6-2-24
Monday.

Module : III

→ Transportation Problem:- (TP)

The objective of transportation problem is ~~the~~ to transport of various amount of a single compodity that as stored at several objects to a number of destination.

The transportation is affected in such way that destinations demand es satisfied within the capacity of orgin. and that total ~~post~~ transportation cost is minimum

factory ↓ \ Showroom	Kochi	Chennai	Delhi	Supply
Hydrabad	11	13	17	250
Mumbai	10	18	14	300
demand	200	225	250	

→ Balanced Transportation Problem (BTP)

If total supply = total demand.

then the given (TP) is balanced. Then the above supp example

Total supply = 550/-

Total demand = 675/-.

∴ total suppy ≠ total demand. ∴ So this transportation is unbalanced.

Factory/Showroom	Kochi	Chennai	Delhi	Kolkata	Supply
Hyderabad	11	13	17	14	250
Mumbai	10	18	14	10	300
Goa	6	7	8	9	400
demand	200	225	275	250	1000

Total supply = 950

" demand = 950

∴ supply = demand

St 1: Check whether Given TP is balanced or unbalanced

St 2: If the problem is unbalanced make it balance, by adding a dummy row / dummy column with cost zero. Find intial feasible solution using north-west corner method or lest cost method or Vogel's Approximation Method.

St 3: check whether the transportation is degenerate or non-degenerate.

St 4: Find optimal solution using MODI method

→ North-west Corner Method:-

This method is to find initial feasible solution of a transportation problem

Step 1: Allocate the cell (1,1) (North-west cell) maximum possible amount which is the minimum of row total and column total. So either a row or a column total get exhosted. So cross of that row or column

Step 2: Consider the new reduced matrix in that matrix allocate to the North-west corner cell maximum possible amount which is the minimum of present row total & column total

Step 3. Repeat the above step untill all avialable quantit are exhosted.

eg:

finding initial feasible solution using northwest corner method?

	D_1	D_2	D_3	D_4	
O_1	$\boxed{200}$ 11	13	17	19	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
	200	225	275	250	950 / 950

so Total supply = Total demand

∴ it balanced.

for the D_1 wants total 200. The O_1 manufactured 250 total.

min (O_1, D_1).

∴ min (200, 250)

∴ it reduced to.

	D_2	D_3	D_4	
O_1	$\boxed{50}$ 13	17	14	50
O_2	18	14	10	300
O_3	24	13	10	400
	225	275	250	

∴ 225 - 50 = 175

	D_2	D_3	D_4	
O_2	$\boxed{175}$ 18	14	10	300
O_3	24	13	10	400
	175	275	250	

then check. it balanced or not. then check min(175, 300)

175 → column

cleanceled.

∴

	D_3	D_4	
O_2	$\boxed{125}$ ~~14~~	~~10~~	~~125~~
O_3	13	10	400
	275	250	

300 - 175 = 125

525

then check balance or not

525 = 525 balanced

min(275, 125)

∴ 125 clear that row.

∴ it changed to:

	D_3	D_4	
O_3	$\boxed{150}$ 13	10	400
	150	250	

275 - 125 = 150

min (150, 400)

∴ Hence

400 - 150 = 250

$\boxed{250}$ 10	250	
250		

125 × 10 = 12500

125 × 14

11 × 2 = 22

∴ final:

	D_1	D_2	D_3	D_4
O_1	$\boxed{200}$ 11	$\boxed{50}$ 13	17	14
O_2	16	$\boxed{175}$ 18	$\boxed{125}$ 14	10
O_3	21	24	$\boxed{150}$ 13	$\boxed{250}$ 10

175 × 18 = 1400 + 175 = 3150

13 × 5 = 65

Then transportation Cost :-

for O_1 to D_1 = 200 × 11 + 50 × 13 + 18 × 175

$O_1 \to D_1$ $\quad$ $O_1 \to D_2$

+ 125 × 14 + 13 × 150 + 250 × 10

= 2500 + 650 + 2200 + 3150 + 1750 + 1950

= 12200/-

Q. Finding inital feasible solution using north-west corner method?

	D_1	D_2	D_3	
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
	7	9	18	

34

34

Ans: Total supply = Total demands. ∴ It is balanced

1. D_1 want total '7' product but O_1 produces only 5 ∴ Min(7, 5)

O_1	$\boxed{5}$ 2	7	4	5

7 - 5 = 2

2. ∴ D_1 wants only '2' the O_2 produces 8.
∴ Min (2, 8)
∴ = 2.
8 - 2 = 6.

