Module: II

→ Simplex Method:.

Steps:-

i) The objective function should be of maximisation type

ii) Express the LPP in the standard form

Q: Max $z = 7x + 5x_2$

subject. $x_1 + 2x_2 \leq 6$

$4x_1 + 3x_2 \leq 12$

$x_1\ x_2 \geq 0$.

Solve the given LPP using simplex method?

Ans:. subject to changed to. with S_1 .....

$x_1 + 2x_2 + S_1 = 6$ — (1)

$4x_1 + 3x_2 + S_2 = 12$ — (2)

& $x_1\ x_2\ S_1\ S_2 \geq 0$.

then add the S_1 & S_2 to the objective.

$\therefore$ Max $z = 7x_1 + 5x_2 + 0S_1 + 0S_2$

iii) Obtain an initial basic feasible solution.

S_1, S_2 → basic variable.

$\therefore x_1 = x_2 = 0$. non basic variables.

then find the value of S_1 and S_2.

$x_1 + 2x + S = 6$

$\therefore S_1 = 6$

$4x_1 + 3x_2 + S_2 = 12$ $\therefore$ $\underline{\underline{S_2 = 12}}$

Step 4: Construction of Simplex table.

Column CB: Coefficient of basic variable in objective function.

Column B: This consist of basic variables.

Column XB: Solution / value of basic variable.

$S_1 = 6$ $S_2 = 12$

→ Constraint with (1)

CB	B	XB	X1	X2	S1	S2	Minimum Ratio (XB/ Incoming vector)
0	S_1	6	1	2	1	0	6
0	S_2	12	4	3	0	1	3
	Z_j		0	0	0	0	
	C_j		7	5	0	0	
	Δ_j		-7	-5			

$Z_j = \sum CB * X_j$ (a) $0 \times 1 + 0 \times 4 = 0$

$0 \times 4 = 0$

$C_j \rightarrow$ coefficient of the complete variable in objective.

$\therefore$ 7 5 0 0

$\Delta j = zj - Cj$

- find the largest negative value in Δj - ie, -7. So the ~~incoming~~ incoming vector is x_1

then find the minimum Ratio.

$\therefore$ 6, 3.

6/1 12/4.

The minimum vector is 3. So, Circle the S_2

-> If all Δj is +ve the present solution is the optimal solution.

If at least one ~~variab~~ value of Δj is -ve the present solution is not optimal.. In that case identify the most negative Δj, in this table the most negative Δj value is -7. -7 lies in the column x_1. So x_1 is the incoming vector, Key column.

Identify the minimum positive ratio. here '3' is minimum positive ratio. So 'S_2' is the leaving vector.

Here '4' is the Key element.

Simplex Table 2

CB	B	XB	X1	X2	S1	S2	Minimum Ratio (XB/ incoming vector.
0	S_1	3	0	5/4	1	-1/4	
7	x_1	3	1	3/4	0	1/4.	
	zj		7	21/4	0	7/4	
	cj		7	5	0	0	
	Δj		0	1/4	0	7/4	

(1) XB = value of basic variable.

now the basic variables are S_1 and x_1

$\therefore$ the non-basic variables are S_2 and x_2.

$\therefore$ $S_2 = x_2 = 0$.

then substitute it in equation, to find the value S_1 and x_1

$\therefore$ (2) -> $4x_1 + 0 + 0 = 12$.

$4x_1 = 12$

$x_1 = 12/4 = 3$

(1) => $3 + 0 + S_1 = 6$ $\therefore$ $S_1 = 6 - 3 = 3$

(2) then values in the odd table with the key value (

$\therefore$ ($4/4 = 1$, $3/4$, $0/4 = 0$ $1/4$)

it in x_1.

$\therefore$

(3) then find the values in the s_1 row.

'for x_1 = value in the odd table + (− value in the key column * below element in the new table)

$\therefore x_1 = 1 + (-1 \times 1)$
$= 0 \times 1 = 0$//

$x_2 = 2 + (-1 \times 3/4)$. $2 - 3/4 = \frac{8-3}{4} = \underline{5/4}$

$s_1 = 1 + (-1 \times 0) = \underline{1}$

$s_2 = 0 + (-1 \times 1/4) = \underline{-1/4}$

(4) find the value of z_j, c_j and Δ_j.

(5) then check Δ_j are +ve or not.

if it is +ve then

$s_1 = 3$
$x_1 = 3$.

other $s_2 = x_2 = 0$.

