transmitter and receiver are equipped with CAP antennas, which is analogous to the MIMO setup for SPD antennas. However, in contrast to the SPD antenna array that delivers finite-dimensional signal vectors, the CAP surface supports a continuous distribution of source currents within the transmitting aperture, giving rise to the generation of an electric radiation field at the receiver aperture. The spatial channel impulse response between any two points on the transceiving surfaces is described by Green's function, which connects the transmitter's current distribution and the receiver's electric field via a spatial integral. Green's function accurately models the EM characteristics in free space and effectively represents the channel response between the transceivers, akin to the channel matrix for SPD-MIMO.

1) DoF: Based on the above considerations, the spatial CAP-MIMO channel can be decomposed into a series of parallel SISO sub-channels by finding the equivalent "SVD" of Green's function [?, Eqn. (27)]. The resultant equivalent "left singular vectors" and "right singular vectors" form two complete sets of orthogonal basis functions, one for the transmitter's aperture and the other for the receiver's aperture. The resultant equivalent "singular values" correspond to the channel gains of the decomposed sub-channels. Alternatively, these "singular values" can be obtained through the eigenvalue decomposition of the Hermitian kernel of Green's function (analogous to the correlation matrix $\mathbf{H}\mathbf{H}^H$ for SPD antennas); see [?, Eqn. (42)] and [?, Section II-C] for more details. The number of non-zero "singular values" of Green's function, or equivalently, the non-zero eigenvalues of its kernel, is defined as the DoF, denoted as DoF. The DoF also signifies the number of SISO sub-channels at a non-zero SNR, each of which supports an independent communication mode within the entire system.

As noted in [?], the far-field LoS CAP-MIMO can support a maximum of one communication mode. Consequently, the DoF of far-field LoS CAP-MIMO is limited to 1. However, in the case of near-field LoS MIMO, the DoF has the potential to approach infinity due to the associated spherical wave propagation [?]. Therefore, we may conclude that the near-field effect significantly enhances the spatial DoFs for CAP-MIMO.

2) EDoF_1: The near-field CAP-MIMO system has the remarkable ability to support infinitely many communication modes. However, it is crucial to recognize that only those modes having significant channel gains can be effectively utilized to convey information. The total number of these effective communication modes is known as the EDoF, i.e., EDoF_1. Several methods have been proposed to determine or approximate the value of EDoF_1, such as analyzing the eigenvalues of the kernel of Green's function [?], employing sampling theory [?], [?], [?], utilizing diffraction theory [?], or leveraging Landau's theorem [?].

Prior research has demonstrated that for near-field LoS CAP-MIMO, the value of EDoF_1 is directly proportional to the product of the transmitter and receiver aperture sizes and inversely proportional to the link distance [?], [?]. On the other hand, for far-field LoS CAP-MIMO, the value of EDoF_1 is limited to 1. These findings highlight the superiority of NFC in terms of enhancing the spatial DoFs.

B. Exploitation of the DoF

To fully exploit the increased EDoFs offered by near-field CAP-MIMO, it becomes essential to determine the left and right singular functions of Green's function and their associated singular values. This task involves solving the eigenvalue problem for the Hermitian kernel [?]. A potential architecture for the CAP-MIMO is illustrated in Fig. ??, which closely resembles that of SPD-MIMO. However, it is important to acknowledge that the computational complexity associated with solving the eigenvalue problem for CAP-MIMO is significantly higher than that for SPD-MIMO. Additionally, the architecture depicted in Fig. ?? requires the use of infinitely many radio-frequency chains.

C. Discussion and Outlook

1) MIMO NLoS Channel: The DoFs of near-field CAP-MIMO have also been investigated in the context of NLoS propagation. In [?] and [?], the authors explored various scattering environments and utilized sampling theory to analyze the EDoF. Their findings revealed that EDoF_1 of NLoS CAP-MIMO is higher than 1 in both the near-field and far-field regions. Moreover, they demonstrated that increasing the effective aperture of the transceivers can lead to further improvements of EDoF_1.

