Exam 3 KEY

MATH 142
Version A

Summer '18

Name (printed): ____________________

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

Name (signature): ____________________ **Section:** ____________________

Instructions:

- You must clear your calculator: MEM (2nd +), Reset (7), cursor right to ALL, All Memory (1), Reset (2).
- There are 17 questions and 9 pages to this exam including the cover sheet. The multiple choice questions are worth 5 points each, and the point values for the problems in the work out section are as indicated. There is no partial credit for the multiple choice problems, but partial credit will be given, if deserved, on the work out problems.
- Clearly circle exactly one answer for each multiple choice question. No partial credit will be given.
- In order to receive full credit on the work out problems, you must show appropriate, legible work.
- You must box or circle your final answer in the work out section.
- Please turn caps bills to the back.
- Please put your cell phone away.
- Please remove any smart watches.
- Disputes about grades on this exam must be handled within ONE WEEK from the day the exam is handed back. After this day, exams will not be re-assessed.
- Your grade on the exam will be written inside on the first page.

GOOD LUCK!

MULTIPLE CHOICE (5 points each)

Implicit Differentiation
Related Rates

1. If $y = x^3 + 4x$ and $\frac{dx}{dt} = 2$, find $\frac{dy}{dt}$ when $x = 5$.

   (a) **158**

   (b) 79

   (c) 83

   (d) 14

   (e) None of the above

Indefinite Integrals

2. Evaluate $\int \frac{12e^{-x} + 14}{2e^{-x}}\, dx$

   (a) $6x - 7e^{-x} + C$

   (b) $\frac{1}{2}\ln|2e^{-x}| + C$

   (c) $\mathbf{6x + 7e^{x} + C}$

   (d) $-12e^{-x} + 14e^{x} + C$

   (e) $6\ln|2e^{-x}| + C$

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3. Given $f(x) = \int_2^x \frac{\sqrt{u^2 + \ln(u) + 39}}{e^u + 25u}\, du$, find $f'(x)$.

   (a) $\mathbf{\frac{\sqrt{x^2 + \ln(x) + 39}}{e^x + 25x}}$

   (b) $\frac{\sqrt{x^2 + \ln(x) + 39}}{e^x + 25x} - \frac{\sqrt{2^2 + \ln(2) + 39}}{e^2 + 50}$

   (c) $\frac{\sqrt{x^2 + \ln(x) + 39}}{e^x + 25x} + C$

   (d) $\ln\left|\frac{\sqrt{x^2 + \ln(x) + 39}}{e^x + 25x}\right|$

   (e) None of the above

Average Value of a Function

4. Calculate the average value of $f(x) = x^2 - 1$ over the interval $[-1, 1]$

   (a) $-\frac{4}{3}$

   (b) 0

   (c) $\frac{2}{3}$

   (d) $\frac{4}{3}$

   (e) $\mathbf{-\frac{2}{3}}$

Properties of Definite Integrals

5. Assume $f(t)$ is continuous and $f > 0$. Which of the following are true about $h(x) = \int_1^x f(t)\,dt$ for $x > 1$?

   (a) Its derivative is continuous.

   (b) It is the area under the curve over the interval $[1, x]$

   (c) It is an antiderivate of f

   (d) (b) and (c)

   (e) **(a), (b), and (c)**

Indefinite Integral

6. Evaluate $\int [x^3 + 2x + 22]\,dx$.

   (a) $\mathbf{\frac{x^4}{4} + x^2 + 22x + C}$

   (b) $3x^2 + 2 + C$

   (c) $3x^2 + 2$

   (d) $\frac{x^4}{4} + x^2 + 22 + C$

   (e) $x^4 + 2x^2 + 22x + C$

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7. Which of the following is an antiderivative of 2^x?

   (a) $\mathbf{\frac{1}{\ln(2)} \cdot 2^x + 2}$

   (b) $2^x + 2$

   (c) $\ln(2) \cdot 2^x$

   (d) 2^{x+1}

   (e) $(x-1) \cdot 2^{x-1}$

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8. Let $f(x) = x^2 + A$. Find the value of A that makes $\int_{-1}^{1} f(x)\, dx$ equal to zero.

