Exam 2 KEY

MATH 142
Version A

Summer '18

Name (printed): ______________________________

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

Name (signature): ____________________ **Section:** ____________________

Instructions:

- You must clear your calculator: MEM (2nd +), Reset (7), cursor right to ALL, All Memory (1), Reset (2).
- There are 17 questions and 8 pages to this exam including the cover sheet. The multiple choice questions are worth 5 points each, and the point values for the problems in the work out section are as indicated. There is no partial credit for the multiple choice problems, but partial credit will be given, if deserved, on the work out problems.
- Clearly circle exactly one answer for each multiple choice question. No partial credit will be given.
- In order to receive full credit on the work out problems, you must show appropriate, legible work.
- You must box or circle your final answer in the work out section.
- Please turn caps bills to the back.
- Please put your cell phone away.
- Please remove any smart watches.
- Disputes about grades on this exam must be handled within ONE WEEK from the day the exam is handed back. After this day, exams will not be re-assessed.
- Your grade on the exam will be written inside on the first page.

GOOD LUCK!

MULTIPLE CHOICE (5 points each)

Limits at Infinity

1. Calculate $\lim_{x\to\infty} \dfrac{5x^2}{4x^2+6x+2}$.

   (a) $\mathbf{\frac{5}{4}}$

   (b) $\frac{5}{10}$

   (c) $\frac{5}{12}$

   (d) 0

   (e) None of the above

Chain Rule

2. Calculate $\dfrac{d}{dx}[2^{e^x} \cdot 2^{x^3} \cdot 2^x]$

   (a) $2^{e^x+3x^2+1}$

   (b) $(e^x+3x^2+1)\cdot 2^{e^x+3x^2+1}\cdot \ln(2)$

   (c) $(\mathbf{e^x}+\mathbf{3x^2}+\mathbf{1})\cdot \mathbf{2^{e^x+x^3+x}}\cdot \ln(\mathbf{2})$

   (d) $e^x\cdot 2^{x^3}\cdot 2^x+3x^2\cdot 2^{e^x}\cdot 2^x+2^{e^x}\cdot 2^{x^3}$

   (e) $e^x\cdot 2^{x^3}\cdot 2^x\cdot\ln(2)+3x^2\cdot 2^{e^x}\cdot 2^x\cdot\ln(2)+2^{e^x}\cdot 2^{x^3}\cdot\ln(2)$

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3. Where are the critical points of $f(x)=\frac{1}{3}x^3+x^2-3x$?

   (a) $\mathbf{x=1,x=-3}$

   (b) $x=1,x=-3,x=0$

   (c) $x=0$

   (d) $x=1,x=0$

   (e) $x=-1$

Concavity/Inflection Points

4. Where is $f(x)=e^{-|x|}$ concave up?

   (a) $\{0\}$

   (b) $(-\infty,\infty)$

   (c) $(-\infty,0)$

   (d) $(0,\infty)$

   (e) $(-\infty,\mathbf{0})\cup(\mathbf{0},\infty)$

Concavity/Inflection Points

5. Where are the inflection points of $f(x) = e^{-x^2}$?

   (a) $\mathbf{x = \frac{1}{\sqrt{2}}, x = -\frac{1}{\sqrt{2}}}$

   (b) $x = 0$

   (c) $x = \frac{1}{\sqrt{2}}$

   (d) $x = \frac{1}{\sqrt{2}}, x = -\frac{1}{\sqrt{2}}, x = 0$

   (e) No inflection points

Limits at Infinity

6. Calculate $\lim_{x \to -\infty} \dfrac{355x^{231} + 81x^{201} + +2x^{120}}{113x^{230} + 6x^{229} + 2x^{228}}$.

   (a) π

   (b) $\frac{355}{113}$

   (c) ∞

   (d) $-\frac{355}{113}$

   (e) $-\infty$

Concavity/Inflection Points

7. Let $f''(x) = \dfrac{x-10}{(x-4)^{31}}$ and $x = 4$ not be in the domain of f. Where is $f(x)$ concave down?

   (a) $(10, \infty)$

   (b) $(4, 10) \cup (10, \infty)$

   (c) $(-\infty, 4) \cup (4, 10)$

   (d) $(-\infty, 10)$

   (e) $\mathbf{(4, 10)}$

Increasing/Decreasing

8. Let $f(x) = \frac{1}{2}x^2 + Ax + 8$. Find the value of A that makes $f(x)$ increasing on $(5, \infty)$ and nowhere else.

