2nd - Sem - Optimization Techniques for Computer Applications

Module-1

#OR:-Is a scientific study of operations for the purpose of making better decisions.

***Features / Characteristics of OR**:

1. **Decision making**:-which helps management to take better decision.
2. **Optimization objective**:-OR attempts to find the best and optimal solution to a problem using OR techniques.
3. **Bad answers to the problems**:-Or cannot give perfect answers or solution to the problem.
4. **Use of computers**:-OR requires a computer to solve the complex mathematical method.
5. **Scientific approach**:-OR uses scientific methods to solve complex problems.

→Definitions of OR:-Some of OR definitions are;

- T.L Saaty:-"OR is threat of giving bad answers to problems which otherwise it have worse answers"
- P.H. Morse:-"OR is a scientific method of providing executive department with a quantitative basis for decision under their control."

***Development /history of OR**:-Developed in the military context during world war II by the british scientists.
-Or helps in finding the best way to utilize the available resources.
-It help to allocate resources such as food,medicine,weapons etc.
-First equipment developed by utilizing OR was a radar for tracking and detecting an aircraft.
-Because of the success of OR on military operations,
-Ir quickly spread to other fields also.

***Importance of OR (Functions of OR)**:-It is a area of knowledge that can make contribution to the solution of the problems in diverse d areas of our interest.
-It greatly helps in tackling the intricate and complex problems of modern business and industry.
→The various functions of OR are;

1. **OR provides a tool for scientific analyser**:-OR provides the effect relationship and risk underlying the business operations in measurable terms.
   -OR replaces the conventional and subjective approach of decision making by an analytical and objective approach.
2. **OR provides solution for various business problems**:-OR techniques are used in the field of production, marketing, finance and other fields.

-For allocating available resources to various products so that in a given time, the profits are maximum or the cost is minimum.

3. **Enables proper development of resources**:-OR renders valuable help in proper deployment of resources.
   -This enables the project
   completion on time.
   -Also provides for determining the probability of completing an event or project by a specified date.
4. **Helps in minimizing waiting and servicing costs**:-OR technique which helps in minimizing total waiting and service coasts.
   -OR enables the management to decide when to buy and how much to buy.
5. **OR assists in choosing an optimum strategy**:-Game theory is specially used to determine the optimum strategy.
   -Game theory enables the businessman to minimize loss by adopting the optimum strategy.
6. **Facilitates the process of decision making**:-Decision theory enables the businessmen to select the best course of action when information is given.

-OR techniques play an important role in various fields of human life.

***Scope / Applications / uses of OR** :-OR has wide scope and it is very important in every day life.
-There is very great impact of OR on engineering, economics, management and other fields.
-Indian organizations like Railways, Indian Airlines etc. are using OR techniques.

1. **In Defence operations**:- In modern warfare the Defence operations are carried out by a number of components namely, Air Force, Army and Navy.
   -It is used to derive max gain for its operations and there is always possibility that strategy can adverse effect on the others.
   -The final strategy is formulated
   by a team of scientists drawn from various disciplines.
   -Both have numbers of actions.
   - who is able to select the best strategy wins the battle.
2. **In Industry**:-The system of modern industries is so complex that the optimum point of operation it cannot judged by an individual.
   -The business environment is always changing and therefore decision once taken also requries change.
3. **In Planning**:- In modern times, for the government it is necessary to have careful planning for economic development of the country.
4. **In Agriculture**:- With the increase in population there is a need to increase agricultural output.
   -But there are a number of restrictions like climatic conditions, available facilities, distribution of water etc.

-This restriction can be done by using OR techniques.

5. **In Transportation**:- OR techniques like **Monte Carlo technique, queuing theory and linear programming** are of great use in transportation activities.
   -Monte Carlo methods can be
   applied to regulate the train arrivals and processing times.
   -Queuing theory can be applied to minimize the passenger's waiting time.
   -Linear programming technique can be used to formulate suitable transportation policy reducing the cost and time of transshipment.

***Phases of OR /process of OR / Methodology of OR:**-Various steps in the application of OR technique are;

1. **Formulating the problem**:-It is very essential that the problem to be clearly defined.
-Formulating a problem consists in identifying, defining and specifying the measures of the components of a decision model.

2. **Constructing the model** :-In OR a model is usually a mathematical model. Mathematical model consists of a set of equations which describe the problem.
-One or more equations or inequalities is required to represent the system.

3. **Deriving solution the model** :-This phase deals with mathematical calculation for obtaining solution to the model.

4. **Testing the validity**:-A model is said to be valid if it can give a reliable prediction of the system's performance.
-A model must have a longer life and must be a good representation of the system

5.**Controlling the solution**:-The solution derived from a model is depending on the variables. So a change in the values of the variables may affect the solution.
-So controls must be established to indicate the limits within which the model and its solution can be considered as reliable.
-When one or more variables change significantly, the solution goes out of control.

6. **Implementing the results**:-The results of OR must be implemented to improve the system performance.
-This phase of OR is executed through the cooperation of both OR experts and those who are responsible for managing and operating the system.

→Tools used in OR :- are ;

- Linear programming.
- Queuing theory.
- Transportation problem.
- Gaming theory and decision making.
- Assignment problem.

