

## Lecture 13: Colors

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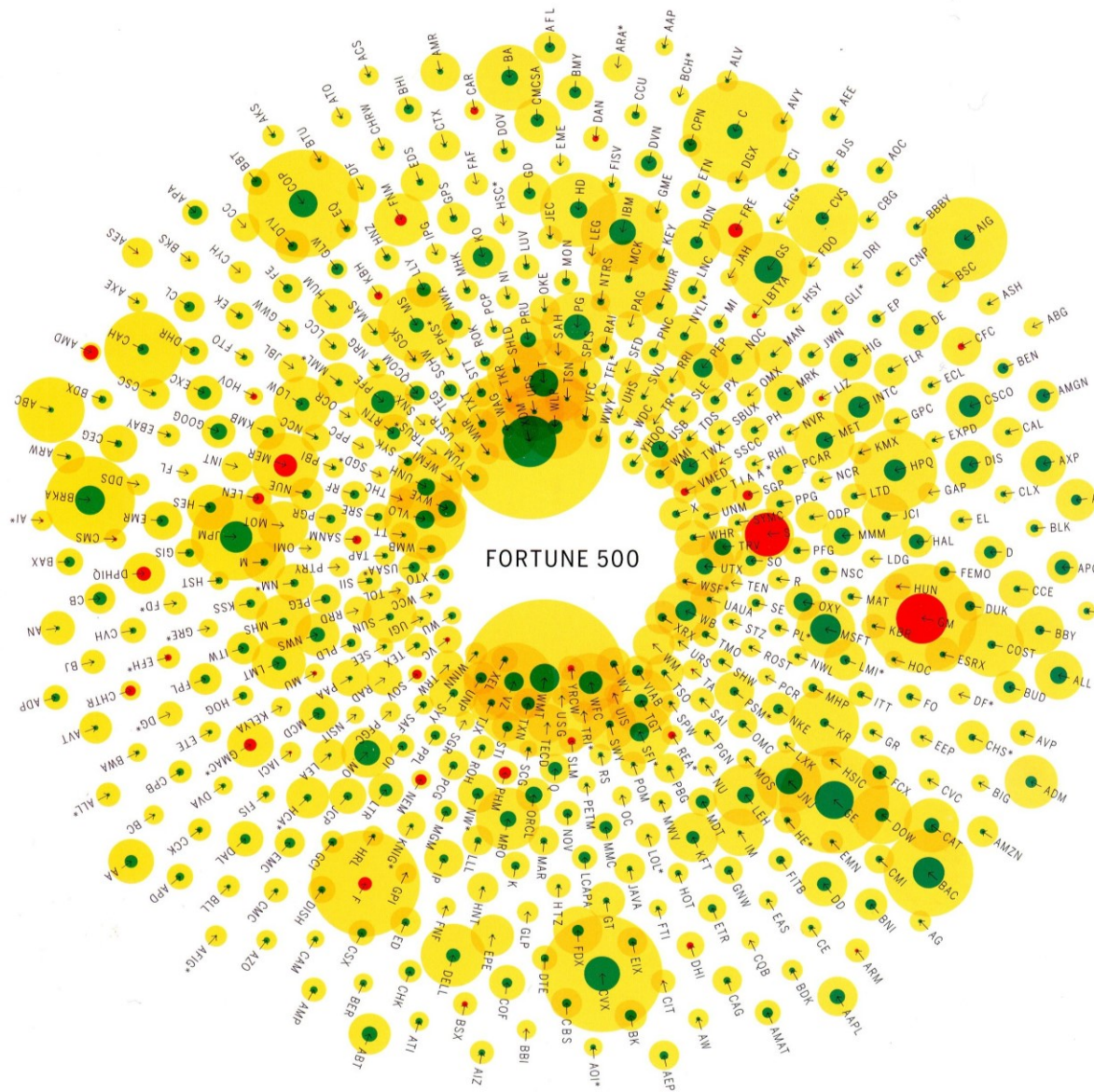


# Outline

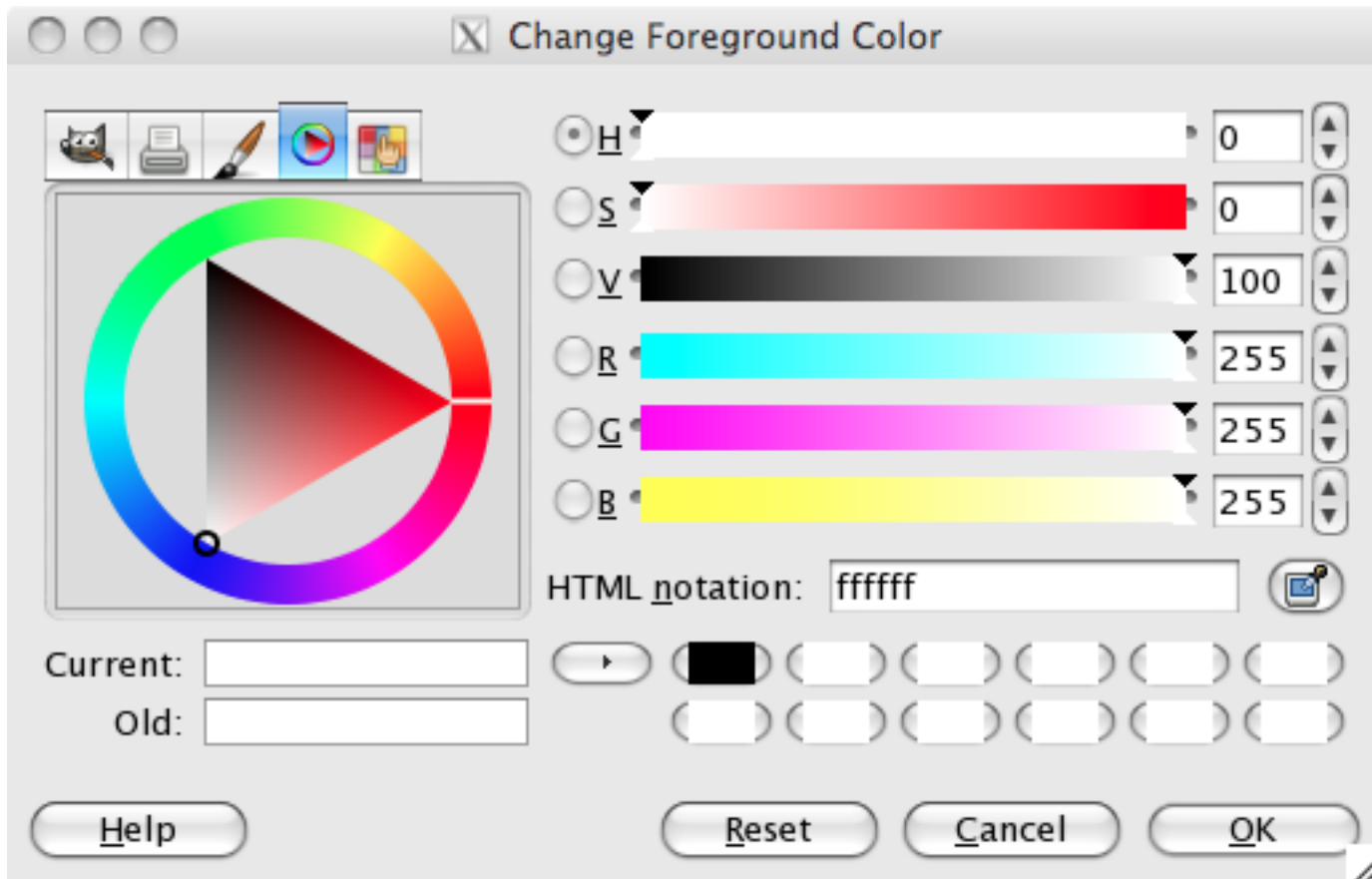
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- Properties of light and colors
- Color models

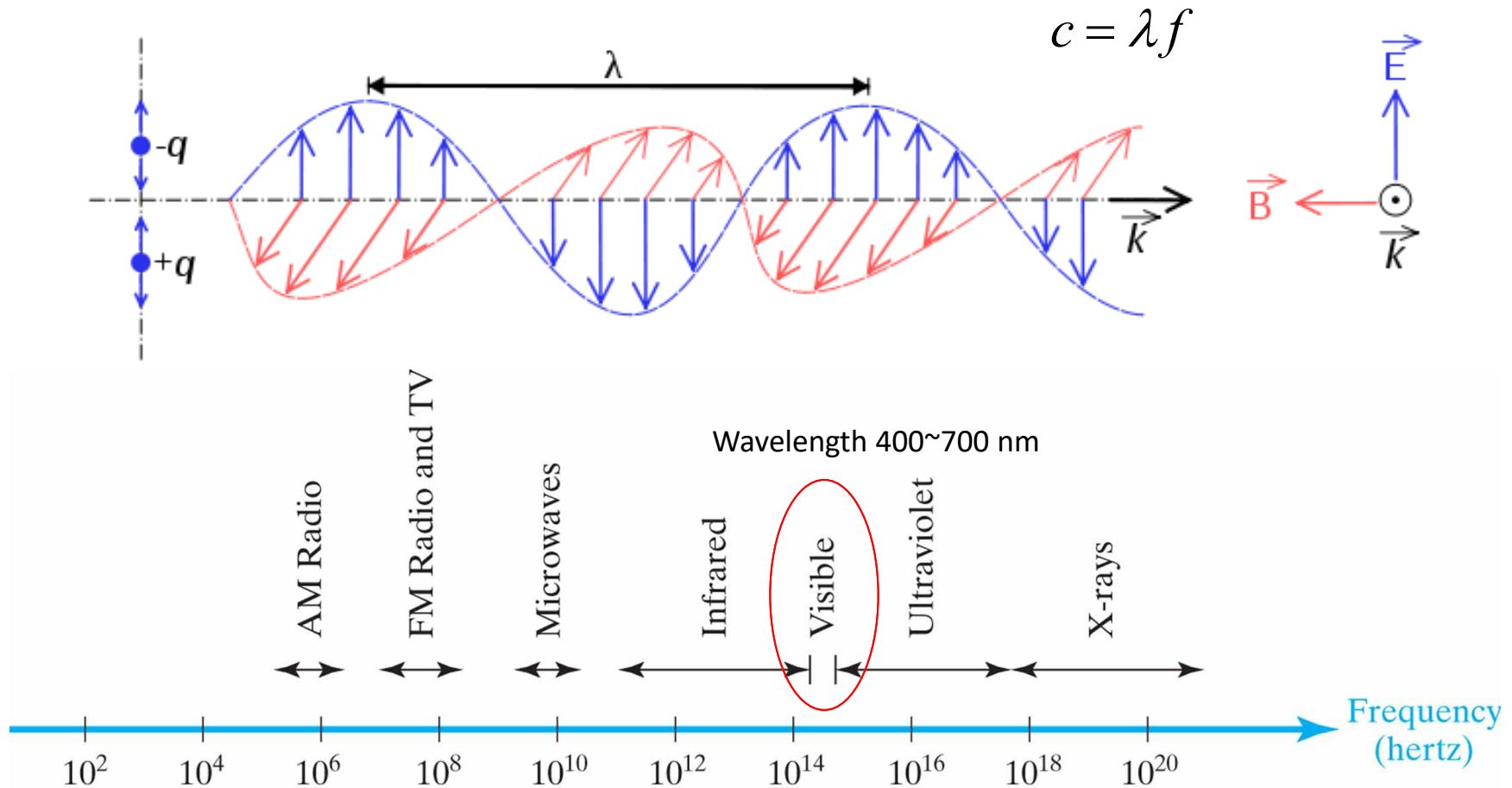




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# Electromagnetic Spectrum



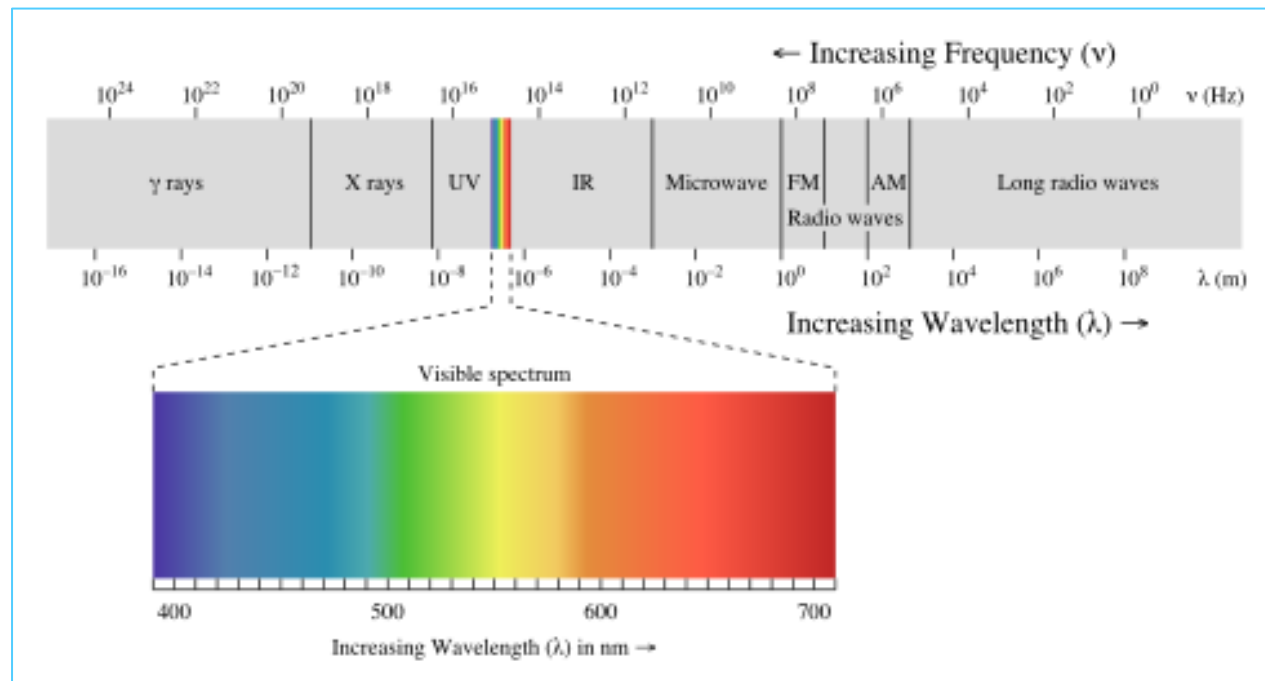
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$\lambda$  : wavelength,  $f$  : frequency,  $c$  : speed of light  
 Wavelength unit is easier to use for visible spectrum.  
 We use wavelength in vacuum.



