① $\cos\theta = x$

$-\sin\theta d\theta = dx$

$\Rightarrow \sin d\theta = -dx$

Now, $\int \frac{-3}{x^2+5x-6}\, dx$

$= \int \frac{-3}{(x+6)(x-1)}\, dx$

Now,

$\frac{-3}{(x+6)(x-1)} = \frac{A}{x+6} + \frac{B}{(x-1)}$

$\Rightarrow -3 = A(x-1) + B(x+6)$

Now,

if $\Rightarrow x = 1$,

$-3 = A.0 + B.7$

$\Rightarrow -3 = 7B$

$\Rightarrow B = -3/7$

if $\Rightarrow x = -6$

$\Rightarrow -3 = A(-6-1) + B.0$

$\Rightarrow -3 = -7A$

$\Rightarrow A = \frac{3}{7}$

Now, $\int \frac{\frac{3}{7}}{x+6} + \frac{(-\frac{3}{7})}{x-1} \, dx$

$\Rightarrow 3/7 \int \frac{1}{x+6} - 3/7 \int \frac{1}{x-1} \, dx$

$= 3/7 (\ln|x+6|) - 3/7 (\ln|x-1|) + c$ (Ans)

~~$= 3/7 (\ln|\cos\theta + 6|) - 1/2 \ln|\cos\theta - 1| + c$~~

~~(Ans)~~

$\Rightarrow 3/7 \left(\frac{\ln|x+6|}{\ln|x-1|}\right)$

$= 3/7 \left(\frac{\ln|\cos\theta + 6|}{\ln|\cos\theta - 1|}\right) + c$

(Ans)

2 $\int \frac{e^t + 1}{e^{3t} - 4e^t} \, dx$

Set $e^t = x$

$e^t \, dx = dx$

Now, $\int \frac{x+1}{x^2(x+2)(x-2)}$

$\Rightarrow \frac{x+1}{x^2(x+2)(x-2)}$, $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} + \frac{D}{x-2}$

~~if,~~

~~$x = 0$~~

$\Rightarrow x+1 = A\,x(x+2)(x-2) + B(x+2)(x-2) +$

$C\,x^2(x-2) + D\,x^2(x+2)$

Now,

if $x = 0$

$$1 = A.0 + B(4) + C.0 + D.0$$

$$\Rightarrow B = -1/4$$

if $x = 2$

$$\Rightarrow 3 = A.0 + B.0 + C.0 + D(4.4)$$

$$\Rightarrow D = 3/16$$

if, $x = -2$

$$\Rightarrow -1 = A.0 + B.0 + C.(4(-4)) + D.0$$

$$\Rightarrow -1 = C(16)$$

$$\Rightarrow C = \frac{1}{16}$$

if, $x = 1$

$$\Rightarrow 2 = -3A + B\,3(-1) + C(-1) + D(3)$$

$\Rightarrow 2 = -3A - 3B - C + 3D$

$\Rightarrow 2 = -3A - 3/4 - 1/16 + 3\ 3/16$

$\Rightarrow 2 = -3A - 1/4$

$\Rightarrow 9/4 = -3A$

$\Rightarrow A = -3/4$

Now, $\int -3/4 \frac{1}{x} - 1/4 \frac{1}{x^2} + 1/16 \frac{1}{|x+2|} + 3/16 \frac{1}{|x-2|}$

~~$-3/4 \frac{x^2}{2} - 1/4 \frac{x^3}{3} + 1/16$~~

$\Rightarrow -3/4 \ln x - 1/4 \frac{x^{-1}}{1} + 1/6 \ln|x+2| + \cancel{3/16 \ln|4}$

$+ 3/16 \ln|x-2|$

$$\Rightarrow -3/4 \ln e^t - 1/4 \frac{1}{e^t} + 1/6 \ln|e^t+2| +$$

$$3/16 \ln|e^t-2|$$