*** Modelling in OR**:-Models play a very important role in OR.
-Modeling is a representation of reality.
-Models provide descriptions and explanations of the operations of the system that they represent.

→Properties (characteristics) of a good model

- It should be simple
- It should be capable of adjustments with new formulations without having any significant change in its frame.
- It should contain very few variables.
- A model should not take much time in its construction.

→Advantages of a model

- It indicates limitation and scope of the problem.
- It enables the use of high-powered mathematical techniques to analyse the problem.
- It helps in finding avenues for new research and improvements in a system.
- It describe problems in less time.

→Disadvantages of a model

- Models are only an attempt to understand an operation and should never be considered as absolute in any sense.
- the validity of any model can only be verified by carrying on experi- ment and relevant data characteristics.

***Types of models** (3 types):-There are three types of models that are commonly used in OR.

1. **Iconic Models**:Physical representation of various aspects of the system
   Eg: blueprint , globe etc.
   -They have the disadvantages also. They are difficult to manipulate for experimental purposes.
   -They cannot be used to study the changes in operation of a system.
   -It is not easy to make any modification or improvement in these models.
2. **Analogue models**: one set of properties is used to represent another set of properties.
   -After the problem is solved, the solution is re-interpreted in terms of the original system.
   -Analogue models are easier to manipulate than iconic models.
3. **Symbolic models (Mathematical models)**: In symbolic models, letters, numbers, and other types of mathematical symbols are used to represent variables and the relationship between them.
   -They are usually the easiest to manipulate experimentally.
   -They usually yield more accurate results, under manipulation.

***Deterministic and Probabilistic models**:-In deterministic models everything is defined and the result are certain.
-In probabilistic models (stochastic models) there is a risk and uncertainty.
-Therefore input and output variables assume probability distributions.

***Static and Dynamic model:**-Models can also be classified as static and dynamic.
-In Static models are those which do not take time into account.
-It assumes that the values of the variables do not change with time during a particular period.
-In dynamic model considers time as one of the important variables.

***Limitations of OR:**- Or tries to find out the optimal solution ,taking all the factor into account.
-When there are large number of factories involved, study of all of them be come difficult or impossible.
-The solution in a problem can be obtained by OR technology ,only if the problem can be quantified.

***General methods of solution for OR models:-**OR models are generally solved by the following methods.

1. **Analytic method**:-In this model all the tools of classical mathematical such as differential calculus and finite difference are used for solving an OR model.
2. **iterative method**:-Whenever the analytic methods(classical method) fail, we use iterative procedure.
   -The classical method may fail because of the complexity of the constraints.
   -In this procedure we start with a trial solution and a set of rules for improving it.
   -The trial solution is improved by the given rules and is then replaced by this improved solution.
   -This process is repeated until either no further improvement is possible or the cost of further calculation cannot be justified.
3. **Monte Carlo technique** :-The basis of Monte Carlo method is random sampling of a variable's possible values.
   -For this technique, some random numbers are required which may be converted into various whose behaviour is known.

#LPP:-generally deals with limited resources(men,machines,materials) to meet given objectives.
-There are certain restrictions on the total amounts of each product made.
-The objective is to optimise the total profit or total cost.

-The term linear means that all the relations in the problem are linear and the term programming refers to a plan of action (Knowing strat and end).

*Definition

- Objective:-To maximise the profit of minimise the cost.
  -Construct the objective function to get total profit or total cost.
- Constrains:-Express in the form of in equalities (<= or >=).
  -The solution to a linear program is to show how much to should produce or purchase and satisfy the constrains.

***Uses of LPP:**-LPP technique is used to achieve the best allocation of available resources. -Available resources may be man-hours, money, machine-hours, raw materials etc.

-To maximize the profit .
-Schedule of orders to minimize the profit.
-Assignment of jobs between workers to have maximum labour productivity.
-Number of crew in buses and train to have minimum operating costs.

***Applications of LP Applications of LP**:-LP is extensively used to solve variety of industrial and management problems.
-Some examples are given below.

- **Product mix**:-The objective is to maximize the profit with by available resources.
- **Product smoothing**.:-Determine the best plan for producing a product with a fluctuating demand.
- **Travelling salesman problem**:- The problem is to find the shortest route for a salesman
  starting from a given city, visiting each of the specified cities and then returning to the original point of departure.
- **Transportation problem:**- Using transportation technique of LP we can determine the distribution system that will minimize total shipping cost from several warehouses to various market locations.
- **Assignment problems**;-The problem of assigning the given number of personnel to different jobs can be solved with the help of assignment model.
  -The objective is to minimize the total time taken or total cost.
- **Rail road industry**:-LP technique can be used to minimize the total crew and engine expenses subject to restrictions on hiring and paying the trainmen, the scheduling of ship capacities of rail road etc.
- **Staffing problem**:-LP method can be used to minimize the total number of employees in restaurant, hospital, police station etc. meeting the staff need at all hours.

→**Advantages of LP**:-Provide best allocation of available resources.
-It makes a scientific and mathematical analysis of the problem situations.
-By using LPP,the decision maker makes sure that he is considering the best solution.

