A New Family of Perfect Polyphase Sequences with Low Cross-Correlation

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Abstract — Spead depectrum multiple access systems demand minimum possible cross-correlation between the sequences within a itself of sequences drawing agood gruto-correlation appropriies. Through a connection between generalise demands sequences and filmentine arrials, ewel presently a family of perfect sequences with some consecutive having a clarger family size, confpared with previous dworks. In painticular, the family size confpared with previous dworks. In painticular, the family size can be equal to the square ropt of the period when the period of the perfect sequences is even before a statistical symbols. Of the perfect sequences of even perfect with above cossecure patient is very above one since the previous works.

Index terms—PerfBerfsequences, uperfect queofcotrelation, townebrtiss—convelations—clawelations, coFtoberitine Falmays, piotyphaseysequences.

I. Introduction

Sequences and their properties have been widely studied in Sequences and their properties have also applied in Sequences and their properties have also are properties and properties in a correlation of their pharacteristics of the communication of the correlation properties thave been used in a communication of the control of the cont

The periodic cross constant and show two complex present the period V(t) = V(

$$R_{\mathbf{u},\mathbf{v}}(\tau) = \sum_{t=0}^{N-1} u(t+\tau)v^*(t), \quad 0 \le \tau < N,$$

where N is appositive integer, # + is taken knowled No and an (t) of (t) is appositive integer, # + is taken knowled No and an (t) of (t) is appositive integer of the fed integer of the interest of the periodic largest conditations function tis reallest integer of the interest interest in all the perfect in all the perfect integer of the integer of

Let S be a set of M sequences of period N. The maximum Let S be a set of M sequences of period N. The maxout-of-phase periodic auto-correlation magnitude is denoted imagnitude of the periodic auto-correlation magnitude. out-of-phase periodic auto-correlation magnitude is denoted in the following periodic auto-correlation magnitude by R_a and defined by $R_a = \max\{|R_{s_i}(\tau)|: s_i \in \mathcal{S}, 0 < \tau < 1 \}$. The maximum periodic cross-correlation magnitude $s_i \in \mathcal{S}, 0 < \tau < N \}$. The maximum periodic cross-correlation magnitude $s_i \in \mathcal{S}, 0 < \tau < N \}$. The maximum periodic cross-is denoted by R_c and defined by $R_c = \max\{|R_{s_i,s_i}(\tau)|: \text{correlation magnitude is denoted by } R_c = \max\{|R_{s_i,s_i}(\tau)|: \text{correlation magnitude is denoted by } R_c = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. A lower bound on $R_{\max} = R_c = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. A lower bound on $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. A lower bound on $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. In the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau < N \}$. And the period of $R_{\max} = \max\{|R_{s_i,s_i}(\tau)|: \text{si} \neq s_j \in \mathcal{S}, 0 \leq \tau$ quences with low correlation. Perfect sequences have ideal autWeomelationt icedaRly =intenthese familiesiliAsother bound sedjednihes Sartvateobound r@htimplieSethet tRsegneviees Auset infeperfact ocquencesionceting, this bound isneafledearf aptilinal Senoth perfects degalbackth Estansive research [Tais) been done 6h. bow 18 generate to primat families of sperfect regules dos (13). is]ca(Pèd [2]n [chtihh] [3èt [3f, pelfe[3],sét)uefdes[?Extensèse wearks (17) (27) (27), (27) (27), (27) (27) (27) (27) (27) (27) (27) (27) (27)of perfect sequencess with optimal cross-correlation is equal (d) p [?] 1, [where p lis the smallest sprifte [divisor, off] the ?period [V]. Recent works the hand bershow prince the stamely esize with be tlanger than porrelation is set emined by the existence of well-studiest combinatoria biobjeths; pircular Morentine awayss How/ever(?)thesewconstructions maked izoncacircular a Florentine

