IRS-Enabled Covert and Reliable Communications: How Many Reflection Elements are Required?

Manlin Wang[‡,†], Bin Xia[†], Yao Yao[†], Zhiyong Chen[†] and Jiangzhou Wang[‡]
[†]Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, China
[‡]School of Engineering, University of Kent, Canterbury, U.K.
Email: [†]{wangmanlin, bxia, sandyyao, zhiyongchen}@sjtu.edu.cn; [‡]j.z.wang@kent.ac.uk

Abstract—Short-packet communications are applied to various scenarios where transmission covertness and reliability are crucial due to the open wireless medium and finite blocklength. Although intelligent reflection surface (IRS) has been widely utilized to enhance transmission covertness and reliability, the question of how many reflection elements at IRS are required remains unanswered, which is vital to system design and practical deployment. The inherent strong coupling exists between the transmission covertness and reliability by IRS, leading to the intractability of the question. To address this issue, the detection error probability at the warder and its approximation are derived first to reveal the relation between covertness performance and the number of reflection elements. Besides, to evaluate the reliability performance of the system, the decoding error probability at the receiver is also derived. Subsequently, the asymptotic reliability performance in high covertness regime is investigated, which provides theoretical predictions about the number of reflection elements at IRS required to achieve a decoding error probability close to 0 with given covertness requirements. Furthermore, Monte-Carlo simulations verify the accuracy of the derived results for detection (decoding) error probabilities and the validity of the theoretical predictions for reflection elements. Moreover, results show that more reflection elements are required to achieve high reliability with tighter covertness requirements, longer blocklength and higher transmission rates.

Index Terms—Covert and reliable transmission, short-packet communications, intelligent reflecting surface

I. Introduction

The fifth generation of mobile standards expands its focus [illegible]

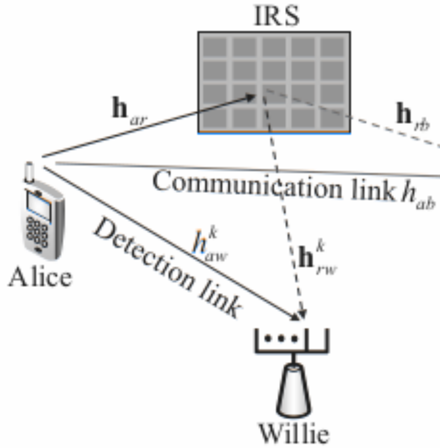

Fig. 1. IRS-enabled short-packet communication system against a multi-antenna warder.

even if the number of antennas at the warder is infinite. Finally, the analytical results are justified by the Monte Carlo simulations, and the impacts of system parameters on the system performance are illustrated, which provides some insights for practical system design.

Notation: Vectors and matrices are denoted by boldface lowercase and uppercase letters, respectively. $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and conjugate transpose of $\mathbf{A}$, respectively. $diag(\mathbf{p})$ denotes a diagonal matrix whose diagonal elements consist of the elements in the vector $\mathbf{p}$. $\|\cdot\|$ and $|\cdot|$ denote the Frobenius norm and the absolute value, respectively. $\mathcal{CN}(\mu,\mathbf{\Sigma})$ denotes the complex Gaussian distribution with mean μ and covariance $\mathbf{\Sigma}$. $\Pr(\cdot)$ denotes the probability of an event. $\mathcal{Q}(x)=\int_x^\infty \frac{1}{\sqrt{2\pi}}\exp\left(-t^2/2\right)dt$ denotes the Q-function. $\Gamma(n)=(n-1)!$ denotes the Gamma function, $\gamma(n,x)=\int_0^x e^{-t}t^{n-1}dt$ denotes the lower incomplete Gamma function, and $\Gamma(n,x)=\int_x^\infty e^{-t}t^{n-1}dt$ denotes the upper incomplete Gamma function.

II. Signal and Channel Model

As shown in Fig. 1, a covert communication scenario is considered, where the transmitter (Alice) desires to deliver messages to the receiver (Bob) while keeping a multi-antenna warder (Willie) unaware of the transmission. Alice and Bob are assumed to be equipped with a single antenna, while Willie is assumed to be equipped with N_w antennas. An IRS with N_r reflection elements is deployed to assist this transmission.

In one transmission round, Alice transmits n covert signals $x[i]\sim\mathcal{CN}(0,P_a)$, $i\in\{1,\ldots,n\}$ to Bob, where P_a denotes the transmit power at Alice. Willie collects the received signals to detect whether the transmission happens or not. We denote the additive white Gaussian noise (AWGN) at Bob and Willie as $n_b[i]\sim\mathcal{CN}(0,\sigma_b^2)$ and $\mathbf{n}_w[i]\sim\mathcal{CN}(0,\sigma_w^2\mathbf{I})$, where σ_b^2 and σ_w^2 are the noise variances at Bob and Willie, respectively.