→**Limitations of LP**:-The major limitation of LPP is that It treats all relationships as linear.
-All parameters in the LP model are assumed to be known constants.
-Many problems are complex since the number of variables and constraints are quite a large number.

*Basic assumptions in LPP (3 marks):-The LPP are solved on the basis of the following assumptions.

1. **Proportionality**. There must be proportionality between objectives and constraints.
2. **Additivity**. Sum of the resources used by different activities must be equal to the total quantity of resources.
3. **Divisibility**. The solution need not be in whole numbers.
4. **Certainty**. Coefficients in the objective function and constraints are completely known and do not change during the period under study.
5. **Finiteness**. Activities and constraints are of finite number.
6. **Optimality**. The solution to a problem is to be optimum (maximum or minimum).

*Formation of LPP (imp):-The following steps are applied in this process.

1. **Step 1**:-Identify the objective as maximization or minimization.
2. **Step 2:**-Find/Form variables (x,y) (decision variables) .create objective function.
3. **Step 3**:-Identify the constraints.
   -express them as linear inequalities or equations (<= or >=).
   (x-decision variable)

→ general form of LPP

▲ general form of LPP

• max/min $Z = c_1x_1 + c_2x_2 + \dots c_nx_n$

• $a_{11}x_1 + \dots a_{1n}x_n \le$ or $=$ or $\ge b_1$ -(1)

• $a_{21}x_1 + \dots + a_{2n}x_n \le$ or $=$ or $\ge b_2$ -(2)

c_j = profit co-efficient
a_{ij} = structure co-efficient
b_j = requirement

Q)

Ex. 1: A manufacturer of furniture makes two products, chairs and tables. Processing of these products is done on two machines A and B. A chair requires 2 hours on machine A and 6 hours on machine B. A table requires 5 hours on machine A and no time on machine B. There are 16 hours of time per day available on machine A and 30 hours on mahcine B. Profit gained by the manufacturer from a chair is Re. 1 and from a table is Rs. 5 respectively. Formulate the problem into a L. P. P. in order to maximise the total profit.

Formation of LPP Q) C6.

	A	B	Rs
chair	2	6	1 Rs
table	5	0	5 Rs
total time	16	30	

Objective fn = $Z = x_1 + 5x_2$

A) decision variable (Table & chair) x_1, x_2

Profit of chair $= 1 \times x_1$

" of table $= 5 \times x_2$

$\therefore$ Total profit $= Z = \cancel{x_1 + 5x_2}$ $x_1 + x_2$

~~$\therefore$ maximise $Z = x_1 + 2x_2$~~

Machine A

Time required of chair $= 2 \times x_1 = 2x_1$

" " of table $= 5 \times x_2 = 5x_2$

$\therefore$ Total time $= 2x_1 + 5x_2$

available time = 16 hrs.

$\therefore 2x_1 + 5x_2 \leq 16$

machine B

Time required of chair $= 6 \times x_1 = 6x_1$

Time " " table $= 0 \times x_2 = 0$

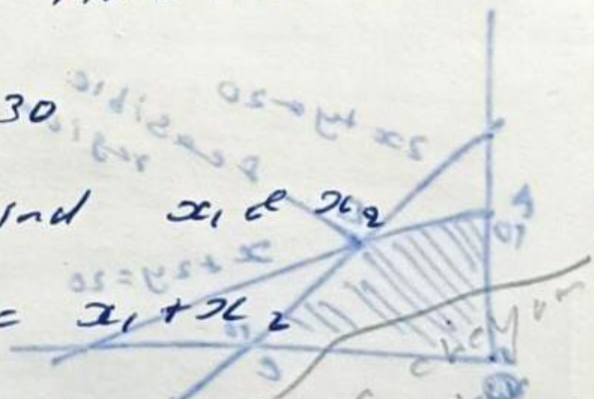
Total $= 6x_1 + 0$

available time $= 30$ hrs.

$\therefore 6x_1 + 0 \leq 30$

$\therefore$ LPP is to find x_1 & x_2

maximise $Z = x_1 + x_2$



subject to.

$$2x_1 + 5x_2 \leq 16$$
$$6x_1 \leq 30$$
$$x_1 \geq 0, \quad x_2 \geq 0$$

Note :always do problems max.
-if the given problem is min then convert it into max.

min = - max

Q) (imp) An animal feed company must produce 250 kgs of a mixture consisting of ingredients A and B daily. A costs Rs.13 per kg and B Rs. 84 per kg. No more than 80 kgs of A can be used and at least 60 kgs of B must be used .Formulate a mathematical model to the problem.

Answers:

***Solution to LPP**:-A linear programming problem can be solved in two methods they are;

1. **Graphic method**:- explanation down ill unde.
2. **Simplex method**.:-2nd module ill explain cheythittune

*** Graphic method:**-LPP involving two variables can be solved by graphical method.

-This method is simple and easy to apply..

-But LPP involving more than two variables can not be solved by this method.

-In this method Each constraint is represented by a line.

-If there are many constrains ,many lines are to be drawn.

-This will make the graph difficult to read.

→Steps for solving a LPP by graphic methods are;

1. Form LPP
2. Inequality written as equality.
3. Draw straight line as there equation.
4. Identify feasible region.
5. Locate corner/vertex and find coordinates.

