IRS-Enabled Covert and Reliable Communications: How Many Reflection Elements are Required?

MMahilin\Wangi, Bin Xidi, Yao Yaoi, Zhiyong Cheni and Jilangahou Wangigi

Department of Electronic Engineering, Shanghai Jiao TenggUiinexisity Shilaghgha Chilaina

†Scholalo Eligiqueering, University of Kent, Claute blurgy UKK.

Elenihii: {\wangganhilin, bxia, sandyyao, zhiyongchen}}@sjtueeduunii; i.wang@Rentacackuk

Abstract - Short-packet communications are applied to various soenariosawhere/transmission/scovertnesstand/reliabilitybare/cnucialciduel to tthehopere swireless smedium and finite blocklength. Although intelligent reflection surface (IRS) has been widely utilized to enhance transmission govertness and reliability. The thuestions tilbhowf rhany reflection flelements latt dRS are Fequired remains lunans wered, a which ris ly ital to by istem design and practical deploymenta The linherent Strong houpling regists the lingen title bransmission covertnessicand creliability abyl IRS abelidyngytol RSe question of intractability of o address this listue, the detection error probability at the warder and its approximation are derived first to reveal the relation between to vertness performance land the numbereof peffection:elements. Besides, to evaluate the reliability performance of the system; the decoding error probability at the rbceiveroisi also aderiyed. Subsequentlye the asymptotic reliability Berformandevint high vooyerttiess dieglinies is diffeestigated in which provides stheogetical ipredictions tabout the pumbers of freflection elements at IRS required to achieve addecading elements obability closeirtol do with igiven dovertnesse requipentents lit Furthermore, Montgi Carloo vsirtudations uiverify nthe Facturacy or of Mhet defined resultatifors detectionhe(decoding)y errorhorobabilitiesesants the vialidityon f dihe dheoretical predictions fond effection delements: Moreover, results show that more reflection elements are required th each levet higher cliabilities with tighter acovert person chair emints; longer diloklitugthi tind higher transmissionerates:ments, longer bldriden fårmsd-Coylert and sreliable transmission, short-packet communications, intelligent deflecting trurfacession, short-packet communications, intelligent reflecting surface

I. INTRODUCTION

The fifth generation of mobile standards expands its focus int@massive machine-type-of-omnomication darMT@panddaltrias feliablentow-latency rearriment ation: (uRLLG): (ft) nI maddition tor drankmas sidnalthroughplutterelia bibitymand i lattiney, (trankfistofs) s?onI seculrityois talso aessensiabint lthese secunaries a sining wast lamounts of aconfidential sinformation also transferred disordrean spenawinelessemedistramounts of confidential information ard thensufre high transmission security, scover to communication has received extensive reoncerius, which the revenus the itransmissionabehaviorei fromebeingi detectedebys, worlders pletveThe thformationishicordich limits fof scovering detectations were eStablisheihiforfr?lafirsti-Subsequently, iextrafuncertainty sources have been ntilized bis further (thhance, Solest quantity, exuch asidhet full-displere credeiver handi theile oldaborativeli quimnie and n addition:atouticklentile assolutefidousupling rissice intelligent reflectionativefacen(HRS.) lhas dehitiged tas ta dulttitigeed go technotogynfogcovert,commbinicationslettion surface (IRS) has enRegentlys massivengfforts havenbeegydevotedyte texploiting the aperformance gain by IRS in covert communications. By

joiRtdycoptimizings the effasts shifts: abdRS deed beamformply int the transmitter favorable icommuniSation environments were established from tower trommunication lins@? https://doi.org/10.100/ above feveterns the thransmission probability by ascalso optimized im (2) pand full-duples technology (ovas combined in (2) a Forthe fill of c. IRS assisted the verb communications were extended toroblability networks st? lontin ozthogonal? hultiple fadčesa jsksy terrisn[3]) gudvaarnandeid eelrial [VehFelet lassistedesVREmss[3]) ted coNoteworthilyaidatimanwereMTCndrd tuRelsCirapplications sülchnası-industriahalutomlitijole anderemoteteurgels), stringent latengydandrhilghveltankmissiøtedelijaktilitys åre, also mandatory requirements, ilyherenshort-packECcomhuRicat@a.jiplicatiired sorchinismizeusheialranstnissitionlatencyeiHowevergethe, above gxistingtworks af 24, [[2]] h [2]] an [3] his [3] hn [2] eli [4] ilfocus ren albe system design thased on Shannon information theoretic frameworkswithrinfinited blocklengthizassthuptions, nandithelyalguore Howteansmission beliability teconstraints. [Thus2], the analytical ifflethföd and i se sultst lobt sine der by dibeiger worked aren infelfective amforcamioto behedirectly frapplied at which i packet branschission subsence photon covertness variet rediability, rare species red. r Howeker, itoissimperative Toureveal the behefit brought by IRS for shorts packéti cobrigatifications des guarante efferatismission convertedes diderellyabilityle. More skpeci fically, the agaestion to whose thanly reflectionselementseata IRISt area required i todac Hileweeco vertitanis reliable transmissionals chitical for toractical system Reploye shent-and-ketstilbannopeiraitsines to guarantee transmission co Vertadere santhiselsabeli (n. th/sopapere cafic IRIS et lable de shortpäckot/communication/system/against/a IRolti-antennaiwalder ischönsidered;råndathe relimbee offræguiriediorefleistioni/elerhefus iscapitóvádedy stornguláradteg ntratismids ionst élő vertnessa ándu reliability.a.Specificallis, itosucyalnatch ithep systemuc oldersness band seliability/performance the average detection error probability at the warder and the average decoding error probability at the receiver are theriveds Besidos; de dacilitate system analysis; som cisce approximation elixpréssion Specifie tection terrova probability isystlson proposodsira hilyheldolyditnesseskemariose. Fulrthermore: wet givie a quantitative lailiswen to like question (that how many risflection elements rate beginned to hachieve var decoding reads Probidbility oclosed lite to system given a bosier tress circuir grown to eventioif the restitute of determina at the pseuderlist infinite. Finalby,edhen anightical/eresults are naustified ubthehen Montee Clarkos simulations yeard stheri napabts cofest/stenthparameters on theleytstemeperfortsance rarp i flus trated hiwhich decoding some insightsifor practical system design covertness requirements

