COSE436

Lecture 15:
Texture Mapping

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Outline

- Texture mapping
- Sampling theory
- Anti-aliasing
- Min/Mag filtering

Limitation of Geometric Modeling

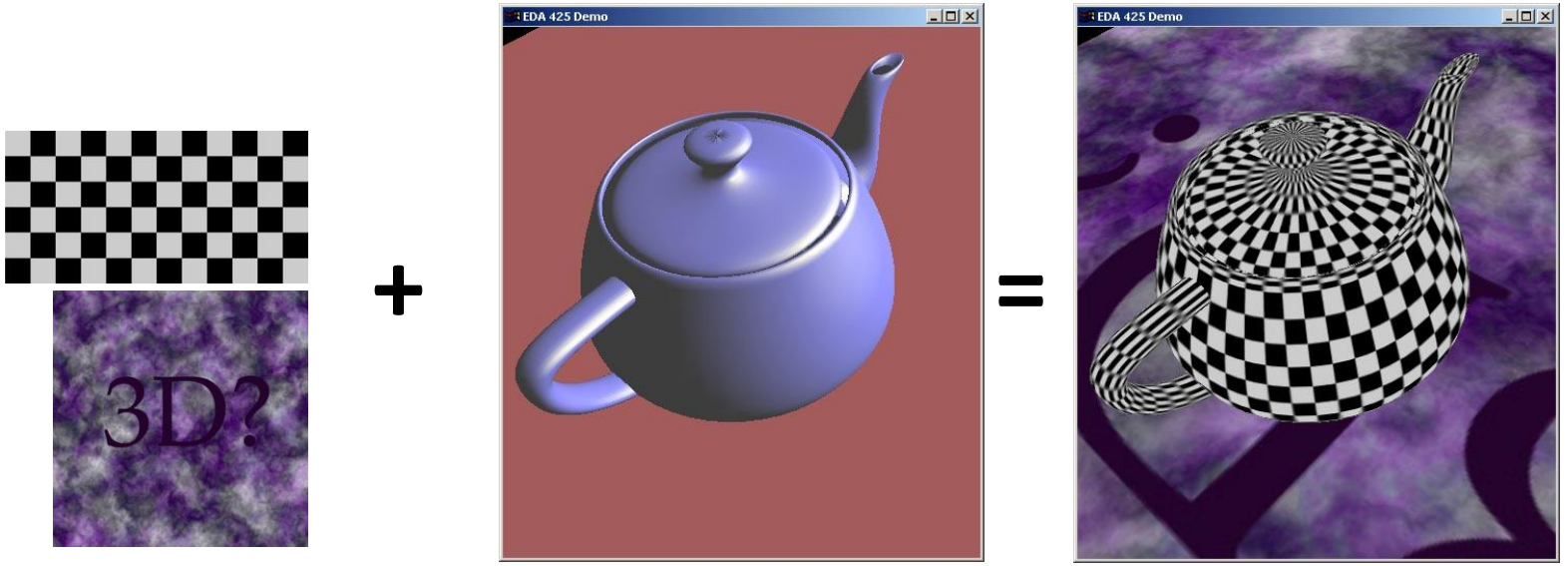
- Requires many polygons to represent complex scenes
  - Clouds
  - Grass
  - Terrain
  - Skin
  - Brick walls
  - ...

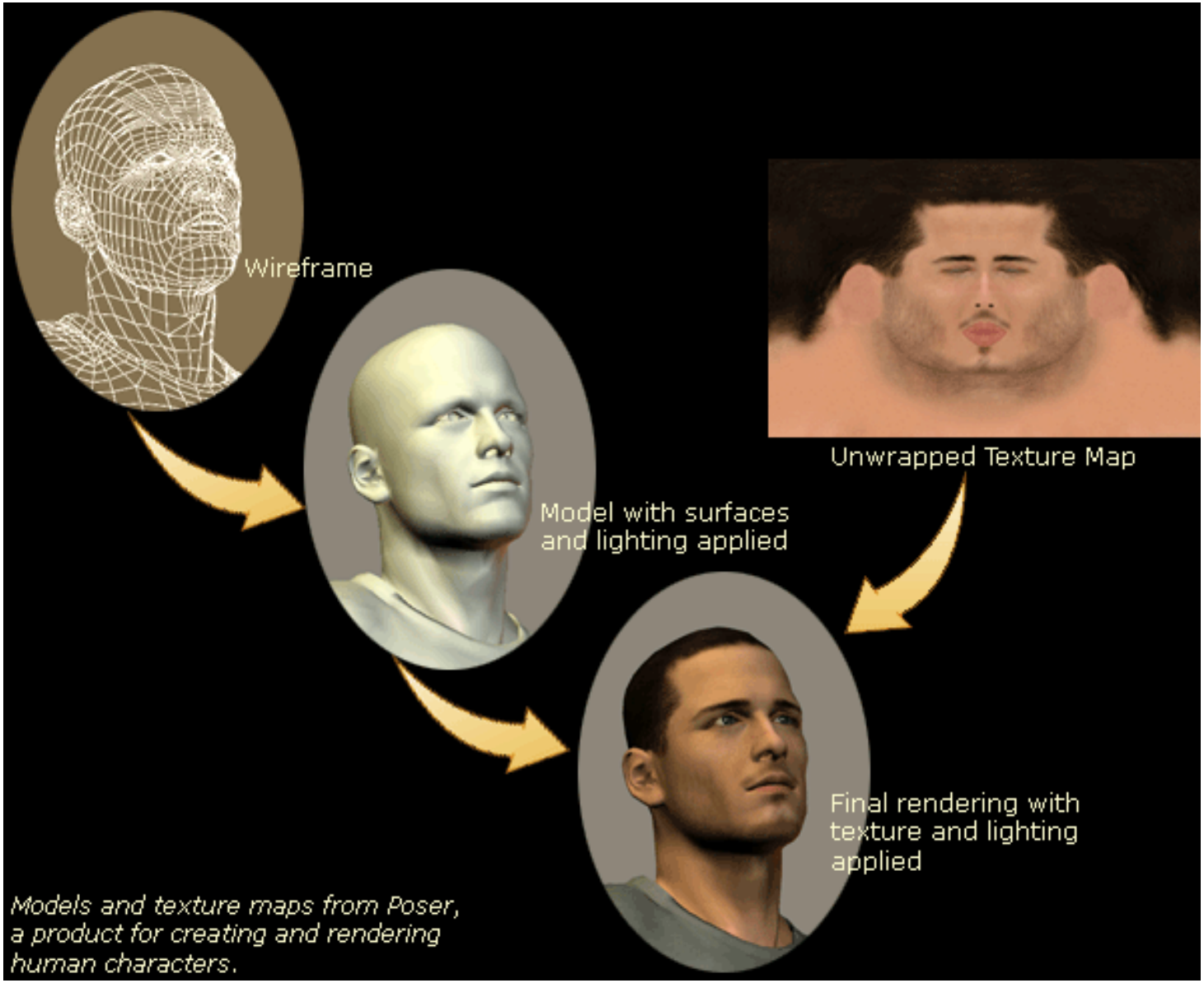
Example: Modeling an Orange

- Methon 1: An orange-colored sphere
  - Too simple
- Method 2: Replace sphere with a complex shape
  - No surface characteristics (e.g., small dimples)
  - Too many polygons to model all dimples
- Method 3: Use picture of orange to paste onto the object
  - Texture mapping
  - Still surface is smooth
- Method 4: Altering the surface normal
  - Bump mapping

Texture Mapping

- Glue *n*-dimensional images onto geometrical objects
  - Different coordinate systems involved

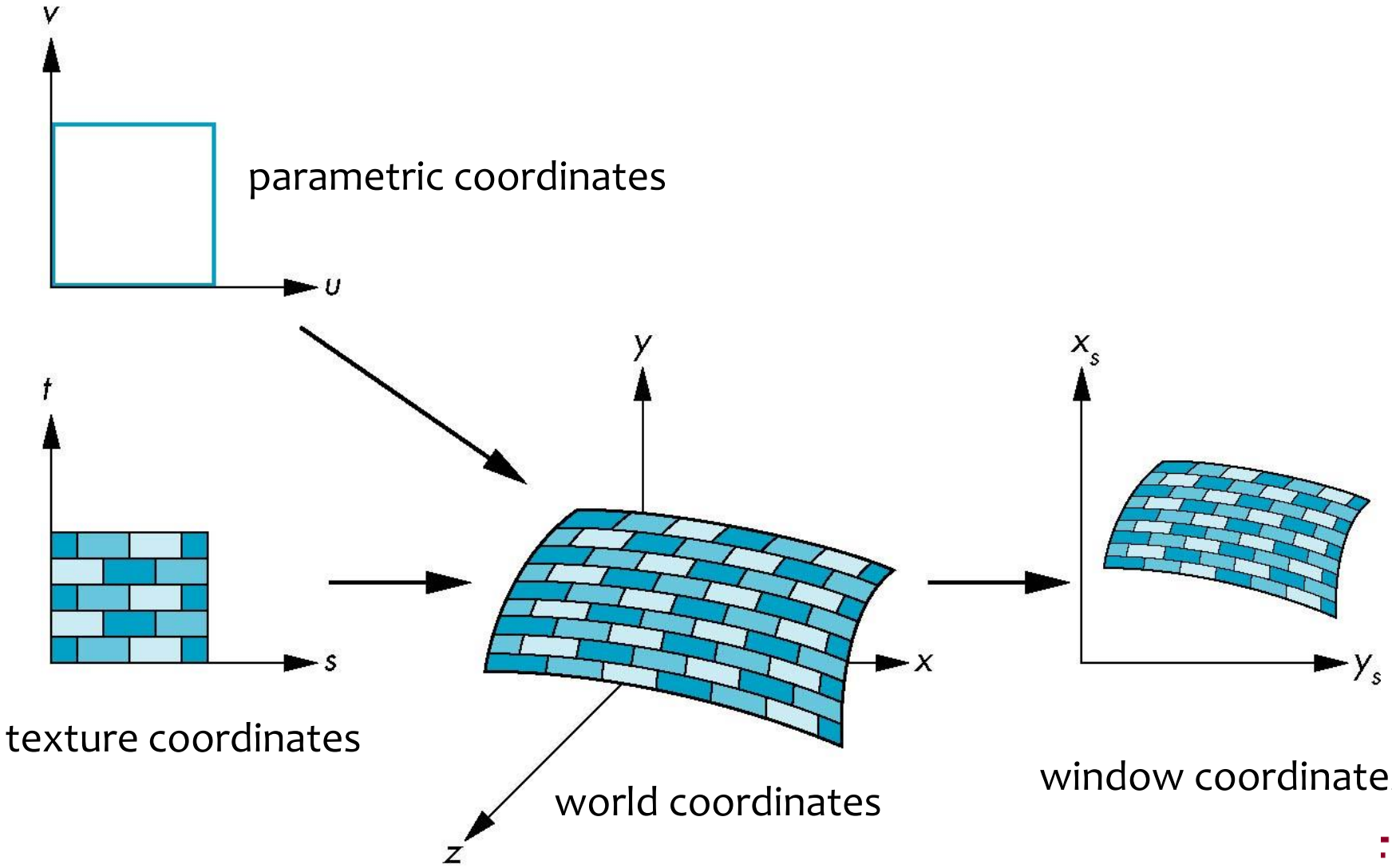




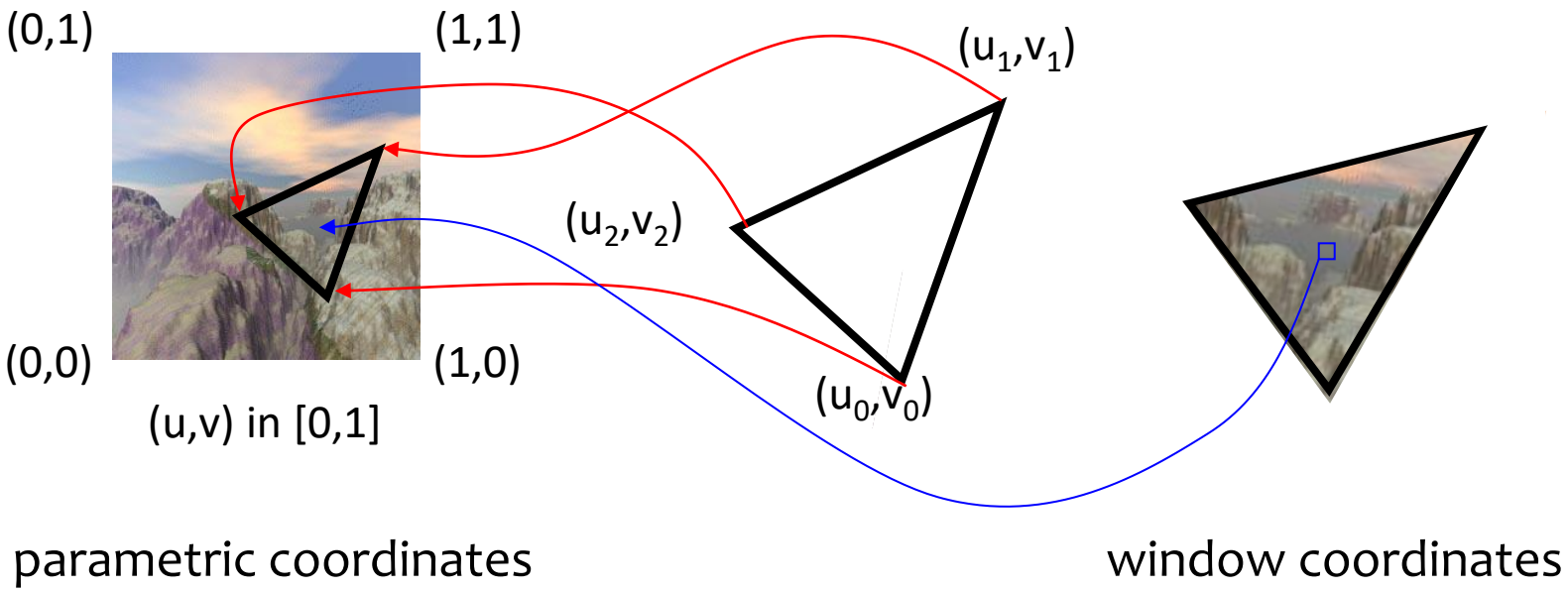
Coordinate Systems

- Parametric coordinates
  - Used to model parametric curves and surfaces
- Texture coordinates
  - Used to identify points in the image to be mapped
- Object or World Coordinates
  - Where the mapping takes place
- Window Coordinates
  - Where the final image is really produced

