Influences of some families of error-correcting codes

Hailey Egan, Jason T. LeGrow[*], Gretchen L. Matthews[†], Jeff Suliga[‡]

Abstract

Binary codes of length n may be viewed as subsets of vertices of the Boolean hypercube $\{0,1\}^n$. The ability of a linear error-correcting code to recover erasures is connected to influences of particular monotone Boolean functions. These functions provide insight into the role that particular coordinates play in a code's erasure repair capability. In this paper, we consider directly the influences of coordinates of a code. We describe a family of codes, called codes with minimum disjoint support, for which all influences may be determined. As a consequence, we find influences of repetition codes and certain distinct weight codes. Computing influences is typically circumvented by appealing to the transitivity of the automorphism group of the code. Some of the codes considered here fail to meet the transitivity conditions required for these standard approaches, yet we can compute them directly.

1 Introduction

Capacity-achieving codes are considered the holy grail by information theorists, since They provide reliable communication at the most efficient rates. Their existence—proven in Shannon's theorem [?]—launched an entire discipline as researchers search for explicit constructions with efficient encoding and decoding algorithms. Recently, algebraic tools have been used by Kudekar, Kumar, Mondelli, Pfister, Şaşoğlu, and Urbanke to demonstrate the capacity-achieving nature of Reed-Muller codes, settling an old problem in coding theory [?]. The authors connect this coding theory question to influences of variables in Boolean functions, a notion introduced by Ben-Or and Linial [?] for collective coin flipping which has since been used in a variety of contexts; see, for instance, the survey [?].

In this paper, we explore influences as they relate to coordinates of error-correcting codes and the ability of a codeword symbol to be recovered from a received word in which erasures have occurred. Following the breakthrough work [?] which relies on the fact that

[*]Jason T. LeGrow is supported in part by the Commonwealth of Virginia's Commonwealth Cyber Initiative (CCI), an investment in the advancement of cyber R&D, innovation, and workforce development. For more information about CCI, visit www.cyberinitiative.org.

[†]G. L. Matthews is supported in part by NSF DMS-1855136 and the Commonwealth Cyber Initiative.

[‡]Support for this project was provided in part by the Commonwealth Cyber Initiative, the Virginia Tech Department of Mathematics, and the MAA Tensor Foundation.

[§]Author list in alphabetical order; see https://ams.org/profession/leaders/CultureStatement04.pdf.