Note: Some of these figures come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

Question: *Can we represent the instantaneous rate of change of $f(x)$ as a function? How do we do this?*

In the previous section we defined the instantaneous rate of change of a function $f(x)$ at a single point, c, as

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}.$$

If we want to know the instantaneous rate of change of f at different points, we can just evaluate the limit with different values of c and check that it exists. This is time consuming and we want to be lazy. Instead, let's treat the c as a variable and take the limit.

Definition (Derivative): If $y=f(x)$, the derivative of $f(x)$, denoted by $f'(x)$, is defined to be

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

if this limit exists.

Recipe for Finding the Derivative of f(x):

1. Find $\frac{f(x+h)-f(x)}{h}$.
2. Simplify.
3. Take the limit as $h\to 0$ of the simplified expression. That is, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

Example: Calculate the derivative of $f(x)=5x+6$.

Example: Calculate the derivative of $f(x) = x^2$.

Example: Calculate the derivative of $f(x) = \frac{1}{x}$, $x \neq 0$.

Example: Calculate the derivative of $f(x) = \sqrt{x}$.

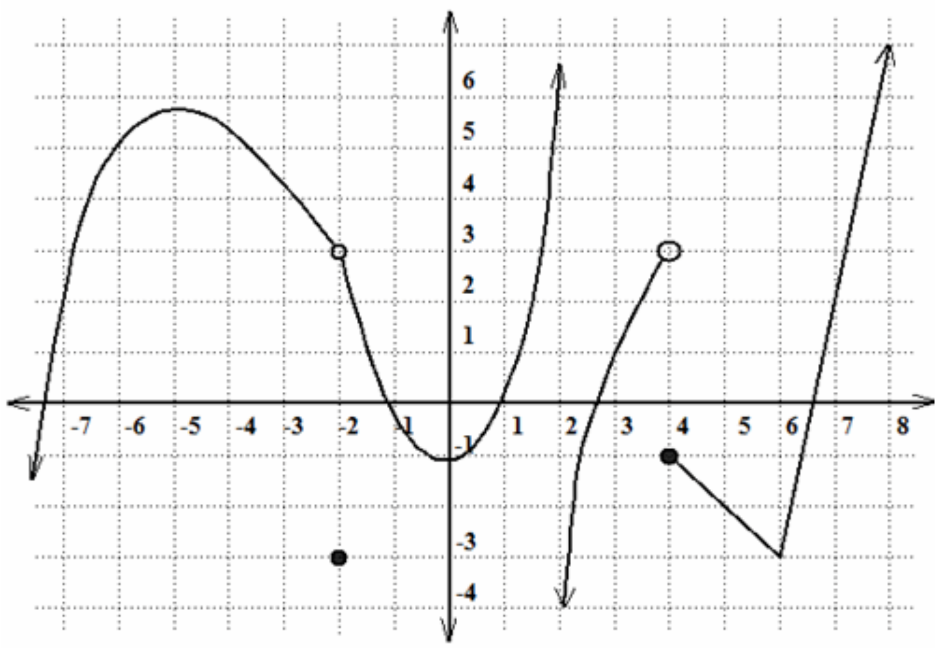
<u>**A Differentiable Function is Continuous:**</u> If $y = f(x)$ has a derivative at $x = c$, then $f(x)$ is continuous at $x = c$.

<u>**When Does the Derivative Fail to Exist?:**</u> The derivative fails to exist in the following circumstances.

1. The graph of the function has a corner.
2. The graph of the function has a vertical tangent.
3. The graph of the function has a break (discontinuity).

Example: Calculate the derivative of $f(x) = |x|$.

Example: Where does the derivative of the function not exist?



Example: Given the graph of the function $f(x)$, where is $f'(x) > 0$? Where is $f'(x) < 0$? Where is $f'(x) = 0$?