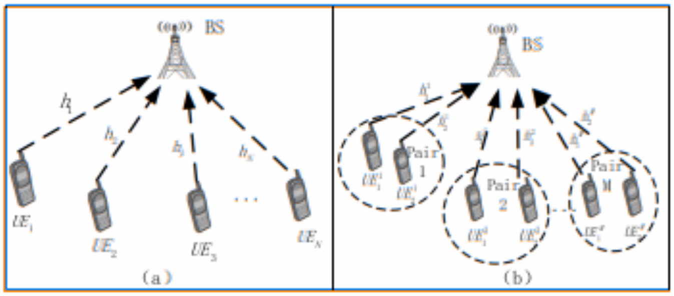

Fig. 1. (a) Uplink RSMA without user pairing. (b) Uplink RSMA with user pairing.

movement status of all platooning vehicles before making control decisions, which needs to minimize the maximum latency for all users. In addition, the complexity of existing algorithms should be further reduced.

In this letter, we optimize the uplink RSMA with the objective of minimizing the maximum latency. Since the optimal decoding order is difficult to find, user pairing is employed to reduce the computational complexity. Then the optimization variables are changed from decoding order and power allocation to bandwidth allocation and power allocation, by making use of the fact that the optimal decoding order of two users can be determined. We derive the closed-form expressions for power and bandwidth allocation, respectively, and the optimal solution of the optimization problem is obtained by bisection method. Finally, the effectiveness of the algorithm is verified by simulation.

II. System Model and Problem Formulation

Without loss of generality, this letter considers a single input single output (SISO) uplink RSMA transmission as shown in Fig. ??, which contains a base station (BS) and N users. The channel gain from the user n to BS is denoted by h_n. Using RSMA uplink transmission, the information x_n of user n is divided into x_{n1} and x_{n2}. The message received by BS can be expressed as $y_{BS} = \sum_{n=1}^{N}\sum_{j=1}^{2}\sqrt{h_n}\cdot x_{nj} + n_0$, where n_0 denotes additive Gaussian white noise. After SIC is performed at BS, the rate of x_{nj} can be expressed as $r_{nj} = B\log_2(1+\frac{h_n p_{nj}}{\sum_{\{(n'\in N, j'\in J)|\pi_{n'j'}>\pi_{nj}\}} h_{n'}p_{n'j'}+\sigma^2 B})$, where B is the bandwidth, p_{nj} is the transmission power allocated to x_{nj}, π_{nj} is the decoding order of x_{nj}, and σ^2 is the power spectral density of the Gaussian noise. So the transmission rate of user n can be expressed as $r_n = \sum_{j=1}^{2} r_{nj}$, and the transmission latency can be write as $t_n = PL_n/r_n$, where PL_n is the package length of user n. In some automatic control systems, the BS needs to obtain information from all users before making control decisions, so we build the following optimization problem to minimize the maximum transmission latency.

$$\begin{aligned} \boldsymbol{P1}: \quad & \min_{\boldsymbol{\pi p}} \max \quad t_n && (1)\\ s.t. \quad & p_{nj} > 0, 1 \le n \le N, j \in [1,2]; && (1a)\\ & \boldsymbol{\pi} \in \boldsymbol{\Pi}; && (1b)\\ & \sum_{j=1}^{2} p_{nj} \le P_n^{max}, 1 \le n \le N. && (1c) \end{aligned}$$

where $\boldsymbol{p} = [p_{11}, p_{12}, p_{21}, ..., p_{n2}]$, $\boldsymbol{\Pi}$ denotes all possible orders of decoding and P_n^{max} is the maximum transmission power of user n.

