Module : IV

Game

→ Assumption of a game.

→ Strategy → planning — pure strategy (a particular method) pre-determine
— Mixed strategy (plan will change,

→ **Pay off matrix:-**

		player B	
		B_1	B_2
player A	A_1	1	2
	A_2	3	4

lost ↓ (player B); gain → (player A)

if A uses strategy A_1 & B uses B_2 strategy - then payoff is $\underline{\underline{2}}$.

ie, A gets a gain of 2.
and B losses 2.

A is maximizing player.
B is minimizing player.

→ Value of the game
maximum value of A - while using their strategy. (ie, A & B)

→ **Maxmin - principle:-**

minimum gain of A using $A_1 = 1$.
" A $A_2 = 3$.

Max of Row minimum $= 3$. ie, it is maximum value.

∴ he uses A_2 strategy.

~~Max~~ MinMax Principle:-

Maximum loss of B using $B_1 = 3$

" $B_2 = 4$.

∴ Min of Column max = 3 // ie minimum value.

ie, he uses B_1 strategy.

This is called Minmax - Maxmin Principle.

- If the maxmin value equal to minmax value then the game is said to have a saddle point.
- The Corresponding strategy called optimum strategies.
- The amount of pay off at saddle point is called value of the game. ie. Eg: 3.

Eg:

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{Row min} \\ \hline A_1 & 0 & 2 & \to 0 \\ A_2 & -1 & 1 & -1 \\ \hline \text{Column Max} \downarrow & 0 & 2 & \end{array}$$

∴ minmax $= \min\{0, 2\} = 0$//

Maxmin $= \max\{0, -1\} = 0$//

∴ value of the game = 0//

→ Zero Sum Game:-

Sum is zero.

Q.

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & 5 & 3 & 2 \\ A_2 & 1 & -2 & 0 \\ A_3 & 8 & -1 & 1 \end{array}$$

Player B, player A. 9

Find row min & colum max.

Ans:

$$\begin{array}{c|ccc|c} & & & & \text{Row min} \\ \hline & 5 & 3 & 2 & \to 2 \\ & 1 & -2 & 0 & -2 \\ & 8 & -1 & 1 & -1 \\ \hline \text{Colum max} \downarrow & 8 & 3 & 2 & \end{array}$$

minmax = min of column max $\{8, 3, 2\}$
$= 2$//

maxmin = max of row min $\{2, -2, -1\}$
$= 2$//

It has saddle point value of the game = 2.

optimum Strategy (A_1, B_3).

→ Mixed Strategy

2 row & 2 co

1) Probability Method:- This method is applied when there is no saddle point and they pay off matrix has two rows and two columns only. The players may adopt mixed strategy with certain probabilities.

$p \rightarrow$ denote probability for A using A_1
$1-p$ " A using A_2.

$q \rightarrow$ " for B using B_1
$1-q \rightarrow$ " " B_2.

~~Dat~~ Player B

$$\text{Player A} \begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & a & b \\ A_2 & c & d \end{array}$$

$$\boxed{P = \frac{d-c}{(a+d)-(b+c)} \qquad q = \frac{d-b}{(a+d)-(b+c)} \qquad V = \frac{ad-bc}{(a+d)-(b+c)}}$$

20-5-24
Monday

Q. Solve

Player B

$$\text{Player A} \begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & -2 & -1 \\ A_2 & 2 & -3 \end{array} \quad ?$$

Ans.

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{row min} \\ \hline A_1 & -2 & -1 & -2 \\ A_2 & 2 & -3 & -3 \\ \hline \text{Column Max} & 2 & -1 & \end{array}$$

minmax = min{column max}
= min{2, -1} = -1

maxmin = max{row min}
= max{-2, -3} = -2

minmax $\neq$ maxmin.
∴ no saddle point.
Mixed Strategy.

2x2 matrix.
∴ probability method.

