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→ Introduction to OR:- Operation Research:

. In the time of 2nd world war.

. Due to Scarcity in that time, group of scientist find a solution to find a optimum solution.

→ C.W. Churchman define

OR as "Operation Research is the application of scientific methods, techniques and tools to problems involving the operations of a system so as to provide those in control of the system with optimum solution to the problem.

→ features of OR:-

i, System Orientation:-
it considered as whole not parts.

ii, continous process:-
It have various steps / phases so it is Continous.

iii, Interdisciplinary team approach:-
field.
Different fields are used this method.

iv, Scientific Approach:
with help of scientific methods.

v. Decision Making:-
for an uncertainty situation, it used for find decision

vi. Bad answer to the problem: it changed from worst to bad

vii. Use of Computers.

→ Phases of OR / Process of OR / Methodology of OR.

1. Identification of the problem
2. Formulating the problem: collect resource.
3. Constructing the model: rough model & create objectives.
4. Selecting appropriate input data:
5. Deriving the solution
6. Testing the validity.
7. Controlling the solution
8. Implementing the result!

→ O.R Techniques / Tools of OR / important OR Model.

- Allocation model: more used in business field. distribute the limited resources
- Sequencing:- reducing the time gap. b/w operation.
- Waiting or Queuing Theory:- used for waiting time reduce. Source provide to us
- Inventory Model:- maintain the stoke
- Decision Theory:-
- Simulation:- creating a dummy one for execute
- Replacement Theory:-

→ linear Programming
LPP includes a set of simultaneous equations or inequality which represents the restrictions related to the limited resources and a linear function which expresses the objective function representing the total profit or ~~loss~~. cost.
This technique which helps us to find the optimum solution for a given problem.

→ General form of LPP

$$\text{Max or Min } z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq \text{ or } \geq \text{ or } = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq \text{ or } \geq \text{ or } = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq \text{ or } \geq \text{ or } = b_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

→ Components of LPP

i, objective , Max or Min $z = c_1x_1 + c_2x_2 \dots + c_nx_n$

ii, Decision variables: $x_1, x_2 \dots x_n$

iii, Constraints: $a_{11}x_1 \pm a_{12}x_2 \pm a_{13}x_3 \dots$

iv Non negative restrictions: $x_1, x_2, x_3 \dots \geq 0$.

→ Basic assumptions of LPP.-

. Proportionality

There must be proportionality between objectives and constraint. for example, if we want to double the output, we ~~coust to~~ have to double the constraints.

2) Addictivity:→ Sum of the resources used by different activities must be ~~used~~ ~~by~~ equal to the total quantity of resources used by each activity for all the resources individually.

3) Divisibility:→ The solution need not be in whole numbers

4) Sertanity:→ coefficience in the objective functions and constraints don't change during the formulation of LPP.

5) finiteness:→ (Non negativity) Activities and constraints are always non negative.

6) Optimality:→ The solution to a problem is to be optimum.

⟹ Standard form of LPP:-

. Max z or Min z. $c_1x_1 + c_2x_2 + \dots c_nx_n$

. subject to.

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$

$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$

$x_1, x_2 \dots x_n \geq 0$.

⟹ Canonical form

. Maximum only,

. less than.

Max z : $c_1x_1 + c_2x_2 + c_nx_2$.

subject to

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$

.

$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$

$x_1, x_2 \dots x_n \geq 0$

Q. Write the given LPP (linear Programming) in the standard form.?

Min $z = 8x_1 + 14x_2$

subject to, $7x_1 - 5x_2 \geq 20$

$12x_1 + 8x_2 = 60$

$x_1, x_2 \geq 0$

The inequality can be changed by introducing a non-negative variable on the LHS of such constraint.

slack variable is to be add the constraint '≤' and surplus variable is to be subtracted if the constraint is to be '≥'.

* For the standard form of LPP the constraint only in equal (=) symbol.

∴ Min $z = 8x_1 + 14x_2$

Subject to,

$7x_1 - 5x_2 - x_3 = 20$

$12x_1 + 8x_2 = 60$

$x_1, x_2, x_3 \geq 0$

Q. Write LPP in standard form?

Max $z = 3x_1 + 5x_2 + 7x_2$

subject to,

$6x_1 - 4x_2 \leq 5$

$3x_1 + 2x_2 + 5x_3 \geq 11$

$4x_1 + 3x_2 \leq 2$

Ans: * no change for objective

∴ $z = 3x_1 + 5x_2 + 7x_2$

* first reduce the '≤' constraint ie, slack.

∴ $6x_1 - 4x_2 \leq 5 \xRightarrow{\text{changed to}} 6x_1 - 4x_2 + x_4 = 5$

$4x_1 + 3x_2 \leq 2 \xRightarrow{\text{changed to}} 4x_1 + 3x_2 + x_5 = 2$

* then reduce the '≥' constraint, ie surplus.