(4)

$\text{Max } z = 7 \times 3 + 5 \times 0 + 3 + 0$

$= 21$

2-4-24

Q. Solve the given LPP using simplex method.

$\text{Min } z = x_1 - 3x_2 + 2x_3$

st, $3x_1 - x_2 + 3x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$.

Ans: objective should be max

$\therefore \text{Max } z = -x_1 + 3x_2 - 2x_3$.

add slack variable

$3x_1 - x_2 + 3x_2 + S_1 = 7$

$-2x_1 + 4x_2 + S_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$.

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$.

then objective became.

$\text{Max } z = -x_1 + 3x_2 - 2x_3 + 0S_1 + 0S_2 + 0S_3$

* Initial basic variables are $S_1\ S_2\ S_3$ → basic variable.

· non basic variables $x_1\ x_2\ x_3 = 0$.

$\therefore S_1 = 7$

$S_2 = 12$

$S_3 = 10$

step : construction of simplex table.

CB	B	XB	X1	X2	X3	S1	S2	S3	Minimum Ratio (XB/ incoming vector.
0	S_1	7	3	-1	3	1	0	0	-7
0	S_2	12	-2	4	0	0	1	0	3
0	S_3	10	-4	3	8	0	0	1	10/3
		Z_j	0	0	0	0	0	0	
		C_j	-1	3	-2	0	0	0	
		Δ_j	1	-3	2	0	0	0	

$Z_j = \Sigma CB * x_j$

C_j = coefficient of complete variable.

$\Delta_j = Z_j - C_j$

check Δ_j is +ve; so find the most -ve column.

$\therefore$ Incoming vector is x_2.

Then find the minimum ratio.

$7/-1$, $12/4$ =

$= -7$ $= 3$

So the leaving vector is S_2.

So key is $\underline{\underline{4}}$

• Simplex table 2.

CB	B	XB	x_1	x_2	x_3	S_1	S_2	S_3	Minimum Ratio (XB/ incoming vector.
0	S_1	10	5/2	0	3	1	1/4	0	4
3	x_2	3	-1/2	1	0	0	1/4	0	-6
0	S_3	1	-5/2	0	8	0	-3/4	1	-2/5
	z_j		-3/2	3	0	0	3/4	0	
	c_j		-1	3	-2	0	0	0	
	Δ_j		-1/2	0	2	0	3/4	0	

∴ Incoming vector is x_1

Then find the minimum ratio.

∴ XB/incoming vector ∴

$10/ \div 5/2 = 10 \times \frac{2}{5} = 4$

$3 \div -\frac{1}{2} = 3 \times \frac{-2}{1} = -6/1 = -6$

$1 \div -\frac{5}{2} = 1 \times \frac{-2}{5} = -2/5$

∴ Key value = $5/2$

∴ Simplex table : 3 :-

CB	B	XB	x_1	x_2	x_3	S_1	S_2	S_3	Minimum Ratio (XB/incoming vector).
-1	x_1	4	1	0	6/5	2/5	1/10	0	
3	x_2	5	0	1	3/5	1/5	3/10	0	
0	S_3	11	0	0	11	1	-1/2	1	
	Z_j		-1	3	3/5	1/5	4/5 ~~8/10~~	0	
	C_j		-1	3	-2	0	0	0	
	Δ_j		0	0	13/5	1/5	8/10	0	

$\therefore$ Δj is +ve.

$\therefore$ find the value of basic variable

$\therefore$ $x_1 = 4$

$x_2 = 5$

$S_3 = 11$

$\therefore$ non-basic variables are $x_3 = S_1 = S_2 = 0$ //

$\therefore$

$$\text{Max } z = -x + 3x_2 - 2x_3 + 0S_1 + 0S_2 + 0S_3$$

$$-4 + 3 \times 5 - 0 + 0 \times 11$$

$$= -4 + 15 + 0 = 11 //$$

$\therefore$ Min $z = -11$ //

Q. Solve the LPP using simplex method ?

$Max\ Z = 6x_1 + 4x_2$

subject to, $-2x_1 + x_2 \leq 2$

$x_1 - x_2 \leq 2$

$3x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$.

Ans: objective should be max.

∴ subject should be equal.

∴ $-2x_1 + x_2 + S_1 = 2$.

$x_1 - x_2 + S_2 = 2$

$3x_1 + 2x_2 + S_3 = 9$.

$x_1, x_2, S_1, S_2, S_3 \geq 0$.

∴ $Max\ Z = 6x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3$

Initial basic variables are $S_1, S_2\ S_3$ - basic variable.

∴ non-basic variables are $x_1\ x_2\ x_3 = 0$.