$a = -2 \quad b = -1 \quad c = 2 \quad d = -3$.

(i) ∴ $P = \frac{d-c}{(a+d)-(b+c)} = \frac{-3-2}{(-2+-3)-(-1+2)} = \frac{-5}{(-5)-(1)} = \frac{+5}{+6} = 5/6$

(ii) $1-P = 1 - 5/6 = \frac{6-5}{6} = 1/6$

(iii) $q = \frac{d-b}{(a+d)-(b+c)} = \frac{-3- -1}{(-5)-(1)} = \frac{-3+1}{-6} = \frac{-2}{-6} = 1/3$

(iv) $1-q = 1 - \frac{1}{3} = \frac{3-1}{3} = 2/3$

(v) $V = \frac{ad-bc}{(a+d)-(b+c)} = \frac{-3\times-2 - 2\times-1}{-6} = \frac{6+2}{-6} = 8/-6 = -4/3$

→ Graphical Method:-

→ Solution for $2 \times n$ games.

- Shade the lower portition and consider the highest point.

Q.
$$\begin{array}{c|c} & \begin{matrix} B_1 & B_2 & B_3 & B_4 & B_5 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \end{matrix} & \begin{bmatrix} 2 & -4 & 6 & -3 & 5 \\ -3 & 4 & -4 & 1 & 0 \end{bmatrix} \end{array}$$
?

Ans: find the minmax & maxmin.

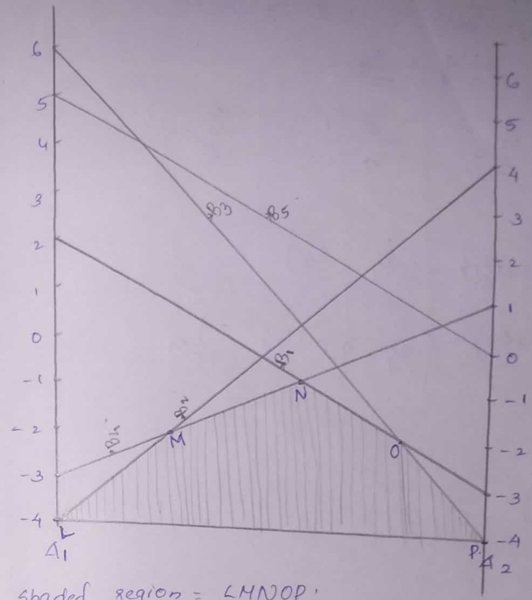
$$\begin{array}{c|c|c} & \begin{matrix} B_1 & B_2 & B_3 & B_4 & B_5 \end{matrix} & \text{row min} \\ \begin{matrix} A_1 \\ A_2 \end{matrix} & \begin{bmatrix} 2 & -4 & 6 & -3 & 5 \\ -3 & 4 & -4 & 1 & 0 \end{bmatrix} & \begin{matrix} -4 \\ -4 \end{matrix} \\ \text{Column max} & \begin{matrix} 2 & 4 & 6 & 1 & 5 \end{matrix} & \end{array}$$

∴ minmax = min{column max}
$\quad$ min{2, 4, 6, 1, 5} = 1

maxmin = max{row min}
$\quad$ max{-4 -4} = -4

minmax ≠ maxmin.

∴ no saddle point.



Shaded region = LMNOP.

highest Point = N.

B_4 & B_1 meet at N.