Texture Mapping



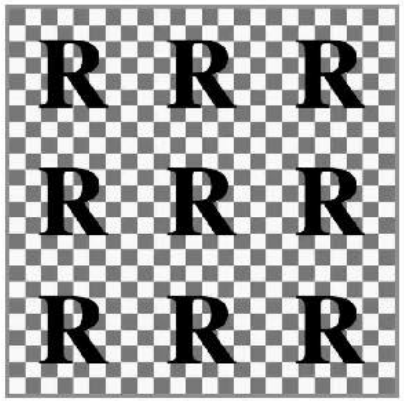
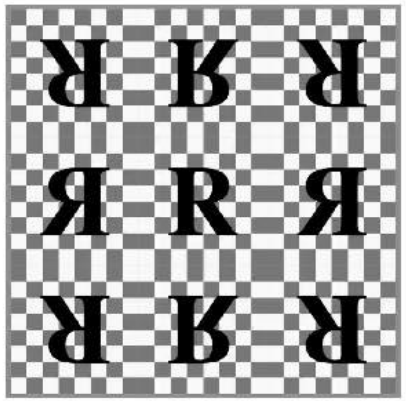
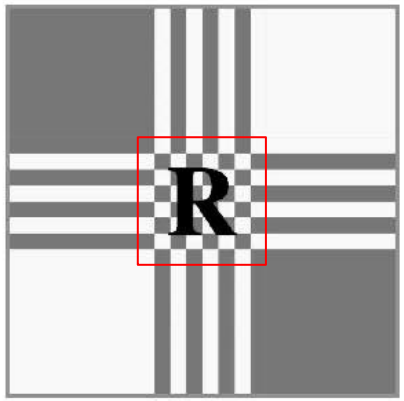
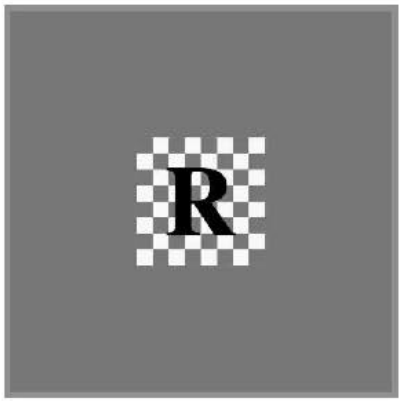
Texture Coordinate



Outside Range

- What if (u,v) > 1.0 or < 0.0?
- Repeat
  - Integer part is dropped (e.g., (1.2, 2.3)->(0.2,0.3))
- Mirror
  - Continuity across edges
- Clamp, Border

Akenine-Möller

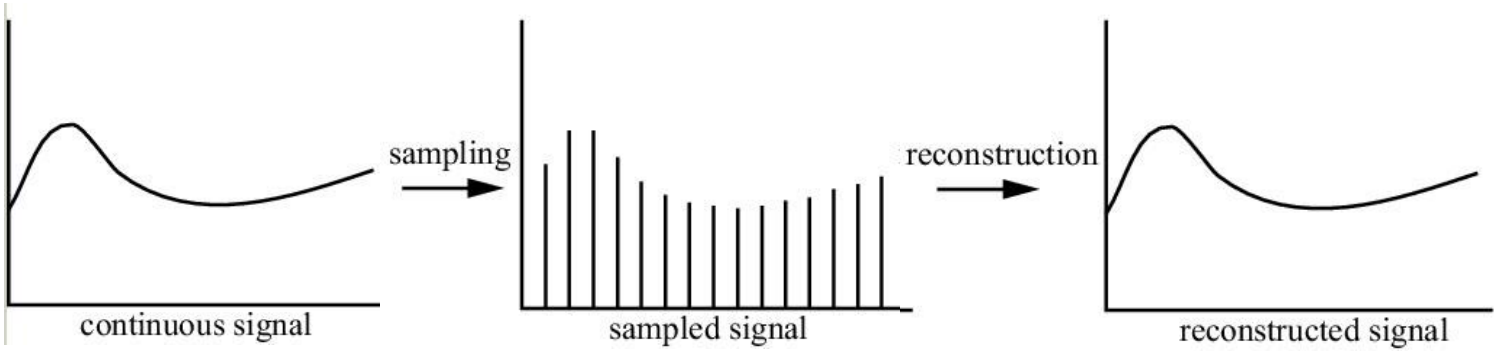





Issues in Texture Mapping

- Sampling problems
  - Texture map and rendered image are not 1:1 mapping
  - Aliasing, magnification, minification

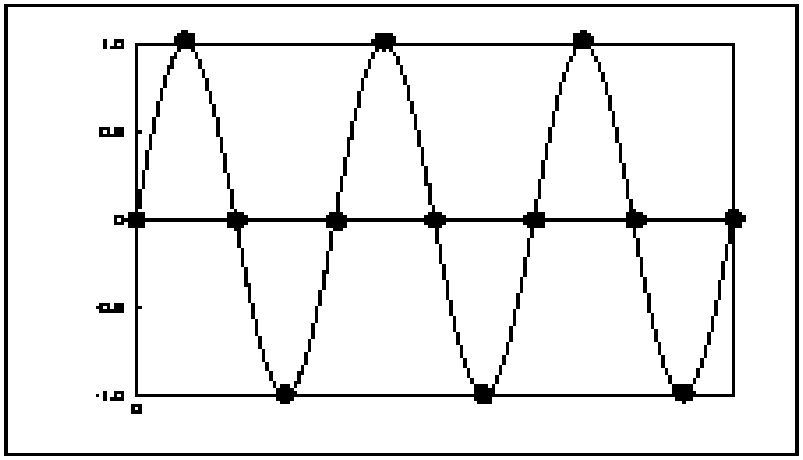
Sampling and Reconstruction

- Sampling
  - From continuous to discrete
- Reconstruction
  - From discrete to continuous

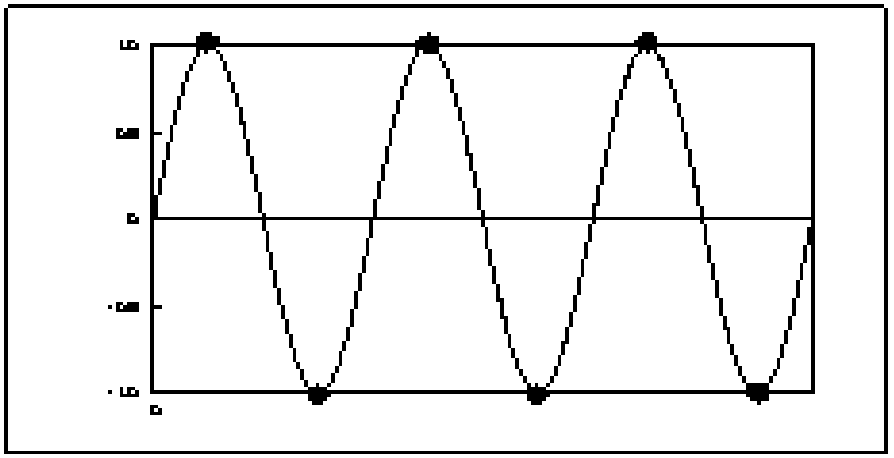


Nyquist Rate

- Sampling rate must be greater than 2x max freq.
  - If not, *Aliasing*!



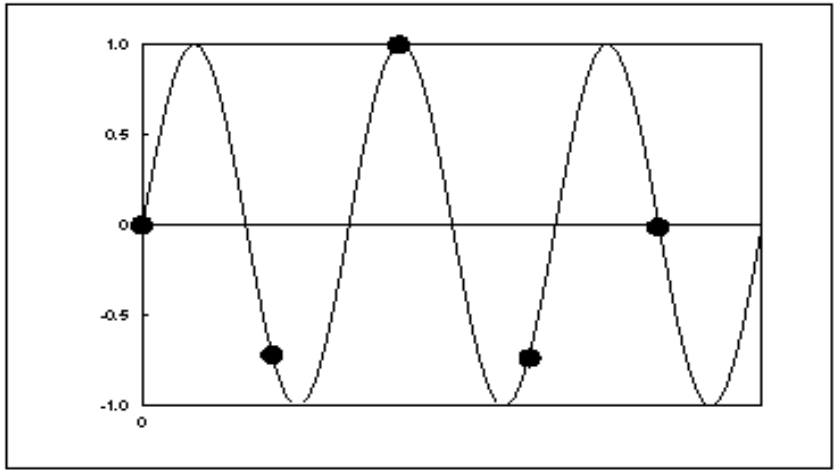
4x max freq



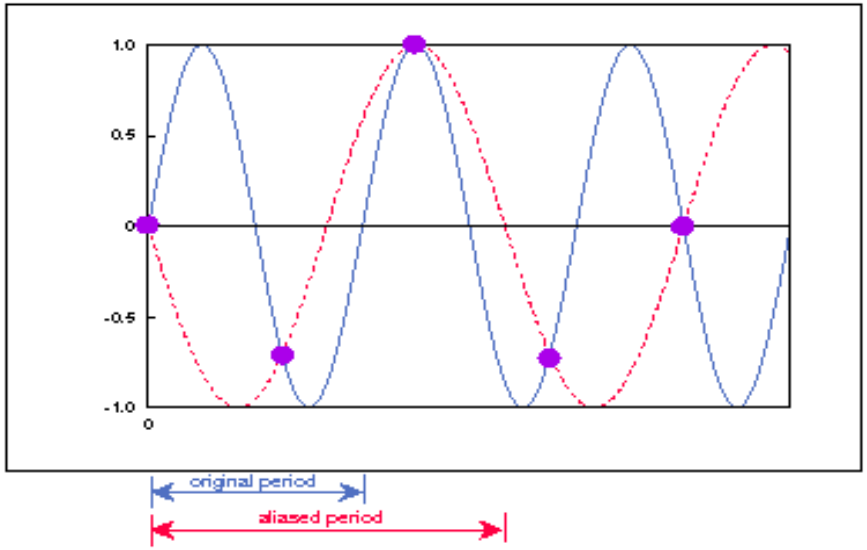
2x max freq

Nyquist Rate

- Sampling rate must be greater than 2x max freq.
  - If not, *Aliasing*!