The wireless channels from Alice to Bob (h_{ab}), from Alice to the k-th antenna of Willie ($h_{aw,k}$), from Alice to IRS ($\mathbf{h}_{ar}$), from IRS to Bob ($\mathbf{h}_{rb}$), and from the IRS to the k-th antenna of Willie ($\mathbf{h}_{rw,k}$) are subject to the quasi-static Rayleigh fading [?]. The channel coefficients remain constant during one transmission round, and are independently and identically distributed among different rounds. Specifically, $h_{ab}\sim\mathcal{CN}(0,\lambda_{ab})$, $h_{aw,k}\sim\mathcal{CN}(0,\lambda_{aw})$, $\mathbf{h}_{ar}\sim\mathcal{CN}(0,\lambda_{ar}\mathbf{I})$, $\mathbf{h}_{rb}\sim\mathcal{CN}(0,\lambda_{rb}\mathbf{I})$, and $\mathbf{h}_{rw,k}\sim\mathcal{CN}(0,\lambda_{rw}\mathbf{I})$. The instantaneous channel state information (CSI) of legitimate links (including the Alice-Bob link and Alice-IRS-Bob link) is available at Alice and IRS by channel estimation [?]. In addition, since Willie does not cooperate with Alice, only statistical CSI of warder-related links (including the Alice-IRS-Willie link and Alice-Willie link) can be estimated at Alice through the local oscillator power inadvertently leaked from the receiver radio frequency frontend signal [?]. Besides, the instantaneous CSI of warder-related links is assumed available at Willie from the worst perspective for covert communication.

III. Covertness Performance Analysis

To detect the presence of covert communications, Willie must distinguish between the following two hypotheses

$$\mathbf{y}_w[i]=\begin{cases}\mathbf{n}_w[i], & \mathcal{H}_0\\ \mathbf{h}_w x[i]+\mathbf{n}_w[i], & \mathcal{H}_1\end{cases} \tag{1}$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the null hypothesis that Alice does not transmit and the alternative hypothesis that Alice transmits, respectively. $\mathbf{y}_w[i]$ is the received signal at Willie, and $\mathbf{h}_w=[h_{w,1},\ldots,h_{w,N_w}]^T$ is the equivalent channel from Alice to Willie, where $h_{w,k}=h_{aw,k}+\mathbf{h}_{rw,k}^H\mathbf{\Theta}\mathbf{h}_{ar}$, $k\in\{1,\ldots,N_w\}$. $\mathbf{\Theta}=diag\left(e^{j\theta_1},\ldots,e^{j\theta_{N_r}}\right)$ is the reflection matrix at IRS, where $\theta_i\in[0,2\pi)$, $i\in\{1,\ldots,N_r\}$ denotes the phase of i-th reflection element.

Considering the worst perspective for covert communication, when Willie knows the instantaneous CSI and the reflection coefficients at IRS, the maximum ratio combiner can be adopted as an optimal detector [?]

$$T=\frac{1}{n}\sum_{i=1}^{n}\left|\mathbf{h}_w^H\mathbf{y}_w[i]\right|^2\underset{\mathcal{D}_0}{\overset{\mathcal{D}_1}{\gtrless}}\tau^*, \tag{2}$$

where T is the average power of each received signal at Willie, $\mathcal{D}_1$ and $\mathcal{D}_0$ denote the binary decisions that infer whether Alice transmits or not, respectively. In addition, τ^* denotes the optimal detection threshold, which is given by [?], [?]

$$\tau^*=\frac{\sigma_w^2\left(\sigma_w^2+\|\mathbf{h}_w\|^2P_a\right)}{P_a}\ln\left(1+\frac{\|\mathbf{h}_w\|^2P_a}{\sigma_w^2}\right). \tag{3}$$

Suppose there is no prior knowledge for Willie about when Alice will transmit. The prior probability of either hypothesis is equal. Mathematically, the detection error probability ξ at Willie is defined as $\xi=\Pr(\mathcal{D}_1\mid\mathcal{H}_0)+\Pr(\mathcal{D}_0\mid\mathcal{H}_1)$, where $\Pr(\mathcal{D}_1\mid\mathcal{H}_0)$ denotes the false alarm probability, and $\Pr(\mathcal{D}_0\mid\mathcal{H}_1)$ denotes the missed detection probability ξ^* with optimal detection threshold τ^* is expressed as [?]

$$\xi(\tau^*) = \Pr(T > \tau^* | H_0) + \Pr(T < \tau^* | H_1) = 1 - \frac{1}{\Gamma(n)}\left(\gamma\left(n, \frac{n\tau^*}{\|\mathbf{h}_w\|^2 \sigma_w^2}\right) - \gamma\left(n, \frac{n\tau^*}{\sigma_w^2 + \|\mathbf{h}_w\|^2 P_a}\right)\right). \quad (4)$$

Since Alice only knows the statistical CSI of warder-related links, the average detection error probability with statistical CSI is adopted as the covertness performance of the system.