∴ $S_1 = 2$

$S_2 = 2$

$S_3 = 9$

* Construct the simplex table:

CB	B	XB	x1	x2	~~x3~~	S1	S2	S3	Minimum Ratio (XB/incoming vector)
0	S_1	2	-2	1		1	0	0	-1
0	S_2	2	1	-1		0	1	0	2
0	S_3	9	3	2		0	0	1	3
	Z_j		0	0		0	0	0	
	C_j		6	4		0	0	0	
	Δ_j		-6	-4		0	0	0	

Most -ve Δ_j value is -6. So select that column. $\therefore$ x_1 is incoming vector

least +ve value

Max value = 12

$\boxed{R_1 \to R_1 + 2R_2}$

$-2 + 2\times 1 = 0$

$\boxed{R_3 \to R_3 - 3R_2}$

Simplex table: 2

CB	B	XB	x1	x2	S1	S2	S3	Minimum Ratio (XB/incoming vector)
0	S_1	6	0	-1	1	2	0	-6
6	x_1	2	1	-1	0	1	0	-2
0	S_3	3	0	5	0	-3	1	3/5
		Z_j	6	-6	0	6	0	
		C_j	6	4	0	0	0	
		Δ_j	0	-10	0	6	0	

'5' is key value.

Simplex table 3:

CB	B	XB	x1	x2	S1	S2	S3	Minimum Ratio XB/incoming vector.
0	S_1	33/5	0	0	1	7/5	1/5	
6	x_1	13/5	1	0	0	2/5	1/5	
4	x_2	3/5	0	1	0	-3/5	1/5	
		Z_j	6	4	0	0	2	
		C_j	6	4	0	0	0	
		Δ_j	0	0	0	0	2	

Big-M-Method:-

- It is Max.

The LPP should be in the standard form. (Max, =)

- Artifical variable are in corperated only for Computational process, they have no physical meaning.
- Artifical variable are introduced, when the constraints are of the type greater than or equal

$\leq + S_1 \Rightarrow =$

$\geq - S_2 + A_1 \quad =$

$= + A_2$

- Coefficient of artifical variable is $-M$

Q. How to identify highest value from expression containing 'M'.

1) $7-4M$, $-6M+6$, $2/3-9M$ highest value.

2) $5-6M$, $6-6M$, $10-5M$

same value so min $\{5, 6\}$ $\therefore$ $5-6M$ most -ve value.

→ Solve by big-M-method ?

$$\text{Max } z = 2x_1 + x_2 + 3x_3$$

Subject to,

$$x_1 + x_2 + 2x_3 \le 5$$
$$2x_1 + 3x_2 + 4x_3 = 12$$
$$x_1, x_2, x_3 \geq 0$$

Ans: for $\le$: $+S_1$, $=$: $+A_1$

$$\therefore \quad x_1 + x_2 + 2x_3 + S_1 = 5 \quad \text{— (1)}$$
$$2x_1 + 3x_2 + 4x_3 + A_1 = 12 \quad \text{— (2)}$$

$$x_1, x_2, x_3, S_1, A_1 \geq 0$$

∴ objective = $\text{Max } z = 2x_1 + x_2 + 3x_3 + 0S_1 - MA_1$

CB	B	xB	x1	x2	x3	S1	A1	Minimum Ratio (xB/incoming vector).
(1)↓		(2)↓						(8)↓
0	S_1	5	(3)→ 1	1	2	1	0	5/2 = 2.5
−M	A_1	12	(4)→ 2	3	4	0	1	3
	(5)→ z_j	~~-12M~~	−2M	−3M	−4M	0	−M	
	(6)→ c_j		2	1	3	0	−M	
	(7)→ Δ_j		−2M−2	−3M−1	−4M−3	0	0	

∴ '2' is the key element.

Simplex table 2.

CB	B	XB	x_1	x_2	x_3	S_1	A_1	Minimum Ratio. XB/Incoming Vector.
3	x_3	$5/2$	$1/2$	$1/2$	1	$1/2$	0	5
-M	A_1	2	0	1	0	-2	1	2
	z_j	[illegible]	$3/2$	$3/2 - M$	3	$3/2 + 2M$	-M	
	c_j		2	1	3	0	-M	
	Δ_j		$-1/2$	$1/2 - M$	0	$3/2 + 2M$	0	

most -ve Value

$5/2 \div \frac{1}{2}$

$\frac{5}{2} \times \frac{2}{1} = 5$ $\frac{3}{2} - \frac{2}{1} = \frac{3-4}{2} = -1/2$ $\frac{3}{2} - M - 1$ $\frac{3}{2} - \frac{1}{1}$ $\frac{3-2}{2} = 1/2 - M$

The Key value is '1' from the A_1 as row.
In the next simplex table The A_1 is changed to x_2. and also the (A_1) ~~etc~~ column is also removed from the table.

