Note: This review does not encompass all of the expected background, but should give an idea of the bare minimum. For more background please refer to Chapter 1 of your book *Calculus: Applications and Technology*, 3rd ed., by Tomastik.

Question: *What should you know already?*

Factoring Trick: $a^2 - b^2 = (a+b)(a-b)$

Example: Factor $f(x) = x^2 - 64$.

Absolute Value:

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Example: Graph $|x|$.

Definition (Function): Let D (domain) and R (range) be two nonempty sets. A function f from D to R is a rule that assigns to each elements x in D one and only one element $y = f(x)$ in R.

Domain: The domain is the set of x values such that the output, $f(x)$, makes sense and is real. Think not $f(x) = \frac{0}{0}$, $f(x) = \frac{1}{0}$, or $f(x) =$ imaginary number.

Example: Find the domain of $f(x) = \sqrt{5x - 20}$.

Example: Find the domain of $f(x) = \frac{x^2-64}{x^2-6x-16}$.

Rational Functions: Any function that is the quotient of two polynomials, i.e. $R(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials is called a rational function.

Vertical Line Test: A graph in the xy-plane represents a function of x if and only if every vertical line intersects the graph in at most one place.

Example: Draw a graph that could represent a function and another graph that could not represent a function.

Point Slope Formula of the Line: The slope of the line passing through the points (x_0, y_0) and (x, y) is given by

$$m = \frac{y - y_0}{x - x_0}$$

Example: Draw a graph Illustrating the Point Slope Formula.

Vertex of Quadratic Functions: Let $q(x) = ax^2 + bx + c$, $a \neq 0$. We can place this in the standard form $q(x) = a(x-h)^2 + k$. The point (h,k) tells us the location of the vertex (min or max of the parabola).

$$h = -\frac{b}{2a}, \qquad k = c - \frac{b^2}{4a}$$

Example: Draw a concave up and concave down parabola and mark where (h,k) should be.

Composition of Functions: Given two functions $f(x)$ and $g(x)$, it's possible to compose them as $f(g(x))$ or $g(f(x))$. Most functions you will encounter can be thought of this way.

Example: Let $f(x) = -x^2$ and $g(x) = e^x$. What is $f(g(x))$? What is $g(f(x))$?

Other Things to Review:

1. **Exponential Functions:** $a^x, \quad a > 0$
2. **Laws of Exponential Functions**
   - $a^{x+y} = a^x \cdot a^y$
   - $a^{-x} = \frac{1}{a^x}$
   - $a^{-x}a^y = a^{y-x}$
   - $a^{-x}b^x = \left(\frac{b}{a}\right)^x$
   - $a^0 = 1$
   - $a^x > 0$
   - $(a^x)^y = a^{xy}$
3. **Logarithmic Functions:** $\log_a(x)$

4. **Laws of Logarithmic Functions**

    - $\log_a(xy) = \log_a(x) + \log_a(y)$
    - $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
    - $\log_a(b^x) = x\log_a(b)$
    - $\log_a(1) = 0$

5. **Power Functions:** ax^r

6. **Piecewise-defined Functions:** $f(x) = \begin{cases} g(x), & \text{if } x \geq a \\ h(x), & \text{if } x < a \end{cases}$

Mathematical Models of Cost, Revenue, and Profits

Definition (Fixed Costs): Fixed costs are those that do not depend on the amount of production.

Definition (Variable Costs): Variable costs are those that depend on the amount of production.

Definition (Cost): Cost is the sum of the fixed and variable costs.

Linear Cost Model:

- Assume that cost m of manufacturing one unit is the same no matter how many units are produced.
- Variable cost = (number of units produced) $\times$ (cost of a unit) $= mx$.
- $C(x)$ = Cost = Variable Cost $+$ Fixed Cost $= mx + b$.

Linear Revenue Model:

- Assume the price p of a unit is the same no matter how many units are produced.
- $R(x)$ = (price per unit) $\times$ (number of units sold) $= px$

Profit: Profit = Revenue - Cost

Example: Chuck Norris is creating a new type of Snake Oil that he plans to sell at a price of $\$25/gallon$. Let the total variable costs of manufacturing Snake Oil be $\$5/gallon$ and the fixed cost of the Snake Oil factory be $1000. What is the total cost, revenue, and profit from his new business?

Demand: $p(x) = -c \cdot (\text{number of units produced and sold}) + d = -cx + d$

- How do we find out c and d?
- Typically given (number of units produced and sold) at two different price points per unit. Apply Point Slope Formula to get $p(x) = -cx + d$.

Example: Wolf Cola's marketing research has determined that consumers are willing to purchase 100 cans of Wolf Cola each day at a price of $\$2/can$. At a price of $\$1/can$ consumers are willing to purchase 300 cans of Wolf Cola per day.

1. Determine the daily demand equation for Wolf Cola assuming a linear relation between price and quantity.

2. Determine the daily revenue function.

3. Determine when the revenue is maximized. (Hint: Use Vertex of Quadratic Functions Formula.)