Theorem 1. The average detection error probability at Willie with optimal detection threshold can be derived as

$$\bar{\xi}(\tau^*) = 1 - \frac{\pi}{B\Gamma(n)} \sum_{b=1}^{B}\left[\gamma\left(n, \frac{n(\sigma_w^2 + \tan\theta_b P_a)}{\tan\theta_b P_a} \ln\left(\frac{\sigma_w^2 + \tan\theta_b P_a}{\sigma_w^2}\right)\right) - \gamma\left(n, \frac{n\sigma_w^2}{\tan\theta_b P_a} \ln\left(\frac{\sigma_w^2 + \tan\theta_b P_a}{\sigma_w^2}\right)\right)\right] \frac{\sqrt{\theta_b\left(\frac{\pi}{2} - \theta_b\right)} f(\tan\theta_b)}{\cos^2\theta_b}, \quad (5)$$

where B is the parameter of Gaussian-Chebyshev Quadrature, $\theta_b = \frac{\pi}{4}\left(1 + \cos\frac{(2b-1)\pi}{2B}\right)$, and $f(x) = \frac{e^{-\frac{x}{\lambda_w}} x^{N_w - 1}}{\lambda_w^{N_w}\Gamma(N_w)}$ with $\lambda_w = \lambda_{aw} + N_r \lambda_{rw} \lambda_{ar}$.

Proof. We denote $X_k = \mathbf{h}_{rw,k}^H \Theta \mathbf{h}_{ar}$, which approximately follows the complex Gaussian distribution, i.e., $X_k \sim \mathcal{CN}(0, N_r \lambda_{rw} \lambda_{ar})$, when N_r is sufficiently large [?]. The tightness of this approximation is validated by [?] and Section VI even N_r is relatively small. Thus, $\|\mathbf{h}_w\|^2$ follows the Gamma distribution with the probability density function (PDF) $f(x) = \frac{e^{-\frac{x}{\lambda_w}} x^{N_w - 1}}{\lambda_w^{N_w}\Gamma(N_w)}$ [?]. By substituting (??) into (??), the average detection error probability can be expressed as

$$\bar{\xi}(\tau^*) = 1 - \int_0^\infty \left[\gamma\left(n, n\left(\frac{\sigma_w^2}{xP_a} + 1\right)\ln\left(1 + \frac{xP_a}{\sigma_w^2}\right)\right) - \gamma\left(n, \frac{n\sigma_w^2}{xP_a}\ln\left(1 + \frac{xP_a}{\sigma_w^2}\right)\right)\right]\frac{f(x)}{\Gamma(n)}dx. \quad (6)$$

By substituting $x = \tan\theta$ into (??) and applying Gaussian-Chebyshev Quadrature into the above integral expression [?], (??) can be obtained, and the proof is completed. □

Due to the complicated form of (??), it is intractable to guide the system design. To address this issue, we derive a tractable approximation of the detection error probability as follows.

Lemma 1. In high covertness scenarios with moderate blocklength (such as $n \geq 50$), the average detection error probability at Willie can be approximated as

$$\bar{\xi}(\tau^*) \approx 1 - \sqrt{\frac{n}{2\pi}} \frac{N_w P_a \lambda_w}{\sigma_w^2}. \quad (7)$$

Proof. The detection error probability is approximated as

$$\xi(\tau^*) \overset{(a)}{\approx} 1 - \frac{e^{-n} n^n}{\Gamma(n)} \frac{P_a \|\mathbf{h}_w\|^2}{\sigma_w^2} \overset{(b)}{\approx} 1 - \sqrt{\frac{n}{2\pi}} \frac{P_a \|\mathbf{h}_w\|^2}{\sigma_w^2}, \quad (8)$$

where step (a) is obtained by the linear approximation in high covertness scenarios [?], and step (b) is due to $\lim_{n\to\infty} \frac{e^{-n} n^n}{\Gamma(n)} = \sqrt{\frac{n}{2\pi}}$ with relative error less than 2×10^{-3} when $n \geq 50$.