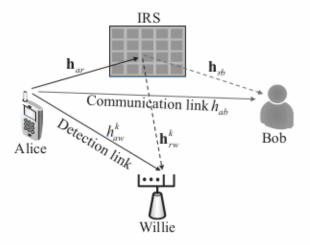


Fig. 1. IRSeenableh dishtropaçketkoommunicutiona tiyate nyatgainst gainstlitis antehna wanden warder.

everbitation receives and intermess at the crosser is isolatice Riwelens than an appticulate reliters, resinctified y by Atheandoux & Gendre sine uleations se and cleorituparet are fiscoste ar par a resters ervet be avaticis) particular accuraço il arstratrix, whose prayidan stemeins entries of the telements of the superor p. | . | and | . | den Note at hen Fronze and matrice the abdonnet edature baldinge tivery.cest (41.2) unenotes the etomplex consistant distribution Aith dramty, that tearminage and performations are no obabit Av propagatively, dog (p) derrotes a diagonel/partial library die or any light party consist of the delimination Campust our Right and be denote the Frobenius convioud the absolut Gallina respective and Crant Langue complex of austian pler tribution with mean function covariance Σ . $\Pr(\cdot)$ denotes the probability of an event. $\mathcal{Q}(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-t^2/2\right) dt$ denotes the Qffingtional (ND Channel dynoses the Gamma function, $\gamma(n,x) = \int_0^x e^{-t}t^{n-1}dt$ denotes the lower incomplete Gailling function, and communication scenario is considered where the transmitter (Alice) desires to deliver messages to the receiver (Bob) while keeping a multi-antenna warder (Willie Hunsieum) of the transmissione Alice and Bob are as samed to be equipped with a single antenna while. Willie is assumed, to the equipped with N. Athlebahs Ant IRS with Mesrellection elementsels/deployed to/hisistkthis/transmissionanharone transmission itomodu. Alica transmitsen towertusignals Alice and Bol Pare installmed to block about the denotes the transmitt power istic Alicess Willied collects outperived isignals tordetectswhether. Shertransmission chappens our not is Werdenote the:additivenwhiterGaussian_noise (AWGN) at Bob and Willie as Into the Ganathier branching Alice V (Quert It), where or and all are the noise (ariances at Hob and Willie, Bespectively) PaThenwirelesh channels of topo Adic cato Blobe (hW) Il from l'Adice to the kith dantegral of (Williet (bt., wheften Alice teal R.S. (s), oh from dRS to rBob Werderand from dRS to the hikedGanteina ofisWi(lAW(H.N.), at altobsubjectWidlithessquasisstaticARagleigh fading [7]. The Achannel I does florients remain generative during one transmission round, Widliage rindependently and identically distributedr(desk) chmonglstifferentAbundso Specifically), http:// $\Delta N(0)$, that he hintly antegral $\Delta A(0)$ ΔW illich $(h_{\alpha(0)}, k)$ CN(0), ΔA is

hy JRSC/M(0) λfullin alfRShtq, Bob CM(0, λεηςI) fr dihe lin state taneous I channely statef information...(CSI) aref segitimateolinks (including ith RAyleeg Bob dink; afrid Alice-IRS+Bob dinkfi cicats semethavailable at dRSrby charmelrestination (2) in the addition; simberWidlier threa not obepeiratel with Adibe, to dly istatistical Gist dffwarder-related Enks ifinallyding the ANGOJRS-Willie link @M (Alice: Willie, link@ AárOba, e3f) mhtgd-at (Alice, throlighathd local, oscillatof (howert inall vertentbaleaked) from the elective radio fretjoen (C Stontendesignal fe LinBesides: l theingstantableous GSb of nyarded-related/little/sis lass unjects avaidable dataWillide from IBeSworst: berspective i foatdover (Communication, since Willie does not cooperate with Alice, only statistical CSI of warder HelaGOVEREN ESSCREBEGRMANGEA NASANISie link and Alige Willie link) can be estimated at Alice through thus local neggislator weeven the radvertently leaked from the receiver radio frequency frontend signal [?]. Besides, the instantaneous CSI of wanter-related laks is assumed available at Willie from the worst perspective for covert communication.

where \mathcal{H}_0 and \mathcal{H}_1 denote the null hypothesis that Alice does not transmit and the alternative hypothesis that Alice transmits, respectively. The presence receiver signal attwinner, and W_w^i is the reflection of the alternative hypothesis that Alice transmits, respectively. The presence receiver signal attwinner, and W_w^i is the reflection of the will be where $h_{w,k} = h_{w,k} + h_{w,k$

