

## Lecture 12: Clipping

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Won-Ki Jeong

(wkjeong@korea.ac.kr)



# Overview

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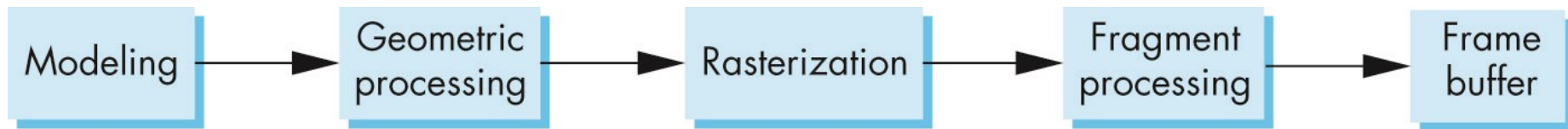
- At end of the geometry processing, vertices have been assembled into primitives
- Must clip out primitives that are outside the view frustum
  - Algorithms based on representing primitives by lists of vertices
- Must find which pixels can be affected by each primitive
  - Fragment generation
  - Rasterization or scan conversion



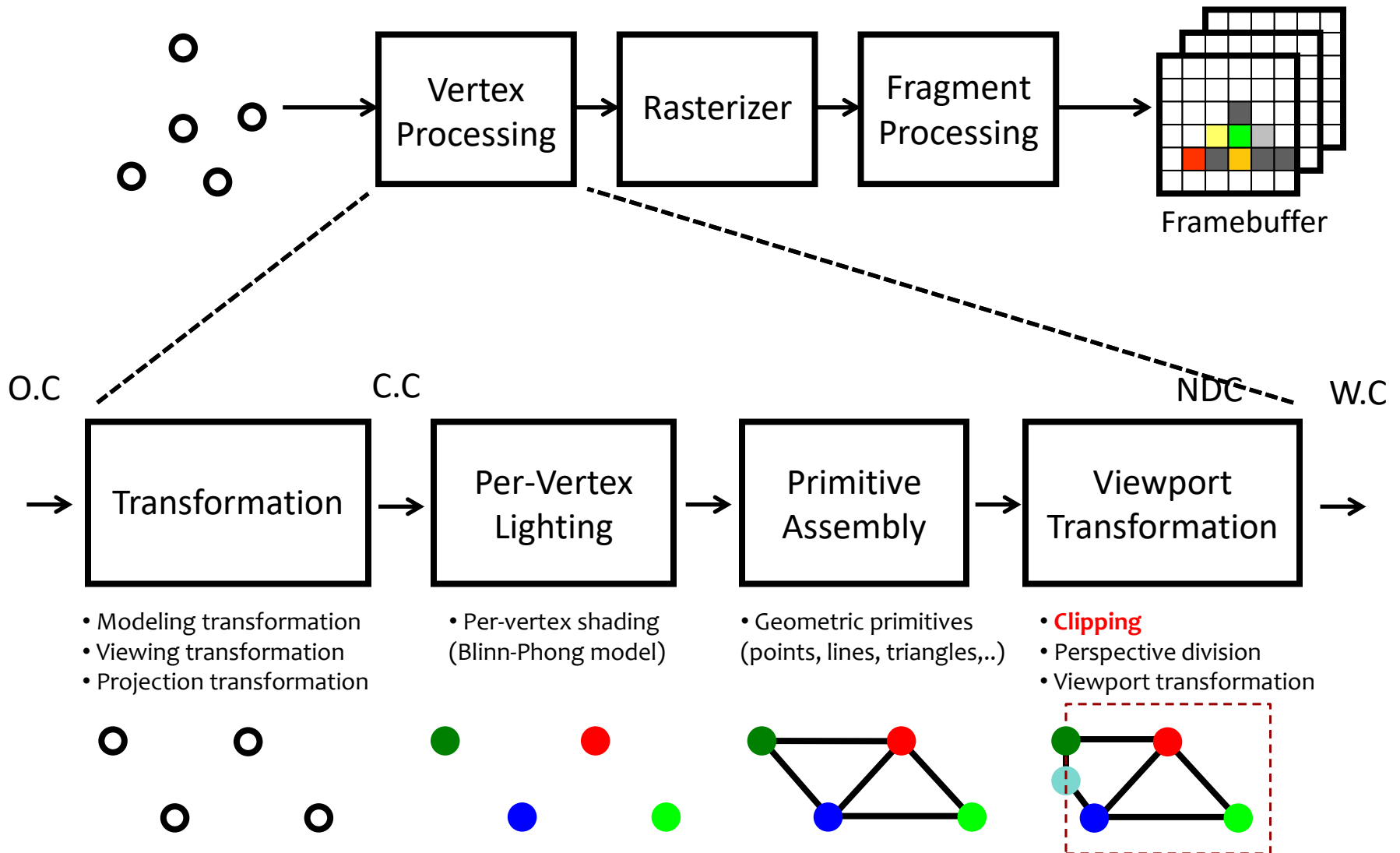
# Required Tasks

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- Transformations
- Clipping
- Rasterization or scan conversion
- Some tasks deferred until fragement processing
  - Hidden surface removal
  - Antialiasing