6. Substitute value in objective function (Z).
7. Choosing optimal values.

Q)

Q maximize $z = x + 3y$. subject to the constrains $x \geq 0$, $y \geq 0$, $2x + y \leq 20$, $x + 2y \leq 20$

A) max $z = x + 3y$

$x = 0$, $y = 0$

$2x + y = 20$.

$x + 2y = 20$

• $2x + y = 20$

put $x = 0$.

$x = 0$, $y = 20$

$= (0, 20)$

put $y = 0$

$2x = 20$, $x = 10$

$(10, 0)$

• $x + 2y = 20$

put $x = 0$

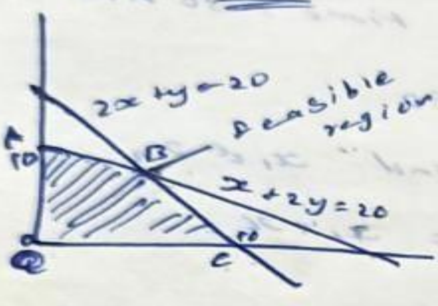
$y = 10$

$(0, 10)$

put $y = 0$

$y = 0$, $x = 20$

$(20, 0)$



feasible solution

$O(0, 0)$

$A(= 0, 10)$

$C(= 10, 0)$

coordinate of B is obtained by solving the eqn

① $2x + y = 20$ & ② $x + 2y = 20$

multi eq ① by 2

$$4x + \cancel{y}\ 2y = 40$$
$$x + 2y = 20$$
$$3x + 0 = 20$$

$\therefore x = \frac{20}{3} = 6.6$

$6.6 + 2y = 20$

$2y = 20 - 6.6$

$2y = 13.4 \quad y = 6.7$

$x = 6.6$ & $y = 6.7$

$(6.6, 6.7)$

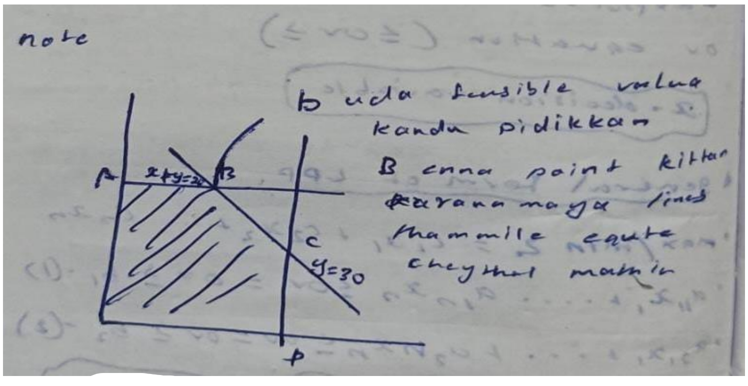
substituting these value in $x + 3y$

point	x	y	$x + 3y$
O	0	0	$0 + 0 = 0$
A	0	10	$0 + 3 \times 10 = 30$
B	6.6	6.7	$6.7 \times 3 \times 6.7 = 26.7$
C	10	0	$10 \times 3 \times 0 = 10$

max value of $z = 30$ at point A.

$\therefore$ the optimum sol

$x = 0$, $y = 10$ & maximum value of $x + 3y$ is 30



Q)Max P =2 x+ 3y

Subject to x+y ≤30,

y ≥5,

0 ≤ y ≤ 15,

0 ≤ x ≤ 20,

X-y≥0

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Q) Minimize Z =X1 + X2

Subject to 2 X1 + X2 ≥4, X1 +7 X2 ≥ 7, X1, X2 ≥ 0

"Check below pdf note page :4 to 5

Q)Max P = 2x+3y

Subject to x + 2y ≤ 2, 4x+3y ≥ 12 and x , y ≥0

"Check below pdf note page :5 to 6

q) "Check below pdf note page :7 to 12

→Some special cases of solutions of graphic method:-

- **Infeasible**:-No feasible solution.

  -If the feasible region of a LPP is empty .
- **Unbounded**:-variable can be made infinitely large without violating constraints.
- **Alternative**:-There are more than one optimal solution.

Module -2

***simplex method**:-It was originally developed by G.B. Dantzing ,an american mathematician.

-Simplex method is a linear programming technique in which we start with a certain solution which is feasible.

-We improve this solution in a number of consecutive steps until we arrive at an optimal solution.

-For arriving at the solution of LPP by this method, the constraints and the objective function

are presented in a table known as **simplex table**.

-The simplex method is an iterative (step by step) procedure in which we proceed in systematic steps from an initial basic feasible solution to another basic feasible solution and finally arrive at an optimal solution.

-The simplex algorithm consists of the following steps.

1. **Find a trial basic feasible solution of the LPP.**
2. **Test whether it is an optimal solution or not.**
3. **If not optimal, improve the first basic feasible solution by a set of rules.**
4. **Repeat the steps 2 and 3 till an optimal solution is obtained.**

→**Feasible solution**:-It is the set of values of the variables which satisfy all the constraints and non-negative restrictions of the problem.

→**Optimal (Otimum) solution**:-it optimizes the objective function Z of the

Problem or it must satisfy optimal function.

