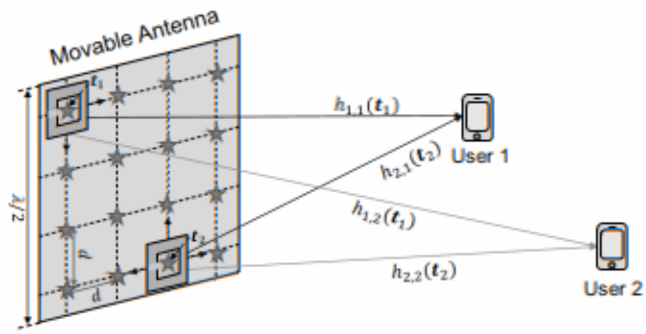

Fig. 1. Transmission from $M = 2$ movable antenna elements with $N = 16$ possible discrete positions to $K = 2$ users (★ symbols represent feasible antenna positions).

horizontal or vertical movement by a fixed increment d in each step [?] [?], the transmitter area of the MA-enabled communication system is quantized [?][2]. We collect the N possible discrete positions of the MAs in set $\mathcal{P} = \{\mathbf{p}_1, \cdots, \mathbf{p}_N\}$, where the distance between the neighboring positions is equal to d in horizontal or vertical direction, as shown in Fig. ??. Here, $\mathbf{p}_n = [x_n, y_n]$ represents the n-th candidate position with horizontal coordinate x_n and vertical coordinate y_n. In other words, the feasible set of the position of the m-th MA element, $\mathbf{t}_m$, is given by $\mathcal{P}$, i.e., $\mathbf{t}_m \in \mathcal{P}$. In the considered MA-enabled MIMO system, the physical channel can be reconfigured by adjusting the positions of the MA elements. The channel vector between the m-th MA element and the K users is denoted by $\mathbf{h}_m(\mathbf{t}_m) = [h_{m,1}(\mathbf{t}_m), \cdots, h_{m,K}(\mathbf{t}_m)]^T$ and depends on the position of the m-th MA element $\mathbf{t}_m$, where $h_{m,k}(\mathbf{t}_m) \in \mathbb{C}$ denotes the channel coefficient between the m-th MA element and the k-th user. Next, we define a matrix $\hat{\mathbf{H}}_m = [\mathbf{h}_m(\mathbf{p}_1), \cdots, \mathbf{h}_m(\mathbf{p}_N)] \in \mathbb{C}^{K\times N}$ to collect the channel vectors from the m-th MA element to all K users for all N feasible discrete MA locations. Then, $\mathbf{h}_m(\mathbf{t}_m)$ can be expressed as

$$\mathbf{h}_m(\mathbf{t}_m) = \hat{\mathbf{H}}_m \mathbf{b}_m, \tag{1}$$

where $\mathbf{b}_m = \left[b_m[1], \cdots, b_m[N]\right]^T$. Here, $b_m[n] \in \{0, 1\}$ with $\sum_{n=1}^{N} b_m[n] = 1$ is a binary variable defining the position of the m-th MA element. For the considered MA-

[2]The value of step size d depends on the precision of the employed electromechanical devices and may vary in different MA-enabled systems.

the following, we detail the formulation and solution of the primal and master problems in the i-th iteration of the GBD algorithm, followed by an explanation of the overall GBD algorithm.

1) Primal Problem: Using the discrete variables $\mathbf{B}^{(i-1)}$ and $\mathbf{y}^{(i-1)}$ obtained from the master problem in the $(i-1)$-th iteration, the primal problem in the i-th iteration is given by

$$\begin{aligned} \underset{\mathbf{X},\mathbf{W},\mathbf{U},\mathbf{V}}{\text{minimize}} \quad & \sum_{k\in\mathcal{K}} \|\mathbf{w}_k\|_2^2 \\ \text{s.t.} \quad & \text{C1a, C1b, C2b} \\ & \text{C2a:} \begin{bmatrix} \mathbf{U} & \mathbf{X} & \mathbf{B}^{(i-1)} \\ \mathbf{X}^H & \mathbf{V} & \mathbf{W}^H \\ (\mathbf{B}^{(i-1)})^T & \mathbf{W} & \mathbf{I}_K \end{bmatrix} \succeq \mathbf{0}. \end{aligned} \tag{18}$$