Considering the worst perspective for covert communiwhere T is the average power of each received signal at Willie Personal Production when Willie knows the listantianeous SI and the Personal Production of the bisary decisions that infer whether Personal Production at IRS, the maximum ratio combiner Alice bransmits or not respectively the maximum ratio combiner Alice bransmits or not reputate detection addition, τ^* denotes the optimal detection threshold, which is given by [?], [?]

the optimal detection threshold, which is given by [?], [?] $\tau^* = \frac{\sigma_w^2 \left(\frac{T_2}{\sigma_w^2 + r_0} \right| \sum_{i=1}^n \left| \mathbf{h}_{P_a}^H \right\rangle_w^{[i]} \right|^2 \underset{\mathcal{D}_0}{\geqslant \tau^*} \tau^*}{\ln \left| \mathbf{h}_w \right|^2 P_a} \quad (2)$ where T is the average power of each received signal at Willie \mathcal{D}_1 and \mathcal{D}_0 denote the binary decisions that infer whether Alice transmits or not, respectively. In addition, Alice will transmit. The prior probability of either hypothesis τ^* denotes the optimal detection threshold, which is given is equal? Mathematically, the detection error probability ξ at Willie is defined as $\xi = \Pr\left(\mathcal{D}_1 \mid \mathcal{H}_0\right) + \Pr\left(\mathcal{D}_0 \mid \mathcal{H}_1\right)$, where $\Pr\left(\mathcal{P}_1 \mid \mathcal{H}_0\right)$ denotes the missed detection probability, and $\Pr\left(\mathcal{P}_0 \downarrow \mathcal{H}_1\right)$ denotes the missed detection probability [3]. Thus, the minimum detection error probability ξ^* with optimal detection ethershold proofs expressed as dr? Willie about when Alice will transmit, $\Pr\left(\mathcal{P}_1 \mid \mathcal{H}_0\right) + \Pr\left(\mathcal{D}_0 \mid \mathcal{H}_1\right)$, where $\Pr\left(\mathcal{P}_1 \mid \mathcal{H}_0\right)$ and $\Pr\left(\mathcal{P}_0 \mid \mathcal{H}_1\right)$ denotes the missed detection probability of either hypothesis is equal. Mathematically, the detection error $\Pr\left(\mathcal{P}_1 \mid \mathcal{H}_0\right) + \Pr\left(\mathcal{D}_0 \mid \mathcal{H}_1\right)$ where $\Pr\left(\mathcal{P}_0 \mid \mathcal{H}_0\right) + \Pr\left(\mathcal{D}_0 \mid \mathcal{H}_1\right)$ where $\Pr\left(\mathcal{P}_0 \mid \mathcal{H}_1\right)$ denotes the missed detection probability, and $\Pr\left(\mathcal{P}_0 \mid \mathcal{H}_1\right)$ has defined as $\frac{1}{2} = \frac{1}{2} \Pr\left(\mathcal{P}_1 \mid \mathcal{H}_0\right) + \Pr\left(\mathcal{D}_0 \mid \mathcal{H}_1\right)$ where $\Pr\left(\mathcal{P}_0 \mid \mathcal{H}_1\right)$ denotes the missed detection error $\Pr\left(\mathcal{P}_0 \mid \mathcal{H}_1\right)$ and $\Pr\left(\mathcal{P}_0 \mid \mathcal{H}_1\right)$ denotes the missed detection error $\Pr\left(\mathcal{P}_1 \mid \mathcal{H}_0\right)$ and $\Pr\left(\mathcal{P}_0 \mid \mathcal{H}_1\right)$ denotes the missed detection $\Pr\left(\mathcal{P}_0 \mid \mathcal{H}_1\right)$ denotes t

prSindei lAtice onThknows the statistical CSI to fiwarder-pelated hither the saydragotidateletion entrop throbability, with estatistical **ESI** adopted as the covertness performance of the system.

Theorem 1. The average detection error probability at Willie with optimal detection threshold can be derived as

$$\overline{\xi(\tau^1)} = \underbrace{\frac{1}{\Gamma^1(n)} \underbrace{\left(\overline{\gamma_u} \left(n, \frac{B}{h_1} \frac{n \tau^*}{n} \right) \left(\sigma_{u_l}^2 \left(\frac{1}{h_1} \frac{\sin \theta_b P_a}{n} \right) \frac{n \overline{\tau}_{\sigma_w}^2 + \tan \theta_b P_a}{\cos w} \right) \left(\overline{h_w} \frac{1}{w} \frac{\partial \varphi}{\partial w} \left(\frac{1}{h_w} \frac{\partial \varphi}{\partial w} \frac{\partial \varphi}{\partial w} \right) \right)}_{h_1} \left(\frac{\partial \varphi}{\partial w} \frac{\partial \varphi}{\partial$$

Since Adice only knows, the stablishical CSI and warderrelated thinksp the average detection errors probability with statistical CSI is adopted as the covertness performanted owhere Stisnthe parameter of Gaussian-Chebyshev Quadrathree $\theta_{\text{tem}} = \frac{\pi}{1}$. The everage detection $(x) = \frac{e^{-\frac{\lambda_w}{\lambda_w}} x^{N_w-1}}{22^{10}}$ with Williedwith Optimala detection threshold can be derived

Proof. We denote $X_k = \mathbf{h}_{rw,k}^H \boldsymbol{\Theta} \mathbf{h}_{ar}$, which approximately follows the acomplex Gaussian-this gibution of a cany $k_B P_a$ when $N_r an Sufficiently (large <math>\sigma_r^2$?). The tightness of this approximation is validated by [?] and Section $(VI \text{ even } N_m)$ is relatively, small $(\frac{1}{4} \text{ has}) / (\frac{1}{4} \text{ has})$ follows the Gamma, Astribution with the probability density function

