Note: Some of these examples come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

Question: *Do we always have to go through the hassle of taking limits? Are there tricks for calculating derivatives of the most common functions?*

What are some of the most common functions?:

1. Constants: $f(x) = c$

   Example: $f(x) = 5$, $f(x) = 10^7$

2. Powers of x: $f(x) = x^n$

   Example: $f(x) = x^2$, $f(x) = x^{-3}$, $f(x) = x^{\frac{1}{4}}$

3. $f(x) = e^x$

4. $f(x) = \log_e(x) = \ln(x)$

Derivative of a constant function: For any constant c,

$$\frac{d}{dx}(c) = 0.$$

Intuitively: A constant function never changes?

Example: Calculate the derivative of $f(x) = 10$.

Derivative of a power of x: If n is any real number,

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

Example: Calculate the derivative of $f(x) = x^2$.

Example: Calculate the derivative of $f(x) = \frac{1}{x}$, $x \neq 0$.

Example: Calculate the derivative of $f(x) = \sqrt{x}$.

Derivative of e^x:

$$\frac{d}{dx}(e^x) = e^x.$$

Derivative of $\ln(x)$: If $x > 0$, then

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}.$$

Question: Based on the rules above, can we calculate the derivative of polynomials like $f(x) = 5x^2$ or $g(x) = 7x^3 + 9x + 1$? Why or why not? What do we need?

Remember: Derivatives are just limits!

Recall Limit Rule 1: $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x) = cL$

Example: Simplify $\lim_{h\to 0} c\left(\frac{f(x+h)-f(x)}{h}\right)$ using Limit Rule 1.

Derivative of a Constant Times a Function: If $f'(x)$ exists, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)].$$

Example: Calculate the derivative of $f(x) = 5x^2$.

Recall Limit Rule 2: $\lim_{x\to a}(f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = L \pm M$

Example: Simplify $\lim_{h\to 0} \frac{f(x+h)+g(x+h)-(f(x)+g(x))}{h}$ using Limit Rule 2.

Derivative of a Sum or Difference: If $f'(x)$ and $g'(x)$ exist, then $\frac{d}{dx}[f(x) \pm g(x)]$ exists, and

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)].$$

Example: Calculate the derivative of $g(x) = 7x^3 + 9x + 1$.

More Derivative Practice

Example: Calculate the derivative of $f(x) = x^2 + 6\ln(x) + 12e^x$.

Example: Calculate the derivative of $f(x) = \ln(x^6)$.

Example: Calculate the derivative of $f(x) = e^{80}$.

Example: Calculate the derivative of $f(x) = x^{-3} + \ln(x) + 10$.

Applications of the Derivative

Remember: $f'(x)$ is the slope of the function $f(x)$ which is the slope of the tangent line at x.

Example: Find the equation of the tangent line to the curve $f(x) = x^4 + 5x^2 - x$ at $(1,5)$.

Example: Find the equation of the tangent line to the curve $f(x) = x^2 - 12x$ at $x = 5$.

Example: Find the points where the tangent lines are horizontal for $f(x) = 2x^3 + 3x^2 - 12x + 9$.