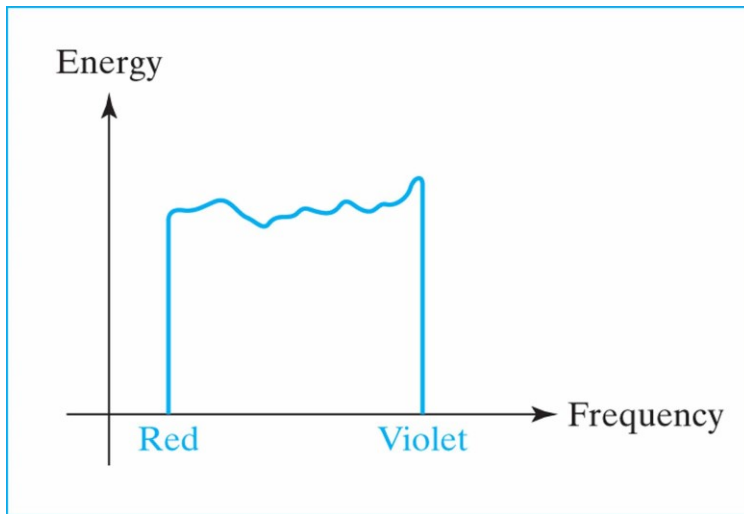
# Spectral Color

- Each frequency (wavelength) in visible spectrum corresponds to a single color

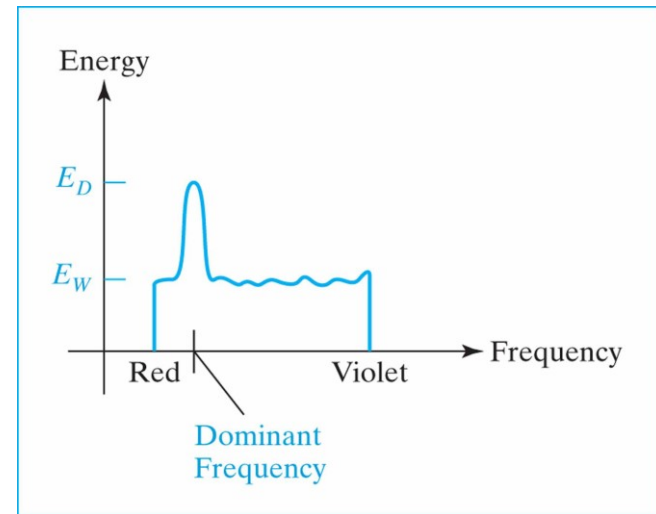


# Color of Light

- Light source emits all frequencies within visible range
- Dominant frequency determines the color of light



white light



red light

# Color Characteristics

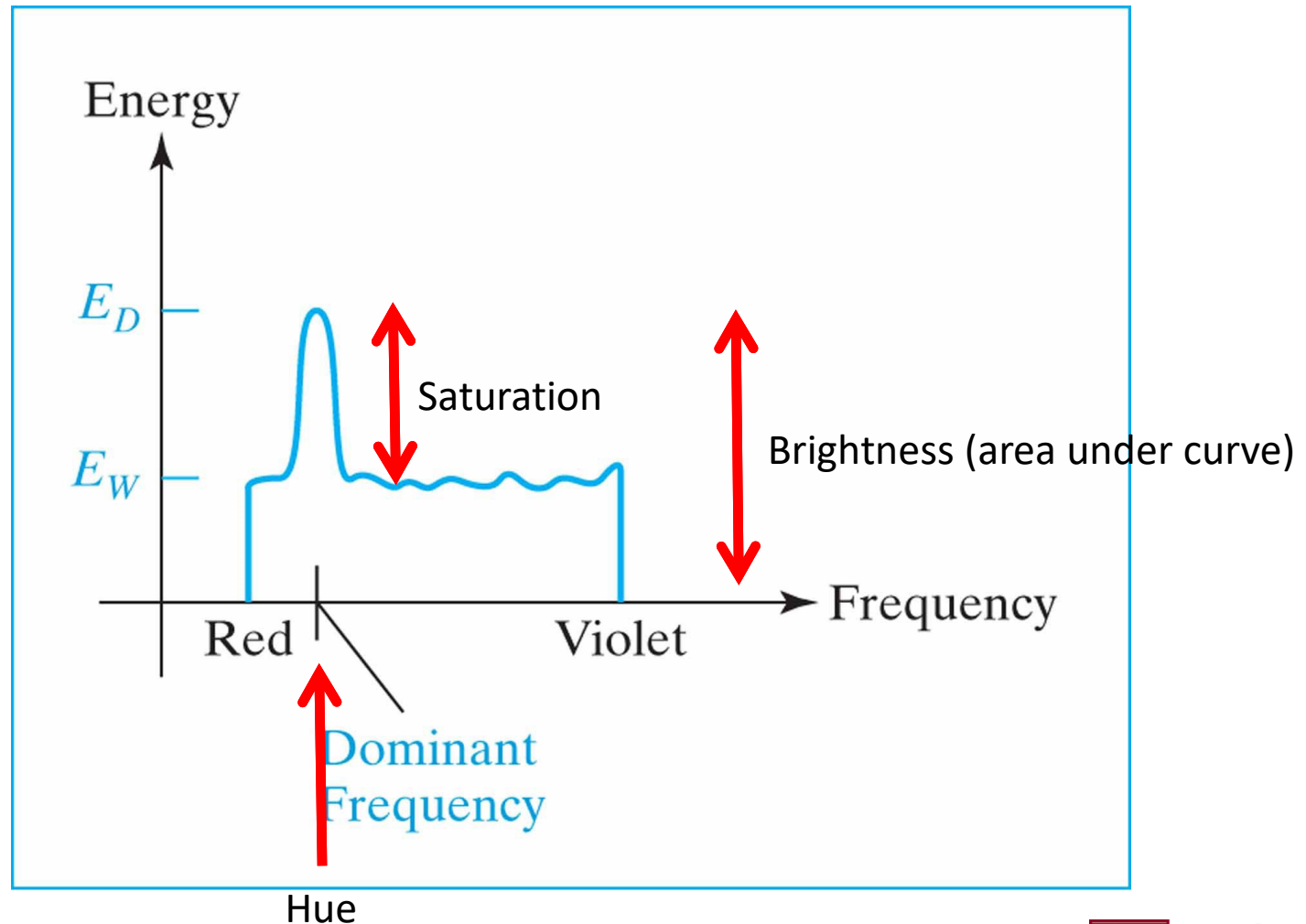
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- Hue (color)
  - Dominant frequency
- Brightness (luminance)
  - Total energy of the light
- Saturation (purity)
  - How close to a pure spectral color
- Chromaticity
  - Hue & Saturation





# Color Characteristics



# Perceiving Colors

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- Interaction between light and materials
  - Reflection
    - Red apple reflects red spectrum of light
  - Transmission
    - Green glass transmits green spectrum of light
- Photoreceptors in the eye
  - Retina (rods & cones)

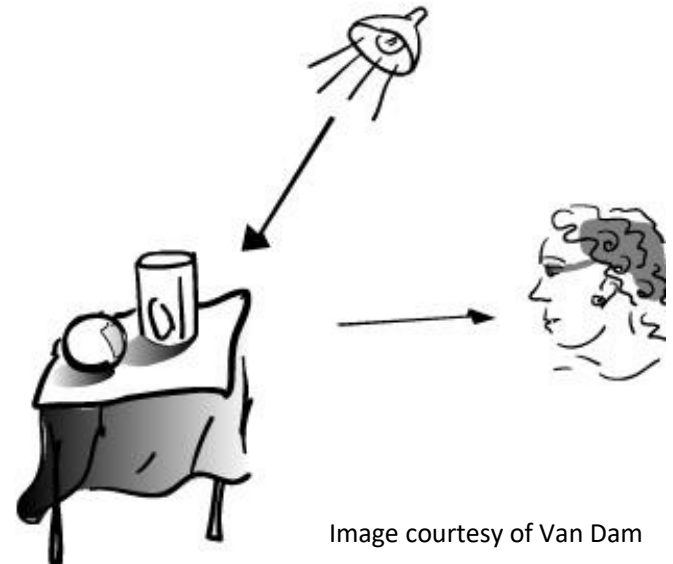


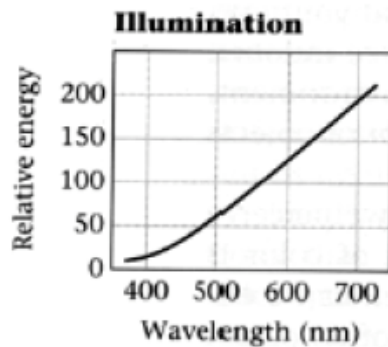
Image courtesy of Van Dam



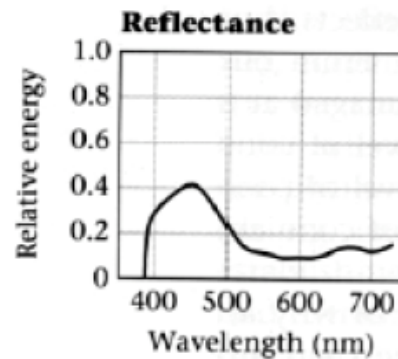
# Reflectance Spectra



## Wavelength-by-wavelength Multiplication

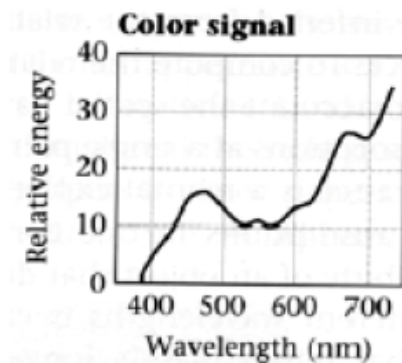


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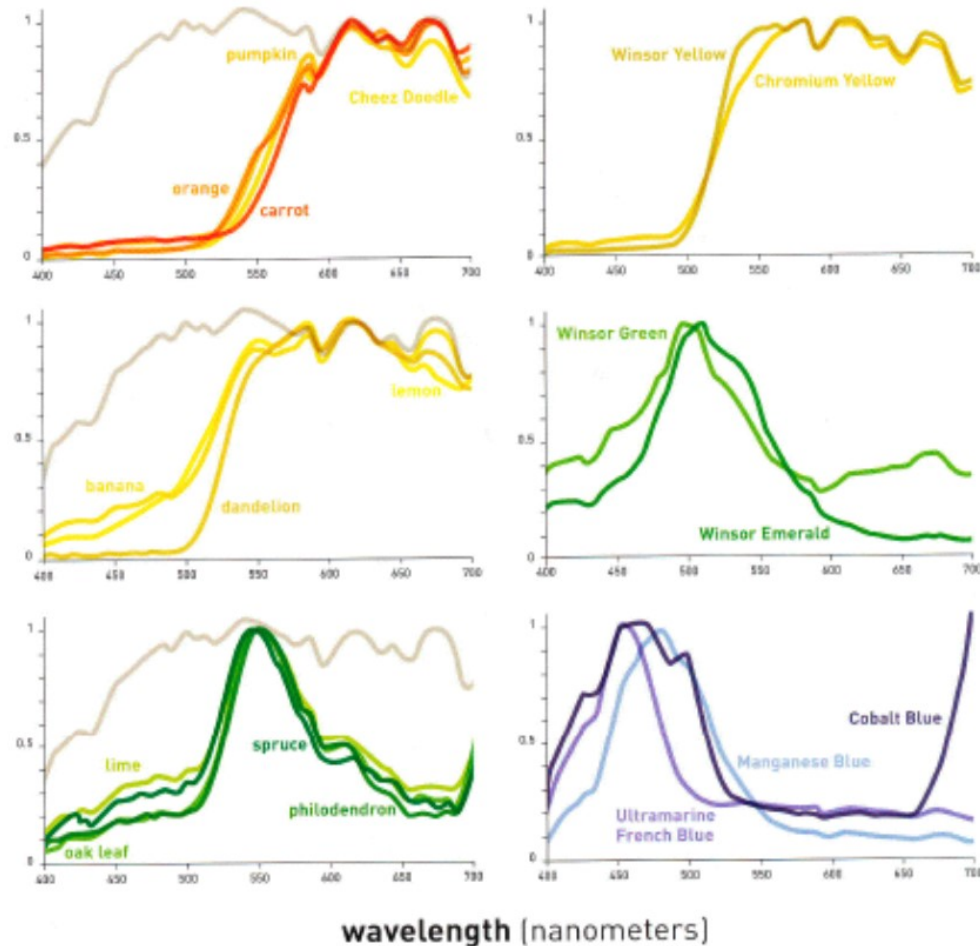


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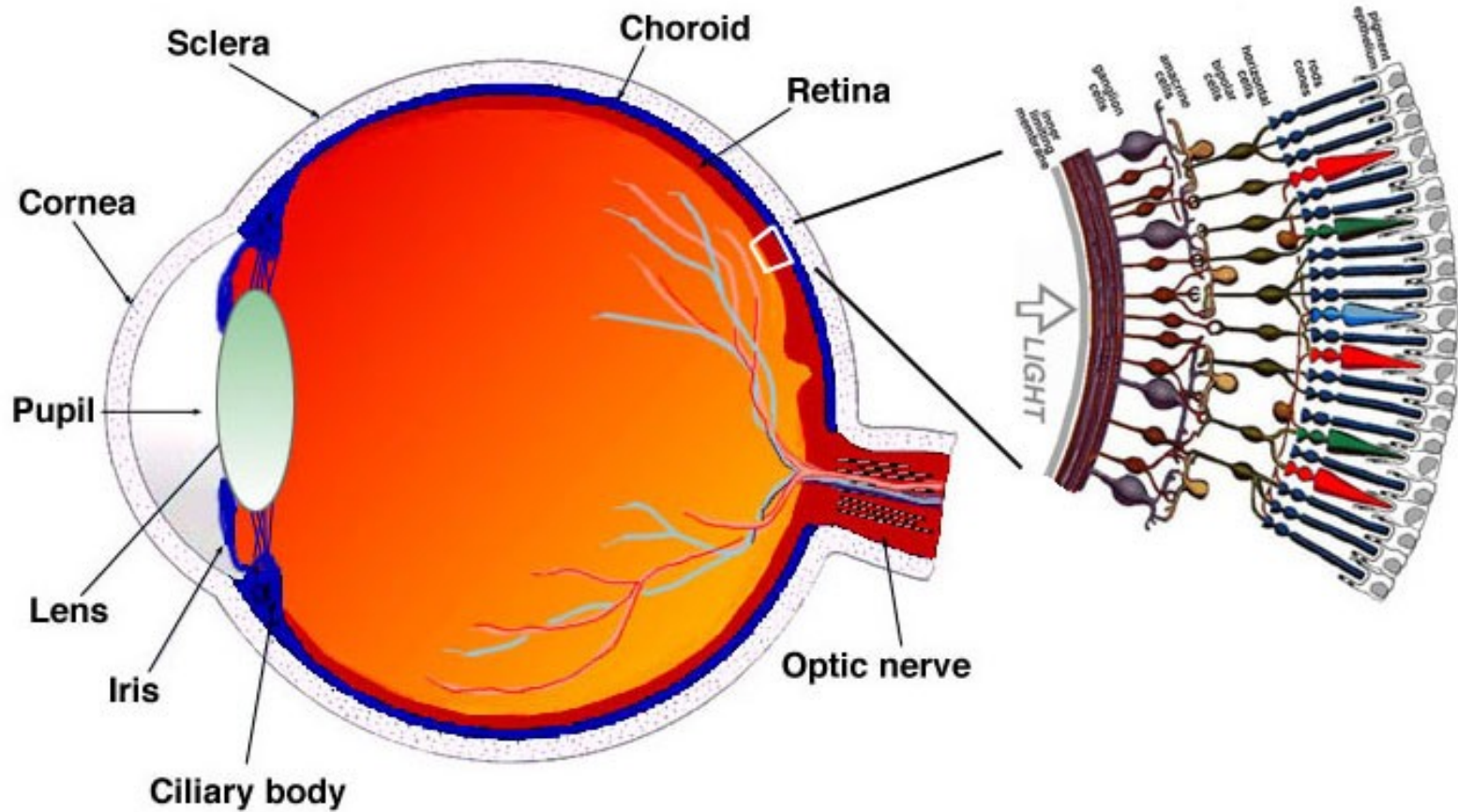
## Spectrum of Reflected Light



