Note: Some of these figures come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

Some of you made it apparent that some additional examples would be helpful for the Section 5.1 Webassign problems.

Example 1: Find the critical points of $f(x) = \dfrac{1}{|x|}$.

$$f(x) = \frac{1}{|x|} = \begin{cases} -\frac{1}{x} & \text{if } x < 0 \\ \frac{1}{x} & \text{if } x > 0. \end{cases}$$

The domain of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$ with a vertical asymptote at $x = 0$.

$$f'(x) = \frac{d}{dx}\frac{1}{|x|} = \begin{cases} \frac{d}{dx}\left[-\frac{1}{x}\right] & \text{if } x < 0 \\ \frac{d}{dx}\left[\frac{1}{x}\right] & \text{if } x > 0 \end{cases} = \begin{cases} \frac{1}{x^2} & \text{if } x < 0 \\ -\frac{1}{x^2} & \text{if } x > 0 \end{cases}$$

This function is increasing for $x < 0$ and decreasing for $x > 0$. Note that $x = 0$ is technically not a critical point, but the sign of the derivative flips as we cross the vertical asymptote.

Example 2: Find the critical points of

$$f(x) = \begin{cases} -x+1 & \text{if } x < 0 \\ x-1 & \text{if } x \geq 0. \end{cases}$$

The domain of $f(x)$ is $(-\infty, \infty)$ with a jump discontinuity at $x = 0$.

$$f(x) = \begin{cases} \frac{d}{dx}[-x+1] & \text{if } x < 0 \\ \frac{d}{dx}[x-1] & \text{if } x > 0. \end{cases} = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0. \end{cases}$$

This function is decreasing for $x < 0$ and increasing for $x > 0$. Note that $x = 0$ is a critical point because $x = 0$ is in the domain.

Example 3: Find the critical points of

$$f(x) = \begin{cases} -x+1 & \text{if } x < 0 \\ x-1 & \text{if } x > 0. \end{cases}$$

The domain of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$ with a jump discontinuity at $x = 0$.

$$f(x) = \begin{cases} \frac{d}{dx}[-x+1] & \text{if } x < 0 \\ \frac{d}{dx}[x-1] & \text{if } x > 0. \end{cases} = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0. \end{cases}$$

This function is decreasing for $x < 0$ and increasing for $x > 0$. Note that $x = 0$ is NOT a critical point because $x = 0$ is not in the domain.

What is special about these examples?

1. All three of these examples have a discontinuity.
2. All three of these examples have a sign flip in the derivative.
3. Only Example 2 has a critical point.
4. Example 1 and Example 3 have a gap in their domain at $x = 0$.

Example 4: Find where $f(x)$ is increasing given

$$f'(x) = \frac{x-4}{(x-5)^7}$$

and $x = 5$ is not in the domain.

•Just need to check the signs in the numerator and denominator.

•In the Numerator: $x-4 = 0$ at $x = 4$ and for x in $(-\infty, 4)$, $x-4$ is negative. For x in $(4, \infty)$, $x-4$ is positive.

•In the Denominator: $(x-5)^7 < 0$ where $x-5 < 0$ since 7 is an odd power. $(x-5)^7 > 0$ where $x-7 > 0$ since 7 is an odd power. For x in $(-\infty, 5)$ the denominator is negative. For x in $(5, \infty)$ the denominator is positive.

Based on the numerator and denominator sign flips, we can break the problem into three regions:

$$(-\infty, 4)$$

$$(4, 5)$$

$$(5, \infty)$$

For $(-\infty, 4)$ the numerator is negative and the denominator is negative. Thus $f'(x) > 0$ in $(-\infty, 4)$.
For $(4, 5)$ the numerator is positive and the denominator is negative. Thus $f'(x) < 0$ in $(4, 5)$.
For $(5, \infty)$ the numerator is positive and the denominator is positive. Thus $f'(x) > 0$ in $(4, 5)$.

What is special about Example 4?
It illustrates a strategy for functions with discontinuities. Notice $x = 4$ and $x = 5$ are the important points we need to be careful around. This is because $x = 4$ is a critical point and $x = 5$ is a discontinuity in the derivative and a gap in the domain.

Add: "Locate where $f(x)$ has gaps in the domain" to the procedure for finding where a function is increasing or decreasing. You're going to want to check either side fo the gap.
•It appears that your book does not address this case, but your Webassign does.