



Lecture 6: OpenGL Transformation

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Outline

- OpenGL transformation
- Virtual trackball



Outline

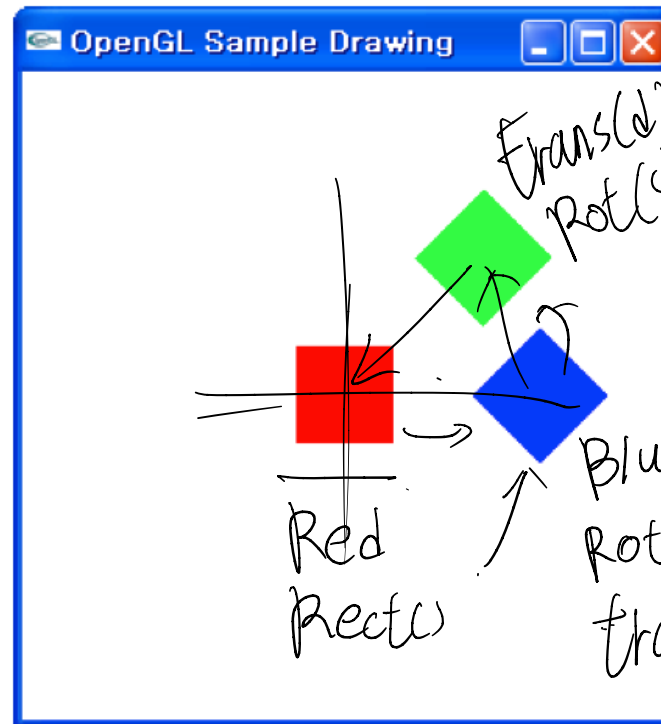
- OpenGL transformation
- Virtual trackball



Modeling Transformation \Rightarrow object transform.

= Object.

- Local to global coordinate transform
- Rotate, Translate, Scale



이렇게 하고

- Global to 3D viewing coordinate transform

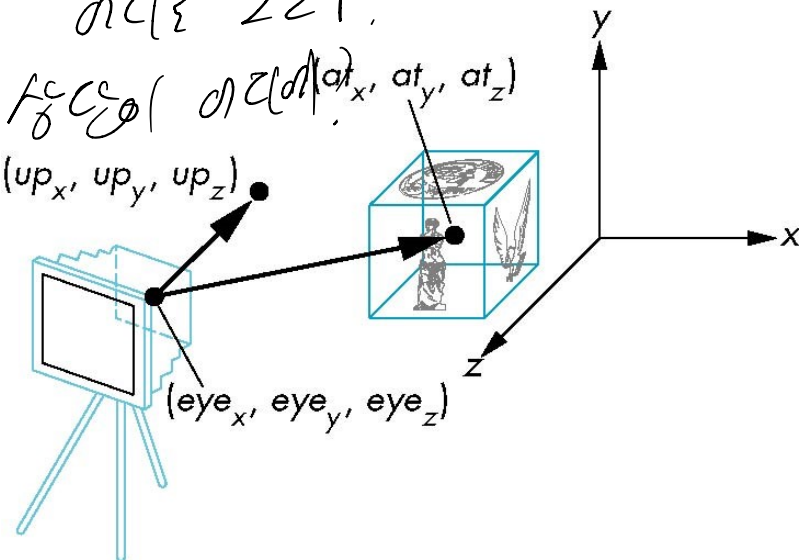
- Set eye position and viewing direction

- LookAt($eyex/y/z$, $atx/y/z$, $upx/y/z$)

- eye x/y/z : eye position (x,y,z) \rightarrow $\frac{1}{2}$ of $|\sigma_z|$?