# Reflectance Spectra

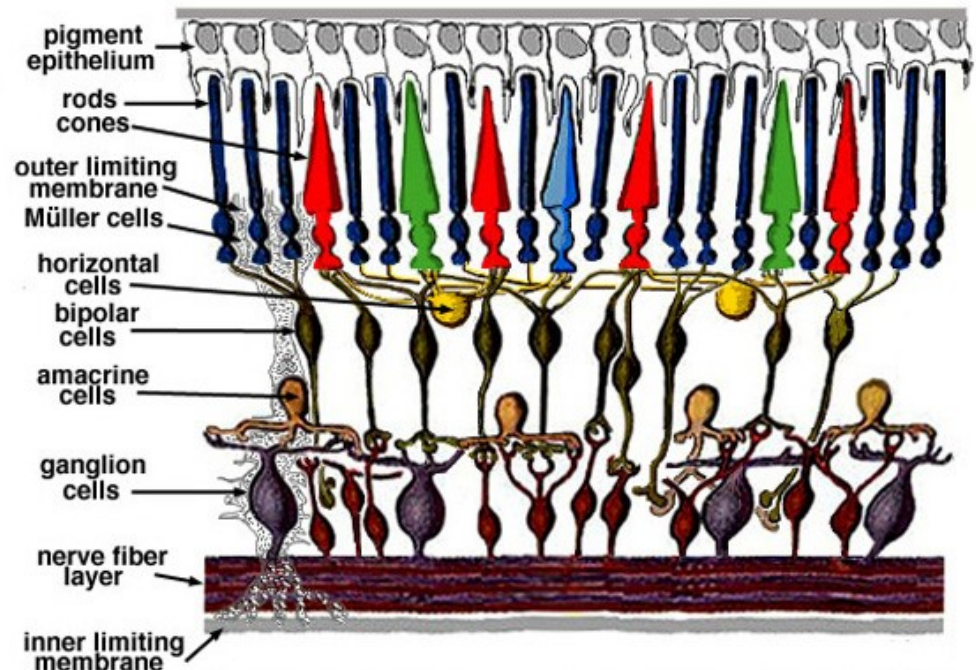


# The Eye



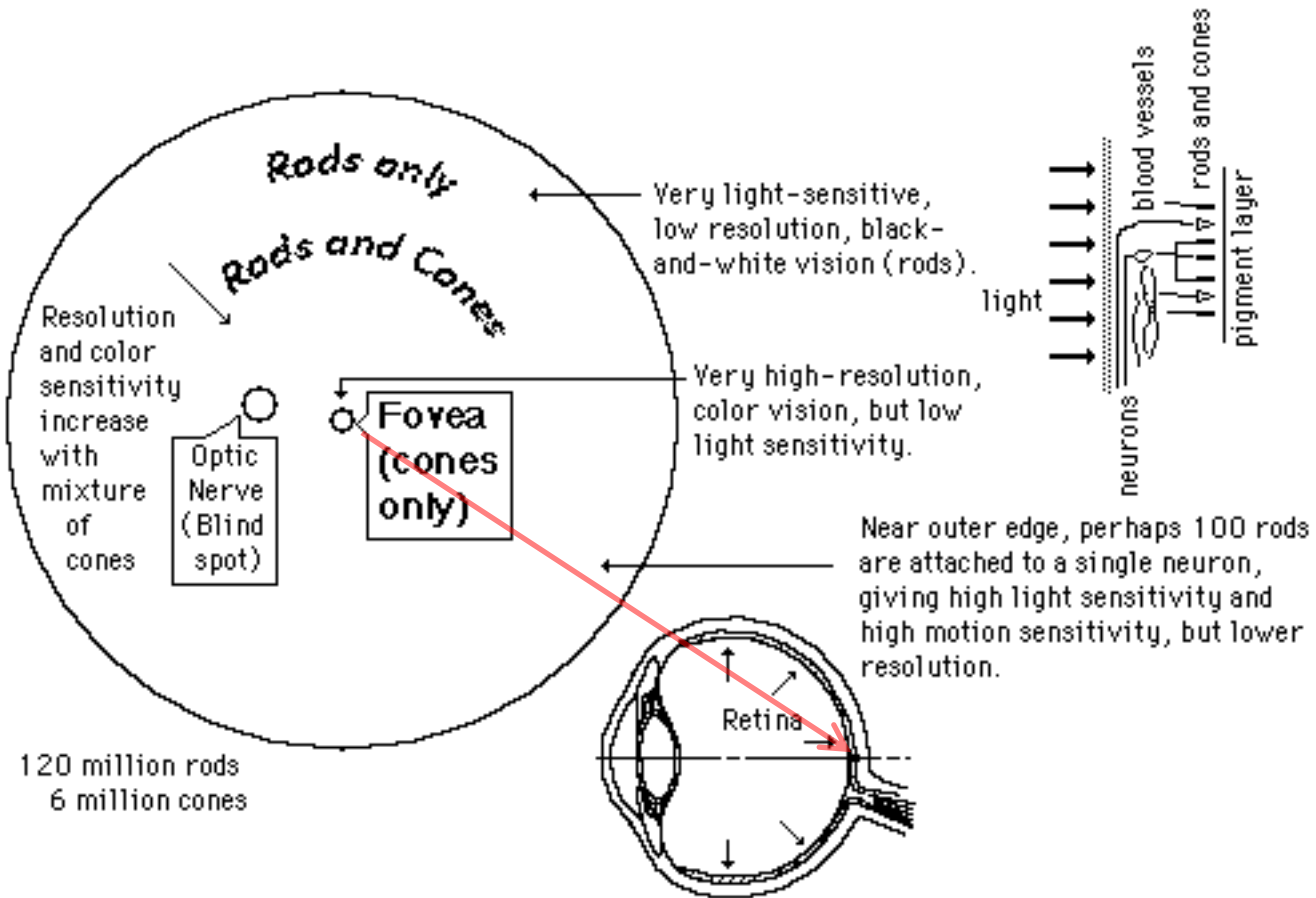
# Rods and Cones

- Rods
  - 120 million cells
  - Monochromatic, night vision
- Cones
  - 6~7 million cells
  - Color sensitive
  - Three types of cones
    - Red (64%)
    - Green (32%)
    - Blue (2%)

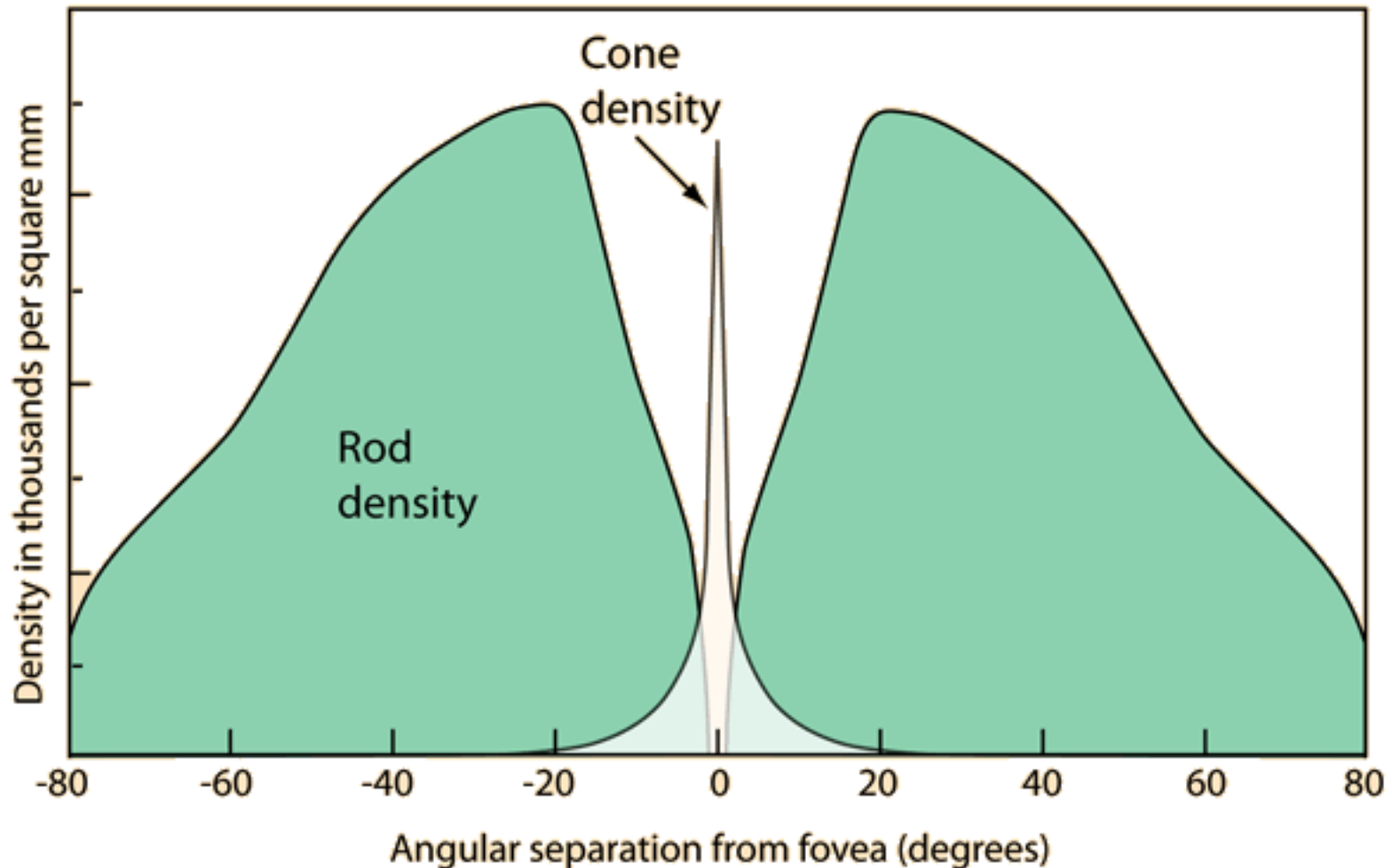




# The Retina



# Rod and Cone Density

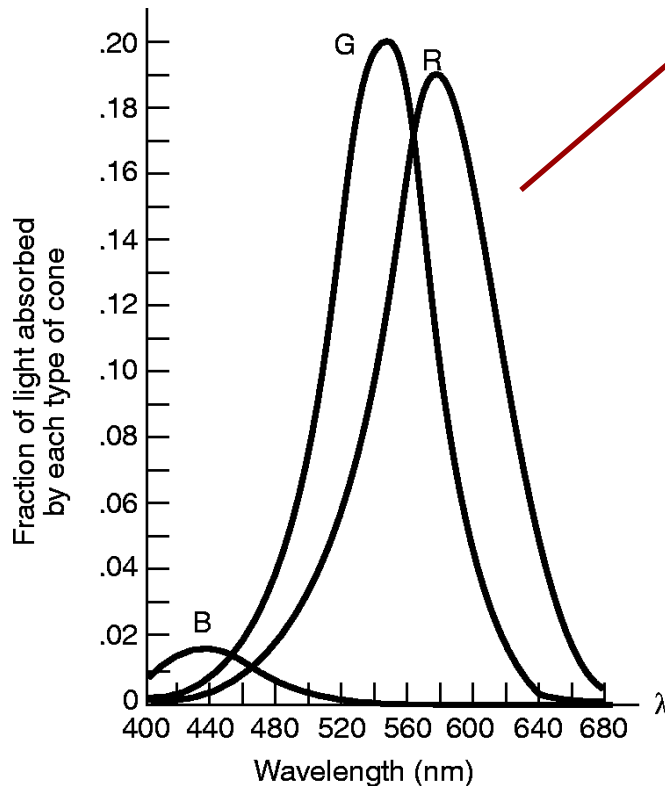
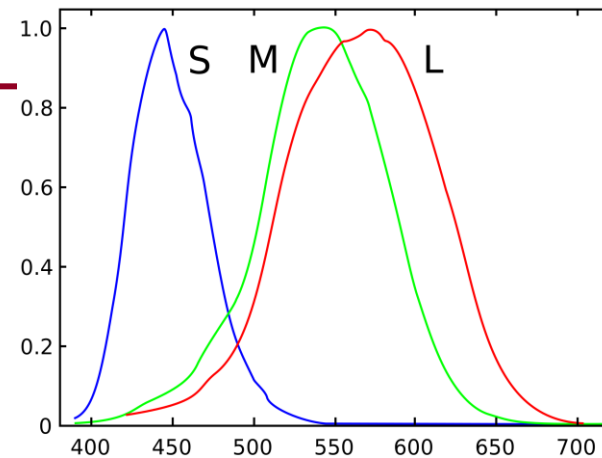




