Note: Some of these figures come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

Question: *What does it mean in terms of the first derivative for a function to have a min or max?*

In the Review Notes we used the Quadratic Vertex Formula to find the maximum/vertex of a quadratic Revenue function. In Quiz 2 we showed that the (x,y) coordinates where $q'(x) = 0$ for $q(x) = ax^2 + bx + c$, $a \neq 0$ were the same as those found using the Quadratic Vertex Formula. We also saw on Quiz 2 that the slope flipped signs as we passed across the vertex of a quadratic function. It turns out that the Quadratic Vertex Formula always gives (h,k) coordinates that correspond to a max or a min of a quadratic function since the vertex of a quadratic function is always a max or min.

In this section we're going to see if the "derivative equal to zero" and "derivative sign flipping" properties around the max or min of a quadratic function generalize to functions that aren't quadratic.

Example: Sketch $h(x) = 5 - x^2$. Where do the tangent lines of $h(x)$ have positive slope. Where do the tangent lines of $h(x)$ have negative slope? Where does the tangent line of $h(x)$ have slope zero?

<u>**Test for Increasing of Decreasing Functions:**</u> If for all $x \in (a,b)$, $f'(x) > 0$, then $f(x)$ is increasing on (a,b). If for all $x \in (a,b)$, $f'(x) < 0$, then $f(x)$ is decreasing on (a,b).

Example: Use derivatives. Let $h(x) = 5x + 6$. Where is $h(x)$ increasing? Where is $h(x)$ decreasing?

Example: Use derivatives. Let $h(x) = 5 - x^2$. Where is $h(x)$ increasing? Where is $h(x)$ decreasing? Where is it doing neither?

Example: Sketch $f(x) = |x|$. Where is $f(x)$ increasing? Where is $f(x)$ decreasing? Where is it doing neither?

<u>Critical Value:</u> A value $x = c$ is a critical value for a function $f(x)$ if

1. c is in the domain of the function $f(x)$ and
2. $f'(c) = 0$ or $f'(c)$ does not exist.

Example: Find the critical values of $f(x) = 5x^3 + 15x^2 + 10$.

Example: Find the critical values of $f(x) = \ln(x)$.

Example: Find the critical values of $f(x) = |x|$.

Connect the Dots: Mark the points $a = (0,2)$ and $b = (5,-2)$. Place your pen/pencil on a. Without lifting your writing utensil or crossing the x-axis, try to connect the points. Can you connect the points?

Intuitively: For a continuous function to go from positive values to negative values, it must

____.

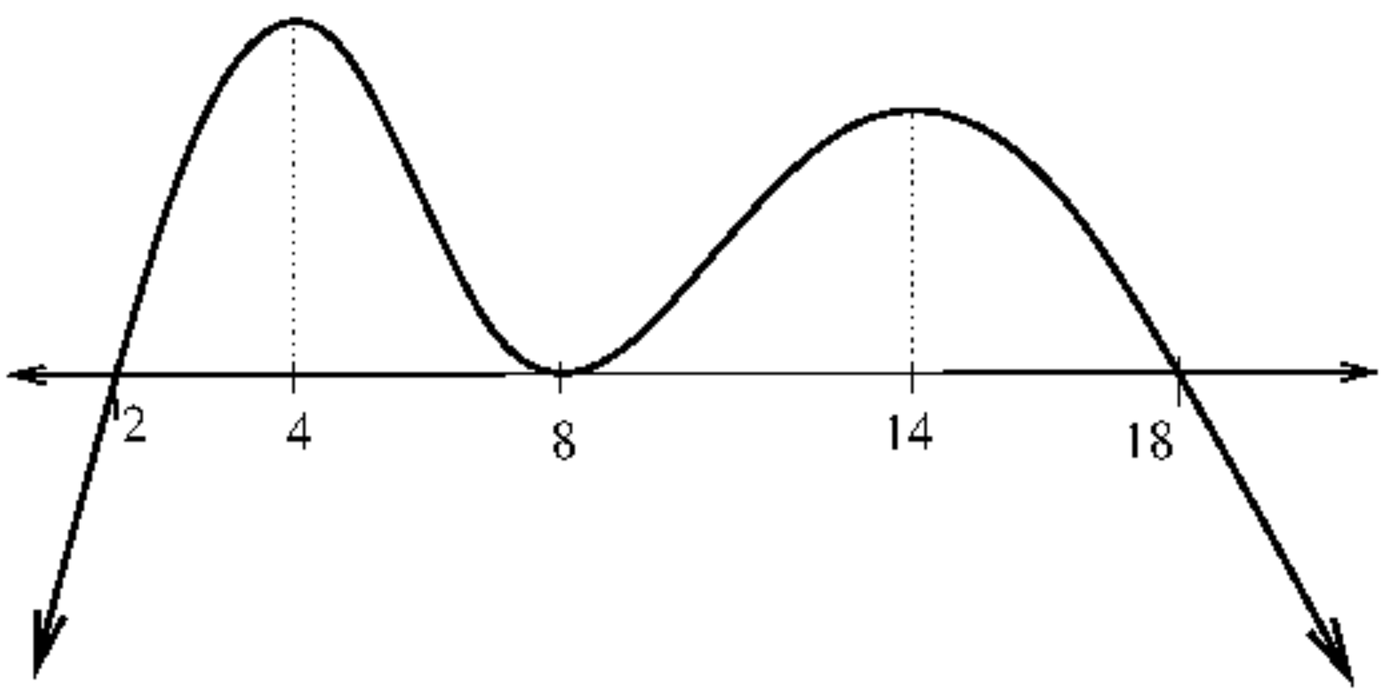
Constant Sign Theorem: If $f(x)$ is continuous on (a,b) and $f(x) \neq 0$ for any x in (a,b), then either $f(x) > 0$ for all x in (a,b) or $f(x) < 0$ for all x in (a,b).

