CAPE Unit 1 Pure Mathematics June 2013 Solutions

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1. (a) Given: p, q are two propositions

To construct a truth table for:

(i) $p \to q$

(ii) $\sim(p \wedge q)$

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{p \to q}$	$\boldsymbol{p \wedge q}$	$\boldsymbol{\sim(p \wedge q)}$
T	T	T	T	F
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T
		(i)		(ii)

(b) Given: binary operation $\oplus$

$y \oplus x = y^2 + x^2 + 2y + x - 5xy$

To solve $2 \oplus x = 0$

$2 \oplus x = 2^2 + x^2 + 2(2) + x - 5x(2) = 0$

$= 4 + x^2 + 4 + x - 10x = 0$

$= x^2 - 9x + 8 = 0$

$= (x - 8)(x - 1) = 0$

$x = 8, 1$

(c) To use math induction to prove that $5^n + 3$ is divisible by 2 for all $n \in N$

Let $P(n)$ be the proposition that $(5^n + 3) = 2m$ where m is a whole number.

Testing $\boldsymbol{P(1)}$

$5^1 + 3 = 8$

$8 = 2(4)$

$\therefore P(1)$ is true

Assume $\boldsymbol{P(k)}$ is true

That is, assume $5^k + 3 = 2q \;\; q \epsilon N$

Show $\boldsymbol{P(k) \to P(k+1)}$

That is, we need to show that $5^{k+1} + 3 = 2r$, if $5^k + 3 = 2q$ where $r \epsilon N$

Proof:

$5^{k+1} + 3 \equiv 5^{k+1} + 3 + 12 - 12$

$\equiv 5^{k+1} + 15 - 12$

$\equiv 5(5^k + 3) - 12$

$\equiv 5(2q) - 12$

$\equiv 2(5q - 6)$

$\therefore 5^{k+1} + 3 = 2r$ where $r = 5q - 6$

Thus, $5^{k+1} + 3$is divisible by 2 if $5^k + 3$ is divisible by 2.

$\therefore P(k) \Rightarrow P(k+1)$

Since $P(1) \to P(2)$, and $P(k) \Rightarrow P(k+1)$, then $P(n)$ is true for all $n \in N$

(d) Let $f(x) = x^3 - 9x^2 + px + 16$

(i) Given $(x+1)$ is a factor of $f(x)$

To show that $p = 6$

$x^3 - 9x^2 + px + 16 \equiv (x+1)Q$

Let $x = -1$

$(-1)^3 - 9(-1)^2 + p(-1) + 16 = 0$

$-1 - 9 - p + 16 = 0$

$-p = -6$

$p = 6$

(ii) To factorize $f(x)$ completely

$x^3 - 9x^2 + px + 16 \equiv (x+1)(Ax^2 + Bx + C)$

$x^3 - 9x^2 + 6x + 16 \equiv (x+1)(Ax^2 + Bx + C)$

From inspection:

$A = 1, C = 16$

$\therefore x^3 - 9x^2 + 6x + 16 \equiv (x+1)(x^2 + Bx + 16)$

Equating the terms in x on both sides of the equation

$+6x = 16x + Bx$

$6x = (16 + B)x$

$6 = 16 + B$

$-10 = B$

$\therefore x^3 - 9x^2 + 6x + 16 \equiv (x+1)(x^2 - 10x + 16)$

$\equiv (x+1)(x-8)(x-2)$

(iii) Hence to solve $f(x) = 0$

$f(x) = x^3 - 9x^2 + 6x + 16 = (x+1)(x-8)(x-2) = 0$

$\therefore x = -1, 8$ or 2

2. (a) Let $A = \{x : x \in \mathbb{R}, x \geq 1\}$

$f : A \to R$

$f(x) = x^2 - x$

To show $f(x)$ is one to one

Let $a, b \in \mathbb{R},\ a, b \geq 1$

Then $f(a) = a^2 - a$

$f(b) = b^2 - b$

$f(a) = f(b) \Rightarrow a^2 - a = b^2 - b$

$a^2 - b^2 - a + b = 0$

$(a+b)(a-b) - (a-b) = 0$

$(a-b)[(a+b) - 1] = 0$

$\therefore$ Either $a - b = 0$

$a = b$

Or $(a+b) - 1 = 0$

However, since $a \geq 1, b \geq 1$

$a + b - 1 = 0$ has no solution

Hence, $a = b$

That is if $a^2 - a = b^2 - b, (f(a) = f(b))$
$a = b$
$\therefore f$ is one to one.