→**Basic feasible solution**:-A feasible solution to a LPP in which the vectors associated to non-zero variables are linearly independent is called a basic feasible solution.

→**Slack variables**:-If a constraint has a sign ≤ then in order to make it an equality we have to add some variable to the left hand side (L.H.S).

eg: Consider the constraint $2x_1 + x_2 \leq 600$.

-To convert the constraint into equation we add s_1 to L.H.S

-then we have $2x_1 + x_2 + s_1 = 600$. Then s_1 is the slack variable.

→**Surplus variable**:- If a constraint has a sign ≥ then in order to make it an equality we have to subtract some variable from the L.H.S. The variables which are subtracted from the L.H.S of the constraints to convert them into equalities are called **surplus variables**.

eg:Consider the constraint $2x_1 + x_2 \geq 600$.

-To convert the constraint into equation we subtract s_2 to L.H.S

-then we have $2x_1 + x_2 - s_2 = 600$. Then s_2 is the surplus variable.

***constructing a simplex table**:-Simplex table consists of rows and columns.

-If there are m original variables and n introduced variables (slack, surplus or artificial variables) then there will be **3 + m+ n columns** in the simplex table.

B (1)	C_B (2)	x_B (3)	x_1 (4)	x_2 (5)	s_1 (6)	s_2 (7)	Repl. Ratio $\theta = x_B \div x_1$

-First column (B) contains the basic variables.

-Second column (C_B) shows the coefficients of the basic variables in the objective function. -Third column (X_B) gives the values of basic variables.

-Each of next m+n columns contains coefficients of variables in the constraints.

→Basis (B):-The variables whose values are not restricted to zero in the current basic solution, are listed in the first column of the simple table known as basis.

→Basic variables:- The variables which are listed in the basis are called basic variables (X_B).

→Vector:- Any column or row of a simplex table is called a vector.

-so we have X_1 and X_2 are vector.

→Unit vector. A vector with one element 1 and all other elements zero is a unit vector.

$$\text{Eg. } \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix} \text{ are unit vectors.}$$

→Net evaluation (Δj):- **Δj = Zj - Cj**

*Simplex algorithm

1. Formulate the problem int a LPP.
2. Convert the constraints into equations (equal form).
3. Draw simple table
4. Conduct test optimality (find Δj) **Δj = Zj - Cj**
5. Find incoming and out coming vectors.
6. The element which is at the intersection of incoming vector and outgoing vector is called the key element.

   -We mark this element in $\boxed{}$ rectangle box.

7. Now test the above improved basic feasible solution for optimality as in step 4. If the solution is not optimal then repeat steps 5 and 6 until an optimal solution is finally obtained.

$$\mathbf{Z_j = C_B \times (x_i S_1 + x_i S_2)}$$

NOTE: A minimisation problem can be converted into a maximisation problem by changing the sign of coefficients in the objective function.

∴ To Min: $Z = 4x_1 - 2x_2 + x_3$, we can Max: $Z' = -4x_1 + 2x_2 - x_3$ subject to the same constraints.

>Problem

Ex 1: Solve the L. P. P

Maximise $Z = 7x_1 + 5x_2$

Subject to

$$x_1 + 2x_2 \le 6 \quad \text{.................. (1)}$$
$$4x_1 + 3x_2 \le 12 \quad \text{.............. (2)}$$
$$x_1, x_2 \ge 0$$

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Q "Check below pdf note page :14 to 19

#Artificial variable:-Are fictitious variables.

-introduced when the constraints are of the form ≥ or =.

-or They are incorporated only for computational purpose or Big M method.

Eg. $2x_1 + 3x_2 + x_3 \ge 10$ can be written as $2x_1 + 3x_2 + x_3 - s_1 + A_1 = 10$. Here s_1 is the surplus variable, A_1 is the artificial variable.

$$\boxed{\ge\ = -S_1 + A_2}$$

Eg 2:

$3x_1 + x_2 + 2x_3 = 6$ can be written as $3x_1 + x_2 + 2x_3 + A_1 = 6$, A_1 is the artificial variable

$$\boxed{= +A_2}$$

***Artificial variable technique:**-Constraints of some LPP may have or = signs. In such problems even after introducing surplus variable, the simplex table may not contain identity matrix or unit vectors.

-So to get identity matrix in the simplex table, we introduce artificial variables to the constraints as per requirement.

-By introducing artificial variables, we are able to get initial basic feasible solution. (the basic variables are those which have unit vectors in the simplex table).

-This technique of LPP in which artificial variables are used for solving is known as ar**tificial variable method.**

-Such problems are solved by **Big - M Method**.

***Big -M Method:**-Big -M Method is a modified simplex method for solving a LPP.

-This method can be applied in minimization problems and maximization problems.

Q) **Example 1: Select the highest negative value from 7 – 4M, –6M + 6**
$\frac{2}{3} - 9M$

Ans: Coefficients of M are –4, –6, –9

So highest negative is –9, $\therefore\ \frac{2}{3} - 9M$ is highest negative.

Example 2: Select the highest negative from 5 – 6M, 6 – 6M, 10 – 5M

Ans: 5 – 6M is the highest negative. bcz const is small.

