HW 1 Solution

1. Let S be the event that a man in U.S. smokes
   L be the event that a man get lung Cancer

$$P(S|L) = \frac{P(L|S)P(S)}{P(L|S)P(S) + P(L|S^c)P(S^c)}$$

we have $P(L|S) = 23\,P(L|S^c)$

$$= \frac{P(L|S)\,P(S)}{P(L|S)\,P(S) + \frac{1}{23}P(L|S)\,P(S^c)} = \frac{P(S)}{P(S) + \frac{1}{23}P(S^c)}$$

$$= \frac{0.216}{0.216 + (1-0.216)/23} \simeq 0.864 \quad \text{(don't smoke!)}$$

2.

a) $$P(K|R) = \frac{P(R|K)P(K)}{P(R|K)P(K) + P(R|K^c)P(K^c)} \quad \left(\begin{array}{l} P(R|K)=1 \\ P(R|K^c)=\frac{1}{n} \end{array}\right.$$

$$= \frac{p}{p + \frac{1-p}{n}}$$

b) $$p + (1-p)/n \le p + 1 - p = 1$$

Since $p + (1-p)/n \le 1$, $P(K|R) \ge p$

$p + (1-p)/n = 1$ when $p = 1$ or $n = 1$, HW 1

3. Let A be the event that the chosen coin lands H all 7 times, and B be the event that the chosen coin is double-headed.

$$P(B|A) = \frac{\overset{1}{P(A|B)}\,\overset{\frac{1}{100}}{P(B)}}{\underbrace{P(A|B)}_{1}\,\underbrace{P(B)}_{\frac{1}{100}} + \underbrace{P(A|B^c)}_{\frac{1}{2^7}}\,\underbrace{P(B^c)}_{\frac{99}{100}}} = \frac{128}{227}$$

4.

Let A be the event that the chosen coin lands H all 7 times

D be the event that one of 100 coins is double Headed

C be the event that the chosen one is double headed

a) $$P(D|A) = \frac{P(A|D)P(D)}{P(A|D)P(D) + P(A|D^c)P(D^c)}$$

$$P(D) = P(D^c) = \frac{1}{2} \qquad P(A|D^c) = \frac{1}{2^7}$$

$$P(A|D) = \underbrace{P(A|D,C)}_{=1}\ \underbrace{P(C|D)}_{=\frac{1}{100}} + \underbrace{P(A|D,C^c)}_{=\frac{1}{2^7}}\ \underbrace{P(C^c|D)}_{=\frac{99}{100}}$$

$$= \frac{1}{100} + \frac{1}{2^7} \cdot \frac{99}{100}$$

$$P(D|A) = \frac{229}{329}$$

b) $$P(C|A) = P(C|A,D)P(D|A) + \underbrace{P(C|A,D^c)P(D^c|A)}_{=0}$$

$$P(C|A,D) = \frac{\overset{1}{P(A|C,D)}\ \overset{\frac{1}{100}}{P(C,D)}}{\underbrace{P(A|C,D)}_{1}\ \underbrace{P(C,D)}_{\frac{1}{100}} + \underbrace{P(A|C^c,D)}_{\frac{1}{2^7}}\ \underbrace{P(C^c,D)}_{\frac{99}{100}}}$$

$$= \frac{128}{229}$$

$$\therefore P(C|A) = \frac{128}{229} \cdot \frac{229}{329} = \frac{128}{329}$$

5\.

Let A be the event that initial marble is green

B be the even that taken out is green

C be the event that the remaining one is green

$$P(C \mid B) = \underbrace{P(C \mid B, A)}_{1} P(A \mid B) + \underbrace{P(C \mid B, A^c)}_{0} P(A^c \mid B)$$

$$= P(A \mid B)$$

$$P(A \mid B) = \frac{P(B \mid A) P(A)}{P(B)} = \frac{1 \cdot \frac{1}{2}}{P(B \mid A) P(A) + \underbrace{P(B \mid A^c)}_{\frac{1}{2}} \cdot \underbrace{P(A^c)}_{\frac{1}{2}}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{4}} = \frac{2}{3}$$ //

6\. Let M be the event that A's blood type matches the guilty one.

A be the event that A is guilty

B " " B "

a) $$P(A \mid M) = \frac{P(M \mid A) P(A)}{P(M \mid A) P(A) + P(M \mid B) P(B)} = \frac{1 \cdot \frac{1}{2}}{1 \cdot \frac{1}{2} + \frac{1}{10} \cdot \frac{1}{2}} = \frac{10}{11}$$

b) Let C be the event that B's blood type matches

$$P(C \mid M) = P(C \mid M, A) P(A \mid M) + P(C \mid M, B) P(B \mid M)$$

$$= \frac{1}{10} \cdot \frac{10}{11} + 1 \cdot \frac{1}{11} = \frac{2}{11}$$ //

7.

a) $P(A>B \mid A>C) = \dfrac{P(A>B, A>C)}{P(A>C)} = \dfrac{\frac{1}{3}}{\frac{1}{2}} = \dfrac{2}{3}$

b) dependent!

$P(A>B) = \frac{1}{2}$ but $P(A>B \mid A>C) = \frac{2}{3}$ //

8. Let W_i be the event of winning the i-th game $(i=1,2)$

a) $P(W_1) = \frac{1}{3}(0.9 + 0.5 + 0.3) = \dfrac{17}{30}$

b) $P(W_2 \mid W_1) = \dfrac{P(W_2 \cap W_1)}{P(W_1)}$

$P(W_2 \cap W_1) = \frac{1}{3}(0.9^2 + 0.5^2 + 0.3^2) = \dfrac{23}{60}$

$P(W_2 \mid W_1) = \dfrac{\frac{23}{60}}{\frac{17}{30}} = \dfrac{23}{34}$ //

9. Let C be the event that Calvin wins the game

a) Consider first two games

(C win, C win) → Calvin wins $\equiv C$

$\left.\begin{array}{l}\text{(C win, H win)} \\ \text{(H win, C win)}\end{array}\right\}$ game starts again

(H win, H win) → Calvin looses

$\therefore P(C) = \underbrace{P(C \mid \text{C wins only once among 2})}_{\text{starts over}} P(\text{C wins only once among 2})$

$+ P(\text{C win \& C win}) = 2p(1-p)P(C) + p^2$

$\therefore P(C) = \dfrac{p^2}{1-2p(1-p)}$ //

10.

$$P(B|E) = P\left(\dot{\bigcup_{k=1}^{n}} (B \cap A_k) \,|\, E\right) = \sum_{k=1}^{n} P(B \cap A_k) | E)$$

$$P(B \cap A_k | E) = \frac{P(B \cap A_k \cap E)}{P(E)}$$

$$= \frac{P(B | A_k \cap E) P(A_k \cap E)}{P(E)}$$

$$= P(B | A_k, E) P(A_k | E)$$

$$P(B|E) = \sum_{k=1}^{n} P(B | A_k, E) P(A_k | E)$$

11. $P(B^c | A) = 1 - P(B|A) = 1 - P(B) = P(B^c)$

$\therefore$ A and B^c are indep. so dose A^c and B

$P(B^c | A^c) = 1 - P(B | A^c) = 1 - P(B) = P(B^c)$

Here A^c and B^c are indep. as well