Exam 1 KEY

MATH 142 Summer '18

Version A

Name (printed): ____

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

Name (signature): ______________________ **Section:** ______________________

Instructions:

- You must clear your calculator: MEM (2nd +), Reset (7), cursor right to ALL, All Memory (1), Reset (2).
- There are 20 questions and 9 pages to this exam including the cover sheet. The multiple choice questions are worth 4 points each, and the point values for the problems in the work out section are as indicated. There is no partial credit for the multiple choice problems, but partial credit will be given, if deserved, on the work out problems.
- Clearly circle exactly one answer for each multiple choice question. No partial credit will be given.
- In order to receive full credit on the work out problems, you must show appropriate, legible work.
- You must box or circle your final answer in the work out section.
- Please turn caps bills to the back.
- Please put your cell phone away.
- Please remove any smart watches.
- Disputes about grades on this exam must be handled within ONE WEEK from the day the exam is handed back. After this day, exams will not be re-assessed.
- Your grade on the exam will be written inside on the first page.

GOOD LUCK!

MULTIPLE CHOICE (4 points each)

1. What is the derivative of $g(x) = \dfrac{5x^2}{e^x}$?

   (a) $\dfrac{10x}{e^x}$

   (b) $\dfrac{10xe^x + 5x^2e^x}{(e^x)^2}$

   (c) $\mathbf{\dfrac{10xe^x - 5x^2e^x}{(e^x)^2}}$

   (d) $\dfrac{5x}{e^x}$

   (e) None of the above

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2. Find the domain of $f(t) = \sqrt{5-t} + \sqrt{t}$

   (a) $(-\infty, \infty)$

   (b) $(-\infty, 5] \cup [0, \infty)$

   (c) $(-\infty, 0)$

   (d) $(0, 5)$

   (e) $\mathbf{[0, 5]}$

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3. What is the derivative of $f(x) = x\ln(x) - x$?

   (a) $\ln(\mathbf{x})$

   (b) 0

   (c) $1 - x$

   (d) $\frac{1}{x}$

   (e) x^2

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4. Calculate the limit $\lim_{x\to 1} \dfrac{x^3 - x}{x^2 - 3x + 2}$.

   (a) 1

   (b) The limit does not exist.

   (c) -1

   (d) 0

   (e) $\mathbf{-2}$

Infinite Limits

5. Where are the vertical asymptotes of $f(x) = \frac{x}{x^3-15x^2+56x}$.

   (a) $x = 0$

   (b) $x = 7, x = 8, x = 0$

   (c) $\mathbf{x = 7, x = 8}$

   (d) $x = 0, x = 7$

   (e) No vertical asymptotes

Average Rate of Change

6. Varys swims 1000 miles from Westeros to Meereen in 24 hours. What is Varys's average speed?

   (a) **41**.7 mph

   (b) 43.1 mph

   (c) 40.6 mph

   (d) 39.8 mph

   (e) 45 mph

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7. Let $f(x) = x^3 + 12$. What is the slope of the secant line that passes through the graph of $f(x)$ at $x = -1$ and $x = 3$?

   (a) 27

   (b) $\frac{26}{4}$

   (c) **7**

   (d) $\frac{10}{4}$

   (e) 9

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8. Let $h(x) = \dfrac{x \cdot f(x) - 2f(x)}{x-2}$. Let $\lim_{x\to 2} f(x) = 5$. What is $\lim_{x\to 2} h(x)$?

   (a) 10

   (b) Limit does not exist

   (c) $\frac{0}{0}$

   (d) **5**

   (e) 2

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9. Let $f(x) = x^2 + Ax + 5$. Find the value of A that makes $f'(2) = 2$.

   (a) $A = 4$

   (b) $\mathbf{A = -2}$

   (c) $A = 7$

   (d) $A = -4$

   (e) $A = 2$

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10. Let $f(x) = \ln(x)$. Find the equation of the tangent line to $f(x)$ at $x = e^2$.

   (a) $\mathbf{y = \frac{x}{e^2} + 1}$

   (b) $y = ex + e^2$

   (c) $y = \frac{x}{e^2} + 1 - e^2$

   (d) $y = -\frac{x}{e^2} + 1$

   (e) $y = \frac{x}{e}$

WORK-OUT

Estimating Limits

11. (8 points total) The graph of the function $g(x)$ is given on the right.

    State the value of each quantity. If it does not exist, write DNE.