$3x_1 + 2x_2 + 5x_3 \geq 11 \xrightarrow[\text{to}]{\text{changed}} 3x_1 + 2x_2 + 5x_3 - x_6 = 11$

∴ Max $z = 3x_1 + 5x_2 + 7x_2$

Subject to, $6x_1 - 4x_2 + x_4 = 5$

$4x_1 + 3x_2 + x_5 = 2$

$3x_1 + 2x_2 + 5x_3 - x_6 = 1$

$x_1\ x_2\ x_3\ x_4\ x_5\ x_6 \geq 0$

Q. Write the LPP in Canonical form ?

$$\text{Max } z = 8x_1 + 5x_2$$

Subject to,

$$8x_1 - 2x_2 \geq 23$$
$$7x_1 + 9x_2 = 10$$
$$x_1,\ x_2 \geq 0$$

Ans: no change for objective.

$$\text{Max } z = 8x_1 + 5x_2$$

for canocial form all are in less than form. So greater than can be changed to less than by Multiply by (-1)

$$\therefore\ 8x_1 - 2x_2 \geq 23 \xrightarrow[\text{changed to}]{} $$

$$-8x_1 + 2x_2 \leq -23.$$

* for 2nd constraint it split into less than greater than

$$7x_1 + 9x_2 = 10 \xrightarrow[\text{changed to}]{} \begin{array}{l} 7x_1 + 9x_2 \geq 10 \\ 7x_1 + 9x_2 \leq 10 \end{array}$$

again greater than changed to less than with (-1)

$$-7x_1 - 9x_2 \leq -10$$

$\therefore$ Max $z = 8x_1 + 5x_2$

Subject to,

$$-8x_1 + 2x_2 \leq -23$$
$$7x_1 + 9x_2 \leq 10$$
$$-7x_1 - 9x_2 \leq -10$$
$$x_1,\ x_2 \geq 0$$

HW

Q. Write the LPP in Canonical form ?

$$\text{Min } z = 2x_1 + 3x_2$$

Subject to,

$$x_1 + x_2 = 5$$
$$5x_1 - 2x_2 \geq 3$$
$$x_1,\ x_2 \geq 0$$

Ans. · Maximum
· less than

Min changed to Max by Multiply (-1).

$$\therefore\ \text{Max } z = -2x_1 - 3x_2$$

$$x_1 + x_2 = 5 \longrightarrow \begin{array}{l} x_1 + x_2 \geq 5 \\ x_1 + x_2 \leq 5 \end{array}$$

changed to

$$-x_1 - x_2 \leq -5$$

$$\cdot\ 5x_1 - 2x_2 \geq 3 \longrightarrow -5x_1 + 2x_2 \leq -3$$

Q. Write the LPP in canonical form?

Max $Z = 8x_1 + 5x_2$

Subject to,

$8x_1 - 2x_2 \geq 23$

$7x_1 + 9x_2 = 10$

$x_1, x_2 \geq 0$

Ans: no change for objective.

Max $Z = 8x_1 + 5x_2$

for canonical form all are in less than form. So greater than can be changed to less than by Multiply by (-1)

$8x_1 - 2x_2 \geq 23 \xrightarrow{\text{changed to}}$

$-8x_1 + 2x_2 \leq -23$.

* for 2nd constraint it split into less than greater than

$7x_1 + 9x_2 = 10 \xrightarrow{\text{changed to}}$ $7x_1 + 9x_2 \geq 10$, $7x_1 + 9x_2 \leq 10$

again greater than changed to less than with (-1)

$-7x_1 - 9x_2 \leq -10$

$\therefore$ Max $Z = 8x_1 + 5x_2$

Subject to,

$-8x_1 + 2x_2 \leq -23$

$7x_1 + 9x_2 \leq 10$

$-7x_1 - 9x_2 \leq -10$

$x_1, x_2 \geq 0$

HW

Q. Write the LPP in canonical form?

Min $Z = 2x_1 + 3x_2$

Subject to, $x_1 + x_2 = 5$

$5x_1 - 2x_2 \geq 3$

$x_1, x_2 \geq 0$

Ans: • Maximum

• less than

Min changed to Max by Multiply (-1).

Max $Z = -2x_1 - 3x_2$

$x_1 + x_2 = 5 \longrightarrow$ $x_1 + x_2 \geq 5$, $x_1 + x_2 \leq 5$

changed to

$-x_1 - x_2 \leq -5$

• $5x_1 - 2x_2 \geq 3 \longrightarrow -5x_1 + 2x_2 \leq -3$

$\therefore$ Max $z = -2x_1 \ldots$

subject to, $x_1 + x_2 \leq 5$

$-x_1 - x_2 \leq -5$

$-5x_1 + 2x_2 \leq -3$

$x_1, x_2 \geq 0$

⇒ Applications of Operations Research:-

Todays, almost all fields of business and government utilizing the benefits of Operations Research. There are voluminous of applications of Operations Research. Although it is not feasible to cover all applications of OR in brief. The

→ Allocation and Distribution in Projects:-

Optimal allocation of resources such as men material machines, time and money to projects

- Determination and deployment of proper workforce.
- Project scheduling, monitoring and control

→ Production and facilities planning

- factory size and location decision.
- Estimation of number of facilities required.
- Preparation of forecasts for the various inventory items and computation of economic order quantities and reorder levels.
- Scheduling and sequencing of production runs by proper allocation of machines.
- Transportation loading and unloading.
- Warehouse location decision
- Maintenance policy decision.

→ Programmer Decisions:-

- What, when and how to purchase to minimize procurement cost.
- Bidding and replacement policies.