(b) Let $f(x) = 3x + 2$
$g(x) = e^{2x}$
To find

(a) $f^{-1}(x)$ and $g^{-1}(x)$
From inspection $f^{-1}(x) = \frac{x-2}{3}$
Or,
let $y = 3x + 2$
Interchange *x* and *y*
$x = 3y + 2$
Make y the subject
$x - 2 = 3y$
$y = \frac{x-2}{3}$
$\therefore f^{-1}(x) = \frac{x-2}{3}$

Let $y = e^{2x}$
Interchange *x* and *y*
$x = e^{2y}$
Make *y* the subject
$\ln x = 2y$
$y = \frac{1}{2}\ln x$
$\mathrm{y} = \ln x^{\frac{1}{2}}$
$\therefore g^{-1}(x) = \ln x^{\frac{1}{2}}$

(b) (i) To find $fg(x)$
$f(x) = 3x + 2$
$g(x) = e^{2x}$
$fg(x) = 3e^{2x} + 2$

(ii) To show $(fg)^{-1} = g^{-1}f^{-1}$
$g^{-1}(x) = \ln x^{\frac{1}{2}}$
$f^{-1}(x) = \frac{x-2}{3}$
$g^{-1}f^{-1} = \ln\left(\frac{x-2}{3}\right)^{\frac{1}{2}}$

$fg(x) = 3e^{2x} + 2$
Let $y = 3e^{2x} + 2$
Interchange *x* and *y*

$x = 3e^{2y} + 2$

Make y the subject

$\frac{x-2}{3} = e^{2y}$

$\text{Ln}\left(\frac{x-2}{3}\right) = 2y$

$\frac{1}{2}\ln\left(\frac{x-2}{3}\right) = y$

$\ln\left(\frac{x-2}{3}\right)^{\frac{1}{2}} = y$

$\therefore (fg)^{-1}(x) = \ln\left(\frac{x-2}{3}\right)^{\frac{1}{2}} = g^{-1}f^{-1}(x)$

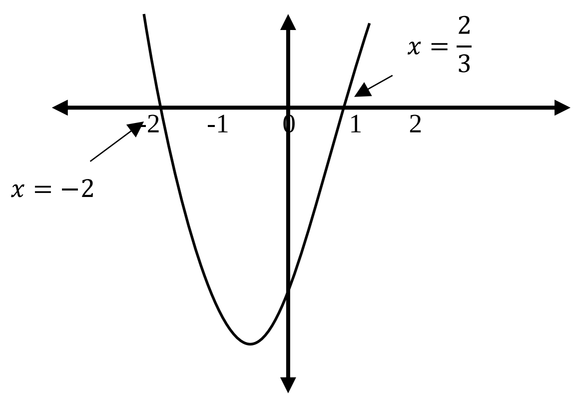
(c) To solve

$3x^2 + 4x + 1 \le 5$

$3x^2 + 4x + 1 \le 5 \Rightarrow 3x^2 + 4x - 4 \le 0$

$(3x-2)(x+2) \le 0$

Sketching $(3x-2)(x+2)$



$\therefore 3x^2 + 4x - 4 \le 0$ for $-2 \le x \le \frac{2}{3}$

Or $3x^2 + 4x + 1 \le 5$ for $-2 \le x \le \frac{2}{3}$

(ii) To solve $|x+2| = 3x + 5$

Method 1

Either $x + 2 = -(3x+5)$

$x + 2 = -3x - 5$

$4x + 2 = -5$

$4x = -7$

$x = -\frac{7}{4}$

Checking answer

$RHS = 3\left(-\frac{7}{4}\right) + 5 = -\frac{21}{4} + \frac{20}{4} = -\frac{1}{4}$