arrays, that producer the idds by dtlargex families of anyolist ddied formbddaperiodsobWhen the pleniddoisentien; the conkleuctions vické families of isize based on circular Florentine arrays than this dupertive despose lacgustruction of perfect sequences with power brelation based or or brischen art Florentine arriags This constitutions in the constitution of perfect seduences wither R. we=provose where ship in the operior feof shousequences;tlThlofamity relationer dended on the registerical of Florentine arrays; swhich is ogretated than allowin the tordvious sy forks il 4 no fparticular sethue numbert bf Poerfe2 t/sequences Nith thw cress-dorfetationecambees. AT for fiverily/VsiTables22mblates the above previous works it works it was sults hich is greater than that in the previous works. In particular, the number of perfect sequences with low cross-correlation can be \sqrt{N} Ar Elementine Taking ?? relates the above previous works to oua fieshits $\cdot n$ (circular) Tuscan-k array has m rows and n columns such that 1) each row is a permutation of n symbols and 2) for any two symbols a and b, and for each t from 1 to k, A. Florentine arrays there is at most one row in which b occurs t steps (circularly) An $m \times n$ (circular) Tuscan-k array has m rows and to the right of a. In particular, a (circular) Tuscan-(n-1) array n columns such that 1) each row is a permutation of is referred to as a (circular) Florentine array. When m = n, we n symbols and 2) for any two symbols a and b, and call them (circular) Tuscan squares and (circular) Florentine for each t from 1 to k, there is at most one row in squares, respectively. which b occurs t steps (circularly) to the right of a. In For each positive integer $n \geq 2$, we denote F(n) the particular, a (circular) Tuscan-(n-1) array is referred to maximum number such that an F(n) × n Florentine array as a (circular) Florentine array. When $m=n_{F_c(n)}$ we call exists and $F_c(n)$ the maximum number such that an $F_c(n) \times n$ them (circular) Tuscan squares and (circular) Florentine circular Florentine array exists. By definition, $F(n) \geq F_c(n)$

them (circular) Tuscan squares and (circular) Florentine circular Florentine array exists. By definition, $F(n) \ge F_c(n)$ squares, respectively. For all n, because any circular Florentine arrays are also For each positive integer $n \ge 2$, we denote F(n) the Florentine arrays maximum number such that an $F(n) \times n$ Florentine array Lemma and F(n) the maximum number such that an F(n) F(n) eircular Florentine, analy exists. By definition, F(n) F(n) = F(n) F(n) and F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F(n) and F(n) = F(n) and F(n) = F(n) are also F(n) = F(n) and F(n) = F

 $F_{\text{emma}}(n) = n - 1$ when n is a prime.

Lemma (2) = 1 when n is even, and

- (2) F(n) ≤ K_c(n)d≤ n − 1, where p is the smallest prime
- (2) ₱(th)r≥fp(napd1) for all n, and
- (3) $F(n) \ge n-11$ and $\Phi(n \text{ is } 1) \Rightarrow \text{rime } 1$ when n is a prime,

Lemma 2. [?]

- (4) $F(n) \geq p-1$ and $F(n-1) \geq p-1$, where p is the
- (2) smallest prime divisor of n. $F(n) \ge F_c(n+1)$ for all n, and

(3)NoTe(th)at the fact that F(n)- \geq) $F_{\overline{c}}(n+1)$ for all m is subsciause, any $E_{\overline{c}}(d+1)$ circular Florentine rows on n+1 symbols can lead to the same numbed of (ows dn) \geq symbols by educating the one symbols in each cody. With this fact and the lower bound on

F_cNoteonbeautherine boths $F(E(n)) \supseteq H_c$ and F(n) for all p_F , is where sp is such F_c (mall ds) prime a divisor rotion. It who leaves that F_c (n) observante and F_c (ne such community when we is no prime bols by deleting any one symbol in each row. With this fact and the lower both S_c and S_c and S_c and S_c are considered and S_c and S_c are considered and S_c and S_c are considered as S_c .

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the multiplication table mod (n+1), where n will be prime. Exhaustive search for Florentine arrays quare exists. Taylor [?] gave a table of all possible values of F(n) for $1 \le n \le 2$ which was later updated by Hopg winds Song [?]. (See Table ??) For more works on Tuscan arrays, essentially is from the multiplication table mod (n+1), where $n \ne 1$ is prime. Exhaustive search for Florentine arrays has been done by many researchers. Taylor [?] gave a table of all possible values of F(n) [?] gave a table of all possible values of F(n) for $1 \le n \le 32$, which was later updated by Hopg where $1 \le n \le 32$, which was later updated by Hopg where $1 \le n \le 32$.