- at x/y/z : viewing direction

– up x/y/z : up vector $u = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ at_x, at_y, at_z



2012년 12월 20일

하루의 마음과 영혼을 위한 기도

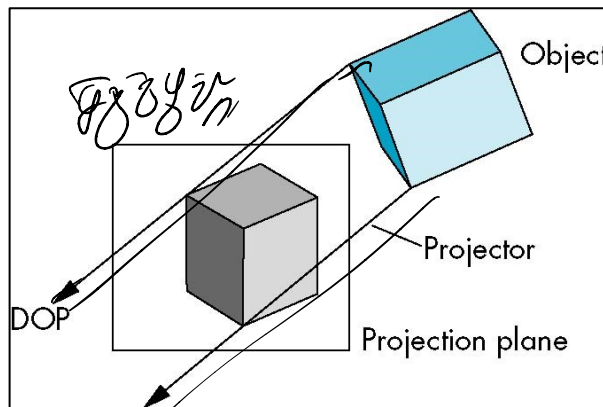
2020년 1월 20일

Projection Transformation

P. V. M.
↓ ↓ ↓
프로젝션 뷰 모델
↓
오브젝트

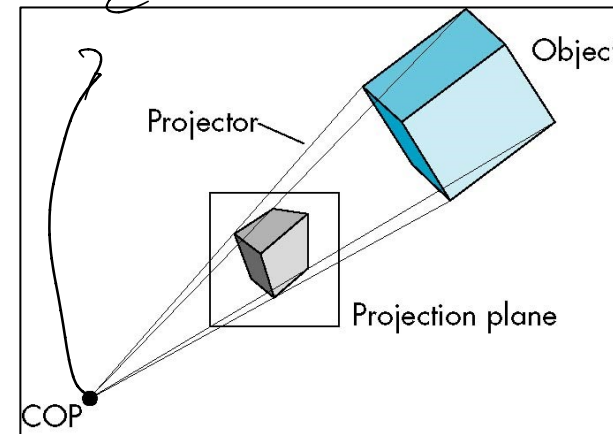
- 3D to 2D viewing coordinate transform
- Define clipping volume ↳ 뷰가 보여야 함,
뷰가 안 보여야 함인가?
- Projection types

- Orthogonal 직교 = 수직 (가장)
- Perspective 원근 = 관상각, 한 지점 모음.



Orthogonal projection

직교



Perspective projection

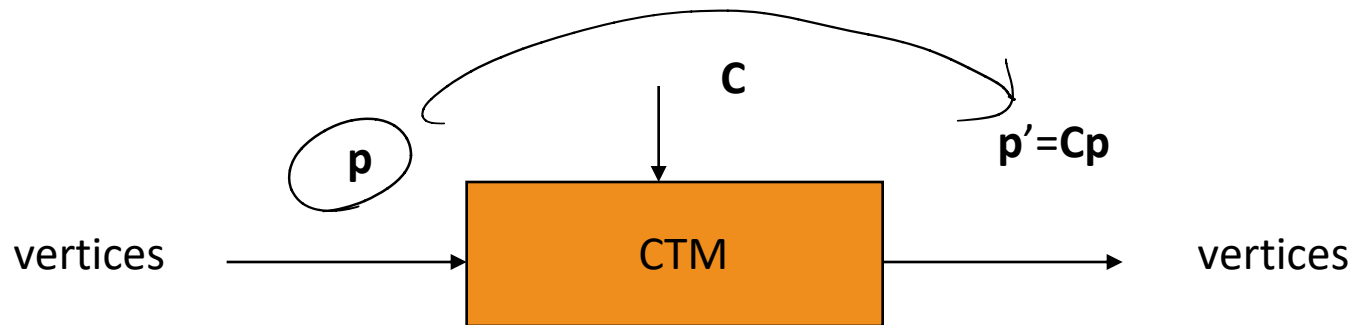
원근

Current Transformation Matrix

CTM

- Conceptually there is a 4 x 4 homogeneous coordinate matrix, the *current transformation matrix* (CTM) that is part of the state and is applied to all vertices that pass down the pipeline
- The CTM is defined in the user program and loaded into a transformation unit

변환 행렬 (p, M, V 가 정해짐)



CTM Operations

General Graphics.
OpenGL.

Load an identity matrix: $\mathbf{C} = \mathbf{I}$

Load an arbitrary matrix: $\mathbf{C} = \mathbf{M}$

Load a translation matrix: $\mathbf{C} = \mathbf{T}$

Load a rotation matrix: $\mathbf{C} = \mathbf{R}$

Load a scaling matrix: $\mathbf{C} = \mathbf{S}$

Postmultiply by an arbitrary matrix: $\mathbf{C} = \mathbf{CM}$ Eq 27.26, $C = C \cdot M$

Postmultiply by a translation matrix: $\mathbf{C} = \mathbf{CT}$

Postmultiply by a rotation matrix: $\mathbf{C} = \mathbf{CR}$

Postmultiply by a scaling matrix: $\mathbf{C} = \mathbf{CS}$

CTM

↓

$C = C \cdot M$



Matrix Order is Reversed

- Example: Rotation about a fixed point

- Start with identity matrix: $\mathbf{C} = \mathbf{I}$

- Move fixed point to origin: $\mathbf{C} = \mathbf{C}\mathbf{T} = \mathbf{T}$.