   (a) $A = 0$

   (b) $\mathbf{A = -\frac{1}{3}}$

   (c) $A = \frac{2}{3}$

   (d) $A = \frac{1}{3}$

   (e) $A = -\frac{2}{3}$

Properties of Definite Integrals

9. Assume f is continuous. Given $f(x) \leq 159$, which of the following must be true?

   (a) $\int_{1}^{4} f(x)\, dx < 476$

   (b) $\mathbf{\int_{-1}^{1} f(x)\, dx \leq 318}$

   (c) $\int_{1}^{4} f(x)\, dx \leq 476$

   (d) $\int_{-1}^{1} f(x)\, dx < 318$

   (e) None of the above.

Implicit Differentiation

10. Let $y^2x+\ln|y|=22x$. Calculate $\dfrac{dy}{dx}$.

(a) $\mathbf{\dfrac{22-y^2}{2yx+\frac{1}{y}}}$

(b) $\dfrac{22-y^2}{\frac{1}{y}}$

(c) $\dfrac{22x-y^2}{\frac{1}{y}}$

(d) $\dfrac{22x-y^2}{2yx+\frac{1}{y}}$

(e) $\dfrac{22x-y^2}{2x+1}$

WORK-OUT

Properties of Definite Integrals

11. (7 points) Let $\int_0^3 7f(x)\,dx=63$ and $\int_0^3 4g(x)\,dx=68$. Calculate $\int_0^3 f(x)-g(x)\,dx$. Show your work.

$$\int_0^3 7f(x)\,dx=63 \implies \int_0^3 f(x)\,dx=9$$
$$\int_0^3 4g(x)\,dx=68 \implies \int_0^3 g(x)\,dx=17$$
$$\int_0^3 f(x)-g(x)\,dx=\int_0^3 f(x)\,dx-\int_0^3 g(x)\,dx=9-17=-8$$

Integral Applications

12. (8 points) Find $y(t)$ using the following information.

$$\frac{dy}{dt}=t^5-7e^t; \qquad y(0)=-9.$$

Show your work.

Calculate the indefinite integral:

$$y(t)=\int t^5-7e^t\,dt=\frac{t^6}{6}-7e^t+C$$

Find C using $y(0)$:

$$-9=y(0)=\frac{0^6}{6}-7e^0+C=-7+C$$

$$\implies C=-2$$

Put everything together:

$$y(t)=\frac{t^6}{6}-7e^t-2$$

Substitution

Indefinite Integral

13. (7 points) Evaluate $\int \frac{e^{19x}}{e^{19x}+2}\,dx$. Show your work.

Use Substitution

$$\int \frac{e^{19x}}{e^{19x}+2}\,dx = \int \frac{1}{e^{19x}+2}\cdot (e^{19x}\,dx)$$

$u = e^{19x}+2$

$$\frac{du}{dx} = 19e^{19x} \implies \frac{du}{19} = e^{19x}\,dx$$

$$\int \frac{e^{19x}}{e^{19x}+2}\,dx = \int \frac{1}{e^{19x}+2}\cdot (e^{19x}\,dx) = \int \frac{1}{u}\cdot (\frac{du}{19}) = \frac{1}{19}\int \frac{1}{u}\,du = \frac{1}{19}\ln|u|+C = \frac{1}{19}\ln|e^{19x}+2|+C$$

Riemann Sums

Estimating Distance Traveled

14. (8 points) Given a velocity function $v(t) = \frac{420}{t}+42$ ft/s. Use the interval $[3,7]$ with $n=4$ subintervals to calculate the left-hand and right-hand sums and estimate the distance traveled. Show your work.

Break $[3,7]$ **into 4 subintervals of equal size:**

$[3,7] = [3,4]\cup[4,5]\cup[5,6]\cup[6,7]$

Left Endpoints = $\{3,4,5,6\}$

Right Endpoints = $\{4,5,6,7\}$

Length of Subintervals = $\Delta x = \frac{7-3}{4} = 1$

Left Sum = $f(3)\Delta x + f(4)\Delta x + f(5)\Delta x + f(6)\Delta x = (42+\frac{420}{3})\cdot 1 + (42+\frac{420}{4})\cdot 1 + (42+\frac{420}{5})\cdot 1 + (42+\frac{420}{6})\cdot 1$