1.25x max freq



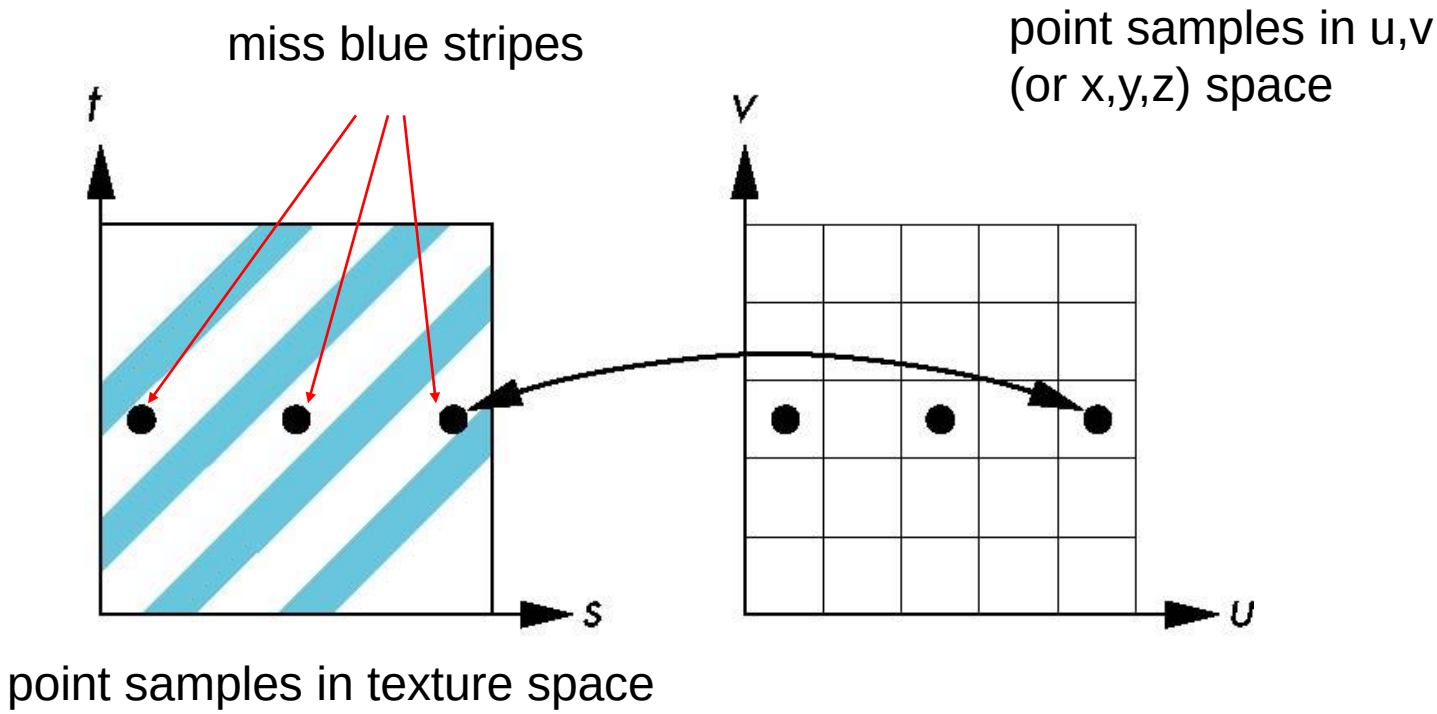
Aliasing

What is Frequency in Graphics?

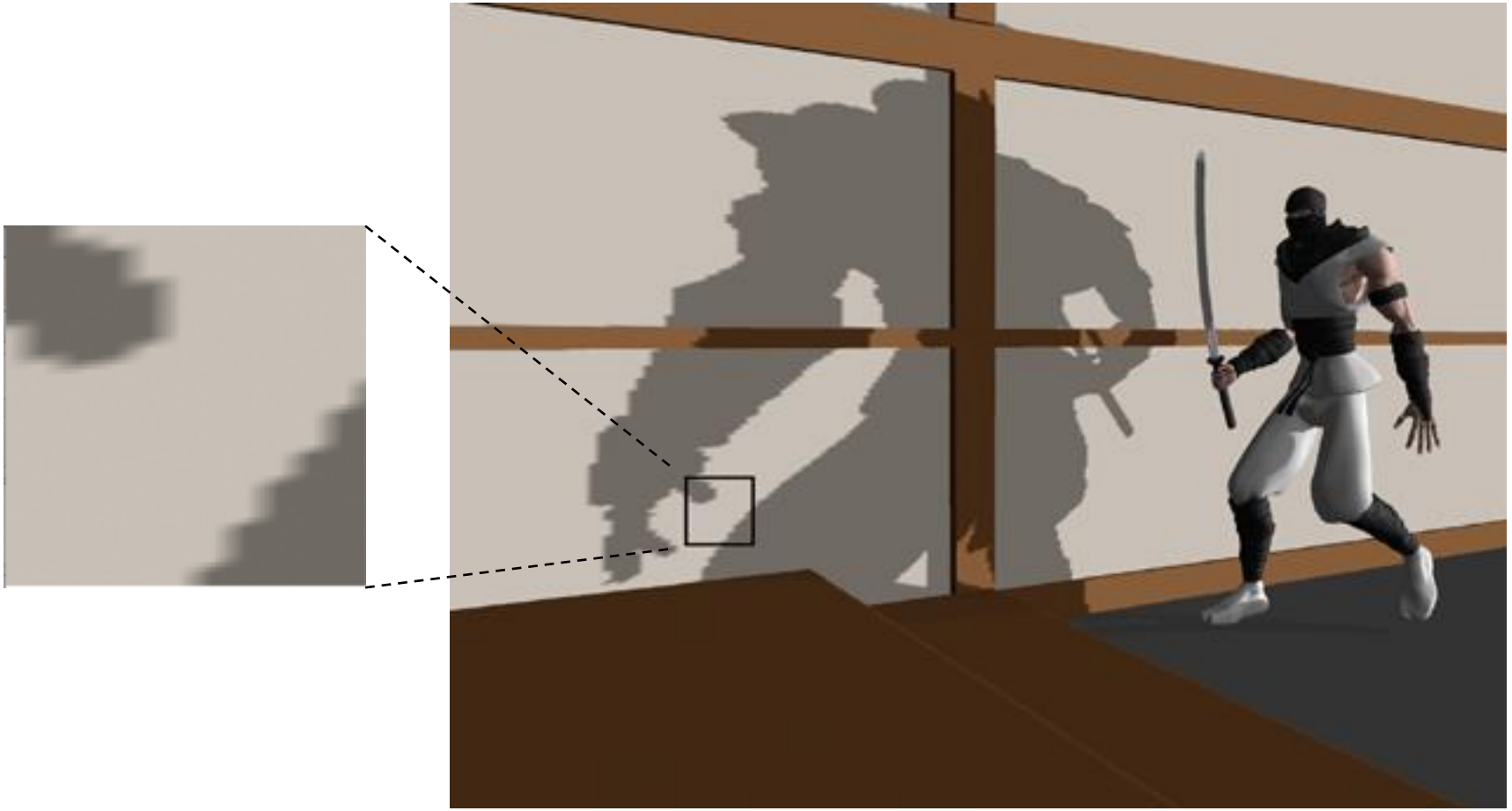
- Frequency implies
  - How fast the signal changes
  - Smoothness
- Sampling in graphics is prone to *aliasing*
  - Artifact due to discrete sampling from continuous function
  - Sharp edges & corners of polygons
  - Shadow boundaries