   (a) $\lim_{t \to 3^-} g(t) = \mathbf{4}$

   (b) $\lim_{t \to 3^+} g(t) = \mathbf{7}$

   (c) $\lim_{t \to 3} g(t) = \mathbf{DNE}$

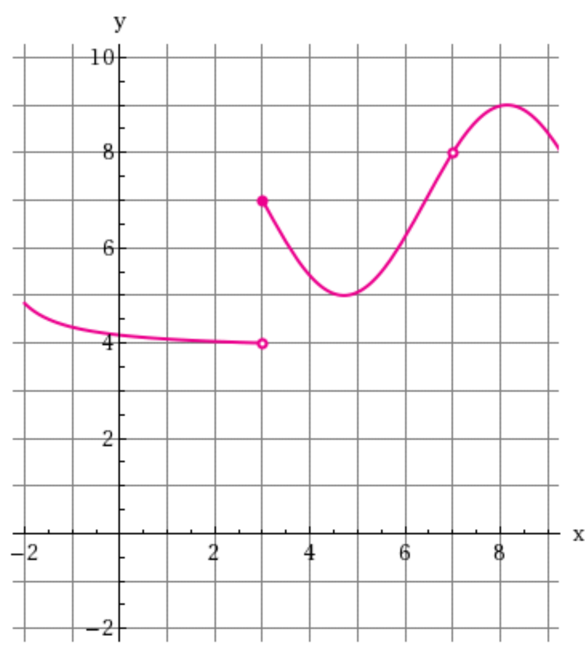
   (d) $\lim_{t \to 7^-} g(t) = \mathbf{8}$

   (e) $\lim_{t \to 7^+} g(t) = \mathbf{8}$

   (f) $\lim_{t \to 7} g(t) = \mathbf{8}$

   (g) $g(7) = \mathbf{DNE}$

   (h) $g(3) = \mathbf{7}$



Applications of the Derivative

12. (6 points total) Jessica Jones jumps off the ground with a velocity of 32 ft/s. Her height in the air (in feet) t seconds later is given by $y = 32t - 16t^2$.

    (a) (3 points) When is her instantaneous velocity equal to 0?

    $$\mathbf{\frac{dy}{dt} = 32 - 32t = 0 \implies t = 1.}$$

    (b) (3 points) She lands when $t = 2$. What is her instantaneous velocity when $t = 2$?

    $$\mathbf{y'(2) = 32 - 32(2) = -32 \implies velocity = -32ft/s.}$$

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13. (6 points) Find the values of x where the slope of $f(x) = 2x^2$ is equal to the slope of $g(x) = -8x^2 + 10x$.

$$\mathbf{f'(x) = 4x}$$
$$\mathbf{g'(x) = -16x + 10}$$
$$\mathbf{f'(x) = g'(x) \implies 4x = -16x + 10 \implies x = \frac{1}{2}}$$

Calculating Limits to a Number

14. (8 points) Evaluate the limit, if it exists: $\lim_{x\to 2} \frac{\frac{1}{2} - \frac{1}{x}}{2x - x^2}$. Show your work algebraically.

$$\mathbf{\lim_{x\to 2} \frac{\frac{1}{2} - \frac{1}{x}}{2x - x^2} = \lim_{x\to 2} \frac{\frac{x}{2x} - \frac{2}{2x}}{2x - x^2} = \lim_{x\to 2} \frac{\frac{x-2}{2x}}{2x - x^2} = \lim_{x\to 2} \frac{\frac{x-2}{2x}}{x(2-x)} = \lim_{x\to 2} \frac{x-2}{2x^2(2-x)} = \lim_{x\to 2} -\frac{1}{2x^2} = -\frac{1}{8}}$$

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15. (6 points total) Slurm Cola marketing estimates that the people of Earth will purchase 1,000,000 cans of Slurm per day at a price of \$3/can. At a price of \$1/can the people of Earth are willing to purchase 4,000,000 cans of Slurm per day.