$= (42+\frac{420}{3}) + (42+\frac{420}{4}) + (42+\frac{420}{5}) + (42+\frac{420}{6}) = (42+140)+(42+105)+(42+84)+(42+70)$

$= 182+147+126+112 = 567$ **ft**

Right Sum = $f(4)\Delta x + f(5)\Delta x + f(6)\Delta x + f(7)\Delta x = (42+\frac{420}{4})\cdot 1 + (42+\frac{420}{5})\cdot 1 + (42+\frac{420}{6})\cdot 1 + (42+\frac{420}{7})\cdot 1$

$= (42+\frac{420}{4}) + (42+\frac{420}{5}) + (42+\frac{420}{6}) + (42+\frac{420}{7}) = (42+105)+(42+84)+(42+70)+(42+60)$

$= 147+126+112+102 = 487$ **ft**

Related Rates

15. (8 points) A 50 foot ladder leaned up against a wall. The bottom of the ladder begins to slip away from the wall. If the top of the ladder is sliding down the wall at a rate of 8 ft/s at a height of 30 feet, how fast is the bottom of the ladder moving? Show your work.

$x^2+y^2=50^2=2500$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$\implies \frac{dx}{dt}=-\frac{y}{x}\frac{dy}{dt}$

$y=30$ **ft and** $\frac{dy}{dt}=-8$ **ft/s, but we still need** x

$x^2=2500-30^2=1600$

$\implies x=40$ **ft**

Pluggin eveything in gives:

$$\frac{dx}{dt}=-\frac{30\ ft}{40\ ft}\cdot(-8\ ft/s)=6\ ft/s$$

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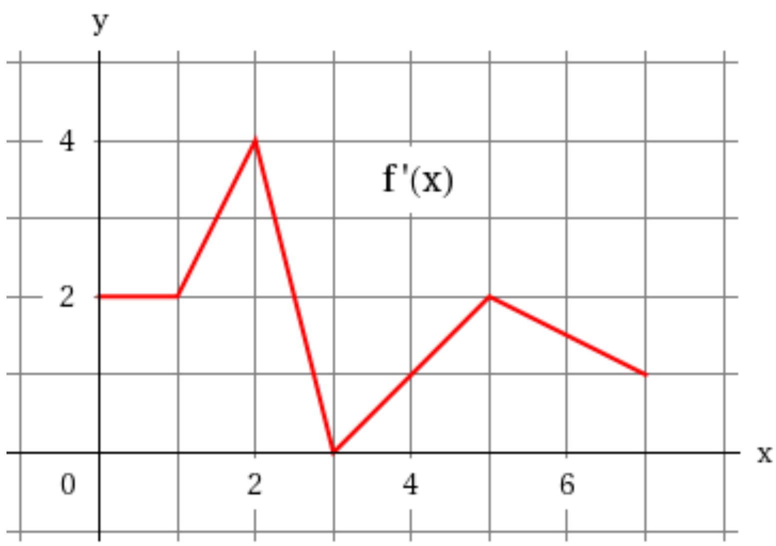
16. (6 points) Use the given graph of the derivative f' of a continuous function f over the interval $(0,7)$ and the fact that $f(0)=9$ to compute $f(3)$ and $f(4)$. Show your work.



Using Fundamental Theorem of Calculus:

$f(3)-f(0)=\int_0^3 f'(x)\,dx=$ **Area Under Curve on** $[0,3]=2\cdot1+2\cdot1+\frac{1}{2}\cdot1\cdot2+\frac{1}{2}\cdot1\cdot4=7$

$f(3)=7+f(0)=7+9=16$

$f(4)-f(3)=\int_3^4 f'(x)\,dx=$ **Area Under Curve on** $[3,4]=\frac{1}{2}\cdot1\cdot1=\frac{1}{2}$

$f(4)=\frac{1}{2}+f(3)=\frac{1}{2}+16=16.5$

Evaluating a Definite Integral

17. (6 points) Evaluate $\int_0^B [x^3 + 3x]\, dx$ by hand.

$$\int_0^B x^3 + 3x\, dx = \frac{x^4}{4} + \frac{3}{2}x^2 \Big|_0^B = \frac{B^4}{4} + \frac{3}{2}B^2 - \frac{0^4}{4} + \frac{3}{2}0^2 = \frac{B^4}{4} + \frac{3}{2}B^2$$