확률 및 랜덤과정_23년 2학기_ 중간고사_정원주

1. There are three coins in your pocket, two fair coins coin(P(H)=1/2) and one biased coin with P(H)=1/4. You pick one randomly and flip it.

(a) what is the probability that you see a Head?
(b) When you observed a Head, What is the probability that you picked the fair coin?
(c) When you observed a Head, what is the probability that the coin will land head again if you toss it a second time.

2. There are 100 guests at a party wearing hats. The host keeps the hats and gives them back to the guests at random. What is the average of guests who get their own hat back?

3. Consider a random variable X with the following PDF

$f(x) = xe^{-\frac{x^2}{2}}$ for x>0 and f(x) = 0 else

(a) Find CDF of X
(b) Let $Y = 1 - e^{-\frac{x^2}{2}}$. Find the mean of Y.
(c) Find the mean of X. (Suggestion for an easy way, use the fact that the variance of standard Gaussian is 1)

4. There is a coin with unknown bias. Knowing that the number of heads are 3 in cousecutive 5 flips. Find the probability that the first three flips are heads.

5. Consider i.i.d. Bernoulli trials X1, X2, ... with Xi ~ Bern(1/3) and Y1, Y2, ... with Yi ~ Bern(1/2). Assuming Xi and Yi are independent. Find the probability that their first successes are simultaneous.

6. Let U ~ Unif(0,1) and X=min(U,1-U). Find the PDF of X.

7. Let X and Y be i.i.d. exponential random variables with λ = 1.
Let L = max(X,Y) - min(X,Y). Find the mean of L.

8. Let X be a random variable with the MGF Mx(t).

(a) When Mx(t) = $e^{at} + b$ form, determine b.
(b) Find the 2023th moment of X, when Mx(t) = $e^{at} + b$
(c) Find the variance of X, when Mx(t) = e^{at^2+bt}