# On the Performance Tradeoff of an ISAC System with Finite Blocklength

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Abstract - Hitteggatedekensing: gnd ncommunication i (ISAC) Afas been byroposed pascal promising paradigm righthe future wintless networks; wherekthe spectral and drandward resources are shared to eprovidel a oconsiderable operformance ogain: "It rise egsentialt to understand thowersensing candecommunicationm(S&C) iinfl(fe&ces eathenther etchguider the practical algorithm land osystem adel signe in dSA(A, iIn18hiC.phper,isveajnvestigatevthtigperfohmance fradeoff-ebetwhenf S&t@eim\_S&sthglesinput]esinglesoutput-(SISO) ISACO system: withefinitet blücktengtheldnugahtieularar weupresent thessysteinemodelrandothel ISACt scheoeCafter which the whitethroratradeoff tisidntföduced odsithe performance metrica Thin Whiderived they achievability about your bounds of ord theorate extorertradeoff,edfeteletining ithet boundary of of the joint S&C performance. Furthermore, we develop the asymptotic analysis at large blocklength regime, where the performance tradeoff between S&C is proved to wanish as the blocklength tends to infinity. Finally, our theoretical analysis is consolidated by simulation results.

Index Terms:—Integrated sensing and communication, finite blocklength, rate-error tradeoff, asymptotic analysis is

### I.I.Introduction

Recent years shave writtessed the Irapid pite velophorm of t.56f to 6 hoologids give shodered evil civil and itariyitapplications to class the Highest land in the work as the order of the generations, which tends, to the severe temperature of the generation hands and each tender are sources of the generation in the severe temperature of the generation is the order of the generation in the severe the performance of the generation is the severe of the generation in the severe of the generation is the severe of the generation in the severe of the generation is the severe of the generation of the severe of the generation of the severe o

There exist many crucial problems in the ISAC wesearch including the theoretical alranework, the thysique protocol and the dignal gracessings algorithmish filse. The smissiportane tand challengthe given congatherings there has a cherication of the tion of the repair and challengthe given congatherings there has a cherication of the signal sharinghin signal space from the signal sharinghin signal space and of the signal sharinghin signal space and the signal sharinghin signal space and the signal strains and the signal sharinghin signal space and the space space and spac

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In [?], the authors characteriziz the heapacity it is distinctions to trade of a distribution state that the its AC systems where sensing refering orders are the stimulation stated in the solute dependent dependent dependent is appopulated as proposed in proposed by depicture and advertised of pegon benefit appopulation are evident in the solution of the sense of the following and the solution of the sense of the solution of t

Although progress has been madel innermor of stabilshing the theoretical from that of ISACS After hexists in teammen that altion their above board is udited. Mexisting strong two keeps the for the forthan remark capacity passithe aperformance metric for the forthan remark capacity passithe aperformance metric for the continuous automorphism of the which spiritually introduced in the stability of the weather that the wide the keeps happened in the stability of the continuous performance as performing tess is introduced by the length spiritual superformance of the contraction of the continuous spendial by the contraction of the contra

In this paper, we characterize the performance undeoff between S&C iria \$1860 A&AS ys seint with within fluid cklehgth. I that he section figure operations are the espetially another including the USAG schemes and performance merries; where the trace who region is defined to evaluate the distribution. Then he section tracked the other sententially, and derivers be orders within and error variate of purific for hide the asymptotic darkly sift as performent to smooth the order of variables us the blocked guidence as selected section by the theoretical leastly also the theoretical leastly are concludes this paper ical experiments. Finally, Section V concludes this paper.

# III. SYSTEM MODEL

In this section we first present our signal ignodel and ISAG 
ISAC hes after which they be forthance metrics nor communication.

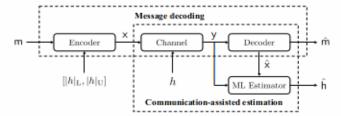


Fig. 1: The block kidgram of of corproposed siSACS action be which both sixts is father the sages depoding stop and phenebrimum cation assisted astistation step.

tion and sensing are introduced to characterize the rate-error transparation and sensing are introduced to characterize the constder rais is the constder rais in the constder rais is the constder rais in the constder rais is the constder rais in the constder rais in the constder rais in the constant raise and the constder raise raise