- Rotate: $\mathbf{C} = \mathbf{C}\mathbf{R} = \mathbf{T} \cdot \mathbf{R}$.

- Move fixed point back: $\mathbf{C} = \mathbf{C}\mathbf{T}^{-1} = \mathbf{T} \cdot \mathbf{R} \cdot \mathbf{T}^{-1}$.

- Result: $\mathbf{C} = \mathbf{T}\mathbf{R}\mathbf{T}^{-1}$ which is **backwards!** 뒤집혀있다!

$$\overset{\mathbf{I}}{(\mathbf{C} \cdot \mathbf{T})} \cdot \mathbf{R}$$

= 2번 뒤집

$$\Rightarrow (\mathbf{C} \cdot \mathbf{T} \cdot \mathbf{R}) \cdot \mathbf{T}^{-1} \Rightarrow \mathbf{C} = \mathbf{T} \cdot \mathbf{R} \cdot \mathbf{T}^{-1}$$



Correct Matrix Order

We want $\mathbf{C} = \mathbf{T}^{-1} \mathbf{R} \mathbf{T}$

so we must do the operations in the following order

$$\left(\begin{array}{l} \mathbf{C} = \mathbf{I} \\ \mathbf{C} = \mathbf{C} \mathbf{T}^{-1} \\ \mathbf{C} = \mathbf{C} \mathbf{R} \\ \mathbf{C} = \mathbf{C} \mathbf{T} \end{array} \right)$$

↑

Each operation corresponds to one function call in the program

한번씩씩씩

Note that **the last operation specified is the first executed** in the program



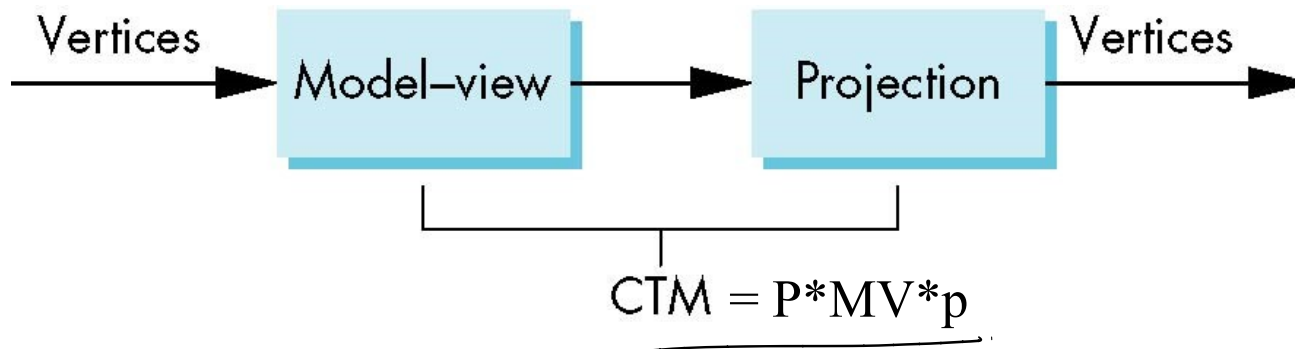
CTM in OpenGL with Shader

- The CTM in OpenGL is defined in the user program and passed down to the vertex processor
 - Vertex shader will multiply the matrix to vertex coordinates
- Programmers should create and manage them in their own applications
- Matrix / vector class can help
 - mat.h, vec.h (textbook source code)
 - Error corrected version of vector class (vec_fixed.h) is included in assign_1 skeleton code



ModelView/Projection Matrix

- In OpenGL, the **model-view** matrix is used to
 - Position the camera
 - Can be done by rotations and translations but is often easier to use the lookAt function in mat.h
 - Build models of objects
- The **projection** matrix is used to define the view volume and to select a camera lens



Basic Matrix Functions (mat.h)

