Efficient Spectrum Sharing Between Coexisting OFDM Radar and Downlink Multiuser Communication Systems

JiJiaZZhu, Ylifeng Xiong, Mentber, IEEE, and Xiaojunn Jidigg

Abstract - This spaper einvestigates the throblemblefold is joint carrierriandapowerwallocaltiont in the toexistence coforadar and multi-uséti-commonication systems. Specific ally, cificouly, résearch scenario, the base station (BS) torivides information transmission tervices for multiple assers while tenturing that lift interference that separatefradar system-will not affect the stadar's illorotal fforction: To this end, welpropose a subcariser and wower allocation scheme base down orthogonal frequency divisid non ultiple access (OFDM). The original problem consisting finy blving multivariate fractional programming and binary of ariables is highly moniconvex. Due to its recomplexity globy relax other bin arye constraints by sintroducing an plenalty/sterinopprovided that the optimal solution is not affected. Then; by ciptegrating multiple power cvariables (into one matrix) the biriginal vproblem like reformulated as in that ti-rigito fractional programming (FP) problem, and finally a quadratic transform's employed to higher the non-bortiex problem as sequence of convex problems: The numerical results andicaté the performance stradeoffibetween the taulti-lisetecthim infration system and the tradar system, land notably that the performance of the communication system (is) not lint proved with answer liberease in the presence of isadart interference/ibeyond/or certainsthresholdar Ehiscprof idesaa isseful insighb (youtheaenergy) tefficient oddsighiof (the system useful insight for the energy-efficient design of the system.

Index Terms—Radar-communication coexistence, sum rate,

resource allocation, OFDM.

resource allocation, OFDM.

I. INTRODUCTION

Radar-Communication Coexisterica (RCC) has become the treRd.ofr-futurenmirelesso system:idevelop(netitC)ast:botheradae and communications systems slevel opiniowards shigher frequency bardas, alarge orantenria a timay sy attelum i hi attoria a tioria fneneasingly) simislar sing than dwaren architectures d channel uch arauteristicsonnid grigmate processing of Mar [24], hard War Thush Both highs ordality elvireless: tecistimunical isigns dryicese sainly feliable radar 3ensing capabili the glare ensitred with less notivates the study of reisource dall-dealibre (RAr) seespegially) abelispectrum is haring Between or water idation and raffare systems { !! [RA], es/Specifynt shasing:tequires/arjudiclous/allocation/tomittigate interferences untimopt fiffize? resource utilization for RCC. Due to Sheedifferenceaini the remeironal pjurdoses and lberformance métrigsteofn conferencie ation candoradar esystems a thère tiper for-Marke Pannoto behoptliffered assing then find titility plunctions Insteaderonershould maximize the performance of radar (resp. exatmunication) psystem, a under a the tebratraintaix of lensuring confinulnication (respicted an) function and resource budget the pe Tio accomplish the desired, objectives a cutirent system mathe ods can sheabroadly classified intorthmecatlego(iespThedins) flesigiostrategyreisoarradaniclentric design. It achieves coexistenCo betweenplish two byekimitinglifur interference of the carder systemdenethe bookistinglyconaminicationosystem (2) telegotiela Thimflast velosigonstrutigations centrial approaches davigabeds proposed in several recent studies; lwhich cailing for althing atthg radarfeinterferentlee throughyatepnioni (knowledge ing croceiver designof(?}ys[0]m[?]], To this send the third, categorynimolous

jointly/coptimizingether/coexisting psystems/driens/ure/shateboth sombuspication and rada operformance are satisfactory (2); through a priori knowledge or receiver design [?], [?], [?]. Total this epaper level considerespectrum of varing the tweent as single blasecostationingnel/saeradato system thising othe @FiDMnsystem whereather basef estations serve saniship to wonlinual calibn lisers simultaneously. We have noticed that in related research on RCe this paines name, outlier one typecraling is substantial and the existence of MGDAserviseure wherered in the case torgoing optimization, Chilirurpication meniods our agentalis not only focuses of the performance reade-on between sugar and communication Systems, but also esources the orithmized of that one we this enti we design atheree sive and practical RAS in effectionary in the hadare stier performance white hadare stier the tradares in the holds. min-user sum rate. The main contributions of this agorithm off between radar fand communication systems, but also ers the multi-user situation. To this end, we design an effective and practical RAscheme to satisfy the radar sharing between radar and communication systems, we system performance while maximizing the total multi-user sum focus on resource allocation when multi-user communi-cation systems and radar sensing coexist.

- The optimization problem we formulated is highly nonconvex due to the inclusion of coupling variables and
 - bitfärgenarfablasthWextkinigatesthechinanykvariablecusing alcontiguousveformulation:that yields identical solutions, and thea transformethallneast onverheroblem into accequential iconvers problem by apadastic inguesformation.
 - The numerical simulationless wits proventable effective physical sorther algorithm. Interestingly sive observe that under the nadab performance be on Weight random the time cycle allocated mainication summater does not timp to be a seldle allocated power ogsown beyond to accutain the shold, this eproprides usefulninsights quention temperaticion blessighty in apparatical applications.

The heminderical this plater is regulated as follows: In Section ?? We describe the stylen industrial optimization formulation. The subsample and power allocation significant described for Section?? We revuluate the special and contract the proposed argorithm by sillustrials in Section? In this my, we conclude the physical section ?? Over the proposed argorithm section provides useful insights for energy-efficient design in practical applications.