Thus, the average detection error probability is derived as

$$\bar{\xi}(\tau^*) \approx \int_0^\infty \left(1 - \sqrt{\frac{n}{2\pi}} \frac{P_a x}{\sigma_w^2}\right) f(x) dx = 1 - \sqrt{\frac{n}{2\pi}} \frac{N_w P_a \lambda_w}{\sigma_w^2} + g_1(P_a), \quad (9)$$

where $g_1(P_a) = \sqrt{\frac{n}{2\pi}} \frac{\lambda_w P_a}{\Gamma(N_w)\sigma_w^2} \Gamma\left(N_w + 1, \frac{\sigma_w^2}{\lambda_w P_a}\sqrt{\frac{2\pi}{n}}\right) - \frac{1}{\Gamma(N_w)}\Gamma\left(N_w, \frac{\sigma_w^2}{\lambda_w P_a}\sqrt{\frac{2\pi}{n}}\right)$, which can be ignored compared with other terms due to $\lim_{P_a \to 0} \frac{g_1(P_a)}{P_a} = 0$. □

The concise expression (??) implies that the $\bar{\xi}(\tau^*)$ is linear to $\sqrt{n}$, N_w and P_a in high covertness scenarios, facilitating the performance analysis in Section V. Besides, the tightness of (??) is also validated in Section VI.

IV. Reliability Performance Analysis

When Alice transmits, the received signal at Bob is

$$y_b[i] = \left(h_{ab} + \mathbf{h}_{rb}^H \Theta \mathbf{h}_{ar}\right) x[i] + n_b[i]. \quad (10)$$

Based on the received signal (??), Bob can decode the messages. However, the decoding error can not be ignored due to finite blocklength, which is given by [?]

$$\delta(\gamma_b) = Q\left(\frac{\ln 2\sqrt{n}\left(\log_2(1+\gamma_b) - R\right)}{\sqrt{1 - (\gamma_b + 1)^{-2}}}\right), \quad (11)$$

where $\gamma_b = \left|h_{ab} + \mathbf{h}_{rb}^H \Theta \mathbf{h}_{ar}\right|^2 P_a / \sigma_b^2$ denotes the received SNR at Bob, and R is the transmission rate measured by bits per channel use (bpcu). Since the decoding error probability (??) is affected by fading channels, the average decoding error probability $\bar{\delta}$ is adopted to evaluate the reliability performance.

IRS can adjust its reflection coefficients at each transmission round to maximize γ_b, where the optimal phase is given by $\theta_i^* = \arg(h_{ab}) + \arg(h_{rb,i}) - \arg(h_{ar,i})$, $h_{ar,i}$ and $h_{rb,i}$ denote the channel from Alice to the i-th element at IRS, and from the i-th element to Bob, respectively. Thus, the instantaneous SNR can be expressed as $\gamma_b = X^2 P_a / \sigma_b^2$ where $X = |h_{ab}| + \sum_{i=1}^{N_r} |h_{rb,i}||h_{ar,i}|$. According to [?, Lemma 1], X can be approximated by a generalized Gamma distribution with PDF $f_X(x) = \beta^\alpha x^{\alpha-1} e^{-\beta x} / \Gamma(\alpha)$, where $\alpha = \Psi^2/\Phi$, $\beta = \Psi/\Phi$, $\Psi = \alpha_d/\beta_d + N_r \alpha_r/\beta_r$, $\Phi = \alpha_d/\beta_d^2 + N_r \alpha_r/\beta_r^2$, $\alpha_d = \frac{\pi}{4-\pi}$, $\beta_d = \frac{2\sqrt{\pi}}{(4-\pi)\sqrt{\lambda_{ab}}}$, $\alpha_r = \frac{\pi^2}{16-\pi^2}$

and $V_r = \frac{4\xi}{(16-\pi^2)\sqrt{\lambda_{ar}\lambda_{rb}}}$. Then, the PDF of δ_b is derived as

$$f_{\gamma_b}(t) = \frac{\beta^{\alpha} e^{-\beta\sqrt{\frac{\sigma_b^2 t}{P_a}}}}{2\Gamma(\alpha)\, t} \left(\frac{\sigma_b^2 t}{P_a}\right)^{\frac{\alpha}{2}} \quad (12)$$

$$\overline{\delta} = \int_0^\infty \delta(t) f_{\gamma_b}(t)dt \overset{(a)}{\approx} \int_{\varpi-\frac{1}{2\nu}}^{\varpi+\frac{1}{2\nu}} F_{\gamma_b}(t)dt \quad (13)$$

[illegible]

where step (a) is due to the linear approximation of Q-function [?], $\varpi = 2^R - 1$, $\nu = \sqrt{\frac{n}{2\pi(4^R-1)}}$, $F_{\gamma_b}(t) = \frac{1}{\Gamma(\alpha)}\gamma\left(\alpha, \beta\sqrt{\frac{\sigma_b^2 t}{P_a}}\right)$ and $g(t)$ = [illegible]

V. Theoretical Predictions about Required Reflection Elements

In this section, the reliability performance is derived under the covertness requirement $\overline{\xi}(\tau^*) \geq 1-\varepsilon$, where ε is predetermined value to evaluate the covertness level. Besides, we provide the theoretical predictions about the number of reflection elements required to achieve a decoding error probability close to 0 with given covertness requirements.

