27/05/24
Monday
10 → 4pm

Assignment Problem:-

Assignment Problem is a special case of transportation problem in which the objective is to assign the number of orgin to equal number of task at a minimum cost

person \ Jobs	1	2	3	...	n
1	c_{11}	c_{12}	c_{13}	...	c_{1n}
2	c_{21}	c_{22}	c_{23}	...	c_{2n}
3	.	.	.		
⋮					
n	c_{n1}	c_{n2}	c_{n3}	...	c_{nn}

c_{ij} = Cost for assigning i^{th} person to the j^{th} job.

$$x_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ person is assigned to } j^{th} \text{ job} \\ 0, & \text{otherwise} \end{cases}$$

LPP

Objective = Min $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to,

Row total $\sum_{j=1}^{n} x_{ij} = 1$, $i \le i \le n$

Colum " $\sum_{i=1}^{n} x_{ij} = 1$, $i \le j \le n$

$x_{ij} \geq 0$.

→ Hungarian Method to solve assignment Problem.

Q. Find the minimum cost to the given A.P

	A	B	C	D
J_1	18	26	17	11
J_2	13	28	14	26
J_3	38	19	18	15
J_4	19	26	24	10

Step 1: row Subtraction - Subtract the smallest element of each row in the table from every element of that row.

∴

7	15	6	0
0	15	1	13
23	4	3	0
9	16	14	0

Step 2: Column Subtraction:- Subtract the smallest element of each column from every element of that column.

7	11	5	0
0	11	0	13
23	0	2	0
9	12	13	0

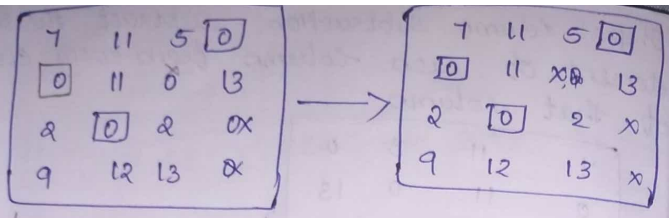
Step 3: row scanning:- mark the zero, if it is the only zero of the row and cross all other zeros of the column having marked zeros.

7	11	5	[0]
0	11	0	13
23	[0]	2	~~0~~
9	12	13	~~0~~

→ mask ↓ then cancel

→ more than one ignore it

Step 4: Column scanning:- Mark the zero, if it is the only zero of the column & cross of all the other zero's of the row having marked zero.



↓ → zero cross
marked

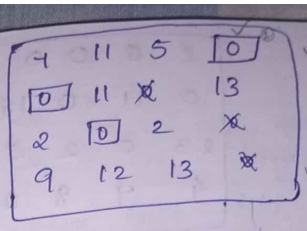
After the above operations, if it has an assignment on every row & every column then we get the solution

Otherwise, draw the minimum number of horizontal & vertical lines to cover all the zero's atleast once. for this the following steps are used.

i, Mark ☑ on all ~~the~~ the rows for which assignments have not being ~~done~~ may

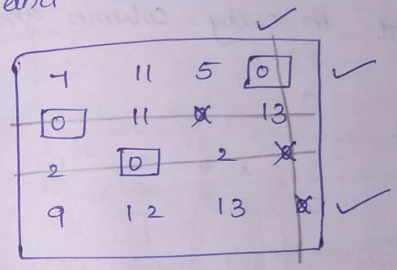
ii, Mark ☑ on columns which have zero's on marked rows

iii, Mark ☑ on row which have assignment in marked columns.