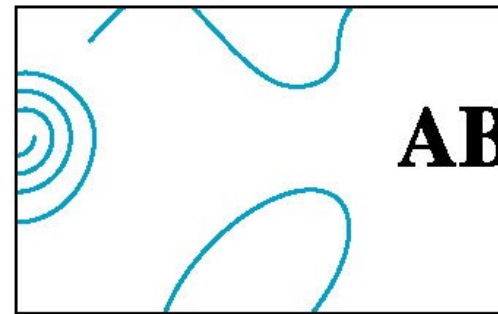
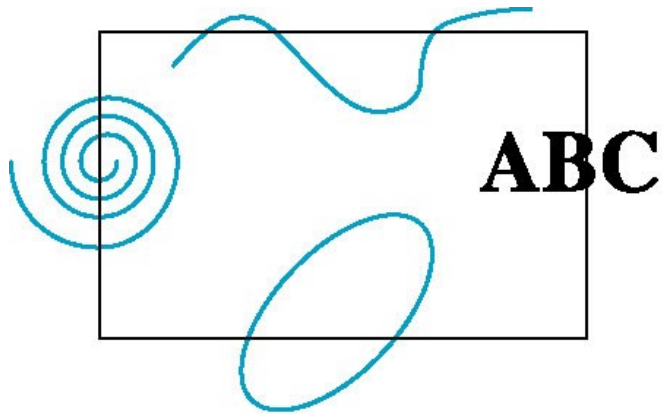
# Clipping



# Clipping

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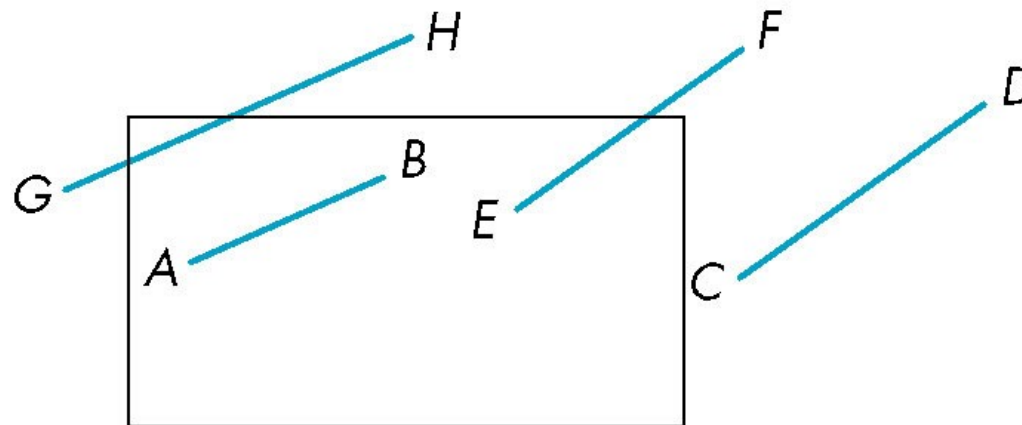
- 2D against clipping window
- 3D against clipping volume
- Easy for line segments polygons
- Hard for curves and text
  - Convert to lines and polygons first



# Clipping 2D Line Segments

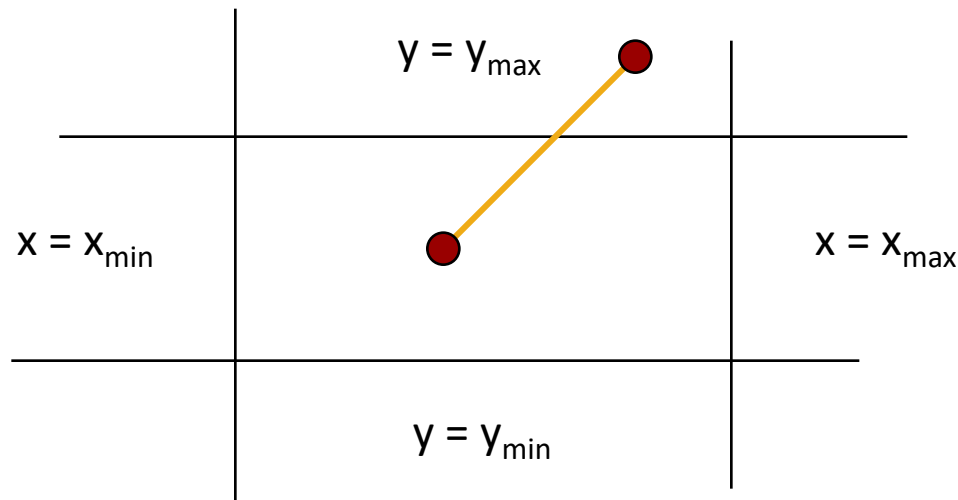
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- Brute force approach: compute intersections with all sides of clipping window
  - Inefficient: one division per intersection



