Introdução à integração

3

Então,

$$\int \operatorname{sen}^{3} x \, dx = \operatorname{sen}^{2} x \cdot (-\cos x) - \int -\cos x \cdot 2 \operatorname{sen} x \cos x \, dx$$

$$= -\operatorname{sen}^{2} x \cos x + 2 \int \cos^{2} x \operatorname{sen} x \, dx$$

$$= -\operatorname{sen}^{2} x \cos x - 2 \frac{\cos^{3} x}{3} + c.$$

6.6 EXERCÍCIOS

Resolver as seguintes integrais usando a técnica de integração por partes.

1.
$$\int x \sin 5x \, dx$$

$$2. \int \ln (1-x) \ dx$$

3.
$$\int t e^{4t} dt$$

4.
$$\int (x+1) \cos 2x \ dx$$

5.
$$\int x \ln 3x \, dx$$

6.
$$\int \cos^3 x \ dx$$

$$7. \quad \int e^x \cos \frac{x}{2} \ dx \quad .$$

8.
$$\int \sqrt{x} \ln x \ dx$$

9.
$$\int \csc^3 x \ dx$$

10.
$$\int x^2 \cos a x \ dx$$

11.
$$\int x \csc^2 x \ dx$$

12.
$$\int \operatorname{arc} \cot g \, 2x \, dx$$

13.
$$\int e^{ax} \sin bx \ dx$$

$$14. \int \frac{\ln (ax + b)}{\sqrt{ax + b}} dx$$

15.
$$\int x^3 \sqrt{1-x^2} \ dx$$

$$16. \quad \int \ln^3 2x \ dx$$

17.
$$\int arc tg a x dx$$

18.
$$\int x^3 \sin 4x \ dx$$

19.
$$\int (x-1)e^{-x} dx$$

$$20. \quad \int x^2 \ln x \ dx$$

21.
$$\int x^2 e^x dx$$

22.
$$\int arc sen \frac{x}{2} dx$$

23.
$$\int (x-1) \sec^2 x \ dx \qquad \Rightarrow$$

24.
$$\int e^{3x} \cos 4x \ dx$$

25.
$$\int x^n \ln x \ dx, \quad n \in N$$

26.
$$\int \ln (x^2 + 1) dx$$

27.
$$\int \ln (x + \sqrt{1 + x^2}) dx$$

28.
$$\int x \operatorname{arc} \operatorname{tg} x \, dx$$

29.
$$\int x^5 e^{x^2} dx$$

30.
$$\int x \cos^2 x \ dx$$

31.
$$\int (x+3)^2 e^x dx$$

32.
$$\int x \sqrt{x + 1} \ dx$$

33.
$$\int \cos(\ln x) dx$$

34.
$$\int arc \cos x \ dx$$

35.
$$\int \sec^3 x \ dx$$

36.
$$\int \frac{1}{x^3} e^{1/x} dx$$
.

SEÇÃO 6.6

1.
$$\frac{-x}{5}\cos 5x + \frac{1}{25}\sin 5x + c$$

3.
$$\frac{e^{4t}}{4}\left(t-\frac{1}{4}\right)+c$$

c 2.
$$(x-1) \ln (1-x) - x + c$$

5.
$$\frac{x^2}{2} \left[\ln 3x - \frac{1}{2} \right] + c$$

4.
$$\frac{(x+1)}{2}$$
 sen $2x + \frac{1}{4}$ cos $2x + c$

7.
$$\frac{2}{5} e^x \left[\sin \frac{x}{2} + 2 \cos \frac{x}{2} \right] + c$$

6.
$$\cos^2 x \sin x + \frac{2 \sin^3 x}{3} + c$$

8. $\frac{2}{3} x \sqrt{x} \ln x - \frac{4}{9} x \sqrt{x} + c$

9.
$$-\frac{1}{2}\operatorname{cosec} x \operatorname{cotg} x + \frac{1}{2}\ln|\operatorname{cosec} x - \operatorname{cotg} x| + c$$

10.
$$\frac{x^2}{a} \sin ax + \frac{2x}{a^2} \cos ax - \frac{2}{a^3} \sin ax + c$$

11.
$$-x \cot x + \ln |\sin x| + c$$

12. x arc cotg
$$2x + \frac{1}{4} \ln (1 + 4x^2) + c$$

13.
$$\frac{b e^{ax}}{a^2 + b^2} \left[-\cos b x + \frac{a}{b} \sin b x \right] + c$$
 14. $\frac{2}{a} \sqrt{ax + b} \left[\ln (ax + b) - 2 \right] + c$

14.
$$\frac{2}{a}\sqrt{ax+b}$$
 [ln $(ax+b)-2$] +

15.
$$-\frac{x^2}{3}(1-x^2)\sqrt{1-x^2} - \frac{2}{15}(1-x^2)^2\sqrt{1-x^2} + c$$

16.
$$x \left[\ln^3 2x - 3 \ln^2 2x + 6 \ln 2x - 6 \right] + c$$

16.
$$x [\ln^3 2x - 3 \ln^2 2x + 6 \ln 2x - 6] + c$$
 17. $x \text{ arc tg } a x - \frac{1}{2a} \ln (1 + a^2 x^2) + c$

$$x^3$$
 3 2 3x 3

$$18. - \frac{x^3}{4}\cos 4x + \frac{3}{16}x^2 \sin 4x + \frac{3x}{32}\cos 4x - \frac{3}{128}\sin 4x + c$$

20.
$$\frac{x^3}{3} \left[\ln x - \frac{1}{3} \right] + c$$
 21. $e^x \left[x^2 - 2x + 2 \right] + c$

19. $-x e^{-x} + c$

22. x arc sen
$$\frac{x}{2} + \sqrt{4 - x^2} + c$$

23.
$$(x-1)$$
 tg $x + \ln |\cos x| + c$

24.
$$\frac{4}{25} \left[e^{3x} \sin 4x + \frac{3}{4} e^{3x} \cos 4x \right] + c$$

25.
$$\frac{x^{n+1}}{n+1} \left[\ln x - \frac{1}{n+1} \right] + c$$

28.
$$\frac{x^2}{2}$$
 arc tg $x - \frac{1}{2}x + \frac{1}{2}$ arc tg $x + c$

26. $x \ln (x^2 + 1) - 2x + 2 \operatorname{arc} \operatorname{tg} x + c$

29.
$$e^{x^2} \left[\frac{x^4}{4} - x^2 + 1 \right] + c$$

27. $x \ln (x + \sqrt{1 + x^2}) - \sqrt{1 + x^2} + c$

30.
$$\frac{1}{4} \left[x^2 + x \operatorname{sen} 2x + \frac{1}{2} \cos 2x \right] + c$$

31.
$$e^x [x^2 + 4x + 5] + c$$

32.
$$\frac{2}{3} x(x+1) \sqrt{x+1} - \frac{4}{15} (x+1)^2 \sqrt{x+1} + c$$

33.
$$\frac{1}{2}x \cos(\ln x) + \frac{1}{2}x \sin(\ln x) + c$$
 34. $x \arccos x - \sqrt{1 - x^2} + c$

34. x arc cos
$$x - \sqrt{1 - x^2} + c$$

35.
$$\frac{1}{2} \left[|\sec x | \tan x + |\sin x| + |\cos x| \right] + c$$
 36. $-\frac{1}{x} e^{1/x} + e^{1/x} + c$