On the one hand, $\overline{\xi}(\tau^*)$ decreases with N_r as shown in (??), which implies that the allowed transmit power P_a also decreases with N_r to meet the covertness requirement $\overline{\xi}(\tau^*) \geq 1-\varepsilon$. On the other hand, $\overline{\delta}$ decreases with N_r and P_a as shown in (??). Therefore, the reliability guarantee by IRS is unclear but crucial, and the question that how many reflection elements are required to achieve a decoding error probability close to 0 with covertness requirements is significative.

To address this issue, we first derive the reliability performance in high covertness scenarios when $N_r \to \infty$ as follows.

Theorem 2: *When $N_r \to \infty$, the achievable average decoding error probability with given ε in high covertness scenarios is*

$$\lim_{N_r\to\infty} \overline{\delta} = \begin{cases} 1, & r_{rw} < r_1 \\ \frac{(2\varpi\nu+1)}{2} - \sqrt{\frac{2\pi}{n}} \cdots, & r_1 \leq r_{rw} < r_2 \\ 0, & r_2 \leq r_{rw} \end{cases} \quad (14)$$

where $r_{rw} = \lim_{N_r\to\infty} \frac{N_r}{N_w}$ denotes the ratio of the number of reflection elements at IRS to that of antennas at Willie, $r_1 = \sqrt{\frac{\pi}{n}} \frac{16\lambda_{rw}\sigma_b^2}{\cdots}\left(\varpi - \frac{1}{2\nu}\right)$ and $r_2 = \sqrt{\frac{\pi}{n}} \frac{16\lambda_{rw}\sigma_b^2}{\cdots}\left(\varpi + \frac{1}{2\nu}\right)$.

Proof: See Appendix ??. □

TABLE I
Achievable Decoding Error Probability

Element Number	$N_w, N_r\to\infty$, $r_{rw} < r_1$	$N_w, N_r\to\infty$, $r_1 \leq r_{rw} < r_2$	$N_w, N_r\to\infty$, $r_2 \leq r_{rw}$
$\overline{\delta}$	1	(??)	0
Element Number	$N_w\to\infty$, N_r finite	$N_r\to\infty$, N_w finite	N_r, N_w finite
$\overline{\delta}$	1	0	(??)

Theorem 2 gives the theoretical predictions about the number of reflection elements required to achieve high-reliability performance ($\overline{\delta} \to 0$) with $N_w \to \infty$. Specifically, when $N_r \geq r_2 N_w$, the decoding error probability approaches 0. However, when $N_r < r_1 N_w$, the decoding error probability approaches 1. Besides, when $r_1 N_w \leq N_r < r_2 N_w$, the decoding error probabilities approach values between 0 and 1. Moreover, it shows that the covertness requirements ε, the blocklength n and the transmission rate R all affect the critical factors r_1 and r_2. Specifically, the tighter the covert requirement, the longer the blocklength and the higher the transmission rate, the more reflection elements are required to guarantee high-reliability performance.

When the number of reflection elements at IRS (antennas at Willie) is finite, the decoding error probability can be obtained by substituting the maximum allowed transmit power $P_a = \sqrt{\frac{2\pi}{n}} \frac{\cdots}{N_w \lambda_{rw}}$ by (??) into (??). Finally, we summarize the achievable reliability of IRS-enabled short-packet communication systems under covertness requirements in Table I.

VI. Numerical Simulation

In this section, numerical results are provided to evaluate the performance of IRS-enabled short-packet communications. For illustration, we assume that Alice, Bob, Willie, and the IRS are located at (0,0), (10m,0), (12m,-5m), and (10m,1m) in a two-dimensional plane, respectively. The fading parameters are expressed as $\lambda_{ij} = \beta_0 \left(\frac{d_{ij}}{d_0}\right)^{-\alpha_{ij}}$, $ij \in \{ab, ar, rb, aw, rw\}$, where d_{ij} denotes the distance between nodes i and j, $d_0 = 1$(m) is the reference distance, and $\beta_0 = -30$(dB) denotes the channel power gain at the reference distance. And α_{ij} is the path loss exponents with $\alpha_{ar} = \alpha_{rw} = 2.2$, $\alpha_{ab} = \alpha_{aw} = 3.5$, $\alpha_{rb} = 2$. Besides, $\sigma_b^2 = \sigma_w^2 = -60$ (dBm), $R = 0.2$ (bpcu) and $n = 200$. All the simulation results presented are obtained by averaging over 10^6 channel realizations.