T2	1	1 1	on a	10	21	(9), (9), 21
For mo	ore ₂ w	orks ₂ on	1280	an agrays.	22	[1], [22.
	307	DI ² E I	13	.12, 13	23	22,23
	4	DIAE I	14	ssipie viii	es ₂₄ pr	$r_{6}^{(n)}$, $r_{24}^{(n)}$
	ñ	F(n)	1,5	$7, F(n)^{15}$	25	$6, F(n)^{25}$
	- 6	6	16	16	26	6,:::,26
	2	6	17	16 ₅₂ 17	37	$6, \cdot 22, 27$
	8	3	18	12,13	28	22,23
	- 9	§.	19	7,18,1914	38	6,28,2924
	140	1,0	29	9 ;:::; 2 9	39	$6, \frac{30}{25}, 25$
	- 6	6	16	16	26	$6, \cdots, 26$
	7	в	17	16 17	97	6 97

denotes the ring of integers modulo n_{29} the rows are indexed as 1 to m_{10} By definition, each row is appermutation over \mathbb{Z}_n , denoted by β_i for $1 \leq i \leq m$. These permutations have the following property. \mathbb{Z}_n denotes the ring of integers modulo n. The rows are Lemma 3s Fet $m \leq p_i$ if for $1 \leq i \leq m$. These permutations have the following property. \mathbb{Z}_n denotes the ring of integers modulo n. The rows are Lemma 3s Fet $m \leq p_i$ it finite we have the following property.

An another than the property of $m \leq p_i$ and $m \leq p_i$ for $m \leq p_i$ for $m \leq p_i$. These permutations have the following property $m \leq p_i$ for $m \leq p_i$

Then $\mathbb{Z}_{(i,j)}^{\mathcal{B}}$ for 2 and the bounded that $i \neq j$ and $l \in \mathbb{Z}_n$,

Proof. Let addition be in \mathbb{Z} and let $\delta(x) = \mathbb{1}(x \geq n)$ where $\mathbb{1}$ is the indicator function. Then δ indicates whether argument **Theorem 1** Theorem 1 Theorem 2 Theorem 2 Theorem 3 The

PrFof.aliyet $\in Cliptiond$ $b \neq in$ Let $t_i \text{ tild} \in C'(b(x)) \text{ and } t \text{ life}(x') \neq in$ where $t_i \text{ tild} \in C(t_i) \neq in$ (if $t_i \text{ tild}$). Without it is is constality; $t_i \text{ tild} \in C(t_i) \neq in$ is $t_i \text{ tild} \in C(t_i)$. Which out it is is constality; $t_i \text{ tild} \in C(t_i)$.

For any $l \in \mathbb{Z}_n$ and $i \neq j$, let $t, t' \in \mathcal{N}^l_{(i,j)}$ and $t \neq t'$. First we prove that $\delta(t+l) \neq 0$ and $\delta(t+l) \neq 0$. Without loss of generality, let $\delta(t') \neq \delta(t') \neq \delta(t'$

We assume that $g(t) \pm l g(t' + l) = (f(t' + l) + l) = (g(t' + l) + g(t' + l) = (g(t' + l) + g($