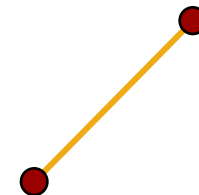
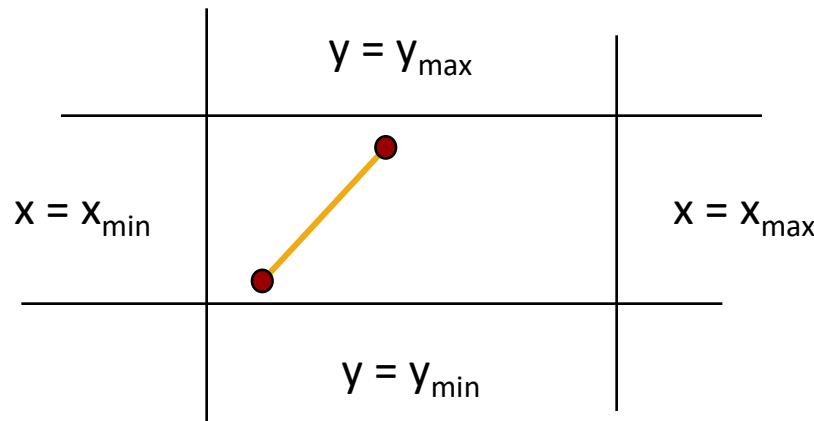
# Cohen-Sutherland Algorithm

- Idea: eliminate as many cases as possible without computing intersections
- Start with four lines that determine the sides of the clipping window



# The Cases

- Case 1: both endpoints of line segment inside all four lines
  - Draw (accept) line segment as is
- Case 2: both endpoints outside all lines and on same side of a line
  - Discard (reject) the line segment

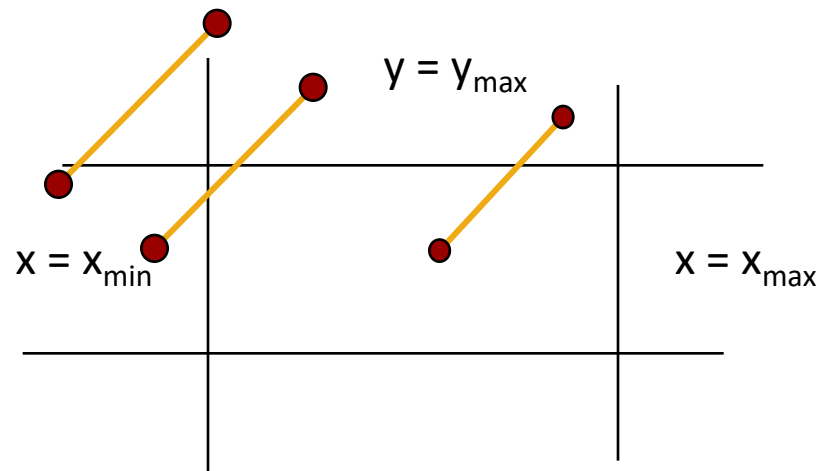




# The Cases

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- Case 3: One endpoint inside, one outside
  - Must do at least one intersection
- Case 4: Both outside
  - May have part inside
  - Must do at least one intersection



# Defining Outcodes

- For each endpoint, define an outcode

$$b_0 b_1 b_2 b_3$$

$b_0 = 1$  if  $y > y_{\max}$ , 0 otherwise

$b_1 = 1$  if  $y < y_{\min}$ , 0 otherwise

$b_2 = 1$  if  $x > x_{\max}$ , 0 otherwise

$b_3 = 1$  if  $x < x_{\min}$ , 0 otherwise

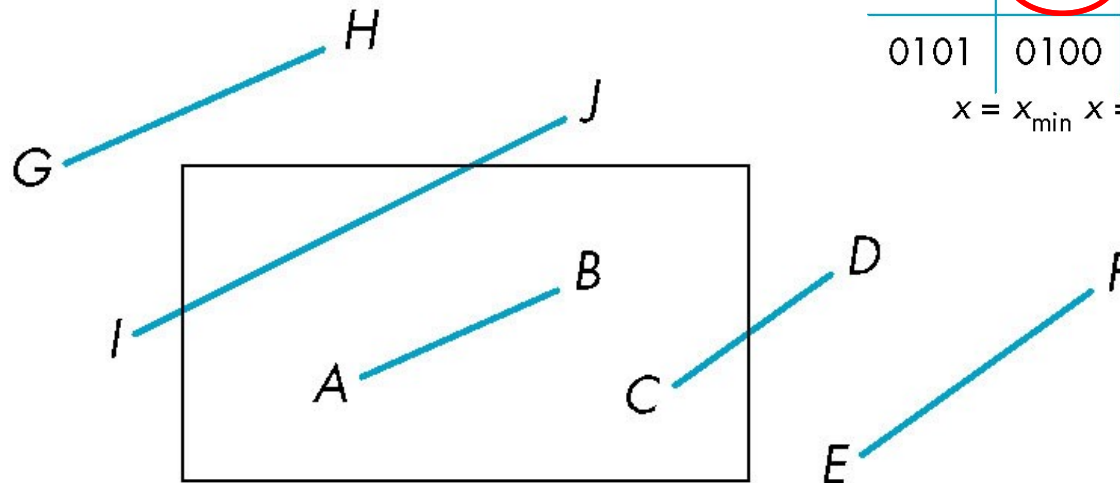
1001	1000	1010	$y = y_{\max}$
0001	0000	0010	
0101	0100	0110	$y = y_{\min}$
$x = x_{\min}$		$x = x_{\max}$	