(PDF) $f(x) = \frac{e^{-\frac{\lambda_w}{\lambda_w} x^N_w - 1}}{N_w}$ [?] By substituting (??) into (??), where B is the parameter of Gaussian-Chebyshev Quadrathe average detection error probability can be expressed as $\frac{\pi}{4}$ ($1 + \cos \frac{(x_0 + 1)\pi}{2B}$), and $f(x) = \frac{e^{-\frac{\lambda_w}{\lambda_w} x^N - 1}}{N_w} \Gamma(N_w)$ with $\frac{\lambda_w}{\sqrt{\pi}} (\frac{\pi}{2}) \frac{\lambda_w}{\sqrt{\pi}} \Gamma(N_w) \frac{\sigma^2}{\sqrt{\pi}} + 1 \ln(1 + \frac{x P_a}{\sigma^2})$ Proof. We denote $X_k = \mathbf{h}_{rw,k}^H \frac{\sigma^2}{\sqrt{\pi}} (\mathbf{h}_{ar}, \text{ which approximate})$

follows the confplex Gantsian distribution, i.e., $X_k \sim$ tightness of this approximation is validated by [?] and by substituting $x = \tan \theta$ into (??) and applying Gaussian-Section VI aven N is relatively small. Thus, the Gamma distribution with the probability density (??) can be obtained, and the proof is completed. [2] function (PDF) $f(x) = \frac{e^{N_w x}}{e^{N_w x}}$ [?]. By substituting Due to the complicated form of (??), it is intractable to (??) the average detection error probability can be expressed as tractable approximation of the detection error probability as follows:

Lemma (I.*) = 1 igh cover thes $\left(\frac{\sigma_w^2}{s_E P_n ario} + 1\right) \ln \left(1 + \frac{x P_a}{w}\right)$ block length (such as no > 50), the average detection error proba- $\begin{array}{c} \text{bility at} \quad \text{Willienetic} \quad \text{in} \quad \text{be} \quad \text{approximated (is)} \\ \frac{1}{N_w} P_a \lambda_w \\ \frac{1}{N_w} P_a \lambda_w \end{array}$

By substituting $x = \tan \theta$ into (???) and applying Gaussian-Pluelfy:The detection enrointrobability (?? intempleximated as [?], (??) can be obtained, and the proof is completed. \square $\xi(\tau^*) \approx 1 - \frac{e^{-n} n}{\Gamma(n)} \frac{P_a \| \mathbf{\hat{h}}_w \|}{\sigma^2} \approx 1 - \sqrt{\frac{n}{2\pi} \frac{P_a \| \mathbf{\hat{h}}_w \|}{\sigma^2}}$, (8) Due to the complicated form of (??), it is intractable to

where steps(a) is obtained by the dinear approximation in thigh covertables aconarios (B) and stop (b) eistitue to olimn (ha Bility as $\frac{1}{2\pi}$ with relative error less than 2×10^{-3} when $n \ge 50$.

Lemma 1. In high coveruses scenarios with moderate blocklyingth (such as $n \geq 50$), the average detection error

Where $f_i(P_d)$ etection $\sqrt{\frac{2\pi}{2\pi}} \frac{p_0 A_i - p_0}{p_0 A_i} \frac{p_0 A_i}{p_0} \frac{p_0 A_i}{p_0 A_i} \frac{p$ $\begin{array}{l} \underset{\Gamma(N_w)}{\operatorname{mated}} \operatorname{Las}(N_w, \frac{\sigma_w^2}{\lambda_w^2 P_n} \sqrt{\frac{2\pi}{p}}), \text{ which can be ignored compared} \\ \underset{\Gamma(n)}{\operatorname{with}} \operatorname{cother} \text{ terms} \underset{\Gamma(n)}{\operatorname{due to}} \operatorname{log}(\frac{\mathbf{h}_w}{P_\sigma^{*0}}) = 0 \sqrt{\frac{n}{2\pi}} \frac{P_a \|\mathbf{h}_w\|^2}{\sigma_w^2}, \end{array} \tag{E}$

where consist ϕ expression (24) implies that the ϕ ϕ is linear to 1/2h Novandhess inchigh covertness (scenarios) facilitating the performance analysis in Section V Besides, the tightness of $(??)^{\Gamma(n)}_{n \ge 30}$ validated in Section VI.

Thus I VheReletare tile ve Perre i en a North Analysis derived as When Alice transmits, the received signal at Bob is

 $\frac{\sigma_{ab}^2}{P_a}\sqrt{\frac{2\pi}{g}}y_b[i] = (h_{ab} + \mathbf{h}_{rb}^H \Theta \mathbf{h}_{ar}) x[i] + n_b[i].$ (10) $\bar{\varepsilon}$ Based on the necetived signal (22), Bob can the code the messages. However, the decoding ento $\bar{\tau}$ can get be ignored due to finite blocklength, which is given by [?] (9)

where $g_1(P_0) = Q \left(\frac{\sqrt{\frac{2\pi}{2\pi}} \sqrt{\frac{1}{N_w} |P_0|} \left(1 \left(\frac{N_w}{N_w} \right) + 1 \frac{N}{N_w} \right)^{\frac{3}{2}} P_v \sqrt{\frac{2\pi}{n}} \right) |11 \rangle$ $\frac{1}{\Gamma(N_w)} \Gamma\left(N_w, \frac{\sigma_w^2}{\lambda_w P_0} \sqrt{\frac{2\pi}{n}}\right), \text{ which Can be ignored compared}$ witherether $\pm e^{ih}_{ns} \oplus \ln_{rb}^{H} \Theta \lim_{r \to \infty} |^{2} \frac{g_{n}(P_{b}^{L})}{dP_{b}^{L}} = \text{denotes the received}$