2) EDoF_2: The concept of EDoF_2 has been extended to CAP-MIMO channels upon replacing the channel matrix by Green's function [?, Eqn. (8)]. Closed-form formulas of EDoF_2 have been derived for near-field CAP-MIMO [?], [?], specifically for the LoS channel. The analysis reveals that while EDoF_2 of FFC is limited to 1, EDoF_2 of NFC is inversely proportional to the link distance. These findings underscore the advantage of NFC in terms of EDoF_2. However, it is essential to acknowledge that there are currently no studies proving that the channel capacity of CAP-MIMO satisfies $\mathsf{EDoF}_2 \cdot [\log_2(\frac{E_b}{N_0}) - \log_2(\frac{E_b}{N_0}_{\min})]$ in the low-SNR regime. As a result, EDoF_2 remains a heuristic concept for CAP-MIMO, lacking precise physical interpretations. Further research is needed to establish a more rigorous and practical understanding of EDoF_2 in the context of CAP-MIMO.

3) EDoF_3: The concept of EDoF_3 is also applicable to CAP-MIMO. It is evident that EDoF_3 of a far-field LoS channel cannot exceed 1, while for near-field LoS CAP-MIMO, EDoF_3 can be higher than 1. However, it is important to note that due to the lack of closed-form expressions for the channel capacity of CAP-MIMO, calculating the exact value of EDoF_3 for near-field CAP-MIMO becomes intractable [?]. Therefore, further investigations are required to address this aspect and gain a deeper understanding of EDoF_3 in the context of CAP-MIMO.

4) Summary and Outlook: A detailed comparison among DoF, EDoF_1, EDoF_2, and EDoF_3 is summarized in Table ??. Taken together, these four DoF-related metrics possess different physical meanings and scopes of application. As such, they should be appropriately utilized based on the practical demands of NFC. While the existing results have been primarily focused on EDoF_1, it is crucial to develop a comprehensive mathematical framework for calculating the upper limits of EDoF_2 and EDoF_3 under both LoS and NLoS scenarios. This avenue represents a potential direction for future research.

TABLE II: Summary of DoF-related metrics for MIMO NFC supported by CAP antennas.

Metric		DoF	EDoF_1	EDoF_2	EDoF_3
Definition		[?]	[?]	[?]	[?]
Value Range		[illegible]	$\in \mathbb{Z}^+$ [illegible]	[illegible]	[illegible]
SNR Ranges		High-SNR region	Low&Medium-SNR region	Unknown	All SNR ranges
Relation with Sub-Channels		Number of sub-channels with a non-zero SNR	Number of dominant sub-channels	No direct relation with the number of sub-channels	Number of equivalent sub-channels
CAP-MIMO Far-Field (Calculation)	LoS	1	1	1	≤ 1
	NLoS	Obtained from solving the eigenvalue problem, ≥ 1	Obtained from solving the eigenvalue problem, ≥ 1	The exact expression is unknown, ≥ 1	$\frac{d}{da}C(\mathsf{SNR}\cdot 2^a)\|_{a=0}$
CAP-MIMO Near-Field (Calculation)	LoS	Obtained from solving the eigenvalue problem, ≥ 1	$\propto A_t A_r$, $\propto d^{-2}$, ≥ 1 [?], [?]	$\propto d^{-1}$, ≥ 1 [?], [?]	$\frac{d}{da}C(\mathsf{SNR}\cdot 2^a)\|_{a=0}$
	NLoS	Obtained from solving the eigenvalue problem, ≥ 1	$\propto A_t A_r$, [?], [?]	The exact expression is unknown, ≥ 1	$\frac{d}{da}C(\mathsf{SNR}\cdot 2^a)\|_{a=0}$

$A_{t/r}$ is the effective aperture of the transmitter/receiver; d is the link distance between the transmitter and receiver

III. DoFs Achieved in CAP-MIMO

Utilizing CAP antennas presents a promising technique of improving the performance of MIMO systems having limited apertures. In contrast to SPD-MIMOs, which involve a large number of discrete antennas having specific spacing, CAP-MIMO adopts an infinite number of antennas with infinitesimal spacing. This section investigates the spatial DoFs in near-field CAP-MIMO.