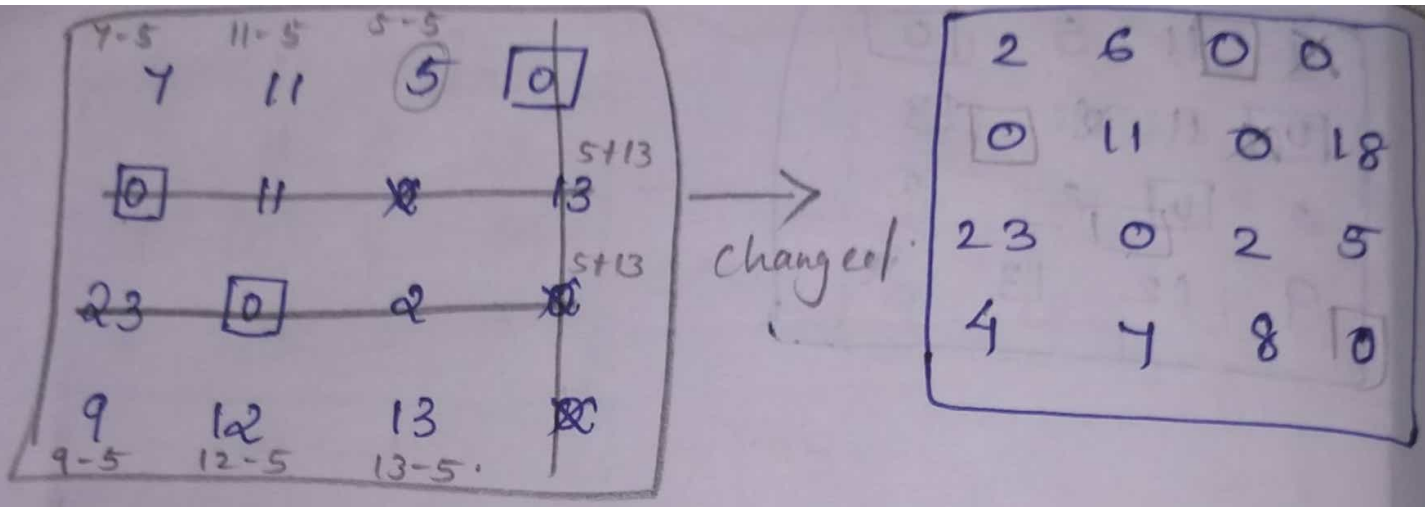
zero

step5: Draw the line through unmarked rows and marked column to cover all the zeros



step 6: Select the smallest uncovered element subtract it from all the elements that are uncovered and add it to every element that lies at the intersection of two lines and leave the remaining elements unchanged.

∴ '5' is smallest element.



Step 4: Now reapply the above steps until we get assignment in every column and every row.

$J_2 \rightarrow A = 13$
$J_3 \rightarrow B = 19$
$J_4 \rightarrow D = 10$

Min Cost = $17+13+19+10 = \underline{\underline{59}}$

Q. Solve A.P to find the minimum cost?

	J_1	J_2	J_3	J_4
A	32	26	35	38
B	27	24	26	32
C	28	22	25	34
D	10	10	16	16

26, 24, 22, 10

Smallest element of each row then subtract with other element..

6	0	9	12
3	0	2	8
6	0	3	12
0	0	6	6

Smallest element of each then subtract with other element..

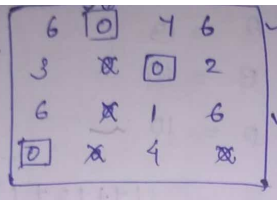
6	0	7	6
3	~~0~~	0	2
6	~~0~~	1	6
0	~~0~~	4	~~0~~

→ assume

→ ignore

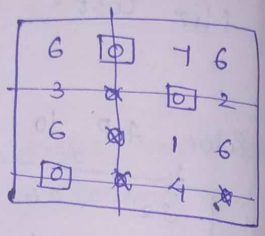
Then

row & column scanning



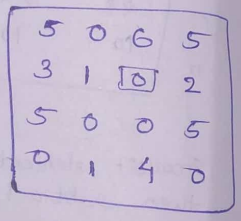
Mark ☑

then draw lines.
for unmarked row is, 2, 4
marked colum, 2.



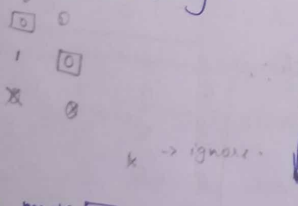
then find min of unmarked
∴ ie, 1

then it subtract with unmarked & add with intersect point.