O_2	$\boxed{2}$ 3	3	1	8 → 6
O_3	5	4	7	7
O_4	1	6	2	14
	2	9	18	

3. Next D_2 wants total '9' but the O_2 manufactured only 6.
∴ Min (9, 6)
= 6.
9 - 6 = 3

	D_2	D_3	
O_2	$\boxed{6}$ 3	1	6
O_3	4	7	7
O_4	6	2	14
	9 → 3	18	

4. In D_2 want '3' product the O_3 manufactured 7 products.

∴ min (3, 7)
= 3.
= 7 - 3 = 4

	D_2		
O_3	$\boxed{3}$ 4	7	7
O_4	6	2	14
	3	18	

5. The D_3 wants total 18 products. But the O_3 produces only 4.

∴ min (18, 4)
= 4.

	D_3	
O_3	$\boxed{4}$ 7	4
O_4	2	14
	18 - 4 = 14	

6. The wants total 14 and the O_3 produces total 14.

O_4	$\boxed{14}$ 2	14
	14	

∴ Finial:

$\boxed{5}$ 2	7	4
$\boxed{2}$ 3	$\boxed{6}$ 3	1
5	$\boxed{3}$ 4	$\boxed{4}$ 7
1	6	$\boxed{14}$ 2

∴ total cost = 5×2 + 2×3 + 6×3 + 3×4 + 4×7 + 14×2
= 10 + 6 + 18 + 12 + 28 + 28
= 102

→ Least Cost Method

Step 1:

Q. Finding initial feasible solution using least cost method ?

	D_1	D_2	D_3	
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
	7	9	18	

1st the least cost ... from ...

ie '1' . so min (row total, column total)

Choose any

∴ min (8, 18)

Min is 8 → so ~~char~~ cancel that row.

∴ 18 − 8 = 10

2	7	4	5
3	3	$\boxed{8}$ 1	8
5	8	7	7
$\boxed{7}$ 1	6	2	14
7	9	18 → 18 − 8 = 10	

(2) then select next '1'

∴ min (14, 7)

min is 7

Cancel column.

row 14 − 7 = 7

(3) ∴

7	$\boxed{3}$ 4	5
8	7	7
6	$\boxed{7}$ 2	7
9	10 → 10 − 7 = 3	

minimum is 2. select that column.

∴ min (7, 10)

minimum is 7 so cancel that row.

then 10 − 7 = 3

(4) then min is 4 so check min (5, 3)

minimum is 3 . So cancel that column

then (row value) 5 − 3 = 2

then

7	2
$\boxed{7}$ 1	7
9.	

5 − 3 = 2

(5) then minimum is 4 . so min (7, 9) minimum is 7. . so cancel that row.

9 − 7 = 2

∴

$\boxed{2}$ 7	2
2	

Cost = 2×7 + 3×4 + 8×1 + 7×4 + 7×2 + 7×1

= 83

23-2-24

→ **Vogel's Approximation Method (VAM)**

• find initial solution by VAM.

	D_1	D_2	D_3	D_4	supply
S_1	1	5	3	3	34
S_2	3	3	1	2	15
S_3	0	2	2	3	12
S_4	2	7	2	4	19
demand	21	25	17	17	

∴ total supply = 80

Total demand = 80 .

Total supply = Total demand ∴ It is balanced

2. find penality of each row and each column

[Penalty is the difference btw smalles and 2nd smallest element of a row (column)]

find penality of each row

*1	5	3	3	1 - 3 = |2|
3	*3	*1	*2	2 - 1 = 1
* 0	* 2	* 2	* 3	0 - 2 = |2|
2	7	2	4	2 - 2 = 0

penality of column

1 - 0 = 1, 2 - 3 = 1, 2 - 1 = 1, 3 - 2 = 1

* consider the largest penality:
  2, 1, 2, 0, 1, 1, 1, 1
The largest penality is 2. it is in 1st and 3rd row.
* select either 1st row or 3rd
* Select the cell containing least cost in 1st row

∴ Min (row 1, C total 1)

min (34, 21)

= 21

∴ cancel column.