A. Calculation of the DoF

We consider a scenario where both the transmitter and receiver are equipped with CAP antennas, which is analogous to the MIMO setup for SPD antennas. However, in contrast to the SPD antenna array that delivers finite-dimensional signal vectors, the CAP surface supports a continuous distribution of source currents within the transmitting aperture, giving rise to the generation of an electric radiation field at the receiver aperture. The spatial channel impulse response between any two points on the transceiving surfaces is described by Green's function, which connects the transmitter's current distribution and the receiver's electric field via a spatial integral. Green's function accurately models the EM characteristics in free space and effectively represents the channel response between the transceivers, akin to the channel matrix for SPD-MIMOs.

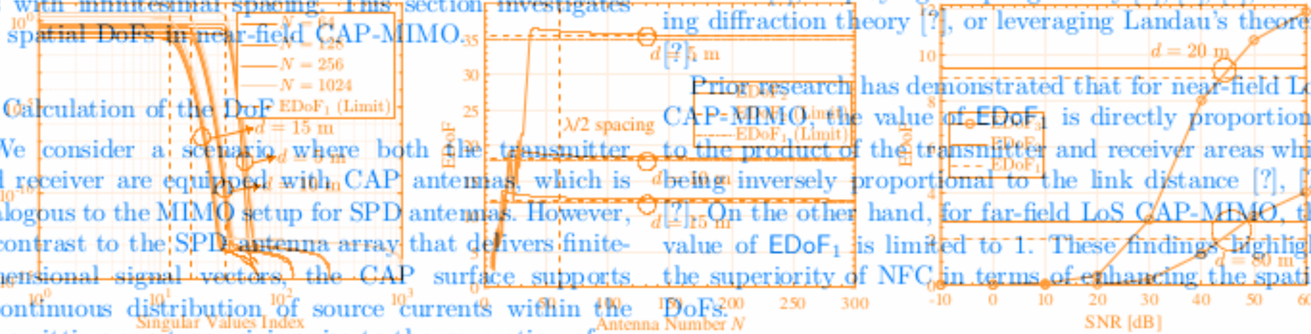
1) DoF: Based on the above considerations, the spatial CAP-MIMO channel can be decomposed into a series of parallel SISO sub-channels by finding the equivalent "SVD" of Green's function [?, Eqn. (27)]. The resultant equivalent "left singular vectors" and "right singular vectors" form two complete sets of orthogonal basis functions, one for the transmitter's aperture and the other for the receiver's aperture. The resultant equivalent "singular values" correspond to the channel gains of the decomposed sub-channels. Alternatively, these "singular values" can be obtained through the eigenvalue decomposition of the Hermitian kernel of Green's function (analogous to the correlation matrix $\mathbf{H}\mathbf{H}^{\mathsf{H}}$ for SPD antennas), see [?, Eqn. (32)] and [?, Section II-C] for more details. [illegible]

[illegible]

the associated spherical wave propagation [?]. Therefore, we may conclude that the near-field effect significantly enhances the spatial DoFs for CAP-MIMO.

2) EDoF_1: The near-field CAP-MIMO system has the remarkable ability to support infinitely many communication modes. However, it is crucial to recognize that only those modes having significant channel gains can be effectively utilized to convey information. The total number of these effective communication modes is known as the EDoF, i.e., EDoF_1. Several methods have been proposed to determine or approximate the value of EDoF_1, such as analyzing the eigenvalues of the kernel of Green's function [?], employing sampling theory [?], [?], [?], utilizing diffraction theory [?], or leveraging Landau's theorem [?].

Prior research has demonstrated that for near-field LoS CAP-MIMO, the value of EDoF_1 is directly proportional to the product of the transmitter and receiver areas while being inversely proportional to the link distance [?], [?], [?]. On the other hand, for far-field LoS CAP-MIMO, the value of EDoF_1 is limited to 1. These findings highlight the superiority of NFC in terms of enhancing the spatial DoFs.