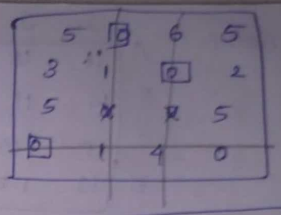
then again row & column subtraction. ie, same.

the row scanning.

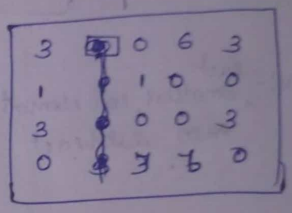


then mark ☑
un marked row = 2

then draw line
for unmarked row ☑ - 4th row
marked column ☑ - 2, 3.

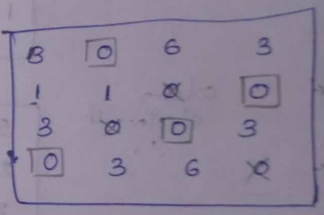


then the min = 2.
then it subtract with unmarked & add with intersect point



then again row & column subtract. ie, Zero - no. = no.

then row and column scanning.



∴ optimal solution

A → J_2 = 26
B → J_4 = 32
C → J_3 = 25
D → J_1 = 10.

Q. Solve.

	J_1	J_2	J_3	J_4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

?

Ans:- Each Smallest row element then subtract

6	0	4	1
3	0	4	4
3	3	1	0
1	0	2	3

" column elementwise

5	0	3	1
2	0	3	4
2	3	0	0
0	0	2	3

then row scanning then column scanning.

5	0	3	1
2	~~0~~	3	4
2	3	0	~~0~~
0	~~0~~	2	3

then mark ✓

- Un marked row - 2nd the two in 2nd column → 1st row.

then draw line.

unmarked row = 4th, 3rd

marked column - 1, 3, 4.

2.

5	0	3	1
2	~~0~~	3	4
2	3	0	~~0~~
0	~~0~~	2	3

then min = 1.

4	0	2	0
1	0	2	3
1	~~2~~	0	~~0~~
0	1	2	3

∴ it is optimal.

A → J_4 = 11

B → J_2 = 11

C → J_3 = 13

D → J_1 = 13

∴ 11 + 11 + 13 + 13

= 48

28-5-24

Q. Given below is a matrix showing the profit for different Jobs done through different machines. find an assignment which will maximize the total profit.

	M_1	M_2	M_3	M_4
J_1	51	53	54	50
J_2	47	50	48	50
J_3	49	50	60	61
J_4	63	64	60	61

?

Ans: highest element = 64. then subtract

Convert this prblm to minimization by subtracting each element of the matrix from the largest value.

∴ new table.

	M_1	M_2	M_3	M_4
J_1	13	11	10	14
J_2	17	14	16	14
J_3	15	14	4	3
J_4	1	0	4	3

then row subtraction → 10; 14, 3; 0

then column subt

3	1	0	4
3	0	2	0
12	11	1	0
1	0	4	3

the column subtra 1, 0, 0, 0.

∴ →

2	1	0	4
2	0	2	0
11	11	1	0
0	0	4	3

row scanning

2	1	[0]	4
2	0	2	~~0~~
11	11	1	[0]
0	0	4	3

→ ignore

→ ignore

column scanning

2	1	[0]	4
2	[0]	2	~~0~~
11	11	1	[0]
[0]	~~0~~	4	3

Each row & column have assignment

∴ $J_1 \rightarrow M_3 = 54$

$J_2 \rightarrow M_2 = 50$

$J_3 \rightarrow M_4 = 61$

$J_4 \rightarrow M_1 = 63$

∴ total profit = $54 + 50 + 61 + 63 = 228$

Q. Solve the A.P to maximize the ~~result~~ profit.

	A	B	C	D
J_1	6	4	6	7
J_2	2	6	5	7
J_3	2	3	4	6
J_4	3	1	3	4

?

Max Element = 7.

∴ then subtract it with other elements

1	3	1	0
5	1	2	0
5	4	3	1
4	6	4	3

then row & column subtraction

1	3	1	0
5	1	2	0
4	3	2	0
1	3	1	0

→

column sub.