Consider a SISO ISAC signal model given by
$$\mathbf{y} = h\mathbf{x} + \mathbf{n}$$
(1)

where  $\mathbf{x} \in \mathbb{C}^N$  is the transmitted random communication symbol  $_{\mathbf{X}}$  and  $_{\mathbf{C}}N$  idenotes ather block length  $_{\mathbf{n}}$  The motation  $_{\mathbf{C}}$   $_{\mathbf{n}}^{N}$ denoted and intensionals complex. Englidean Pagage (where the sleperseripVislipmovpohwith:dynplek, WhildiBelarefers.do thehelye dimensional real Euclidean space. We denote hyper RVCrefthe received\signaleanthna€r@a\ Bucdirkularkyssymm\\tridecomplex Gaussian maispewithed congruented in  $\mathcal{A}$ ,  $\mathcal{A} \subset \mathcal{A}$  the  $(0 + \sigma^2 \mathbf{I} \mathbf{I}_{\mathcal{M}})$ . The saalar hom fleis Gaussknown but vdeterministic channel coefficient (determined by the laensing parameters, unkichwis assumed to be constant in all Norman Luces since the sensing parameters such as target positions and velocities remain stable during the communication process in most ISAC systems. The gonfor LSACiontonimultaneously recover the dominunication messageriandi estimate-shensensing I parameters rhas effhong that dualSinuctional signal aneously recover the communication message and estimate the sensing parameters based on the dual-functional signal.

To analyze the performance trade-off between S&C, we first present the mathematical formulation for our ISAC system, which is based on a two-step scheme including message decoding and communication-assisted estimation shown in Fig. ??.

The message-decoding step aims to recover the transmitted to analyze the performance trade-off between SVC, we communication symbol based on the received signal y(x,h), which resembles the communication performance of SOC system. The communication performance of SOC system which is based on a two price use the process of the communication assisted estimation introduced in (2) which includes shown in Fig. ??.

1) A message set  $\mathcal{M} = \{1, 2, \dots, M\}$  with equiprobable The mussages decoding step aims to recover the transmitted to the process of the process

1) A message set  $\mathcal{M} = \{1, 2, ..., M\}$  with equiprobable The mussage decoding step aims to recover the transmitted for multications ymbol has edges the received signal  $\mathbf{y}(\mathbf{x}, h)$  which determines the communication performance of our hard first the partie was applied that  $\mathbf{y}(\mathbf{x}, h)$  which is refined applied that  $\mathbf{y}(\mathbf{x}, h)$  code introduced in [7] which is known to the encoder. The

The confidence and the change coefficient with afform the land a random metallegisth certain prior distribution, while it is mainly determined by the Asterministical by Indiana, sensing managetes in LAC systems. Therefore, the encoder is assumed to have knowledge of the incertainty set of the frame? Therefore, the encoder is assumed to have knowledge of the incertainty set of the frame? Therefore, the encoder is assumed to have knowledge of the incertainty set of the frame? Therefore, while it is also feasible title package and system is confined to a certain set, i.e.,

photoGoih[h], denotesvitielabsolute-walue off-the-complex filmboohatiGuer/dorbent-beencloderlassible expressed the complex number h. Therefore, the encoder can be expressed by  $\mathbb{R}^2 \mapsto \mathcal{X}, \ m \times [|h|_{\mathbb{L}}, |h|_{\mathbb{U}}] \to x_m$ . (2)

Furthermore, the codewords satisfy the power constraint  $X \mapsto X$ ,  $X \mapsto X$ ,

Furthermore,  $t_1 = \frac{N}{2} \sum_{i=1}^{N} \rho_{ij} distribution in the power of the po$ 

- A decoder whæh map the received signal to the mest sage i.e. the constant, ρ is the per-codeword power budget; g: C → M, y → m. (4)
- Purderenore, which everes stars freezeived signal to the message, i.e.,

 $\mathbb{P}\{q(\mathbf{y}) \neq \mathbf{m}\} \leq \epsilon. \tag{5}$   $g: \mathbb{C}^{N} \mapsto \mathcal{M}, \ \mathbf{y} \to m. \tag{4}$ 

where  $\mathbb{P}(X)$  denotes the probability of the random set Furthermore, the decoder satisfies

The communication-assisted estimation step aims to estimation step aims to estimation step aims to estimate a step aims a step aims a step aims to estimate a step aims a step mate the sensing parameters based on the received signal, which where P(X) the sensing performance of our is AC system. For simplicity of analysis, we focus on the estimation of the Tobannol a confficient bit self is nothing paper to while other analysis oftestimating general gensing parameterseit left foreour future works, In particular we director struct the communication ISmbolsestxn= F(x).sTheniove/applyneamaximum-likelihood (ML) sestimator to festimate the channel coefficient since the Mbe estimator than assymptotically tiachiene the cramer Rao lower-bound is? left for somefalgebray we can obtain that, we first reconstruct the communication symbol as  $\hat{\mathbf{x}} = g(\mathbf{y})$ . Then we apply a hraxhmu(x, lk)elthqqq (ML) estimator (6) estimate the channel coefficient since the ML estimator wheresthe potation or achievates the Charmetian transposition of complexervectore and This also AC content at takes full advantage of the communication result to improve the sensing performance of the channel coefficient (which is  $\frac{x}{\|\hat{\mathbf{x}}\|_2^2}$  applied in (6) practical ISAC systems [?].

where the notation  $x^H$  denotes the Hermitian transposi-Bron Performance Metricor x. This ISAC scheme takes full adminiscrabsection covergive a thrief remultive improvement the stitional performance emetrics of communication and veinsings affectly high line rate-berg region is infroduced to Characterize the performance tradeoff.