SNR at Bob, and R is the transmission rate measured by bits perTetrannetciese (topenysiSince the intelecting hetrothorobability Proper affected by fading channels; the average decoding winer brobidative & be adorfeed no revaluate whis retiabelity robif demaines. threselimanjust its reflection coefficients in Sactional Smission round to maximize γ_b , where the optimal phase is given by $\theta_i^* = \arg\left(h_{ab}\right) + \arg\left(h_{rb,i}\right) - \arg\left(h_{ar,i}\right)$, $h_{ar,i}$ and $h_{rb,i}$ dentitienthelichartnehsfroits, Alicer to eined isthrollentent ohr iRS, and from the *i*-th element to Bob, respectively. Thus, the instantaneous SNR can be expressed as $x_{jb}^{[i]} = x_{jb}^{[i]} P_a / \sigma_b^2$ where

Based on the received signal (??). Bob can decode the messages! However, the decoding error can not be ignored Ane are the application where the arbitrarial distribution with PDF $f_X(x) = \beta^{\alpha} x^{\alpha-1} e^{-\beta x} / \Gamma(\alpha)$, where $\alpha = \Psi^2 / \Phi$, $\beta = \Psi / \Phi_{\delta} \Psi_{\gamma b} = \alpha_d \phi \beta_d \frac{1}{2} \frac{1}{2} \frac{2\sqrt{\pi} (\log_b (1\Phi + 2\log_d / B_b^2)}{(4-\pi)\sqrt{\lambda_{ab}}}, (4e_r + 4)^{-2} \frac{\pi^2}{16-\pi^2})$ and $\beta_r = \frac{4\pi}{(4\pi)^2}$. Then, the PDE of γ_b is derived as $\frac{4\pi}{(4\pi)^2} \frac{1}{2} \frac{1}{2} \frac{\pi^2}{(4\pi)^2} \frac{1}{2} \frac{1}{2} \frac{\pi^2}{(4\pi)^2} \frac{1}{2} \frac{1}{2} \frac{\pi^2}{(4\pi)^2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{\pi^2}{(4\pi)^2} \frac{1}{2} \frac{1$

SNR at Bob, and R is the transpission rate measured by bits per channel use (bpcu). Since the decoding error probability (??) is affected by adding channels, the average and thin average decoding terror probability can be durined use reliability performance. ##

IRS_can adjust its reflection coefficients at each transmission round to maximize γ_b , where the optimal phase is given by $\theta_i^* = \arg(h_{ab}) + \arg(h_{rb,i}) - \arg(h_{ar,i}), h_{ar,i}$ $h_{rb,i}$ denotes the channel from Alice to the i-th element at IRS, and from the i-th element to Bob, respectively. Thus, the instantaneous SNR can be expressed as $\gamma_b \stackrel{2\nu}{=} X^2 P_a / \sigma_b^2$ where step (1) is due to linear approximation of C-function $h_{ab} = \frac{1}{2} \sum_{i=1}^{n} \frac{h_{rb,i}}{\omega} h_{rb,i} h_{2Ri} - A_{i}^{coording} \text{ to } \int_{2\pi(4^R-1)}^{7} \frac{1}{2\pi(4^R-1)} h_{rb,i}$ 1], X can be approximated by generalized Gamma This (but ion with $P_{P(\alpha)}^{P(\alpha)} \mathcal{J}_{\lambda}(\alpha) \beta \sqrt{\frac{\sigma_{k}^{2} \ell}{P_{\alpha}}} e^{-1} e^{$ and Vi_r Theoretical Predictions about to Requeriboided as Reflection Elements

In this section, the reliability/\performance is derived under the covertness f requirement $\overline{\xi}(\gamma_t^*)$ $\stackrel{\circ}{=} p^1$ $\stackrel{\circ}{=} \varepsilon$, where $\varepsilon^1 2$ predetermined value to evaluate the covertness level. Besides, wed provide the ordering quediations hallow one hardwine reflection elements required to achieve a decoding error probability close to 0 with given sovertness requirements.

On the one hand $t \in T^*$ degrees dwith N_r as shown in (??), which implies that the allowed transmit power P_a also decreases with N_r to meet the coverness requirement $\overline{\xi}(\tau^{-1/3}) = 1-\varepsilon$. On the other liand, $\overline{\xi}(\tau^{-1/3}) = 1-\varepsilon$. On the other liand, $\overline{\xi}(\tau^{-1/3}) = 1-\varepsilon$. Therefore, the reliability guarantes by IRS is unclear but crucial, and the question that how many reflection elements are required to achieve a devoding enterprobability cluster of the former mass [requirements 28] significative.