0	2	0	0
4	0	1	0
3	2	1	0
0	2	0	0

Row scanning:

0	2	0	0
4	[0]	1	0
3	2	1	[0]
0	2	0	⊗

unmarked row = 1, 4.

Max col = 2, 4.

Case I

[0]	2	⊗	⊗
4	[0]	1	⊗
3	2	1	[0]
⊗	2	[0]	⊗

→ Give [0] to first row cross. then check column case.

case II

⊗	2	[0]	⊗
4	[0]	1	⊗
3	2	1	[0]
[0]	2	⊗	⊗

→ Give 0 first row not assign [2]

Each row & column is assigned

∴ case I: $J_1 \rightarrow M_1 = 6$

$J_2 \rightarrow M_2 = 6$

$J_3 \rightarrow M_4 = 6$

$J_4 \rightarrow M_3 = 3$

$= 6+6+6+3 = 21$

Case II: $J_1 \rightarrow M_3 = 6$

$J_2 \rightarrow M_2 = 6$

$J_3 \rightarrow M_4 = 6$

$J_4 \rightarrow M_1 = 3$

$= 6+6+6+3 = 21$

∴ Maximum profit = 21

we cannot proceed further because rows 1 & column 3 contain 2 un allocated zeros.

→ Travelling Salesman Problem:-

without repeating any city

Q- Find the minimum transportation cost.

	A	B	C	D	E
A	–	4	7	3	4
B	4	–	6	3	4
C	7	6	–	7	5
D	3	3	7	–	7
E	4	4	5	7	–

Ans:

	A	B	C	D	E
A	∞	4	7	3	4
B	4	∞	6	3	4
C	7	6	∞	7	5
D	3	3	7	∞	7
E	4	4	5	7	∞

1) Row & Colum Subtraction

∞	1	3	0	1
1	∞	2	0	1
2	1	∞	2	0
0	0	3	∞	4
0	0	0	3	∞

Row scanning & Column scanning

∞	1	3	[0]	1
1	∞	2	0	1
2	1	∞	2	[0]
0	0	3	∞	4
0	0	[0]	3	∞

→ again Colum scanning

∞	1	3	[0]	1
1	∞	2	0	1
2	1	∞	2	[0]
[0]	0	3	∞	4
0	0	[0]	3	∞

[0] mark

line

Unmarked row – 2nd.

then

smallest of 3 1

∞	0	2	[0]	0
0	∞	1	0	[0]
2	1	∞	3	[0]
[0]	0	3	∞	4
0	0	[0]	4	∞

Again row & colone Subtraction then row & colum Scanning.

∞	0	2	0	0
0	∞	1	0	[0]
2	1	∞	3	[0]
0	0	3	∞	4
0	0	[0]	4	∞

Again row & colum scanning

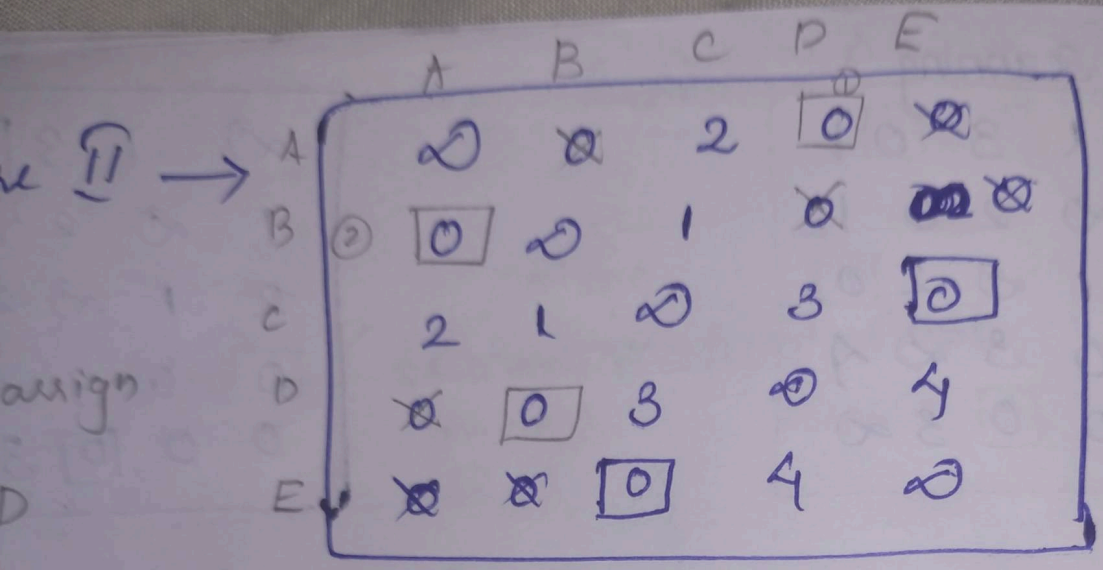
∴ more Scanning is not done case I & case II.