→ Marketing:-

- Advertising budget, allocation
- Product introduction timing.
- Selection of advertising media
- Selection of product mix.

→ Organization Behaviour:

- Selection of personnel, determination of retirement age and skills.
- Recruitment policies and assignment of Jobs.
- Recruitment of Employees.
- Scheduling of training programs.

→ Finance:

- Capital requirements, cash flow analysis
- Credit policies, credit risk etc.
- Investment decision.
- Profit plan for the Company.

→ Research and Development:

- Product introduction Planning
- Control of R&D projects
- Determination of areas for research and development
- Selection of projects and preparation of their budget.

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Q. Write the LPP in standard form.

Max $z = 2x_1 + 5x_2$

Subject to, $3x_1 + 2x_2 \le 6$

$2x_1 + 9x_2 \ge 8$

$x_1, x_2 \ge 0$

Ans:

Max $z = 2x_1 + 5x_2$

$3x_1 + 2x_2 \le 6 \xrightarrow[\text{changed to}]{} 3x_1 + 2x_2 + x_3 = 6$

$2x_1 + 9x_2 \ge 8 \xrightarrow[\text{changed to}]{} 2x_1 + 9x_2 - x_4 = 8$

$x_1, x_2, x_3, x_4 \ge 0$.

Q. Write the LPP in Canonical form?

Max $z = x_1 - 2x_2 - x_3 + x_4$

Subject to, $x_1 + x_2 + x_3 = 14$

$x_1 + x_2 \ge 9$.

Ans: $x_1 + x_2 + x_3 = 14 \xrightarrow[\text{changed to}]{}$ $x_1 + x_2 + x_3 \ge 14$, $x_1 + x_2 + x_3 \le 14$.

changed to

$-x_1 - x_2 - x_3 \le -14$.

$x_1 + x_2 \geq 9 \xrightarrow{\text{Changed to}} -x_1 - x_2 \leq -9$

$\therefore$ Max $z = x_1 - 2x_2 - x_3 + x_4$

$x_1 + x_2 + x_3 \leq 14$

$-x_1 - x_2 - x_3 \leq -14$

$-x_1 - x_2 \leq -9$

$x_1, x_2, x_3, x_4 \geq 0$

⇒ Mathematical formulation of LPP:

Step 1: Identify decision variable

Step 2: Identify objective as Maximize or minimize and express it as linear function.

Step 3: Identify constraints, express it as linear equations or inequalities.

Step 4: Write the non negative restrictions.

Q. A furniture company produces tables and chairs. the production process for each is similar in that both require a certain number of hours of carpentary work and a certain number of labour hours in the painting department. Each table takes 4 hours of carpentary and 2 hr in the painting department. Each chair require 3 hrs of carpentary and 1 hr in the painting department.
During the current production period, 240 hrs of carpentry time are available and 100 hr in painting is available. Each table sold yields a profit 7 and each chair produce is sold for a profit of rupees 5. Formulate the LPP.

	Table	chair
cap	4 hr	3 hr
po	2 hr	1 hr
profit	7/-	5/-

Ans: Step 1:

no. of tables = x_1

no. of chair = x_2.

→ carpentery:

Available time = 240.

required time for table = $4x_1$

chair = $3x_2$.

$\therefore$ total time $= 4x_1 + 3x_2 \leq 240$

→ Painting:

Available time : 100

Required time for table $= 2x_1$

" chair $= 1x_2$

$\therefore$ total time $= 2x_1 + x_2 \leq 100$.

→ Profit:

Profit from table $= 7x_1$

" chair $= 5x_2$.

$\therefore$ Maximize $z = 7x_1 + 5x_2$.

$\therefore$ LPP

$\text{Max } z = 7x_1 + 5x_2$.

Subject to, $4x_1 + 3x_2 \leq 240$

$2x_1 + x_2 \leq 100$.

$x_1, x_2 \geq 0$.

Q. A manufactural of furniture makes 2 products chairs and tables Processing of each products is done on two machine A and B.

A chair requires 2 hr on machine A and 6hr of machine B. A table requires 5 hr on machine A and non-time on machine B. Ther are 16 hr of time per day available on machine A and 30hr on machine B. Profit gain by the manufacture of 1/- and from table is 5/- formulate the problem into LPP inorder to maximize the total profit.

	Table	Chair	Time
A	5	2	16
B	no	6	30
	5/-	1/-	

no. of tables $= x_1$

no. of chair $= x_2$.

for Machine A: total time $= 16$ hr.

$\therefore 5x_1 + 2x_2 \leq 16$

Machine B: total time $= 30$ hr.

$6x_2 \leq 30$ hr.

→ Profit:

$\text{Max } z = 5x_1 + x_2$

$\therefore$ LPP.

$\text{Max } z = 5x_1 + x_2$

Subject to, $5x_1 + 2x_2 \leq 16$

$6x_2 \leq 30$

$x_1, x_2 \geq 0$

Q. An animal feed company must produce 250 kgs of a mixture consisting of ingredients A and B daily. A cost Rs. 13 per kg and B Rs. 84 per kg. No more than 80 kgs of A can be used and at least 60 kg of B must be used. Formulate a mathematical model to the problem?