- Outcodes divide space into 9 regions
- Computation of outcode do not require fp division

# Using Outcodes

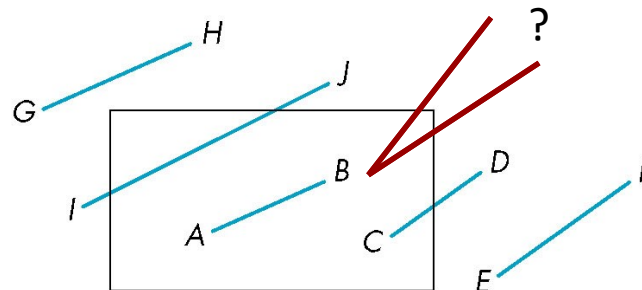
- Consider the 5 cases below
- AB:  $\text{outcode}(A) = \text{outcode}(B) = 0$ 
  - Accept line segment

1001	1000	1010	$y = y_{\max}$
0001	0000	0010	
0101	0100	0110	$y = y_{\min}$
$x = x_{\min}$		$x = x_{\max}$	



# Using Outcodes

- CD: outcode (C) = 0, outcode(D)  $\neq 0$ 
  - Compute intersection
  - Location of 1 in outcode(D) determines which edge to intersect with
  - Note if there were a segment from C to a point in a region with 2 ones in outcode, we might have to do two intersections



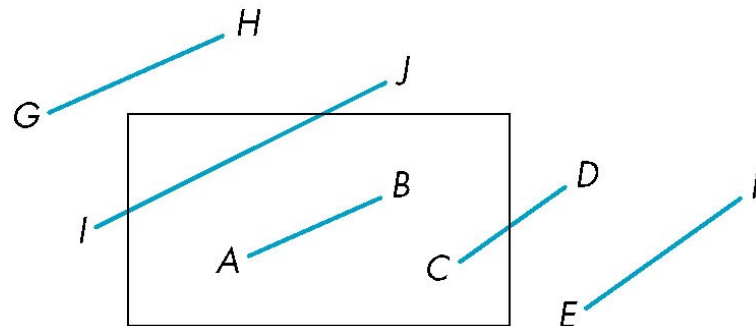
1001	1000	1010	$y = y_{\max}$
0001	0000	0010	
0101	0100	0110	$y = y_{\min}$
$x = x_{\min}$		$x = x_{\max}$	



# Using Outcodes

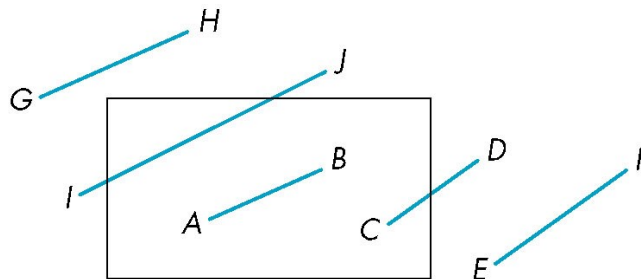
- EF: both non-zero, AND is non-zero
  - Both outcodes have a 1 bit in the same place
  - Line segment is outside of corresponding side of clipping window
  - reject

1001	1000	1010	$y = y_{\max}$
0001	0000	0010	
0101	0100	0110	$y = y_{\min}$
$x = x_{\min}$		$x = x_{\max}$	



# Using Outcodes

- GH and IJ: both non-zero, AND is zero
- Shorten line segment by intersecting with one of sides of window
- Compute outcode of intersection (new endpoint of shortened line segment)
- Re-execute algorithm



1001	1000	1010	$y = y_{\max}$
0001	0000	0010	
0101	0100	0110	$y = y_{\min}$
$x = x_{\min}$		$x = x_{\max}$	

# Example

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1001	1000	1010	$y = y_{\max}$
0001	0000	0010	
0101	0100	0110	$y = y_{\min}$
$x = x_{\min} \quad x = x_{\max}$			