∴ take a matrix with B_1 & B_4.

$$\begin{array}{c|c} & \begin{matrix} B_1 & B_4 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \end{matrix} & \begin{bmatrix} 2 & -3 \\ -3 & 1 \end{bmatrix} \end{array}$$

∴ $a = 2 \quad b = -3 \quad c = -3 \quad d = 1$

$$P = \frac{d-c}{(a+d)-(b+c)} = \frac{1--3}{(1+2)-(-3+-3)} = \frac{4}{3--6} = \frac{4}{9}$$

$$\therefore 1-P = 1-\frac{4}{9} = \frac{9-4}{9} = 5/9$$

$$q = \frac{d-b}{(a+d)-(b+c)} = \frac{1 - -3}{9} = \frac{4}{9}$$

$$\therefore 1-q = 1-\frac{4}{9} = \frac{5}{9}$$

$$v = \frac{ad-bc}{(a+d)-(b+c)} = \frac{2-9}{9} = -\frac{7}{9}$$

A's strategy $= \left(\frac{4}{9}, \frac{5}{9}\right)$

B's strategy $= \left(\frac{4}{9}, 0, 0, \frac{5}{9}, 0\right)$

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Q.

$$\begin{array}{c|ccccc} & B_1 & B_2 & B_3 & B_4 & B_5 \\ A_1 & 6 & 3 & -1 & 0 & -3 \\ A_2 & 3 & 2 & -4 & 2 & -1 \end{array}$$

?

Ans:-

$$\begin{array}{c|ccccc|c} & B_1 & B_2 & B_3 & B_4 & B_5 & \text{rowmin} \\ A_1 & 6 & 3 & -1 & 0 & -3 & -3 \\ A_2 & 3 & 2 & -4 & 2 & -1 & -4 \\ \hline \text{column max} & 6 & 3 & -1 & 2 & -1 & \end{array}$$

minmax = min {column max}

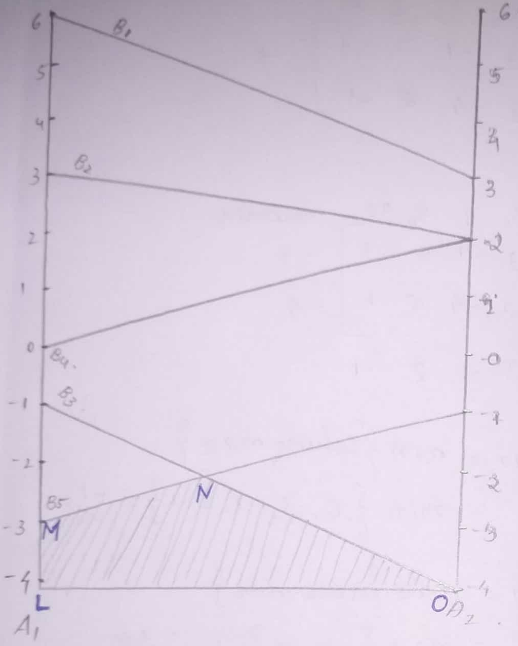
= min {6, 3, -1, 2 -1} = -1//

maxmin = max {rowmin}

= max {-3, -4} = -3//

∴ no saddle point

ie, maxmin ≠ minmax.



Shaded region = LMNO.

highest point = N.

B_5 & B_3 meet at N.

∴ matrix:

$$\begin{array}{c|cc} & B_3 & B_5 \\ \hline A_1 & -1 & -3 \\ A_2 & -4 & -1 \end{array}$$

$$P = \frac{d-c}{(a+d)-(b+c)} = \frac{-1 - -4}{(-1+-1)-(-3+-4)} = \frac{-1+4}{(-2)-(-7)} = \frac{3}{5}$$

$-2 + 7$

$$\therefore 1-p = 1-\frac{3}{5} = \frac{2}{5}$$

$$q = \frac{d-b}{(a+d)-(b+c)} = \frac{-1 - -3}{5} = \frac{-1+3}{5} = \frac{2}{5}$$

$$\therefore 1-q = 1-\frac{2}{5} = \frac{3}{5}$$

$$v = \frac{ad-bc}{(a+d)-(b+c)} = \frac{-1\times -1 - -3\times -4}{5} = \frac{1-12}{5} = \frac{-11}{5}$$

A's strat ~~4/9, 5/9~~ $\left(\frac{3}{5}, \frac{2}{5}\right)$

B's strategy ~~(0, 0, 4/9, 0, 5/9, 0)~~ $\left(0 \quad 0 \quad \frac{2}{5} \quad 0 \quad \frac{3}{5}\right)$

→ Solution for $n \times 2$ Matrix

Shade the upper portion and consider the lowest point.