	A	B
	13 Rs/kg	84 Rs/kg
	80	60
	<80	>60

Ans: let x_1 & x_2 be the units of ingredients A and B respectively.

$A = x_1$
$B = x_2$

• profit / cost is the objective.

$13x_1 + 84x_2$

Cost is ~~to~~ minimum

$$\text{Min } Z = 13x_1 + 84x_2$$

Subject to,

$$x_1 + x_2 \geq 250$$

and

$$x_1 \leq 80$$
$$x_2 \geq 60$$

$$x_1, x_2 \geq 0$$

Q. A company produces two types of hats. Each hat of the first type requires twice as much labour time as the second type. If all hats are of the ~~second type only~~, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second type to 150 and 250 hats. Assuming the profit per hat is Rs 8 for type 1 and Rs. 5 for type 2, formulate the problem as a linear programming model in order to determine the number of hats to be produced of each type so as to maximize the profit.

Ans: • The objective is the Profit so it Maximize.

x_1 = type 1
x_2 = type 2

$$\text{Max } Z = 8x_1 + 5x_2$$

let. time required for type II = t

∴ time required for type I = 2t

total time = 2t + t → if 2 & 2 single

$2x_1t + x_2t$

no additional variables is not allowed

∴ consider t = 1

$\therefore 2x_1 + x_2$

Subjective:

$2x_1 + x_2 \leq 500$

$x_1 \leq 150$

$x_2 \leq 250$.

$x_1, x_2 \geq 0$

→ Solution to LPP

1) Graphical

LPP involving two variable can be solved by graphical method. This method is simple and easy to apply. But LPP involving more than two variables cannot be solved using this method.

Each constraint is represented using lines

→ Steps for solving LPP using graphic method:-

1) Formulate the problem into a LPP. (Linear Prog. Problem)

2) Each inequality in the constraint may be written as equality.

3) Draw straight lines corresponding to the equation obtained in step two.

4. Identify the feasible region.

feasible region is the area which satisfy all the constraints

5. The vertices of feasible region are to be located and their ~~#~~ coordinates are ~~two~~ to be measured

6. Calculate the value of the objective function at each vertex

7) The solution is the coordinates of the vertex, which obtimizes the objective function.

Q. Solve the following LPP using the graphical method.

Max $Z = x + 3y$

Subject to,

$2x + y \leq 20$

$x + 2y \leq 20$

$x \geq 0$

$y \geq 0$

2. Equality

$\therefore 2x + y = 20$

$x + 2y = 20$

$x = 0$

$y = 0$

$\rightarrow 2x + y = 20$

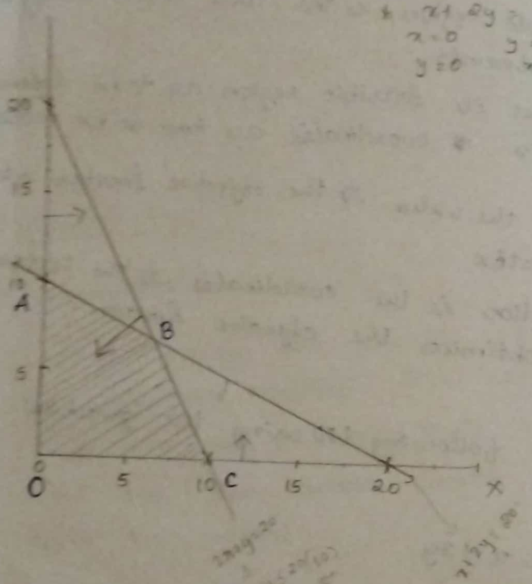
$x = 0 \quad y = 20 \quad (0, 20)$

$y = 0 \quad x = 10 \quad (10, 0)$

$\rightarrow x + 2y = 20$

$x = 0 \quad y = 10 \quad (0, 10)$

$y = 0 \quad x = 20 \quad (20, 0)$



ABCO is the feasible ~~regia~~ solution

the coordinates

A (0, 10) $\qquad$ B = ?

O (0 0)

C (10, 0)

for finding the value of B solve the two equations of line

$\therefore 2x + y = 20 \quad \text{— ①}$

$x + 2y = 20 \quad \text{— ②}$

② × 2. $\quad 2x + 4y = 40 \quad \text{— ③}$

then ③ − ①

$\therefore 2x + 4y = 40 -$

$\quad 2x + 2y = 20$

$\quad 3y = 20$

$\therefore 3y = 20$

$y = 20/3 = 6.7$

$x + 2 \times 6.7 = 20$

$x + 13.4 = 20$

$\therefore x = 6.6$

$\therefore B(6.6, 6.7)$

put values to the $z = x + 3y$

(a) $z = x + 3y \qquad A(0, 10)$

$z = 0 + 3 \times 10$

$z = 30$

$\qquad O(0\ 0)$

• $z = x + 3y$

$z = 0$

$\qquad C(10, 0)$

(c) $z = x + 3y$

$z = 10$

$\qquad B(6.6, 6.6)$

(d) $z = x + 3y$

$z = 6.6 + 3 \times 6.6$

$6.6 + 19.8$

$z = 26.4$

~~Vertice A is~~

Maximum value for vertice A

$x = 0 \quad y = 10$ //

5. A company produces two [illegible] B. There are two different departments through which the articles are processed such as assembly and finishing. The potential capacity of the assembly department 60 hrs a week and that of finishing department is 48 hr's a week. The production of one unit of 'A' requires 4 hr's in assembly and 2 hr's in finishing. Each unit of B requires 2 hr's in assembly and 4 hr in finishing.