- Create an identity matrix

```
mat4 m = Identity();
```

- Fill it with components

```
mat4 m = mat4(0,1,2,3,4,5,6,7,  
              8,9,10,11,12,13,14,15);
```

- By vectors

```
mat4 m = mat4( vec4(0,1,2,3),  
               vec4(4,5,6,7),  
               vec4(8,9,10,11),  
               vec4(12,13,14,15) );
```



Rotation, Translation, Scaling

- Multiply on right by rotation matrix of **theta** in degrees where (**vx**, **vy**, **vz**) define axis of rotation:

```
mat4 r = Rotate(theta, vx, vy, vz)
m = m * r
```

- Also have rotateX, rotateY, rotateZ.
- Do same with translation and scaling:

```
mat4 s = Scale(sx, sy, sz)
mat4 t = Translate(dx, dy, dz);
m = m*s*t;
```



Rotation about A Fixed Point using mat.h

- Fixed point: (4, 5, 6)
- Rotation angle: 45 degrees
- Rotation axis: the line through the origin and the point (1, 2, 3) $(0, 0, 0)$
- Remember that last matrix specified in the program is the first applied

```
mat4 m = Identity();
m = Translate(4.0, 5.0, 6.0) *
    Rotate(45.0, 1.0, 2.0, 3.0) *
    Translate(-4.0, -5.0, -6.0);
```



How to pass matrix to shader?

- GL Shader

```
#version 150
in  vec4 vPosition;
in  vec4 vColor;
out vec4 color;
uniform mat4 model_view;
uniform mat4 projection;
void main()
{
    gl_Position = projection*model_view*vPosition;
    color = vColor;
}
```



How to pass matrix to shader?

- User code (C++)

```
void display( void )
{
    glClear( GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT );
    point4  eye( 0, 0, -100, 1.0 );
    point4  at( 0.0, 0.0, 0.0, 1.0 );
    vec4     up( 0.0, 1.0, 0.0, 0.0 );
    mat4     mv = LookAt( eye, at, up );
    glUniformMatrix4fv( model_view, 1, GL_TRUE, mv );
    mat4     p = Perspective( fovy, aspect, zNear, zFar );
    glUniformMatrix4fv( projection, 1, GL_TRUE, p );
    glDrawArrays( GL_TRIANGLES, 0, NumVertices );
    glutSwapBuffers();
}
```



Outline

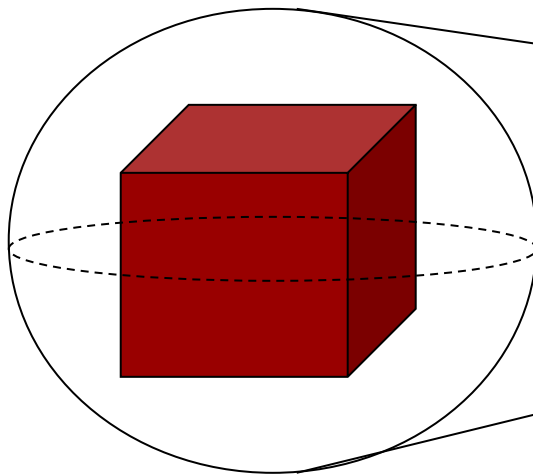
- OpenGL transformation
- Virtual trackball



3D Rotations with Trackball

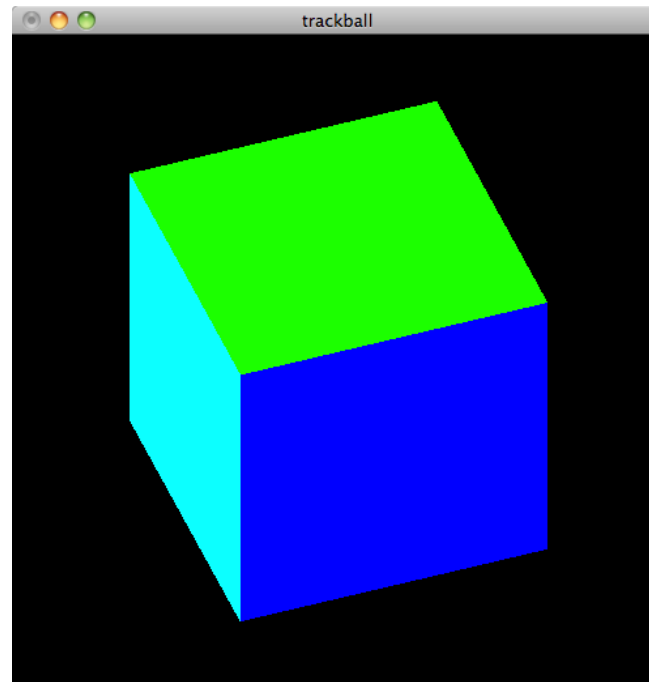
- Imagine the objects are rotated along with a imaginary hemi-sphere