The performance metric for communication systems with the quasi-state challeful state defined as the achievable communication rate for the  $(N,M,\epsilon)$  code i.e.

nication rate for the  $(N,M,\epsilon)$  code, i.e., In this subsection, we give a brief introduction on the traditional postorial metrics M metrics of communication and sensing, after which the rate-error region is introduced to The performance metric for sensing systems is characterized by the mean squared error (MSE) of the ML estimator, i.e., given any  $(N,M,\epsilon)$  code, the channel coefficient is usually mod-

eled as a random variable with certain prior distribution, while it is mainly determined by the determined by the determined by the parameters in ISAC systems. Therefore, the encoder is assumed to have knowledge of the uncertainty set of the channel gain to design where the notation is AC settings, which is also leasible in practical to the code pook in the ISAC settings, which is also leasible in practical to the care and on variable x.

When posts the communication and sensing performance are taken into quasideration have defined the rate arrow region value communication rate for the  $(N,M,\epsilon)$  code, i.e.,

 $\begin{array}{c}
\mathcal{F}(N,\epsilon) = \overline{\log}\left\{ (R,e) : \exists (N,M,\epsilon) \text{ code} \right\} \\
R = \overline{\mathbb{Q}} : \exists (N,M,\epsilon) \text{ code}.
\end{array} \tag{9}$ 

 $R = \frac{\log M}{N}: \exists (N,M,\epsilon) \text{ code.} \tag{7}$  which collects all the feasible pairs of the communication rate and sensing error metricled by sinsing westernois. Then not provided by the receiver of the representation of sensing when the offer one meets certain minimum requirements, which characterizes the performance tradeoff. Therefore, we focus on determining the boundary of the rate-error region in the following sections of this paper, where the notation of the rate error with respect to the random variable  $\mathbf{x}$ .

When both the communication and sonsing performance are taken into consideration, we define the rate-error Where B denotes the minimum requirements for the sensing performance, and  $R^{\star}$  denotes the rate-error tradeoff. When D tends to infinity, the rate-error tradeoff approaches the maximal achievable rate of the quasi-static channel regardless of sensing which collects all the feasible pairs of the communication which collects all the feasible pairs of the communication performance, i.e., rate and sensing error achieved by the  $(N, M, \epsilon)$  code. Then the boundary Rof (N(ND) reveals, the optimal perfor) mance of communication or sensing when the other one which day been inidely, investigated in whe communication the Errornance tradeoff. Therefore, we focus on determinin Remarkolin In most existing researches on the performance tradgoffs between S &C, then influence of the blocklength Nand the probability of decoding error  $\epsilon$  are not taken into convidention since the Restination orrow is independent 10f the decoder outputs where communication and sensing are separately performed the their occiner and utbentransmitter, the spectivelyel?hrl?lar[2], How ever) when soluble functionalities are refluite that the receiver concurrently, such as in the intelligent webiculab networks: and other knoperative applications, existing tradentil analysiseis inapplicable Asowikibe, shown later, the blocklength and the probability of decoding error induce a tighter connection for S&C and require in depth rate-error which has been widely investigated in the communication

theory [?]HI?] R[A] re-ERROR TRADEOFF ANALYSIS

Remark 1: In most existing researches on the performance cracker of the characterized by rather nontradeoff in our ISAC systems. Note the probability of decoring error are moverable of the active the characteristic of the existing error are moverable of the active blocklength and the probability of decoring growers and environment of the active blocklength and comprehensive bound of the performance tradeoff between S&C and discuss the asymptotic property of the rection of the performance tradeoff between S&C and discuss the asymptotic property of the rection of the performance tradeoff between the blocklength when the blocklength and athar reported the subsection decoring the active blocklength and the probability of the subsection decoring the active blocklength and the probability of the property of the active blocklength and the probability of the property of the active blocklength and the probability of the property of the active blocklength and the probability of the property of the property of the active points. The active blocklength and the probability of the property of the pr

of Rh(Nura:DinaFurthbimorblewhenewegondides the maximal communication intearegardless los sensing performance dit lis proved that [31] decoding error. We derive the achievability (lower) and converse (upper)  $pound_of R^*$  to characterizeRthe (NorthrenRice (Nactorf betweeRcs)(N, tahde) disches the asymptotic property of these two bounds when the which implies that the approximation is asymptotically tight. Dlocklength N tends to infinity.