To address this issue, we first derive the reliability performance in high coverages scenarios when Nand \infty all follows:

Theorem 2 When Nr , A continue the average decoding error probability with given ε in high covertness scenarios is

V. Theoretical Predictions about Required Reflection

 $\lim_{t\to\infty} \overline{\delta} = \begin{cases} \frac{(2\varpi\nu+1)}{n} - \sqrt{\frac{2\pi}{n}} \frac{g_1 g_2 g_3 N_{r_b} \nu_{r_w}}{n}, & r_1 \leqslant r_{rw} \leqslant r_2 \end{cases} \tag{14}$ In this section, the reliability performance is derived under the covertness requirement $\overline{\xi}(\tau^*)^{\frac{n}{2}} \geq 1$ where ε is predetermined value to evaluate the covertness level besides, we provide the theoretical predictions about reflection elements at IRS to that of antennas at Willie, $r_1 \equiv 1 + \frac{n}{n} \frac{16\lambda_{rw}\sigma_b}{16\lambda_{rw}\sigma_b}$ of reflection elements required to achieve a decorrection probability disserts $\frac{n}{n} \frac{16\lambda_{rw}\sigma_b}{16\lambda_{rw}\sigma_b}$ of reflection elements $\frac{n}{n} \frac{16\lambda_{rw}\sigma_b}{16\lambda_{rw}\sigma_b}$ of reflectio

Provinescent Appendix ??. On the one hand, $\bar{\xi}(\tau^*)$ decreases with N_r as shown in Theorem 21 gives the theoretical predictions shout the numhero of creflection velements required to achieve shigh reliability performance (On the owith hand of decrespecifically when $N_{d} \approx \sin 2N_{H}$, in the recording force of a probability, approaches 10. However action of the control of the appressives elemBesidesse when ited to schieves at allowding decoding/satot/probabilities/approach/values/between-floand L. Moreoverveit shows that the covertness requirements ε , the Thocklength m_i and the transmission crate R_{ball} $raffeet_i$ the exitical factors in thand vaveSpecifically altestighten the covert requirement, the longer the blocklength and the higher the transmission rate, the more reflection elements are required to Theorem 12 When it was performance. achievable average guarantee high-reflability performance. dewdingner numbelabilitmexital givenenin aliaks quetemas at While) is finite, the decoding error probability can be obtained by substituting the maximum allowed transmit power $P_a = \frac{2\pi}{1000} \sqrt{\frac{2\pi}{3000}}$ by (??) $\frac{1}{1000}$ to (??) Finally, we summarize the achievable reliability of IRS-enabled short-packet communication systems under covertness requirements in Table I.

where $r_{rw} = \underset{N,r\to\infty}{\text{VME}} \underset{\text{NP}}{\text{ME}} \underset{\text{Richarts}}{\text{Richarts}}$ of the number of reflection elements at IRS to that of antenias section, numerical results are provided to evaluate the performance of IRS enabled shortspacket communications. For interior we also that Alice, Bob, Willie, and the

TABLE II
ACHIEVORDIE DECORDING ERROR PROBABILLYTY

$N_w, N_r \rightarrow \infty$	$N_w, N_r \rightarrow \infty$	$N_w, N_r \rightarrow \infty$
$N_{rm}, N_{rr1} > \infty$	$r_1 \underline{N}_{ur,r}\underline{N}_{r} \leftrightarrow r_2$	$N_{2\nu} \leq N_{rest} \infty$
$r_{ru} < r_1$	$r_1 \leq r_{gyw} < r_2$	$r_2 \leq r_{rw}$
$N_w \rightarrow \infty$	N _r (38)0	N_r, N_w ,
$NN_{\rm e}$ finite:	$N_{\rm e}N_{\rm e}$ finite	$Mini,teV_w$,
$N_{r,q}$ finite	N_{w_0} finite	finite
1	0	(??)
	$r_{ru_1} < r_1$ $N_w \rightarrow \infty$ NN_w finite	$r_{ru} < r_1$ $r_1 \le r_2 < r_2$ $N_w \to \infty$ $N_r (330)$ $N_N \text{ finite}$ $N_N \text{ finite}$

Proof. See Appendix ??.

Theorem 2 gives the theoretical predictors about the number of reflection elements required to achieve highreliability performance $(\delta \to 0)^2$ with $N_w \to \infty$ Specifically, when $N_r \ge N_w$, the decoding error probability approaches 0. However, when $N < r_1 N_w$, the decoding error probability approaches 1. Besides, when $r_1N_w \leq$ $N_r < r_2 N_u$ the decoding error probabilities approach values between 0 and 1. Moreover, it shows that the covertness requirements the blocklength and the transmission rate R all affect the critical factors \mathbf{r}_1 and r_2 . Specifically, the tighter the covert requirement, the longer the blockle Tethnand the higher the transmission rate, the more reflection elements are required to guarantee highreliabilitya per formenor probability and decoding error probability versashmentalismin pureber of reflection elements at IRS (antennas at Willie) is finite, the decoding error probability can be obtained by substituting the maximum allowed IRS are located at (0.0), (10m,0); (12m,-5m), and (10m,1m) in a two-dimensional plane, respectively. The fading parameters storexpressed as nammade (tight) systeinis Schelera coveraves sure. where the telephone between nodes i and j, $d_0 =$ 1(m) is the reference distance, and $\beta_0 = -30 (\mathrm{dB})$ denotes the channel power gain at the reference distance. And α_{ij} is the pain loss exponent will $\alpha_{ar} = \frac{-30 (\mathrm{dB})}{2}$ are $\alpha_{ab} = \frac{-30 (\mathrm{dB})}{2}$. systuate the performance of 2 IRS-enabled whom packet foncuvunigations2060Aillüstratiõuativa assuuseptelehtedisre Bothin Willie averaging over 1000 charated realizations. (10m, 0), (12m Figs.) 2 and (10m) m) in a two dimensional plane bernetivelyage betetrion error probability and average des coding Emprophability are investigated uprom Fig. her (a) dit dan be seen that the curves with numerical simulations (the analytical corpression (22), and the approximation corpression (22) refinside regardifuther our beautifuther of the chief chief is relatively, small, such as n/s = 16.1 From Fig. 2. (b), it can be seen that, the fesults obtained by (22) are close to those oblined By =numeri¢alcsimulations; exon if thehaumbetaeforeflection elementeds are attividational llusure as generally such case Mag = o4e In 10fditional the $detection_0(decoding)$ error probabilities decrease with P_a and Nasincomore energy is received by Willie (Bob) when IRS is equipped with energe raffection elements: The sards all slivelidate the results given ging Section blib implying that the proposed approximations (22) and (22) can abe tadopted vas contentness and-ireliability);performance ametrics;duextor their noticiseness and tightness nation expression (??) coincide, even if the nulnlFigsoB (aflantd)(b)e therempacts roflatievelumber of, antehnas