Aliasing

- Point sampling can lead to aliasing errors



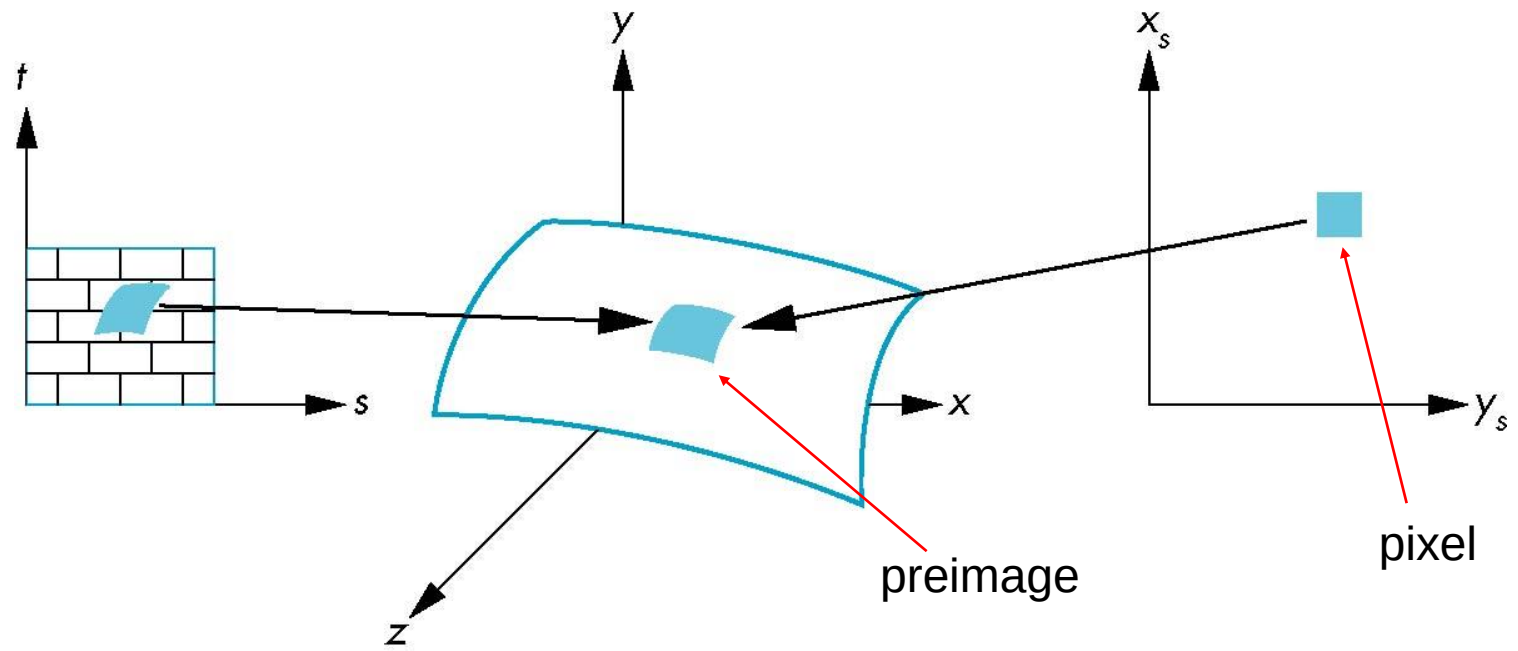
Aliasing



GPU Gems

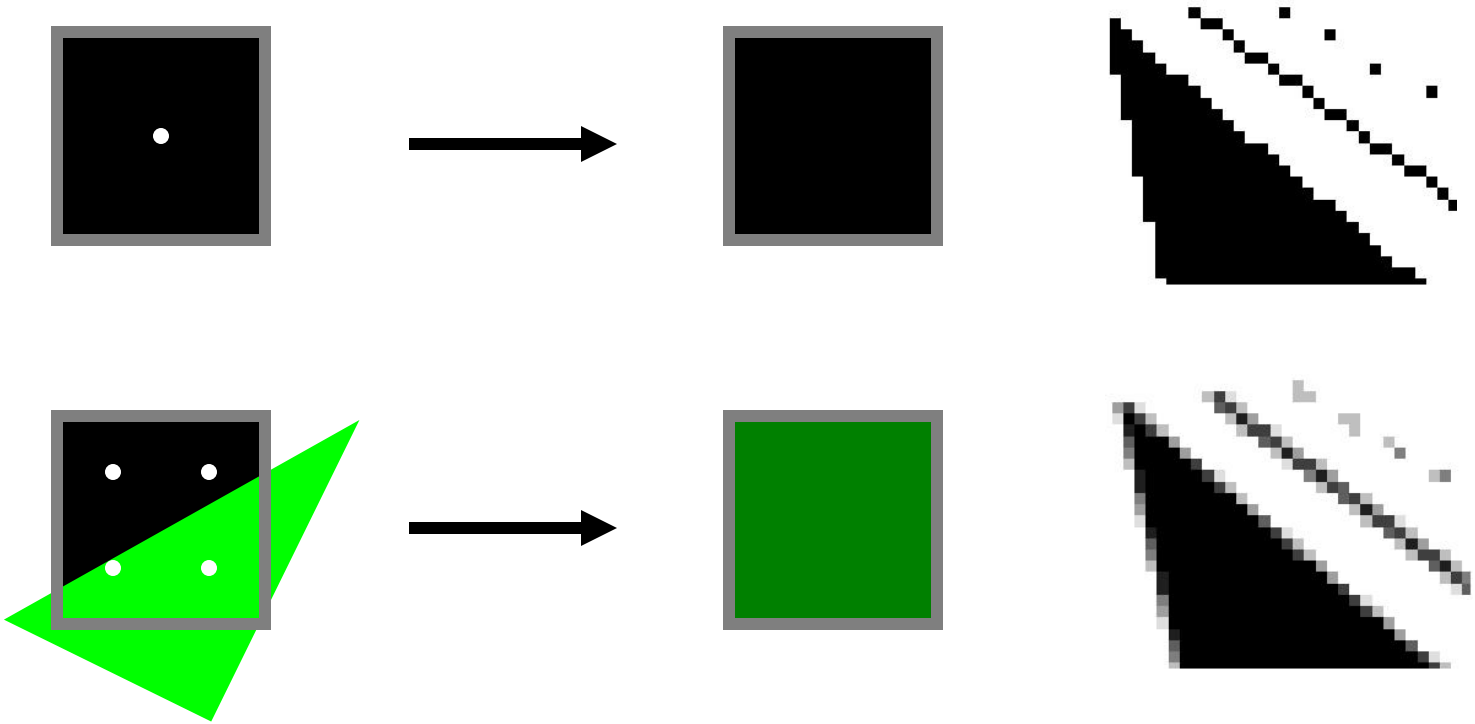
Antialiasing: Area Averaging

- Region instead of point



Antialiasing: Supersampling

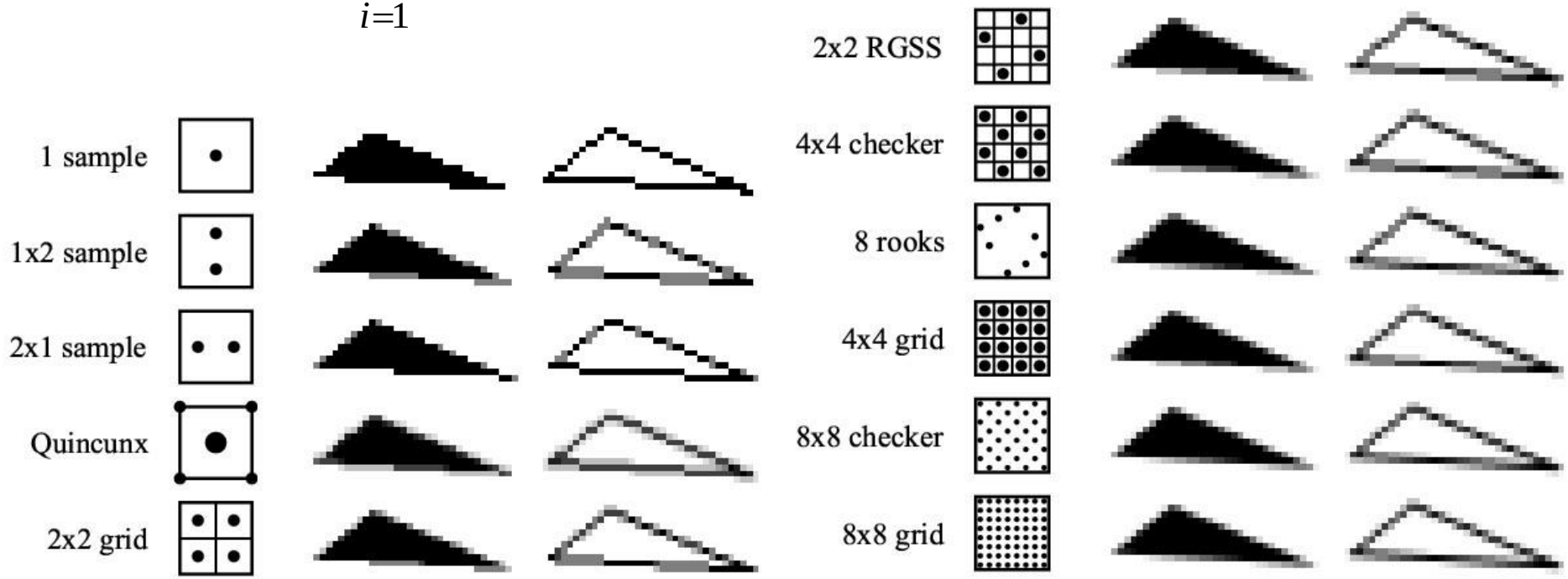
- Use more than one sample per pixel



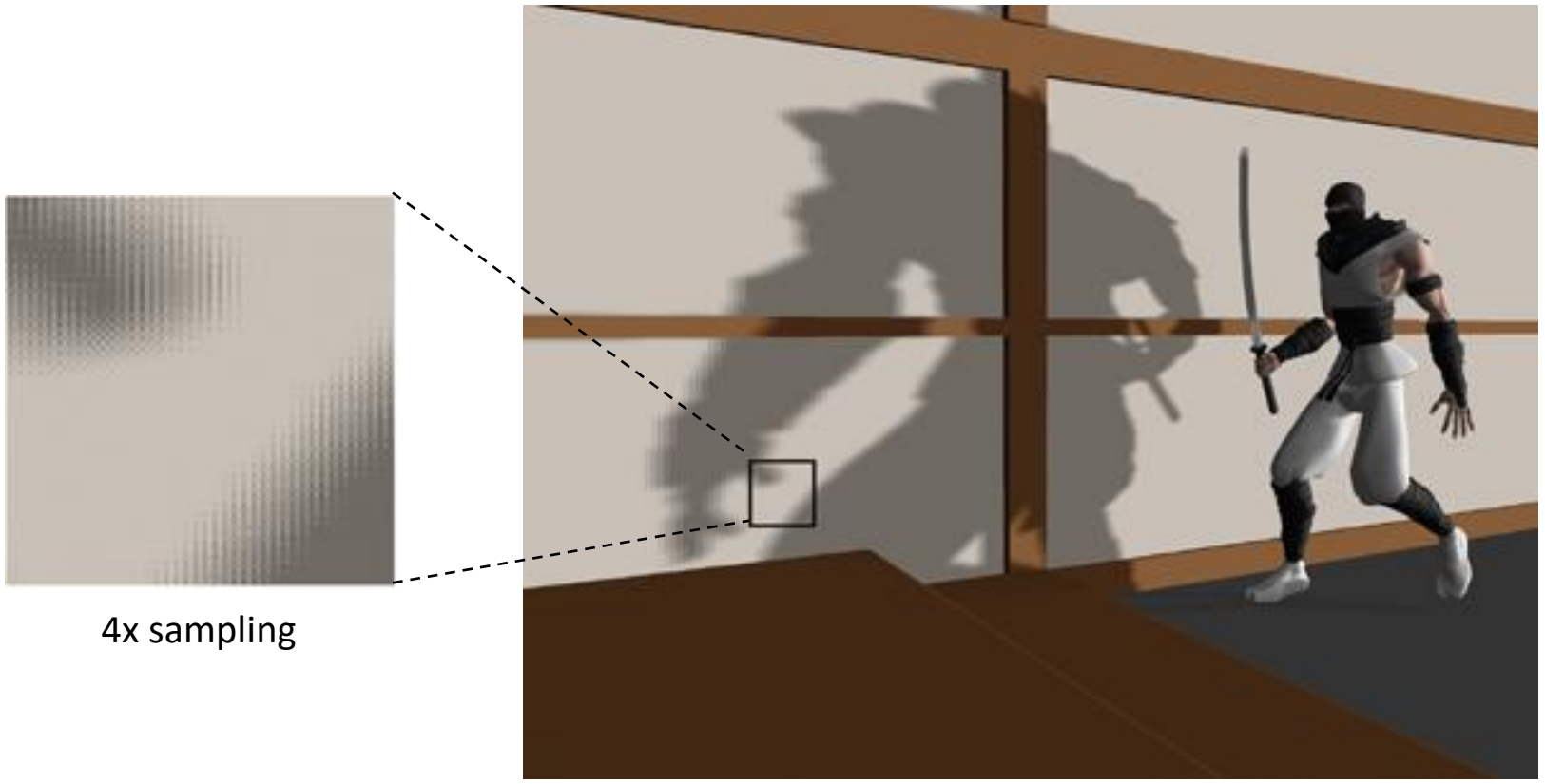
Different Supersampling

- w are the weights in [0,1]
- c is the color sample inside pixel

$$\mathbf{p}(x,y) = \sum_{i=1}^{n} w_i \mathbf{c}(i,x,y)$$



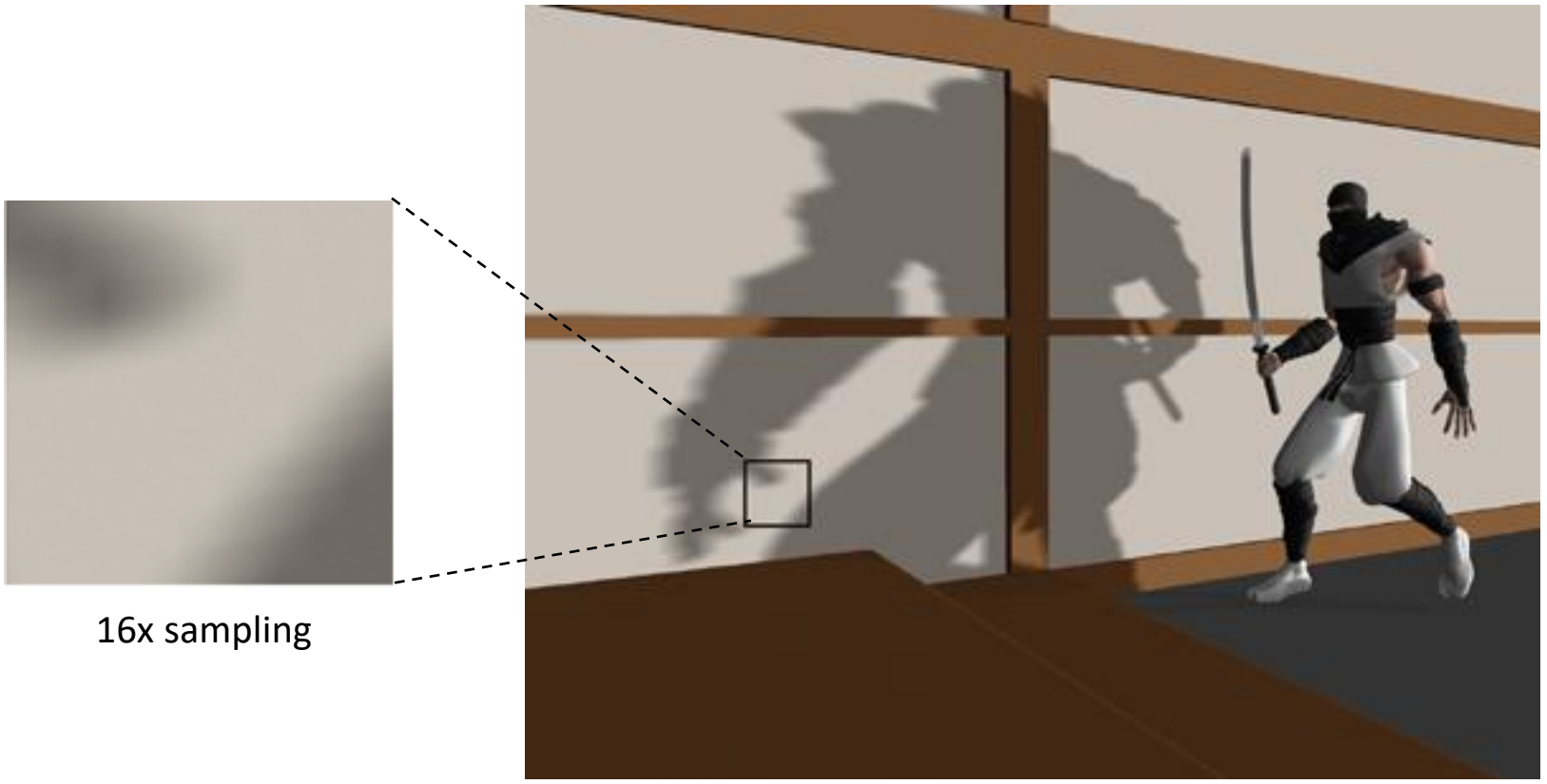
Antialiasing



4x sampling

GPU Gems

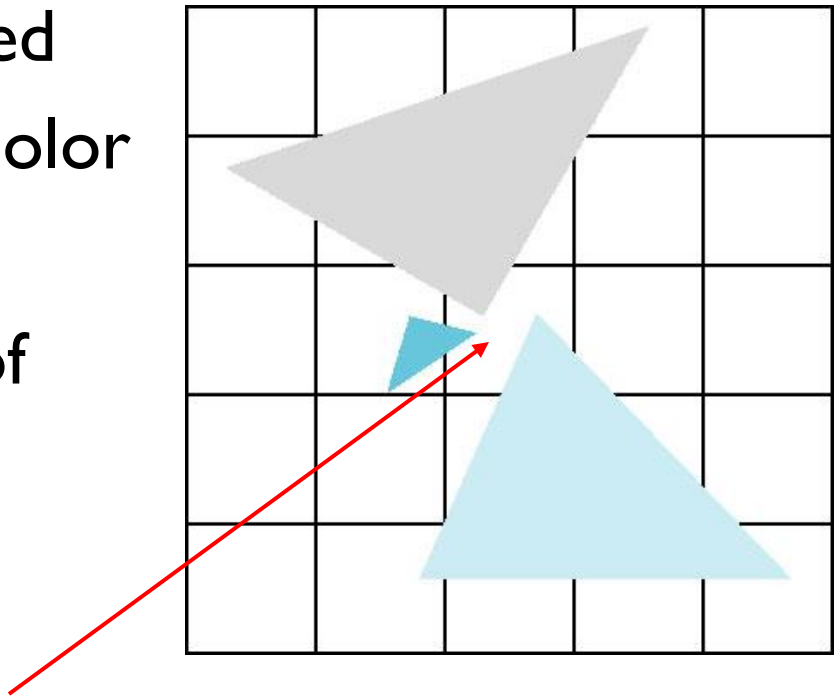
Antialiasing



16x sampling

Polygon Aliasing

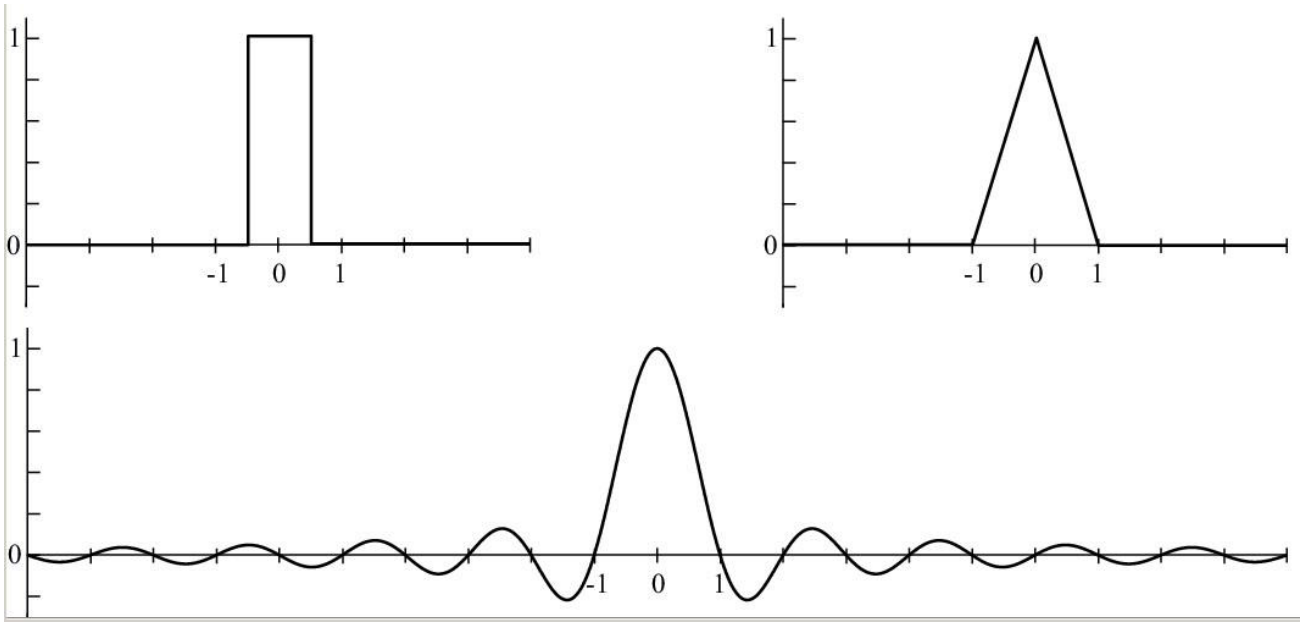
- Aliasing problems can be serious for polygons
  - Jaggedness of edges
  - Small polygons neglected
  - Need compositing so color of one polygon does not totally determine color of pixel



