Note: Some of these figures come from your Webassign practice and your textbook *Calculus: Applications & Technology*, 3rd ed., by Tomastik.

Question: *What can lines tell us about functions?*

Equation of the Line:

$$y = \underbrace{m}_{\text{slope}} x + \underbrace{b}_{\text{y-intercept}}$$

Point Slope Formula of the Line: The slope of the line passing through the points (x_0, y_0) and (x, y) is given by

$$m = \frac{y - y_0}{x - x_0}.$$

Example: Calculate the slope of $y = 5x + 6$ using the point slope formula and the points at $x = 1$ and $x = 10$.

Another Way to Use Point Slope Formula of the Line: If we are given the slope ,m, of the line and a point, (x_0, y_0), on the line, we can get the equation of the line by rearranging the point slope formula:

$$y = m(x - x_0) + y_0.$$

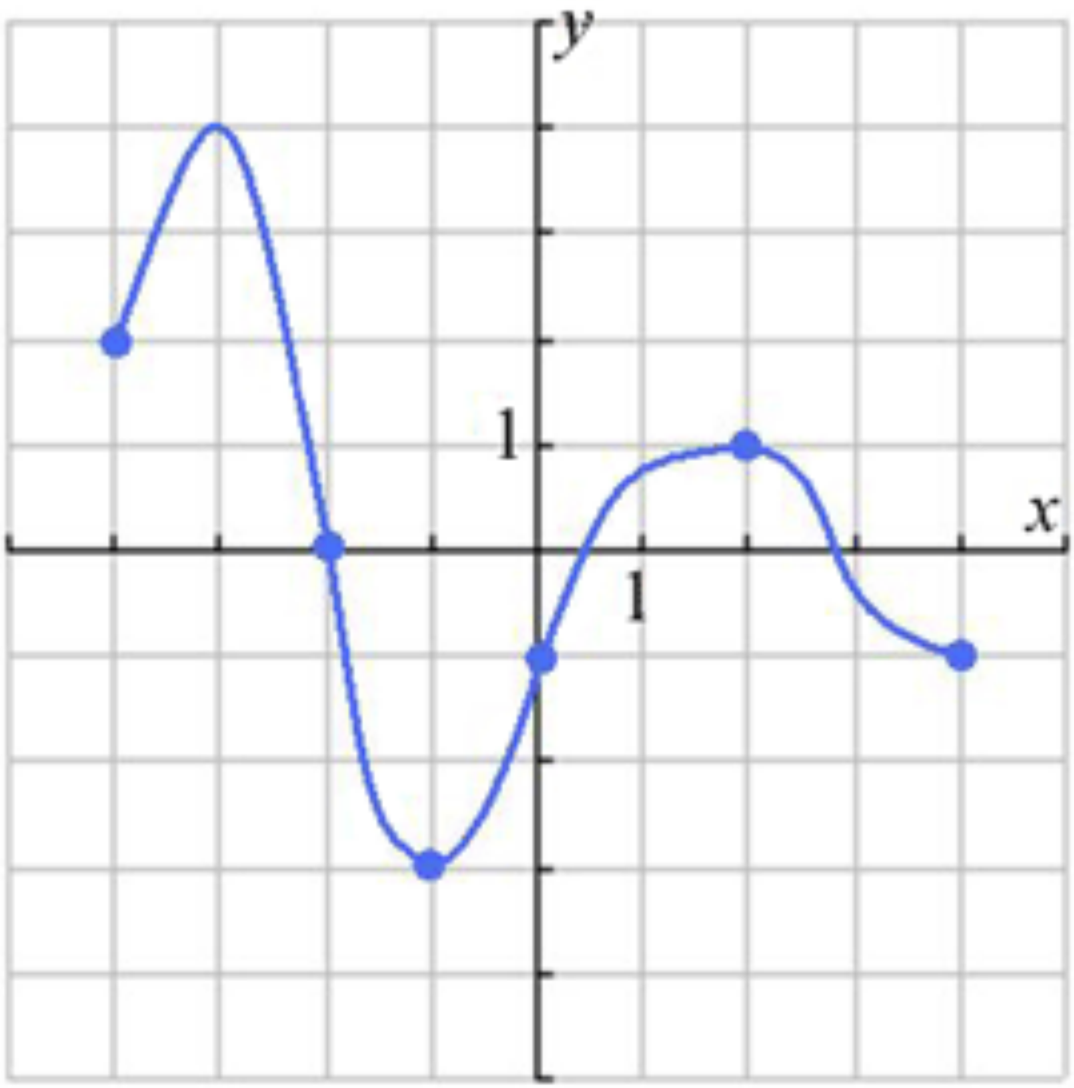
Definition (Average Rate of Change): The average rate of change of $y = f(x)$ with respect to x from a to b is the quotient

$$\frac{\text{change in y}}{\text{change in x}} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Example: You're sitting in a car and pass mile marker 15 and notice the clock reads 12:00pm and you fall asleep. You wake up an hour later at 1:00pm and notice you are passing mile marker 85. What is the average speed the car has been going?

<u>**Average Rate of Change Is Slope of Secant Line:**</u> The average rate of change of $y = f(x)$ from a to b is the slope of the secant line from $P(a, f(a))$ to the point $Q(b, f(b))$.

Example: Calculate the slope of the secant line between two points.