Therefore, we only need to determine the achievability bound for Rehive Dity where the codewords are distributed on the complexishancesphere Swe present the achievablet bound of Then we investigate the relationship between the MSE given by F(22), and the hoasible set of take codewords a Laparticular, given the setulality  $S_{i}^{N}$ , ive introduce the definition of the waximalohiaslas  $\Phi(w, \epsilon, D)$  the rate-error tradeoff with th Defigition el: The enaximal bias. With respective any subset W  $\subseteq$  Sweis defined as  $R^*(N, \epsilon, D)$ . Furthermore, when we consider the maximal communication rate regardless of sensing per 400 m = 400 m = 400 m sensing per 400 m = 400

Recalling the expression in the feasible set of the codewords are distributed in the feasible set of the codewords are distributed as  $V_{N} = V_{N} = V_{N}$ 

Remarki 3E Recalling the expression shown in (??) (14) find that  $\Delta_W$  equals to the maximum  $N_P$  an error of any  $M_R$  entire 3t-A could be purely of purely of the super-boundard must be reasonably and the interpretation of the super-boundary of the interpretation of the interpretat

With Prior with in a Vind the tide MS less also controlled by the internal bias by the Wich set, which provides distributed bias by the Wich birthes distributed bias by the Wich birthes distributed bias by the Wich birthes distributed by the state of the wind and says is. Then we derive the achievability bound for the anti-citien tradeoff with the tightened power constraint. In particular, we first prove that  $R^*(N, \epsilon, D)$  saturates when D exceeds certain threshold. Proposition 2. The state-error tradeoff  $R(N, \epsilon, D)$  satisfies bound of MSE can be divided into two parts: the first part (Nices D) =  $R^*$  (Sensing aperior  $R^*$ ) and  $R^*$  (Nices D) =  $R^*$  (Sensing aperior  $R^*$ ) and  $R^*$  (Nices  $R^*$ ) are where the reconstructed symbol equals to the transmit(165)

where uRfcat(\(\Delta\) is ydenotes The maximal pacitic valots rate the gandless of the sensing patforhande with the tightehed power obitstraintasybichihasibedneinvostigatediini(2); [?]<sub>EL</sub>The thiteshi old  $\partial_{\mathbf{x}}$  edsegiven aby  $\mathbf{x}$  of  $\mathbf{x}$  with  $\hat{\mathbf{x}} \neq \mathbf{x}$ . In the radar-based ISAC systems where the transmitted symbol is known to the estimato  $D_{\text{ni.e-}}$ ,  $N_{\text{ni.e-}} = 0.4$  which coincides with the existing theoretical results.

With Proposition,??? we only need to derive the achievo ability/hound in the assessa Doise On the birde is other in the following uperposition to the following analysis. Then we define whitian hieva barty Would for the rate ether rate arror wadcoffe Rightenell bis lower bounded by particular, we first prove that  $\tilde{R}^{\star}(N, \epsilon, D)$  saturates when  $Q_2$  exceeds certain thres  $\tilde{R}^{\star}(N, \epsilon, D) \ge \max\{R^{\star}_{\text{com}}(N, \epsilon) + \frac{1}{N}, 0\}$  (17) Proposition 12: The rate-error tradeoff  $\tilde{R}^{\star}(N, \epsilon, D)$  satisfies

$$\bar{R}^{\star}(N,\epsilon,D) \cong \bar{R}^{\star}(2^{N}_{2} + \bar{R}^{\star}(2$$

The function  $I_{X}(a,b)$  is the regularized incomplete beta function. The affice A is denotes the maximal achievable rate tion. The affice A is determined by regardless of the sensing performance with the tightened power constraint, which has been Agestigated in [?], [?]. The threshold  $D_{m}^{o}$  is given by

where the maximal 
$$\frac{b_{\text{max}}^{2}}{N\rho} \stackrel{\text{beas}}{=} \frac{|\hat{p}|_{\text{UU}}^{2}}{|\hat{p}|_{\text{UU}}^{2}} \stackrel{\text{def}}{=} \frac{|\hat{p}|_{\text{UU}}^{2}}{\sqrt{N\rho}}$$
. (16)