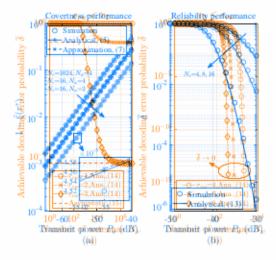


Fig. 2. The achievgbleleteetding error probability versus/theorimgberror patchabilityWidficus the transmit power.

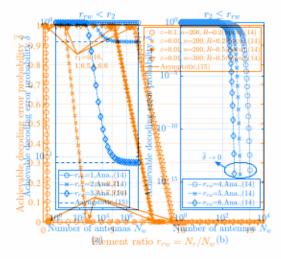


Fig. 3.4.Thm estimable decoding error probabilities as the dumber of antennas at Willie.

and reflection elements (on, the decoding enrotherobability are investigated) where the leaves withe "analidence them estates shall end of the leaves with the fless that by (22) at the latest of th

In Higgst, the aimpacts (if) system parameters on the decoding entemporability reflective stigated an where the number of antenpastath Withie is in the figure of the control of the cont

from 0.1 to 0.01. Similarly, r_1 (r_2) changes from 1.6 (3.1) to 5.1 (7.9), when R changes from 0.20 (becu) to 0.5 (bpcu). In addition r_1 (r_2) changes from r_1 (r_2) changes from r_2 (r_3) r_4 (r_4) r_5 (r_4) r_5 (r_5) r_6 (r_5) r_6 (r_6) r_6 (r_6) r_6 (r_6) r_6 (r_6) when r_6 changes from 200 to r_6 (r_6) r_6 (r_6)

VII. Conclusion

0.3 In this paper, we have provided the theoretical prediction about the equired number of reflection elements to guarantee coveriness and reliability performance in IRS-enabled short-packet communications. To this end, the detection error probability at Willie has been derived firstly for evaluating the covertness performance, where a concise approximation is also provided to facilitate further analysis. Then, the decoding error probability at Bob has been derived for evaluating the reliability performance. Combined with the above system performance metrics, we have analyzed the required number of reflection elements at IRS to achieve high-reliability $i(\delta \rightarrow 0)$ with covertness requirement $\xi(\bar{\tau}^*) > 1 - \xi$ in the asymptotic rogime. Finally, Monte-Carlo simulations bave been provided to evaluate the accuracy of analytical results and the validity of the theoretical predictions. Simulation results have shown that system parameters (s. n. R) significantly affect the required number of refelction elements, which guides the system denumber of antennas at Willie is 10⁶. It can be seen that the results by (??) coincide with the asymptotic ones by (??). Besides, it can be peop that the covertness requirements ε and system parameters nR all affect the critical factors transmit power with bren coverheuser continuent of (1831) to 1.62 \pm 3.1), when ε changes from 0.1 to 0.01. Similarly, r_1 (r_2) changes from 1.6 (3.1) to 5.1 (7.9), when R changes from 1.6 (3.1) to 5.1 (7.9), when R changes from 0.1 to 0.5 (bpcu) In addition, r_1 (r_2) changes and above approximation south (77) we can obtain from 200 to 300. These results demonstrate that more reflection elelinents are required to achieve high reliability $(\delta \to 0)$ with tighter covertness requirements, higher transmission rates and longer blocklength which provides insights for the practical system design.

 $\gamma \left(\sqrt{\frac{N_{T}\pi^{2}}{16U_{\pi}}^{2}} C_{0} \frac{1}{16U_{\pi}} \sin \sqrt{\frac{n}{2\pi}} \frac{N_{T}N_{w} \lambda_{rw} \sigma_{b}^{2}}{\lambda_{rb} s \sigma_{w}^{2}} t \right)$

In this paper, we have provided the heoretical prediction about the required number of reflection elements to guarantee covertiess and reliability performance in IRS-enabled short-packet (constitution) in this end, the detection of functionability, at Willie has and engaged firstly for evaluating the covertness of these functions first covers approximation is also provided to facilitate further analysis. Then, the decoding error probability at Bob has been derived for evaluating the reliability performance. Combined with the abolity system perfordance metrics, we have analyzed the required number of reflection elements at IRS to achieve high-reliability (\$\sqrt{xe}\$0] with covertness requirement \$\sqrt{z}\$ [27] (x) \$\sqrt{x}\$ | \$\sqrt{x

Where step of order situation becomes the order of order to the order of analytical results and the validity of the theorem of predictions. Simulation results have still the convex of the theorem of the theorem of the convex of the system design $x \to \infty$ (1.43). Step (c) holds due to $\lim_{n \to \infty} \sqrt{x}e^{x(1-r)}r^x = 0$.