21	5	3	3	34 - 21 = 13
3	3	1	2	15
0	2	2	3	12
2	7	2	4	19
21	25	17	17	

* Then, again find the penality and largest penality

	D_2	D_3	D_4		
S_1	5	3	3	13	3 - 3 = 0
S_2	3	1 *	2 *	15	2 - 1 = 1
S_3	2 *	2 *	3	12	2 - 2 = 0
S_4	7	17 2 *	4 *	19	4 - 2 = 2
	25	17	17		
	3 - 2 = 1	2 - 1 = 1	3 - 2 = 1		

The largest penality is 2. It is in 4th row.

then find the least cost.

2. ∴ min (4th row, 2nd col)

$D_2 = $ min (19, 17) = 17

∴ cancel column.

19 - 17 = 2

	D_2	D_4		
	5	13 3	13	
S_2	3	2	15	
S_3	2	3	12	
	7	2 4	2	
	25	17		
	3 - 2 = 1	2 - 3 = 1		

Penality =
5 - 3 = 2
3 - 2 = 1
3 - 2 = 1
4 - 7 = 3

largest penality is 3.

∴ 4th row.

min (2, 17)

cancel row.

17 - 2 = 15

Then largest penality is 2. it is on 1st row.

$\therefore \min(13, 15) = 13$ $\therefore$ cancel row.

$15 - 13 = 2$.

$\therefore$

	D_2	D_4		
S_2	3	$\boxed{2}$ 2	15	1 → 13
S_3	2	3	12	1
	25	2.		
	1	1		

1st row. $\min(15, 2) = 2$. Cancel column.

$15 - 2 = 13$

$\therefore$

	D_2	
S_2	3	13
S_3	$\boxed{12}$ 2	12.

$\min(12, 25) = 12$.

$25 - 12 = 13$.

D_2.	
$\boxed{13}$ 3	13
13	

$\boxed{21}$ 1	5	3	$\boxed{13}$ 3
3	$\boxed{13}$ 3	1	$\boxed{2}$ 2
0	$\boxed{12}$ 2	2	3
2	7	$\boxed{17}$ 2	$\boxed{2}$ 4

Cost =

Q. find optimal solution ?

	D_1	D_2	D_3	D_4	Supply
O_1	6	$\boxed{12}$ 3	$\boxed{2}$ 5	$\boxed{8}$ 4	22
O_2	5	9	$\boxed{15}$ 2	7	15
O_3	$\boxed{7}$ 5	7	8	$\boxed{1}$ 6	8
demand	7	12	17	9	45

Total supply = 45

Total demand = 45

∴ it is balanced.

* find initial Solution by VAM.

	D_1	D_2	D_3	D_4					
O_1	6	$\boxed{12}$ 3	$\boxed{2}$ 5	$\boxed{8}$ 4	~~22~~ ~~10~~ 8	2 1	1	1	2
O_2	5	9	$\boxed{15}$ 2	7	~~15~~	3	3	x	x
O_3	5	7	8	6	8	1	1	1	1
	7	12	~~17~~ 2	~~9~~ 1					

0	4	1	2
0	x	3	2
1	x	3	2

O_3	$\boxed{7}$ 5	$\boxed{1}$ 6	~~8~~ 1
	7	1	

Transportation cost = 12×3 + 2×5 + 8×4 + 15×2 + 7×5 + 1×6

= 36 + 10 + 32 + 30 + 35 + 6 $\underline{\underline{= 149}}$

* for allocated cell $U_i + V_j = C_{ij}$.

		V_1 (3)	V_2 (3)	V_3 (5)	V_4 (4)
0	U_1	6	$\boxed{2}$ 3	$\boxed{2}$ 5	$\boxed{8}$ 4
-3	U_2	5	9	$\boxed{15}$ 2	7
2	U_3	$\boxed{7}$ 5	7	8	$\boxed{1}$ 6

$U_1 + V_2 = 3$

$U_1 + V_3 = 5$

$U_1 + V_4 = 4$

$U_2 + V_3 = 2$

$U_3 + V_1 = 5$

$U_3 + V_4 = 6$

$V_2 = 3$

$V_3 = 5$

$V_4 = 4$

$U_2 = 2+5 = 7$

Consider $U_1 = 0$.