All three polygons should contribute to color

Reconstruction

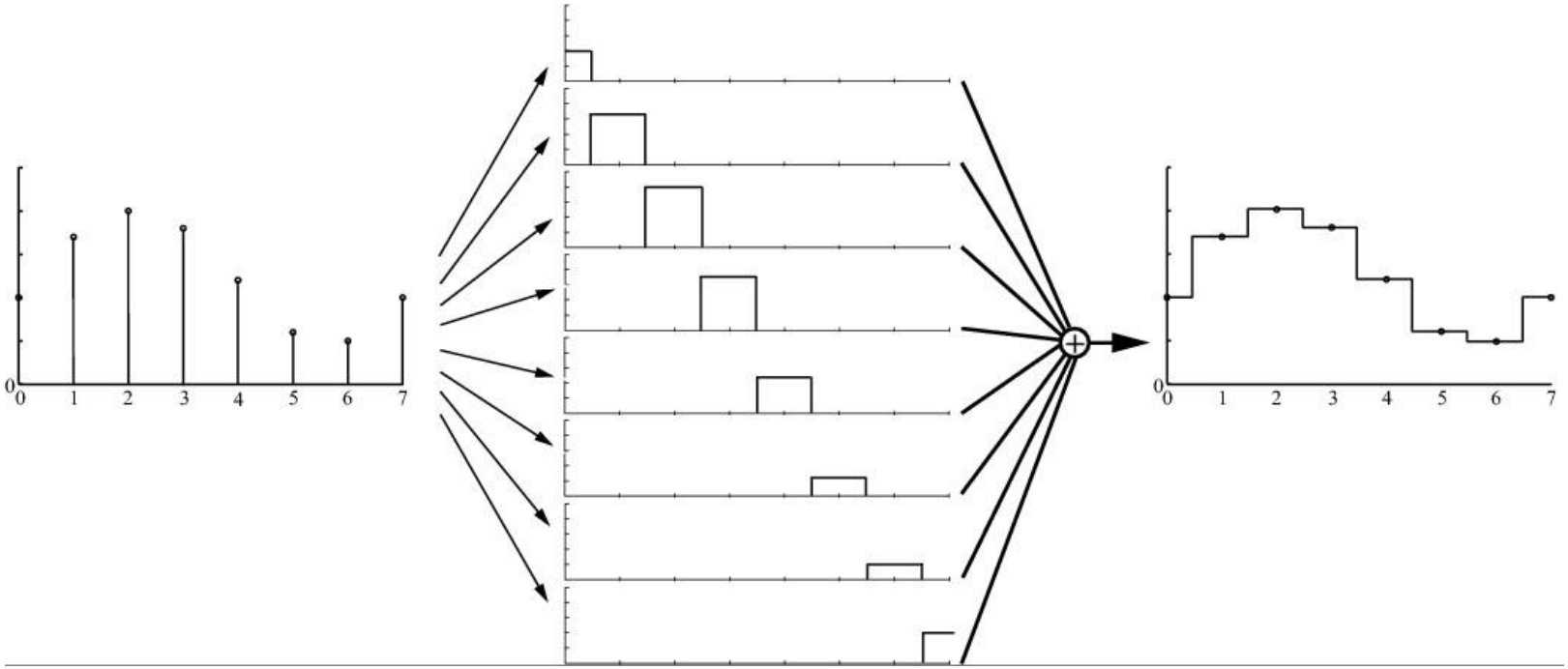
- Basis Filters



$$\frac{\sin \pi x}{\pi x}$$

Box Filter

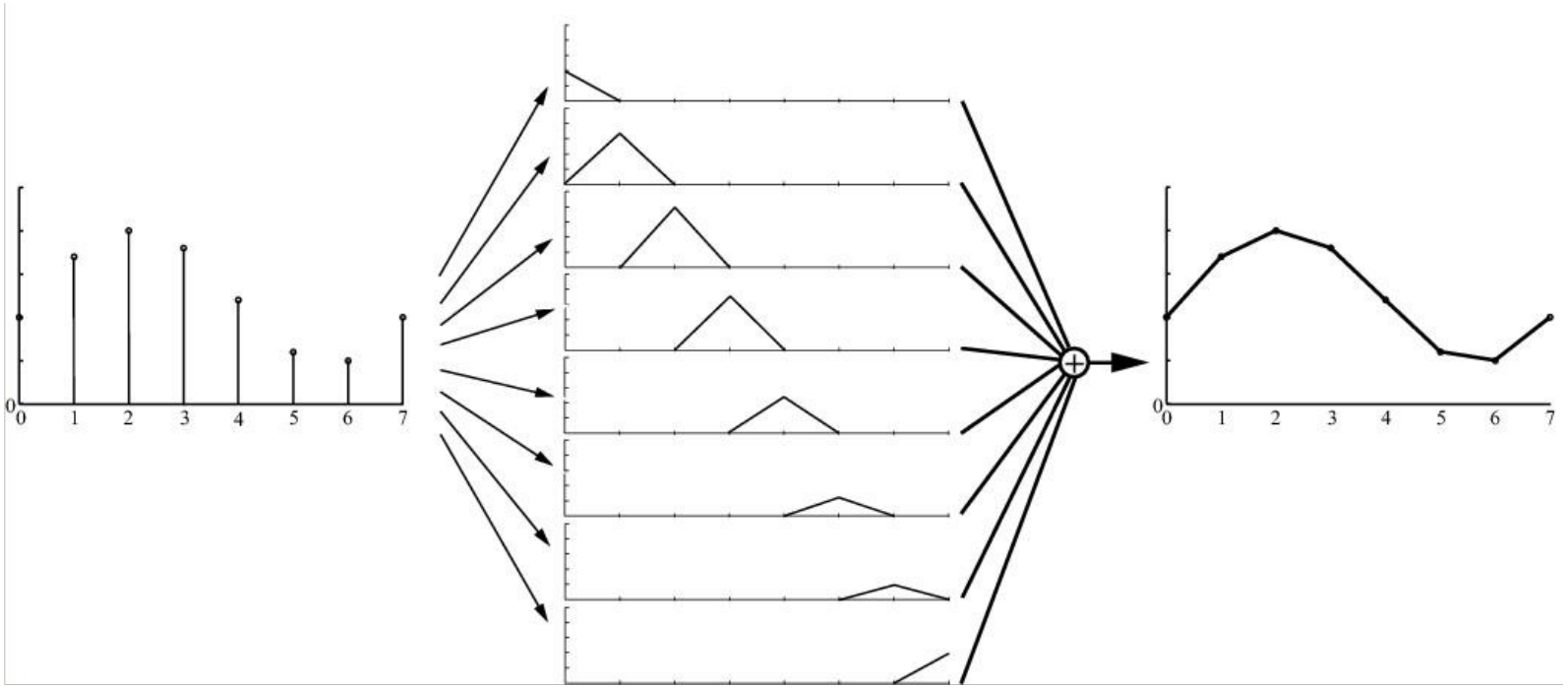
- Nearest neighbor



$$f(x) = \sum_i s(i)B(x-i)$$

Tent Filter

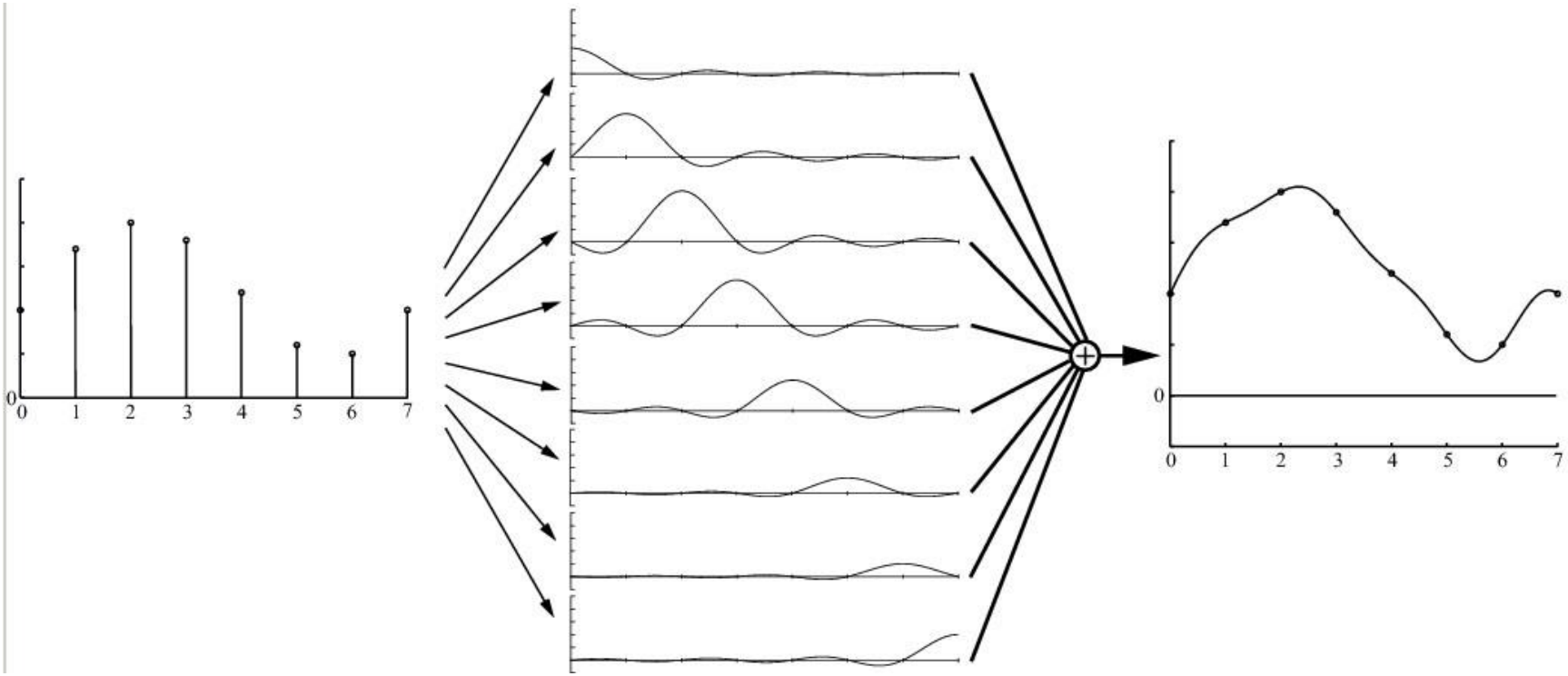
- Linear interpolation



$$f(x) = \sum_i s(i)B(x-i)$$

Sinc Filter

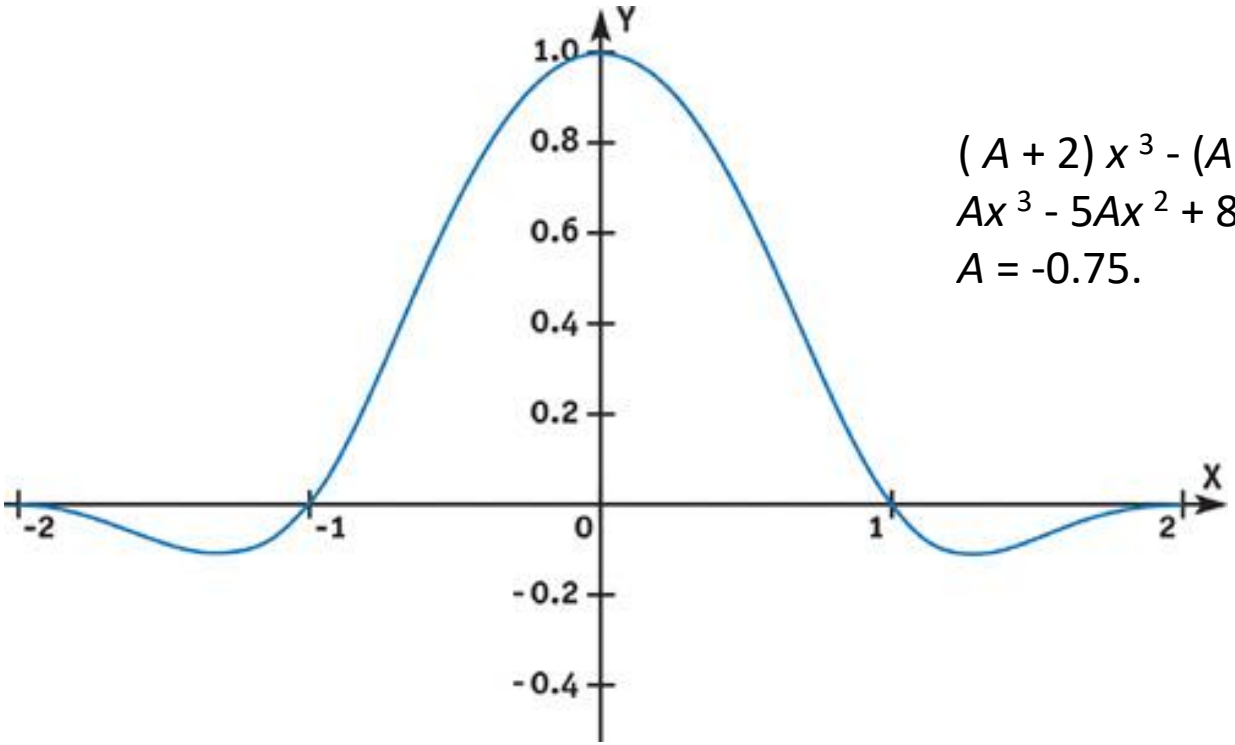
- Theoretically ideal, but not practical



$$f(x) = \sum_i s(i)B(x-i)$$

Cubic Filters

- Smooth and compact support



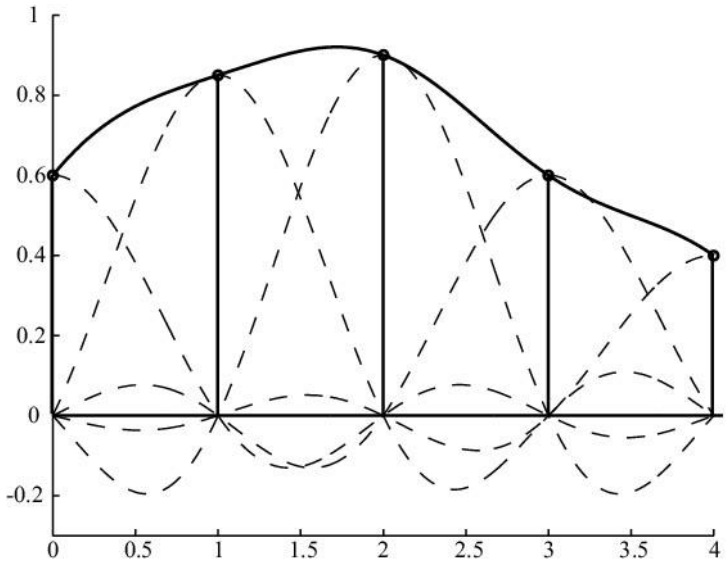
$(A + 2)x^3 - (A + 3)x^2 + 1.0,\ 0 < x < 1$

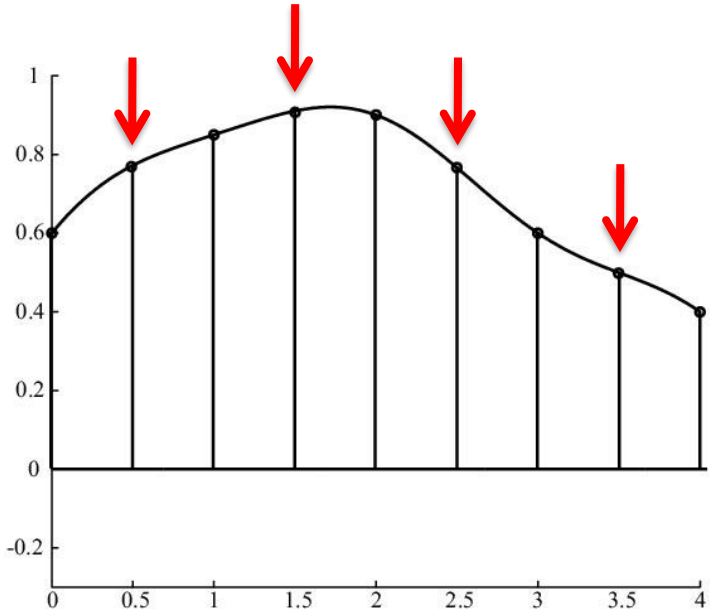
$Ax^3 - 5Ax^2 + 8Ax - 4A,\ 1 < x < 2$

$A = -0.75.$

Texture Magnification

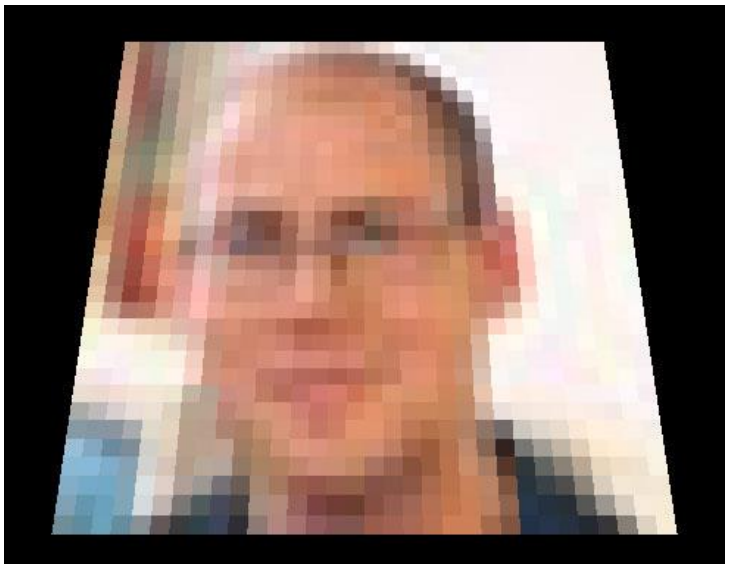
- Hardly get 1:1 mapping
- Reconstruction and resampling