case I

A → B , B → D

D → A ∴ not complete ← E

	A	B	C	D	E
A	∞	[0]	2	0	0
B	0	∞	1	[0]	0
C	2	1	∞	3	[0]
D	[0]	0	3	∞	4
E	0	0	[0]	4	∞



$A \rightarrow D$

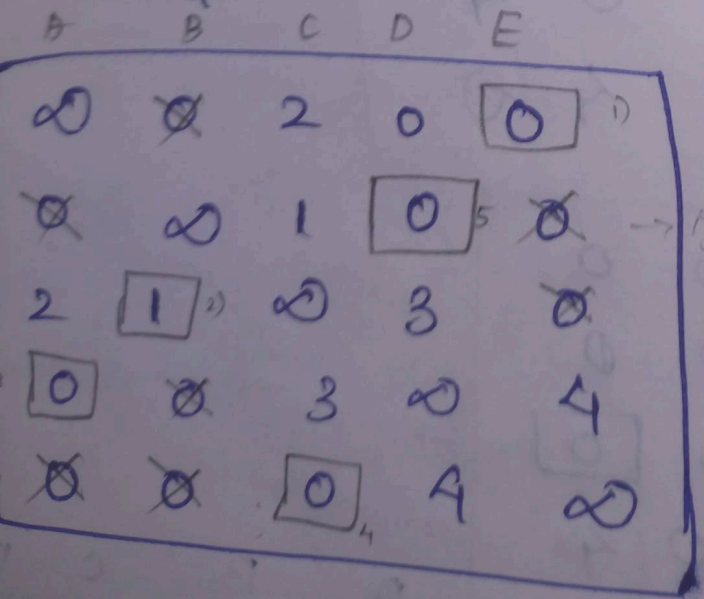
$D \rightarrow B$

$B \rightarrow A$.

get the optimal solution.