II. SYSTEM DESCRIPTIONS AND PROBLEM FORMULATION

The remainder of this paper is organized as follows. A System Descriptions of the system model and the option ??, we describe the system model and the optionide option of the considerior scentariow where bombourisation is described. The considerior scentariow where the continuous form and the ladar system elaptical of the construction with the construction of the

1

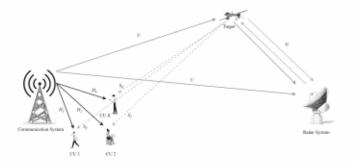


Fig. 11. Diligram for dosexistencis tofr@FDM @@ib Mrid downlink dodumurlicals tionsystemstion systems.

II. System Descriptions and Problem Formulation stationary over the observation period, and perfect channel Statte information idea both the communication and radar channels is obtained in advance. The radar steers its beam As depicted in Fig. 11, we consider a scenario where at the potential target area according to the acquired a priori communication and radar coexist in which both the knowledge, so the radar signal does not directly interfere with communication system and the radar system employ the CUs, but rather indirectly through target, scattering.

OFDM waveforms with N subcarriers. The BS provides serEquethe downlinklingmonningation tays temer CVs took quly receive communication signals from the BS, but also receive interference signalst from radart system in the same, from ency bandardninartispland fords Charaters receive the signal crambe represented aseers its beam at the potential target area according to the acquired a priori knowledge, so the radar signal does not directly interfere with the CUsubit rather indirectly through target scattering.

where the downlindscarper unightion system, CU1 nationly maciyabcamenunisatassignianals Cross and BS. but also versive interference signals from radar asstran justle same for communication systemular; for the power anseited right subcarrier presented $[P_1, P_2, \dots, P_N]^T$ is the transmit power vector for radar system, P_n^r is the power allocated to the subcarries n= h, k fis the channel gain, from the BSk to user 1/2 on subcarrier $n=s_{n,k}$ is the interference channel gain from the radar transmitter to communication receiver for subcarrier moderates the symbols repaired a engage carrier to and k_h The symbol streams x_k pre ptatistically independent with distribution GN(0.1) Truis the radar symbols transmitted on subcarrier w. The symbol streams x^r are statistically independent dent with distribution $GN(0, 1)_{dHLS}$ denotes the additive noise afithet CV to the sassumed to be distributed as he for from the offis point; the achievable idata rate, of is the ontsubcarrier chisngiveraby from the radar transmitter to communication receiver k on subcarrier n, x_k is the symbols transmitted on subcarpier, \underline{n} to $GU_g k$ The symbol streams x_k are statistically independent with distribution $\mathcal{CN}(0,1)$. x_r is the radar symbols transmitted on subcarrier n. The So we can get the total rate of U(k) symbol streams x' are statistically independent with distribution $\mathcal{CN}(0,1)$. m_k denotes the kditive noise of the CU k. It is assumed to be distributed as $\mathcal{CN}(0, \sigma_0^2)$.

To this point, the achievable data rate of CU k on The dataeat the radar receiver with can be expressed as

$$y_r R_{\overline{n}} \sum_{n=1}^{N} (f_n R_r^{\eta} g_{02}^2 + \left(\sum_{k=1}^{K} \frac{h_{n,p}^2 P_c^k}{s_{n,k}^2 P_n^k + \sigma_{n,k}^2} \right) + m_r)$$
(3)

Schere ganischetlehannel gain of tadar system on subcarrier n, u_n is the interference channel gain from the BS to radar receiver on subcarrier $\eta_r |_{\mathcal{W}_r}$ denotes the additive noise at the radar receiver. It is assumed to be distributed as $\mathcal{LN}(0, \sigma_r^2)$. The data at the radar receiver with can be expressed as need to ensure that the signal-to-noise ratio (SINR) of the radar receiver is not lower than a certain specified threshold $y_r = \sum_{n=1}^{N} (x_r P_n^r g_n^2 + \sum_{n=1}^{N} f_{n,k} x_k P_n^s u_n^2 + m_r)$ (4 $SINR = \frac{\sum_{n=1}^{N} g_n^2 P_n^r}{\sum_{n=1}^{N} g_n^2 P_n^r} > \mu.$ (5)

 $SINR \stackrel{n=1}{=}$ where g_n is the $c_{n,n}^{N} = \sum_{k=1}^{N} of_{n,k}^{2} d_{n,k}^{2} d_{n,k}^{2}$ where g_n is the $c_{n,n}^{N} = \sum_{k=1}^{N} of_{n,k}^{2} d_{n,k}^{2}$ where g_n is the $c_{n,n}^{N} = \sum_{k=1}^{N} of_{n,k}^{2} d_{n,k}^{2}$ n, to is the interference channel gain from the BS to rader formulated this problem as an optimization problem and then solved for its optimal operation of the radar function, we need to ensure that the signal-to-noise ratio (SINR) of the radar receiver is not lower than a certain specified R. Optimization Problem Formulation threshold, We choose the sum rate of CUs as the optimization metric,

while ensuring that the SINR of the radar system is above a preset threshold and satisfies the power constraint of the system, etc. The optimization problem is formulated as follows:

To conclude, we have obtained the signal model for the communication system serving prelitiple users and the sigmax mode of the radar system sensing a single target. Next, we formulated this problem as n,k=1 and optimization n,k=1 and then solved for its optimal solution.

$$\sum_{n,k} f_{n,k} \leq 1, \forall n \in [1, 2, ..., N]$$
B. Optimization Problem Formulation

(6c)

We thouse the sum rate of CUs as the optimization metric. while ensuring that the SINR of the radar system is above a present threshold and satisfies the power constraint of the system, etc. The optimization problem is formulated as follows:

$$\sum_{l} P_n^r \leq P_r^{\text{max}}, \qquad \qquad (6f)$$

$$\max_{\substack{n=1\\0\\\frac{1}{2},\frac{1}{2},\frac{1}{2}}} P_n^r \not \lesssim P_r^{\max}, \qquad (6f)$$

$$\max_{\substack{n=1\\0\\\frac{1}{2},\frac{1}{2},\frac{1}{2}}} f_{n,k} \log_2 \left[\underbrace{1+\frac{h_{n,k}^2 P_n^c}{s_{n,k}^2 N_n^2 + \sigma_{c,k}^2}}_{(6g)} \right] \qquad (6g)$$

s.t.
$$0 \le P^r \le 0$$
 R.). $\forall m \in [0, 2, ..., N] \ \forall k \in [1, 2, ..., K]$ (66h)

Constraints (2?), and (??) ensure that each subcarrier is allocated to at most one CU, Constraint (??) represents the minimum SINR for radar sensing. P_c^{max} in (??) and P_r^{max} (6in) (??) arekthenmaximum transmit powers of the communication and radar transmitters; respectively. Constraints (??) and (??) guarantee the transmit powers of communication and radar transmitters cannot go beyond their maximum limits. P_c and P_r represent the peak power constraints of communication subcarriers, respectively. It should be highlighted that constraints (??) and (??) has the effect of preventing the concentration of system power on one or a f@wissbeamier\$??husnal/qilfing-therlosshof-frequencycaliversity advantage tand the officeraseCdf, distance are solution represents chrienisystems \$1\text{NH? for a srawlelf as n son greve in exception of the christian of t interfelenceina (ised the excessive inear powern[2] powhich of practical and inecessary nd radar transmitters, respectively. CoProblemts(??)) is rad (fixed-integers or bacconvex roptimization) problem iandicseemilighyd intractableit fer sparticulary of he word convex combinatoriahiobje/five-funcRonre37)sehe nonconvex ponstraintn (%%) introdof herbinaryi csellection beonstraintn (%%) date the main obstacles of overheaders in old the resignification algorithms Neverthelds (? The spite (the seffe hallenges wim they not a section tratei will be provide a poefficient algorithm wielding neasobtisnal/solutionthe problem (??) uency diversity advantage and the decrease of distance resolution in multi-carrier systemsWAXPMtZATEQNsStemBateBtBateBtAteLQCAFIQOnce caused by excessive peak power?], which is practical and netresthiv section, we reformulation the problem (??) by applyingerP ([?]) Firstly iwed stateghe binary war able timiz to ai comprobles variables and including a general temp articulare that the regumative for their continuence of their continuence of the ther geother two x viperstantibles into some that hinard usel FP on solvet private (27) 270 the main obstacles for the design of the resource allocation algorithm. Nevertheless, despite these ghallangestein thatmatusestjormutatioill provide an efficient algorithm yielding near-optimal solution to problem $(P_n)^2$ an auxiliary variable $w_{n,k} = f_{n,k} P_n \in (0,P_n)^2$ is rintroduced, to makes the problem statement more rougise. $w_{n,k} = P^c$ represents that subcarrier n is allocated to CU k and the corresponding power is P. By allowing $w_{n,k}$ to take applying FP? It still we relax the binary variable f_n to continuous values in $(0, P^c)$, the communication rate (??) may to a continuous variable and introduce a penalty term to be rewritten as ensure that the optimal solution of the (??) is not altered. Then, we merge the two type Vanishle Pinto one matrix

A. Equivalent continuous reformulation Firstly, an auxiliary variable $w_{n,k}^2 w_n f_{n,k} B_n^x \in (0, P_n^c)$ is introduced to make the problem statement more concise. where ne represents, this tapenalty rterm appresenting the interference derm caused by subcarrier multiplexing. In particular oif the constraints (??) and (??) care satisfied the value of the penalty term is zero. In fact, the optimal solutions of the relaxed problem always have zero penalty terms for appropriate choices of η , as indicated by the following proposition: Proposition: U. Optimization, problem (22) and (22) are equivalent for all feasible solutions when $\eta \geq 1/2$.

$$\max_{\substack{w_{n,k}, P_n^r \\ w_{n,k}, P_n^r}} \sum_{k=1}^{K_1} \sum_{n=1}^{N} \frac{h_{n,k}^2 w_{n,k}}{\sum_{n=1}^{K_1} \frac{h_{n,k}^2 w_{n,k}}{\sum_{i=1}^{K_1} \frac{h_{n,k}^2 w_{n,k}}{\sum_{i=1}^{K_1} \frac{h_{n,k}^2 w_{n,k}}{\sum_{i\neq k} \frac{h_{n,k}^2 w_{n,k}}{\sum_{i\neq k} \frac{h_{n,k}^2 w_{n,i}}{\sum_{i\neq k} \frac{h_{n,k}^2 w_$$

where $\eta \sum_{i\neq k}^{K} h_{n,i}^2 w_{n,i}$ is a penalty term representing interference term caused by subcarrier multiplexing. (81) particular, if the constraints (??) and (??) are satisfied, the value of the penalty term is zero. In fact, the optimal solutions of the relaxed problem always have zero penalty term for appropriate choices of η , as indicated by ϕ_{ij} following proposition:

Proposition is Optimization problem (??) and (??) are equivalent for all feasible solutions when $\eta \ge 1/2$. (86)

new of: Sistement total communication transmit power allocated to subcarrier n is W_n^s and $V_n^s = V_n^s V_$ $\delta_{n,k} \triangleq \sum_{i=1}^{K} w_{n,i}$ represent the power allocated to of (Sh) subcarriers. The communication rate of user k on subcarrier

$$\begin{array}{c} n \text{ in } (2^{\frac{K}{2}}) \sum\limits_{k=1}^{K} \inf\limits_{n=1} \text{ evenimum catton rate of user k on subcarrier} \\ n \text{ in } (2^{\frac{K}{2}}) \sum\limits_{k=1}^{K} \inf\limits_{n=1} \text{ evenitten as} \\ R_{n,k} = \log_2 \left(1 + \frac{h_{n,k}^2 w_{n,k}}{s_{n,k}^2 P_n^r + \eta h_{n,k}^2 \delta_{n,k} + \sigma_{c,k}^2}\right) \\ \sum\limits_{n=1}^{P_r} \sum\limits_{n=1}^{P_r} \sum\limits_{k=1}^{K} \frac{h_{n,k}^2 P_n^r + \eta h_{n,k}^2 \delta_{n,k} + \sigma_{c,k}^2}{s_{n,k}^2 P_n^r + \eta h_{n,k}^2 \delta_{n,k}^2 + \frac{1}{2^2 P_n^r + 1} P_n^2 \delta_{n,k}^2 + \frac{1}{2^$$

$$0 \le \deg_2 \left\{ P_{c,b} \forall n \in [1, 2] \frac{W_n - \delta_{n,k}}{\sqrt{k}} \left[\frac{1}{\sqrt{k}} \frac{2}{\sqrt{k}} \frac{1}{\sqrt{k}} \frac{2}{\sqrt{k}} \right] \right\}. \quad (89)$$

LeP usofirstA consider like (scenario) that there i are only stwo users:(Klbc2):eWearauhterested in ithelicondition underwyhich the following holds $\triangleq \sum_{i \neq k}^K w_{n,i}$ represent the power allocated to other subcarriers. The communication rate of users of subcarriers in (??) can be rewritten as

 $R_{n,k} = \log_2 \left(1 + \frac{1}{s_{n,k}^2} \frac{1}{r_{n,k}^2} \frac{h_{n,k}^2 w_{n,k} \delta_{n,1}}{r_{n,k}^2 + \zeta_n h_{n,k}^2 + \delta_n (W_n \sigma_{c,k}^2 \delta_n)} \right)$

namely that it is better not to share the power between the two users, where $Q_n = \frac{1+\frac{s_{n,k}^2}{h_{m,k}^2}P_{n,k}^{r-\frac{1}{p}}\frac{\sigma_{n,k}^{r+1}}{R_{n,1}^2} + \frac{s_{n,k}^2}{h_{n,k}^2}\frac{P_n^r}{h_{m,k}^2}\frac{\sigma_{n,k}^2}{R_{n,1}^2}\frac{P_n^r}{h_{n,k}^2} + \frac{\sigma_{n,k}^29}{h_{n,k}^2},$

and $\zeta_{n,1} \leq \zeta_{n,2}$.

After some arguing the security the holds whenever (??) Holds (K=2). We are interested in the condition under which the fallowing pholding achieved when $\delta_{n,1} = 0$. Next, we wish to investigate the condition-under which $(\ref{eq:total_superior})$ holds for all $\delta_{n,1}$ to this end, it suffices we show that $f'(\delta_{n,1}) \leq 0$ $0, \forall \delta_{n,1} \geq 0$. Taking the derivative of $\delta(\delta_{n,1})$ with respect to $\delta_{n,1}$, we have (??). $\log_2\left(1 + \frac{\delta_{n,1} + \delta_{n,1}}{\zeta_{n,2} + \eta(W_n - \delta_{n,1})}\right)$, (10) Through observation, we can determine that (??) holds.

namelinehatoitisis abetter not to share the power between $\begin{array}{l} \text{the two users, where } \zeta_{n,1} = \frac{s_{n,1}^2}{W_n h_{n,1}^2} 2\delta_{n,1}^r + \frac{\sigma_{c,1}^2}{h_{m,2}^2} \text{ and } \zeta_{n,2} = \\ \frac{-2}{s_{n,2}^2} \eta(W_n - \frac{\delta_{n,1}}{\delta_{n,1}}) + 2\eta \delta_{n,1} + \frac{1}{W_n h_{n,1}^2} 2\delta_{n,1}^r + \frac{\sigma_{c,1}^2}{h_{m,2}^2} \text{ and } \zeta_{n,2} = \\ \frac{s_{n,2}^2}{h_{n,2}^2} P_n^r + (\eta_{h,n,1}^{3c,2} + h_n^2 d_{-n}^2) (\zeta_n \leq (W_2 - \delta_{n,1}) + \zeta_{n,2}) \leq 0, \end{array}$ the Afondinione f((ggbr)a, ≤ve) sevothat≤?(i) will deciral interview