   (a) $A = 0$

   (b) $\mathbf{A = -5}$

   (c) $A = 8$

   (d) $A = 5$

   (e) Impossible

Increasing/Decreasing

9. The derivative of a function f is given by $f'(x) = x^2 + 5x - 6$. Where is $f(x)$ decreasing?

(a) $(\mathbf{-6}, \mathbf{1})$

(b) $(-\frac{5}{2}, \infty)$

(c) $(-\infty, -\frac{5}{2})$

(d) $(-\infty, -6) \cup (1, \infty)$

(e) None of the above.

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10. What value of k makes the following function have no critical point?

$$f(x) = \begin{cases} 2x & x \leq 0 \\ kx & x > 0 \end{cases}$$

(a) $k = -2$

(b) $k = 0$

(c) $\mathbf{k = 2}$

(d) $k = 1$

(e) $k = 3$

WORK-OUT

Second Derivative Test

11. (6 points total) Assume $a < x < b$ and (a, b) is in the domain of $f(x)$.

State whether the point is a location of a relative min, relative max, or neither.

(a) $f'(x) = 0, f''(x) > 0$ **relative min**

(b) $f'(x) = 0, f''(x) < 0$ **relative max**

(c) $f'(x) = 0$, on the interval (a, c) $f'(x) > 0$, on the interval (c, b) $f'(x) < 0$ **relative max**

(d) $f'(x) = 0$, on the interval (a, c) $f'(x) < 0$, on the interval (c, b) $f'(x) > 0$ **relative min**

(e) $f'(x) = 0$, on the interval (a, c) $f'(x) < 0$, on the interval (c, b) $f'(x) < 0$ **neither**

(f) $f'(x) = 0$, on the interval (a, c) $f'(x) > 0$, on the interval (c, b) $f'(x) > 0$ **neither**

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12. (8 points) If 30,000 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Bottom of box is x **by** x

Height of box is y

Box has one x **by** x **bottom and 4** x **by** y **sides**

Area = $x^2 + 4xy = 30,000$

Volume = (Area of bottom)$\times$**(Height) =** $xxy = x^2y$

$y = \frac{30,000}{4x} - \frac{x}{4}$

Volume = $f(x) = x^2(\frac{30,000}{4x} - \frac{x}{4}) = 7500x - \frac{x^3}{4}$

$f'(x) = 7500 - \frac{3}{4}x^2 = 0$

$\implies x^2 = 10000 \implies x = \pm 100$

A negative length box doesn't make sense, so $x = 100$

Largest volume = $7500(100) - \frac{(100)^3}{4} = 750,000 - 250,000 = 500,000cm^3$

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13. (7 points) Find the relative extrema of $f(x) = x^2e^{5x}$ and classify them as relative mins or maxes. Show your work.

$f'(x) = 2xe^{5x} + 5x^2e^{5x} = 0$

$\implies (2x + 5x^2)e^{5x} = 0$

$\implies x(2 + 5x) = 0$

$x = -\frac{2}{5}, 0$

$f'(-1) = 2(-1)e^{-5} + 5(-1)^2e^{-5} = (-2+5)e^{-5} > 0$

$f'(-\frac{1}{5}) = 2(-\frac{1}{5})e^{-1} + 5(-\frac{1}{5})^2e^{-1} = (-\frac{2}{5} + \frac{1}{5})e^{-1} < 0$

$x = -\frac{2}{5}$ **is the location of a relative max**

$f'(1) = 2(1)e^5 + 5(1)^2e^5 = (2+5)e^5 > 0$

$x = 0$ **is the location of a relative min**

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14. (7 points) Find the critical points of $f(x) = \dfrac{1}{x^2}$. Where is $f(x)$ increasing and where is $f(x)$ decreasing. Use interval notation.