# Efficiency

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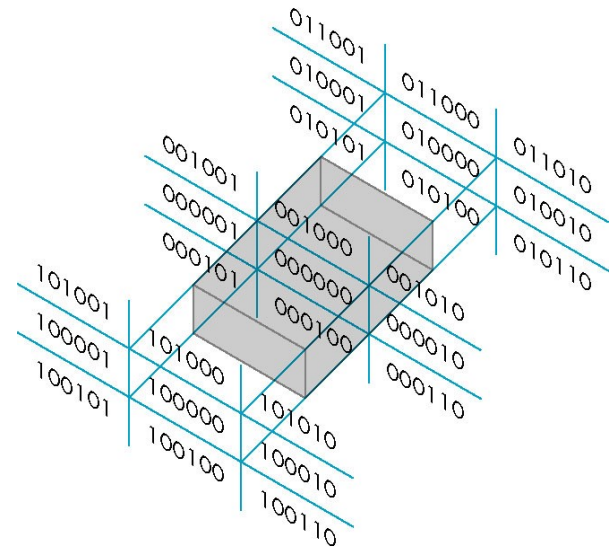
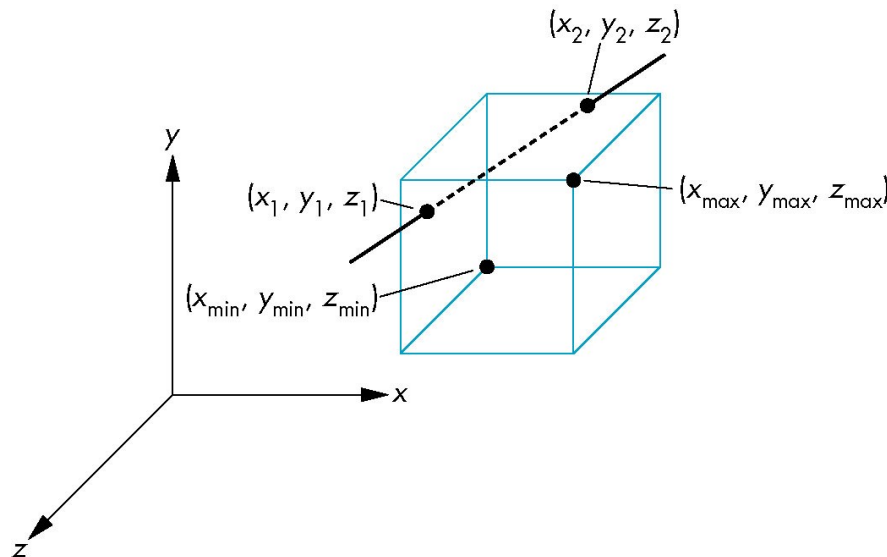
- In many applications, the clipping window is small relative to the size of the entire data base
  - Most line segments are outside one or more side of the window and can be eliminated based on their outcodes
- Inefficiency when code has to be reexecuted for line segments that must be shortened in more than one step



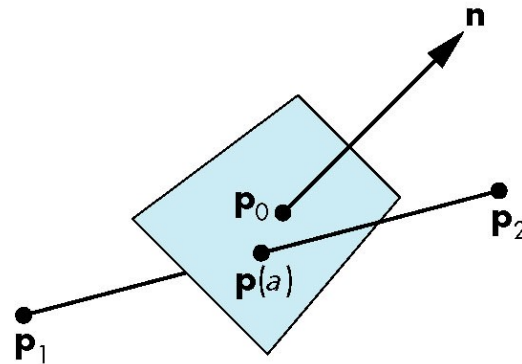


# Cohen Sutherland in 3D

- Use 6-bit outcodes
- When needed, clip line segment against planes



# Plane-Line Intersections



$$p(\alpha) = (1 - \alpha)p_1 + \alpha p_2$$

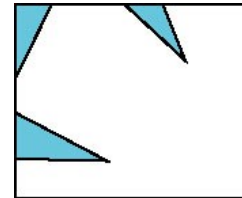
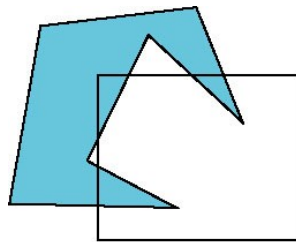
$$n \cdot (p(\alpha) - p_o) = 0$$

$$\alpha = \frac{n \cdot (p_o - p_1)}{n \cdot (p_2 - p_1)}$$

# Polygon Clipping

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- Not as simple as line segment clipping
  - Clipping a line segment yields at most one line segment
  - Clipping a concave polygon can yield multiple polygons