Thus, equality that possibly inaction thio when $f(\delta_n) \to 0$. Next we wish to investigate the condition under which (??) holds for all $\delta_{n,1} \geq 0$. To this end, it suffices to show that $f'(\delta_{n,1}) \leq 0\eta \not\cong \delta_{\overline{n},1}^{-1} + \xrightarrow{\mathcal{C}_{n,1}} \stackrel{\mathbb{C}_{n,1}}{\text{Taking}}$ the derivative $f(\delta_{n,1})$ with respect to $\delta_{n,1}$, we have f(??).

satisfied ds.

SinChronish-ohservation, we grand charming that (ha) holds is sufficient for (3?) no hotel for any $\delta_{n,1} \geq 0$.

- We time stend the resulting the case of Kt 5,13. By viewing $W_{\delta_n,1}$ as the total power (denoted by W_n), we are ainterested in the condition under which the following holds the condition $f'(\delta_{n,1}) \leq 0$, $\forall \delta_{n,1} \geq 0$ will certainly be satisfied. $(1+\frac{\widetilde{W}_n}{\log_2}) \geq \log_2 (1+\frac{\widetilde{W}_n-\delta_{n,2}}{\widetilde{V}_n}) + f'(\delta_{n,1}) \leq 0$. $0, \forall \delta_{n,1} \geq 0 \text{ is}$

$$\eta \ge \frac{\log_2}{2} + \left(\frac{\xi_{n,1} - \zeta_{n,2}}{W_n - 2\delta_{n,1}} (\widetilde{W}_n - \delta_{n,2}) \right), (14)$$

where $\zeta_{n,1}+(1/2)\delta_{n,1}$ and $\zeta_{n,2}+(1/2)\delta_{n,1}$ as the noise plus hiterreference denoted by $\zeta_n=0$, and $\zeta_n=0$ and $\zeta_n=0$ and $\zeta_n=0$ and $\zeta_n=0$ and $\zeta_n=0$ are sufficient for (??) to hold for any $\delta_{n,1}\geq 0$.

After some afgebra, hwe ested that (??) fiolds of the newer (??) Kinking $W_n - \delta_{n,1}$ as the total power (denoted by W_n), we arsainterested in the wood king ander which the afollowing achieved when $\delta_{n,2}=0$. Next, we wish to investigate the condition underwhich (??) holds Wor-all δ_n , δ_n , ≥ 0 . To this end, figuratives to show that $f'(\delta_{n,2}) \leq 0, \delta_n$, δ_n , δ_n , δ_n the derivative of $f(\delta_{n,2})$ with respect to $\delta_{n,2}$, we can derive an inequality equivalent $\log_2\left(1+\frac{\delta_{n,2}}{\widetilde{\zeta}_{n,1}}\widetilde{\zeta}_{n,2}\widetilde{\zeta}_{n,2}^+\eta(\widetilde{W}_n-\delta_{n,2})\right)$

where $\zeta_{n,1} + (1/2) \delta_{n,1}^{n} \ge \frac{1}{2} \text{and} \sqrt{\zeta_{n,2} + 2\delta_{n,2}^{1/2}} 2) \delta_{n,1}$ as the noise plus interference (denoted by $\widetilde{\zeta}_{n,1}$ and $\widetilde{\zeta}_{n,2}$, respectively. Since the term $\frac{\zeta_{n,1}-\zeta_{n,2}}{\overline{W}_n-2\delta_{n,2}}\leq 0$, we may conclude that $\eta\geq \frac{1}{2}$

$$\frac{W_n}{\zeta_{n,1}} \ge f(\delta_{n,1}) = \frac{\delta_{n,1}(W_n - \delta_{n,1}) + (W_n - \delta_{n,1})(\zeta_{n,2} + \eta(W_n - \delta_{n,1})) + \delta_{n,1}(\zeta_{n,1} + \eta\delta_{n,1})}{(\zeta_{n,1} + \eta\delta_{n,1})(\zeta_{n,2} + \eta(W_n - \delta_{n,1}))}$$
(11)

$$f'(\delta_{n,1}) = \frac{-2\eta(W_n - \delta_{n,1}) + 2\eta\delta_{n,1} + W_n - 2\delta_{n,1} - \zeta_{n,2} + \zeta_{n,1}}{(\eta\delta_{n,1} + \zeta_{n,1})(\zeta_{n,1}(W_n - \delta_{n,1}) + \zeta_{n,2})} + \frac{\eta\left(\eta(W_n - \delta_{n,1})^2 + \delta_{n,1}(\eta\delta_{n,1} + \zeta_{n,1}) + \zeta_{n,2}(W_n - \delta_{n,1}) + \delta_{n,1}(W_n - \delta_{n,1})\right)}{(\eta\delta_{n,1} + \zeta_{n,1})(\eta(W_n - \delta_{n,1}) + \zeta_{n,2})^2} - \frac{\eta\left(\eta(W_n - \delta_{n,1})^2 + \delta_{n,1}(\eta\delta_{n,1} + \zeta_{n,1}) + \zeta_{n,2}(W_n - \delta_{n,1}) + \delta_{n,1}(W_n - \delta_{n,1})\right)}{(\eta\delta_{n,1} + \zeta_{n,1})^2(\eta(W_n - \delta_{n,1}) + \zeta_{n,2})}$$
(12)