$$(\beta_l(t')) = \beta_l(t') \pm l(p + q) + l(p + q)$$

We assume that $t/\delta(t+l)cn\delta(t(t+l)=cn)$ It follows that

$$(t't'+t)^t \mod n - ((t+l) \mod n)$$

Then the pair $(\beta_i'(t), \beta_i(t)) \equiv (\beta_j'(t+t)) \mod n)$, $\beta_j((t'+t)) \mod n)$ $\triangleq (a'b)$ with b being the (t'-t)-th step to the right of a appear at two different rows i and j, which contradicts the Then the pair $(\beta_i(t), \beta_i(t)) = (\beta_j((t+t)) \mod n), \beta_j((t'+t)) \mod n)$ of Florentine arrays. Therefore, $\delta(t+t) \neq \delta(t'+t)$ $\delta(t'+t) \pmod n$ with $\delta(t)$ being the $\delta(t'-t)$ -th step to for $\delta(t)$ the right of $\delta(t)$ appear at two different rows $\delta(t)$ and $\delta(t'+t)$ step to the right of $\delta(t)$ appear at two different rows $\delta(t)$ and $\delta(t'+t)$ the right of $\delta(t)$ appear at two different rows $\delta(t)$ and $\delta(t'+t)$ step to the right of $\delta(t)$ appear at two different rows $\delta(t)$ and $\delta(t'+t)$ the definition of Florentine arrays. Therefore, there exist $\delta(t'+t')$ for $\delta(t)$ with $\delta(t'+t)$ are $\delta(t'+t')$ for $\delta(t'+t)$ must share the same value. This there exist $\delta(t'+t')$ and $\delta(\delta(t'+t'))$ must share the same value. This there exist $\delta(t)$ and $\delta(t'+t)$ must share the same value. This there exist $\delta(t)$ and $\delta(t'+t)$ must share the same of the elements same for any $\delta(t'+t')$ and $\delta(t'+t')$ and $\delta(t'+t')$ can not be the $\delta(t'+t')$ and $\delta(t'+t')$ and $\delta(t'+t')$ and $\delta(t'+t')$ can not be the $\delta(t'+t')$ and $\delta(t'+t')$ and $\delta(t'+t')$ can not be the same for any $\delta(t'+t')$ for $\delta(t'+t')$ consequently, we have $\delta(t'+t')$ and $\delta(t'+t')$ can not be the same for any $\delta(t'+t')$ consequently, we have $\delta(t'+t')$ can not be the same for any $\delta(t'+t')$ consequently, we have $\delta(t'+t')$ can not be the same for any $\delta(t'+t')$ consequently, we have $\delta(t'+t')$ can not be the same for any $\delta(t'+t')$ consequently, we have $\delta(t'+t')$ can not be the same for any $\delta(t'+t')$ consequently, we have $\delta(t'+t')$ consequently, we have demonstrating that the bound is fight.

B. For the folyphase sequence array in Table ??, $\mathcal{N}_{(1,2)}^2 = \{3,5\}$, demonstrating that the bound is tight.

A polyphase sequence is a sequence whose elements are all complex protection of the sequence of the sequence x is

a rational/priumberseand-rice=is/a-keddanyestudiosehakenbers doneadn other denstructions of other feet polyphase sequences) Mowe [2] classified: allaknownbperfectdpolyphase—sequences introlifeurhaciasiesen generalised t Frankrist quetices: [31], pgefiert ališed lehiepstike eseggericksw[??] Mileviškil selgdengesn[@erfand pedfech polyphasersequences fassociated swittergendralide Brbent functionss[2], Mow alised proposition surfictions of penfectopolyphasadsequencespandpeonjectured that the unified construction describes (all the (perfect) polyphases equences (that existed construction of perfect polyphase sequences and cofijeneraliziethaFrank usefjudncesstarection classribés përfect pelyphaseo sequencese whichearth from one-dimensional bent fund tionralid over proposed by Kesman Scholtz and Welch (2). Thesplaceucesewere first discovérod by Frahk and Zadoff (2) funthteorasend werte and prosbeing bytheKidentity Spermutation Heilmiller . [Theourse the requences fusion and the little of the state of the stat ease @fa-poifine? N inwhere base also=an Oarbitchry function the

ZentGeneratized Hiank sequences and amore general family, and $\pi^{h(t)}$ defined as follows case of prime N, where h is also an arbitrary function on \mathbb{Z}_N . Generalized Frank sequences Lemma 4. [?] Let N be a positive integer and ω_N be a are a more general family, and are defined as follows.

L(i)n m: be a p? pn utatNn utatNn utatvtatvtatutatvtatutat

Then $\pi s(t)$ appropriation of elements in \mathbb{Z}_N and $totall 0 \le t$ (ii) $t_2 \sigma < t$ as a property sequence of pertod $N_0^2 \cdot \mathbb{Z}_{N^2}$.