The primal problem in (??) is a convex problem, which can be solved by a standard convex programming solver such as CVX [?]. However, the problem (??) is not feasible for all possible discrete variables $\mathbf{B}$ and $\mathbf{y}$. If the primal problem (??) in the i-th iteration is feasible, the optimal solution of (??) is denoted by $\mathbf{X}^{(i)}$, $\mathbf{W}^{(i)}$, $\mathbf{U}^{(i)}$, and $\mathbf{V}^{(i)}$ and iteration index i is included in the index set of the feasible iterations $\mathcal{F}$. Next, let $\mathbf{\Lambda} = \{\mu_k, \nu_k, \mathbf{\Xi}, \xi\}$ denote the collection of Lagrangian multipliers of (??), where $\mu_k \in \mathbb{R}, \nu_k \in \mathbb{R}, \mathbf{\Xi} \in \mathbb{C}^{(N+K+M)\times(N+K+M)}$, and $\xi \in \mathbb{R}$ represent the dual variables for constraints C1a, C1b, C2a, and C2b, respectively. The Lagrangian function of the primal problem is given by

$$\mathcal{L}(\mathbf{X},\mathbf{W},\mathbf{U},\mathbf{V},\mathbf{B}^{(i-1)},\mathbf{\Lambda}) = \sum_{k\in\mathcal{K}} \|\mathbf{w}_k\|_2^2 + f(\mathbf{X},\mathbf{W},\mathbf{U},\mathbf{V},\mathbf{\Lambda}) + 2\mathrm{Re}\left\{\mathrm{Tr}\big(\mathbf{B}^{(i-1)}\mathbf{\Xi}_{31}\big)\right\}, \tag{19}$$

where

$$\begin{aligned} f(\mathbf{X},\mathbf{W},\mathbf{U},\mathbf{V},\mathbf{\Lambda}) = & \sum_{k\in\mathcal{K}} \mu_k \left(\sqrt{\sum_{k'\in\mathcal{K}\setminus\{k\}} |\hat{\mathbf{h}}_k^H \mathbf{x}_{k'}|^2 + \sigma_k^2} - \frac{\mathrm{Re}\{\hat{\mathbf{h}}_k^H \mathbf{x}_k\}}{\sqrt{\gamma_k}} \right) \\ & + \sum_{k\in\mathcal{K}} \nu_k \big(\mathrm{Im}\{\hat{\mathbf{h}}_k^H \mathbf{x}_k\}\big) + \mathrm{Tr}\big(\mathbf{U}\mathbf{\Xi}_{11}\big) + \mathrm{Tr}\big(\mathbf{V}\mathbf{\Xi}_{22}\big), \\ & + 2\mathrm{Re}\left\{\mathrm{Tr}\big(\mathbf{\Xi}_{32}^H\mathbf{W}\big) + \mathrm{Tr}\big(\mathbf{X}\mathbf{\Xi}_{21}\big)\right\}. \end{aligned}$$

Here, the dual matrix $\mathbf{\Xi}$ is decomposed into nine sub-matrices as follows:

$$\mathbf{\Xi} = \begin{bmatrix} \mathbf{\Xi}_{11} & \mathbf{\Xi}_{21}^H & \mathbf{\Xi}_{31}^H \\ \mathbf{\Xi}_{21} & \mathbf{\Xi}_{22} & \mathbf{\Xi}_{32}^H \\ \mathbf{\Xi}_{31} & \mathbf{\Xi}_{32} & \mathbf{\Xi}_{33} \end{bmatrix}, \tag{20}$$

(MILP) problem, which can be solved by employing standard numerical solvers for MILPs, e.g., MOSEK [?]. The optimal solution of (??) in the i-th iteration is denoted as $\mathbf{B}^{(i)}$ and $\mathbf{y}^{(i)}$.