Q.

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & -6 & 7 \\ A_2 & 4 & -5 \\ A_3 & 1 & -2 \\ A_4 & 2 & 5 \\ A_5 & 7 & -6 \end{array}$$

?

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{row min} \\ \hline A_1 & -6 & 7 & -6 \\ A_2 & 4 & -5 & -5 \\ A_3 & 1 & -2 & -2 \\ A_4 & 2 & 5 & 2 \\ A_5 & 7 & -6 & -6 \\ \hline \text{column max} & 7 & 7 & \end{array}$$

$\therefore$ minmax = min{columnmax}

= min{7, 7} = 7

maxmin = max{rowmin}

max{-6, -5, -2, 2, -6}

= 2

minmax $\neq$ maxmin $\therefore$ no saddle point.

M is lowest

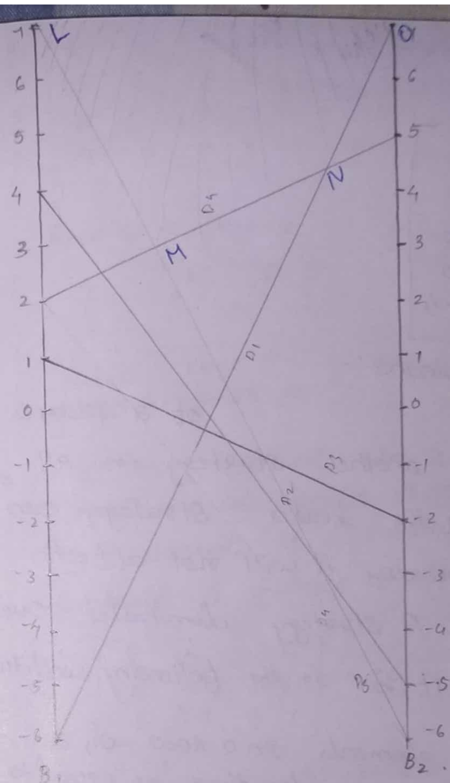
A_4 & A_5. meet at M.

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_4 & 2 & 5 \\ A_5 & 7 & -6 \end{array}$$

$\therefore p = \frac{d-c}{(a+d)-(b+c)} = \frac{-6-7}{(2+-6)-(5+7)} = \frac{-13}{-16}$

$-4-(12)$

$\therefore 1-p = 1-\frac{13}{16} = 3/16$



$q = \frac{11}{16}$ $\quad 1-q = 1-11/16 = 5/16$

$v = \frac{-47}{-16} = 47/16$

A's strategy = (0, 0, 0, 13/16, 3/16)

B's strategy = ($^{9}/_{16}$, $^{7}/_{16}$)

Q. H.W

$$A \begin{array}{c} \\ \end{array}\begin{array}{cc} B_1 & B_2 \\ \end{array}$$

$$\begin{bmatrix} -2 & 5 \\ -5 & 3 \\ 0 & -2 \\ -3 & 0 \\ 1 & -4 \end{bmatrix}$$

Principle of dominance :-

It says that of a player dominate over another strategy in all condition, then the later strategy can be ignored, because it will not affect the solution. A strategy dominates over the other if it is in the following condition.

i, if all the elements in a row of a pay of matrix are less than or equal to corresponding elements of another row. then the later dominate and so former is ignored.