They is firm \(\mathbb{N}^{N_0} \) there are in tombet \(N^{2m} \) perfect sequences \(\text{bf} \), perfect \(N^{2m} \) there are in tombet \(N^{2m} \) perfect sequences. The maximum cross-correlation magnitude of any By Lemma \(\frac{1}{2} \), there are in total \(N \) in the perfect sequences of period \(N^{2m} \). In order to generate an optimal quences of period \(N^{2m} \). In order to generate an optimal ies on perfect sequences with optimal cross-correlation (see set from these sequences, the maximum cross-correlation Table \(\frac{2}{2} \). However, these constructions are trivial when \(N \) magnitude of any two distinct sequences should be \(N \). Is even, which means no pair of perfect sequences of even here exist many studies on perfect sequences with period with optimal cross-correlation (see Table \(\frac{2}{2} \)). However, these next section, we present a family of perfect sequences of constructions are trivial when \(N \) is even, which means no period. \(N^{2} \) based on Lemma \(\frac{2}{2} \), whose maximum cross-correlation has been reported. In next section, we present a The number of sequences in this family can be \(N \) when \(N \) is family of perfect sequences in this family can be \(N \) when \(N \) is family can be

In this section, we build a connection between generalised III. Families of perfect sequences with low Frank sequences and Florentine arrays, which allows us to generate a family of perfect sequences with a large family sizh and so sections, correlation, a connection between generalised Planks sequencial definition a connection between generalised Planks sequencial definition array which allows the theoretical definition of the sequence of periodic lands are exists. Let $A = N\{Be B_2 positive vinites a squence of periodic verifical defined as the property of the section of the sectio$

Theorem 1. The set \mathcal{S} defined by $(??)^{\sigma_1}$ is a family of perfect sequences of size F(N) with $R_c=2N$. where $t=t_1+t_2\cdot N$, $0\leq t_1,t_2< N$, and σ is an arbitrary Remotioning math R_0 and R_0 is a permutation over R_0 , each sequence in R_0 is perfect by Lemma R_0 . For any shift $0\leq \tau<0$ theorem 1. The set R_0 defined by R_0 is a family of perfect R_0 , we rewrite R_0 is a family of perfect R_0 , we rewrite R_0 is a family of perfect R_0 , we rewrite R_0 is a family of perfect R_0 , we rewrite R_0 is a family of perfect R_0 , where R_0 is a family of perfect R_0 , we rewrite R_0 is a family of perfect R_0 .

Proof. Since each β $\oplus A$ if θ_1 permutation over \mathbb{Z}_N , each sequence in \mathcal{S}^{τ} is perfect by Lemma \mathcal{S}^{τ} . For any shift $0 \leq$

TAIABLE IIII Franilities of perfect polyphase sequences with lower ossse or relation on

	RReferences	[[?]]	[2][2]	[1][1]	[7]	[?]	[?]]	[?7]	[[]]	[?]]	[2]	[7][7]	thisi p apeper	
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N.N., ryma add/h are positive integers; Pfs is nod/optime: Q is Qnioddivided integers the smallest prime divisor p of the pariodlest prints the viscoallest thrime relief or of \hat{r} ; is the smallest the maximization of the pariodlest prints and the smallest thrime relief of \hat{r} ; is the smallest thrime relief of \hat{r} . Fo(roulis Florenting range existing sufficient in a fraction of the state of the st F(N) is the maximum number such that an $F(N) \times N$ Florentine array exists;

Let s_i and s_j be two sequences in S, where $1 \le i \ne j \le$ FONN THE CROSS CONFEDENTIAL DEEM SON STATE OF STRIVEN DV. and define

$$R_{\mathbf{s}_{i},\mathbf{s}_{j}}(\tau) = \sum_{t=0}^{N^{2}-1} \begin{cases} i(\theta + \tau^{i} \mathbf{f} s_{j}^{*}(t) \tau_{1} < N, \\ 1 & \text{if } t_{1} + \tau_{1} \ge N. \end{cases}$$

Let s_i and s be wo_s equalities in S^{r_i} where $P \leq t_i \neq 1$ F(N). The cross-correlation between s_i and s_j is given by

$$\begin{split} R_{\mathbf{s}_{i},\mathbf{s}_{j}}(\tau) &= \sum_{N=0}^{N^{2}} \sum_{\substack{N \in \{t_{i}\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_{i}+\tau\} \\ N^{2} = 1}}^{N^{2}} \sum_{\substack{N \in \{t_{i}+\tau\} \\ N^{2} \in \{t_$$