Which is not possible since the $LHS \ge 0$

Or

$x + 2 = 3x + 5$

$-3 = 2x$

$-\frac{3}{2} = x$

Checking

$RHS = 3\left(-\frac{3}{2}\right) + 5 = -\frac{9}{2} + 5 = \frac{1}{2}$

$LHS = \left|-\frac{3}{2} + 2\right| = \frac{1}{2}$

$\therefore x = -\frac{3}{2}$

Method 2

Squaring both sides

$|x+2|^2 = (x+2)^2 = x^2 + 4x + 4$

$\therefore x^2 + 4x + 4 = (3x+5)^2$

$x^2 + 4x + 4 = 9x^2 + 30x + 25$

$0 = 8x^2 + 26x + 21$

$8x^2 + 26x + 21 = (4x+7)(2x+3) = 0$

$\therefore x = -\frac{7}{4}$ or $-\frac{3}{2}$

After testing,

$x = -\frac{3}{2}$

3. (a) (i) To show: $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$RHS: \frac{\frac{2\sin\theta}{\cos\theta}}{\sec^2\theta} \equiv \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}} \equiv \frac{2\sin\theta}{\cos\theta} \cdot \cos^2\theta$

$\equiv 2\sin\theta\cos\theta \equiv \sin 2\theta \equiv LHS$

(ii) Hence, or otherwise, to solve $\sin 2\theta - \tan\theta = 0$ for $0 \le \theta \le 2\pi$

Substituting:

$\frac{2\tan\theta}{1+\tan^2\theta} - \tan\theta = 0$

$\frac{2\tan\theta - \tan\theta - \tan^3\theta}{1+\tan^2\theta} = 0$

$\tan\theta - \tan^3\theta = 0$

$\tan\theta(1 - \tan^2\theta) = 0$

$\therefore \mathit{either}\ \tan\theta = 0$

$\theta = 0, \pi, 2\pi$

Or

$1 - \tan^2\theta = 0$

$\tan^2\theta = 1$

$\tan\theta = \pm 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\therefore \theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$

(b) (i) To express $f(\theta) = 3\cos\theta - 4\sin\theta$ in the form $r\cos(\theta + \alpha)$ where $r > 0$ $\quad 0 \le \alpha \le \frac{\pi}{2}$

Note: $a\cos\theta \pm b\sin\theta = R\cos(\theta \mp \alpha)$ where $R = \sqrt{a^2+b^2}$ and $\tan\alpha = \frac{b}{a}$

$\therefore r = \sqrt{3^2 + (-4)^2} = 5$

$\tan\alpha = \frac{4}{3} \Rightarrow \alpha = 0.927$ radians

$\alpha = 53.1°$

$\therefore 3\cos\theta - 4\sin\theta = 5\cos(\theta + 0.927)$

(ii) Hence to find

(a) Maximum value of $f(\theta)$

Note: the maximum value occurs when $\cos(\theta + 0.927) = 1$

$\therefore$ maximum value is 5

(b) The minimum value of $\frac{1}{8+f(\theta)}$

Note: $-5 \leq f(\theta) \leq 5$

$\therefore$ minimum value of $\frac{1}{8+f(\theta)} = \frac{1}{8+5} = \frac{1}{13}$

(iii) Given: *A*, *B*, *C* angle of a triangle

To show: $\sin A = \sin(B + C)$

$B + C = \pi - A$

$\therefore \sin(B + C) = \sin(\pi - A)$

$\sin(\pi - A) = \sin A$ (Note: The sine of supplementary angles are equal)

$\therefore \sin A = \sin(B + C)$

(c) To show:

$\sin A + \sin B + \sin C \equiv \sin(A + B) + \sin(B + C) + \sin(A + C)$

$\sin A = \sin(B + C)$ ----shown above

Hence, we need to show that

$\sin B + \sin C = \sin(A + B) + \sin(A + C)$

As shown above $A + C = \pi - B$

$\therefore \sin(A + C) = \sin(\pi - B)$ but $\sin(\pi - B) = \sin B$

$\therefore \sin(A + C) = \sin B$

Similarly, $\sin(A + B) = \sin C$

$\therefore \sin A + \sin B + \sin C \equiv \sin(A + B) + \sin(B + C) + \sin(A + C)$

4. (a) Given: circle *C* with equation $x^2 + y^2 - 6x - 4y + 4 = 0$

(i) To show that the centre of *C* is $(3,2)$ and the radius 3.

$x^2 + y^2 - 6x - 4y + 4 = 0$

$x^2 - 6x + 9 + y^2 - 4y + 4 = -4 + 13 = 3^2$

$(x - 3)^2 + (y - 2)^2 = 3^2$

$\therefore$ *C* has centre $(3,2)$ and $r = 3$ units.