예제가 보듯. 3D 객체의 2D 투영된 모습이다.



Virtual Trackball \Rightarrow 마우스로 3D 회전 가능.

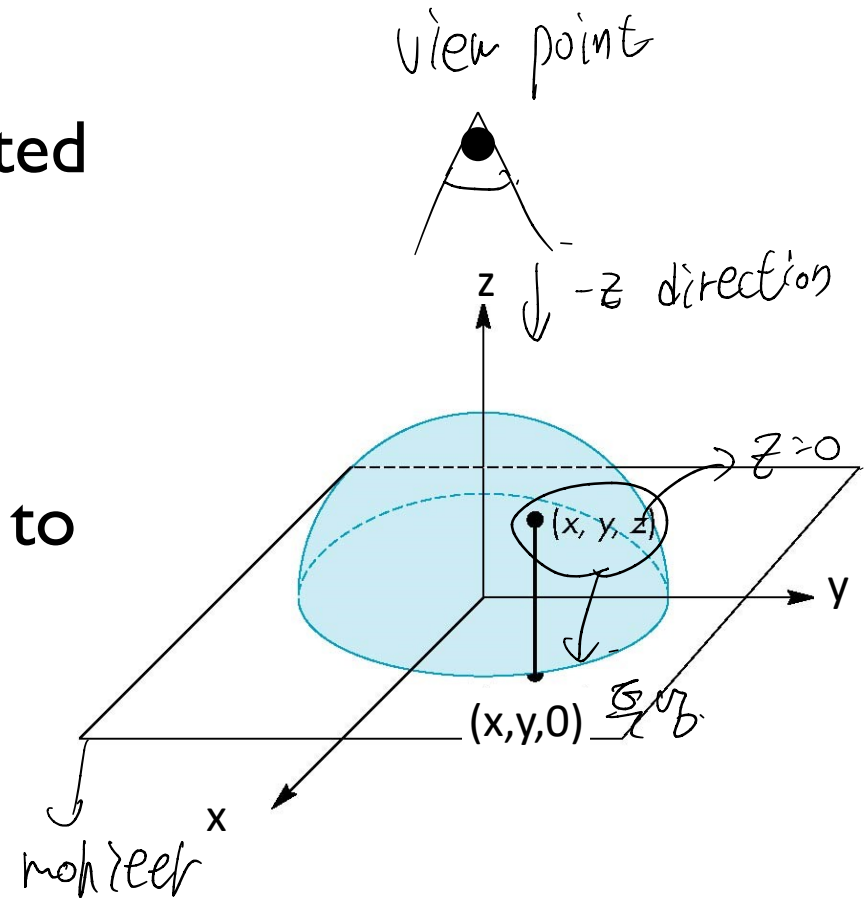
- Allow the user to define 3D rotation using mouse click in 2D windows
- Work similarly like the (hardware trackball) devices



Virtual Trackball

- Superimpose a hemi-sphere onto the viewport
- This hemi-sphere is projected to a circle inscribed to the viewport
- The mouse position is projected orthographically to this hemi-sphere