(a) EDoF_1. (b) EDoF_2. (c) EDoF_3.

Fig. 5: Illustration of EDoFs in SPD-MIMO LoS channels, where the transmitter and receiver are equipped with uniform linear arrays (ULAs), each containing N antennas, and the system operates at a frequency of 28 GHz (with a corresponding wavelength of $\lambda = 1$ cm). The ULAs have an aperture size of 1.37 m. The center of the transmitter is located at the origin of a three-dimensional plane, while the center of the receiver is at $(0, d, 0)$ with d denoting the link distance. The ULAs face each other and are parallel to the z-axis.

B. Exploitation of the DoF

To fully exploit the increased EDoFs offered by near-field CAP-MIMO, it becomes essential to determine the left and right singular functions of Green's function and their associated singular values. This task involves solving the eigenvalue problem for the Hermitian kernel [?]. A potential architecture for the CAP-MIMO is illustrated in Fig. ??, which closely resembles that of SPD-MIMO. However, it is important to acknowledge that the computational complexity associated with solving the eigenvalue problem for CAP-MIMO is significantly higher than that for SPD-MIMO. Additionally, the architecture depicted in Fig. ?? requires the use of infinitely many radio-frequency chains.

C. Discussion and Outlook

1) MIMO NLoS Channel: The DoFs of near-field CAP-MIMO have also been investigated in the context of NLoS propagation. In [?] and [?], the authors explored various scattering environments and utilized sampling theory to analyze the EDoF. Their findings revealed that EDoF_1 of NLoS CAP-MIMO is higher than 1 in both the near-field and far-field regions. Moreover, they demonstrated that increasing the effective aperture of the transceivers can lead to further improvements of EDoF_1.

2) EDoF_2: The concept of EDoF_2 has been extended to CAP-MIMO channels upon replacing the channel matrix by Green's function [?, Eqn. (8)]. Closed-form formulas of EDoF_2 have been derived for near-field CAP-MIMO [?], [?], specifically for the LoS channel. The analysis reveals that while EDoF_2 of far-field LoS CAP-MIMO is limited to 1, EDoF_2 of [illegible]

[illegible]

DoFs introduced by the near-field effect in a multiuser CAP-MIMO setup holds both theoretical and practical significance. As previously mentioned, the practical implementation of near-field CAP-MIMO is computationally intractable. Therefore, it is imperative to explore practical and scalable techniques of CAP-MIMO implementations.

IV. Numerical Results

In this section, we explore the enhanced DoFs and EDoFs offered by MIMO NFC through computer simulations in LoS channel scenarios.

A. SPD-MIMO

Fig. ?? illustrates the DoFs and EDoFs in SPD-MIMO, [illegible]

[illegible] that a shorter link distance results in a higher EDoF_1 [?], showcasing the superiority of NFC in terms of DoF enhancement.

Fig. ?? presents the plot of EDoF_2 for SPD-MIMO in the near-field. We observe that as the number of antennas increases, EDoF_2 of SPD-MIMO converges to its limit, which is equivalent to EDoF_2 of CAP-MIMO [?]. This convergence occurs more rapidly for higher link distances. Remarkably, as depicted in the graph, SPD-MIMO having half-wavelength antenna spacing can achieve nearly the same EDoF_2 as CAP-MIMO. The results in Fig. ?? indicate that the singular values of our system satisfy $\sigma_1 \approx \ldots \approx \sigma_{\mathsf{EDoF}_1} \gg \sigma_{\mathsf{EDoF}_1+1} > \ldots > \sigma_{\mathsf{DoF}}$, where EDoF_1 can be approximated by the value of EDoF_2. This observation aligns with the findings from Fig. ??.

In Fig. ??, we plot EDoF_3 as a function of the SNR. Observe that reducing the link distance enhances EDoF_3, further validating the use of the near-field effect to improve channel capacity. Additionally, we note that in the high-SNR regime, EDoF_3 can exceed EDoF_1 and EDoF_2. This phenomenon arises because the non-dominant sub-channels can also support reliable communications, when sufficient transmit power resources are available.