Using τ to denote the upper bound of latency for all users, we have $t_n \le \tau, 1 \le n \le N$, Then problem $\boldsymbol{P1}$ can be transformed into:

$$\begin{aligned} \boldsymbol{P2}: \quad & \min_{\boldsymbol{\pi p}} \tau && (2)\\ s.t. \quad & r_n \ge \frac{PL_n}{\tau}, 1 \le n \le N, j \in [1,2]; && (2a)\\ & p_{nj} > 0, 1 \le n \le N, j \in [1,2]; && (2b)\\ & \boldsymbol{\pi} \in \Pi; && (2c)\\ & \sum_{j=1}^{2} p_{nj} \le P_n^{max}, 1 \le n \le N; && (2d) \end{aligned}$$

The decoding order $\boldsymbol{\pi}$ in problem $\boldsymbol{P2}$ is a discrete variable, and the condition (2a) is in the form of the sum of two logarithmic functions. We can exhaust $\boldsymbol{\pi}$ and use the successive convex approximation (SCA) algorithm to transform condition (2a) into a convex form to find the approximate solution. However, the decoding order set $\boldsymbol{\Pi}$ contains $(2N)!/2^N$ elements, which means that the complexity of the algorithm using the exhaustive method will become extremely high as the number of users increases. Existing studies have determined the optimal decoding order of RSMA transmissions for two users [?], so we consider pairing every two users to reduce the complexity of the algorithm.

As show in part (b) of Fig. ??, suppose all users are paired into M pairs, and each pair contains two users. According to the simulation test, since the change of channel gain is much larger than package length, the performance of pairing mainly depends on the channel gains, so this pairing is performed according to the order of channel gains [?]. The channel gain of the k-th user in the m-th pair to the BS is denoted by $h_k^m, k \in [1,2], 1 \le m \le M$. Research [?] shows that uplink RSMA transmission of two users can achieve all rate regions by splitting the information of only one user. Without loss of generality, suppose the message of the first user in each pair is split into two parts x_{11}^m and x_{12}^m , and the message of the second user is not split, the optimal decoding order at the BS is x_{11}^m, x_2^m, x_{12}^m.

The following is an analysis for the m-th pair of users. the rate of x_{11}^m, x_2^m and x_{12}^m can be expressed respectively as:

$$r_{11}^m = B\alpha_m \log_2(1+\frac{h_1^m p_{11}^m}{h_2^m p_2^m + h_1^m p_{12}^m + \sigma^2 B\alpha_m}), \quad (3)$$

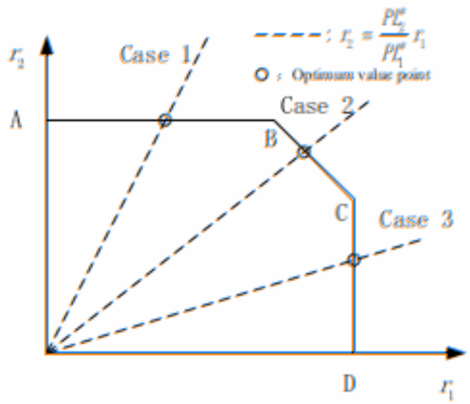

Fig. 2. Rate region of r_1 and r_2 in the same pair when use RSMA

$$r_2^m = B\alpha_m \log_2(1 + \frac{h_2^m p_2^m}{h_1^m p_{12}^m + \sigma^2 B\alpha_m}), \tag{4}$$

$$r_{12}^m = B\alpha_m \log_2(1 + \frac{h_1^m p_{12}^m}{\sigma^2 B\alpha_m}), \tag{5}$$

where α_m is the bandwidth allocation factor with $\sum_{m=1}^{M} \alpha_m \leq 1$, p_{11}^m, p_2^m and p_{12}^m are the power allocated to x_{11}^m, x_2^m and x_{12}^m respectively. So the rate of user 1 in m-th pair is $r_1^m = r_{11}^m + r_{12}^m$ and the transmission latency of user k in the m-th pair is $t_k^m = PL_k^m / r_K^m$, where PL_k^m is the package length of user k in the m-th pair.