Intuitively: The zeros of $f(x)$ give us the subintervals where $f(x)$ is always positive or always negative.

Recipe for Finding Where a Function Is Increasing or Decreasing:

1. Find the critical values
   (a) Locate all values c for which $f'(c) = 0$.
   (b) Locate all values c for which $f'(c)$ does not exist but $f(c)$ does.
2. Find the subintervals on which $f'(x)$ has constant sign (critical points will give subintervals).
3. Select a convenient test value on each subinterval found in the previous step and evaluate $f'(x)$ at this value. The sign of $f'(x)$ at this test value will be the sign of $f'(x)$ on the entire subinterval.
4. Use the test for increasing or decreasing functions to determine whether $f(x)$ is increasing or decreasing on each subinterval.

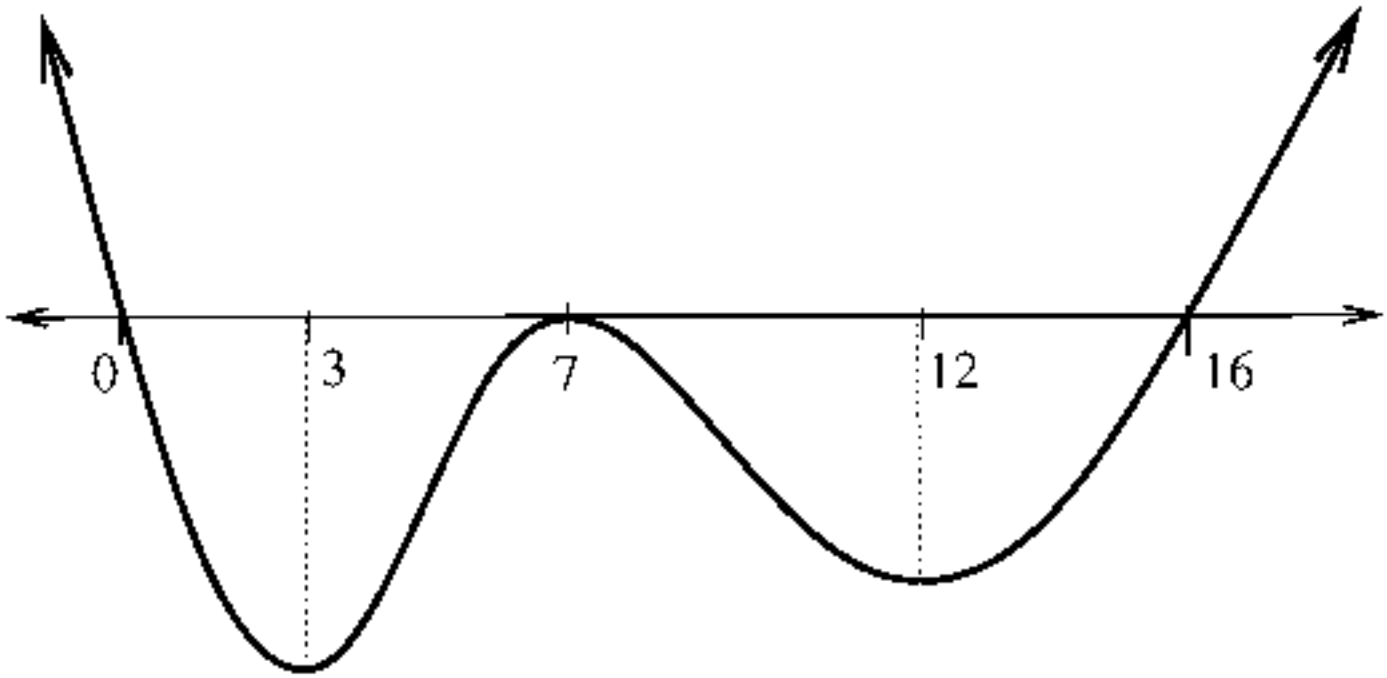
Example: Find where $f(x)$ is increasing and decreasing given the graph of $f'(x)$ below.



<u>Relative Maximum and Relative Minimum:</u> We say that the quantity $f(c)$ is a relative maximum if $f(x) \leq f(c)$ for all x in some open interval (a,b) that contains c. We say that $f(c)$ is a relative minimum if $f(x) \geq f(c)$ for all x in some open interval (a,b) that contains c.

<u>Relative Extremum:</u> We say that $f(c)$ is a relative extremum if $f(c)$ is a relative maximum or minimum.

Example: Find the relative extremum given the graph of $f(x)$ below.



First Derivative Test: Suppose f is defined on (a,b) and c is a critical value in the interval (a,b).

1. If $f'(x) > 0$ for x near and to the left of c and $f'(x) < 0$ for x near and to the right of c, then $f(c)$ is a relative maximum.
2. If $f'(x) < 0$ for x near and to the left of c and $f'(x) > 0$ for x near and to the right of c, then $f(c)$ is a relative minimum.
3. If the sign of $f'(x)$ is the same on both sides of c, then $f(c)$ is not a relative extremum.

Note: Figure 5.9 in Section 5.1 of your book features all of the cases for the first derivative test. I recommend looking at it.

Example: Suppose the domain of $f(x)$ is all real numbers except $x = 6$. Let $f'(x) = \dfrac{x-2}{(x-6)^7}$. Where is $f(x)$ increasing? Where is $f(x)$ decreasing?