(ii) (a) To find the equation of the normal at $(6,2)$

m_{radius} at $(6,2) = \frac{2-2}{6-3} = \frac{0}{3}$

Thus, the radius is horizontal with equation $y = 2$

(b) The tangent is $\perp$ to the normal(radius) and passing through $(6,2)$

Therefore, the equation of the tangent is $x = 6$ which is parallel to the y-axis

(b) Given parametric equations:

$x = t^2 + t$ …….(1)

$y = 2t - 4$ ……(2)

To show that the cartesian equation is $4x = y^2 + 10y + 24$.

From equation (2): $t = \frac{y+4}{2}$

Substituting in (1):

$x = \left(\frac{y+4}{2}\right)^2 + \frac{y+4}{2}$

$x = \frac{y^2+8y+16}{4} + \frac{2y+8}{4}$

$4x = y^2 + 10y + 24$

(c) Given: $A(3,-1,2)B(1,2,-4)C(-1,1,-2)$ are three vertices of a parallelogram *ABCD*

(i) To express $\overrightarrow{AB}, \overrightarrow{BC}$ in the form $x\mathbf{i} + y\mathbf{j} + 3\mathbf{k}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$$\overrightarrow{AB} = \begin{pmatrix}-3\\1\\-2\end{pmatrix} + \begin{pmatrix}1\\2\\-4\end{pmatrix} = \begin{pmatrix}-2\\3\\-6\end{pmatrix} = -2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$$

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$$= \begin{pmatrix}-1\\-2\\4\end{pmatrix} + \begin{pmatrix}-1\\1\\-2\end{pmatrix} = \begin{pmatrix}-2\\-1\\2\end{pmatrix} = -2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

(ii) To show $r = -16\mathbf{j} - 8\mathbf{k}$ is perpendicular to the plane through *A*, *B*, *C*.

$$\mathbf{r} = \begin{pmatrix}0\\-16\\-8\end{pmatrix} \text{ and } \overrightarrow{BC} = \begin{pmatrix}-2\\-1\\2\end{pmatrix}$$

If $\mathbf{r}$ is $\perp$ to the plane then $\mathbf{r} \cdot \overrightarrow{BC} = \mathbf{r} \cdot \overrightarrow{AB} = 0$

$$\mathbf{r} \cdot \overrightarrow{BC} = \begin{pmatrix}0\\-16\\-8\end{pmatrix} \cdot \begin{pmatrix}-2\\-1\\2\end{pmatrix} = 0 + 16 - 16 = 0$$

$$\mathbf{r} \cdot \overrightarrow{AB} = \begin{pmatrix}0\\-16\\-8\end{pmatrix} \cdot \begin{pmatrix}-2\\3\\-6\end{pmatrix} = 0 - 48 + 48 = 0$$

$\therefore$ $\mathbf{r}$ is $\perp$ to the plane.

(iii) Hence to find the Cartesian equation of the plane

Equation of plane: $r \cdot n = a \cdot n$

$$r \cdot \begin{pmatrix}0\\2\\1\end{pmatrix} = \begin{pmatrix}-3\\1\\-2\end{pmatrix} \cdot \begin{pmatrix}0\\2\\1\end{pmatrix}$$ (Note: $\begin{pmatrix}0\\2\\1\end{pmatrix} = -\frac{1}{8}\begin{pmatrix}0\\-16\\-8\end{pmatrix}$ and thus is $\perp$ to the plane.)

$$\begin{pmatrix}x\\y\\z\end{pmatrix} \cdot \begin{pmatrix}0\\2\\1\end{pmatrix} = \begin{pmatrix}-3\\1\\-2\end{pmatrix} \cdot \begin{pmatrix}0\\2\\1\end{pmatrix}$$

$2y + z = 0$

5. (a) (i) Given: $f(x) = \begin{cases} x+2, & x < 2 \\ x^2, & x > 2 \end{cases}$

To find $\lim_{x \to 2} f(x)$

$\lim_{x \to 2^+} f(x) = 2^2 = 4$

$\lim_{x \to 2^-} f(x) = 2 + 2 = 4$

$\therefore \lim_{x \to 2} f(x) = 4$

(ii) determine whether $f(x)$ is continuous at $x = 2$.