B. CAP-MIMO

Fig. ?? [illegible]

TABLE II: Summary of DoF-related metrics for MIMO NFC supported by CAP antennas.

Metric		Degree of Freedom: DoF	Effective Degree of Freedom: EDoF		
		DoF	EDoF_1	EDoF_2	EDoF_3
Definition		[?]	[?]	[?]	[?]
Values Range		$\in \mathbb{Z}^+$, $[1, \infty)$	$\in \mathbb{Z}^+$, $[1, \infty)$	$\in \mathbb{R}^+$, $[1, \infty)$	$\in \mathbb{R}^+$, $(0, \infty]$
SNR Ranges		High-SNR region	Low&Medium-SNR region	Unknown	All SNR ranges
Relation with Sub-Channels		Number of sub-channels with a non-zero SNR	Number of dominant sub-channels	No direct relation with the number of sub-channels	Number of equivalent sub-channels
CAP-MIMO Far-Field (Calculation)	LoS	1	1	1	≤ 1
	NLoS	Obtained from solving the eigenvalue problem, ≥ 1	Obtained from solving the eigenvalue problem, ≥ 1	The exact expression is unknown, ≥ 1	$\frac{d}{da}C(\mathsf{SNR}\cdot 2^a)\big\vert_{a=0}$
CAP-MIMO Near-Field (Calculation)	LoS	Obtained from solving the eigenvalue problem, ≥ 1	$\propto A_t A_r$, $\propto d^{-2}$, ≥ 1, [?], [?]	$\propto d^{-1}$, ≥ 1, [?], [?]	$\frac{d}{da}C(\mathsf{SNR}\cdot 2^a)\big\vert_{a=0}$
	NLoS	Obtained from solving the eigenvalue problem, ≥ 1	$\propto A_t A_r$, [?], [?]	The exact expression is unknown, ≥ 1	$\frac{d}{da}C(\mathsf{SNR}\cdot 2^a)\big\vert_{a=0}$

** $A_{t/r}$ is the effective aperture size of the transmitter/receiver, d is the link distance between the transmitter and receiver

However, it is essential to acknowledge that there are currently no studies proving that the channel capacity of CAP-MIMO satisfies $\mathsf{EDoF}_2 = [\log_2(\frac{E_b}{N_0}) - \log_2(\frac{E_b}{N_0}_{\min})]$ in the low-SNR regime. As a result, EDoF_2 remains a heuristic concept for CAP-MIMO, lacking precise physical interpretations. Further research is needed to establish a more rigorous and practical understanding of EDoF_2 in the context of CAP-MIMO.

3) EDoF_3: The concept of EDoF_3 is also applicable to CAP-MIMO. It is evident that EDoF_3 of a far-field LoS channel cannot exceed 1, while for near-field LoS CAP-MIMO, EDoF_3 can be higher than 1. However, it is important to note that due to the lack of closed-form expressions for the channel capacity of CAP-MIMO, calculating the exact value of EDoF_3 for near-field CAP-MIMO becomes intractable [?]. Therefore, further investigations are required to address this aspect and gain a deeper understanding of EDoF_3 in the context of near-field CAP-MIMO.

4) Summary and Outlook: A detailed comparison of DoFs, EDoF_1, EDoF_2, and EDoF_3 is summarized in Table ??. The results presented in Table ?? primarily pertain to point-to-point CAP-MIMO channels. However, investigating the spatial DoFs introduced by the near-field effect in a multiuser CAP-MIMO setup holds both theoretical and practical significance. As previously mentioned, the practical implementation of near-field CAP-MIMO is computationally intractable. Therefore, it is imperative to explore practical and scalable techniques for CAP-MIMO implementations.

IV. Numerical Results

In this section, we explore the enhanced DoFs and EDoFs offered by MIMO NFC through computer simulations in LoS channel scenarios.