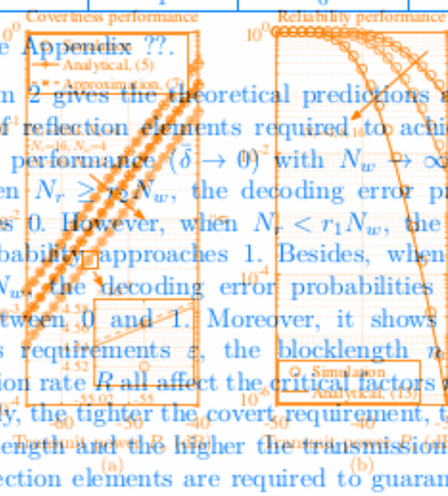

Fig. 2. The average detection error probability and decoding error probability versus the transmit power.

In Figs. 2 (a) and (b), the impacts of the transmit power on the average detection error probability and average decoding error probability are investigated. From Fig. 2 (a), it can be seen that the curves with numerical simulations, the analytical expression (??), and the approximation expression (??) coincide, even if the number of reflection elements is relatively small, such as $N_r = 16$. From Fig. 2 (b), it can be seen that the results obtained by (??) are close to those obtained by numerical simulations. The detection (decoding) error probabilities decrease with P_a and N_r since more energy is received by Willie (Bob) when IRS is equipped with more reflection elements. These results validate the results given in Section III, implying that the proposed approximation (??) can be adopted to the covertness and reliability performance metrics due to their conciseness and tightness.

In Figs. 3 (a) and (b), the impacts of the number of reflection elements at IRS and antennas

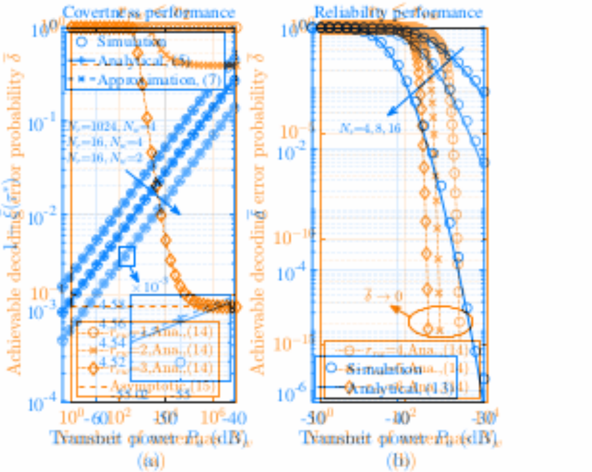

Fig. 2. The average detection error probability and decoding error probability versus the transmit power.

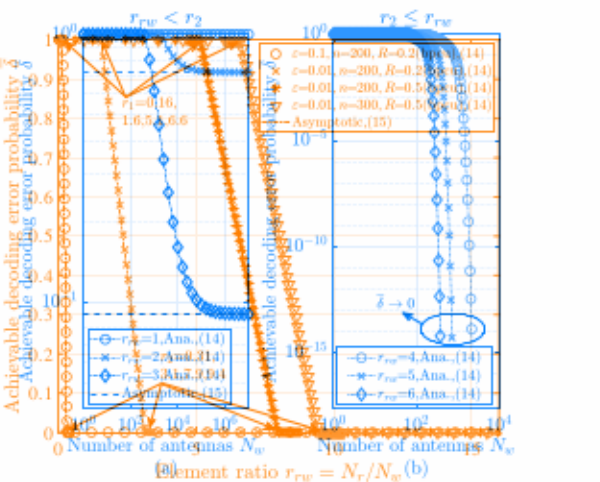

Fig. 3. The achievable decoding error probability versus the number of antennas at Willie.

Fig. 4. The achievable decoding error probability versus the ratio r_{rw}.

$N_r = 16$. From Fig. 2 (b), it can be seen that the results obtained by (??) are close to those obtained by numerical simulations, even if the number of reflection elements is relatively small, such as $N_r = 4$. In addition, the detection (decoding) error probabilities decrease with P_a and N_r since more energy is received by Willie (Bob) when IRS is equipped with more reflection elements. These results validate the results given in Section III, implying that the proposed approximations (??) and (??) can be adopted as covertness and reliability performance metrics due to their conciseness and tightness.