Q)

Ex. 1: Min $Z = 5x_1 + 6x_2$

S. t

$2x_1 + 5x_2 \geq 1500 - S_1 + A_1$

$3x_1 + x_2 \geq 1200 - S_2 + A_2$

$x_1 \geq 0, x_2 \geq 0$

"Check below pdf note page :20 to 21

Note:

$z_j = c_B \times x_i$

M - penality.

Max case = −M

min case = +M

min = $c_j - z_j$

max = $z_j - c_j$

Q "Check below pdf note page :21 to 23

Note: In the case of inequality '≤' add slack variable only. In the case of the inequality '≥' subtract the surplus variable and add artificial variable. In the case of '=', add artificial variable only.

Eg: $2x_1 + x_2 \leq 3$ is $2x_1 + x_2 + s_1 = 3$

$3x_1 + x_2 \geq 4$ is $3x_1 + x_2 - s_1 + A_1 = 4$

$4x_1 + 5x_2 = 2$ is $4x_1 + 5x_2 + A_2 = 2$

Note. The incoming vector corresponds to the highest negative Δ_j for maximisation cases and highest positive Δ_j for minimisation cases.

#Duality in LPP:-Every LPP is associated with another LPP called dual.

-**Original problem is called primal**.

-Dual problems may be defined as mirror image problems of primal problems.

-The final simplex table which gives the solution of the primal also contains the solution of the dual.

-Similarly the final simplex table which gives the solution of the dual, also contains the solution of the primal.

-So when the optimal solution of one of them is known, then that of the other is also known.

-In situations where dual can be solved easily, the solution to the primal can be obtained through its dual.

→How to find the dual of a given primal?

1. If the primal is maximisation, the dual is minimisation or primal is min and dual is max.
2. Primal Constants in the Right Hand side of the constraint are the coefficients in the objective function is dual and vice versa.
3. The transpose of the coefficients in the constraints of primal form the coefficients for the dual.
4. The inequalities in the constraints are reversed.

   Prime > and dual is < or prime is < and dual is >.

Q) check given question is primal or not?

eg: $2x_1 + x_3 \leq 0$

$3x_2 + 4x_2 \leq 0$

$5x_1 + 5x_2 \leq 0$

is prime because same sympol

$2x_1 + x_3 \geq 0$

$4x_1 + 5x_2 \geq 0$

primal because same sympol

$5x_1 + 2x_3 \leq 0$

$4x_1 + 2x_3 \geq 0$

$5x_1 + 3x_2 = 0$

not prime because diff sym

Q) Q $\max z = 3x_1 + x_2 + 2x_3$

subject to

$x_1 + x_2 + x_3 \le 5$

$2x_1 + x_3 \le 10$

$x_2 + 3x_3 \le 15$

A) prime

$\max z = 3x_1 + x_2 + 2x_3$

$x_1 + x_2 + x_3 \le 5$

$\cancel{2x_1 + x_3 \le 10}$

$\cancel{x_2 + 3x}$

$2x_1 + 0x_2 + x_3 \le 10$

$0x_1 + x_2 + 3x_3 \le 15$

$= \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}^T$

dual

$\min z = 5y_1 + 10y_2 + 15y_3$

$y_1 + 2y_2 + 0y_3 \ge 3$

$y_1 + y_3 \ge 1$

$y_1 + y_2 + 3y_3 \ge 2.$

$y_1, y_2, y_3 \ge 0.$

$= \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

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→Characteristics of the dual problem:

- Dual of the dual is primal.
- If the primal (or dual) has a finite optimal solution, then the other also has finite optimal solution.
- Optimal value of objective function (ie. Z) will be same for both primal and dual.
- If one has no feasible solution, the other will also have no feasible solution.

→shadow price:- is also called dual price.

Module -3

 # Transportation Problem:-It is used to minimise the transportation cost.
-Transportation technique can be applied not only to the cost minimization problems, but also to time minimizing problems, distance minimizing problems, profit maximizing problems etc.

→Transportation table:

Denote the origins as $O_1, O_2, \dots, O_m$ and destinations as $D_1, D_2, \dots, D_n$. Let the quantity produced at the origins be respectively $a_1, a_2, \dots, a_m$. Let the requirements in various destinations be respectively $b_1, b_2, \dots, b_n$. The total quantity produced and the total quantity required must be equal. That is, $a_1 + a_2 + \dots + a_m = b_1 + b_2 + \dots + b_n$.

Note: **∑ai = ∑bj** (always equal numbers ayirikkum)

	D_1	D_2	D_j	D_n	Available
O_1	c_{11}	c_{12}	c_{1j}	c_{1n}	a_1
O_2	c_{21}	c_{22}	c_{2j}	c_{2n}	a_2
O_i	c_{i1}	c_{i2}	c_{ij}	c_{in}	a_i
O_m	c_{m1}	c_{m2}	c_{mj}	c_{mn}	a_m
Required	b_1	b_2	b_j	b_n	

-This matrix is known as transportation **table** or **cost effectiveness matrix**.
-Here **total availability = total requirement**.

→**Transportation problem in the form of a LPP** (Mathematical formulation) (3 mark) :-

Let x_{ij} be the number of units transported from the i^{th} origin to the j^{th} destination.