where the maximal bras  $\frac{bras}{N\rho} \stackrel{\text{def}}{\to} \frac{h}{|U|} \stackrel{\text{def}}{\to} \frac{h}{|U|} \stackrel{\text{def}}{\to} \frac{h}{|U|} \stackrel{\text{def}}{\to} \frac{h}{|U|}$ . (16) With Proposition  $\frac{h}{|U|}$ ,  $\frac{h}{|U|} \stackrel{\text{def}}{\to} \frac{h}{|U|} \stackrel{\text{def}}{\to} \stackrel{\text{def}}{\to} \frac{h}{|U|} \stackrel{\text{def}}{\to} \frac{h}{|U|} \stackrel{\text{def}}{\to} \frac{h}{|U|} \stackrel{\text{def}}{\to} \frac{h}{|U|} \stackrel{\text{def}}{\to} \stackrel{\text{def}}{\to} \frac{h}{|U|} \stackrel{\text{def}}{\to} \stackrel{\text{def}}$ Sphoifically, there it is  $R^*(N, \epsilon, D) = 0$  for  $0 \le D \le \frac{\sigma^2}{N_0}$ .

Pr*Skatsh of Proof* For As for the analysis of the achievability bound ffwR guarantog the sensing uperformance through constraining the feasible set of the  $(N, M, \epsilon)$  codewords, which is shown Nn eFig.) 2(a)nalm franticulare) the codewords can take values on the entire hypersphere with  $D^{N} \geq D_{\rm m}$ , which Who there exists  $(R^{\perp})$  is  $(R^{\perp})$  is  $(R^{\perp})$  in  $\sigma^2/N\rho \leq D < D_{\rm nl}$ , we restrict the codewords to the set  $\mathcal{W} \subset \mathcal{S}^N$  with  $\Delta_{\mathcal{W}_2}^{-1}$  with  $\Delta_{\mathcal{W$ performance requirement, where the largest feasible set W is a hyperspherical cap. The coefficient  $\alpha_L$  can be viewed as the area ratio of the hyperspherical cap to the hypersphere. Details

are omitted due to the lack of  $\frac{\text{pace.}\Delta_{W_L}^2}{\text{Remark 4: With Proposition}}$  (19) we can obtain the achievability bound for the rate-error tradeoff  $R^*(N, \epsilon, D)$ . We denote by  $R_{\text{com}}^{\text{L}}(N,\epsilon)$  as the achievability bound of  $\tilde{R}_{com}^{\star}(N,\epsilon)$  determined by Nie expression of which is provided in [?]. Then the achievability bound  $R^{L}(N, \epsilon, D)$  of

 $R^{\star}(N, \epsilon, D)$  is given by Specifically, there exists  $\bar{R}^{\star}(N, \epsilon, D) = 0$  for  $0 \le D \le \frac{\sigma^2}{N\rho}$ . Sketch of Proof: As for the analysis D of D is D. phility hound Lwe Muarantee the sensing performance through constraining the feasible set of the  $(N, M, \epsilon)$ codewords, which is shown in Fig. 2(a). In particular, the codewords can take values on the entire hypersphere with  $D \ge D_{\rm m}$ , which implies that  $R^{\rm L}(N, \epsilon, D) = \tilde{R}_{\rm com}^{\rm L}(N, \epsilon)$ . When we restrict the  $\sigma^2/N\rho \leq D < D_{\rm m}$ , we restrict the collecthis laubsedtionetwe/present that lookyerse bright of the tate-crioritradeoffe Rink, peHormance requirement, where

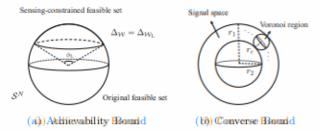


Fig. 2: Geometric illustrations of the additionability and converse bounds. (a)a)FoFothet heh iewlability bitymb, othe de itsingsperform profoD constrains the striginal toudewigid spacel (block sphere) to the spanish settΔφ,f (aed)splsericΔl,cap)e(b)pFor ithel capver(b) bound; the maximal raterial obtained abore packing the Moranoi degion and us phere of cradius reginto (the signal-space adduich is ismalleretsagothe splack, spherb of radiation it but, bigg of them stripeblack ispherical, short longradiations and black spherical shell of radius  $r_1$  and  $r_2$ .

th Note that the MSE set the ISACh system list lower bounded Exefficient  $\gamma_L$  can be yiewed as the area ratio of the hyperspherical SED to the Hypersphere. Details are omitted due to the lack of space  $\|\mathbf{x}\|_2^2$   $\mathbb{E}_{\mathbf{x}}\{\|\mathbf{x}\|_2^2\}$ 