A. SPD-MIMO

Fig. ?? illustrates the DoFs and EDoFs in SPD-MIMO, showcasing the increased DoFs provided by the near-field effect. Specifically, in Fig. ??, we present the singular values of the MIMO channel matrix for different link distances and numbers of antennas. Notably, the DoF of NFC is significantly higher than the value achieved by FFC, surpassing the single DoF threshold. As shown, the singular values exhibit a slow decline until they reach a critical threshold, after which they decrease rapidly. The number of dominant singular values defines the EDoF_1. From Fig. ??, we can infer that as the number of antennas increases, the singular values and thus the channel gains of the decomposed sub-channels experience slight improvements, with EDoF_1 converging rapidly to its upper limit (as calculated in [?]). Additionally, it is noteworthy that a shorter link distance results in a higher EDoF_1 [?], showcasing the superiority of NFC in terms of DoF enhancement.

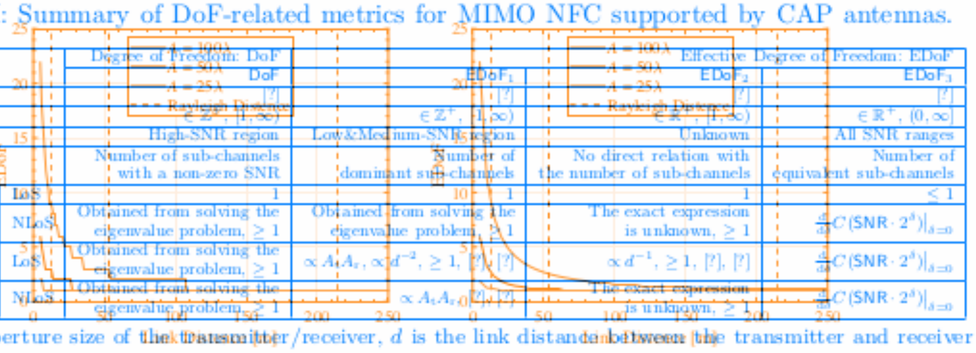
Fig. ?? presents the plot of EDoF_2 for SPD-MIMO in the near-field. We observe that as the number of antennas increases, EDoF_2 of SPD-MIMO converges to its limit, which is equivalent to EDoF_2 of CAP-MIMO [?]. This convergence occurs more rapidly for higher link distances. Remarkably, as depicted in the graph, SPD-MIMO having half-wavelength antenna spacing can achieve nearly the same EDoF_2 as CAP-MIMO. The results in Fig. ?? indicate that the singular values of our system satisfy $\sigma_1 \approx \ldots \approx \sigma_{\mathsf{EDoF}_1} \gg \sigma_{\mathsf{EDoF}_1+1} \geq \ldots \geq \sigma_{\mathsf{DoF}}$, where EDoF_1 can be approximated by the value of EDoF_2. This observation aligns with the findings from Fig. ??.

In Fig. ??, we plot EDoF_3 as a function of the SNR. Observe that reducing the link distance enhances EDoF_3, further validating the use of the near-field effect to improve channel capacity. Additionally, we note that in the high-SNR regime, EDoF_3 can exceed EDoF_1 and EDoF_2. This phenomenon arises because the non-dominant sub-channels can also support reliable communications, when sufficient transmit power resources are available.

B. CAP-MIMO

Fig. ?? presents an analysis of the DoFs in CAP-MIMO systems. Due to the computational complexity associated with calculating the channel capacity of CAP-MIMO [?], we focus on illustrating EDoF_1 and EDoF_2 in this figure, and the numerical results for EDoF_3 are omitted.

In Fig. ?? and Fig. ??, we showcase EDoF_1 and EDoF_2 as functions of the link distance, respectively. To differentiate between the near-field and far-field regions, we mark the Rayleigh distance in both graphs. The figures demonstrate that both EDoF_1 and EDoF_2 can be enhanced by either increasing the aperture sizes of the transceivers or reducing the link distance. These strategies align with



(a) EDoF_1.
(b) EDoF_2.