The inner sum of the last identity above is zero unless $= \sum_{\beta_i(t_{\mp}0+\tau_1)} \omega_{N^2}^{N\cdot\beta_i(t_1+\tau_1)(\tau_2+\delta_{t_1,\tau_1})+\sigma(t_1+\tau_1)-\sigma(t_1)} \\ \beta_i(t_{\mp}0+\tau_1) \equiv \beta_j(t_1) \mod N.$

Since β_i and β_i are who rows from a^2 Florentine array, the above equation Has^{-0} at most two solutions in \mathbb{Z}_N for $\forall \tau_1 \in \mathbb{Z}_N$ and improprimental ?? Latter dentitive have IR zero thics? N for all $0 \le \tau \underset{i}{\sim} (N^2 - 1 \text{ and } i \ne j \text{ mod } N$.

Example 1nd Let, Nre-two and we from 6 af tons neinener agrain provided in Table 37 h Let 4 170 st 9 182 solutions denote the set of permutations from the raws of the Florentine array For simplicity, let 2N=10. Then a set of requences of period 225 is defined as

Example 1. Let N = 6 and a 6×6 Florentine array is provided in Talk 12s, $(t_0)t = 4\omega_{E_0}^{\pi_i(t_0)t_2} \beta_1 \le i \le \beta$ denote the set of permutations from the rows of the Florentine array. where $t = t_1 + t_2 \cdot 6$, $0 \le t_1, t_2 \le 6$, $\pi_i \in \mathcal{A}$ for $1 \le i \le 6$. It For simplicity, let $\sigma = 0$. Then a set of sequences of period is verifiable that 225 is defined as

- each sequence is a perfect sequence of period 36; and $|R_{\mathbf{s}_i,\mathbf{s}_i}[\tau]| \le 12 \frac{\mathbf{s}_i(t)}{50} a_{ij} w_0^{\pi_i(t)} t^2 \le 16 \le 1 \le \frac{6}{2} i \le 6$.

Therefore, the set S. is, a family, of 6-porfect sequences of period B6 isvithr Rablel 2hathich are consistent with Theorem ??.

Given an $F(N) \times N$ Florentine array, we can get a family of Fench generalised Frances feature general or conformation with the rand is a positive integer and PAN) is the maximum intimber such Threnefor(Nt)he NeFlorentine farmy exists Table 20 gives a list of known/results: NotR, that R2, invalid the authorowsisks is equal to the square root of the period, which means optimal cross-

correlation. However, the family size in the previous works is Given an $F(N) \times N$ Florentine array, we can get a family either determined by the smallest prime divisor of the penod of F(N) generalised Frank sequences of period N^2 , where or the existence of circular Florentine arrays. The properties N is a positive integer and F(N) is the maximum number of Florentine arrays in Lemma ?? implies that the family size such that an $F(N) \times N$ Florentine array exists, Table ?? is larger in this paper. Furthermore, the number of rows in a gives a list of known results. Note that R_c in all the other Florentine array for even N, can be equal to N (see Table ??), works is equal to the square root of the period, which which allows us to derive perfect sequences with low crossmeans optimal cross-correlation. However, the family size correlation with family size N. In contrast, the family size in in the previous works is either determined by the smallest all the other works, is equal to one when the period of the prime divisor of the period or the existence of circular sequences is even. sequences is even.
Florentine arrays. The properties of Florentine arrays in

Lemma ?? implies that Concausionize is larger in this pawer denyther mannity by purebers equences in in Florentise extratation wasted Non correlative water v. The samuel of the periodia llegue necto deperiore offe che cum enero ith low renone spravlatine with family of 2018 of three atrast, the family size inamily the other awark this count if one previous house or not blevious enforteriet for are trivial when the period of the perfect sequences is even. In this work, a small compromise on the optimality of the cross-correlation allows us to derive an notVerdarived surfaction of perfect sequences with low crosscorrelation for seven perforentine arrays. The number of the perfect sequences depends on the existence of Florentine arrays. The properties of Florentine arrays assure that the farThis syorkswas supported in part by Innlander Eylkeskome mungous constructions are trivial when the period of the perfect sequences is even. In this work, a small compromise on the optimality of the cross-correlation allows us to derive an non-trivial construction of perfect sequences with low cross-correlation for even period.