$\therefore$ CB	B	XB	x_1	x_2	x_3	S_1	Minimum Ratio (XB/incoming vector)
3	x_3	3/2	1/2	0	1	3/2	3
1	x_2	2	0	1	0	-2	not defined.
		Z_j	3/2	1	3	5/2	
		C_j	2	1	3	0	
		Δ_j	-1/2	0	0	5/2	

$\boxed{R_1 \rightarrow R_1 - \frac{1}{2} R_2}$

$$\boxed{R_1 \rightarrow R_1 - \frac{1}{2} R_2}$$

$\frac{5}{2} - \frac{2}{2}$, $\frac{5}{2} - \frac{1}{1}$ $\frac{5-2}{2} = 3/2$

$\frac{1}{2} - \frac{-2 \times}{2} = \frac{1}{2} + \frac{1}{1} = \frac{1+2}{2} = 3/2$

$\frac{3 \times 1}{2} = 3/3$ $\quad \frac{3}{2} \times 3 + 1 \times -2 =$ $\quad \frac{9}{2} + \frac{-2}{1}$ $\quad \frac{9-4}{2} = 5/2$

$\frac{3}{2} \times 3 + -2$

$\frac{9}{2} - \frac{2}{1}$

$\frac{9-4}{2} = 3/2$

$\frac{3}{2} - \frac{2}{1}$ $\quad \frac{3-4}{2} = -1/2$

Δj contain most negative value.

$\frac{3}{2} \times \frac{2}{1} = 3$ $\quad 3 \times 2 = 6$ $\quad 1 \times -2$ $\quad -2$

$xB/x_1 = 3/2 \quad \frac{1}{2}$

$1/2$ is key value.

∴ Simplex table 4.

CB	B	XB	x_1	x_2	x_3	S_1	Minimum Ratio (XB/incoming vector).
2	x_1	3	1	0	2	3	
1	x_2	2	0	1	0	-2	
	Z_j		2	1	4	4	
	C_j		2	1	3	0	
	Δj		0	0	1	4	

$\frac{3}{2} \times \frac{2}{1}$ $\quad 0 + \quad 1 \times \frac{2}{1}$

All Δj values are +ve.

∴ $x_1 = 3 \quad x_2 = 2$

remaining variables are zero.

$\therefore$ Max $z = 2x_1 + x_2 + 3x_3 + 0S_1 - MA_1$

$= 2\times3 + 2 + 0 + 0 + 0$

$= 6 + 2 = 8$ //

If the value of 'z' contain 'M' then the solution is infeasible.

Dual of LPP

Q Find the dual of

Max $z = 4x_1 + 3x_2$

Subject to,

$-x_1 - 2x_2 \leq 3$

$-x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$.

there ... primal.

First Max changed to Min then values of subjectives is changed to objective & 'x' changed to another variable 'y'

$\therefore$ Min $z = 3y_1 + 2y_1$

then subject coefficient make Matrix is $-x_1 - 2x_2$, $-x_1 + x_2$

$\begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix}$ → then transpose : $\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$

then $x_1 \to y_1$

$-y_1 - y_2$

Ans: Min $z' = 3y_1 + 2y_2$

$A = \begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$

$\therefore$ Min $z' = 3y_1 + 2y_2$

$-y_1 - y_2 \geq 4$

$-2y_1 + y_2 \geq 3$

$y_1, y_2 \geq 0$.

Q. Find the dual of Min $z = 2x_1 + 3x_2$

Subject to, $x_1 + x_2 \geq 10$

$2x_1 + 3x_2 \geq 12$

$x_1, x_2 \geq 0$?

Ans. Max $z' = 10y_1 + 12y_2$

$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \xrightarrow{\text{Transpose}} A^T \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$\therefore$ Max $z' = 10y_1 + 12y_2$

$y_1 + 2y_2 \leq 2$

$y_1 + 3y_2 \leq 3$

$y_1, y_2 \geq 0$.

Q. Find the dual Max $z = 2x_1 + x_2$,

Subject to $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0$.