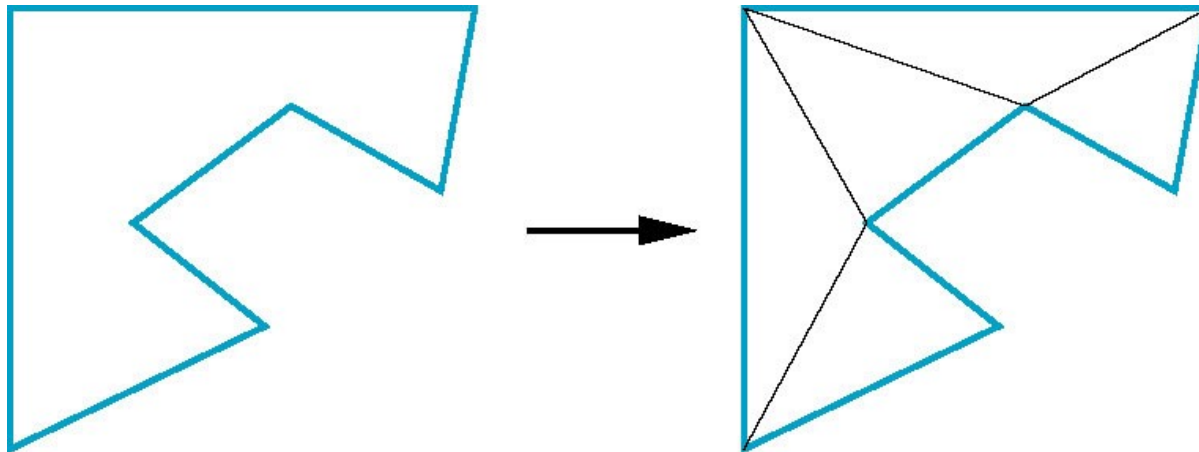


- However, clipping a convex polygon can yield at most one other polygon

# Tessellation and Convexity

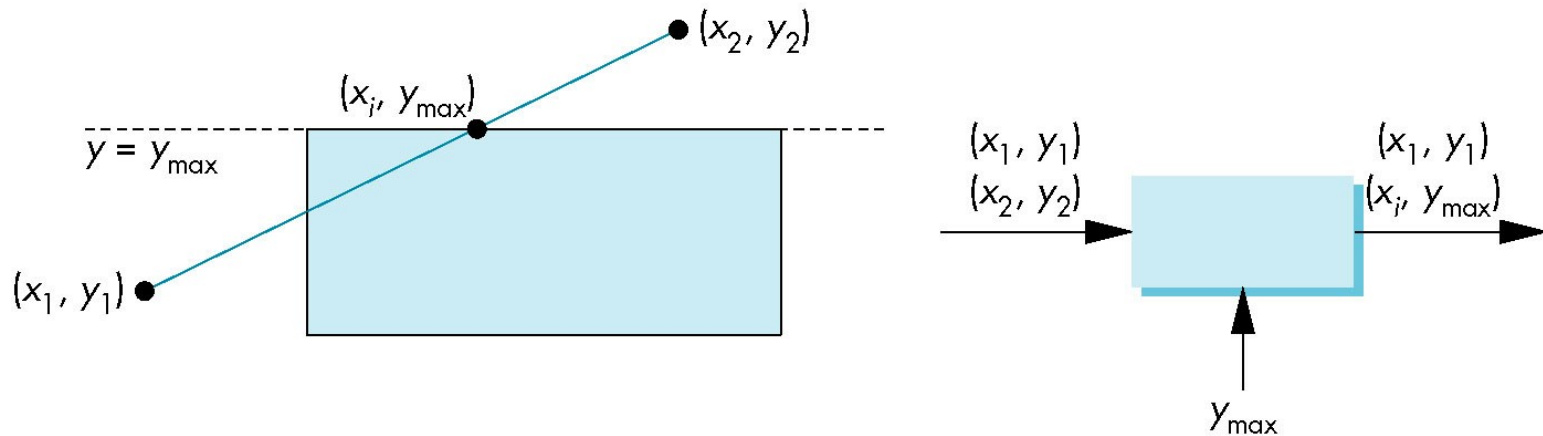
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- One strategy is to replace nonconvex (concave) polygons with a set of triangular polygons (a tessellation)
- Also makes fill easier
- Tessellation code in GLU library



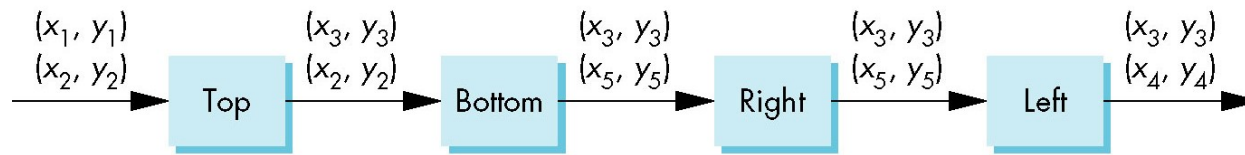
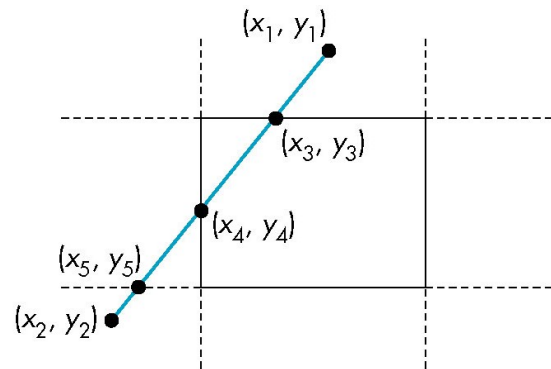
# Clipping as a Black Box

- Can consider line segment clipping as a process that takes in two vertices and produces either no vertices or the vertices of a clipped line segment