special case

another

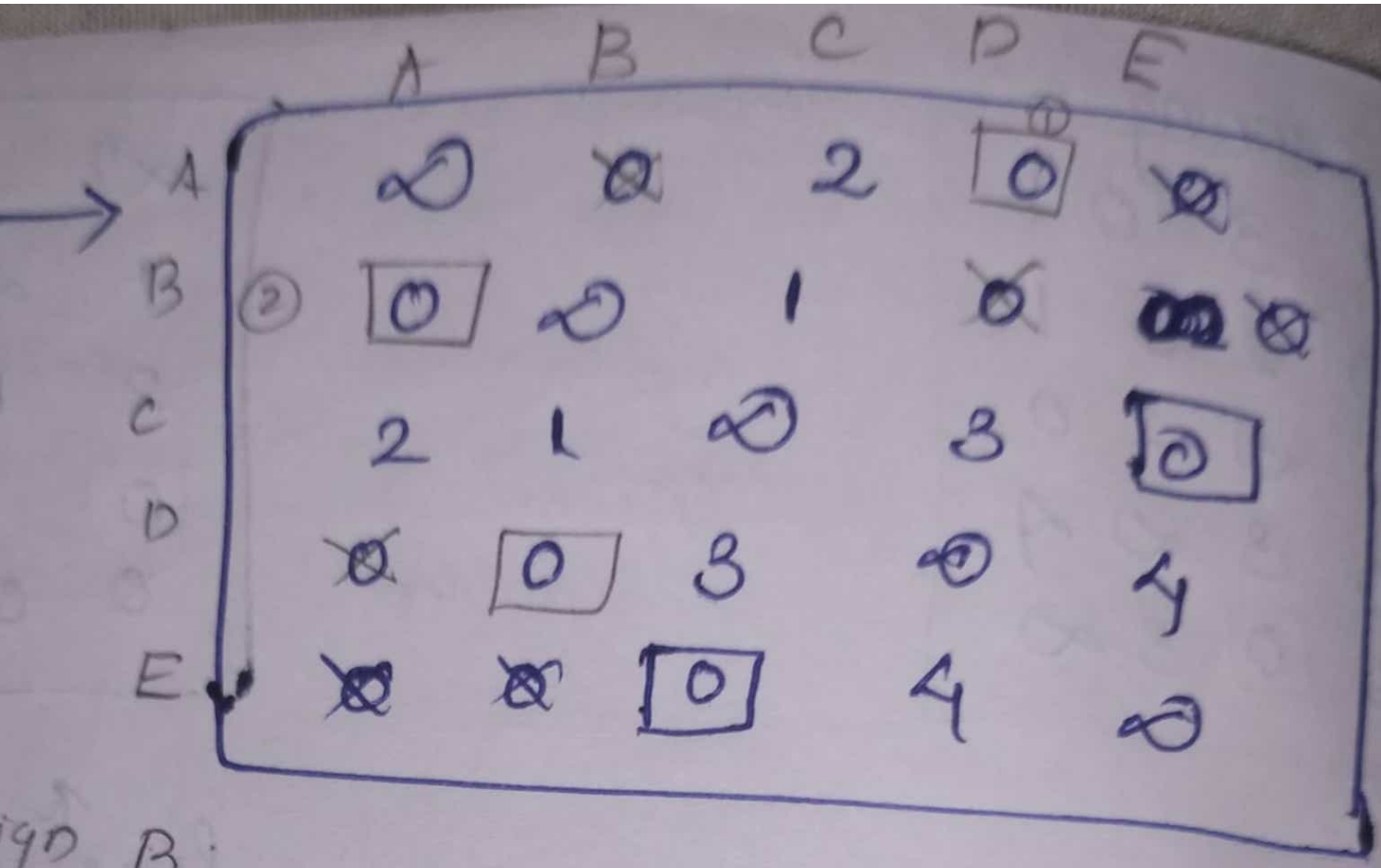


1) 1st zero last (others are already assigned)

2) no zero in 3rd row so select (1).

3) 4th row

4) 5th row



$A \longrightarrow D$

$D \longrightarrow B$

$B \longrightarrow A$.

didn't get the optimal solution.

special case

another

	A	B	C	D	E
A	∞	~~0~~	2	0	[0] 1)
B	~~0~~	∞	1	[0] 3)	~~0~~
C	2	[1] 2)	∞	3	~~0~~
D	[0]	~~0~~	3	∞	4
E	~~0~~	~~0~~	[0] 4)	4	∞

1) 1st row last (other already assigned)

2) no zero in 3rd row so select (1).

3) 4th row

4) 5th row

$\therefore$ A → E E → C C → B B → D

D → A.

$\therefore$ 4 + 5 + 6 + 3 + 3 = 21

(4 + 5 = 9; 6 + 3 + 3 = 12)

Q. solve the game.

$$\begin{bmatrix} -2 & 5 \\ -5 & 3 \\ 0 & -2 \\ -3 & 0 \\ 1 & -4 \end{bmatrix}$$

?

			rowmin
D_1	-2	5	-2
D_2	-5	3	-5
D_3	0	-2	-2
D_4	-3	0	-3
D_5	1	-4	-4
column max	1	5	

rowmin
columnmax.

