

Note: Some of these figures come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

Question: *How do we analyze the long term behavior of a function?*

In Sections 5.1 and 5.2 we used derivative data to describe the behavior of a function in different regions. Many functions are defined on $(-\infty, \infty)$ or for arbitrarily large values of $|x|$. Unfortunately, as humans we can really only check function behavior on small intervals of finite length. Sometimes we'd like to have some idea about how a function behaves for very large values of $|x|$. To do this we can look at limits as $|x|$ goes off to infinity.

Limits at infinity:

1. If $f(x)$ approaches the number L as x becomes large without bound, then we say that L is the limit of $f(x)$ as x approaches ∞ , and we write

$$\lim_{x \rightarrow \infty} f(x) = L$$

2. If $f(x)$ approaches the number K as x becomes a large negative number without bound, then we say that K is the limit of $f(x)$ as x approaches $-\infty$, and we write

$$\lim_{x \rightarrow -\infty} f(x) = K$$

We refer to the lines $y = L$ and $y = K$ as horizontal asymptotes.

Example: Calculate $\lim_{x \rightarrow \infty} \frac{1}{x}$.

Example: Calculate $\lim_{x \rightarrow \infty} \frac{1}{x^n}$ for $n > 0$. (Hint: Use Limit Power Rule)

Example: Calculate $\lim_{x \rightarrow \infty} e^x$

Example: Calculate $\lim_{x \rightarrow \infty} e^{-x}$

Example: Calculate $\lim_{x \rightarrow \infty} x^n$ for $n > 0$

Example: Calculate $\lim_{x \rightarrow \infty} x^5 - x^4$. (Hint: Use Limit Product Rule)

Example: Calculate $\lim_{x \rightarrow \infty} \frac{x^5 - x^4}{x^5}$

Example: Calculate $\lim_{x \rightarrow \infty} \frac{3x^3 - x^2 + 4x}{2x^3 + 4x + 5}$

Example: Calculate $\lim_{x \rightarrow \infty} \frac{99x^{100} - x^{29} + 4x}{4x^{100} + 49x^{77} + 5}$

Example: Calculate $\lim_{x \rightarrow \infty} \frac{509x^{100} - x^{29} + 4x}{44x^{89} + 49x^{77} + 5}$

Example: Calculate $\lim_{x \rightarrow -\infty} \frac{x^{199} + 12x^{29} + 4x}{44x^{200} + 59x^{77} + 5}$

Example: Calculate $\lim_{x \rightarrow \infty} \frac{5 + 3e^x}{10 + 4e^x}$

Example: Calculate $\lim_{x \rightarrow -\infty} \frac{5 + 3e^x}{10 + 4e^x}$