Eg:

$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{bmatrix} 2 & 4 & 3 & 4 \\ 5 & 6 & 3 & 8 \\ 6 & 7 & 9 & 7 \\ 4 & 2 & 8 & 3 \end{bmatrix}$$

Row 4 ≤ Row 3.

$$\begin{array}{cccc} 4 & 2 & 8 & 3 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 6 & 7 & 9 & 7 \end{array}$$

delete row 4.
delete the small row.

$$\therefore \begin{bmatrix} 2 & 4 & 3 & 4 \\ 5 & 6 & 3 & 8 \\ 6 & 7 & 9 & 7 \end{bmatrix}$$

ii → if all the elements in a column of a pay of matrix are greater than equal to correspon-ding elements of a column, then the latter dominate & former is ignored.

Eg: $\begin{bmatrix} 2 & 4 & 3 & 4 \\ 5 & 6 & 3 & 8 \\ 6 & 7 & 9 & 7 \end{bmatrix}$ →

Column 1		Column 4
2	≤	4
5	≤	8
6	≤	7

delete the greater column.

$$\therefore \begin{bmatrix} 2 & 4 & 3 \\ 5 & 6 & 3 \\ 6 & 7 & 9 \end{bmatrix}$$

→ Queueing Theory. -

Essay: Elements of (characteristics) of Queueing System

(1) Input or arrival Process
- Queue Discipline
- Service

Input → size of queue
- finite
- infinite -
- Patter Arrival -
→ deterministic
prob

customer are of 2 type based behavior.

1) Inpatient:
it are of 3 type. Balked Reneged Jockeyed

i (1) Balked: A customer may leave the queue because the queue is too long and the

queue → big → so leave

2) Reneged :

3) Jockeyed.
shift from one queue to another → for speed need.

2) Queue Discipline: -
FIFO :-
LIFO :-
- Service in random order (S
It is rules according to which customer are selected for service.

III . Single Queue & Single Server.
- Several Queue & Single Sever.
- Several Queue & Several Sever. (Railway bookin
- Single Queue & Several Severs.

System Capacity: -
The Source from which customer are generated

Model - I.

This queueing model deals with Queueing system having single service channel. Poisson input, Exponential service. There is no limit on system capacity while customers are served on first in first out basis.

(Average) Expected number of customers in System

$$\boxed{L_s = \frac{\lambda}{\mu - \lambda}}$$

where λ - is the average ... customer arriving per unit of time.

μ - avg. no. of customers completing service per unit of time.

Average (expected) no. of customers in queue.

$$\boxed{L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}}$$

Expected waiting time in the system

$$\boxed{W_s = \frac{1}{\mu-\lambda}}$$

Expected waiting time in the queue.

$$\boxed{W_q = \frac{\lambda}{\mu(\mu-\lambda)}}$$

The server utilization factor (or busy period) or traffic intensity = $\boxed{\rho = \frac{\lambda}{\mu}}$

It is the proportion of time that a server actually spends with the customer.

average length of non-empty queue = $\boxed{\frac{\mu}{\mu-\lambda}}$

Probability of non customers, $P_0 = 1-\rho$

probability of 'n' customers, $P_n = \rho(P_0)$
$\rho^n(1-\rho)$

probability of queue size being greater than or equal to k.

$$P(\geq k) = \rho^k.$$

Q. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Service time on an average is 36 minutes calculate.

i, Average length of non-empty queue

ii, Prob. that queue size exceeds 10.

iii, Avg. waiting time.

Ans: avg. no of customers arriving per unit time

λ = 30 trains per unit time.

= 30 trains per day.

avg. no of customer completing service per day

μ = ______ train per day

1 train is served in 36 minutes.

how many can be served in 1 minutes.

$= \frac{1}{36}$ train.

how many in 1 hr:
$= \frac{1}{36} \times 60 = \frac{10}{6} = 5/3$

how many train can be served in one day

$= \left(\frac{1}{36} \times 60\right) \times 24$ ie, $\frac{5}{3} \times 24 = 5 \times 8 = 40$

1) Avg. length

$$\frac{\mu}{\mu - \lambda} = \frac{\cancel{30}\ 40}{40 - 30} = \frac{40}{10} = 4$$

2) Probability that q size exceeds 10.

$$P(\geq 10) = \rho^{10}$$

$$= \left(\frac{\lambda}{\mu}\right)^{10}$$

$$= \left(\frac{30}{40}\right)^{10} = (.75)^{10}$$

$$= .056$$

• avg. waiting time?

avg. waiting time in queue.