then minmax = min {column max}

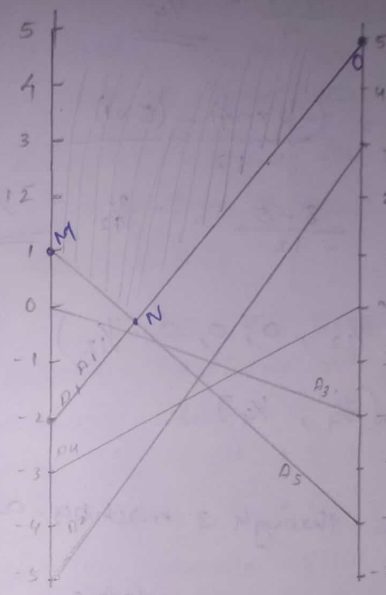
= min {1, 5} = 1//

then maxmin = max {rowmin}

= max {-2, -5, -2, -3, -4}

= -2//

∴ no saddle point.



N is lowest.

A_5 & A_1 meet at N.

$$\begin{bmatrix} -2 & 5 \\ 1 & -4 \end{bmatrix}$$

$$\therefore p = \frac{d-c}{(a+d)-(b+c)} = \frac{-4-1}{(-2+-4)-(5+1)} = \frac{-5}{-6-6} = \frac{-5}{-12} = \frac{5}{12}$$

$$1-p = 1-\frac{5}{12} = \frac{12-5}{12} = \frac{7}{12}$$

$$q = \frac{d-b}{(a+d)-(b+c)} = \frac{-4-5}{-12} = \frac{-9}{-12} = \frac{3}{4}$$

$1-2 = 1-\frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$

$$v = \frac{ad-bc}{(a+d)-(b+c)} = \frac{(-2\times-4)-(5\times1)}{-12}$$

$$= \frac{8-5}{-12} = -\frac{3}{12} = -\frac{1}{4}$$

A's strategy = $(5/12, 0, 0, 0, 7/12)$

B's strategy = $(3/4, 1/4)$

Q. Processing N jobs through 3 machines. Assumptions

1. (x) 3 machines A, B & C available
2. (x) each job is processed in a prescribed order say ABC
3. Exact processing time on machines are given

Procedure:-

Procedure is available is only for special cases where either 1 or both of the following conditions hold

1. Cond. Minimum time on machine A is greater than or equal maximum time on machine B.
2. Cond. Minimum time on machine C is greater than or equal to maximum time on machine B.

Replace the problem with an equivalent problem involving 'n' Jobs and 2 fictitious machines denoted G and H. with processing time $G_i = A_i + B_i$ and

$H_i = B_i + C_i$

if this problem with prescribed order GH is solved, the resultant optimum sequence will also be optimal for orginal problem.

Jobs	Ai	Bi	Ci
1	8	5	4
2	10	6	9
3	6	2	8
4	7	3	6
5	11	4	5

i) A min time = 6
Max time B = 6.
$\therefore 6 \geq 6$ ✓

2) Min of C = 4.
Max of B = 6
$4 \geq 6$ ✗.

Job	$G_i = D_i + B_i$	$H_i = B_i + C_i$
1	8 + 5 = 13	5 + 4 = 9
2	16	15
3	8	10
4	10	9
5	15	9

J_3 J_2 J_4 J_1 J_5

Job	Machine A intime	Machine A outtime	Machine B intime	Machine B outtime	Machine C intime	Machine C outtime
3	0	0+6 = 6	6	6+2 = 8	8	8+8 = 16
2	6	6+10 = 16	Max{8, 16} = 16	16+6 = 22	22	22+9 = 31
4	16	16+7 = 23	Max{22, 23} = 23	23+3 = 26	31	31+6 = 37
1	23	23+8 = 31	31	31+5 = 36	37	37+4 = 41
5	31	31+11 = 42	Max{36, 42} = 42	42+4 = 46	46	46+5 = 51

∴ total elapsed = 51

idle time of A = 51 − 42 = 9

idle time of B = 0→6, 8→16, 22→23, 26→31, 36→42; 46→51 (waiting for C)

∴ 6 + 8 + 1 + 5 + 6 + 5

= 12 + 10 + 8 + 1 = 31

idle time of C = 0→8, 16→22, 41→46

= 8 + 6 + 5 = 19