Texture Magnification

- Zoom in
- Linear (Tent filter) is better than nearest neighbor (box filter)



Nearest neighbor

Linear interpolation

Texture Magnification

- Zoom in
- Cubic is better than linear

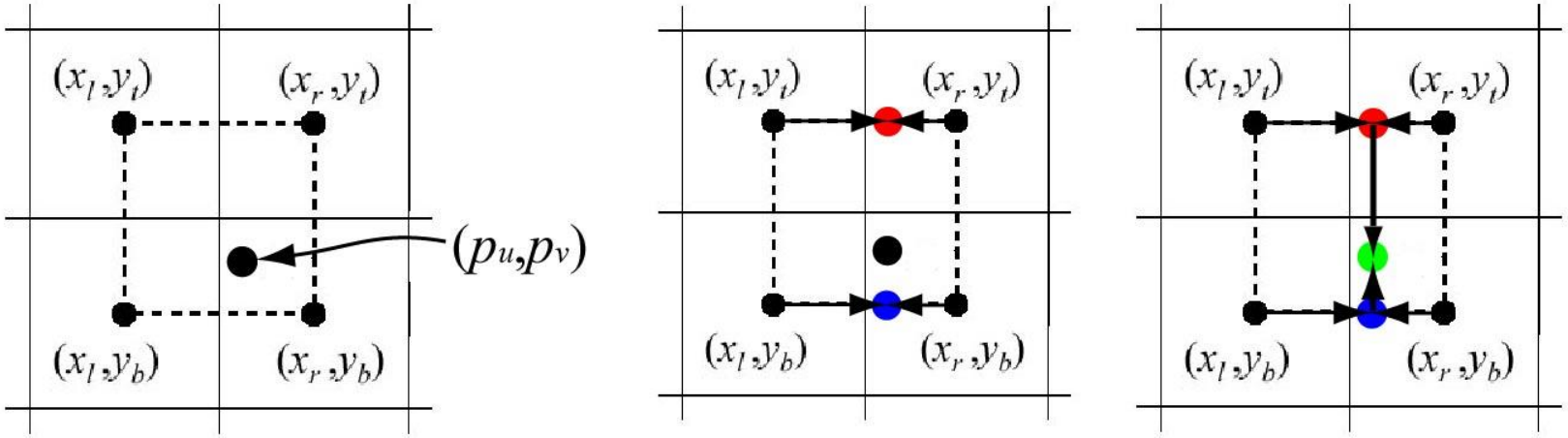
Markus Hadwiger

Magnification using Reconstruction

- Local averaging using filter kernel
  - Nearest neighbor : 1 sample
  - Linear interpolation : 2 samples
  - Cubic interpolation : 4 samples

$$f(x) = \sum_i s(i)B(x-i)$$

Bilinear Interpolation

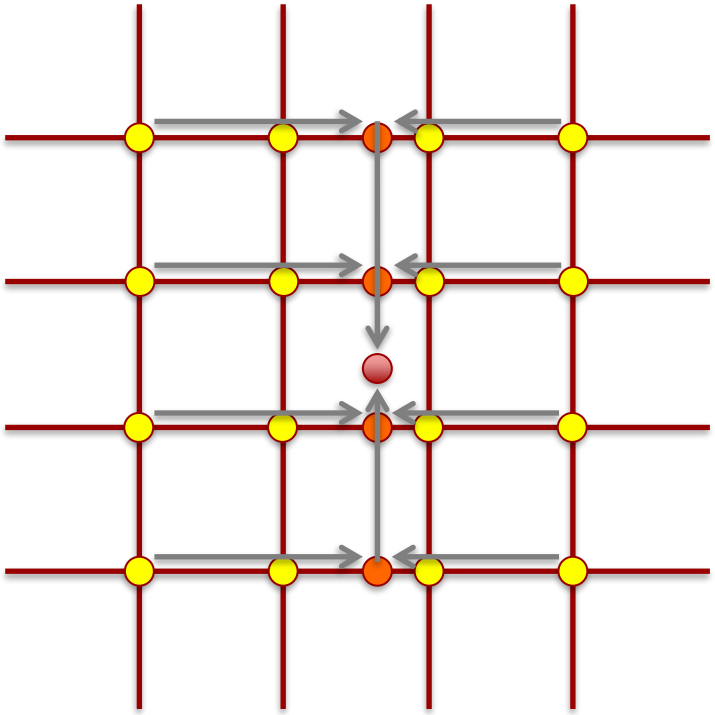


$$(u', v') = (p_u - \lfloor p_u \rfloor, p_v - \lfloor p_v \rfloor).$$

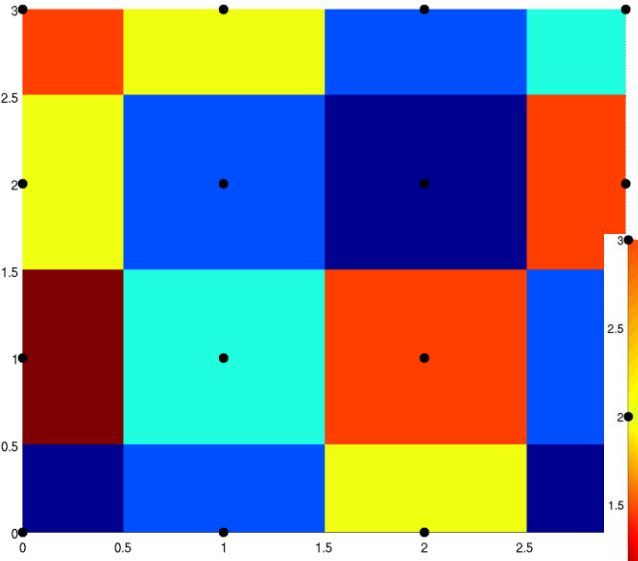
$$\begin{aligned}\mathbf{b}(p_u, p_v) =& (1-u')(1-v')\mathbf{t}(x_l, y_b) + u'(1-v')\mathbf{t}(x_r, y_b) \\ &+ (1-u')v'\mathbf{t}(x_l, y_t) + u'v'\mathbf{t}(x_r, y_t).\end{aligned}$$