Thereforking (With Purpositional? (NuM,?) codeconasolat tean sansoychievaveitäge bowerlconstraint givenenger tradeoff  $R^{\star}(N, \epsilon, D)$ . We denote by  $R_{\text{com}}^{L}(N, \epsilon)$  as the achievability bound of  $\tilde{R}_{\text{com}}^{\star}(N,\epsilon)$  bound of  $\tilde{R}_{\text{com}}^{\star}(N,\epsilon)$  bound of  $\tilde{R}_{\text{com}}^{\star}(N,\epsilon)$  bound of  $\tilde{R}_{\text{com}}^{\star}(N,\epsilon)$ of which is provided in [?]. Then the achievability bound  $R^{\text{II}}$ Therefore, whe Richote by  $R_{\text{IS}}(N_{\text{VEn}}B)$ , the maximal achievable communication rate for the  $(N, M, \epsilon)$  code satisfying power constraints (??) and (??), which is a converse bound for the rate-error (tradeout  $R^*$ ) ( $N, \frac{\epsilon}{\epsilon}, \frac{\epsilon}{N}$ ). Note that the expectation term in (R2) makes) it difficult to obtain the ≥xact expression of  $R^{\cup}(N, \epsilon, D)$ . We provide a approximation of it in (2h) following proposition.

Proposition 4: The maximal achievable communication Bate  $(R_0 \cap (A \cap B))$  used the  $(N, M, \epsilon)$  code satisfying the power constraints (??) and (??) satisfies In this subsection, we present the converse bound of the

rate-error trad  $\underbrace{\log \mathbb{E}(\mathbb{R}^*(N_{\mathbb{U}^c}^{2N})D)}_{\text{Note that the MSEVof the ISAC system is lower-bounded}} R_{\text{com}}^{\text{U}}(N,\epsilon,D) \leq R_{\text{com}}^{\text{U}}(N,\epsilon)$ 

where  $\gamma_{\mathbf{U}}$  is given by  $\gamma_{\mathbf{U}} = \sigma \mathbf{z}_2 / r_1$ . The coefficient  $r_1, r_2$  are given by  $\text{MSE} \geq \mathbb{E}_{\mathbf{x}} \left\{ \frac{\|\mathbf{x}\|_2^2}{\|\mathbf{x}\|_2^2} \right\} \geq \frac{1}{\mathbb{E}_{\mathbf{x}} \{\|\mathbf{x}\|_2^2\}}$ . (22)

Then for any  $(R_N e) \in \mathcal{K}(N, \epsilon)$ , the  $(N_1^2, N_1^2, \epsilon)$  gods must at least satisfy the average power constraint given by

respectively. We denote  $R_{\text{conn}}^{\text{U}}(N,\epsilon)$  as the converse bound of the maximal communication rate regardless of sensing

performance given by Therefore, we denote by  $R^{\mathrm{U}}(N,\epsilon,D)$  the maximal achievable communication  $\epsilon$  tatelographe  $(N, M, \epsilon)$  que satisfying power constraints (??) and (??), which is a where sheb conflicion the isatther smallestlev ff such (Mat, The following tinequality ebolds on term in (??) makes it difficult to obtain the exact expression of  $R^{\mathrm{U}}(N,\epsilon,D)$ . We provide a approximation of it in the following proposition. (27)

where Bisition 4: is the complex by perbalbly ith madius in a tion  poweketalstafiiRtsof??) Accordingsatisthes hypothesis testing theory, the optimal decoder with equiprobable messages is the nRixin(uv), d)kelihood decoder, the Riccoding Digitors of (which are called the Voronoi regions [?]. Then Proposition ? 43 inspired from the idea of sphere packing where the Voronoi region is treated as the hyperball with radius  $r_{\epsilon}$ . As is shown in Fig. 2(b), the first inequality in (??) is obtained by the sphere packing within the hyperspherical shell of radius rat and 23) while the second one is obtained by the sphere packing within the pentire hyperball of tending viv. Detail stare comitted due to thetheknokspace communication rate regardless of sensing personark 5: Proposition ?? provides a converse bound of the rate-error tradeoff  $R^{\star}(N,\epsilon,D)$  which takes the influence of D into consideration NHowever  $g_2R_{\rm p}^{\rm LL}(N,\epsilon,D)$  is nearly independent of D with large N since the term  $\log_2(1-\gamma_{\rm U}^{2N})$ where the orentaries with the blocklength, which equals to the converse bound alicythe lifaximal communication rate regard of the sensing performance given by the RHS of (??), (pc.)  $R^{\mathrm{U}}(N,\epsilon,D) \approx R^{\mathrm{U}}_{\mathrm{com}}(N,\epsilon) = 2\log_2(r_1/r_{\epsilon})$ . Therefore, the Wervarian of a righter to overse bound is both wether dougive ia more accurate characterization on the performance tradeoff betweenestee theory, the optimal decoder with equiprobable messages is the Asymptotic Analysis of the maximum-likelihood decoder, the decoding regions of wHn: this esubsection. Wer present the safying totic Panalysis for the achievability and converse, bounds, of the aster grown tradeoff  $R_{\text{orwhen}}^{\dagger}$  runer the chlocklength  $N_{\text{o}}$  sends to infinity with radius  $r_{\epsilon}$ . As Ascording to the the cretical tanalysis in [2] where the variety bound  $R_{\rm con}^{\rm L}(N_{\rm loc})$  of the maximal communication rate regards less of sensing performance visi preved to satisfye is obtained by the sphere packing within the entire hyperball of radius r<sub>1</sub>. Details are of spade.