Domain of $f(x)$ **is** $(-\infty, 0) \cup (0, \infty)$

$f(x) = x^{-2}$

$f'(x) = -2x^{-3} = -\frac{2}{x^3}$

You may be tempted to say $x = 0$ **is a critical point, but it isn't since it's not in the domain of** f**.**

We just need to check the sign of the derivative of $f(x)$ **at a single point in each subinterval to know**

if it's increasing or decreasing in that subinterval.

$f'(-1) = -\frac{2}{(-1)^3} = 1 > 0$

$f'(1) = -\frac{2}{(1)^3} = -1 < 0$

Increasing: $(-\infty, 0)$

decreasing: $(0, \infty)$

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15. (7 points) Find the absolute min and ablsolute max of $f(x) = 6x^2 - 36x + 1000$ on $(-1, 9)$. If the absolute min or absolute max doesn't exist, write DNE.

Endpoints don't count, but need to check behavior near them.

$\lim_{x \to -1^+} f(x) = f(-1) = 6(-1)^2 - 36(-1) + 1000 = 1042$

$\lim_{x \to 9^-} f(x) = f(9) = 6(9)^2 - 36(9) + 1000 = 1162$

Look for critical points.

$f'(x) = 12x - 36 = 0$

$\implies x = 3$

$x = 3$ **is in** $(-1, 9)$**, so it needs to be checked.**

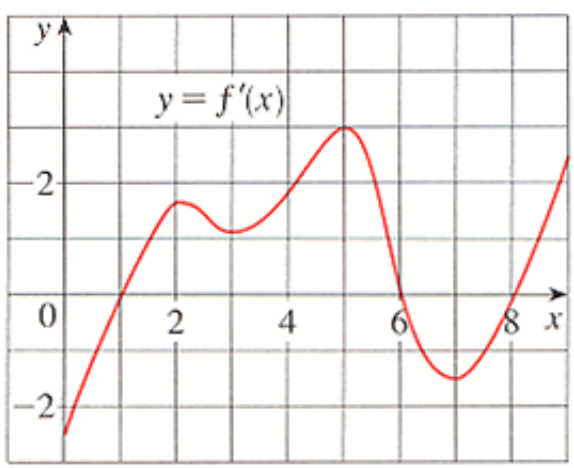
$f(3) = 6(3)^2 - 36(3) + 1000 = 54 - 108 + 1000 = 946$

Absolute min is 946.

Absolute max doesn't exist since an endpoint that isn't in the interval takes the max value.

Graphing Strategy

16. (8 points) Use the given graph of the derivative f' of a continuous function f over the interval $(0, 9)$ to find the following



State your answers in interval notation if appropriate.

(a) Where are the inflection points? $x = 2, 3, 5, 7$

(b) Where is f concave up? $(0, 2) \cup (3, 5) \cup (7, 9)$

(c) Where is f concave down? $(2,3) \cup (5,7)$

(d) Where are the relative extrema of f? $x = 1, 6, 8$

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17. (7 points) Calculate the second derivative of $f(x) = \dfrac{x^2}{4+2x}$.

Using product rule and chain rule.

$$f'(x) = \frac{2x}{4+2x} - \frac{2x^2}{(4+2x)^2} = \frac{8x+4x^2-2x^2}{(4+2x)^2} = \frac{8x+2x^2}{(4+2x)^2}$$

$$f''(x) = \frac{8+4x}{(4+2x)^2} - \frac{4(8x+2x^2)}{(4+2x)^3} = \frac{(8+4x)(4+2x)-4(8x+2x^2)}{(4+2x)^3} = \frac{32}{(4+2x)^3}$$
