Information Sheet 1: Number System

The following part are excerpt from (2)

The technique to represent and work with numbers is called number system. Decimal number system is the most common number system. Other popular number systems include binary number system, octal number system, hexadecimal number system, etc.

As a computer programmer or an IT professional, you should understand the following number systems which are frequently used in computers.

Computer Number Systems and its types

S.No.	Number System and Description
1	Binary Number System Base 2. Digits used : 0, 1
2	Octal Number System Base 8. Digits used : 0 to 7
3	Hexa Decimal Number System Base 16. Digits used: 0 to 9, Letters used : A- F

What are the number systems?

Number systems are the technique to represent numbers in the computer system architecture, every value that you are saving or getting into/from computer memory has a defined number system.

Computer architecture supports following number systems.

- Binary number system
- Octal number system
- Decimal number system
- Hexadecimal (hex) number system

1) Binary Number System

A Binary number system has only two digits that are 0 and 1. Every number (value) represents with 0 and 1 in this number system. The base of binary number system is 2, because it has only two digits. Where 0 is used to represent a condition and 1 is used to represent another condition. It uses 2 as its base.

The Binary System Table

16s	8s	4s	2s	1s	Places
2^4	2^3	2^2	2^1	2^0	Powers
16	8	4	2	1	Values

Let us take for example, 12.

16s	8s	4s	2s	1s	Places
2^4	2^3	2^2	2^1	2^0	Powers
16	8	4	2	1	Values
	1	1	0	0	Number

Step 1:	12÷16=0	The answer is 0 since the divisor is larger that the dividend.
Step 2:	12÷8=**1** rem. 4	The answer is 1 remainder 4. The remainder 4 will be carried to the next step
Step 3:	4÷4=**1**	The remainder is 1. Since there is no remainder zero (0) will be carried over to the next step.
Step 4:	0÷2=**0**	The answer is 0.
Step 5:	0÷1=**0**	

The answer is : 1100

2) Octal number system

Octal number system has only eight (8) digits from 0 to 7. Every number (value) represents with 0,1,2,3,4,5,6 and 7 in this number system. The base of octal number system is 8, because it has only 8 digits.

Octal number system has eight digits – 0, 1, 2, 3, 4, 5, 6 and 7. Octal number system is also a positional value system with where each digit has its value expressed in powers of 8, as shown here –

8^5	8^4	8^3	8^2	8^1	8^0

Decimal equivalent of any octal number is sum of product of each digit with its positional value.

$726_8 = 7\times8^2 + 2\times8^1 + 6\times8^0$

$= 448 + 16 + 6$

$= 470_{10}$

3) Decimal number system

Decimal number system has only ten (10) digits from 0 to 9. Every number (value) represents with 0,1,2,3,4,5,6, 7,8 and 9 in this number system. The base of decimal number system is 10, because it has only 10 digits.

Decimal number system is a base 10 number system having 10 digits from 0 to 9. This means that any numerical quantity can be represented using these 10 digits. Decimal number system is also a positional value system. This means that the value of digits will depend on its position. Let us take an example to understand this.

Say we have three numbers – 734, 971 and 207. The value of 7 in all three numbers is different−

- In 734, value of 7 is 7 hundreds or 700 or 7 × 100 or 7 × 102

- In 971, value of 7 is 7 tens or 70 or 7 × 10 or 7 × 101
- In 207, value 0f 7 is 7 units or 7 or 7 × 1 or 7 × 100

The weightage of each position can be represented as follows –

10^5	10^4	10^3	10^2	10^1	10^0

In digital systems, instructions are given through electric signals; variation is done by varying the voltage of the signal. Having 10 different voltages to implement decimal number system in digital equipment is difficult. So, many number systems that are easier to implement digitally have been developed. Let's look at them in detail.

A numbering system that uses the base 10 and uses the number 0,1,2,3,4,5,6,7,8,9.