Let c_{ij} be the cost of transportation of one unit from the i^{th} origin to the j^{th} destination.

Let a_i be the units available in the i^{th} origin and b_j be he units required in the j^{th} destination.

Then the problem is

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to $\sum_{j=1}^{\boxed{n}} x_{ij} = a_i$, for $i = 1, 2, \dots, m$

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$\sum_{i=1}^{m} x_{ij} = b_j$, for $j = 1, 2, \dots, n$

$x_{ij} \geq 0$ for i, j

→Basic assumptions in transportation problem

1. **∑ai = ∑bj** is always equal.
2. The unit transportation cost from one origin to a destination is certain.
3. Objective is to minimize the total transportation cost.

→Uses of transportation technique

1. To minimize transportation cost from factories to warehouses or from warehouses to markets.
2. To determine lowest cost location for new factory
3. To determine minimum cost production schedule.

*Definitions

→Feasible solution:-A feasible solution to a transportation problem is a set of non-negative individual allocations which satisfy the row and column sum restrictions.
-So the sum of the allocations in the rows must be equal to the availability in that row. -Similarly sum of the allocations in the columns must be equal to the demand in that column.
→Basic feasible solution:-A feasible solution to a m x n transportation problem is said to be a basic feasible solution if the total number of allocations is **exactly equal to m + n+ 1**.
→Optimal solution:- A feasible solution is said to be optimal if it minimizes the total transportation cost.
→Non degenerate basic feasible solution:-A feasible solution of a m x n transportation problem is said to be non degenerate basic feasible solution if

- The number of allocations is equal to m + n - 1
- The allocations are in independent positions.

***Loops in transportation table** (non-independent position**)**:-Allocations are said to be in independent positions, if it is impossible to increase or decrease any allocation without either changing the position of the allocation or violating the rim requirements.
-Therefore when the allocations are in independent positions, it is impossible to travel from any allocation back to itself through a series of horizontal or vertical jumps.

Table I

5	3	$\boxed{2}$ 2
4	$\boxed{3}$ 7	2
5	$\boxed{7}$ 6	$\boxed{3}$ 2

In table I, the allocations are in independent positions.

Table II

5	$\boxed{8}$ 3	$\boxed{1}$ 2
4	7	2
5	$\boxed{7}$ 6	$\boxed{3}$ 2

In table II, the allocations are not in independent positions.

Table III

$\boxed{8}$ 5	3	$\boxed{2}$ 2	3
$\boxed{5}$ 4	7	2	$\boxed{2}$ 4
5	6	$\boxed{5}$ 2	$\boxed{2}$ 1

In table III, the allocations are not in independent positions.

***Initial (basic) feasible solution**:-The initial feasible solution can be obtained either by inspection or by some rules.

-The commonly used methods for finding initial feasible solution are

1. **North west corner rule**
2. **Lowest cost entry method (matrix minima method)**
3. **Vogel's approximation method (unit cost penalty method)**

***North west corner rule**:-This method is used to find initial feasible solution.

->__Procedure__:-

	Destinations			
Origins	D_1	D_2	D_3	Supply
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	

-First check that the given problem is balanced or not.

-If the total demand and total supply is Equal then the problem is balanced.

-If it is not then the problem is unbalanced.

-here total demand and total supply =34.

-Therefore this problem is balanced.

- Allocate to cell (1, 1), minimum of 5 and 7 (row total and column total), which is 5. -Then O_1 row total is exhausted, since the supply of is completely met. So cross off row O_1.

	D_1	D_2	D_3	Supply
O_1	~~$\boxed{5}$ 2~~	~~7~~	~~4~~	5 ✗
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7 (2)	9	18	

- Consider the reduced matrix after deleting O_1 row.
  -Allocate to cell (1, 1), minimum of 8 and 2, which is 2.
  -Then column is exhausted, and it is crossed off.

	D_1	D_2	D_3	
O_2	[2] 3	3	1	8 (6)
O_3	5	4	7	7
O_4	1	6	2	14
	2 X	9	18	

- Consider the reduced matrix after deleting D_1 column.
  -Allocate to cell (1, 1), minimum of 6 and 9, which is 6.
  -Then row is exhausted,and it is crossed off.

	D_2	D_3	
O_2	[6] 3	1	6 X
O_3	4	7	7
O_4	6	2	14
	9 (3)	18	

- Consider the reduced matrix after deleting D_2 row.
  -Allocate to cell (1, 1), minimum of 7 and 3, which is 3.
  -Then column is exhausted, and it is crossed off.

	D_2	D_3	
O_3	[3] 4	7	7 (4)
O_4	6	2	14
	3 X	18	

- Consider the reduced matrix after deleting D_2 column.
  -Allocate 4 to cell (1, 1) and 14 to cell (2, 1).

