Note: Some of these figures come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

Question: *How do we use derivative and asymptotic behavior to sketch a function?*

In Sections 5.1 and 5.2 we saw how derivatives behavior can be used to interpret the behavior of functions. In Section 5.3 we used limits to see long term behavior of a function. In this section we're going to use all of this data to help us sketch the function.

Checklist for Graphing a Function:

A. Use $f(x)$ to

   1. Determine the domain of a function and the intervals on which the function is continuous.

   2. Determine whether the function is symmetric about the y-axis or the origin.

   3. Find all vertical asymptotes.

   4. Find all horizontal asymptotes. Find where the function crosses the axes.

B. Use $f'(x)$ to

   1. Find the critical values.

   2. Find the intervals where the function is increasing or decreasing.

   3. Find all relative extrema.

C. Use $f''(x)$ to

   1. Find intervals where the graph of the function is concave up and concave down.

   2. Find all inflection points.

D. (Final step.) Use steps A, B, C, and the values of f at the critical values and inflection values to graph.

We refer to the lines $y = L$ and $y = K$ as horizontal asymptotes.

Example: Sketch the graph of $\dfrac{x^2}{x^2-4}$.

Example: Sketch the graph of $x^4 - 6x^2 + 4$.