<u>The Decimal System Table</u>

1000s	100s	10s	1s	Places
10^3	10^2	10^1	10^0	Powers
1000	100	10	1	Values

The Equivalent of the numbers in the powers row is located at the places row. Simply, put, the decimal system table starts form $10^{0,}$ $10^{1,}$ $10^{2,}$ $10^{3,}$ 10^4 and so on. $10^1 = 1$, $10^2 = 10$, and $10^3 = 1000$.

Let us take for example. 25.

1000s	100s	10s	1s	Places
10^3	10^2	10^1	10^0	Powers
1000	100	10	1	Values
		2	5	Number

The number 25 is treated as (2)(10)+(5)(1)=25. The Decimal System is what we use but the computer uses different number system called binary system.

4) Hexadecimal number system

A Hexadecimal number system has sixteen (16) alphanumeric values from 0 to 9 and A to F. Every number (value) represents with 0,1,2,3,4,5,6, 7,8,9,A,B,C,D,E and F in this number system. The base of hexadecimal number system is 16, because it has 16 alphanumeric values. Here A is 10, B is 11, C is 12, D is 13, E is 14 and F is 15.

Table of the Numbers Systems with Base, Used Digits, Representation, C language representation:

Number system	**Base**	**Used digits**	**Example**	**C Language assignment**
Binary	2	0,1	(11110000)2	int val=0b11110000;
Octal	8	0,1,2,3,4,5,6,7	(360)8	int val=0360;

Decimal	10	0,1,2,3,4,5,6,7,8,9	(240)10	int val=240;
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9, A,B,C,D,E,F	(F0)16	int val=0xF0;

Number System Conversion

Decimal to Binary

Decimal numbers can be converted to binary by repeated division of the number by 2 while recording the remainder. Let's take an example to see how this happens.

Example: 43_{10} = _____ $_2$

		Remainder	
2	43		
2	21	1	MSB
2	10	1	↑
2	5	0	↑
2	2	1	↑
2	1	0	↑
	0	1	LSB

The remainders are to be read from bottom to top to obtain the binary equivalent.

Answer: $\mathbf{43_{10} = 101011_2}$

Example #2

Steps:

- ✓ Step 1 – Divide the decimal number to be converted by the value of the new base.
- ✓ Step 2 – Get the remainder from Step 1 as the rightmost digit (least significant digit) of new base number.
- ✓ Step 3 – Divide the quotient of the previous divide by the new base.
- ✓ Step 4 – Record the remainder from Step 3 as the next digit (to the left) of the new base number.
- ✓ Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.

The last remainder thus obtained will be the Most Significant Digit (MSD) of the new base number.
Example: Decimal Number: 29_{10} = _____ $_2$

Calculating Binary Equivalent

Step	Operation	Result	Remainder
Step 1	29 / 2	14	1
Step 2	14 / 2	7	0
Step 3	7 / 2	3	1
Step 4	3 / 2	1	1
Step 5	1 / 2	0	1

As mentioned in Steps 2 and 4, the remainders have to be arranged in the reverse order so that the first remainder becomes the Least Significant Digit (LSD) and the last remainder becomes the Most Significant Digit (MSD).

Answer: (Decimal Number) 29_{10} = (Binary Number) 11101_2.

Decimal to Octal

Decimal numbers can be converted to octal by repeated division of the number by 8 while recording the remainder. Let's take an example to see how this happens.

Example: 473_{10} = ______ $_8$

		Remainder	
8	473		
8	59	1	MSD
8	7	3	↑
	0	7	LSD

Reading the remainders from bottom to top,

Answer: $\mathbf{473_{10} = 731_8}$

Decimal to Hexadecimal

Decimal numbers can be converted to octal by repeated division of the number by 16 while recording the remainder. Let's take an example to see how this happens.

Example: 423_{10} = _____ $_{16}$

		Remainder
16	423	
16	26	7
16	1	A
	0	1

Reading the remainders from bottom to top we get,

Answer: $\boldsymbol{423_{10} = 1A7_{16}}$

Binary to Decimal

Steps

- ✓ Step 1 – Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system).
- ✓ Step 2 – Multiply the obtained column values (in Step 1) by the digits in the corresponding columns.
- ✓ Step 3 – Sum the products calculated in Step 2. The total is the equivalent value in decimal.