Alternative Form of Average Rate of Change: The average rate of change,

$$\frac{f(b)-f(a)}{b-a},$$

can be rewritten as

$$\frac{f(a+h)-f(a)}{h},$$

where $h=b-a$.

Definition (Average and Instantaneous Velocity): Suppose $s=s(t)$ describes the position of an object at time t. The average velocity from c to $c+h$ is

$$\text{average velocity} = \frac{s(c+h)-s(c)}{h}.$$

The instantaneous velocity (or velocity) $v(c)$ at time c is

$$v(c) = \lim_{h\to 0}(\text{average velocity}) = \lim_{h\to 0}\frac{s(c+h)-s(c)}{h}$$

if this limit exists.

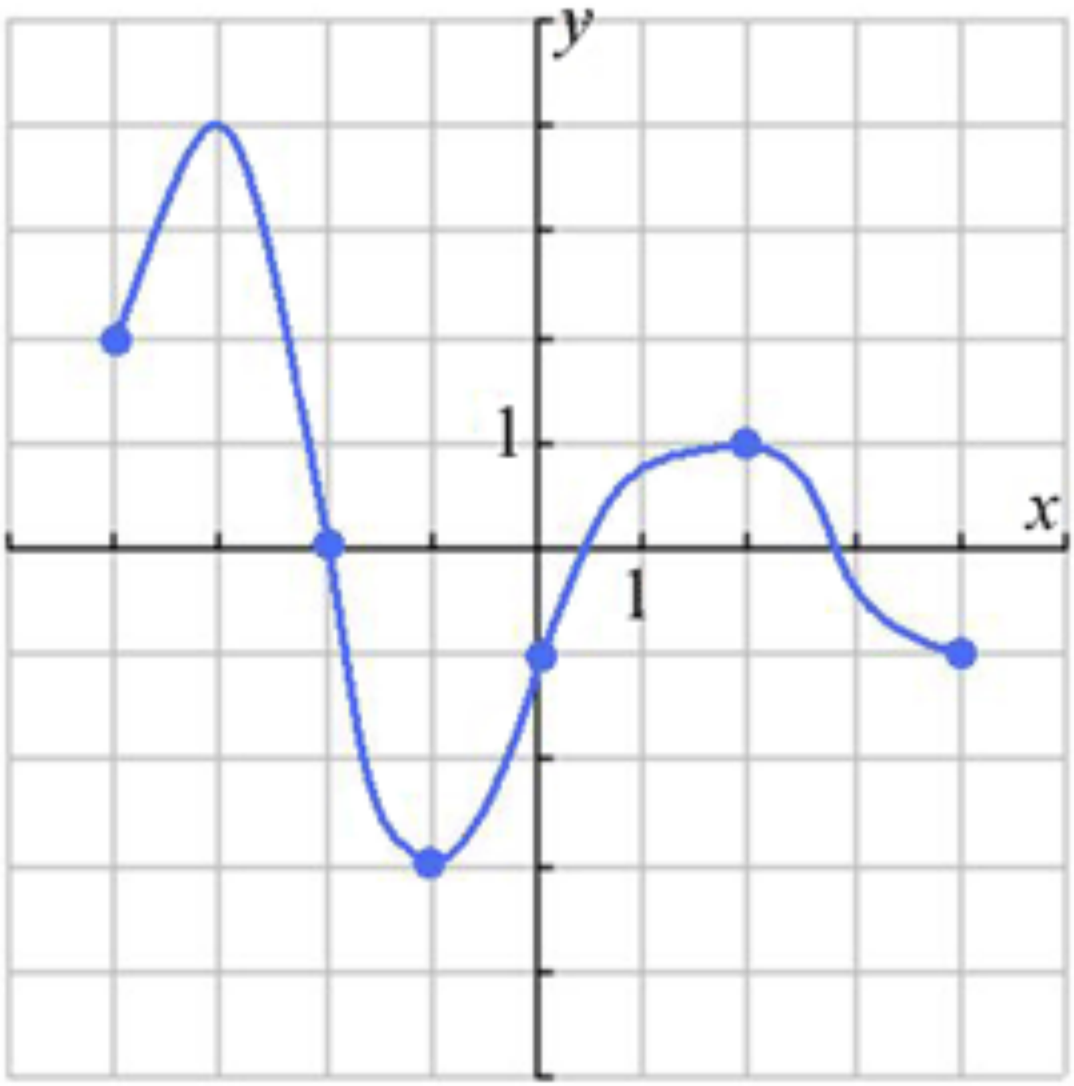
Intuitively (Velocity): If you measure the distance traveled in your car over shorter and shorter time intervals, you'll get a better estimate of the current speed of the car.

Definition (Tangent Line): The tangent line of the graph of $y=f(x)$ at $x=c$ is the line through the point $(c, f(c))$ with slope

$$m_{\tan}(c) = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}.$$

Instantaneous Rate of Change and the Slope of the Tangent Line: If the instantaneous rate of change $f(x)$ with respect to x exists at a point c, then it is the slope of the tangent line at that point.

Example: Choose two points on the curve, draw the tangent line at those points. At which point is the rate of change higher in magnitude?



Example: Consider $y = 5x - x^2$

1. Find the slope fo the tangent line at $(1,4)$.
2. Find the equation of the tangent line using part 1.