    (a) (2 points) Determine the daily demand equation for Slurm Cola assuming a linear relation between price and quantity.
    Use Point Slope Formula

    $$\mathbf{m = \frac{1-3}{4000000 - 1000000} = -\frac{2}{3000000}}$$
    $$\mathbf{p(x) = -\frac{2}{3000000}(x - 1000000) + 3 = -\frac{2}{3000000}x + \frac{11}{3}}$$

    (b) (2 points) Determine the daily revenue as a function of the number of cans sold per day.
    Daily Revenue = (Price per can) × (Cans sold per day)

    $$\mathbf{R(x) = p(x)x = \left(-\frac{2}{3000000}x + \frac{11}{3}\right)x = -\frac{2}{3000000}x^2 + \frac{11}{3}x}$$

    (c) (2 points) Determine the number of cans that should be sold per day to maximize daily revenue.
    If $q(x) = ax^2 + bx + c$, the coordinates (h, k) of the max are at

    $$\mathbf{h = -\frac{b}{2a} \qquad k = c - \frac{b^2}{4a}}$$
    $$\mathbf{a = -\frac{2}{3000000}, \qquad b = \frac{11}{3}, \qquad c = 0}$$
    $$\mathbf{h = -\frac{\frac{11}{3}}{-\frac{4}{3000000}} = \frac{11000000}{4}}$$

    Should sell $\frac{11,000,000}{4}$ cans per day.

Continuity

16. (5 points) Determine the value of k that makes

$$f(x) = \begin{cases} (x-k)^2 & x \le 2 \\ 6x+4 & x > 2 \end{cases}$$

continuous.
Both halves of piecewise defined function are continuous. Need to make them match at $x = 2$. Set two functions equal at $x = 2$ and solve for k

$$\mathbf{(2-k)^2 = 6(2)+4 \implies 2-k = \pm 4 \implies k = 6, -2}$$

Either Solution is acceptable.

Applications of the Derivative

17. (5 points) Let $f(x) = x^2 e^x$. Calculate where the tangent line is horizontal.
Tangent line is horizontal where derivative is zero.

$$\mathbf{f'(x) = 2xe^x + x^2e^x = 0 \implies e^x(2x+x^2) = 0 \implies 2x+x^2 = x(2+x) = 0 \implies x = -2, 0}$$

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18. (5 points) Let $h(x) = f(x) \cdot g(x)$. Let $f(2) = 7$, $f'(2) = 1$, $g(2) = 5$, and $g'(2) = 14$. Compute $h'(2)$.

$$\mathbf{h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)}$$
$$\mathbf{h'(2) = f'(2) \cdot g(2) + f(2) \cdot g'(2)}$$
$$\mathbf{h'(2) = 1 \cdot 5 + 7 \cdot 14 = 103}$$

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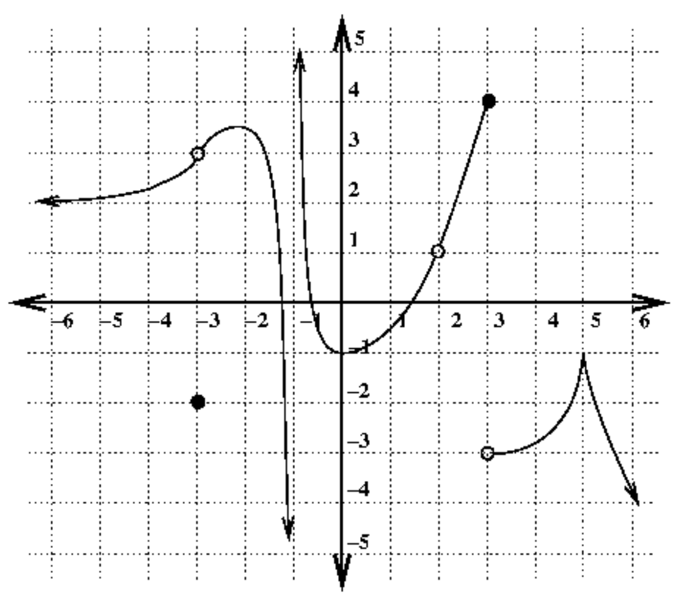
19. (5 points) The graph of the function $f(x)$ is given on the right.

List all points where the derivative of $f(x)$ does not exist.

$$\mathbf{x = -3, -1, 2, 3, 5}$$



Computing Derivatives Algebraically

20. (6 points) Differentiate $f(x) = 3x^2$ algebraically using limits.

$$\mathbf{f'(x) = \lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h} = \lim_{h\to 0} \frac{3(x^2+2xh+h^2) - 3x^2}{h} = \lim_{h\to 0} \frac{3x^2+6xh+3h^2-3x^2}{h} = \lim_{h\to 0} \frac{6xh+3h^2}{h}}$$

$$\mathbf{= \lim_{h\to 0} 6x + 3h = 6x}$$