Bicubic Interpolation

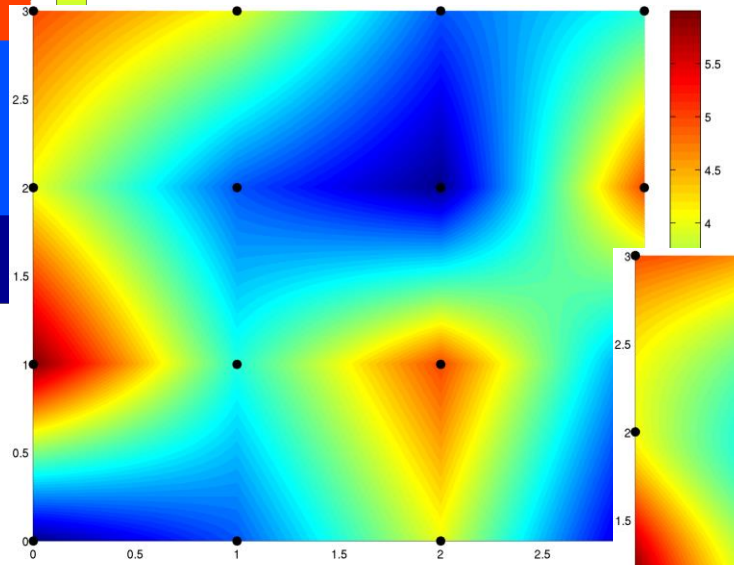
$$\mathrm{CINT}_x(p_{-1},p_0,p_1,p_2) = \frac{1}{2}\begin{pmatrix} -x^3+2x^2-x \\ 3x^3-5x^2+2 \\ -3x^3+4x^2+x \\ x^3-x^2 \end{pmatrix} \cdot \begin{pmatrix} p_{-1} \\ p_0 \\ p_1 \\ p_2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} x((2-x)x-1) \\ x^2(3x-5)+2 \\ x((4-3x)x+1) \\ (x-1)x^2 \end{pmatrix} \cdot \begin{pmatrix} p_{-1} \\ p_0 \\ p_1 \\ p_2 \end{pmatrix}$$



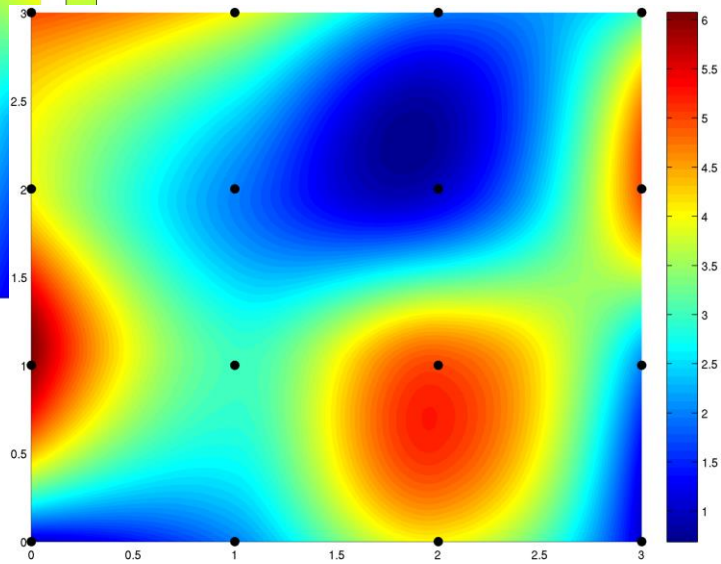
Comparison



Nearest neighbor



Bilinear interpolation

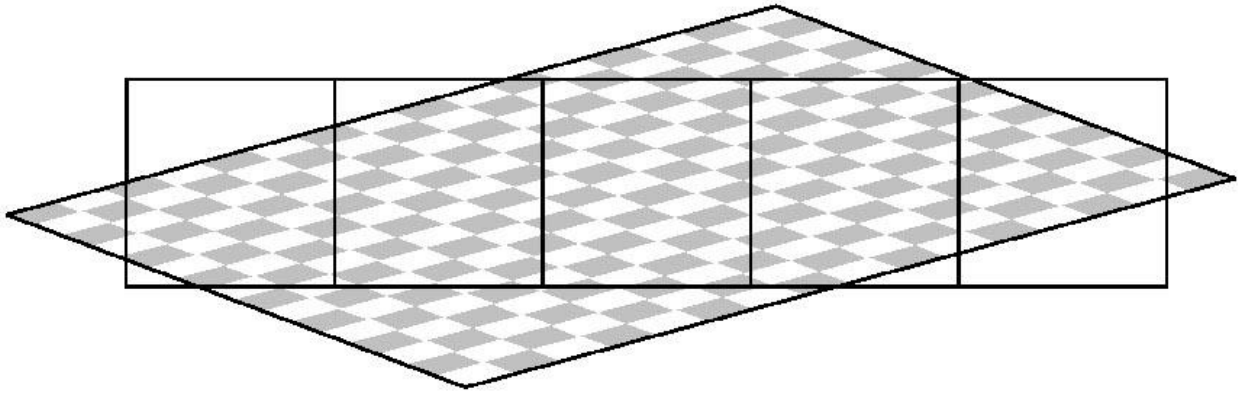


Bicubic interpolation

Wikipedia

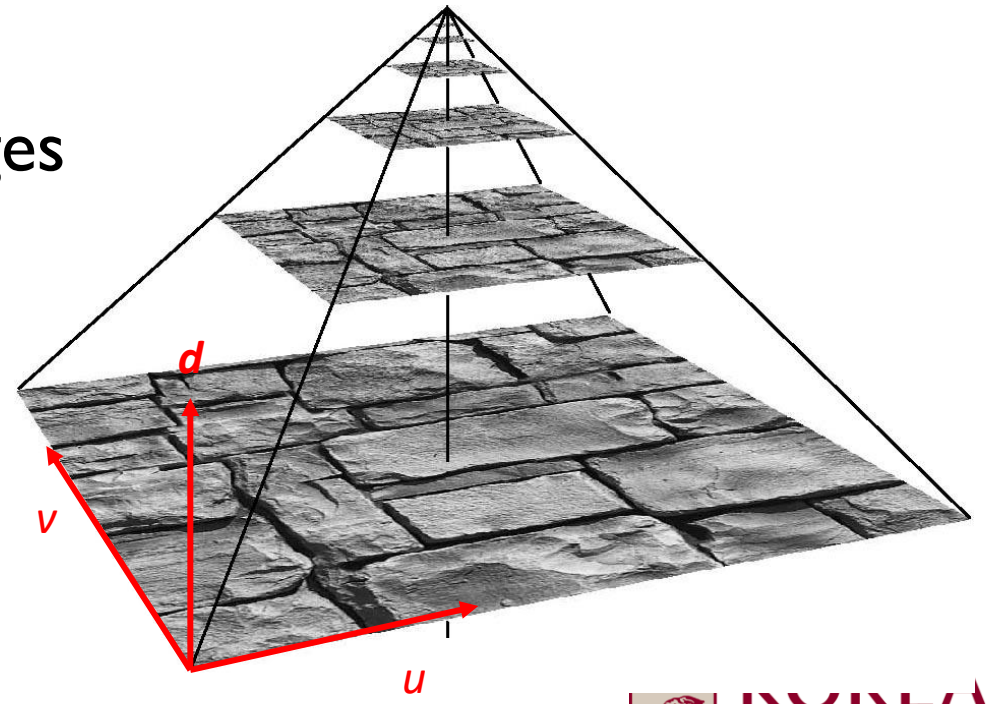
Texture Minification

- Zoom out
- Need filter to reduce aliasing
- Average of texels inside a pixel
  - Still too expensive to do real-time



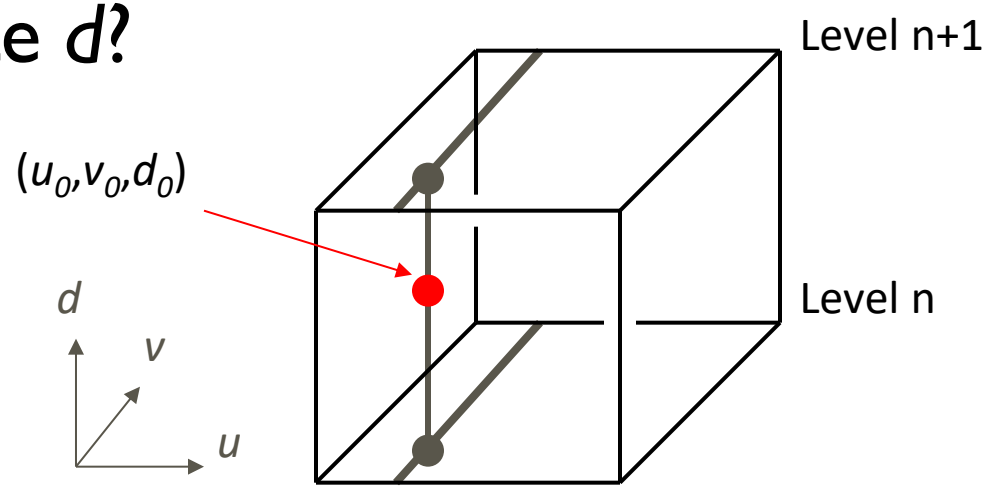
Mipmapping

- Pre-filtered image pyramid
- 4-to-1 average
- Process
  - Compute *d*
  - Pick two closest images
  - Bilinear interpolation

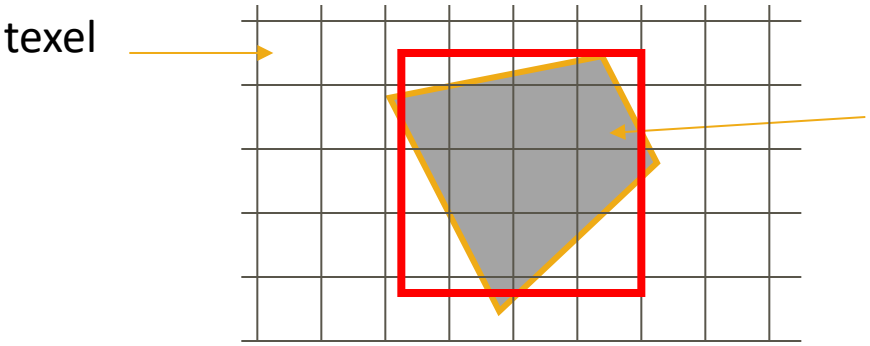


Mipmapping

- Interpolate between bilinear values
  - Trilinear interpolation
- Constant time filtering
  - 8 texels per sampling
- How to compute *d*?