In Figs. 3 (a) and (b), the impacts of the number of antennas and reflection elements on the decoding error probability are investigated, where the curves with "Ana." denote the results obtained by (??). It can be seen that the results by (??) tend to the asymptotic ones by (??) as N_w ...

from 0.1 to 0.01. Similarly, r_1 (r_2) changes from 1.6 (3.1) to 5.1 (7.9), when R changes from 0.2 (bpcu) to 0.5 (bpcu). In addition, r_1 (r_2) changes from 5.1 (7.9) to 6.6 (9.4), when n changes from 200 to 300. These results demonstrate that more reflection elements are required to achieve high reliability ($\bar{\delta} \to 0$) with tighter covertness requirements, higher transmission rates and longer blocklength, which provides insights for the practical system design.

VII. Conclusion

In this paper, we have provided the theoretical prediction about the required number of reflection elements to guarantee covertness and reliability performance in IRS-enabled short-packet communications. To this end, the detection error probability at Willie has been derived firstly for evaluating the covertness performance, where a concise approximation is also provided to facilitate further analysis. Then, the decoding error probability at Bob has been derived for evaluating the reliability performance. Combined with the above system performance metrics, we have analyzed the required number of reflection elements at IRS to achieve high-reliability ($\bar{\delta} \to 0$) with covertness requirement $\xi(\tau^*) \geq 1-\varepsilon$ in the asymptotic regime. Finally, Monte-Carlo simulations have been provided to evaluate the accuracy of analytical results and the validity of the theoretical predictions. Simulation results have shown that system parameters (ε, n, R) significantly affect the required number of refelction elements, which guides the system design.

Appendix A: Proof of Theorem 2

In high covertness scenarios with $N_r \to \infty$, the allowed transmit power with given covertness requirement ε is ... By substituting the allowed transmit power and above approximations into (??), we can obtain

$$\lim_{N_r \to \infty} \bar{\delta} = f_2(\cdot) \cdots \qquad (15)$$

where $f_2(t)$ = ... and $f_3(t)$ = ...

We denote functions $f_4(x, r) = \frac{\Gamma(r)}{\Gamma(x)}$ and $f_5(x, r) = \frac{\Gamma(r)}{\Gamma(x)}$, and derive some properties of these functions first.

For the case with $r < 1$, we can obtain that

$$\lim_{x \to \infty, r < 1} f_4(x, r) = \lim_{x \to \infty, r < 1} \frac{\Gamma(r)}{\Gamma(x)} \qquad (16)$$

Finally, Monte-Carlo simulations have been provided to evaluate the accuracy of analytical results and the validity of the theoretical predictions. Simulation results have shown that system parameters (ε, n, R) significantly affect the required number of refelction elements, which guides the system design.

Appendix A
Proof of Theorem 2

In high covertness scenarios with $N_r \to \infty$, the allowed transmit power with given covertness requirement ε is $P_r \approx \sqrt{\frac{2\pi}{n}}\frac{\varepsilon\sigma^2}{N_rN_w\lambda_{rw}\lambda_{ar}}$. Besides, when $N_r \to \infty$, $\alpha \approx \frac{N_r\pi^2}{16-\pi^2}$, $\beta \approx \frac{4\pi}{(16-\pi^2)\sqrt{\lambda_{ar}\lambda_{rb}}}$. By substituting the allowed transmit power and above approximations into (??), we can obtain

$$\lim_{N_r\to\infty}\bar{\delta}=f_2\left(\varpi-\frac{1}{2\nu}\right)+\left(\sqrt{\frac{2\pi}{n}\frac{N_r\pi^2\varepsilon\sigma_w^2\lambda_{rb}\nu}{16\sigma_b^2N_w\lambda_{rw}}}-\frac{(2\varpi\nu+1)}{2}\right)\times\left(f_3\left(\varpi+\frac{1}{2\nu}\right)-f_3\left(\varpi-\frac{1}{2\nu}\right)\right), \tag{15}$$

where $f_2(t)=\frac{\Gamma\left(\frac{N_r\pi^2}{16-\pi^2},\frac{4\pi}{(16-\pi^2)}\sqrt{\sqrt{\frac{n}{2\pi}}\frac{N_rN_w\lambda_{rw}\sigma_b^2}{\lambda_{rb}\varepsilon\sigma_w^2}t}\right)}{\Gamma\left(\frac{N_r\pi^2}{16-\pi^2}\right)}$ and $f_3(t)=\frac{\Gamma\left(\frac{N_r\pi^2}{16-\pi^2},\frac{4\pi}{(16-\pi^2)}\sqrt{\sqrt{\frac{n}{2\pi}}\frac{N_rN_w\lambda_{rw}\sigma_b^2}{\lambda_{rb}\varepsilon\sigma_w^2}t}\right)}{\Gamma\left(\frac{N_r\pi^2}{16-\pi^2}\right)}$.