3) Overall Algorithm: The overall procedure of the proposed optimal algorithm is summarized in Algorithm 1. Before the first iteration, we set index i to zero and initialize $\mathbf{B}^{(0)}$ and $\mathbf{y}^{(0)}$ to a feasible solution. In the i-th iteration, we begin by solving problem (??). If the problem is feasible, we generate the optimality cut for the master problem in (??) based on the intermediate solution $\mathbf{X}^{(i)}$, $\mathbf{W}^{(i)}$, $\mathbf{U}^{(i)}$, $\mathbf{V}^{(i)}$, and their corresponding Lagrangian multiplier set $\mathbf{\Lambda}^{(i)}$. Furthermore, we update the upper bound $\mathrm{UB}^{(i)}$ of (??) with the objective value $\sum_{k\in\mathcal{K}} \left\|\mathbf{w}_k^{(i)}\right\|_2^2$ obtained in the current iteration. If problem (??) is infeasible, we turn to solve the feasibility-check problem in (??) to generate the feasibility cut for the master problem. Subsequently, we optimally solve the master problem in (??) using a standard MILP solver. The objective value of the master problem provides a performance lower bound $\mathrm{LB}^{(i)}$ for the original optimization problem in (??). By following this procedure, we iteratively reduce the gap between the LB and UB in each iteration. Based on [?, Theorem 2.4], the proposed GBD-based algorithm is guaranteed to converge to the globally optimal solution of (??) in a finite number of iterations for a given convergence tolerance $\Delta \geqslant 0$. Although the worst case computational complexity of the proposed GBD-based algorithm scales exponentially with the number of MA elements, in our simulation experiments, the proposed GBD method converged in significantly fewer iterations than an exhaustive search.

IV. Numerical Results

In this section, we evaluate the performance of the proposed optimal algorithm via numerical simulations. We consider a system where the BS is equipped with $M = 4$ MA elements to provide communication service for $K = 4$ single-antenna users. The carrier frequency is set to 5 GHz, i.e., the wavelength is $\lambda = 0.06$ m. The transmitter area is a square area of size $l\lambda \times l\lambda$, where l is the normalized transmitter area size at the BS. Due to the properties of the MA driver, the transmitter area is quantized into discrete positions with equal distance d as shown in Fig. 1. The minimum distance $D_{\min}$ is set to 0.015 m. The users are randomly distributed, and their distance to the BS is uniformly distributed between 20 m to 100 m. The noise variance of each user is set to -80 dBm, $\forall k \in \mathcal{K}$. As

Algorithm 1 Optimal Resource Allocation Algorithm

Set iteration index $i = 0$, initialize upper bound $\mathrm{UB}^{(0)} \gg 1$, lower bound $\mathrm{LB}^{(0)} = 0$, the set of feasible iterations indices $\mathcal{F} = \varnothing$, the set of infeasible iterations indices $\mathcal{I} = \varnothing$, and convergence tolerance $\Delta \ll 1$, generate a feasible $\mathbf{B}^{(0)}$.
repeat
    Set $i = i + 1$
    Solve (??) for given $\mathbf{B}^{(i-1)}$, and $\mathbf{y}^{(i-1)}$.
    if the primal problem (??) is feasible **then**
        Update $\mathbf{X}^{(i)}, \mathbf{W}^{(i)}, \mathbf{U}^{(i)}$, and $\mathbf{V}^{(i)}$ and store the corresponding objective function value of $\sum_{k\in\mathcal{K}} \left\|\mathbf{w}_k^{(i)}\right\|_2^2$
        Construct $\mathcal{L}(\mathbf{X}, \mathbf{W}, \mathbf{U}, \mathbf{V}, \mathbf{B}, \mathbf{\Lambda}^{(i)})$ based on (??)
        Update the upper bound of (??) as $\mathrm{UB}^{(i)} = \min\left\{\mathrm{UB}^{(i-1)}, \sum_{k\in\mathcal{K}} \left\|\mathbf{w}_k^{(i)}\right\|_2^2\right\}$, and update $\mathcal{F}$ by $\mathcal{F} \cup \{i\}$
    else
        Solve (??), update $\widetilde{\mathbf{X}}^{(i)}$, $\widetilde{\mathbf{W}}^{(i)}$, $\widetilde{\mathbf{U}}^{(i)}$, $\widetilde{\mathbf{V}}^{(i)}$
        Construct $\widetilde{\mathcal{L}}(\mathbf{X}, \mathbf{W}, \mathbf{U}, \mathbf{V}, \mathbf{B}, \widetilde{\mathbf{\Lambda}}^{(i)})$ based on (??)
        Update $\mathcal{I}$ by $\mathcal{I} \cup \{i\}$
    end if
    Solve the relaxed master problem (??) and update $\eta^{(i)}$, $\mathbf{B}^{(i)}$, and $\mathbf{y}^{(i)}$.
    Update the lower bound as $\mathrm{LB}^{(i)} = \eta^{(i)}$
until $\mathrm{UB}^{(i)} - \mathrm{LB}^{(i)} \leqslant \Delta$