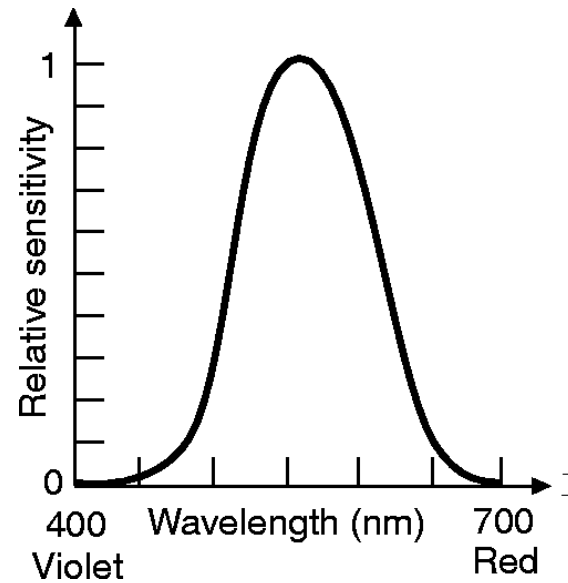
# Primary Colors

- Tristimulus theory
  - Three types of cone

Short, Middle, Long wavelength types



Normalization

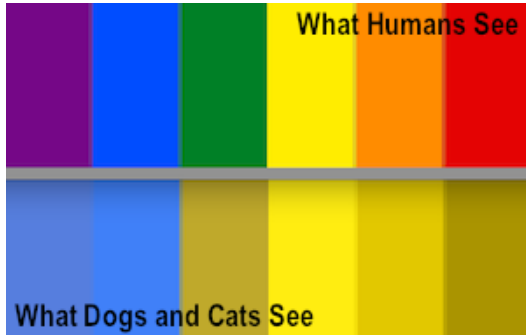


Luminous-Sensitivity of human eye



# Hunters vs. Gatherer

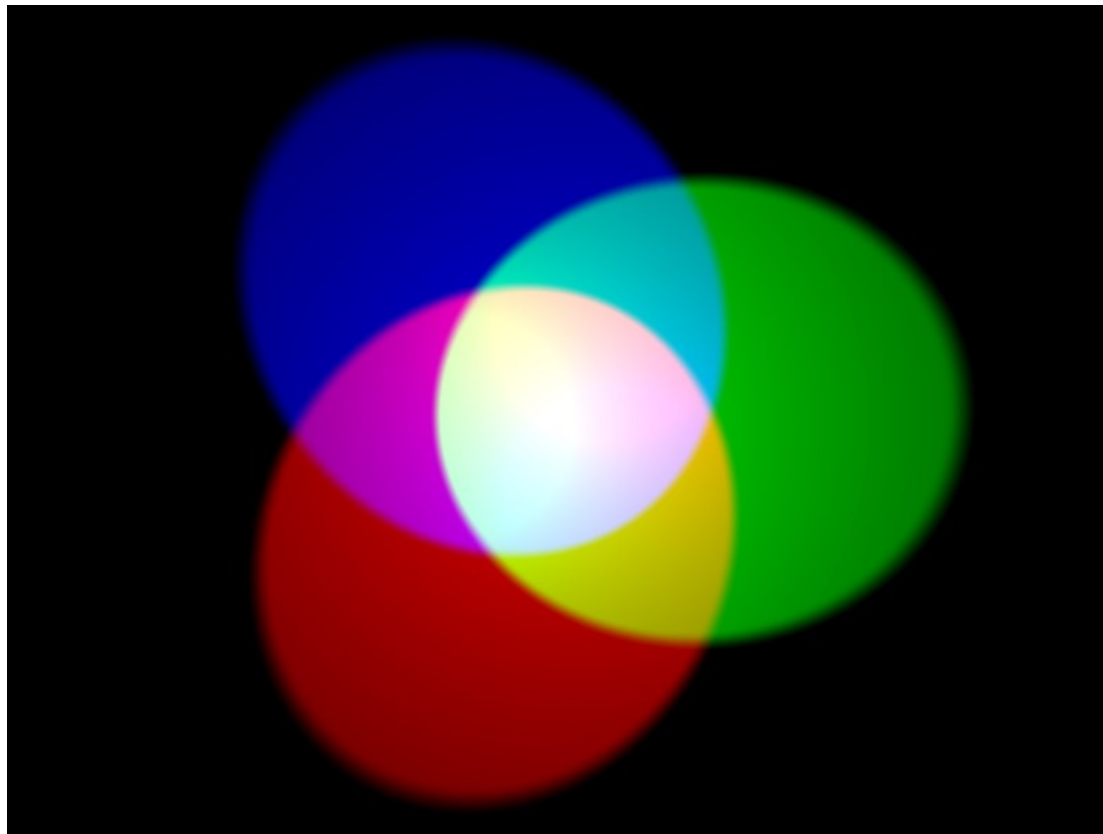
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# Color Mixing

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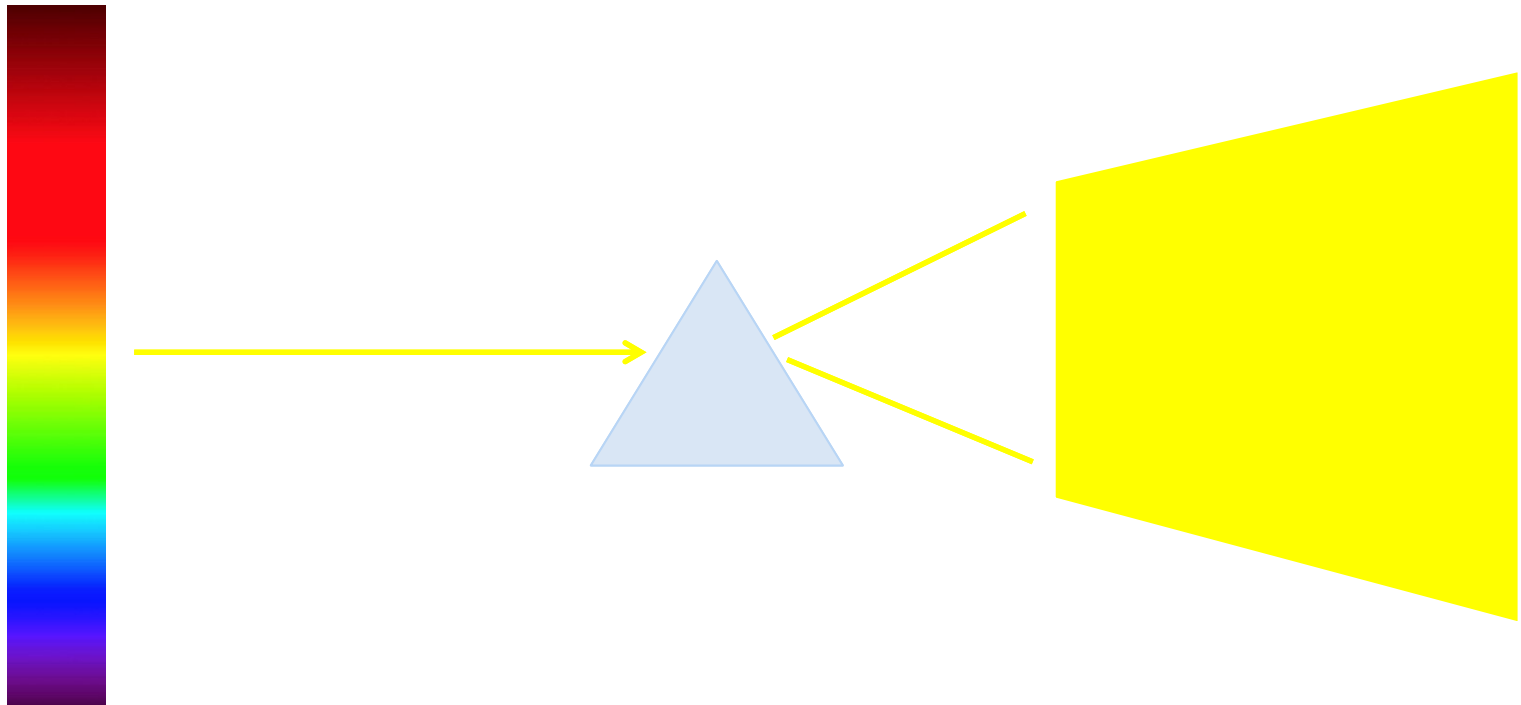
- Yellow = Red + Green



# Color Mixing

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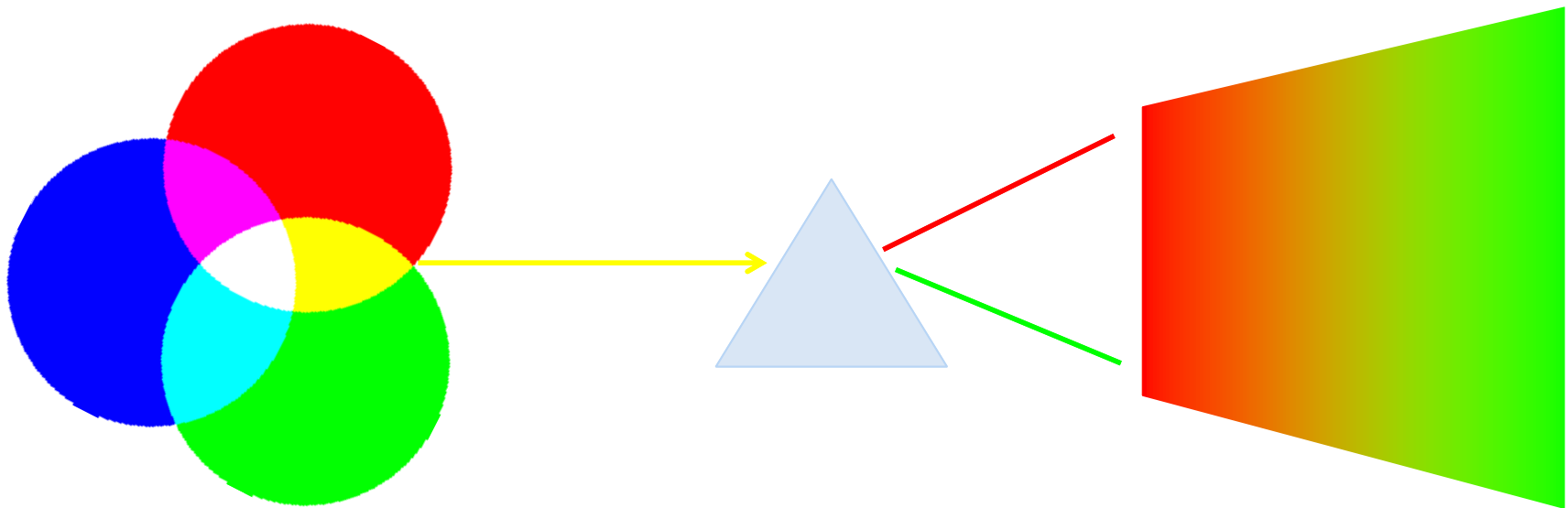
- Spectral Yellow



# Color Mixing

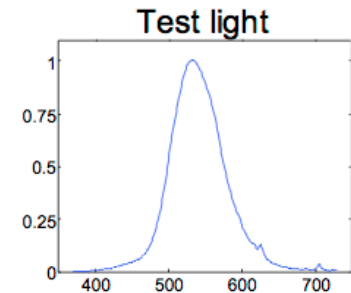
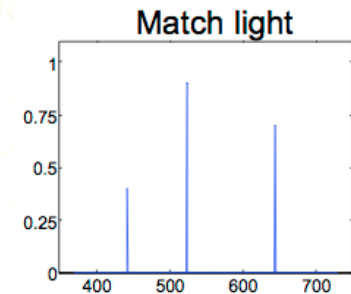
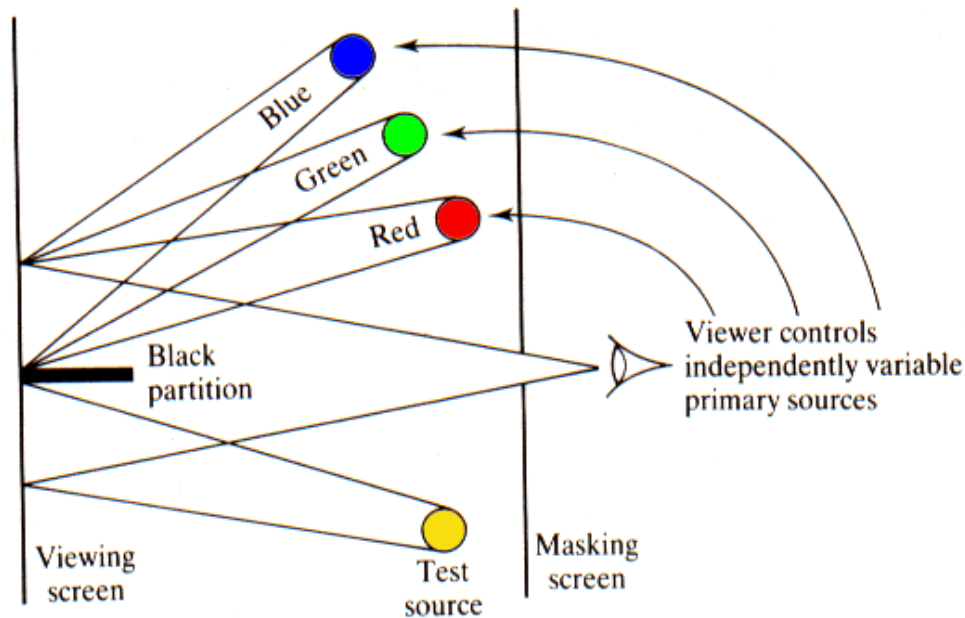
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- Yellow as Red & Green mixture



# Color Matching Experiment

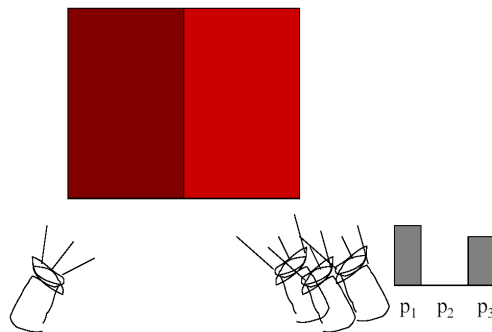
- Need a precise way to describe colors
  - Use monochromatic (single wavelength) primaries
    - 700nm (red), 546.1nm(green), 435.8nm(blue)



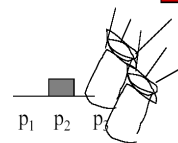
Find matching parameters for each spectral color!

# Color Matching Experiment

- Not all visible colors can be expressed using R/G/B primary colors
  - -R required



We say a “negative” amount of  $p_2$  was needed to make the match, because we added it to the test color’s side.



The primary color amounts needed for a match:

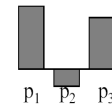
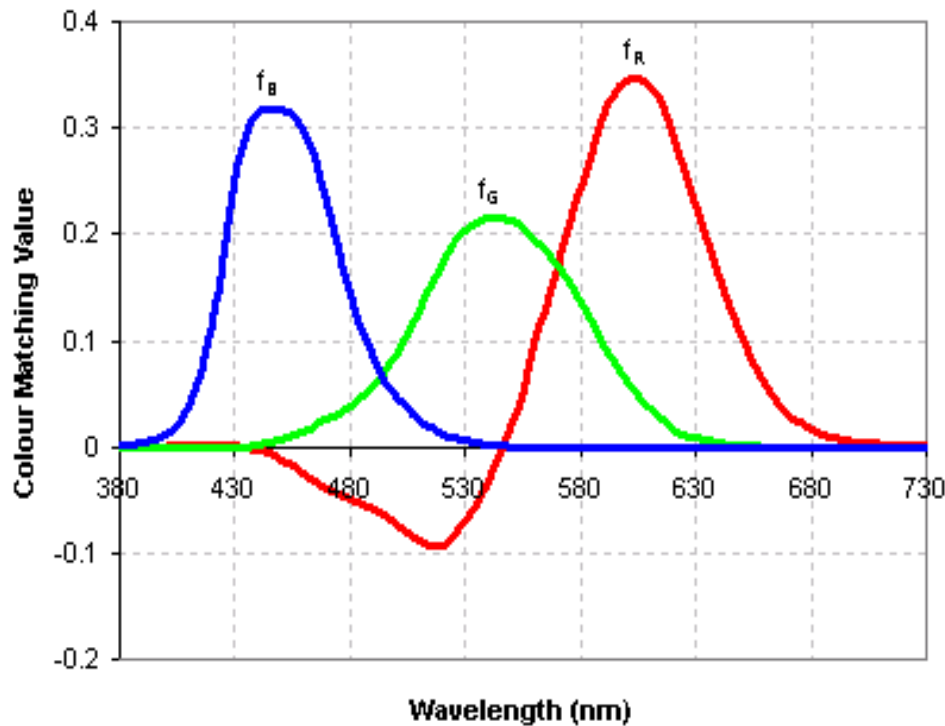