Compute d for Mipmapping



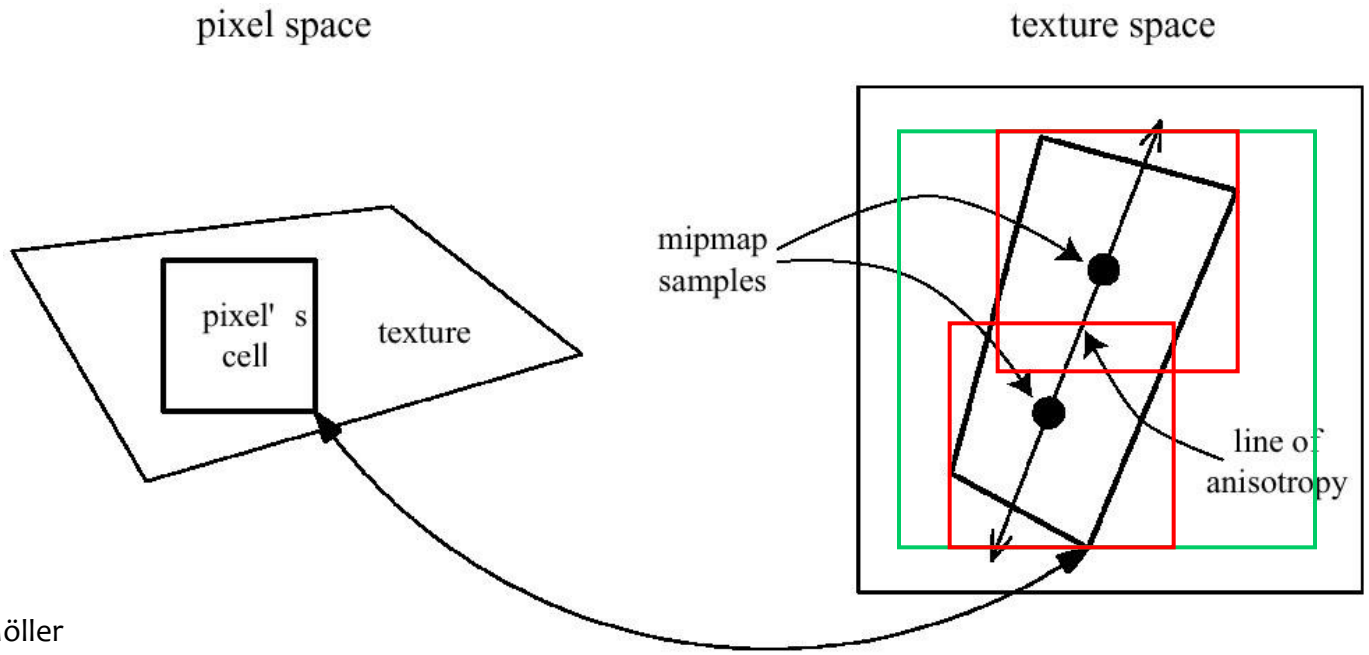
$$A = \text{approximative area of quadrilateral}$$

$$b = \sqrt{A}$$

$$d = \log_2 b$$

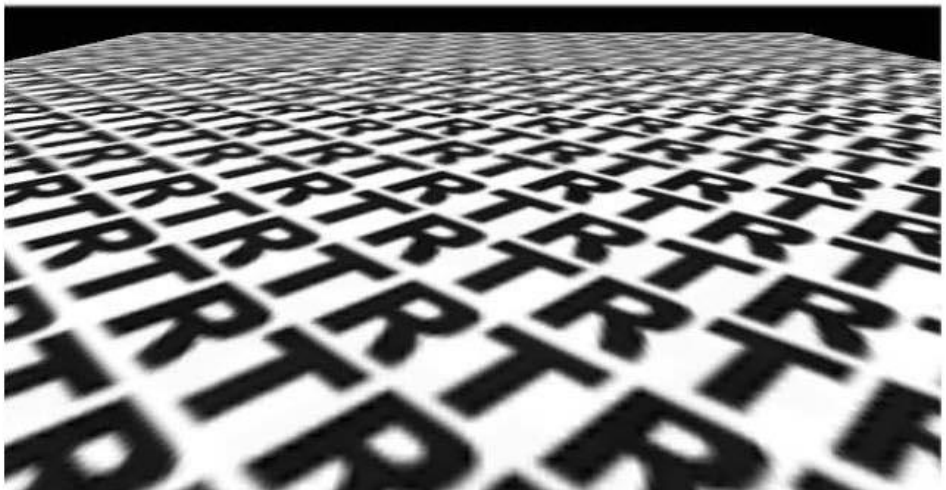
- Approximate quad with square
- Problem: overblur

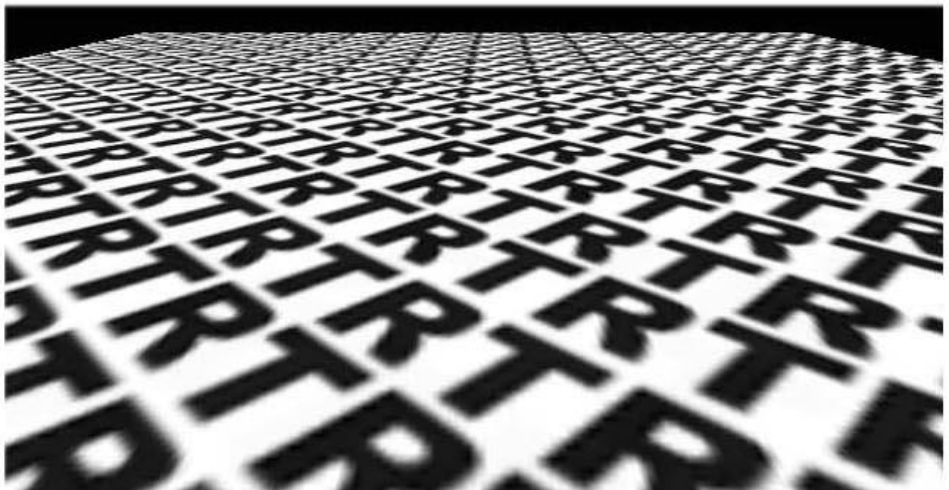
Anisotropic Texture Filtering

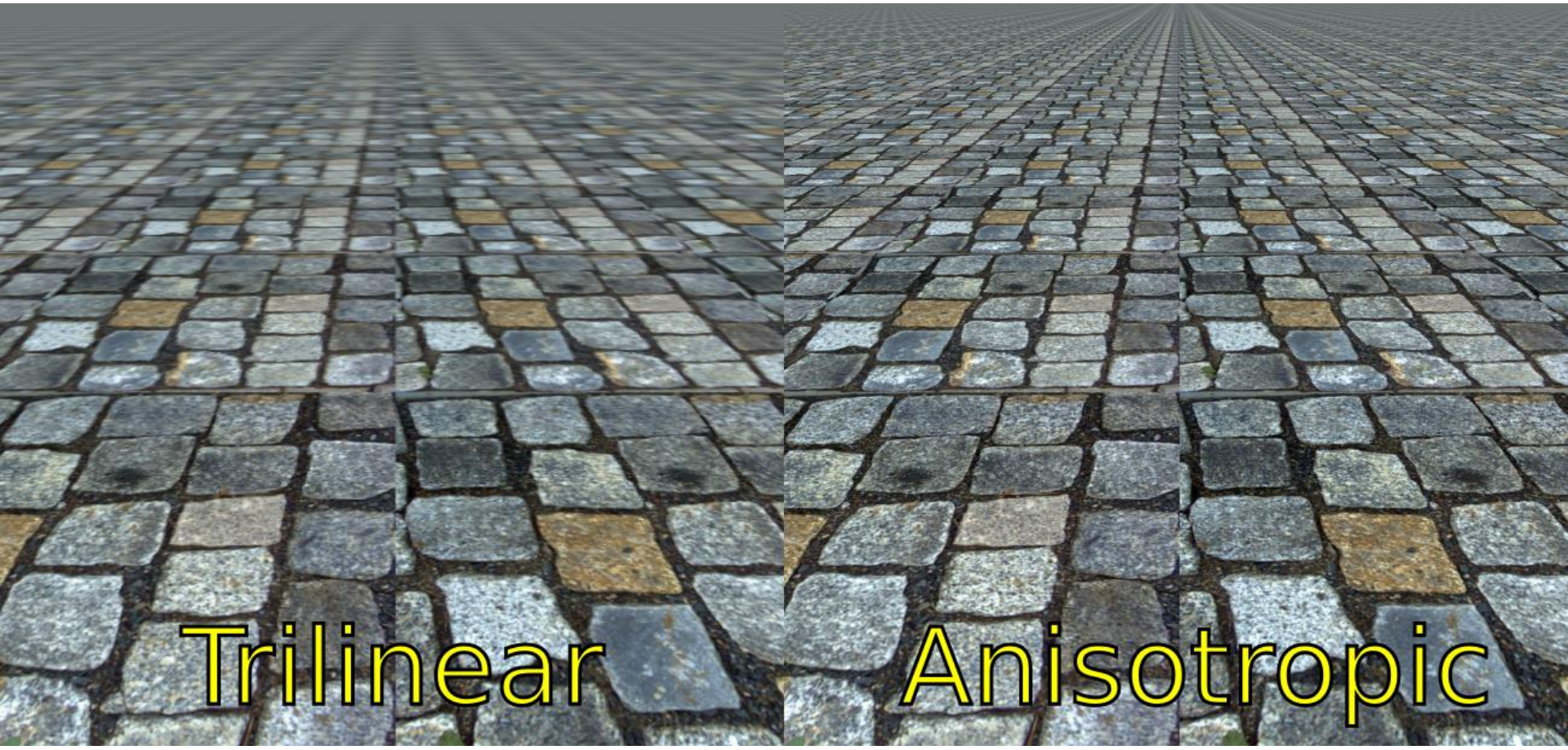


16 samples

Akenine-Möller







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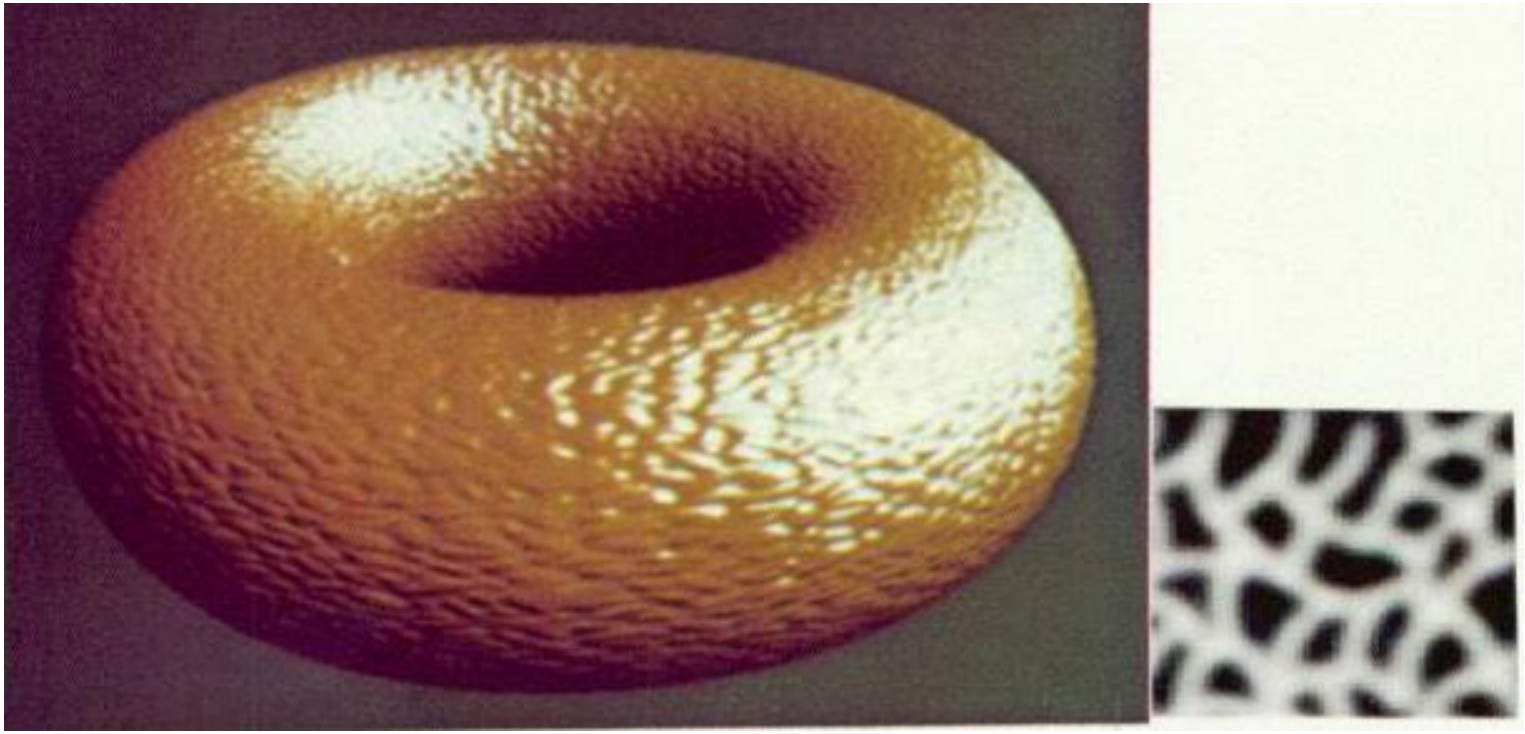
Questions?



First bump mapping image by Jim Blinn in his paper "Simulation of Wrinkled Surfaces" published in 1978