$$\frac{\eta\left(\eta(W_{n}-\delta_{n,1})^{2}+\delta_{n,1}(\eta\delta_{n,1}+\zeta_{n,1})+\zeta_{n,2}(W_{n}-\delta_{n,1})+\delta_{n,1}(W_{n}-\delta_{n,1})\right)}{(\eta\delta_{n,1}+\zeta_{n,1})(\eta(W_{n}-\delta_{n,1})+\zeta_{n,2})^{2}}-\frac{\eta\left(\eta(W_{n}-\delta_{n,1})^{2}+\delta_{n,1}(\eta\delta_{n,1}+\zeta_{n,1})+\zeta_{n,2}(W_{n}-\delta_{n,1})+\delta_{n,1}(W_{n}-\delta_{n,1})\right)}{(\eta\delta_{n,1}+\zeta_{n,1})^{2}(\eta(W_{n}-\delta_{n,1})+\zeta_{n,2})}\leq 0.$$
(13)

$$\frac{\widetilde{W}_n}{\overline{\zeta}_{n,1}} \ge f(\delta_{n,2}) = \frac{\delta_{n,2}(\widetilde{W}_n - \delta_{n,2}) + (\widetilde{W}_n - \delta_{n,2})(\widetilde{\zeta}_{n,2} + \eta(\widetilde{W}_n - \delta_{n,2})) + \delta_{n,2}(\widetilde{\zeta}_{n,1} + \eta\delta_{n,2})}{(\widetilde{\zeta}_{n,1} + \eta\delta_{n,2})(\widetilde{\zeta}_{n,2} + \eta(\widetilde{W}_n - \delta_{n,2}))}$$
(16)

is Affreignus 129bro hote for about (22) (holds whenever (?Byhelfistoying the method of mathematical induction, the previous arguithents are any ben reused to henow ality is not a reused to henow ality is not a reused. putter achieved when sors is he ver better wish tain vertigates that candition power towhickers? have thereor allocating power the abumal choice > 0. Taking other derivative optimization with 1567? the fee? are can decide whe inversality 19 quivalent

$$\eta \geq \frac{1}{2} + \frac{\widetilde{\zeta}_{n,1} - \widetilde{\zeta}_{n,2}}{\widetilde{W}_{n} - 2\delta_{n,2}}.$$
(17)

B. Sequential convex relaxation:

SirAithfoughexue have refaxed the chinary considered into top minuous mariable othe existence of coupling variables $w_{n,k}$ and P_n^r Bin e(2) omakes lit still to donf convex problem in Torcsolve (22), palternating optimization is a common solution omethod. Blyofixing one wariable and optimizing another Nariable the ariginal (problem his decomposed vinto (two sub-problem heThe disadventage one this limethed is shatighe udecomposed stube problem is stillea non-convex optimization problem, which has highs computational complexity and its phillicult (10) obtain (10) aptimalisalation/Inspired by [?] we combine the variables $w_{n,k}$ and P_n^r to be optimized into matrix variable P, avoiding the Sprocess and colle-mating continuization, and only need to update the matrix variables to get the solution of the problem. confidentially any able tingle existence of Compling variables $w_{h,k}^{r,k}e_{k}$ nd θ_{n}^{r} in (?4) makeseit.st θ_{k} a non-convex problem. To solve (??) alternating optimization is a common solution $[\frac{1}{2}]$ and $[\frac{1}{2}]$ is a $[\frac{1}{2}]$ fixing one variable and optimizing another variable) as The original problem is decomposed into two sub-problems. The disadvantage of this method is that the decomposed sub-problem is still a Ron-convex on timiza tion problem, which see pigh computational complexity

and is difficult to obtain the optimal solution. Inspired by [?], we combine the variables $\psi_{n,k} \in \mathcal{P}_n^r$ to be optimized into matrix variable P, avoiding the process of alternating optimization, and only need to update the matrix variables where v_{ne} is Nutrion ensugational precion $v_n(j) = 1$ when j = nand $v_n(j) = 0$ otherwise. Then, (??) can be rewritten as