The optimization of the power and decoding order in $\boldsymbol{P2}$ can be translated into the optimization of the bandwidth and power allocation in $\boldsymbol{P3}$ as follows:

$$\boldsymbol{P3}: \quad \min_{\boldsymbol{\alpha p}} \quad \tau \tag{6}$$

$$s.t. \quad r_k^m \geq \frac{PL_k^m}{\tau}, 1 \leq m \leq M, k \in [1, 2]; \tag{6a}$$

$$p_{kj}^m > 0, 1 \leq m \leq M, k, j \in [1, 2]; \tag{6b}$$

$$\sum_{m=1}^{M} \alpha_m \leq 1; \tag{6c}$$

$$\sum_{j=1}^{2} p_{kj}^m \leq P_{kmax}^m, 1 \leq m \leq M, k, j \in [1, 2]; \tag{6d}$$

III. RESOURCE ALLOCATION ALGORITHM FOR PAIRED RSMA

Since the expression of transmission rate is a monotonically increasing function of power and bandwidth, we can use contradiction to prove that the optimal solution of $\boldsymbol{P3}$ is obtained when and only when all users have the same latency, i.e., $t_1^1 : t_2^1 : t_1^2 : ... : t_2^M = 1 : 1 : 1 : ... : 1$, in other words, the ratio of optimized rates is $r_1^1 : r_2^1 : r_1^2 : ... : r_2^M = PL_1^1 : PL_2^1 : PL_1^2 : ... : PL_2^M$. Thus, the rates of two users in the same pair have $r_2^m = \frac{PL_2^m}{PL_1^m} r_1^m$.

Fig. ?? shows the achievable rate region for two users using RSMA transmission, when the decoding order is x_{11}^m, x_2^m and x_{12}^m, all points on the rate region can be reached:

For the line AB, the power allocation is:

$$0 \leq p_{11}^m \leq P_{1max}^m, \quad p_{12}^m = 0, \quad p_2^m = P_{2max}^m, \tag{7}$$

User 2 has the maximum rate as:

$$r_2^m = B\alpha_m \log_2(1 + \frac{h_2^m P_{2max}^m}{\sigma^2 B\alpha_m}). \tag{8}$$

For the line BC, the power allocation is:

$$p_{11}^m + p_{12}^m = P_{1max}^m, \quad p_2^m = P_{2max}^m, \tag{9}$$

The sum rate of the two user is:

$$r_1^m + r_2^m = B\alpha_m \log_2(1 + \frac{h_1^m P_{1max}^m + h_2^m P_{2max}^m}{\sigma^2 B\alpha_m}). \tag{10}$$

For the line CD, the power allocation is:

$$p_{11}^m = 0, \quad p_{12}^m = P_{1max}^m, \quad 0 \leq p_2^m \leq P_{2max}^m, \tag{11}$$

User 1 has the maximum rate as :

$$r_1^m = B\alpha_m \log_2(1 + \frac{h_1^m P_{1max}^m}{\sigma^2 B\alpha_m}). \tag{12}$$

According to the above analysis, the optimal power allocation of problem $\boldsymbol{P3}$ is the intersection of line $r_2^m = \frac{PL_2^m}{PL_1^m} r_1^m$ and the rate region. As shown in Fig. ??, the intersection point may exist three cases.

Case 1: The intersection point is on AB, and according to (??), (??), (??) and (6a), we have:

$$B\alpha_m^{AB} \log_2(1 + \frac{h_2^m P_{2max}^m}{\sigma^2 B\alpha_m^{AB}}) = \frac{PL_2^m}{\tau}, \tag{13}$$

$$p_{11}^m = \frac{(2^{\frac{PL_1^m}{\tau B\alpha_m^{AB}}} - 1)(h_2^m P_2^m max + \sigma^2 B\alpha_m^{AB})}{h_1^m}. \tag{14}$$

Case 2: The intersection point is on BC, and according to (??), (??), (??) and (6a):

$$B\alpha_m^{BC} \log_2(1 + \frac{h_1^m P_{1max}^m + h_2^m P_{2max}^m}{\sigma^2 B\alpha_m^{BC}}) = \frac{PL_1^m + PL_2^m}{\tau}, \tag{15}$$

$$p_{12}^m = \frac{h_2^m P_{2max}^m}{h_1^m (2^{\frac{PL_2^m}{\tau B\alpha_m^{BC}}} - 1)} - \frac{\sigma^2 B\alpha_m^{BC}}{h_1^m}, \tag{16}$$