Remark 5: Proposition ?? provides a converse bound for any  $\epsilon \in (0,1/2)$ . Note that there exists  $\lim_{N \to \infty} \frac{1}{N} = \frac{1}{N}$ 

Therefore, the asymptotic expression of the converse bound  $R^{\mathrm{U}}(N.\epsilon.D)$  is given by

C. Asymptotic Analysis  $\lim_{N\to\infty} R^{\rm U}(N,\epsilon,D) = \lim_{N\to\infty} R^{\rm U}_{\rm com}(N,\epsilon) = \log_2(1+\frac{N\rho|h|_{\rm U}^2}{2})$  In this subsection, we present the asymptotic analysis for the achievability and converse bounds of the rate-error tradeoff this whether the believe that analysis, we find that the performance tradeoff between S&G signishes, as the objection of the implementation for this phenomenoles considering ISAG system where the (or all sty) code consists of two parts: the first part of length  $\sqrt{N}$  is fixed as the pilot that while the other part of length  $N - \sqrt{N}$ 8

for any  $\epsilon \in (0,1/2)$ . Note that there exists  $\lim_{N\to\infty} D_{\rm m} = 4\epsilon |h|_{\rm U}^{2.5}$  according to (??). We can obtain  $\lim_{N\to\infty} R^{\rm L}(N,\epsilon,D) = \lim_{N\to\infty} \tilde{R}_{\rm com}^{\rm L}(N,\epsilon) = \log_2(1+\frac{N}{\sigma^2}) \log_2(1+\frac{N}{\sigma^2})$  for any  $D>4\epsilon |h|_{\rm U}^2$  and  $\epsilon \in (0,1/2)$ . Then we focus on the asymptotic analysis for the converse bound  $\frac{2}{N^{\rm H} \ln N^{\rm D}} \tilde{e} D$   $\frac{10}{N^{\rm D}} \tilde{R}$  ccording to analysis to that in [??], there exists  $\frac{1}{N^{\rm D}} \tilde{e} = 1$ ,  $\forall \epsilon \in (0,\frac{1}{2})$ . (30) Therefore, the asymptotic expression of the converse bound  $\frac{N^{\rm H}(N,\epsilon,D)}{N^{\rm D}}$  is given by  $\lim_{N\to\infty} R^{\rm H}(N,\epsilon,D) = \lim_{N\to\infty} R^{\rm U}_{\rm SO}(N,\epsilon) = \log_2(4+\frac{N\rho_{\rm S}^{\rm H}h|_{\rm U}^2}{\sigma^2})$  (31)

Fig. 3: The achievability and converse bounds for the rate-error tradeoff with varying code blocklength.

According to the above theoretical analysis, we find that the performance tradeoff between S&C vanishes as the bleade as the Communication data: The seising pintermatee tion for this revenum snow a consider a mile A B system/vln/re the (Nis the MSE consists of two parts the first part of leogt line Philis fixed anithearily datas while there have go length Which Unipites and the henconymunication of the Tibaysensingererformennne offathis WillAGreystfocktength Art Note that the prior data or length whith win the Millende the charmen conflicted too Pateins of figure that he wint the Millende the charmen conflicted to Pateins of figure that the second of the constraint of the conflicted that the second of the our principle in the procession of the pilot data with large block length Via Note that the expressions benetel acide validing tained using eithe boungi bellener on tinic gating t rate asymptotically since there exists limy when we have Worefine that the Ski Swieger or the channel campled when the black of sensing, the interval [[ $h|_{V}$ ,  $|h|_{U}$ ] approaches to the true Utahnien gain it has which carso that the that the tachic exprinty sions of the achievability and converse bound depend on the range of the channel gain, i.e.,  $|h|_{L}$  and  $|h|_{U}$ . When we have plure Recharte Portor kinewRedger of the elfannel (32) under the assistance of sensing, the interval  $[|h|_L, |h|_U]$ AMBIBACHES to 1980 trute Chain/fel gasin | 1982 Whith SANS hispites Shappon channels apacity of converse bound coincides, i.e.,

 $\lim_{N\to\infty} R^{L}(N, \epsilon, N) = C \qquad (32)$ 

In this section, we perform some simulation experiments to workstindate our theoretical bounds and casculate the rate error region in the rate error region in the region.