Fig. 4: Illustration of EDoFs in CAP-MIMO LoS channels, where the transmitter and receiver are equipped with continuous linear arrays of the same aperture size A, and the system operates at a frequency of 28 GHz (with a corresponding wavelength of $\lambda = 1$ cm). The center of the transmitter is located at the origin of a three-dimensional plane, while the center of the receiver is at $(0, d, 0)$ with d denoting the link distance. The linear arrays face each other and are parallel to the z-axis.

commonly employed techniques for supporting NFC. A notable observation from the comparison of Fig. ?? and Fig. ?? is that the curves for EDoF_1 follow similar trends to those of EDoF_2, corroborating the findings from Fig. ??. The numerical results presented in Fig. ?? and Fig. ?? collectively underscore the substantial impact of near-field effects on augmenting the DoFs and EDoFs. These findings contribute valuable insights to the understanding and design of NFC techniques.

V. Conclusions and Promising Research Directions

In this article, we have conducted an in-depth investigation into the performance of MIMO NFC from a DoF perspective. We began by elucidating the spatial DoFs achievable in near-field SPD-MIMO and exploring how these increased DoFs can be exploited for enhancing the channel capacity. Next, we analyzed and compared three DoF-related performance metrics, namely EDoF_1, EDoF_2, and EDoF_3, to their far-field counterparts to demonstrate the superiority of NFC in terms of spatial multiplexing and channel capacity. To further explore the potential of MIMO NFC, we extended these results to CAP-MIMO to determine the upper limit of performance. We have deepened the understanding of the augmented spatial DoFs offered by the near-field effect, with the hope of inspiring further innovations in this field. There are still numerous open research problems in this area, which are summarized from three aspects:

- DoF-Based Information-Theoretic Limits: The DoF is a significant information-theoretic metric for [illegible] channel capacity. [illegible] essential insights for system design. Additionally, the pursuit of capacity-approaching transmission schemes for NFC from a DoF perspective represents a valuable endeavor.
- DoF-Based Performance Analysis: Although our analysis has concentrated on point-to-point MIMO NFC, extending our investigations to multiuser scenarios holds the potential of offering valuable insights into the spatial DoFs in more complex communication setups, presenting a promising avenue for future research. Additionally, the heuristic nature of EDoF_2 and the computational challenges in calculating EDoF_3 for CAP-MIMO necessitate further research efforts to derive precise physical interpretations and practical implications for these metrics.
- DoF-Inspired Beamforming Design: Effective beamforming designs are crucial for fully harnessing the increased DoFs offered by NFC. However, the computational and hardware complexities, particularly in the context of CAP-MIMO implementation, pose significant challenges. Therefore, there is a pressing need to explore scalable and computation/hardware-efficient beamforming techniques that can exploit the benefits of augmented DoFs in practical NFC scenarios.

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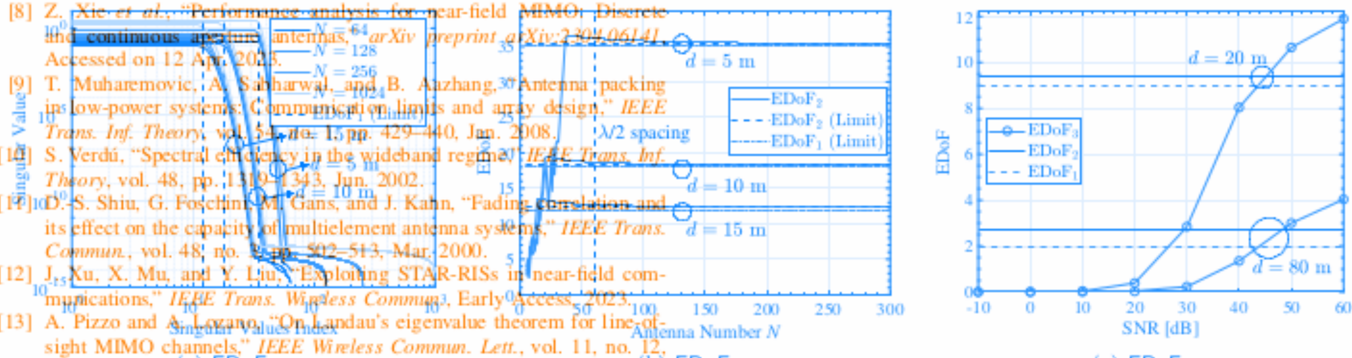
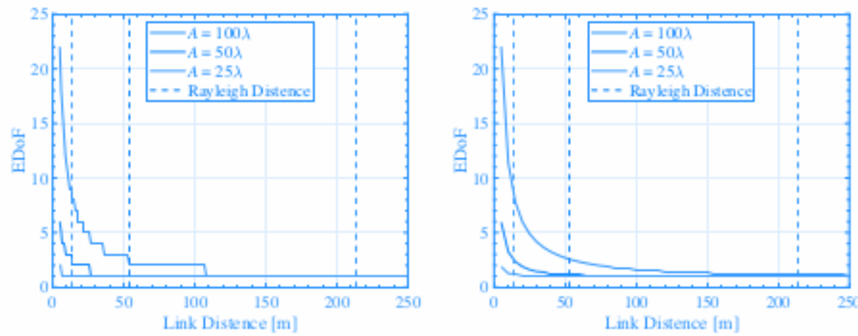