The r_{12}^m can be obtained by taking p_{12}^m into (??), then $r_{11}^m = PL_1^m / \tau - r_{12}^m$, and according to (??) we can obtain p_{11}^m:

$$p_{11}^m = \frac{(2^{\frac{r_{11}^m}{B\alpha_m^{BC}}} - 1)(h_2^m P_{2max}^m + h_1^m p_{12}^m + \sigma^2 B\alpha_m^{BC})}{h_1^m}. \tag{17}$$

Case 3: The intersection point is on CD, and according to (??), (??) and (6a):

$$B\alpha_m^{CD} \log_2(1 + \frac{h_1^m P_{1max}^m}{\sigma^2 B\alpha_m^{CD}}) = \frac{PL_1^m}{\tau}, \tag{18}$$

$$p_2^m = \frac{(2^{\frac{PL_2^m}{\tau B\alpha_m^{CD}}} - 1)(h_1^m P_1^m max + \sigma^2 B\alpha_m^{CD})}{h_2^m}. \tag{19}$$

According to (??), (??) and (??), the expression of α_m^{AB},α_m^{BC} and α_m^{CD} are given in (??), (??) and (??). Where $W(\cdot)$ is the Lambert-W function which satisfies $W(xe^x) = x$. Note that the Lambert-W function has multiple solutions here, and the appropriate solution should be chosen.

$$\alpha_m^{AB} = \frac{-\ln 2 h_2^m P_{2max}^m PL_2^m}{\tau B h_2^m P_{2max}^m W(\frac{-\ln 2 PL_2^m \sigma^2}{\tau h_2^m P_{2max}^m} e^{\frac{-\ln 2 PL_2^m \sigma^2}{\tau h_2^m P_{2max}^m}}) + \ln 2 PL_2^m \sigma^2 B}, \tag{20}$$

$$\alpha_m^{BC} = \frac{-\ln 2 (h_1^m P_{1max}^m + h_2^m P_{2max}^m)(PL_1^m + PL_2^m)}{\tau B (h_1^m P_{1max}^m + h_2^m P_{2max}^m) W(\frac{-\ln 2 (PL_1^m + PL_2^m)\sigma^2}{\tau (h_1^m P_{1max}^m + h_2^m P_{2max}^m)} e^{\frac{-\ln 2 (PL_1^m + PL_2^m)\sigma^2}{\tau (h_1^m P_{1max}^m + h_2^m P_{2max}^m)}}) + \ln 2 (PL_1^m + PL_2^m)\sigma^2 B}, \tag{21}$$

$$\alpha_m^{CD} = \frac{-\ln 2 h_1^m P_{1max}^m PL_1^m}{\tau B h_1^m P_{1max}^m W(\frac{-\ln 2 PL_1^m \sigma^2}{\tau h_1^m P_{1max}^m} e^{\frac{-\ln 2 PL_1^m \sigma^2}{\tau h_1^m P_{1max}^m}}) + \ln 2 PL_1^m \sigma^2 B}, \tag{22}$$

With the closed-form expressions for bandwidth allocation and power allocation, we can solve the problem ***P3*** by the bisection method. For each given τ, a set of α_m^{AB}, α_m^{BC} and α_m^{CD} can be calculated according to (??), (??) and (??), and the power allocation corresponding to each case can be obtained by (??), (??), (??) and (??), and then the condition (6d) is used to judge which case occurs. The specific algorithm is shown in Algorithm 1.

Algorithm 1 User pairing-based power allocation algorithm.