- $V_2 = 3 - U_1 = 3 - 0 = 3$ //
- $U_1 + V_3 = 5$, $\underline{V_3} = 5 - 0 = 5$ //
- $U_1 + V_4 = 4$ $\underline{V_4} = 4 - U_1 = 4$ //
- $U_2 + V_3 = 2$ $\underline{U_2} = 2 - 5 = -3$ //
- $U_3 + V_4 = 6$ $\underline{U_3} = 6 - 4 = 2$ //
- $U_3 + V_1 = 5$ $\underline{V_1} = 5 - 2 = 3$ //

$V_1 = 3$

$V_2 = 3$

$V_3 = 5$

$V_4 = 4$

$U_1 = 0$

$U_2 = -3$

$U_3 = 2$

For non-allocated cell:

$$d_{ij} = c_{ij} - (u_i + v_j)$$

$d_{11} = 6 - (0+3) = 3$ //

$d_{21} = 5 - (-3+3) = 5$ //

$d_{22} = 9 - (-3+3) = 9$ //

$d_{24} = 7 - (-3+4) = 7 - 1 = 6$ //

$d_{32} = 7 - (2+3) = 7 - 5 = 2$ //

$d_{33} = 8 - (2+5) = 1$ //

all $d_{ij} > 0$.

∴ minimum transportation cost = 149

Q. Find optimal Solution?

	D_1	D_2	D_3	D_4	supply
O_1	11	20	7	8	50
O_2	21	16	10	12	40
O_3	8	12	18	9	70
demand	30	25	35	40	

* Total Supply = 160 //

Total demand = 130

∴ It is unbalanced.

* In order to make it balance a dummy destination is introduced in the transportation table with zero cost. The excess supply (30) is entered as the column total of the dummy destination.

	D_1	D_2	D_3	D_4	D_5	
O_1	11	~~20~~	$\boxed{25}$ 7	$\boxed{25}$ 8	0	~~50~~ 25
O_2	21	16	$\boxed{10}$ 10	12	$\boxed{30}$ 0	~~40~~ 10
O_3	$\boxed{30}$ 8	$\boxed{25}$ 12	18	$\boxed{15}$ 9	0	~~70~~ ~~35~~ 15
	30	25	~~35~~ 10	~~40~~ 25	30	

7	1	1	1	1
10	2	2	2	2
8	1	1	9	×

3	4	3	1	0
3	4	3	1	×
3	×	3	1	×
×	×	3	1	×
×	×	3	4	×

∴ Transportation Cost = $25 \times 7 + 25 \times 8 + 10 \times 10 + 30 \times 0 + 30 \times 8 + 25 \times 12 + 15 \times 9$

=

for allocated cell $u_i + v_j = c_{ij}$.

		v_1 (8)	v_2 (12)	v_3 (8)	v_4 (9)	v_5 (-2)
-1	u_1	11	20	[25] 7	[25] 8	0
2	u_2	21	16	[10] 10	12	[30] 0
0	u_3	[30] 8	[25] 12	18	[15] 9	0

∴ $u_1 + v_3 = 7$ — (5) $-1 + v_3 = 7$, $v_3 = 8$ //

$u_1 + v_4 = 8$ — (4) $u_1 + v_4 = 8$, $u_1 = -1$ $(8-9)$

$u_2 + v_3 = 10$ — (6) $u_2 = 10 - 8 = 2$ //

$u_2 + v_5 = 0$ — (7) $v_5 = 0 - 2 = -2$ //

$u_3 + v_1 = 8$ — (1) $v_1 = 8$ //

$u_3 + v_2 = 12$ — (2) $v_2 = 12 - 0 = 12$ //

$u_3 + v_4 = 9$. — (3) $0 + v_4 = 9$ ∴ $v_4 = 9$ //

Consider u_3 as 0.

∴

* For non-allocated cell $d_{ij} = c_{ij} - (u_i + v_j)$

$d_{11} = 11 - (-1 + 8) = 4$ //

$d_{12} = 20 - (-1 + 12) = 9$ //

$d_{15} = 0 - (-1 + -2) = +3$ //

$d_{21} = 21 - (2 + 8) = 11$

$d_{22} = 16 - (2 + 12) = 2$ //

$d_{24} = 12 - (2 + 9) = 1$ //

$d_{33} = 18 - (0 + 8) = 10$ //

$d_{35} = 0 - (0 + -2) = 2$ //

all $d_{ij} > 0$

minimum transportation cost = _____

→ for a loop!