Image courtesy of Van Dam

# RGB Color Matching Functions

- **R** (700 nm), **G** (546.1 nm), **B** (435.8 nm)



$$R = k \int I(\lambda) f_R(\lambda) d\lambda$$

$$G = k \int I(\lambda) f_G(\lambda) d\lambda$$

$$B = k \int I(\lambda) f_B(\lambda) d\lambda$$

$$C(\lambda) = RR + GG + BB$$

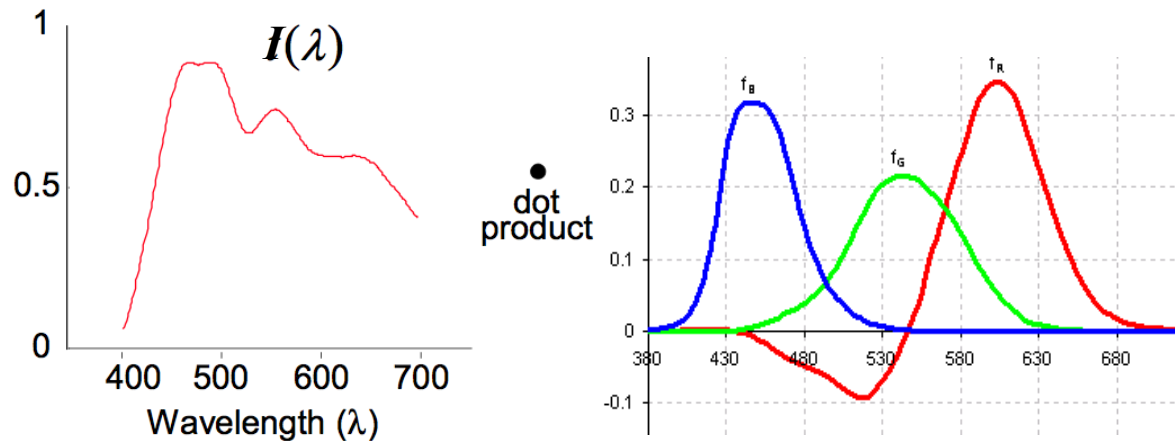
$(R, G, B)$  is uniquely defined per primary

Image courtesy of Van Dam



# RGB Color Matching Function

- Inner product of spectral radiance function and color matching function



$$\begin{aligned}
 R &= k \int I(\lambda) f_R(\lambda) d\lambda \\
 &= G = k \int I(\lambda) f_G(\lambda) d\lambda \\
 B &= k \int I(\lambda) f_B(\lambda) d\lambda \\
 C(\lambda) &= RR + GG + BB
 \end{aligned}$$

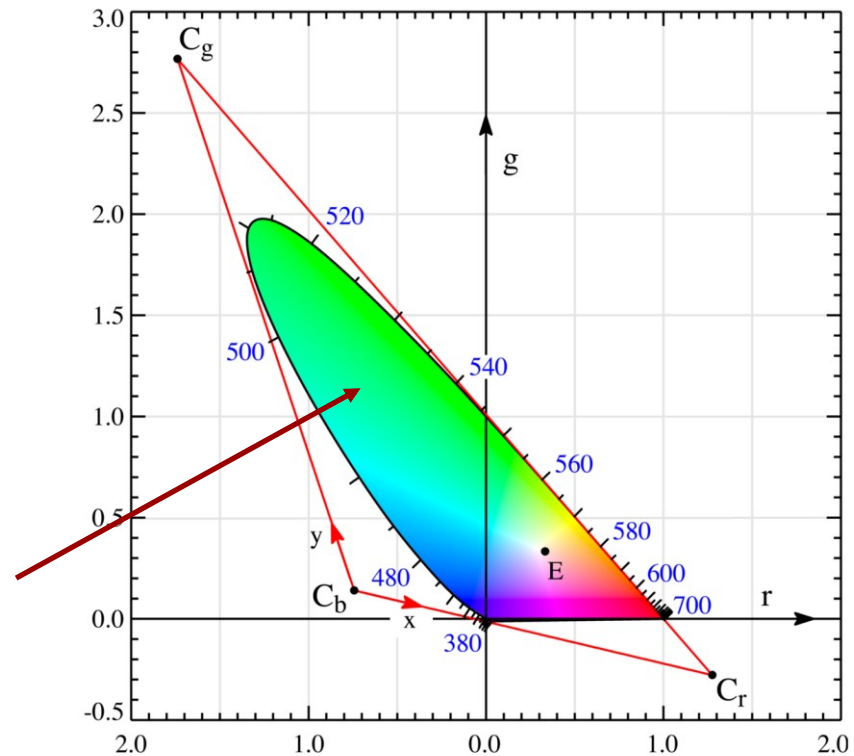
# CIE rg Chromaticity Space

$$r = \frac{R}{R + G + B},$$

$$g = \frac{G}{R + G + B},$$

$$b = \frac{B}{R + G + B}.$$

Outside of triangle  $[1,0][0,0],[0,1]$   
(negative weight)



- Can we design new primaries?

# Construction of XYZ Color Space

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- New primaries  $X, Y, Z$  (and normalized  $x, y, z$ )
- Goals
  - New color matching functions should be non-negative everywhere
    - Historical reason, positive computations are easier
  - $Y$  represents photopic luminous efficient function
    - Perceived brightness over different wavelength
  - White point :  $x = y = z = 1/3$
  - All visible colors should be within the  $x$ - $y$  triangle of  $[1,0], [0,0], [0,1]$



# CIE Standard Primaries

- XYZ system
  - Coordinate transform
  - Synthetic primaries

$$C(\lambda) = XX + YY + ZZ$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

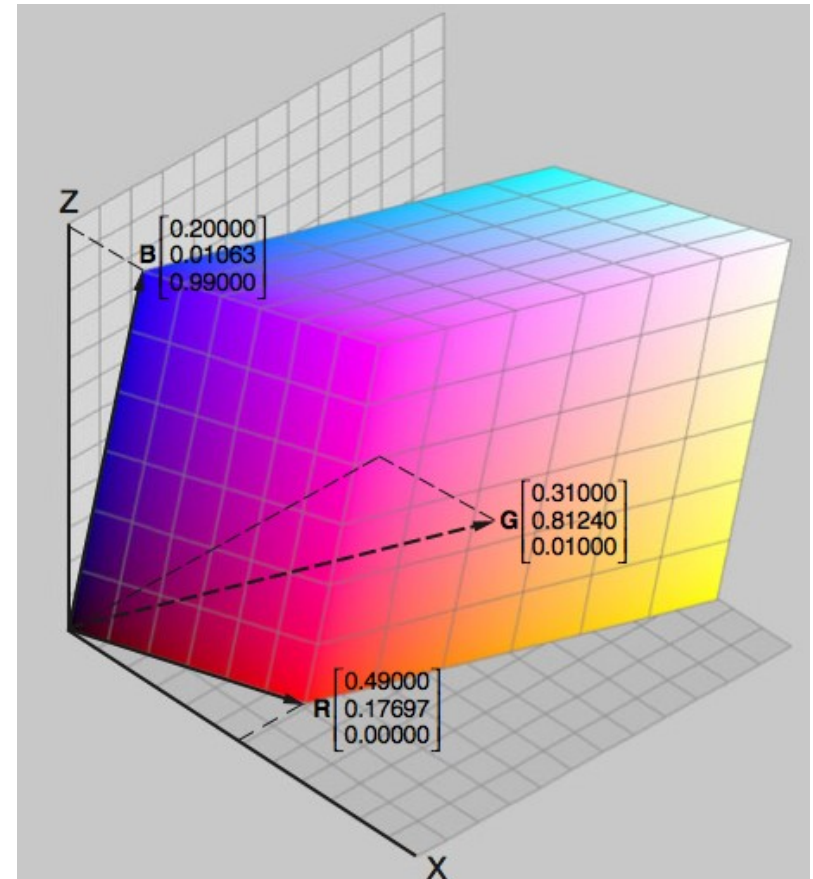
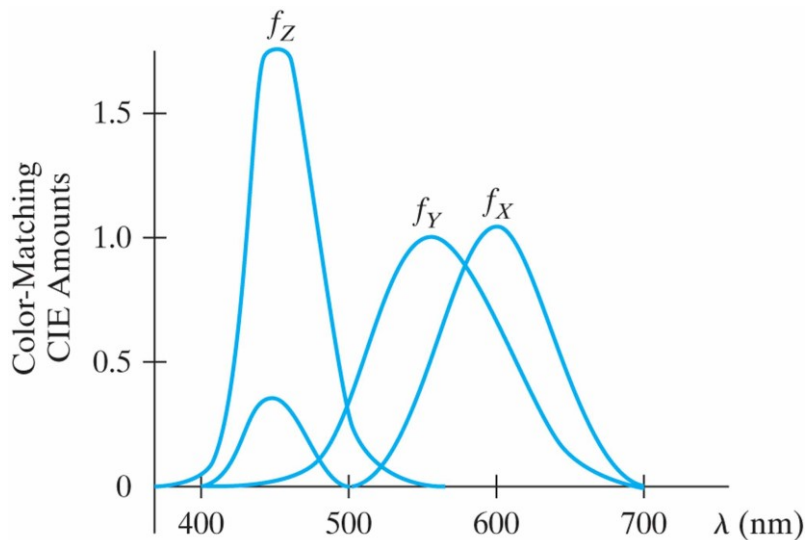
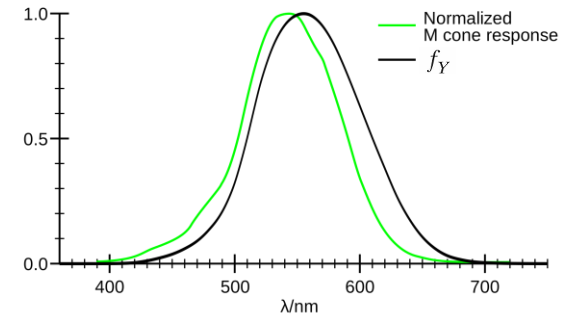


Image courtesy of Hoffmann

# CIE Primary Matching Functions

- Positive matching functions
- XYZ color model
  - xy (chromatic) & Y(luminance)



$$X = k \int I(\lambda) f_X(\lambda) d\lambda$$

$$Y = k \int I(\lambda) f_Y(\lambda) d\lambda$$

$$Z = k \int I(\lambda) f_Z(\lambda) d\lambda$$

$$C(\lambda) = X\mathbf{X} + Y\mathbf{Y} + Z\mathbf{Z}$$

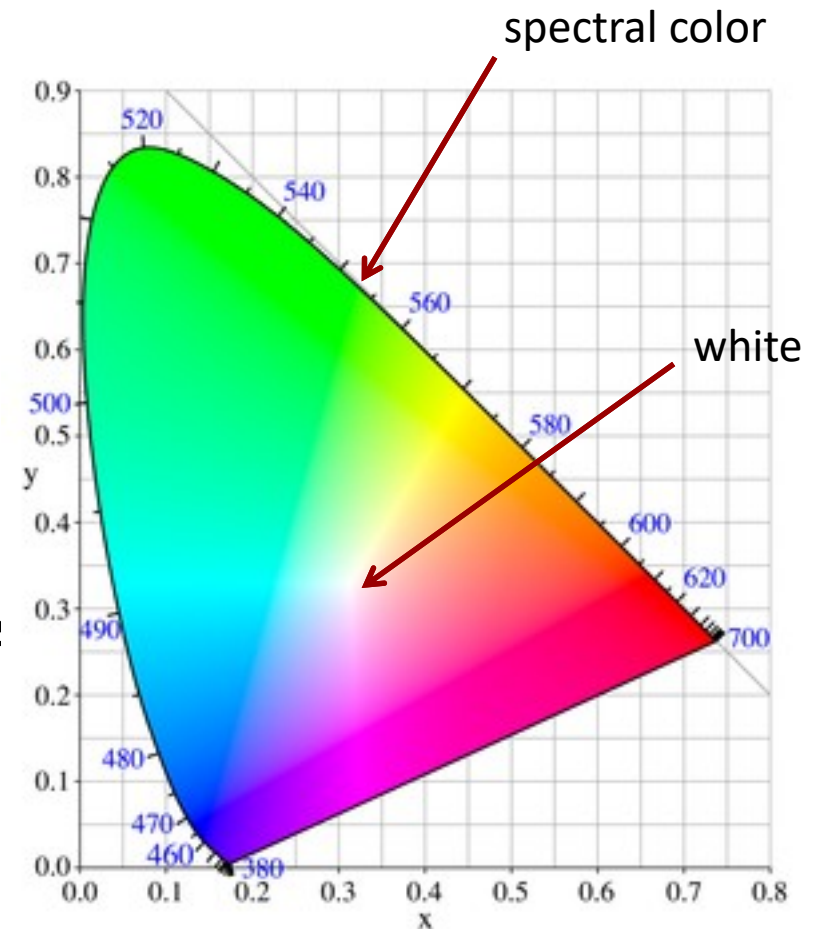
$$x = \frac{X}{X+Y+Z}, y = \frac{Y}{X+Y+Z}, z = \frac{Z}{X+Y+Z}$$

$$X = \frac{x}{y}Y, Z = \frac{z}{y}Y, z = 1 - x - y$$

Image courtesy of Hearn

# CIE Chromaticity Diagram

- 2D x/y plot
- No luminance
  - Colors with same chromaticity map to same point
- Useful for
  - Comparing color gamut for different primaries
  - Identifying complementary colors
  - Determine purity and dominant wavelength for a given color