1: Initialize upper and lower bound τ_{ub}, τ_{lb}, tolerance ε.
2: **while** $\tau_{ub} - \tau_{lb} > \varepsilon$ **do**
3: set $\tau = \frac{\tau_{ub}+\tau_{lb}}{2}$
4: **for** m=1:M **do**
5: calculate α_m^{AB}, α_m^{BC} and α_m^{CD} respectively according to (??), (??) and (??).
6: Calculate the power allocation for each case according to (??), (??), (??) and (??) respectively.
7: **if** The power allocation of case 1 satisfies (6d) **then**
8: $\alpha_m = \alpha_m^{AB}$.
9: **else if** The power allocation of case 2 satisfies (6d) **then**
10: $\alpha_m = \alpha_m^{BC}$.
11: **else if** The power allocation of case 3 satisfies (6d) **then**
12: $\alpha_m = \alpha_m^{CD}$.
13: **else**
14: set $\tau_{lb} = \tau$, break and jump to step 2.
15: **end if**
16: **end for**
17: **if** $\sum_{m=1}^{M} \alpha_m \leq 1$ **then**
18: set $\tau_{ub} = \tau$.
19: **else**
20: set $\tau_{lb} = \tau$.
21: **end if**
22: **end while**
23: Output τ, α_m, p_{11}^m, p_{12}^m, and p_2^m.

The complexity of Algorithm 1 in each iteration is to check which case satisfies the power constraint (6d), which introduces the complexity of $\mathcal{O}(M)$ according to (??), (??), (??) and (??)-(??). In addition the complexity of the bisection method with accuracy ε is $\mathcal{O}(\log_2(1/\varepsilon))$ [?], so the total complexity of Algorithm 1 is $\mathcal{O}(M \log_2(1/\varepsilon))$.

IV. SIMULATION RESULT AND DISCUSSIONS

We simulate the proposed algorithm in this section. N users are uniformly distributed within a radius of 200 m from the BS, and the path loss model is $128.1+37.6\log_{10} d$ (d is in km). The bandwidth is set to 1 MHz, and the noise power spectral density is $\sigma^2 = -174$ dBm/Hz [?]. Each user has the same power limit P_{max} and randomly generates a packet of 50-1200 bytes. The algorithms for the comparison include unpaired RSMA [?], paired NOMA [?] and unpaired NOMA. All simulation results were performed on an Intel Core i9-13900KF CPU @ 5.8 GHz and 32G RAM using MATLAB R2023a.

Fig. ?? shows the transmission latency of each scheme with different power limits for the number of users $N = 4$, where unpaired RSMA exhausts all decoding orders to obtain the optimal solution. As P_{max} increases, the latency decreases for all schemes. It can be seen that RSMA outperforms NOMA, regardless of pairing or non-pairing, the reason is that NOMA is an extreme special case of RSMA, while RSMA can handle the resource allocation and decoding methods between different packets more flexibly to obtain better performance. In addition, the proposed paired RSMA resource allocation algorithm gives a closed-form solution for power and bandwidth allocation, which significantly reduces the computational complexity and has not much performance loss compared to the unpaired RSMA.

Since the algorithmic complexity of the exhaustive method grows exponentially as the total number of users increases, we choose a suboptimal decoding order [?] to compare the latency performance at different total number of users in Fig. ??, and the power limit is set to $P_{max} = 23$ dBm. It can be seen that RSMA always outperforms NOMA, regardless of paired or unpaired. In addition, the paired schemes are more advantageous when the total number of users is small, and as the total number of users increases, the performance of the unpaired schemes will outperform the paired schemes due to the fact that more number of users means more number of pairs and therefore less bandwidth is allocated to each pair. Overall, the proposed paired-based RSMA resource allocation algorithm greatly reduces the complexity and achieves similar performance to unpaired RSMA.

To further evaluate the benefits of the proposed algorithm in terms of complexity, Table ?? gives the simulation the closed-form expressions of three possible values of the optimal power and bandwidth allocation are derived. Then the maximum latency τ is determined by the bisection method, and for each given τ, the corresponding three power and bandwidth allocations are derived. The optimal power allocation scheme is determined by judging which one satisfies the power constraint. Results show that the proposed low-complexity algorithm based on user pairing significantly reduces the user complexity and also achieves similar performance to unpaired uplink RSMA.

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