$d_{11} = -2$

$d_{12} = 9$

$d_{15} = 11$

$d_{21} = 8$

$d_{22} = 1$

$d_{23} = -3$

$d_{33} = 8$

$d_{35} = 3$ //

→ Select the cell having most -ve d_{ij} value and draw a loop, has same starting and end point, a loop move rowwise & column wise alternatively and move only to allocated cell (except the starting point)

only straight lines are used.

. starting +10 then -ve, then +ve and -ve

11	20	[35] -ve 7	[15] +ve 8	0
21	16	+ve 10	[10] 12 -ve	[30] 0
[30] 8	[25] 12	18	[15] 9	0

then -ve allocations are min (35, 10) = 10

∴ add 10 to -ve sign in loop and subtract -ve

[25] -1	[25]
[10]	[0]

Q. A Company manufacturing 'AC' has two plants located at Mumbai and Kolkatta with weekly capacity of 200 and 100 units. The company supplies two ~~for~~ 4 showrooms situated at Ranchi, Delhi, Lucknow and Kanpur. Which are at demand of 75, 100, 100, 30 units. The cost of transporation per unit is given below.

	Ranchi	Delhi	Lucknow	Kanpur	
Mumbai	90	90	100	100	200
Kolkata	50	70	130	85	100
	75	100	100	30	

Total supply = 300

Total demand = 305

∴ it is unbalanced.

Add dummy row. , with 5 unit supply.

∴

	90	[75] 90	[95] 100	[30] 100	~~200~~ ~~105~~ 30
	[75] 50	[25] 70	130	85	~~100~~ 25
	0	0	[5] 0	0	5
	75	~~100~~ 75	~~100~~ 95	30	

0	0	10	10
20	20	15	15
0	×	×	×

50	70	100	85
40	20	30	15
×	20	30	15
×	20	×	15

∴ Transportation cost = 75 × 90 + 95 × 100 + 30 × 100 + 75 × 50 + 25 × 70 + 5 × 0

= 6750 + 9500 + 3000 + 3750 + 1750 + 0 = 24750

* for allocated cell $u_i + v_j = c_{ij}$.

		70	90	100	100
		v_1	v_2	v_3	v_4
0	u_1	90	[75] 90	[95] 100	[30] 100
-20	u_2	[75] 50	[25] 70	130	85
-100	u_3	0	0	[5] 0	0

$u_1 + v_2 = 90$ | $v_2 = 90 - 0 = 90$

$u_1 + v_3 = 100$ | $v_3 = 100 - 0 = 100$

$u_1 + v_4 = 100$ | $v_4 = 100$

$u_2 + v_1 = 50$ | $v_1 = 50 - -20 = 70$

$u_2 + v_2 = 70$ | $u_2 = 70 - 90 = -20$

$u_3 + v_3 = 0$ | $u_3 = 0 - 100 = -100$

∴ consider $u_1 = 0$.

∴ $u_1 = 0$ $u_2 = -20$ $u_3 = -100$

$v_1 = 70$, $v_2 = 90$, $v_3 = 100$ $v_4 = 100$

* for non-allocated cell.

$d_{ij} = c_{ij} - (u_i + v_j)$.

$d_{11} = 90 - (0 + 70) = 20$

$d_{23} = 130 - (-20 + 100) = 130 - 80 = 50$

$d_{24} = 84 - (-20 + 100) = 84 - 80 = 4$

$d_{31} = 0 - (-100 + 70) = 0 - -30 = 30$

$d_{32} = 0 - (-100 + 90) = 0 - -10 = 10$

$d_{44} = 0 - (-100 + 100) = 0 - 0 = 0$

all $d_{ij} > 0$

$\therefore$ minimum transportation cost = <u>24750</u>

1-3-24

Q. Given $x_{13} = 50$, $x_{14} = 20$, $x_{21} = 55$, $x_{31} = 30$

$x_{32} = 35$, $x_{34} = 25$

It is an optimal solution for given T.P. If not find optimal solution?