	D_3	
O_3	[4] 7	4
O_4	[14] 2	14
	18	

- Total transportation cost
  = (5 2) + (2 3) +(6 3) +(3 4) +(4 7) +(14 2)
  = 10 + 6 + 18 + 12 + 28 + 28
  = 102

	D_1	D_2	D_3	Supply
O_1	[5] 2	7	4	5
O_2	[2] 3	[6] 3	1	8
O_3	5	[3] 4	[4] 7	7
O_4	1	6	[14] 2	14
Demand	2	9	18	

Q) "Check below pdf note page :27 to 28

***Lowest cost entry method**:-Here we take least number in the matrix.

eg:Find the initial feasible solution to the following transportation problem by lowest cost entry method.

	W_1	W_2	W_3	Supply
F_1	2	7	4	5
F_2	3	3	1	8
F_3	5	4	7	7
F_4	1	6	2	14
Demand	7	9	18	

Answer. The lowest cost is 1 in cells (2, 3) and (4, 1). Select one of these, say (2, 3). Allocate to cell (2, 3), minimum of 8 and 18 (row total and column total), which is 8. Then F_2 row total is exhausted. So cross off the row F_2.

	W_1	W_2	W_3	Supply
F_1	2	7	4	5
F_2	3	3	8	1	8 X
F_3	5	4	7	7
F_4	1	6	2	14
Demand	7	9	18 (10)	

The lowest cost in the reduced matrix is 1 in the cell (3, 1). Allocate to this cell, minimum of 7 and 14, which is 7. Then W_1 column total is exhausted. So cross off the column W_1.

	W_1	W_2	W_3	
F_1	2	7	4	5
F_3	5	4	7	7
F_4	7	1	6	2	14 (7)
	7	9	10	
	X			

The lowest cost in the reduced matrix is 2 in the cell (3, 2). Allocate to this cell, minimum of 7 and 10, which is 7. Then F_4 row total is exhausted. So cross off the row F_4.

	W_2	W_3	
F_1	7	4	5
F_3	4	7	7
F_4	6	7	2	7 X
	9	10 (3)	

Allocate to the cell (2, 1), minimum of 7 and 9, which is 7. Then F_3 row total is exhausted. So cross off the row F_3.

	W_2	W_3	
F_1	7	4	5
F_3	$\boxed{7}$ ~~4~~	~~7~~	7 X
	9 (2)	3	

Finally allocate 2 to the cell (1, 1) and 3 to the cell (1, 2).

	W_2	W_3	
F_1	$\boxed{2}$ 7	$\boxed{3}$ 4	5
	2	3	

Total transportation cost =
$= (2 \times 7) + (3 \times 4) + (8 \times 1) + (7 \times 4) + (7 \times 1) + (7 \times 2) = 14 + 12 + 8 + 28 + 7 + 14 = 83.$

	W_1	W_2	W_3	Supply
F_1	2	$\boxed{2}$ 7	$\boxed{3}$ 4	5
F_2	3	3	$\boxed{8}$ 1	8
F_3	5	$\boxed{7}$ 4	7	7
F_4	$\boxed{7}$ 1	6	$\boxed{7}$ 2	14
Demand	7	9	18	

Note:before starting the problem check than the problem is balanced or not.

***Vogel's approximation method:**-Find the initial feasible solution for the transportation problem by Vogel's method.

Q) "Check below pdf note page :29 to 32

***Optimal solution for transportation problem:-**By applying Vogel's method or lowest cost entry method or north west corner rule, an initial feasible solution is obtained.

-The next step is to examine whether the solution is optimal or not.

-For this we conduct test of optimality.

-For this we use the modified distribution method **(MODI method**) (important annu)

***MODI method** (important annu)

"Check below pdf note page :32 to 38

"Here we use vogel's method "

*Degeneracy in T.P:- google

***Unbalanced transportation problems**:- A transportation problem is said to be unbalanced if the sum of all available amounts is not equal to the sum of all requirements in all destinations together.

-That is, $\sum a_i \neq \sum b_j$.

-An unbalanced transportation problem is converted into a balanced transportation problem, by introducing a fictitious source or destination.

-The cost of transportation corresponding to it is taken to be zero.

Q) "Check below pdf note page :38 to 40

***maximization in T.P**:-A transportation problem in which the objective is to maximize, can be solved by converting it into a minimization problem.

-For this select the highest value and subtract all other values from this highest value.

-Then the given problem becomes minimization problem.

Eg. Solve the following transportation problem to maximize profit.

Profit in Rs./unit distribution

	A	B	C	D	Supply
I	15	51	42	33	23
II	80	42	26	81	44
III	90	40	66	60	33
Demand	23	31	16	30	

Answer. This is a maximization problem. Convert it into a minimization problem. For this select the highest profit, which is 90. Subtract each profit from 90. The modified matrix is,

	A	B	C	D	Supply
I	75	39	48	57	23
II	10	48	64	9	44
III	0	50	24	30	33
Demand	23	31	16	30	

Then by using Vogel's method find initial basic feasible solution and by using MODI method, find optimal solution as before, in the minimization problem.

#Assignment Problem:-Assignment problem is a special case of transportation problem, in which the objective is to assign a number of origins (persons) to the equal number of destinations (tasks) at a minimum cost.

-For example, a department has four persons available for assignment and four jobs to fill.

-Then the interest is to find the best assignment which will be in the best interest of the department.

-The assignment problem can be stated in the form of n x n matrix called **cost of effectiveness matrix**.

*Travelling salesman problem -imp