For the case with r = Approximation betain that

Proof of Theorem 2. In high dimerthese soonarios with $X_{\bullet}^{(x,x)}(a)$ the allowed transmit $\frac{1}{2}$ power with given covertness (requirement ε is where $\frac{2\pi}{p_{\bullet}}(a)\sqrt{\frac{n^2}{2}}$ sindestines $\Gamma(a)$ and $\frac{2\pi}{2}$ per $\frac{2$

For the case with f > 1, $(x_1 + 3)$.

For the case with f > 1, $(x_2 + 2)$. $\lim_{N_r \to \infty} \delta = f_2(\varpi - \frac{1}{2\nu}) + (\sqrt{\frac{2\pi}{n}} \frac{N_r \pi}{16\sigma_b^2 N_{w}} \frac{N_r \pi}{N_w}) \frac{1}{e^{-t}} \frac{1}{2}$ $1 - \lim_{x \to \infty} \frac{f_4(x, \eta)}{\Gamma(x)} = \lim_{x \to \infty} \frac{1}{r^2 \nu} \frac{1}{\sqrt{16\sigma_b^2}} \frac{1}{r^2 \nu} \frac$

By substituting x $= \frac{16-\pi x}{16-\pi x}(1-r)\frac{\pi}{x}$ $= \frac{4}{\pi}\sqrt{\frac{N_w \ln r}{N_v \ln r}\sigma_b^2} \frac{(rx)}{\sqrt{\frac{n}{2}}} + \frac{1}{2\nu} \frac{1}{2\nu} \frac{1}{\sqrt{\frac{n}{2}}} \frac{(rx)}{\sqrt{\frac{n}{2}}} + \frac{1}{2\nu} \frac{1}{2\nu} \frac{1}{\sqrt{\frac{n}{2}}} \frac{(rx)}{\sqrt{\frac{n}{2}}} + \frac{1}{2\nu} \frac{1}{2\nu} \frac{1}{\sqrt{\frac{n}{2}}} \frac{(rx)}{\sqrt{\frac{n}{2}}} + \frac{1}{2\nu} \frac{1}{2\nu} \frac{1}{2\nu} \frac{1}{\sqrt{\frac{n}{2}}} \frac{(rx)}{\sqrt{\frac{n}{2}}} + \frac{1}{2\nu} \frac{$

$$\lim_{x \to \infty, r=1} f_4(x, r) = 1 - \lim_{x \to \infty} \frac{\Gamma(x, x)}{\Gamma(x)} \stackrel{(a)}{=} \frac{1}{2}, \quad (17)$$

where step (a) holds since $\lim_{x\to\infty} \Gamma(x,x) = \sqrt{\frac{\pi}{2}}x^{x-\frac{1}{2}}e^{-x}$ according to the resurgence property of the incomplete Gamma function by [?, (1.2)] and the asymptotic property of Gamma function by [?, (1.43)].

For the case with r > 1, we can obtain that

$$1 - \lim_{x \to \infty, r > 1} f_4(x, r) = \lim_{x \to \infty, r > 1} \int_{rx}^{\infty} \frac{t^{x-1} e^{-t}}{\Gamma(x)} dt$$

$$\stackrel{(a)}{<} \lim_{x \to \infty} \frac{(rx)^x e^{-rx}}{\Gamma(x)} \stackrel{(b)}{<} \lim_{x \to \infty} \frac{\sqrt{x} r^x e^{x(1-r)}}{\sqrt{2\pi}} \stackrel{(c)}{=} 0,$$

$$(18)$$

where the step (a) holds since $g_3(t) = t^{x-1}e^{-t} < g_4(t) = (rx)^{x+1}e^{-rx}t^{-2}$ for $t \ge rx, r > 1, x \to \infty$, and $\int_{rx}^{\infty} t^{-2}dt = \frac{1}{rx}$. Besides, steps (b) and (c) holds for the same reason as elaborated in the case with r < 1.

To sum, $\lim_{x\to\infty} f_4(x,r) = 0$ with r < 1, $\lim_{x\to\infty} f_4(x,r) = \frac{1}{2}$ with r = 1, and $\lim_{x\to\infty} f_4(x,r) = 1$ with r > 1. Similarly, we can obtain $\lim_{x\to\infty} f_5(x,r) = 0$ with r > 1, $\lim_{x\to\infty} f_5(x,r) = \frac{1}{2}$ with r = 1, and $\lim_{x\to\infty} f_5(x,r) = 1$ with r < 1.

By substituting $x = \frac{N_r \pi^2}{16 - \pi^2}$ and $r = \frac{4}{\pi} \sqrt{\frac{N_w \lambda_{rw} \sigma_b^2}{N_r \lambda_{rb} \varepsilon \sigma_w^2}} \sqrt{\frac{n}{2\pi}} \left(\varpi + \frac{1}{2\nu}\right) \left(\frac{4}{\pi} \sqrt{\frac{N_w \lambda_{rw} \sigma_b^2}{N_r \lambda_{rb} \varepsilon \sigma_w^2}} \sqrt{\frac{n}{2\pi}} \left(\varpi - \frac{1}{2\nu}\right)\right)$ into $f_4(x,r)$ and $f_5(x,r)$, results in Theorem ?? are obtained.