Specifically, we defin $\begin{aligned} & \underset{\boldsymbol{\alpha}_{n,k}}{\text{Specifically,}} & \underset{\boldsymbol{\alpha}_{n,k}}{\text{we defin}} \sum_{k=2}^{K} \boldsymbol{e}_{k} (\boldsymbol{P}) & [\boldsymbol{0}_{k-1}; 1; \boldsymbol{0}_{K+1}]^{T} \\ & \boldsymbol{\alpha}_{n,k} & = \frac{h_{n,k}^{2}}{\sigma_{c}^{2}} \boldsymbol{e}_{k}, & \boldsymbol{\beta}_{n,k} & = \frac{g_{n}^{2}}{k-2} \boldsymbol{e}_{k}, & \boldsymbol{\xi}_{n} & = \frac{g_{n}^{2}}{\sigma_{r}^{2}} \boldsymbol{e}_{K+1}, & \boldsymbol{\gamma}_{n} & = \\ [\frac{u_{n}^{2}}{\sigma_{r}^{2}}, \dots, \frac{u_{n}^{2}}{\sigma_{r}^{2}}, 0]^{T} & \text{and} & \boldsymbol{P} & \boldsymbol$ 1) $\times N$ matrix, $\mathbf{w}_k = [w_{1,k}, w_{2,k}, \dots, w_{N,k}]^T$, we rewrite (?) Problem (?) are mains a challenging non-convex problem due to the strong interdependence of the transmit power levels of different subcarriers, as reflected in the interference terms of the SINK. We take the quadratic transform is proposed in [?] to address the multiple-ratio FP problems. By performing a quadratic transform on each SINR Pelm, we obtain the following reformulation $\sum_{n=1}^{N} (\gamma_n^T P v_n + 1)$

where v_n is N-dimensional (PCV), $v_n(j) = 1$ when j(213)and $v_n(j) = 0$ otherwise. Then, (??) can be rewritten as s.t. (??)(??) - (??) (21b) (21b)

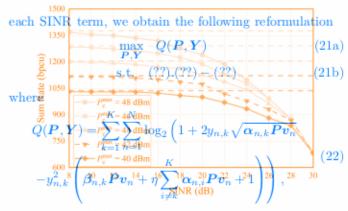
where
$$\max_{\boldsymbol{P}} \sum_{k=1}^{K} R_k(\boldsymbol{P})$$
 (20a)
$$Q(\boldsymbol{P}, \boldsymbol{Y}) = \sum_{k=1}^{K} \sum_{n=1}^{K} \log_2^{n} \left(P^n + 2y_{n,k}^{(n)} \sqrt{\alpha_{n,k} \boldsymbol{P} \boldsymbol{v}_n} \right)$$
 (20b)

$$Q(\mathbf{P}, \mathbf{Y}) = \sum_{k=1}^{8} \sum_{n=1}^{8} \log_2 \left(\mathbf{f}^2 + 2y_{n,k}^{(2)} \sqrt{\alpha_{n,k} \mathbf{P} v_n} \right)$$
(20b)

Problem (3) Propins a challer ing non-convex problem due to the strong interdependence of the transmit power levels of different subcarriers, as reflected in the where $y_{n,k} = V_{n,k}$ is the auxiliary variable introduced by interference terms of the SINE. We take the quadratic the quadratic transform is proposed in [2] to address the multiple-ratio FPWpnupdetesynRyanduPoinningiterativedfashiontralibecoptimal

TABLE II SSIDULATION PARAMETERS

Panametérs s	Váldes
Number off subcamicies:	11288
Camien frequency	2241 GHz
Cell radius	8900 m
noise variance σ_{obb}^{2}	1055d B B
noise variance σ_{cdd}^2 noise variance σ_{cd}^2	1055d B B
Maximum transmitipqwere Poporax	500 dBm
Maximum transmitipqwere Proporac	445 d Bin n
Maximum subleanticiep qwere P_cP_c	300dBhn
Maximum subleassiciep qwere P_iP_r	300dBm
Shadowing distribition	Llogenormalal
Shadowing standard deletration	88dBB
Pattitlessmodelel	WINNERPHI[?]?]



Where Sym_k are $v[Y]_k$, S_k in the difficulty wards B_k^{pp} introduced by the quadratic transform for each CU k on subcarrier y_k for fixed P is

We update
$$y_{n,k}$$
 and P in an iterative fashion. The optimal $y_{n,k}$ for fixed P is $\sqrt{\alpha_{n,k}^T P v_n}$ (23)
$$y_{n,k} = \frac{1}{\beta_{n,k}^T P v_n + \sqrt{\alpha_{n,k}^T P v_n} + 1}$$

Then, finding the optimal P for fixed $y_{n,k}$ is a convex problem and can be solved by off-the-shelf convex optimization solvers. Then, finding the optimal P for fixed $y_{n,k}$ is a convex Algorithm 1. Joint Design Algorithm Problem and can be solved by off-the-shelf convex optimization and can be solved by off-the-shelf convex optimization.

Inpution, solvers, u_n , g_n , η , m_k , m_r , p^c , p^r .

Output: Communication power p^c . Radar power p^r . Algorithm 1: Joint Design Algorithm 1: Initialization: Initialize p^r , p^r and η to feasible values.

Repeat $h_{n,k}$, $s_{n,k}$, u_n , g_n , η , m_k , m_r , p^c , p^r .

Outpluts alvan Robbiant (32) power p^c , Radar power p^r . Initial Likipitate Phibylisolving pre-reformulated sible values. Repeat convex optimization problem (??) for fixed $y_{n,k}$. until 1 converge problem (??).

 Update P by solving the reformulated convex optimization problem (??) for fixed y_{n,k}. until convergently. SIMULATIONS RESULTS

We consider a scenario where one BS serves 5 CUs are randomly distributed within the cell. The main simulation parameters are listed in simulations. Results

Wg. channel 51sC Uspan) versus middle Schumel 51sC Uspan) versus middle Sc

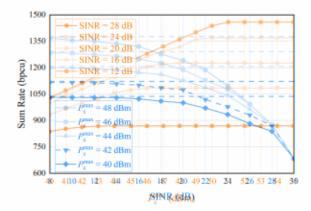


Fig. 3: Sum rate versus the Makhillad power of the velocities and system with different radar SINR constraints.