If profits are rupees 8/- for each unit of 'A' and rupees 6/- for each unit of B. Formulating the mathematical LPP.

	A	B	
assembly	4	2 hr	= tot - 60
finishing	2	4 hr	- 48 hr
profit	8/-	6/-	

∴ let article A be x_1 and article B be x_2.

∴ it is a profit

∴ $\text{Max } z = 8x_1 + 6x_2$

subject to,

$$4x_1 + 2x_2 \leq 60$$

$$2x_1 + 4x_2 \leq 48$$

$$x_1, x_2 \geq 0$$

Q. Solve using graphical method

$\text{Max } Z = 60x_1 + 40x_2$

subject to,

$$2x_1 + x_2 \leq 60$$

$$x_1 \leq 25$$

$$x_2 \leq 35$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

it changed to equalities

∴ $2x_1 + x_2 = 60$

$x_1 = 25$

$x_2 = 35$

$x_1 = 0$

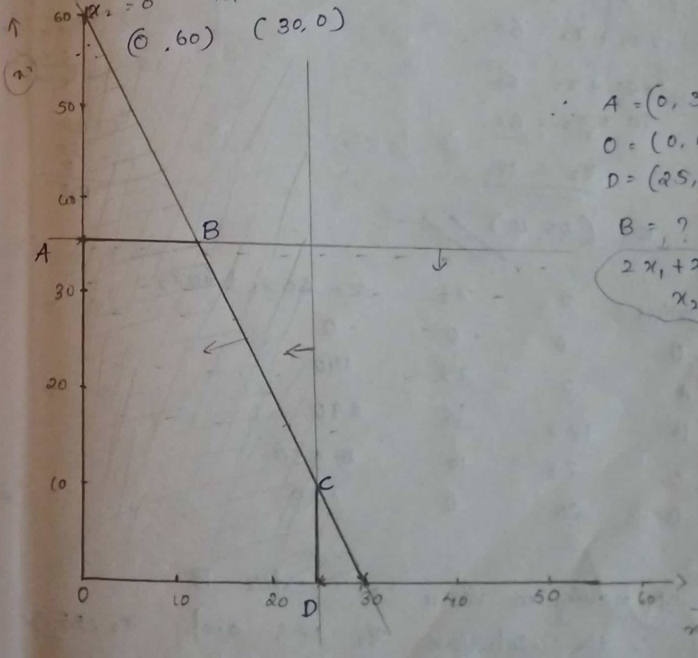
$x_2 = 0$.

then $2x_1 + x_2 = 60$.

let $x_1 = 0$ ∴ $x_2 = 60$.

$x_2 = 0$ $x_1 = 30$.

(0, 60) (30, 0)



∴ A = (0, 35)

O = (0, 0)

D = (25, 0)

B = ?

$2x_1 + x_2 = 60$

$x_2 = 35$

$\therefore 2x_1 + x_2 = 60$

$x_2 = 35$

$\therefore 2x_1 = 60 - 35$

$2x_1 = 25$

$x_1 = 25/2 = 12.5$

$\therefore B = (12.5, 35)$

C? $x_1 = 25$

$2x_1 + x_2 = 60$

$2 \times 25 + x_2 = 60$

$50 + x_2 = 60$

$\therefore x_2 = 10$

$\therefore C = (25, 10)$

vertices	x_1	x_2	$z = 60x_1 + 40x_2$
O	0	0	0
A	0	35	140
B	12.5	35	890
C	25	10	550
D	25	0	150

the maximum value at vertice B.

$\therefore$ the solution $x_1 = 12.5$ and $x_2 = 35$

(1)
25
6×
150+
400

(1) (3)
12.5×
6
750
(40
790

(2)
35
4
140

Q. Solve Graphically the following LPP?

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Min $z = 3x_1 + 5x_2$
subject to,
$-3x_1 + 4x_2 \geq 12$
$2x_1 - x_2 \geq -2$
$2x_1 + 3x_2 \geq 12$
$x_1 \leq 4$, $x_2 \geq 2$
$x_1, x_2 \geq 0$.

Step 1: Inequality changed to Equality.

$\therefore 3x_1 + 4x_2 = 12$

$2x_1 - x_2 = -2$

$2x_1 + 3x_2 = 12$

$x_1 = 4 \quad x_2 = 2$.

