Note: Some of these figures come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

Question: *If we have derivative rules for adding and subtracting functions, are there any rules for other arithmetic operations like dividing and multiplying?*

In the previous section we managed to find derivative rules for functions multiplied y a constant and for adding and subtracting functions. The rules came from the Limit Rule 1 and Limit Rule 2 in the Section 3.1 Notes. Let's see if we can find more derivative rules related to the Limit Rules in Section 3.1.

Recall Limit Rule 3: $\lim_{x\to a}(f(x)\cdot g(x)) = \left(\lim_{x\to a} f(x)\right)\cdot\left(\lim_{x\to a} f(x)\cdot g(x)\right) = L\cdot M.$

<u>**Product Rule:**</u> If both $f'(x)$ and $g'(x)$ exist, then $\frac{d}{dx}(f(x)\cdot g(x))$ exists and

$$\frac{d}{dx}[f(x)\cdot g(x)] = f(x)\cdot\frac{d}{dx}g(x) + g(x)\cdot\frac{d}{dx}f(x)$$

Example: Calculate the derivative of $h(x) = x^2$ using the product rule.

Example: Calculate the derivative of $h(x) = x^2 e^x$.

Example: Calculate the derivative of $h(x) = \frac{1}{x}\ln(x)$, $x \neq 0$ using the product rule.

Recall Limit Rule 4: $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$ if $\lim_{x\to a} g(x) = M \neq 0$.

Quotient Rule: If both $f'(x)$ and $g'(x)$ exist, then $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ exists and

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot \frac{d}{dx}f(x) - f(x)\cdot \frac{d}{dx}g(x)}{[g(x)]^2}$$

Example: Calculate the derivative of $h(x) = \frac{\ln(x)}{x}$, $x \neq 0$ using the quotient rule.

Example: Calculate the derivative of $h(x) = \frac{x^2+3x+2}{x^2}$, $x \neq 0$ using the quotient rule.