half - sphere



2차원 점 (x, y) 를 3차원 점 $(x, y, 0)$ 으로 변환

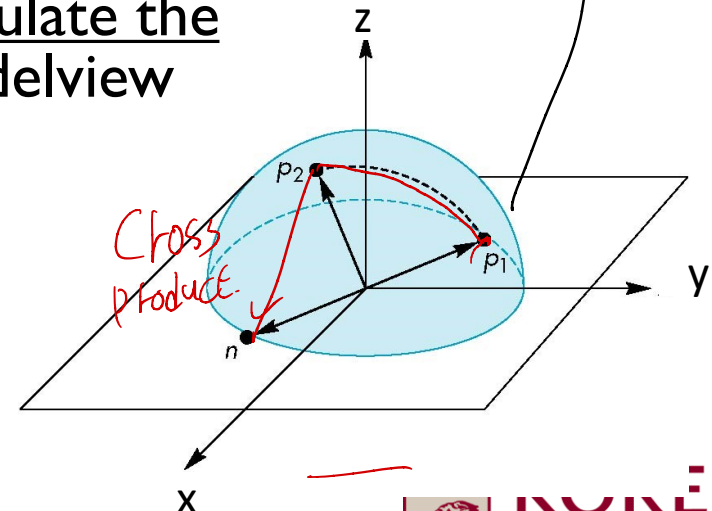
Virtual Trackball

- Click! →
- Keep track the previous mouse position and the current position
 - Calculate their projection positions (p_1 and p_2) to the virtual hemi-sphere
 - We then rotate the sphere from p_1 to p_2 by finding the proper rotation axis and angle
 - You should also remember to accumulate the current rotation to the previous modelview matrix

클릭한 지점의 위치를
반경 구에 맞춰
유사가능.

estimate

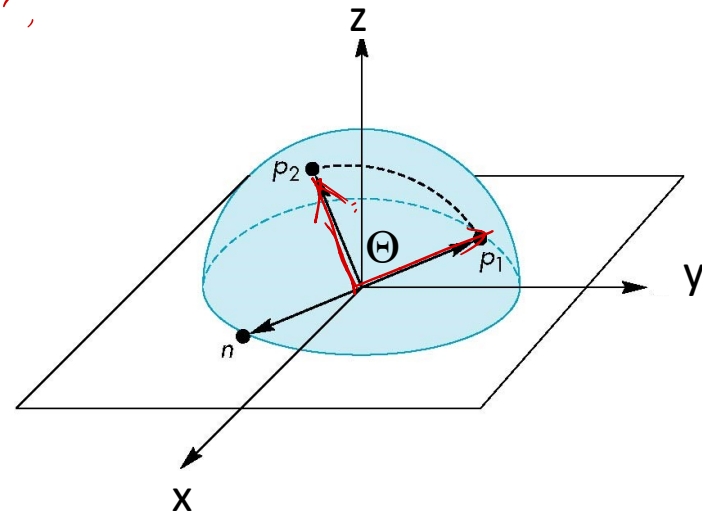
p_1, p_2 를 가라앉히는 거임.



Virtual Trackball

- The axis of rotation is given by the normal to the plane determined by the origin, p_1 , and p_2

$$\mathbf{n} = \mathbf{p}_1 \times \mathbf{p}_2$$

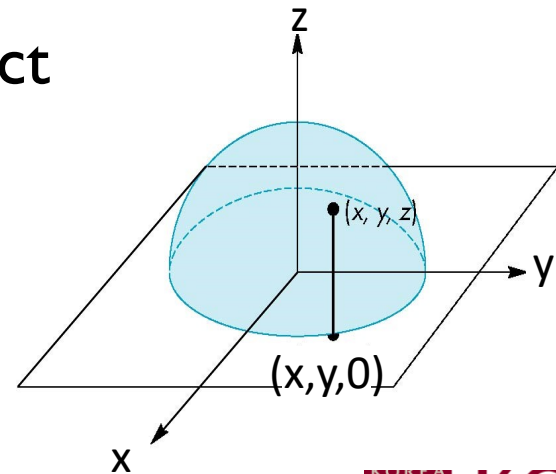


Virtual Trackball

- How to calculate p1 and p2?
- Assuming the mouse position is (x,y) , then the sphere point P also has x and y coordinates equal to x and y
- Assume the radius of the hemi-sphere is r . Then the z coordinate of P is

$$\sqrt{r^2 - x^2 - y^2}$$

- If a point is outside the circle, project it to the nearest point on the circle (set z to 0 and renormalize (x,y))



glut Callback Functions

- glutMouseFunc(mouseButton)
 - Mouse click (UP/DOWN)

Handwritten notes: "마우스 버튼" (mouse button) with an arrow pointing to mouseButton; "콜백 함수" (callback function) with an arrow pointing to the function name; "UP/DOWN" with "올라" (up) and "내려" (down) written below it.
- glutMotionFunc(mouseMotion)
 - Mouse move / *detect*.
 - You need to count only when mouse is moving while mouse button is pressed

