

**Quiz 5 (100 points)****Due in class 6/20/2018****Name:**

*NOTE: YOU MUST SHOW YOUR WORK TO RECEIVE FULL CREDIT. REMEMBER TO BOX YOUR FINAL ANSWER(S).*

1. (100pts) When you walk around outside, the Earth seems fairly flat, but we know the Earth is round (If you're a Flat Earther, too bad.). The purpose of this quiz is to use implicit differentiation to see how much the Earth deviates from being flat by comparing the difference between where you would end up walking along a tangent line (flat situation) versus walking on the surface of the Earth for greater and greater distances.

Let  $x^2 + y^2 = 4000^2 = 16,000,000$  be the equation of a circle that is roughly the size of Earth.

- (a) Calculate the derivative of  $y$  using implicit differentiation.
- (b) Compute the equation of the tangent line at  $(2000\sqrt{2}, 2000\sqrt{2})$
- (c) Find the  $x$  and  $y$  on the tangent line where the distance between  $(x, y)$  and  $(2000\sqrt{2}, 2000\sqrt{2})$  is 10 miles and  $x > 2000\sqrt{2}$ . (Hint  $10 = D = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ .  $x_0 = 2000\sqrt{2}$ ,  $y_0 = 2000\sqrt{2}$ ,  $y = y'(x_0, y_0) \cdot (x - x_0) + y_0$ . Plugging all of this into  $D$  gives you a function of  $x$ . Solve for  $x$  and use  $y = y'(x_0, y_0) \cdot (x - x_0) + y_0$  to find  $y$ .)
- (d) Walking 10 miles on the circle puts you at the coordinates  $(x, y) = (2835.489, 2821.347)$ . Calculate the distance between this point and the location you found in part (c). What percentage of 10 is this distance?
- (e) Walking 100 miles on the circle puts you at the coordinates  $(x, y) = (2898.247, 2756.840)$ . Walking 100 miles on that tangent line puts you at  $(2050\sqrt{2}, 1950\sqrt{2})$ . Calculate the distance between these points. What percentage of 100 is this distance?
- (f) Walking 1000 miles on the circle puts you at the coordinates  $(x, y) = (3440.262, 2040.734)$ . Walking 1000 miles on that tangent line puts you at  $(2500\sqrt{2}, 1500\sqrt{2})$ . Calculate the distance between these points. What percentage of 1000 is this distance?