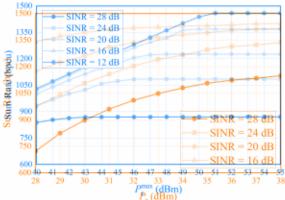


Fig. 3. Sum rate versus the maximum power of the communication Fig. 4. Sum rate versus the maximum power of single radar subcarner with system with different radar SINR constraints.

skithy then corigouitification psystem, dandg as ithe misnimum ISINR requirestry the stadar of stada in SPeaRes. The interference to the communication by stempwill the configurator of serious. The statted line in the figure shows the communication vater in the labsenbe ofinidamint & Personal ticard beyseen that when there is no eradar threefetenege the sunt bare is thigheightan the stampatel when the radge interference his present with santa-total communication powery Oricitie orther thand, twhen the rad arf SINRs becomes large, The crotal brossenum bation power has little of feet on the total rate, and they tested to begine same, the sum rate when the radar intFiger22) shows the sum rately errore the total communication power.vihert ISENTher: [hazid,6v20r24h28fadBrahlNr,nbecem45 dBm. We observe unumbeasion provendeasist as effect out communication production contents sundetherifferent SINR constraints. However, an interesting at sulvis that although the dotal communication power listins with ng. the 2sum, 2ate24028 ergls tond #dfistant beyondBan cettainobower thresholde Thenlarger thereadar SINRies total lowerntheightesholds will thereathis indicateliffeatutadan RINRistonistrain Hoprevent suthintsums trate fromt increasing tunboundedly oas I thoutotaln powers increases: This execut timplies that the power of the genantubication system-should/be reasonablly allheatedgender-given-ra@iNSINR chastraints the achievacpowerlphis inflony indicates that radar SING, c82shows threweins rate sensus a the firmax imurnaping subcarricleraldarapowler Rotwhen Repair imc50edBm andisPressalt dBplidSigtl?2tintlicates that the maximum power of isingly sadar subcarrièreis positivelly collelated with the genievable sum Diffe

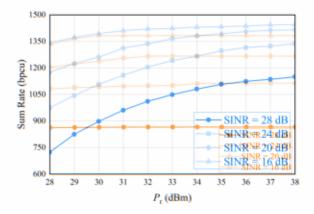


Fig. 51. Sümuratea versossatte thaximum i power provinglef coinglimication subcarrice with different dans INE DNR trainstraints.

of communication system, which is the case under different SINR constraints. The conclusion that can be drawn is that the smaller the constraint on radar SINR, the weaker the impact of the change in maximum power of single radar subcarrier on the sum rate.

In contrast to Fig. ??, Fig. ?? shows the impact of changing the maximum transmit power of a single communication subcarrier on the sum rate while keeping the remaining gaydariables fixed. Overall, the change in the maximum transmission power of a single communication subcarrier has less impact on the sum rate compared to the impact of changing the maximum transmission power of a single change subcarrier.

Fig. 5. Sum rate versus the maximum power of single communication subcarrier with different a GON CARL SHONG ints.

In this paper, we have investigated the power allocation constraints to achieve power, parsimony, problem, in the spectrum coexistence of radar and communities. The problem is the sum rate versus the maximum percation systems, where we jointly allocate the communication subcarrier radar power. When Problem is the maximum power and radar transmission power to maximize the sum rate of Cus under the constraint of radar sensor single radar subcarrier is positively correlated with the ingolf performance. Introduction problem achievable sum rate of communication system, which is the containing binary variables is transformed into an equivalent case under different SINE constraints. The conclusion that optimization problem with only continuous valued variables, and be drawn is that the smaller the constraint on radar and then the computationally tedious alternating optimization power of single radar subcarrier on the sum rate. The single radar subcarrier on the sum rate does not continuous to the change in maximum is replaced by an FP optimization in vector form. Simulation power of single radar subcarrier on the sum rate does not continuously the maximum transmit power of a single expecially, the interesting result that he sum rate does not communication subcarrier has less impact on the sum rate while keeping increase with the total power beyond certain thresholds can the remaining variables fixed. Overall, the change in the beuserier has less impact on the sum rate compared to the impact of changing the maximum transmission power of a single communication subcarrier has less impact on the sum rate compared to the impact of changing the maximum transmission power of a single communication subcarrier has less impact on the sum rate compared to the impact of changing the maximum transmission power of a single communication subcarrier has less impact on the sum rate compared to the impact of changing the maximum transmission power of a single radar subcarrier.

V. Conclusions

In this paper, we have investigated the power allocation problem in the spectrum coexistence of radar and communication systems, where we jointly allocate the communication transmission power and radar transmission power to maximize the sum rate of CUs under the constraint of radar sensing performance. Through proper reformulation, the problem containing binary variables is transformed into an equivalent optimization problem with only continuous-valued variables, and then the computationally tedious alternating optimization is replaced by an FP optimization in vector form. Simulation results exhibit the effectiveness of the algorithm and show the trade-off between communication rate and radar SINR. Especially, the interesting result that the sum rate does not increase with the total power beyond certain thresholds can be useful for the design of energy-efficient RCC systems.