We denote functions $f_4(x,r)=\frac{\gamma(x,rx)}{\Gamma(x)}$ and $f_5(x,r)=\frac{\Gamma(x,rx)}{\Gamma(x)}$, and derive some properties of these functions first.

For the case with $r<1$, we can obtain that

$$\lim_{x\to\infty,r<1}f_4(x,r)=\lim_{x\to\infty,r<1}\int_0^{rx}\frac{t^{x-1}e^{-t}}{\Gamma(x)}dt \overset{(a)}{\leqslant}\lim_{x\to\infty,r<1}\frac{(rx)^xe^{-rx}}{2\Gamma(x)}\overset{(b)}{\leqslant}\lim_{x\to\infty,r<1}\frac{\sqrt{x}e^{x(1-r)}r^x}{2\sqrt{2\pi}}\overset{(c)}{=}0, \tag{16}$$

where step (a) holds due to the convexity of $g_2(t)=t^{x-1}e^{-t}$ on $(0,rx]$ with $r<1, x\to\infty$, and $\int_0^{rx}g_2(t)dt \leqslant \frac{rx(g_2(0)+g_2(rx))}{2}$. Step (b) holds due to the asymptotic property of Gamma function as $\lim_{x\to\infty}\Gamma(x)\le e^{-x}x^{x-\frac{1}{2}}\sqrt{2\pi}$ by [?, (1.43)]. Step (c) holds due to $\lim_{x\to\infty}\sqrt{x}e^{x(1-r)}r^x=0$.

For the case with $r=1$, we can obtain that

$$\lim_{x\to\infty,r=1}f_4(x,r)=1-\lim_{x\to\infty}\frac{\Gamma(x,x)}{\Gamma(x)}\overset{(a)}{=}\frac{1}{2}, \tag{17}$$

where step (a) holds since $\lim_{x\to\infty}\Gamma(x,x)=\sqrt{\frac{\pi}{2}}x^{x-\frac{1}{2}}e^{-x}$ according to the resurgence property of the incomplete Gamma function by [?, (1.2)] and the asymptotic property of Gamma function by [?, (1.43)].

For the case with $r>1$, we can obtain that

$$\begin{aligned}1-\lim_{x\to\infty,r>1}f_4(x,r)&=\lim_{x\to\infty,r>1}\int_{rx}^{\infty}\frac{t^{x-1}e^{-t}}{\Gamma(x)}dt\\&\overset{(a)}{<}\lim_{x\to\infty}\frac{(rx)^xe^{-rx}}{\Gamma(x)}\overset{(b)}{<}\lim_{x\to\infty}\frac{\sqrt{x}r^xe^{x(1-r)}}{\sqrt{2\pi}}\overset{(c)}{=}0,\end{aligned} \tag{18}$$

where the step (a) holds since $g_3(t)=t^{x-1}e^{-t}<g_4(t)=(rx)^{x+1}e^{-rx}t^{-2}$ for $t\geqslant rx, r>1, x\to\infty$, and $\int_{rx}^{\infty}t^{-2}dt=\frac{1}{rx}$. Besides, steps (b) and (c) holds for the same reason as elaborated in the case with $r<1$.

To sum, $\lim_{x\to\infty}f_4(x,r)=0$ with $r<1$, $\lim_{x\to\infty}f_4(x,r)=\frac{1}{2}$ with $r=1$, and $\lim_{x\to\infty}f_4(x,r)=1$ with $r>1$. Similarly, we can obtain $\lim_{x\to\infty}f_5(x,r)=0$ with $r>1$, $\lim_{x\to\infty}f_5(x,r)=\frac{1}{2}$ with $r=1$, and $\lim_{x\to\infty}f_5(x,r)=1$ with $r<1$.

By substituting $x=\frac{N_r\pi^2}{16-\pi^2}$ and $r=\frac{4}{\pi}\sqrt{\frac{N_w\lambda_{rw}\sigma_b^2}{N_r\lambda_{rb}\varepsilon\sigma_w^2}}\sqrt{\frac{n}{2\pi}}\left(\varpi+\frac{1}{2\nu}\right)$ $\left(\frac{4}{\pi}\sqrt{\frac{N_w\lambda_{rw}\sigma_b^2}{N_r\lambda_{rb}\varepsilon\sigma_w^2}}\sqrt{\frac{n}{2\pi}}\left(\varpi-\frac{1}{2\nu}\right)\right)$ into $f_4(x,r)$ and $f_5(x,r)$, results in Theorem ?? are obtained.