$\rightarrow 3x_1 + 4x_2 = 12$

($x_1 = 0 \quad \therefore x_2 = 3$) $\quad 4x_2 = 12$, $x_2 = 3$

($x_2 = 0 \quad \therefore x_1 = 4$) $\quad 3x_1 = 12$, $x_1 = 4$

(b) $2x_1 - x_2 = -2$

($x_1 = 0 \quad -x_2 = -2$

$\therefore x_2 = 2$)

($x_2 = 0 \quad 2x_1 = -2$

$x_1 = -1$) $\quad x_1 = -1$

(c) $2x_1 + 3x_2 = 12$

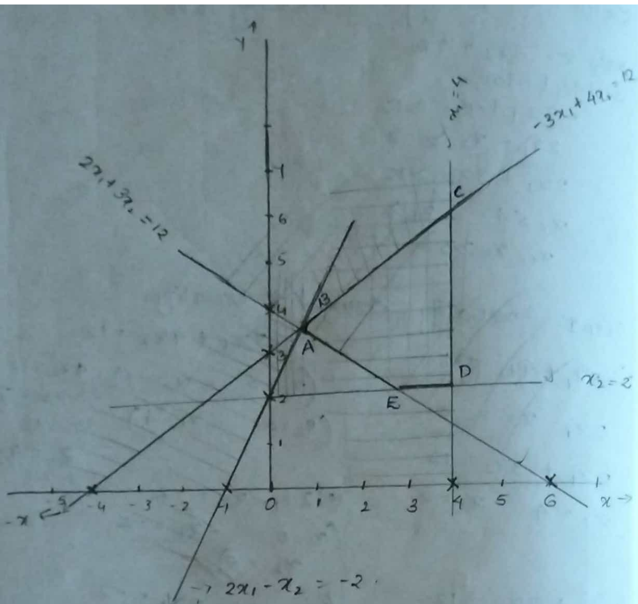
$x_1 = 0 \quad 3x_2 = 12$

$x_2 = 4$

$x_2 = 0 \quad 2x_1 = 12$

$x_1 = 6$

(d)



(c) $D = (4, 2)$

$A = B = C = E = ?$

- $E = 2x_1 + 3x_2 = 12$

  $x_2 = 2$

  $\therefore x_1 = 2x_1 + 3x_2 = 12$

  $2x_1 + 6 = 12$

  $2x_1 = 6$

  $x_1 = 3$

$\therefore (3, 2)$

(c) $C = x_1 = 4$

$-3x_1 + 4x_2 = 12$

$\therefore x_2 = -3 \times 4 + 4x_2 = 12$

$4x_2 = 24$

$x_2 = 24/4 = 6$

$\therefore C (4, 6)$

(c) $B = -3x_1 + 4x_2 = 12$

$2x_1 - x_2 = -2 \quad \times 4$

$= 8x_1 - 4x_2 = -8 \quad +$

$-3x_1 + 4x_2 = 12$

$5x_1 = 4$

$x_1 = 4/5 = 0.8$

$\therefore x_2 =$

$2 \times 0.8 - x_2 = -2$

$1.6 + 2 = x_2$

$= 3.6$

$\therefore B (0.8, 3.6)$

(c) $A = 2x_1 + 3x_2 = 12$

$2x_1 - x_2 = -2 \quad \times 3$

$6x_1 - 3x_2 = -6 \quad +$

$2x_1 + 3x_2 = 12$

$8x_1 = 6$

$x_1 = 6/8 = 0.75$

$2x_1 - x_2 = -2$

$x_1 = 0.75$

$2 \times 0.75 - x_2 = -2$

$1.5 - x_2 = -2$

$x_2 = 1.5 + 2$

3.5

$A (0.75, 3.5)$

$\therefore$ find the nimimum valued vertice.

So.

vertice	x_1	x_2	Min $z = 3x_1 + 5x_2$
A	0.75	3.5	$2.25 + 17.5 = 19.75$
B	0.8	3.6	$2.4 + 18 = 20.4$
C	4	6	$12 + 30 = 42$
D	4	2	$12 + 10 = 22$
E.	3	2	$9 + 10 = 19$

vertice 'E' coordinates have minimium value.

$\therefore (x, y) = (3, 2)$

vertice	x_1	x_2	Min $z = 3x_1 + 5x_2$
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vertice 'E' coordinates have minimium value.

$\therefore (x, y) = (3, 2)$

14-3-24

Q. Solve the given LPP using graphical Method?

Max $z = 2x + 3y$

subject to,

$x + y \leq 30$

$y \geq 5$

$0 \leq y \leq 15$

$0 \leq x \leq 20$

$x - y \geq 0$.

- Step 1: Inequality changed to equality.

* $0 \leq y \leq 15$ → split into $y \geq 0$ and $y \leq 15$

* $0 \leq x \leq 20$ → split into $x \geq 0$ and. $x \leq 20$.

- $x - y \geq 0$ = $x - y = 0$

$\therefore x = 0 \quad y = 0$.

then give Max value of x ie, 20.

$x = 20 \quad y = 20$

$\therefore (0\ 0) \quad (20\ 20)$.

- There is no feasible region. So the solution is infeasible.