in [?], [?], [?], the channel coefficient $h_{m,k}(\mathbf{p}_n)$ between the m-th MA element and the k-th user at $\mathbf{p}_n$ is modeled as follows[3],

$$h_{m,k}(\mathbf{p}_n) = \mathbf{1}_{L_p}^T \mathbf{\Sigma}_k \mathbf{g}_k(\mathbf{p}_n), \tag{28}$$

where $\mathbf{1}_{L_p}$ is the all-one field response vector (FRV) at the k-th user equipped with a single fixed-antenna. Diagonal matrix $\mathbf{\Sigma}_k = \mathrm{diag}\{[\sigma_{1,k}, \cdots, \sigma_{L_p,k}]^T\}$ contains the path responses of all $L_p = 16$ channel paths from the transmitter area to the k-th user. All path response coefficients $\sigma_{l_p,k}, \forall l_p \in \{1, \cdots, L_p\}$ are independently and identically distributed and follow complex Gaussian distribution $\mathcal{CN}(0, L_0 D_k^{-\alpha})$, where L_0, D_k, and $\alpha = 2.2$ denote the large-scale fading at reference distance $d_0 = 1$ m, the distance from BS to the k-th user, and the

[3]The adopted field-response channel model in [?] leverages the amplitude, phase, and angle of arrival/departure information on each multipath component under far-field condition to characterize the general multipath channel for MA-enabled systems.

path loss exponent, respectively. $\mathbf{g}_k(\mathbf{p}_n)$ denotes the transmit FRV between the k-th user and the MA at position $\mathbf{p}_n$, which is given by [?], [?]

$$\mathbf{g}_k(\mathbf{p}_n) = \left[e^{j\rho_{k,1}(\mathbf{p}_n)}, \cdots, e^{j\rho_{k,L_p}(\mathbf{p}_n)}\right]^T, \tag{29}$$

where $\rho_{k,l_p}(\mathbf{p}_n) = \frac{2\pi}{\lambda}\left((x_k - x_1)\cos\theta_{k,l_p}\sin\phi_{k,l_p} + (y_k - y_1)\sin\theta_{k,l_p}\right)$ represents the phase difference of the l_p-th channel path between $\mathbf{p}_n$ and the first position $\mathbf{p}_1$. θ_{k,l_p} and ϕ_{k,l_p} denote the elevation and azimuth angles of departure of the l_p-th channel path for the k-th user, respectively, and follow the probability density function $f_{\mathrm{AoD}}(\theta_{k,l_p}, \phi_{k,l_p}) = \frac{\cos\theta_{k,l_p}}{2\pi}$, $\theta_{k,l_p} \in [-\pi/2, \pi/2]$, $\phi_{k,l_p} \in [-\pi/2, \pi/2]$. In this work, we set the step size d as 0.01 m for both the proposed scheme and the baseline schemes, as per the experimental setup in [?]. In addition, we consider also a step size d of $\lambda/2 = 0.03$ m to investigate the effect of step size on system performance.