# Color Gamut

- Range of color represented by primaries
- Straight line or polygon

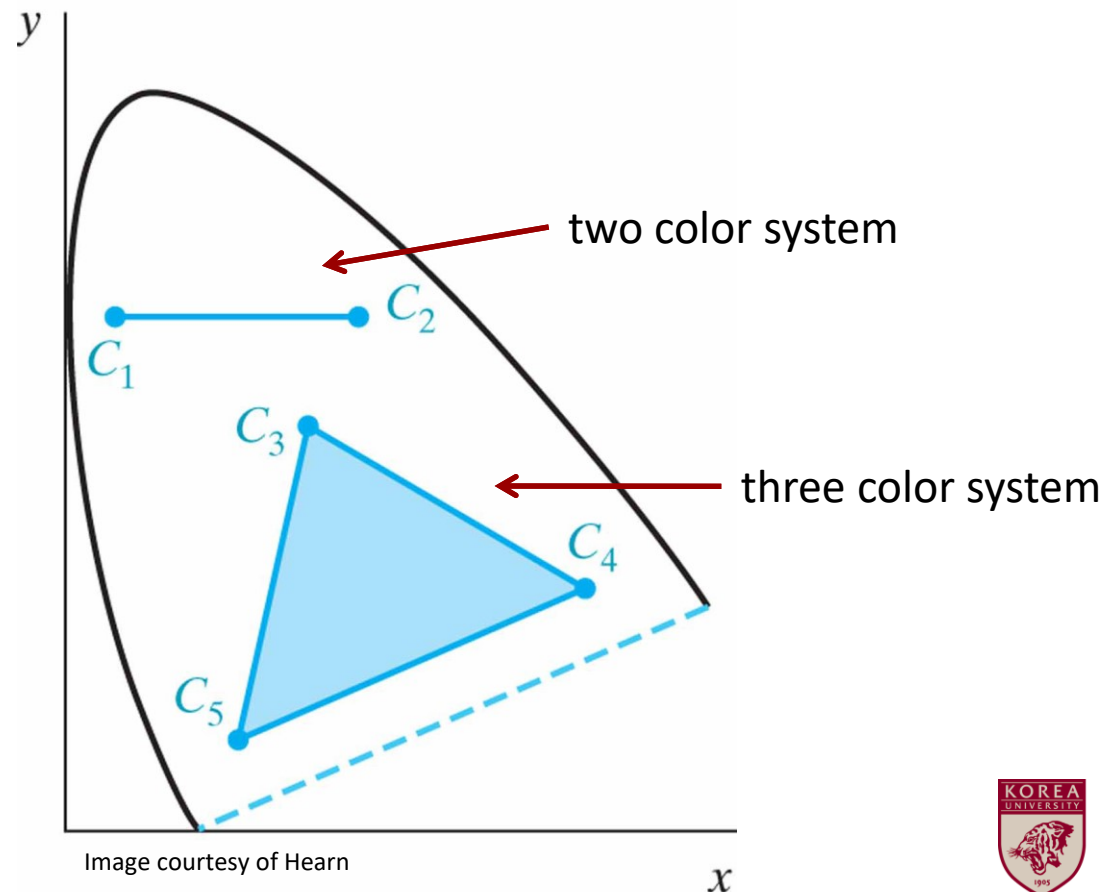


Image courtesy of Hearn

# Complementary Colors

- Two colors collinear with white

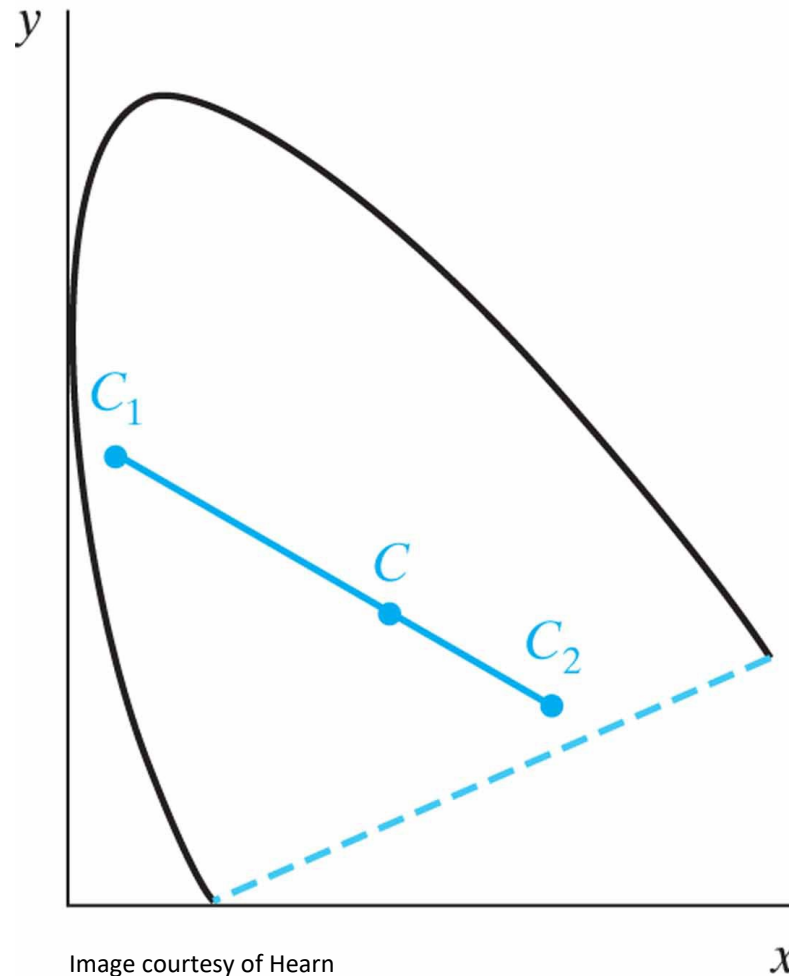
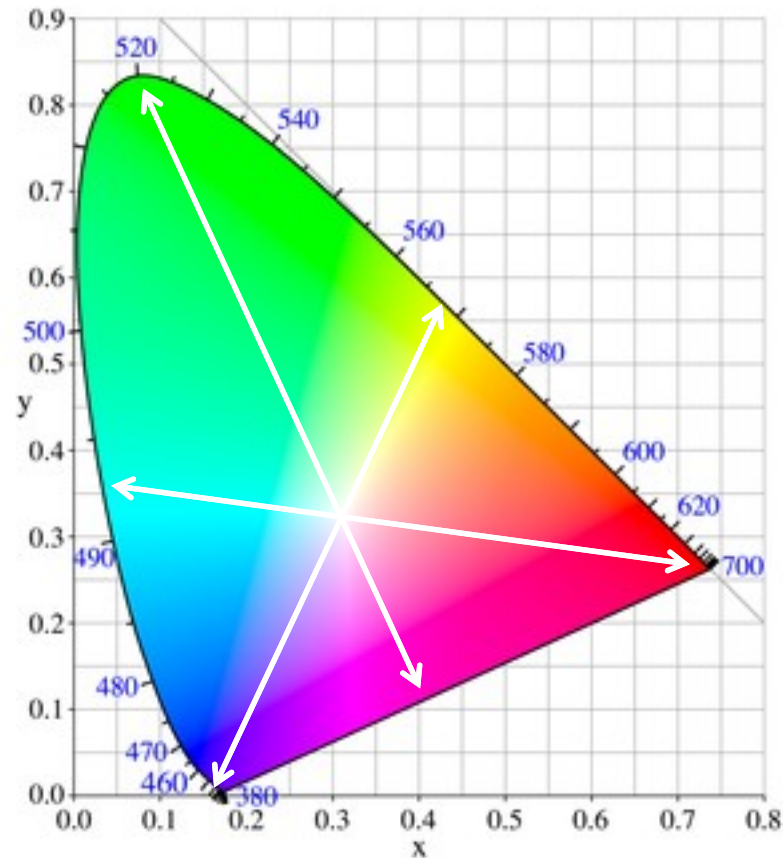


Image courtesy of Hearn





# Complementary Colors



Yellow		Blue Violet
Yellow Green		Violet
Purple		Green
Blue Green		Red
Blue		Orange
Violet		Yellow

# Dominant Wavelength and Purity

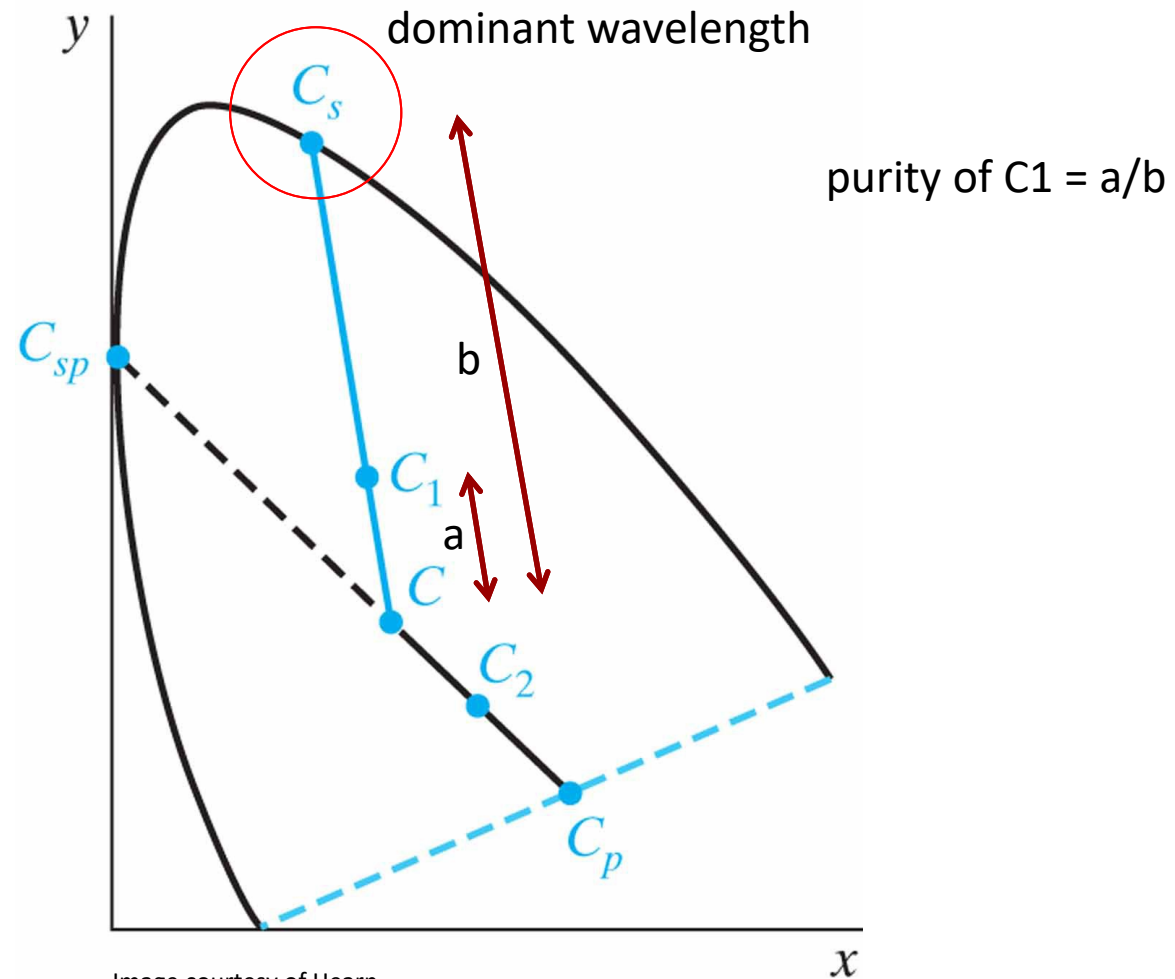


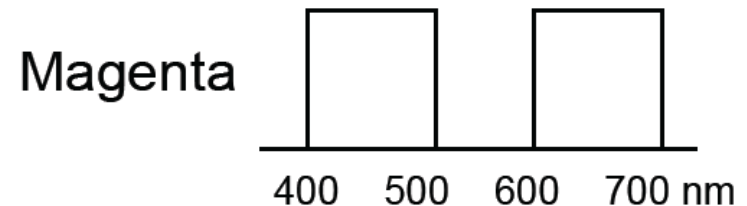
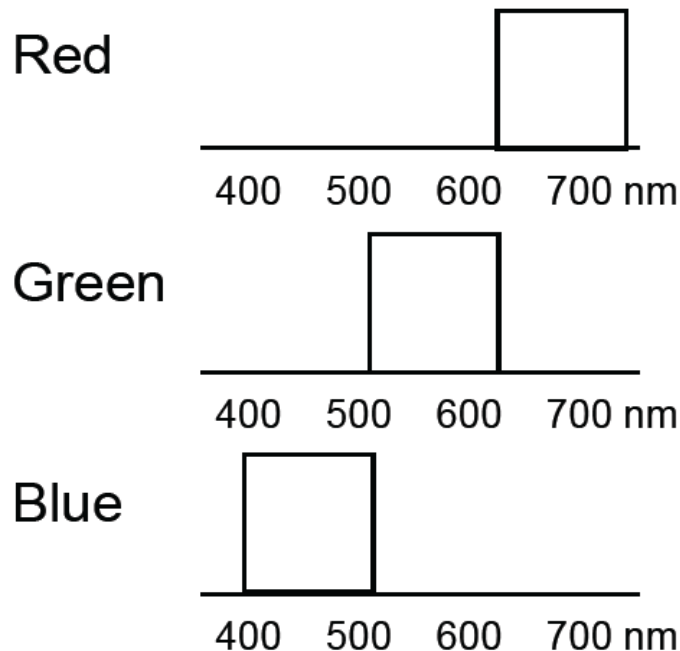
Image courtesy of Hearn

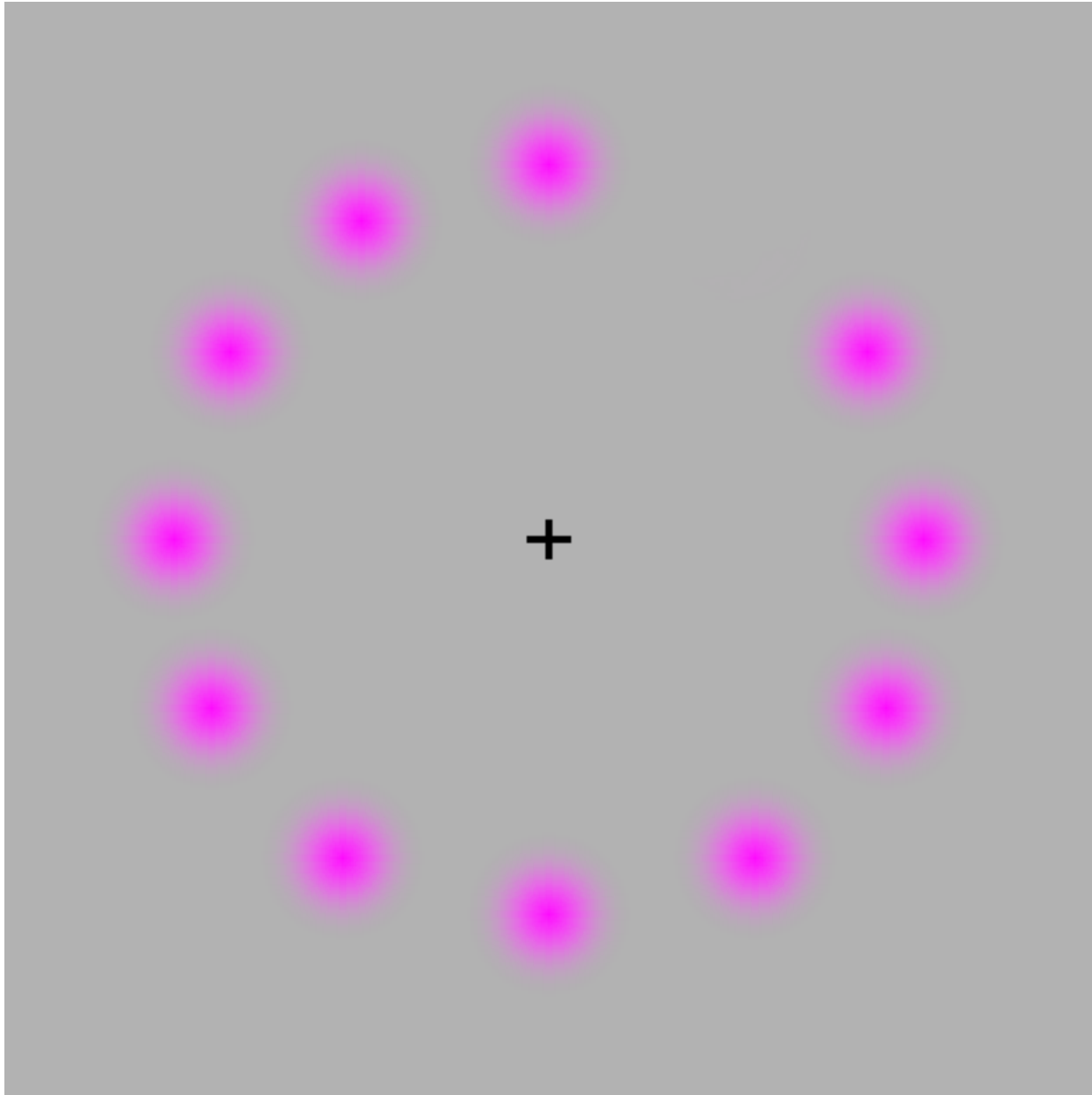
C2: Since there is no wavelength for  $C_p$ , take complementary ( $C_{sp}$ ) to compute dominant wavelength