Ans: Min $z' = 10y_1 + 6y_2 + 2y_3 + y_4$

$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & -1 \\ 1 & -2 \end{bmatrix} \xrightarrow{\text{Transpose}} A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & -2 \end{bmatrix}$

∴ Subject to

$y_1 + y_2 + y_3 + y_4 \geq 2$

$2y_1 + y_2 - y_3 - 2y_4 \geq 1$

$y_1, y_2, y_3, y_4 \geq 0$ //

→ Dual Simplex Method :-

- objective function should be maximization type
- All the $\geq$ constraints should convert to $\leq$ by multiplying -1.
- Transform this $\leq$ constraints into = by adding slack variables.
- find initial feasible solution.

(i) object = max
constraint '='
$\geq \longrightarrow$ changed to =
$\geq * -1 \Rightarrow \leq \rightarrow$
$\leq + S_1 = =$

Simplex
$\leq + S_1$

Big Method
$\leq + S_1$
$\geq - S_1 + A_1$
$= + A_1$

Q. Solve by dual simplex method ?

$\text{Min } z = 5x_1 + 6x_2$

Subject, $x_1 + x_2 \geq 2$

$4x_1 + x_2 \geq 4$

$x_1, x_2 \geq 0$

Ans: $\text{Min } z \xrightarrow[\text{changed}]{} \text{to}$ $\quad \text{Max } z = -5x_1 - 6x_2$

$x_1 + x_2 \geq 2 \longrightarrow \times -1 \qquad -x_1 - x_2 \leq 2 \longrightarrow -x_1 - x_2 + S_1 = 2$

$4x_1 + x_2 \geq 4 \longrightarrow * -1 \qquad -4x_1 - x_2 \leq 4 \longrightarrow -4x_1 - x_2 + S_2 = 4$

∴

$\text{Max } z = -5x_1 - 6x_2 + 0S_1 + 0S_2$

Subject to,

$-x_1 - x_2 + S_1 = -2$

$-4x_1 - x_2 + S_2 = -4.$

$x_1, x_2, S_1, S_2 \geq 0$ //

basic variable = S_1, S_2
non basic variable = x_1, x_2
$S_1 = -2$
$S_2 = -4$.

CB	B	XB	x_1	x_2	S_1	S_2
0	S_1	-2	-1	-1	1	0
0	S_2	-4	-4 key value	-1	0	1
	Z_j	0	0	0	0	0
	C_j		-5	-6	0	0
	Δ_j		5	6	0	0
	Δ_j/ Key row		5/-4 -1.2	6/-1 -6.		

Test for optimality:-

- if all values of xB ≥ 0 and $\Delta j \geq 0$ then the current solution is optimal solution.
- If any xB value is negative, select the most negative xB and this is the leaving vector / key row.
- find maximum ratio = Δj/Key row (key row < 0)
- find Max {max ratios}, this will be incoming vector/ Key column.
- find the new solution table until we get all values of xB and Δj positive.

key row is most negative value

⑥ → only find the Δj / Key row for the negative values of that row of most negative value

⑨ then check the least negative value / maximum value

ie, -1.2

CB	B	xB	x_1	x_2	S_1	S_2
0	S_1	-1	0	-3/4	1	-1/4
-5	x_1	1	1	1/4	0	-1/4
	z_j		-5	-5/4	0	5/4
	c_j		-5	-6	0	0
	Δj		0	19/4 = 4.7	0	5/4
	Δj/ key row			19/-3 = -6.3		-5

③ → value changed to zero.

$R_1 \to R_1 + \frac{R_2}{-4}$

(0) $-2 + \frac{-4}{-4} = -2+1 = -1$

(0) $-1 + \frac{-4}{-4} = -1+1 = 0$

$-1 + \frac{-1}{-4} = \frac{-1}{1} + \frac{1}{4} = \frac{-4+1}{4} = -3/4$

(1) $1 + \frac{0}{-4} = 0 + 1/-4 = -1/4$

$0 + 1/-4 = -1/4$

Key row = S_1 ie, -1

for -ve S_1 value

Δj / Key row

$-3/4 \div \frac{19}{4} = \frac{-3}{4} \times \frac{4}{19} = \frac{19}{4} \times \frac{4}{-3} = 19/-3$

$\frac{5}{4} \div \frac{-1}{4} = \frac{5/4}{-1/4} = -5$

∴ Max value = -5

∴ last column. ∴ Key value = -1/4.

CB	B	xB	x_1	x_2	S_1	S_2
0	S_2	4	0	3	-4	1
-5	x_1	2	1	1	-1	0
	z_j		-5	-5	5	0
	c_j		-5	-6	0	0
	Δj		0	1	5	0
	Δj/ Key row					

all xB and Δj values are ≥ 0. So it is optimal.