Note: Some of these figures come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

## **Question:** How do we analyze the long term behavior of a function?

In Sections 5.1 and 5.2 we used derivative data to decribe the behavior of a function in different regions. Many functions are defined on  $(-\infty,\infty)$  or for arbitrarily large values of |x|. Unfortunately, as humans we can really only check function behavior on small intervals of finite length. Sometimes we'd like to have some idea about how a function behaves for very large values of |x|. To do this we can look at limits as |x| goes off to infinity.

## Limits at infinity:

1. If f(x) approaches the number L as x becomes large without bound, then we say that L is the limit of f(x) as x approaches  $\infty$ , and we write

$$\lim_{x \to \infty} f(x) = L$$

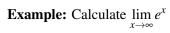
2. If f(x) approaches the number K as x becomes a large negative number without bound, then we say that K is the limit of f(x) as x approaches  $-\infty$ , and we write

$$\lim_{x \to -\infty} f(x) = K$$

We refer to the lines y = L and y = K as horizontal asymptotes.

**Example:** Calculate  $\lim_{x\to\infty}\frac{1}{x}$ .

**Example:** Calculate  $\lim_{x\to\infty} \frac{1}{x^n}$  for n>0. (Hint: Use Limit Power Rule)



**Example:** Calculate 
$$\lim_{x\to\infty} e^{-x}$$

**Example:** Calculate 
$$\lim_{x\to\infty} x^n$$
 for  $n>0$ 

**Example:** Calculate 
$$\lim_{x\to\infty} x^5 - x^4$$
. (Hint: Use Limit Product Rule)

**Example:** Calculate  $\lim_{x \to \infty} \frac{x^5 - x^4}{x^5}$ 

**Example:** Calculate  $\lim_{x\to\infty} \frac{3x^3 - x^2 + 4x}{2x^3 + 4x + 5}$ 

**Example:** Calculate  $\lim_{x \to \infty} \frac{99x^{100} - x^{29} + 4x}{4x^{100} + 49x^{77} + 5}$ 

**Example:** Calculate 
$$\lim_{x\to\infty} \frac{509x^{100} - x^{29} + 4x}{44x^{89} + 49x^{77} + 5}$$

**Example:** Calculate 
$$\lim_{x \to -\infty} \frac{x^{199} + 12x^{29} + 4x}{44x^{200} + 59x^{77} + 5}$$

**Example:** Calculate  $\lim_{x\to\infty} \frac{5+3e^x}{10+4e^x}$ 

**Example:** Calculate  $\lim_{x \to -\infty} \frac{5 + 3e^x}{10 + 4e^x}$