# Magenta (Pink)

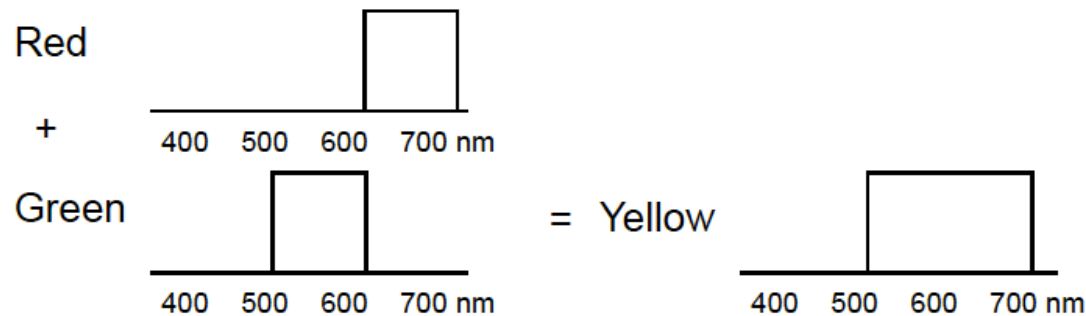
- Not a spectral color
  - No corresponding single wavelength
  - (Full spectrum (white) - green spectrum) = addition of red and blue spectra





# Additive Colors

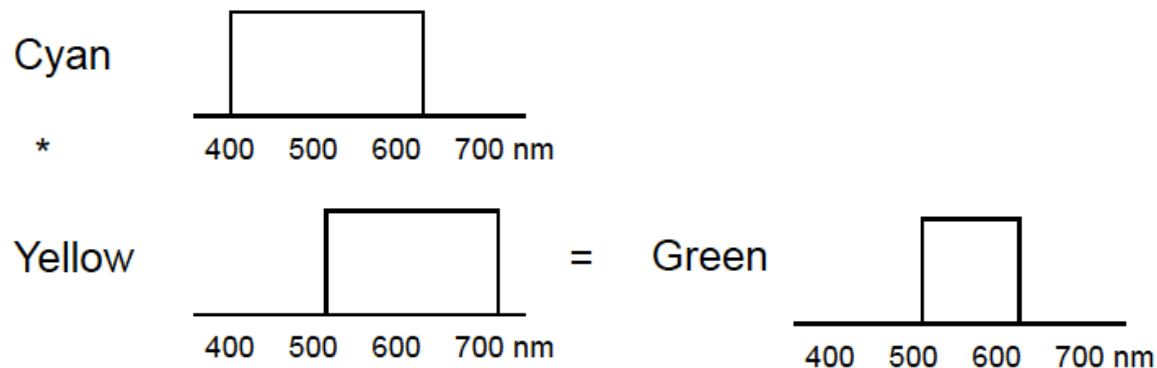
- Colors are combined by adding their spectra
- Light



RGB

# Subtractive Colors

- Colors are combined by multiplying their spectra
- Ink, paint



# RGB Color Model

- Additive

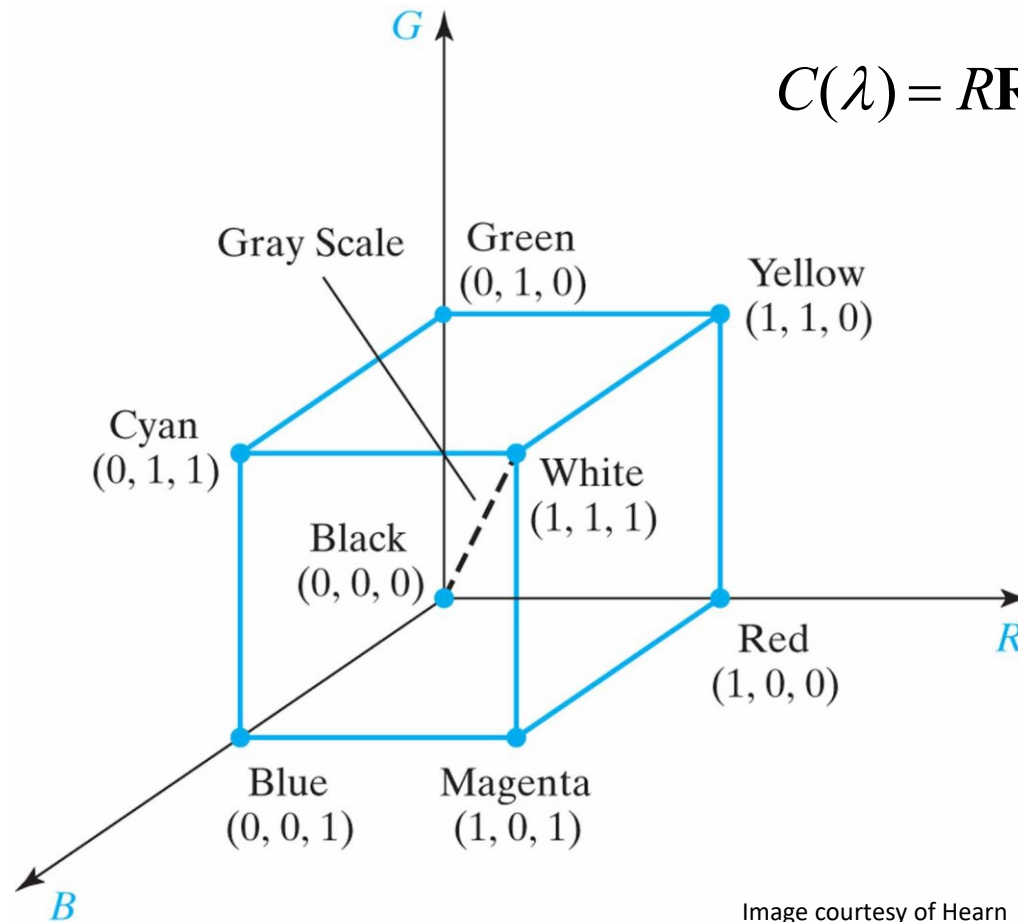


Image courtesy of Hearn



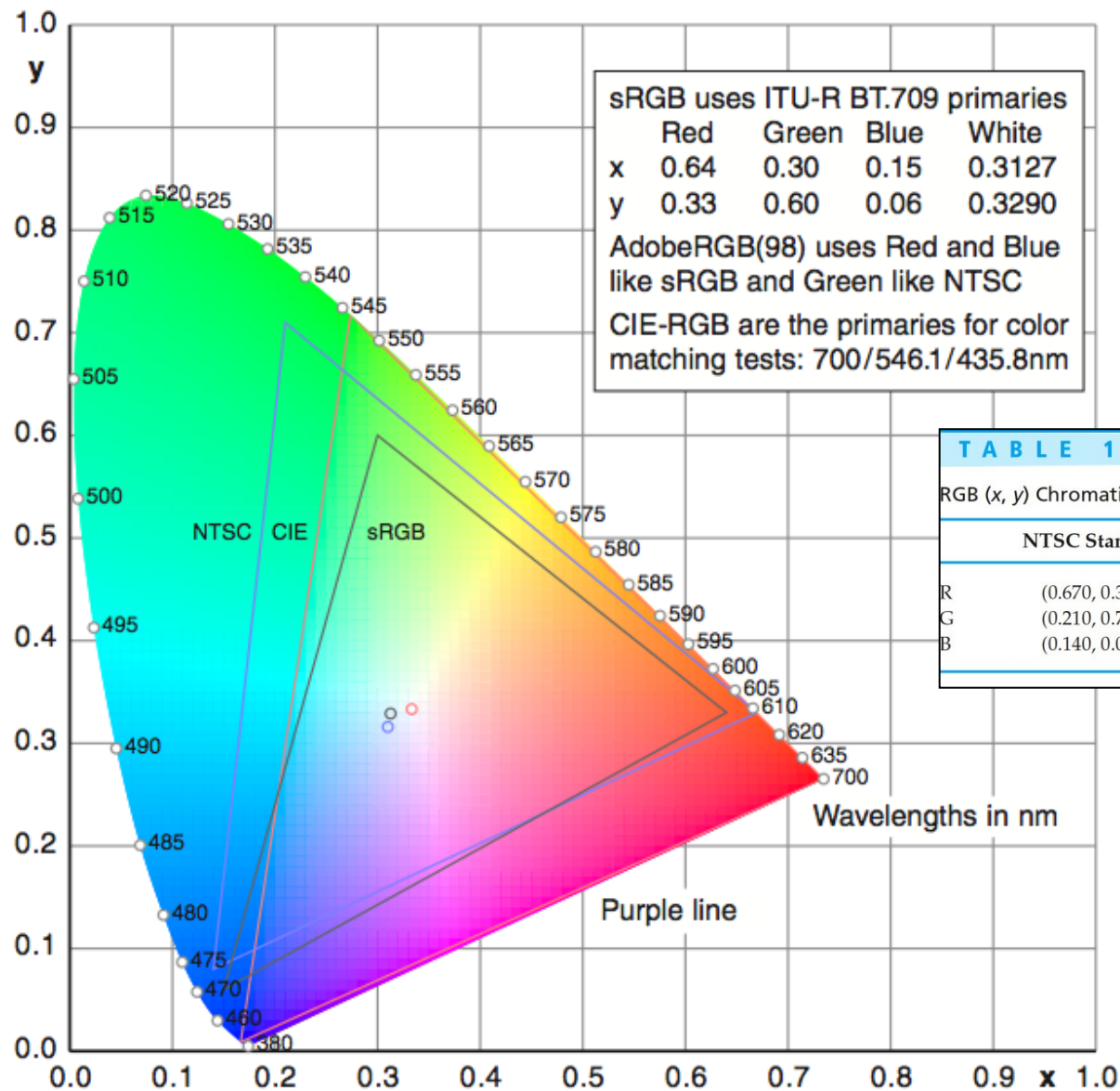


TABLE 19 - 1

RGB (x, y) Chromaticity Coordinates

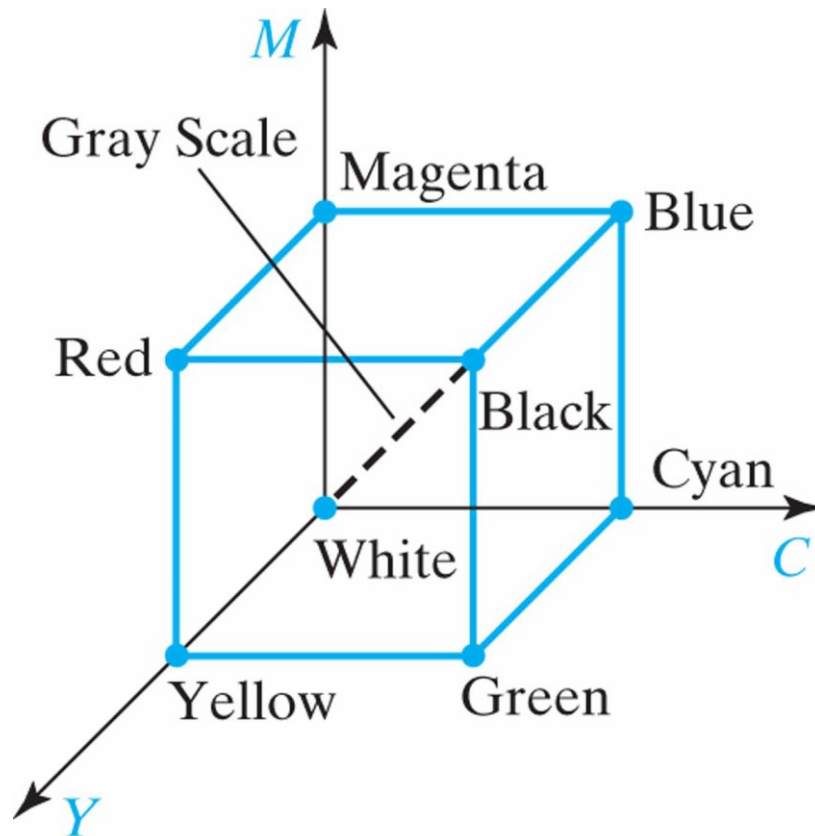
	NTSC Standard	CIE Model
R	(0.670, 0.330)	(0.735, 0.265)
G	(0.210, 0.710)	(0.274, 0.717)
B	(0.140, 0.080)	(0.167, 0.009)

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# CMY Color Model

- Subtractive



$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} C \\ M \\ Y \end{bmatrix}$$

# CMYK Color Model

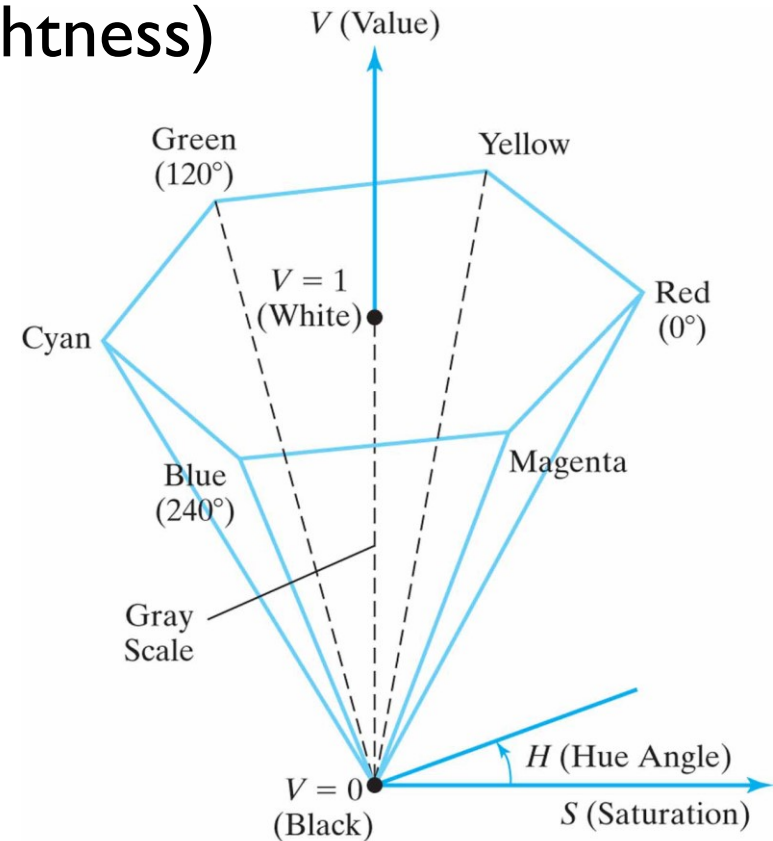
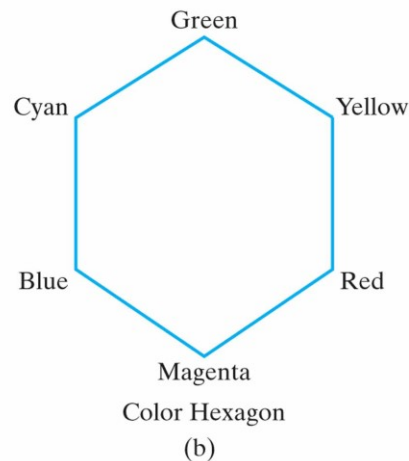
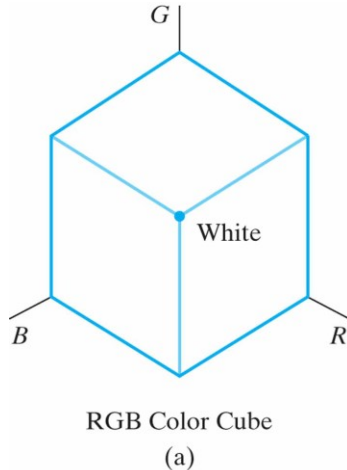
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- Use K for richer black
  - Black by CMY is dark gray
  - Use less ink
- $K = \min(C, M, Y)$   
 $C' = C - K$   
 $M' = M - K$   
 $Y' = Y - K$   
(one of  $C', Y', M'$  will be 0)