Mouse Callback Example

own func

```
void mouseButton(int button, int state, int x, int y)
{
    if (button==GLUT_LEFT_BUTTON) switch(state)
    {
        case GLUT_DOWN:
            startMotion(x,y);
            break;
        case GLUT_UP:
            stopMotion(x,y);
            break;
    }
}
```



glutMotionFunc Example

```

Void mouseMotion(int x, int y)
{
    float curPos[3],
    dx, dy, dz;

    /* compute position on hemisphere */
    trackball_ptov(x, y, winWidth, winHeight, curPos);

    if(trackingMouse)
    {
        /* compute the change in position
           on the hemisphere */
        dx = curPos[0] - lastPos[0];
        dy = curPos[1] - lastPos[1];
        dz = curPos[2] - lastPos[2];
    }
    :
}

```

Handwritten notes in red:

- Under `trackball_ptov`: `2D` →
- Under `winWidth`: `3D` →
- Under `winHeight`: `2, y, z`
- Next to the position change calculations: `dx, dy, dz` (circled)

Vertical ellipsis (three dots) indicating continuation of code.

Handwritten notes in red:

- 마지막에
- 무엇을
- 추가할
- 것인가
- 고민



glutMotionFunc Example

```
void trackball_ptov(int x, int y, int width, int height,
    float v[3])
{
    float d, a;

    /* project x,y onto a hemi-sphere centered within width,
    height */
    v[0] = (2.0F*x - width) / width;
    v[1] = (height - 2.0F*y) / height;
    d = (float) sqrt(v[0]*v[0] + v[1]*v[1]);
    v[2] = (float) cos((M_PI/2.0F) * ((d < 1.0F) ? d : 1.0F));
    a = 1.0F / (float) sqrt(v[0]*v[0] + v[1]*v[1] + v[2]*v[2]);
    v[0] *= a;
    v[1] *= a;
    v[2] *= a;
}
```



glutMotionFunc Example

⋮

```
if (dx || dy || dz)
{
```

```
    /* compute theta and cross product */
```

```
    angle = 90.0 * sqrt(dx*dx + dy*dy + dz*dz);
```

```
    axis[0] = lastPos[1]*curPos[2] -  
              lastPos[2]*curPos[1];
```

```
    axis[1] = lastPos[2]*curPos[0] -  
              lastPos[0]*curPos[2];
```

```
    axis[2] = lastPos[0]*curPos[1] -  
              lastPos[1]*curPos[0];
```

```
    /* update position */
```

```
    lastPos[0] = curPos[0];
```

```
    lastPos[1] = curPos[1];
```

```
    lastPos[2] = curPos[2];
```

```
}
```

```
}
```

```
glutPostRedisplay();
```

```
}
```

→ cross product
P₁ × P₂



Update Rotation Matrix

- Order is important! *→ revers.*
- rot * cmt for right-side multiplication

우에서 좌로.

*타는 1221
← ←
cmt * rot * ?*

```
mat4 cmt; // current matrix
```

...

```
mat4 rot = Rotate(alpha, vx, vy, vz);
```

```
cmt = rot*cmt;
```



Notes about Glew

glew

- glewInit() must be called before any OpenGL command and after glutDisplayFunc()

```
int main(int argc, char **argv)
{
    glutInit(&argc, argv)
    glutInitDisplayMode(...)
    ...
    glutCreateWindow(...)

    glewInit(); ←
    glutMainLoop();
    return 1;
}
```



glew

- glew.h must be included before glut.h

```
#include <GL/glew.h>
```

```
#include <GL/glut.h>
```

```
....
```



glutPostRedisplay() \Rightarrow refresh screen

- Update framebuffer
 - Call display callback function registered by `glutDisplayFunc()`
- Do not call `glutPostRedisplay()` inside display callback function! (= `glut display func`)
 - Infinite loop
- Call when you need to manually refresh screen
 - After mouse or keyboard events



glutSwapBuffers() / double buffering

- Swap back and front buffers
- Render target must be back buffer [✓] double check!
 - `glDrawBuffer(GL_BACK)`
- Call only once per each render pass
 - Only at the end of the display callback function



Questions?



Refraction Effect using Vertex Shader (Wikipedia)