$f(x)$ is continuous at $x = 2$ if and only if $f(2) = 4$

However, based on the information given we must conclude that $f(x)$ is not defined at $x = 2$.

Hence $f(x)$ is discontinuous at $x = 2$.

(b) Given: $y = \frac{x^2+2x+3}{(x^2+2)^3}$

To show: $\frac{dy}{dx} = \frac{-4x^3-10x^2-14x+4}{(x^2+2)^4}$

Applying the quotient rule:

$$\frac{dy}{dx} = \frac{(x^2+2)^3(2x+2) - (x^2+2x+3)3(x^2+2)^2(2x)}{(x^2+2)^6}$$

$$= \frac{(x^2+2)^2[(x^2+2)(2x+2) - 6x(x^2+2x+3)]}{(x^2+2)^6}$$

$$= \frac{2x^3+2x^2+4x+4-6x^3-12x^2-18x}{(x^2+2)^4}$$

$$= \frac{-4x^3-10x^2-14x+4}{(x^2+2)^4}$$

(c) Given: $x = 1 - 3\cos\theta$

$y = 2\sin\theta$

To find: $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$

$\frac{dy}{d\theta} = 2\cos\theta \qquad \frac{dx}{d\theta} = 3\sin\theta$

$\therefore \frac{dy}{dx} = \frac{2\cos\theta}{3\sin\theta} = \frac{2}{3}\cot\theta$

(d) (i) Given: $y = x^2 + 3$ -----(1)

$y = 4x$ ----------(2)

To find: the coordinates of the point of intersection of the two functions

From (2): $y = 4x$

Substituting in (1): $4x = x^2 + 3$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1, 3$

When $x = 1$, sub in (2): $y = 4(1) = 4$

When $x = 3$, sub in (2): $y = 4(3) = 12$

$\therefore$ the points of intersection are: $(1,4)$ and $(3,12)$.

(ii) To find the area of the shaded region.

$$A = \int_1^3 [4x - (x^2 + 3)]dx$$

$$A = \int_1^3 (4x - x^2 - 3)dx$$

$$= \left[\frac{4x^2}{2} - \frac{x^3}{3} - 3x\right]_1^3$$

$$= \left[2x^2 - \frac{x^3}{3} - 3x\right]_1^3$$

$$= \left[2(3)^2 - \frac{3^3}{3} - 3(3)\right] - \left[2(1)^2 - \frac{1^3}{3} - 3(1)\right]$$

$$= (18 - 9 - 9) - \left(2 - \frac{1}{3} - 3\right)$$

$= 0 - \left(-1\frac{1}{3}\right) = 1\frac{1}{3}$ or $\frac{4}{3}$ unit^2

6. (a) (i) To find $\int x(1-x)^2\,dx$

Using the substitution: $u = 1 - x$

$u = 1 - x \rightarrow du = -dx$

$x = 1 - u$

$$\therefore \int x(1-x)^2 dx \equiv -\int (1-u)u^2 du$$

$$= -\int (u^2 - u^3)dx$$

$$= -\left[\frac{u^3}{3} - \frac{u^4}{4}\right] = \frac{u^4}{4} - \frac{u^3}{3} + c$$

$$= \frac{1}{4}(1-x)^4 - \frac{1}{3}(1-x)^3 + c$$

(ii) Given: $f(x) = 2\cos t$; $g(t) = 4\sin 5t + 3\cos t$

To show: $\int [f(t) + g(t)]dt = \int f(t)dt + \int g(t)\,dt$

$$\int f(t)dt = 2\sin t + c_1$$

$$\int g(t)dt = -\frac{4}{5}\cos 5t + 3\sin t + c_2$$

$$\int f(t)dt + \int g(t)dt = 2\sin t + 3\sin t - \frac{4}{5}\cos 5t + c_3$$

$$= 5\sin t - \frac{4}{5}\cos 5t + c_3$$

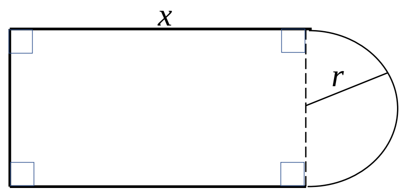
$$\int [f(t) + g(t)]dt = \int (5\cos t + 4\sin 5t)dt$$

$$= 5\sin t - \frac{4}{5}\cos 5t + c_4$$

$$\therefore \int f(t)dt + \int g(t)dt \equiv \int [f(t) + g(t)]dt$$

Note: $c_4 = c_3$

(b) (i) Given: Diagram with perimeter 600 m



To show: $r = \frac{600-2x}{2+\pi}$

$600 = 2x + 2r + \pi r$

$2r + \pi r = 600 - 2x$

$r(2 + \pi) = 600 - 2x$

$$r = \frac{600 - 2x}{2 + \pi}$$

(ii) To find *x* such that the area A is a maximum.