# HSV Color Model

- Intuitive color selection
  - Hue, saturation, value (brightness)
  - Rearrangement of RGB



# HSV Color Model

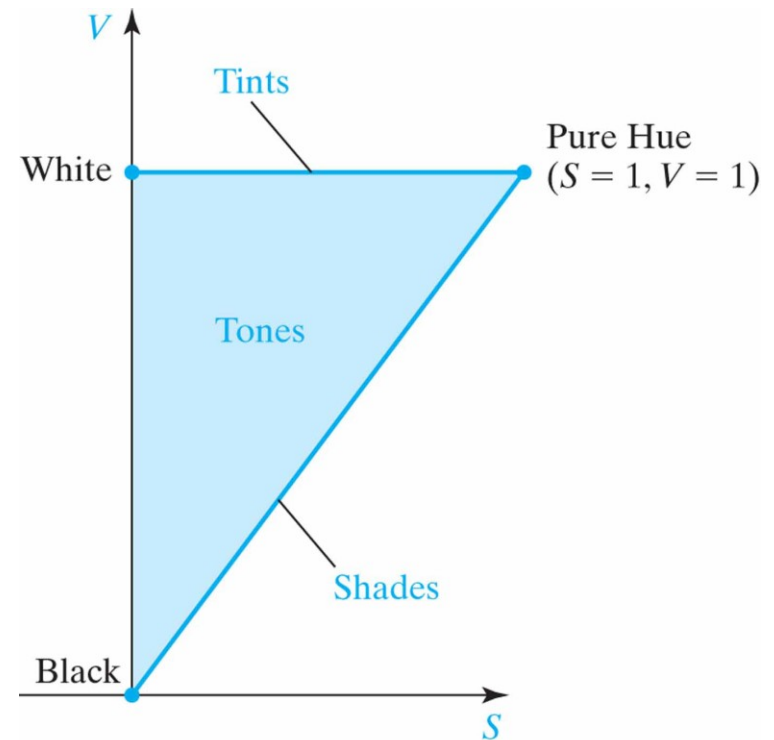
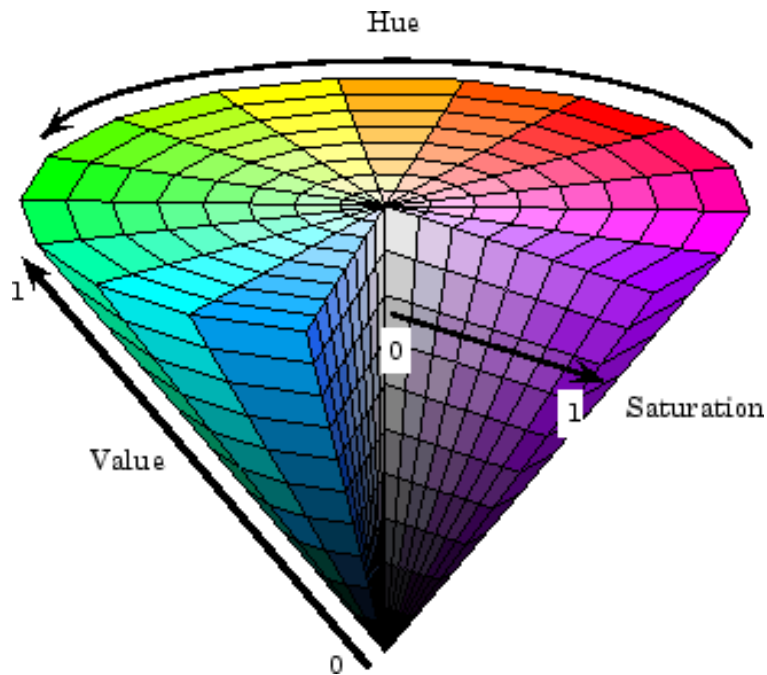


Image courtesy of Hearn

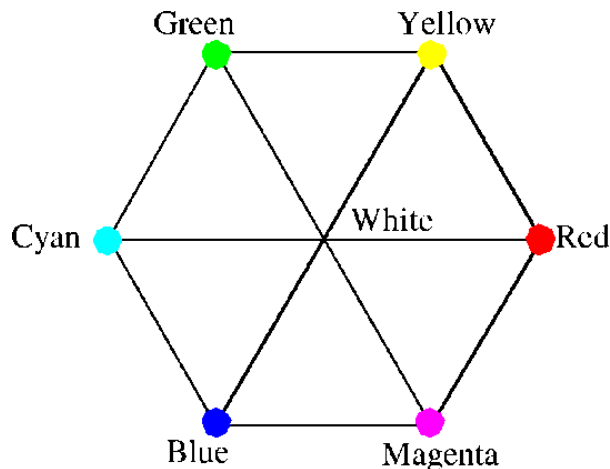


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# HSV Color Models

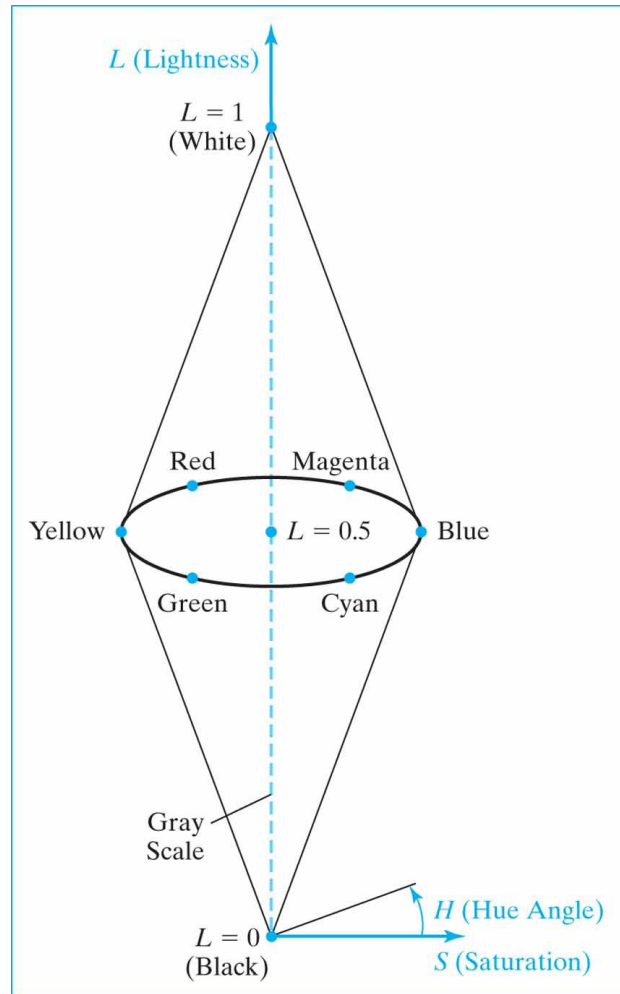
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- Colors on  $V=I$  plane are not perceptually uniform (not equally bright)
- Complementary colors are  $180^\circ$  opposite
- $S=I$  is not pure spectral color
  - HSV color space is a subset of CIE space



# HSL Color Model

- Similar to HSV except double-cone model



White:  $L=1$

Black:  $L=0$

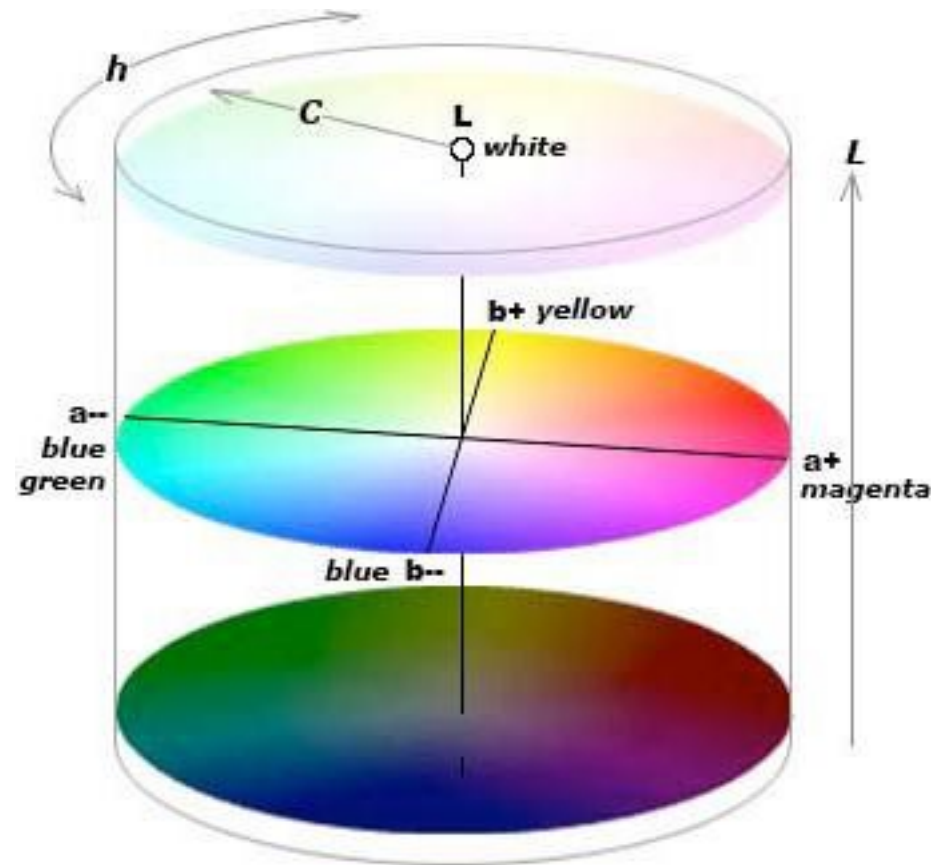
Pure color:  $L=0.5, S=1$

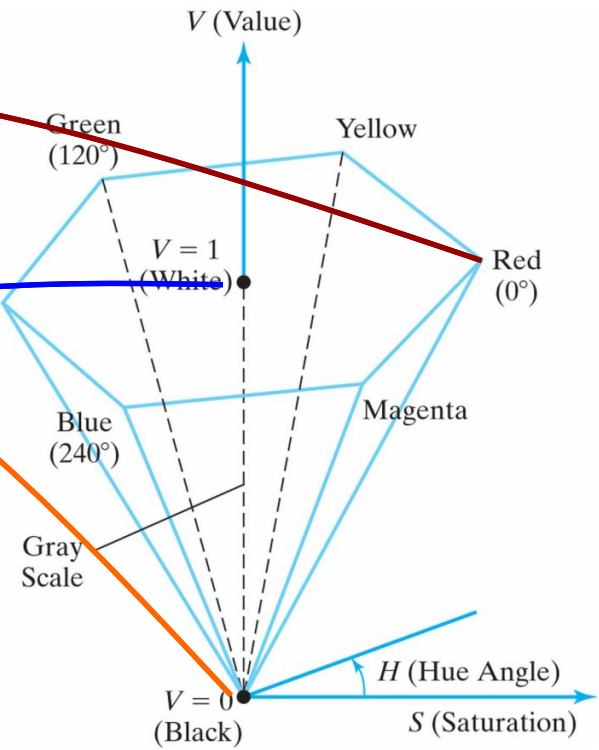
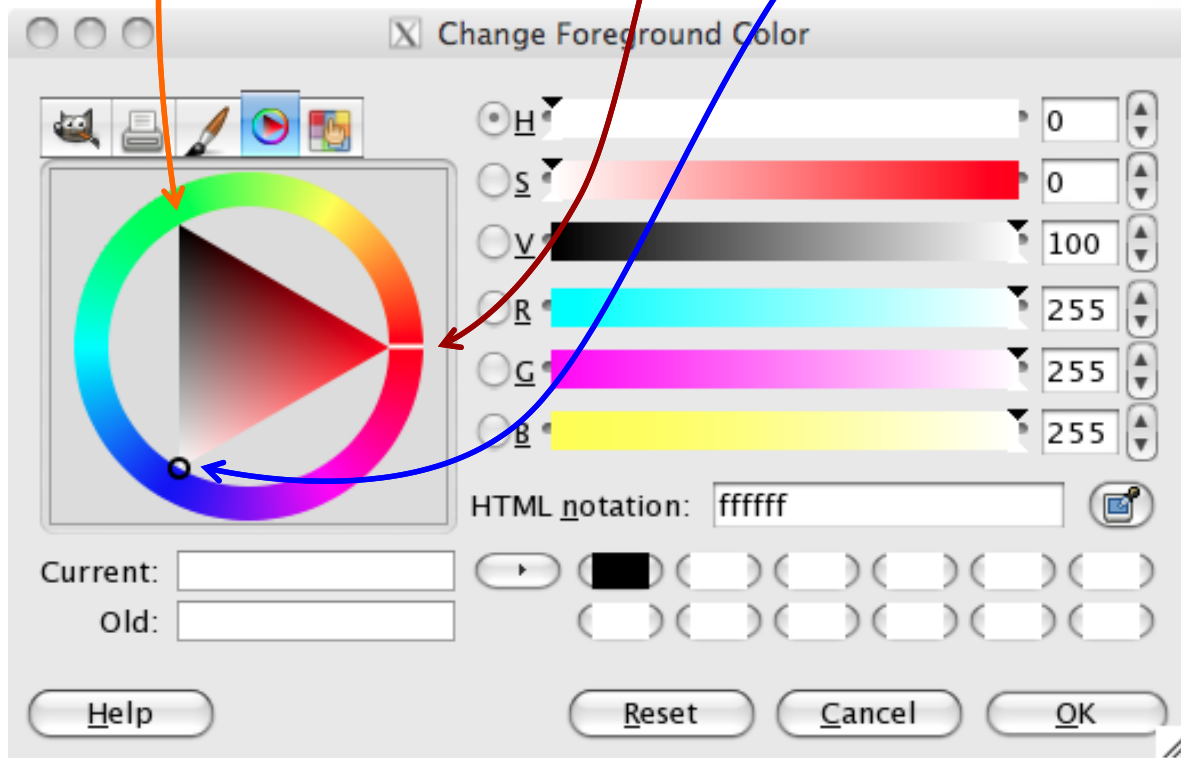
Image courtesy of Hearn



# LAB / HCL Color Model

- Perceptually uniform
  - L : luminance
  - a : green to red
  - b : blue to yellow
- HCL is a cylindrical transformation of LAB







# Questions?