First, we verify the effectiveness of the achievability and converse bounds derived for the rate-error tradeoff R\*I(Mhis B): (Considerations) the converse bounds derived for the rate-error tradeoff R\*I(Mhis B): (Considerations) the converse blands and the anoise alariante the set easy post region durmerical spectively. The channel gain is assumed to be longered the effective fixed five fixed in the additional littly and converse bounds so fithing at formal to the longered fixed fix

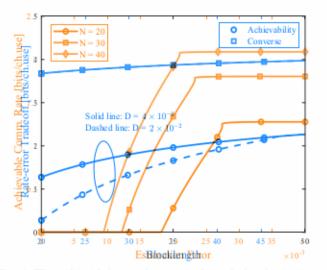


Fig. 4: The archievable little sembrongions with thels any ingeblocklength tradeoff with varying code blocklength

sensing performance is set to be  $D=4\times 10^{-2}$  and  $2\times 10^{-2}$ , variance are set as  $\rho=10$  and  $\sigma=1$ , respectively. The variance set as p=10 and  $\sigma=1$ , respectively. The channel gain is assumed to belong to the set  $|h| \in [1,1.5]$ . The conding blinking and conversed bhandle of onverse the punds always outperform the achievability bounds, which is invariant with the sensing performance of since the loss term almost vanishes 1 according to our theoretical analysis. As for the achievability-bounds, the grate-error tradeoff increases as D increases since the sensing constraint is relaxed. When W is large enough, the S&C performance is decoupled which always outperform the achievability bounds, which is implies that the two achievability bounds converge to the same  $K^{-1}$  vanishes according to our theoretical analysis. As Then we calculate the rate error region  $\mathcal{F}(N,\epsilon)$  numerically which are based on the achievability hound singe it can provide a-more accurate sharacterization of the performance tradeoff hetween S&G than the converse bound. The system parameters are sets the same as above . The achievable rative fror region is In the achievability bound sinAccording too Fige 2? mwee findulate as Dadnoreasus at the aphievable matece/trachminisetoveber 2&rC attafirste Wherer 12 bxcgeds Thethreshold-bigggrethan are/sNipt]thesachiexable rate Rhstartshte increase since the achievability bound requires the feasible ckodegeord-elected/blarge=e200gb, to carsy information. As AD moves close to Dm whence efficient Dapproaches 1/2 according to: (22) PHowevers it is witches from file 2, to when to  $D_{converse}$  past  $D_{co}$  hothich gleads to the  $4/N_{\rho}$  sharp increased in the craft-error strade offershowns inc Figh 2 2acThem that boundary iscinnation of a Delindicating that the LS&Cepenformanca is decoupled on. As D moves close to  $D_{\rm m}$ , the coefficient  $\gamma_{\rm L}$ ap Furthernsore/ The carea diff the crate-error I region in ite assist with thenblocklength whesing the vperformance, tradeoff between S&C //Anishespininthe-larger blockletigth regimed-Asf the oblocks Teigth? NT lends the liminidar this boundary t of the laubievable tates ethor Segion capproaches the borizontal line with height logo(itther)\u00e4\u00fc\u00e4

## V. CONCLUSION

This paper provides a characterization of the performance tradeoff between S&C in a SISO ISAC system with finite blocklength where the rate-error tradeoff is introduced as the performance metric. In particular, we derive the achievability and converse bounds for the rate-error tradeoff, after which the asymptotic analysis is performed to show that the performance tradeoff vanishes as the blocklength tends to infinity. Finally, our theoretical results are verified by the numerical experiments. Future work will focus on obtaining tighter bounds for the rate-error tradeoff as well as the extension to MIMO ISAC systems. The contributions of this paper give insights to the understanding of the fundamental tradeoff and the future system design in ISAC.

Fig. 4: The achievable rate-error region with the varying blocklength

Estimation Error

with the blocklength N, since the performance tradeoff between S&C vanishes in the large blocklength regime. As the blocklength N tends to infinity, the boundary of the achievable rate-error region approaches the horizontal line with height  $\log_2(1 + N\rho|h|_L^2/\sigma^2)$ .

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