Q. Assignment.

Min $Z = 4x_1 + 2x_2$

Subject to,

$x_1 + 2x_2 \geq 2$

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1, x_2 \geq 0.$

(0, 1)

• $x + 2x_2 = 2 \rightarrow x_1 = 0.$

$\therefore 2x_2 = 2.$

$x_2 = 1$

• $x_2 = 0.$

$x_1 = 2.$ $\therefore (2, 0)$

• $3x_1 + x_2 = 3. \rightarrow x_1 = 0$, $x_2 = 3.$ (0, 3)

$x_2 = 0.$

$3x_1 = 3$

$x_1 = 1$ (1, 0)

• $4x_1 + 3x_2 = 6. \rightarrow x_1 = 0.$ $3x_2 = 6$

$x_2 = 6/3 = 2$ (0, 2)

• $x_2 = 0.$ $4x_1 = 6.$

$x_1 = 6/4 = 1.5$ (1.5, 0)

A = ? (2, 0)

B = ? $x_1 + 2x_2 = 2$ $\times 4.$

$4x_1 + 3x_2 = 6.$

$\therefore 4x_1 + 8x_2 = 8.$ −

$(4x_1 + 3x_2 = 6)$

$5x_2 = 2.$

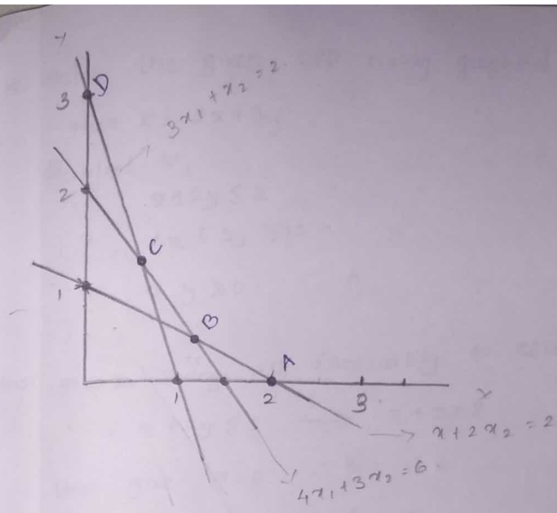
$\therefore 5x_2 = 2.$

$x_2 = 2/5 = .4$

$\therefore x_1 = 2 \times .4 = 2.$

$x_1 + .8 = 2.$

$2 - .8 = 1.2$



$\therefore$ B (1.2, .4)

C = ? $3x_1 + x_2 = 3.$ $\times 3.$

$4x_1 + 3x_2 = 6.$

$\therefore 9x_1 + 3x_2 = 9$ −

$4x_1 + 3x_2 = 6$

$5x_1 = 3$

$x_1 = 3/5 = 0.6$

$x_2 + 3 \times .6 = 3.$

$x_2 = 3 - 1.8$

$x_2 = 1.2$

$\therefore (0.6, 1.2)$

D = (0, 3)

Its an unbounded solution.

Q. Solve the given LPP using graphical Method?

Max $z = 2x + 3y$

subject to,

$x + 2y \leq 2$

$4x + 3y \geq 12$.

$x, y \geq 0$. ?

Ans: step 1: change all inequality to equality.

$\therefore$ • $x + 2y \leq 2 \longrightarrow x + 2y = 2$.

then give $x = 0$. $\therefore y =$

$2y = 2$

$y = 2/2 = 1$ //

* $(0, 1)$

* $(2, 0)$

$y = 0$. $x = 2$.

* $4x + 3y \geq 12 \longrightarrow 4x + 3y = 12$.

$x = 0$. $3y = 12$

$y = 12/3 = 4$ //

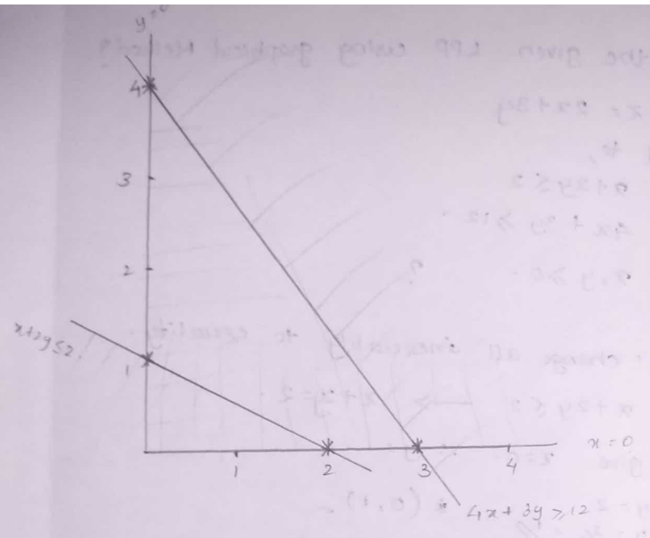
put $x, y = 0$.

$\therefore$ * $(0, 4)$

$y = 0$ $4x = 12$ $x = 12/4 = 3$ //

$(3, 0)$

$x = 0$ $y = 0$.



∴ No feasible region.

← Ans

* Assignment

Min $z = 4x_1 + 2x_2$

subject to,

$x + 2x_2 \geq 2$

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1, x_2 \geq 0$

Q. A firm makes 2 product X and Y. has a total production capacity of 9 tones per day, X and Y require the same production capacity. the firm has a permanent contract to supply at least two tones of X and at least 3 tones of Y per day to another company. Each tone of X requires 20 machine hr/s production time. and Each tone of Y requires 50 machine hr/s. production time, the daily maximum possible number of machine hr/s is 360 all the first output can be solved and the profit made is rupees 80/- per tone of X. and rupees 120/- per tone of Y. It is required to determine schedule for maximum profit and calculate this profit.