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$= \frac{30}{40(40-30)} = \frac{3\cancel{0}}{40 \times 1\cancel{0}} = \frac{3}{40} = .015$$

Q. A customer arrives at one window drive in a bank according to poission distribution with mean 10 per hr.

Ans: $\lambda = 10$ pr. hr.

μ = ______

1 $\longrightarrow$ 5 minutes.

1/5 —— 1 minutes

$\frac{1}{5} \times 60$ —— 60 minutes

$\therefore$ 12 pr. hr//

i) prob. that arriving customer can drive directly in the space in front of window.

$= P(0\text{ cars})$ or $P(1\text{ car})$ or $P(2\text{ cars})$.

$= P_0 + P_1 + P_2$.

$= P_0 = 1-\rho$

$P_1 = \rho^1 P_0$

$P_2 = \rho^2 P_0$.

$P_0 + P_1 + P_2 = P_0 + \rho^1 P_0 + \rho^2 P_0$

$= P_0 \left[1 + \rho + \rho^2\right]$

$= (1-\rho)\left[1 + \rho + \rho^2\right]$

$= \left(1 - \frac{10}{12}\right)\left(1 + \frac{10}{12} + \left(\frac{10}{12}\right)^2\right) = \frac{2}{12}$.

$= \frac{2}{12}\left[\frac{144 + 120 + 100}{144}\right] = \underline{.42}$

iii) P(the customer has to wait outside)

1- P (customer can park the car

$1 - .42 = .58$//

iii, $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{12(12-10)} = \frac{10}{12\times 2}$

$= \frac{5}{12} = \underline{\underline{.417}}$

Q. A belt snapping for conveyors in an open cast mine occur at the rate of 2 per ~~unit~~. shift. There is only one hot plate available for vulcanissing and it can vulcanise on an averge 5 belts snap per shift

a) what is the probability that when a belt snaps, the hot plate is readility available.

b) what is the average number of belts in the system ?

c) What is the waiting time of an arrival?

d) what is the avg. waiting time plus vulcanising time.

$\lambda = 2 \qquad \mu = 5$.

a) Pro. of facility being empty

or

$p(0) = 1 - p = 1 - \frac{\lambda}{\mu}$

$= 1 - \frac{2}{5} = 3/5$.

b) $L_s = \frac{\lambda}{\mu - \lambda} = \frac{2}{5-3} = 2/3$

c) $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{2}{5(5-2)} = 2/15$ shift

d) $W_s = \frac{1}{\mu-\lambda} = \frac{1}{5-2} = 1/3$ shift.

Q.

- single mechanic.

$\lambda = 4$ cus an hr. six min.

$\mu =$ $1 \longrightarrow 6$ min.

$1/6 \longrightarrow 1$ min

$\frac{1}{6} \times 60^{10} \longrightarrow 60$ min

$\therefore \mu = 10$

a) $p(0) = 1 - \rho = 1 - \frac{\lambda}{\mu} = 1 - \frac{4}{10}$ $\frac{2}{5}$

$= \frac{5-2}{5} = 3/5$

b) ~~$P_4 + P_3 + P_2 + P_1$~~

$P_1 = \rho^1 P_0$

or

$(1 - P_0) = 1 - \frac{3}{5} = \frac{5-3}{5} = 2/5$

c) $L_s = \frac{\lambda}{\mu - \lambda} = \frac{4}{6} = .67$

$W_s = \frac{1}{\mu - \lambda} = \frac{1}{6} = .167$