# Pipeline Clipping of Line Segments

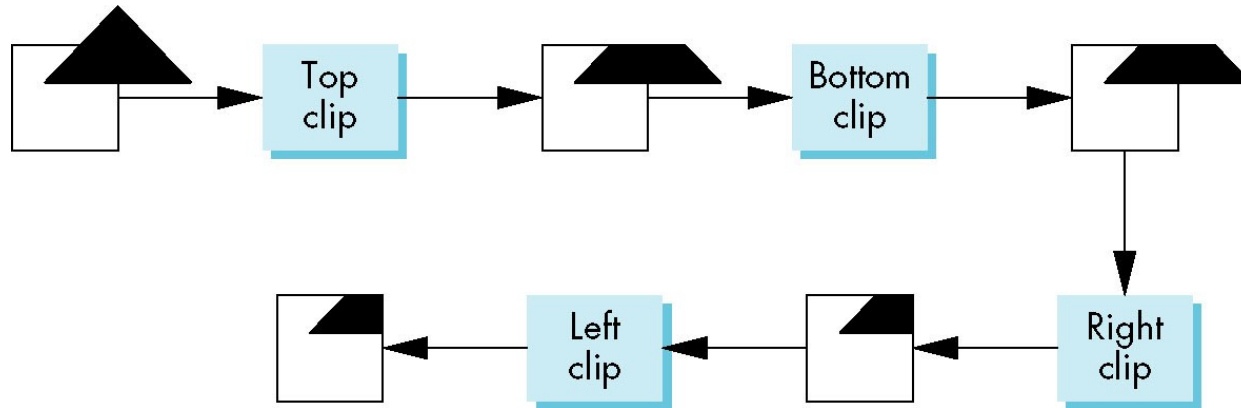
- Clipping against each side of window is independent of other sides
  - Can use four independent clippers in a pipeline



Sutherland & Hodgeman Algorithm



# Pipeline Clipping of Polygons

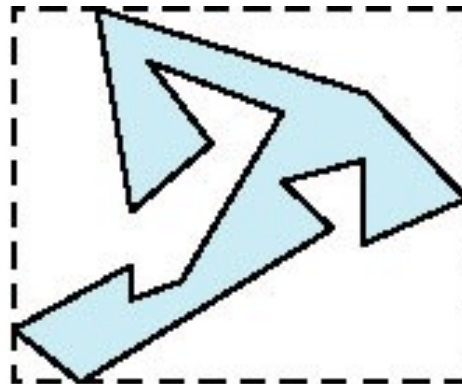


- Three dimensions: add front and back clippers
- Strategy used in SGI Geometry Engine
- Small increase in latency

# Bounding Boxes

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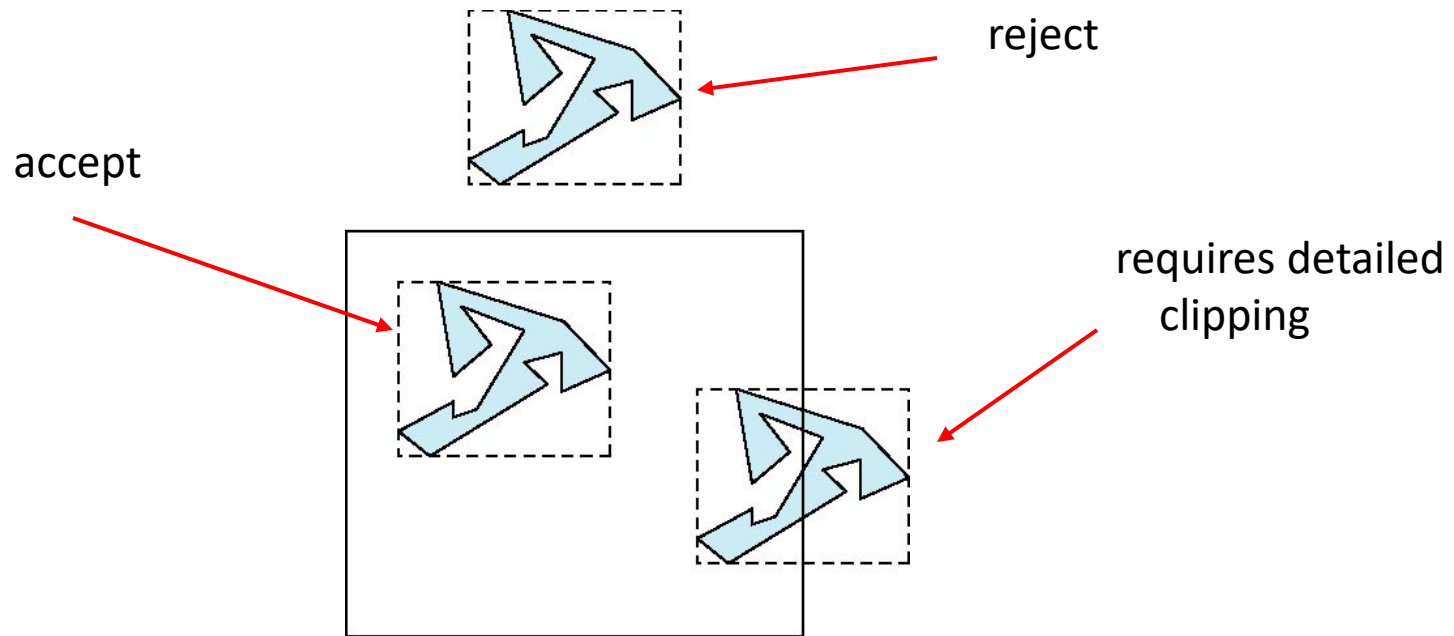
- Rather than doing clipping on a complex polygon, we can use an *axis-aligned bounding box* or *extent*
  - Smallest rectangle aligned with axes that encloses the polygon
  - Simple to compute: max and min of x and y