	X	Y	
supply	2	3	
time	20 m hr/s	50 m/s	360
total	9		

Max $z = 80x + 120y$

∴ subject to, $x + y \le 9$

$x \geq 2$

$y \geq 3$

$20x + 50y \le 360$

$x, y \geq 0$ //

Step 2: inequality changed to Equality.

$\therefore x+y=9$

$x=2$

$y=3$

$20x+50y=360$

then find coordinates

1) $x+y=9$ $\begin{pmatrix} x=0 \\ y=9 \end{pmatrix}$

$(x=9,\ y=0)$

2) $20x+50y=360$ $(0,\ 7.2)$

$x=0$ $50y=360$

$y=\frac{360}{50}=7.2$

$y=0$ $20x=360$ $(18,\ 0)$

$x=\frac{360}{20}=18$

then graph →

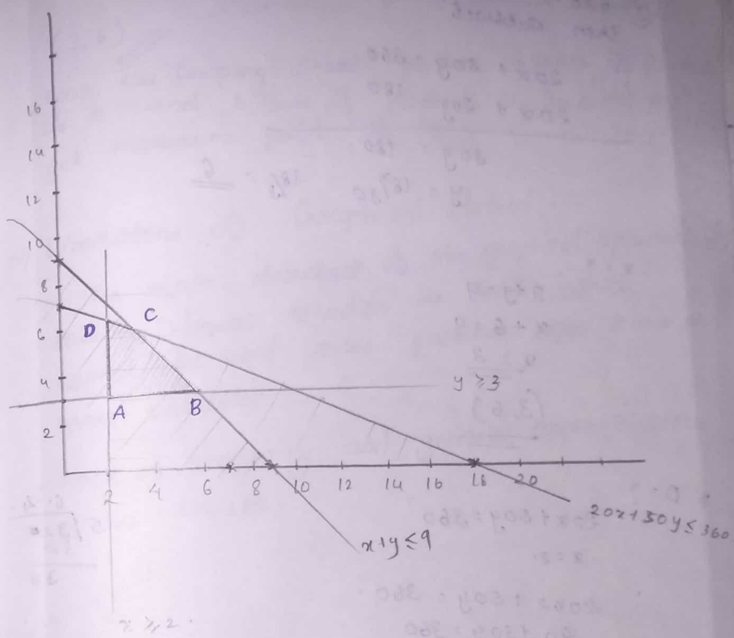
then coordinates of the shaded part

A $(2,3)$

B = ?

C = ?

D = ?



B = ?

$x+y=9$

$y=3$

$\therefore x+3=9$

$x=6$ $(6,3)$

C = ? $20x+50y=360$

$x+y=9$

② × 20 then subtract.

$\therefore$ $20x + 50y = 360$ –
$20x + 20y = 180$

$30y = 180$
$y = 180/30 = 18/3 = \underline{6}$

$\therefore x = ?$
$x + y = 9$
$x + 6 = 9$
$\underline{x = 3}$

$\therefore$ $(3, 6)$

* D = ?
$20x + 50y = 360$
$x = 2$.
$\therefore$ $20 \times 2 + 50y = 360$.
$40 + 50y = 360$.
$50y = 320$
$y = \underline{6.4}$

$(2, 6.4)$

Max $z = 80x + 120y$.

Vertices.	x	y	
A	2	3	$80 \times 2 + 120 \times 3 = 520$
B	6	3	$80 \times 6 + 120 \times 3 = 840$
C	3	6	$80 \times 3 + 120 \times 6 = 960$
D	2	6.4	$80 \times 2 + 120 \times 6.4 = 928$

$\therefore$ the Maximum value for z for the vertice (3, 6)//

Hence the company should produce 3 tones of products of x and 6 tones of products y. In order to get maximum profit of 960//

→ Limitations of Graphical Method:-

The main drawback of the graphical approach of solving linear equation is that which cannot be used solve problems with 3 ~~me~~ or more variables.

• Lack of accuracy and general approximation of the results.

21-3-24

→ Feasible Solution:-

A set of values of the variables which satisfy all the constains and all non-negative restrictions of the variables, is known as a feasible solution

→ Infeasible solution:-

A LPP is said to be infeasible of there is no solution that satisfies all the constraints

→ Unbounded solution:-

LPP is said to have unbounded solution if its solution can be made infinitly large without violating an of its constraints in the problem.

→ Basic Varable:-

$x = 0 \rightarrow$ non basic

$x \neq 0 \rightarrow$ basic varable

Let 'n' denote number of variables 'm' denotes number of constraints.

If 'm' is less than 'n' then $n-m$ varables are equal to zero. Then the remaining variables can take any values other than zero's, are called Basic varables.

→ Basic feasible Solution:-

Value of basic varable that satisfy the constraints as well as the non-negative restriction is called basic feasible Solution.

Eg: $x_1 + 2x_2 + S_1 = 6$

$4x_1 + 3x_2 + S_2 = 2$.

$n = 4$ $m = 2$

$m < n$

$n - m$ varable $= 2$.

$x_1, x_2 = 0 \Rightarrow$ non basic varables.

$S_1, S_2 \Rightarrow$ basic varable.

→ Degenerate Basic feasible solution:

The value of atleast one basic varable becomes zero, we say the solution is degerate basic feasible solution.