Note: Some of these figures come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

Question: *What can we do with derivatives and how are they used in practice?*

Chapter 3 was dedicated to the theoretical underpinnings necessary for derivatives. Chapter 4 was dedicated to rules for calculating derivatives. This note is dedicated to different real world uses of the derivative rules we've learned so far.

Interpretations of Derivatives:

1. Instantaneous rate of change
2. Slope of the tangent line
3. Marginals (cost revenue, and profit)

What can we do with a derivative?:

1. Calculate the instantaneous rate of change
2. Calculate the slope of the tangent line
3. Determine whether a function is increasing or decreasing (going up or down)
4. Locally approximate a function (Linear Approximation)
5. Model nature (Differential Equations)
6. Find the maximum or minimum of a function (Optimization)

Finding the Marginal Profit: Suppose the the cost and revenue functions are given by $C(x)$ and $R(x)$, respectively, where x is measured in pounds of nails and $C(x)$ and $R(x)$ are given in cents. $C'(x)$ is referred to as marginal cost. $R'(x)$ is referred to as marginal revenue. Suppose the marginal revenue for 1000 pounds of nails is 52 and the marginal cost for 1000 pounds of nails is 50. Find the marginal profit, $P'(x)$, for 1000 pounds of nails abd interpret the solution.

Calculate the marginal cost, marginal revenue, and marginal profit problem: A quadratic model for cost in thousands of dollars for a large cotton gin plant is given by

$$C(x) = 0.026689x^2 + 21.7397x + 377.3865,$$

where x is the annual quantity of bales in thousands produced if annual production is below 50,000 bales. Revenue was estimated at \$63.25 per bail.

1. Find the marginal cost.
2. Find the marginal revenue.
3. Find the marginal profit.

Calculate the instantaneous rate of change problem: Let Sally's position in space be given by the function $f(t) = 4t$. What is Sally's velocity?

Determine whether a function is increasing or decreasing problem: Elandt-Johnson and Johnson determined that the survival curve for white males is

$$y = 0.99e^{-0.001t - 0.0001t^2},$$

where t is measured in years. Is this curve increasing or decreasing?

Example: The demand equation for a certain product is given by

$$p = \frac{1}{\sqrt{1+x^2}},$$

where x is the number of items sold and p is the price in dollars. Find the instantaneous rate of change of p with respect to x.

Example: The Gompertz growth curve used in modeling tumor growth is given by

$$P(t) = ae^{-be^{-kt}}.$$

Show that $P'(t)$ is always positive, i.e. the tumors keep growing.

Example: Suppose that the concentration c of a drug in the blood t hours after it is taken orally is given by

$$c = \frac{3t}{2t^2+5}.$$

What is the rate of change of c with respect to t?

Example: Suppose the profit equation is given by

$$P(x) = xe^{-\frac{1}{2}x^2}.$$

Find the marginal profit and where the marginal profit is zero.

Derivative Approximation: If h is small, then

$$f'(c) \approx \frac{f(c+h) - f(c)}{h}$$

or

$$f(c+h) \approx f(c) + f'(c)h.$$

Tangent Line Approximation: If $y = f(x)$ is differentiable at $x = c$, then for values of x near c,

$$f(x) \approx f(c) + f'(x)(x - c).$$

Example: Find an approximation to $\sqrt{4.1}$ using the tangent line approximation.

Example: Find an approximation to $\sqrt{8.9}$ using the tangent line approximation.

Example: Let $P(x)$ be the profit function for a firm with x being the number of units produced and sold. If $P'(1000) = 2$, what is the approximate profit from selling the 1001st unit?

Example of a Differential Equation:

$$f'(x) = 5.$$