				Supply
6	1	$\boxed{50}$ 9	$\boxed{20}$ 3	50+20 = 70
$\boxed{55}$ 11	5	5	8	55
$\boxed{30}$ 10	$\boxed{35}$ 12	4	$\boxed{25}$ 7	30+35+25 = 90
55+30 = 85	35	50	25+20 = 45	

		10 v_1	12 v_2	13 v_3	7 v_4
-4	u_1	6	1	$\boxed{50}$ 9	$\boxed{20}$ 3
1	u_2	$\boxed{55}$ 11	5	5	8
0	u_3	$\boxed{30}$ 10	$\boxed{35}$ 12	4	$\boxed{25}$ 7

(i) for allocated cell

$u_1 + v_3 = 9$	$v_3 = 9 - -4 = 13$
$u_1 + v_4 = 3$	$u_1 = 3 - 7 = -4$
$u_2 + v_1 = 11$	$u_2 = 11 - 10 = 1$
$u_3 + v_1 = 10$	$v_1 = 10$
$u_3 + v_2 = 12$	$v_2 = 12$
$u_3 + v_4 = 7$	$v_4 = 7$

Consider $u_3 = 0$.

for non-allocated cell:

$$d_{ij} = c_{ij} - (u_i + v_j)$$

$d_{11} = 6 - (-4 + 10) = 6 - 6 = 0$

$d_{12} = 1 - (-4 + 12) = 1 - 8 = -7$

$d_{22} = 5 - (1 + 12) =$

$d_{23} = 5 - (1 + 13) =$

$d_{24} = 8 - (1 + 7) =$

$d_{33} = 4 - (0 + 7) =$

solution is not optimal as not all d_{ij} is positive.

so again find optimal solution.

$\boxed{70}$ 6	1	9	3	~~70~~	2	×	×	×
11	$\boxed{35}$ 5	$\boxed{20}$ 5	8	~~55~~ 20	0	0	3	×
$\boxed{15}$ 10	12	$\boxed{30}$ 4	$\boxed{45}$ 7	90 ~~60~~ 15	3	3	3	3
~~85~~ 15	~~35~~	~~50~~ 30	~~45~~					

4	4	1	4
1	7	1	1
1	×	1	1

Transportation cost = $10 \times 6 + 35 \times 5 + 20 \times 5 + 15 \times 10$

$+ 30 \times 4 + 45 \times 7$

$= \underline{1280}$

		10	4	4	7
		v_1	v_2	v_3	v_4
-4	u_1	$\boxed{10}$ 6	1	9	3
1	u_2	11	$\boxed{35}$ 5	$\boxed{20}$ 5	8
0	u_3	$\boxed{15}$ 10	12	$\boxed{30}$ 4	$\boxed{45}$ 7

$u_1 + v_1 = 6$

$u_2 + v_2 = 5$

$u_2 + v_3 = 5$

$u_3 + v_1 = 10$

$u_3 + v_3 = 4$

$u_3 + v_4 = 7$

$u_1 = 6 - 10 = -4$

$v_2 = 5 - 1 = 4$

$u_2 = 5 - 4 = 1$

$v_1 = 10$

$v_3 = 4$

$v_4 = 7$

for allocated cell $c_{ij} = u_i + v_j$

consider $u_3 = 0$.

for non-allocated cell :-

$d_{12} = 1 - (-4 + 4) = 1$

$d_{13} = 9 - (-4 + 4) = 5$

$d_{14} = 3 - (-4 + 7) = 0$

$d_{21} = 11 - (1 + 10) = 0$

$d_{24} = 8 - (1 + 7) = 0$

$d_{32} = 12 - (0 + 4) = 8$

for all $d_{ij} \geq 0$

$\therefore$ minimum transportation cost = $\underline{1280}$

H.W.

Q. solve.

6	1	9	3	70
11	5	2	8	55
10	12	4	7	70
85	35	50	45	

Step 1: check it is balanced or not

$\therefore$ total suppy = 195

total demand = 215

It is not balanced.

$\therefore$ $215 - 195 = 20$

so add an row.

	D_1	D_2	D_3	D_4					
O_1	$\boxed{65}$ 6	$\boxed{5}$ 1	9	3	~~70~~ 5	2	2	2	x
O_2	11	$\boxed{30}$ 5	2	8	~~55~~ 25	3	3	3	3
O_3	10	12	4	7	70	3	3	3	3
O_4	$\boxed{20}$ 0	0	0	0	~~20~~	0	x	x	x
	~~85~~ 65	~~35~~ 30	50	45					
1st	6	1	2	3					
2nd	5	4	2	4					
3rd	x	4	2	4					
4th	x	7	2	1					

O_2	[25] 2		8	25
O_3	[25] 7		[45] 7	~~70~~ 45
	~~50~~ 25		45	

(6)

3

2 1

		~~0~~	-5	-8	-5	
		V_1	V_2	V_3	V_4	
6	U_1	[65] 6	[5] 1	9	3	70
10	U_2	11	[30] 5	[25] 2	8	55
12	U_3	10	12	[25] 4	[45] 7	70
0	U_4	[20] 0	0	0	0	20
		85	35	50	45	

for allocated cell:

consider $U_1 = 0$.