We consider three baseline schemes for comparison. For baseline scheme 1, the MA elements are fixed at M positions that satisfy the minimum distance constraint and are chosen randomly from $\mathcal{P}$. The beamforming vectors $\mathbf{w}_k$ are obtained by solving the beamforming problem for fixed antenna arrays employing semidefinite relaxation [?]. For baseline scheme 2, we adopt the AS technique, where the BS is equipped with a $2 \times M$ uniform planar array (UPA) with fixed-position antennas spaced by $\lambda/2 = 0.03$ m such that the channels corresponding to different antennas are statistically independent. We solve the beamforming problem for all possible subsets of M antenna elements and selected the optimal subset that minimizes the BS transmit power. Note that the antenna spacing cannot be adjusted in baseline scheme 2. For baseline scheme 3, we design a suboptimal iterative algorithm based on AO. Specifically, in each iteration, beamforming matrix $\mathbf{W}$ is optimized for a fixed binary decision matrix $\mathbf{B}$ obtained in the last iteration. Afterwards, $\mathbf{B}$ is updated in a block-coordinated-descent (BCD) manner for the fixed $\mathbf{W}$ obtained in the current iteration.

Fig. ?? shows the average BS transmit power required for the considered schemes versus the users' minimum required SINR values $\gamma_k = \gamma$, $\forall k$, where the transmitter area of the MA at the BS is $2\lambda \times 2\lambda = 0.12$ m $\times$ 0.12 m. For the considered SINR range and $d = 0.01$ m, the proposed scheme requires on average approximately 120 iterations to obtain the global optimal solution, which is much faster than an exhaustive search over all possible positions of the $M = 4$ MAs. It is observed that as the minimum required SINR value

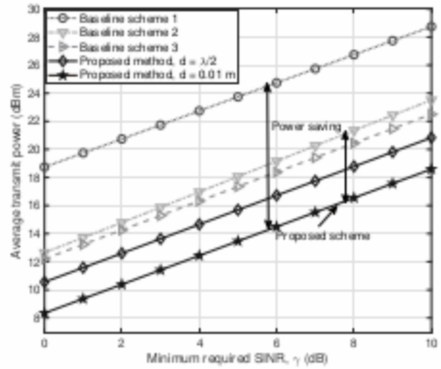

Fig. 2. Average BS transmit power versus the minimum required SINR of the users.

increases, the BS consumes more transmit power to satisfy the more rigorous quality-of-service requirements of the users. Furthermore, we also observe that the proposed scheme outperforms the three baseline schemes for the entire range of γ. In particular, for baseline scheme 1, the BS is equipped with an antenna array with fixed antenna positions, i.e., the spatial correlation of the transmit antenna array is not optimal. Although baseline scheme 2 employs AS to increase the spatial DoFs at the BS, which leads to a performance improvement compared to baseline scheme 1, the spatial resolution ($\lambda/2$) is quite coarse due to the fixed-position antenna setting. As for baseline scheme 3, the adopted AO algorithm optimizes the positions of the MA elements and the beamforming matrix, leading to a 5 dB gain compared to baseline scheme 1. However, as the AO-based algorithm is generally suboptimal due to its local search, it may get trapped in stationary points, resulting in a 4 dB performance gap compared to the proposed optimal solution, which highlights the significance of fully exploiting the DoFs provided by the proposed MA-enabled system. Moreover, increasing the step size of the electromechanical device to $d = \lambda/2$ leads to a performance loss of roughly 2 dB, indicating the trade-off between the transmit power and the MA control precision in MA-enabled systems.

Fig. ?? depicts the relationship between the BS transmit power and the normalized transmitter area size for $\gamma = 10$ dB. We observe that for the proposed scheme and the AO-based scheme, the BS transmit power decreases as the normalized transmitter area size increases. This can be attributed to the fact that a larger transmitter area allows higher flexibility for the positioning of the MA elements for shaping the desired spatial correlation,