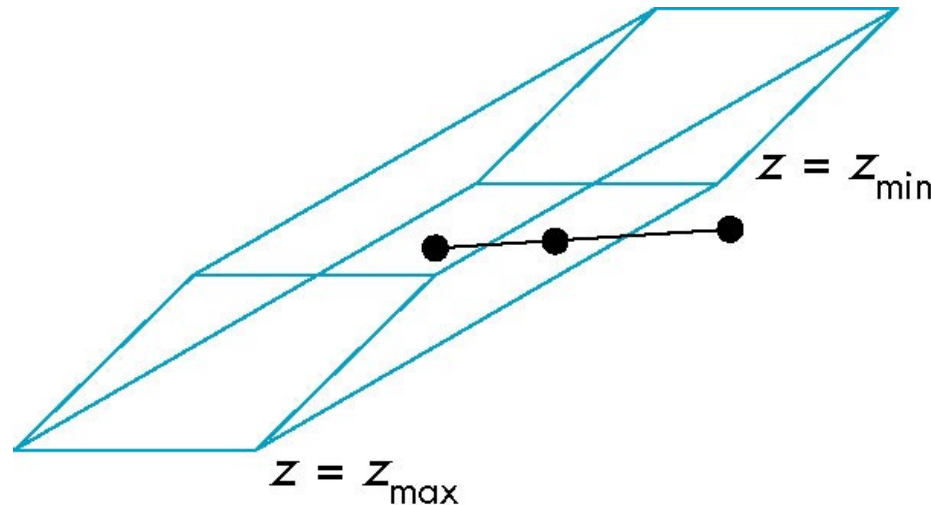
# Bounding Boxes

- Can usually determine accept/reject based only on bounding box

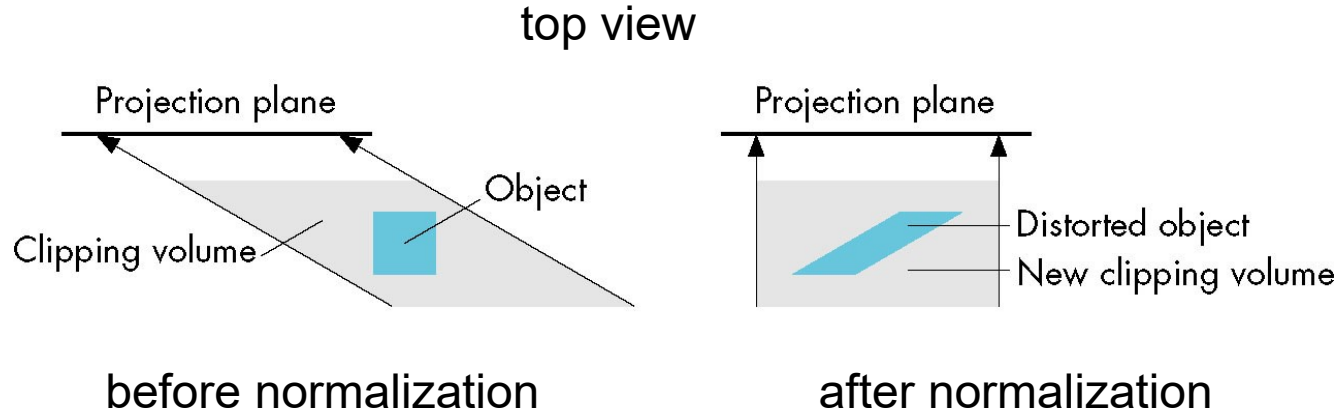


# Clipping and Normalization

- General clipping in 3D requires intersection of line segments against arbitrary plane
- Example: oblique view



# Normalized Form



Normalization is part of viewing (pre clipping)  
but after normalization, we clip against sides of  
right parallelepiped

Typical intersection calculation now requires only  
a floating point subtraction, e.g. is  $x > x_{\max}$  ?

# Projection Matrix

- Orthogonal Projection
  - Near, far, left, right, top, bottom are w.r.t eye coordinate

$$M_p = ST = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bottom} & 0 & -\frac{top + bottom}{top - bottom} \\ 0 & 0 & -\frac{2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# Projection Matrix

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- Perspective Projection
  - Near, far, left, right, top, bottom are w.r.t eye coordinate

$$\begin{pmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$



# Clipping in Homogeneous Coordinate

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- Clipping happens in *clip coordinate*
- Why not clipping in normalized device coordinate?
  - $(x, y, z, w) \rightarrow (x/w, y/w, z/w) \rightarrow (\pm 1, \pm 1, \pm 1)$
  - What if  $w = 0$ ?



# Clipping in Homogeneous Coordinate

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- Clipping plane:  $x=\pm w, y=\pm w, z=\pm w$
- Example: clip to  $x=-w$  (left)
  - Homogeneous coordinate:  $x/w=-1$
  - Homogeneous plane:  $w + x = 0$
  - Intersection between line  $P_1P_2$  and plane  $x+w=0$ 
    - $P=(1-u)P_1 + uP_2$  on  $x+w=0$
    - $[(1-u)w_1+uw_2]+[(1-u)x_1+ux_2]=0$
    - $u=(w_1+x_1)/\{(w_1+x_1)-(w_2+x_2)\}$



# Questions?

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