(a) EDoF_1. (b) EDoF_2. (c) EDoF_3.

Fig. 3: Illustration of EDoFs in SPD-MIMO LoS channels, where both transmitter and receiver are equipped with uniform linear arrays (ULAs), each containing N antennas, and the system operates at a frequency of 28 GHz (with a corresponding wavelength of $\lambda = 1$ cm). The ULAs have an aperture size of 1.37 m. The center of the transmitter is located at the origin of a three-dimensional plane, while the center of the receiver is at $(0, d, 0)$ with d denoting the link distance. The ULAs face each other and are parallel to the z-axis.



(a) EDoF_1. (b) EDoF_2.

Fig. 4: Illustration of EDoFs in CAP-MIMO LoS channels, where the transmitter and receiver are equipped with continuous linear arrays of the same aperture size A, and the system operates at a frequency of 28 GHz (with a corresponding wavelength of $\lambda = 1$ cm). The center of the transmitter is located at the origin of a three-dimensional plane, while the center of the receiver is at $(0, d, 0)$ with d denoting the link distance. The linear arrays face each other and are parallel to the z-axis.

commonly employed techniques for supporting NFC. A notable observation from the comparison of Fig. ?? and Fig. ?? is that the curves for EDoF_1 follow similar trends to those of EDoF_2, corroborating the findings from Fig. ??.

The numerical results presented in Fig. ?? and Fig. ?? collectively underscore the substantial impact of near-field effects on augmenting the DoFs in MIMO systems. These findings contribute valuable insights to the understanding and design of NFC technologies.

V. Conclusion and Promising Research Directions

In this article, we have conducted an in-depth investigation into the performance of MIMO NFC from a DoF perspective. We began by elucidating the spatial DoFs achievable in near-field SPD-MIMO and exploring how these increased DoFs can be exploited for enhancing the channel capacity. Next, we analyzed and compared three DoF-related performance metrics, namely EDoF_1, EDoF_2, and EDoF_3, to their far-field counterparts for demonstrating the superiority of NFC in terms of spatial multiplexing and channel capacity. To further explore the potential of MIMO NFC, we extended these results to CAP-MIMO to determine the upper limit of performance. We have deepened the understanding of the augmented spatial DoFs offered by the near-field effect, with the hope of inspiring further innovations in this field. There are still numerous open research problems in this area, which are summarized from three aspects.

- DoF-Based Information-Theoretic Limits: The DoF is a significant information-theoretic measure directly related to channel capacity. Exploring the DoF to characterize the fundamental information-theoretic limits of NFC, including deriving the achievable DoF region, can provide essential insights for system design. Additionally, the pursuit of capacity-approaching transmission schemes for NFC from a DoF perspective represents a valuable endeavor.
- DoF-Based Performance Analysis: Although our analysis has concentrated on point-to-point MIMO NFC, extending our investigations to multiuser scenarios holds the potential of offering valuable insights into the spatial DoFs in more complex communication setups, presenting a promising avenue for future research. Additionally, the heuristic nature of EDoF_2