$U_1 + V_1 = 6$ — $U_1 = 6 - 0 = 6$//

$U_1 + V_2 = 1$ — $V_2 = 1 - 6 = -5$//

$U_2 + V_2 = 5$ — $U_2 = 5 - {}^-5 = 10$//

$U_2 + V_3 = 2$ — $V_3 = 2 - 10 = -8$//

$U_3 + V_3 = 4$ — $U_3 = 4 - {}^-8 = 12$//

$U_3 + V_4 = 7$ — $V_4 = 7 - 12 = {}^-5$//

$U_4 + V_1 = 0$ — $U_4 = 0 - 0 = 0$//

for non-allocated cell:

$d_{13} = 9 - (6 + {}^-8) = 9 - {}^-2 = 11$//

$d_{14} = 3 - (6 + -5) = 3 - 1 = 2$//

$d_{21} = 11 - (10 - 0) = 11 - 10 = 1$//

$d_{24} = 8 - (10 + {}^-5) = 8 - 5 = 3$//

$d_{31} = 10 - (12 + 0) = 10 - 12 = {}^-2$//

$d_{32} = 12 - (12 + {}^-5) = 12 - 7 = 5$//

$d_{42} = 0 - (0 + {}^-5) = 5$//

$d_{43} = 0 - (0 + {}^-8) = 0 - {}^-8 = 8$//

$d_{44} = 0 - (0 + -5) = 5$//

select the cell having most negative (-ve) dij value and draw a loop.

[65] -ve 6	[5] +ve 1	9	3	70
11	[30] -ve 5	[25] 2 +ve	8	55
10	12	[25] 4 -ve	[45] 7	70
[20] 0	0	0	0	20

select -ve allocate value
min (65, 30, 25)
= 25//

add 25 to +ve sign and subtract 25 from -ve sign

[40] 6	[30]		
	[5]	[50]	
[25]		[0]	

		V_1	V_2	V_3	V_4
6	U_1	$\boxed{40}$ 6	$\boxed{30}$ 1	9	3
10	U_2	11	$\boxed{5}$ 5	$\boxed{50}$ 2	8
10	U_3	$\boxed{25}$ 10	12	4	$\boxed{45}$ 7
0	U_4	$\boxed{20}$ 0	0	0	0

$u_1 + v_1 = 6$	→ consider $v_1 = 0$ $\therefore U_1 = 6 - 0 = 6$
$u_1 + v_2 = 1$	$V_2 = 1 - 6 = -5$
$U_2 + V_2 = 5$	$U_2 = 5 - -5 = 10$
$U_2 + V_3 = 2$	$V_3 = 2 - 10 = -8$
$U_3 + V_1 = 10$	$U_3 = 10 - 0 = 10$
$U_3 + V_4 = 7$	$V_4 = 7 - 10 = -3$
$U_4 + V_1 = 0$	$U_4 = 0 - 0 = 0$

* for non allocated cell:-

$d_{13} = 9 - (6 + -8) = 9 - -2 = 11$

$d_{14} = 3 - (6 + -3) = 3 - 3 = 0$

$d_{21} = 11 - (10 + 0) = 1$

$d_{24} = 8 - (10 + -3) = 8 - 7 = 1$

$d_{32} = 12 - (10 + -5) = 12 - 5 = 7$

$d_{33} = 4 - (10 + -8) = 4 - 2 = 2$

$d_{42} = 0 - (0 + -5) = 0 - -5 = 5$

$d_{43} = 0 - (0 - 8) = 8$

$d_{44} = 0 - (0 + -3) = 0 - -3 = 3$

$\therefore$ for all $d_{ij} \geq 0$

$\therefore$ minimum transportation cost

$= 40 \times 6 + 30 \times 1 + 5 \times 5 + 50 \times 2 + 25 \times 10 + 45 \times 7 + 20 \times 0$

$= 240 + 30 + 25 + 100 + 250 + 315 + 0$

$= \underline{\underline{960}}$