Example

Binary Number – 11101_2 = ______ $_{10}$

Step	Binary Number	Decimal Number
Step 1	11101_2	$((1 \times 24) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))10$
Step 2	11101_2	$(16 + 8 + 4 + 0 + 1)10$
Step 3	11101_2	29_{10}

Answer: (Binary Number) 11101_2 = (Decimal Number) 29_{10}

Binary to Octal and Vice Versa

Steps

- ✓ Step 1 – Divide the binary digits into groups of three (starting from the right).
- ✓ Step 2 – Convert each group of three binary digits to one octal digit.

Example

Example: Binary Number – $\mathbf{10101_2}$ = _______ $_8$

Calculating Octal Equivalent –

Step	Binary Number	Octal Number
Step 1	10101_2	010 101
Step 2	10101_2	2_8 5_8
Step 3	10101_2	25_8

Answer: (Binary Number) 10101_2 = (Octal Number) 25_8

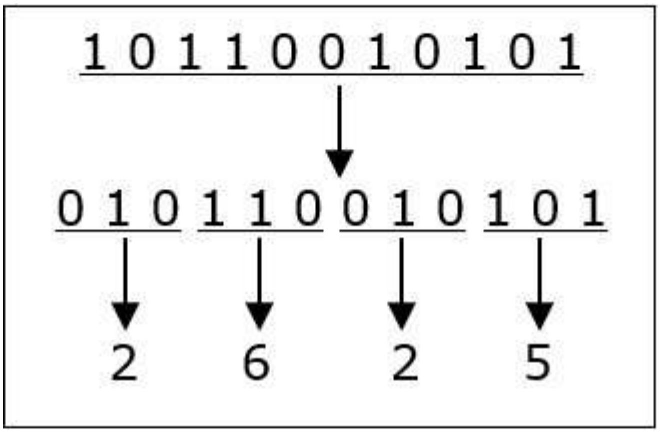
Binary to octal: (Other way of solving)

To convert a binary number to octal number, these steps are followed –

- *Starting from the least significant bit, make groups of three bits.*
- *If there are one or two bits less in making the groups, 0s can be added after the most significant bit*
- *Convert each group into its equivalent octal number*

Let's take an example to understand this.

Example: 10110010101_2 = _______$_8$



Answer: 1011001010_{12} = 2625_8

Octal to Binary

To convert an octal number to binary, each octal digit is converted to its 3-bit binary equivalent according to this table.

Example: 54673_8 = ____________ $_2$

Octal Digit	0	1	2	3	4	5	6	7
Binary Equivalent	000	001	010	011	100	101	110	111

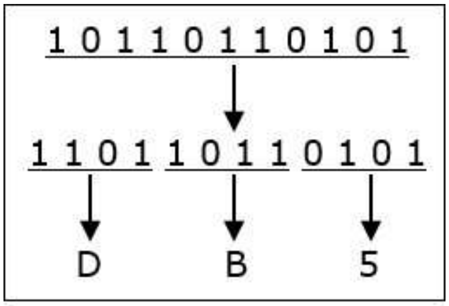
Answer: 54673_8 = 101100110111011_2

Binary to Hexadecimal

To convert a binary number to hexadecimal number, these steps are followed –

- *Starting from the least significant bit, make groups of four bits.*
- *If there are one or two bits less in making the groups, 0s can be added after the most significant bit.*
- *Convert each group into its equivalent octal number.*

Let's take an example to understand this.



$10110110101_2 = DB5_{16}$

To convert an octal number to binary, each octal digit is converted to its 3-bit binary equivalent.

References:

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2. Dennis T. Fajarito. 2010. Computer Fundamentals.
3. Number System Conversion - Tutorialspoint. 2020. Tutorials Point. Retrieved August 19, 2020 from https://www.tutorialspoint.com/computer_logical_organization/number_system_conversion.htm