$$A = (x)(2r) + \frac{1}{2}\pi r^2$$

$$A = \frac{x(2)(600 - 2x)}{2 + \pi} + \frac{\pi(600 - 2x)^2}{2(2 + \pi)^2}$$

$$= \frac{2}{2 + \pi}[600x] - \frac{4x^2}{2 + \pi} + \frac{\pi}{2(2 + \pi)^2}[600 - 2x]^2$$

$$\frac{dA}{dx} = \frac{1200}{2 + \pi} - \frac{8x}{2 + \pi} + \frac{2\pi}{2(2 + \pi)^2}[600 - 2x][-2]$$

$$= \frac{1200}{2 + \pi} - \frac{8x}{2 + \pi} - \frac{2\pi}{(2 + \pi)^2}[600 - 2x]$$

$$= \frac{1200}{2 + \pi} - \frac{1200\pi}{(2 + \pi)^2} - \frac{8x}{2 + \pi} + \frac{4\pi x}{(2 + \pi)^2}$$

At a stationary point, $\frac{dy}{dx} = 0$

$$\therefore x\left[\frac{4\pi}{(2 + \pi)^2} - \frac{8}{2 + \pi}\right] = \frac{1200\pi}{(2 + \pi)^2} - \frac{1200}{2 + \pi}$$

$$x\left[\frac{4\pi}{(2 + \pi)^2} - \frac{8(2 + \pi)}{(2 + \pi)^2}\right] = \frac{1200\pi}{(2 + \pi)^2} - \frac{1200(2 + \pi)}{(2 + \pi)^2}$$

$$x\left[\frac{4\pi - 16 - 8\pi}{(2 + \pi)^2}\right] = \frac{1200\pi - (1200)(2) - 1200\pi}{(2 + \pi)^2}$$

$$\therefore x = -\frac{2400}{(2 + \pi)^2} \cdot \frac{(2 + \pi)^2}{-4\pi - 16}$$

$$= \frac{2400}{4\pi + 16} = 84.014 \ldots$$

$$\simeq 84m$$

$$\frac{d^2A}{dx^2} = 0 - 0 - \frac{8}{2 + \pi} + \frac{4\pi}{(2 + \pi)^2} = \frac{-16 - 8\pi + 4\pi}{(2 + \pi)^2} < 0$$

Hence the stationary point is a maximum

(iii) Given $y = -x\sin x - 2\cos x + Ax + B$

To show: $y'' = x\sin x$

$y' = (-x)(\cos x) + (\sin x)(-1) + 2\sin x + A$

$= -x\cos x - \sin x + 2\sin x + A$

$= -x\cos x + \sin x + +A$

$y'' = (-x)(-\sin x) + (\cos x)(-1) + \cos x$

$= x\sin x - \cos x + \cos x$

$= x\sin x$

(iv) Hence to determine the specific solution of the differential equation

$y'' = x\sin x$

Given $x = 0$ when $y = 1$ and $x = \pi$ when $y = 6$

$y'' = x\sin x \Rightarrow y = -x\sin x - 2\cos x + Ax + B$ (see above)

When $x = 0, y = 1$ (substituting in $y = -x\sin x - 2\cos x + Ax + B$)

$\therefore 1 = -(0)\sin 0 - 2\cos 0 + A(0) + B$

$1 = -2 + B$

$B = 3$

When $x = \pi,\ \ y = 6$

$6 = -\pi\sin\pi - 2\cos\pi + A\pi + B$

$6 = -\pi(0) - 2(-1) + A\pi + B$

$6 = 2 + A\pi + 3$

$\frac{1}{\pi} = A$

$\therefore y = -x\sin x - 2\cos x + \frac{x}{\pi} + 3$