* Solve the following LPP using graphical method.

i) Maximize $z = 2x + 3y$.

Subject to

$x+y \leq 30$

$y \geq 5$

$0 \leq y \leq 15$

$0 \leq x \leq 20$

$x-y \geq 0$

$x, y \geq 0$

Ans:-

$x+y = 30$ ——(1)

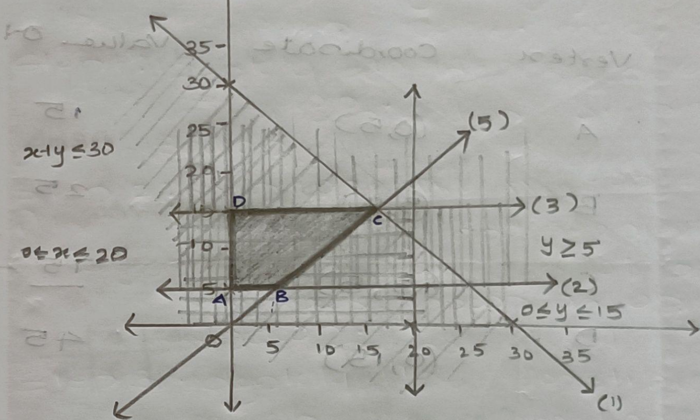
x	0	30
y	30	0

$y = 5$ ——(2)

$0 = y = 15$ ——(3)

$0 = x = 20$ ——(4)

$x = y$ ——(5)



A – (0, 5)

B is the intersecting point of (2) and (5)

$y = 5$ ——(2)

$x = y$ ——(5)

$\therefore$ B – (5, 5)

C is the intersecting point of (1) & (5)

$x + y = 30$ ——(1)

$x = y$ —— (5)

$\frac{30}{2} = 15$

$x + y = 30$

$15 + 15 = 30$

$\therefore C = (15, 15)$

$D = (0, 15)$

Vertex	Coordinate	Value of Z
A	(0,5)	15
B	(5,5)	25
C	(15,5)	$\underline{75}$ —— Max Value
D	(0,15)	45

$Z = 2x + 3y$

$A = 2\times0 + 3\times5 = 15$

$B = 2\times5 + 3\times5 = \underline{\underline{25}}$

$C = 2\times15 + 3\times5 = \underline{75}$

$D = 2\times0 + 3\times15 = \underline{\underline{45}}$

Solution of given LPP is $x = 15$ and $y = 15$.

* Solve the following LPP using graphical method. Minimize $C = 3x + 5y$ subject to

$-3x + 4y \le 12$

$2x - y \ge -2$

$2x + 3y \ge 12$

$x \le 4$

$y \ge 2$

$x, y \ge 0$

Ans:-

$-3x + 4y = 12$ —— (1)

x	0	-4
y	3	0

$2x + 3y = 12$ —— (2)

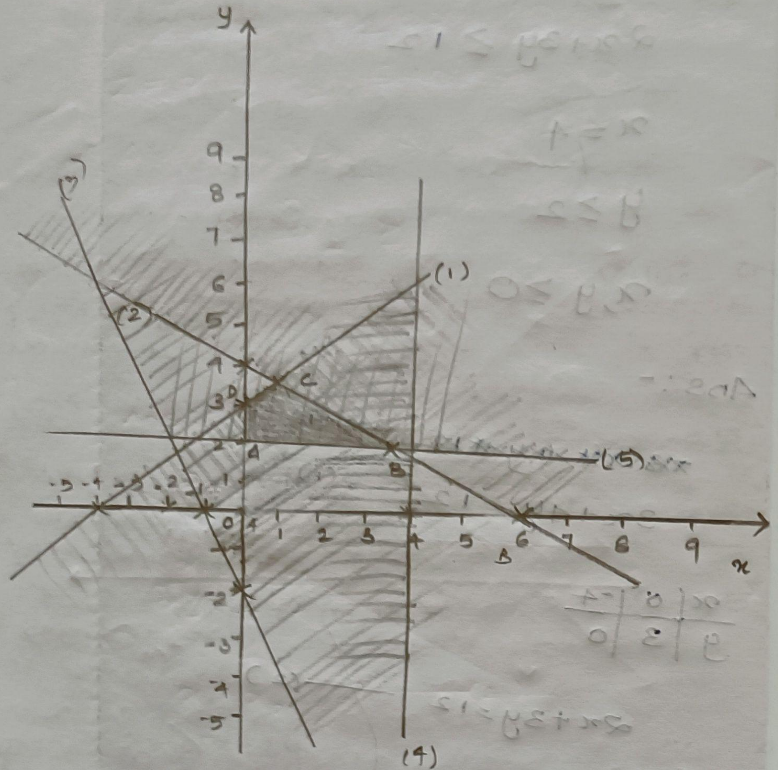
x	0	6
y	4	0

$2x - y \ge -2$ —— (3)

x	0	-1
y	-2	0

$x = 4$ —— (4) $\quad$ $y = 2$ —— (5)

$x, y = 0$ —— (6)



A – (0,0)

B – (6,0)

C – (0.75, 3.5)

(2)-(3)

$$2x + 3y = 12$$
$$2x - y = -2$$
$$4y = 14$$

$$y = 3.5$$

Put y in (2)

$$2x + 3y = 12$$
$$2x + 3(3.5) = 12$$
$$2x = 12 - 10.5$$
$$2x = 1.5$$
$$x = 0.75$$

$C = 3x + 5y$

D – (0, 3)

Vertex	coordinate	value of z
A	(0,0)	0
B	(6,0)	18
C	(0.75, 3.5)	19.75
D	(0, 3)	15

The solution of given LPP is $x = 0,\ y = 0$

* Solve the following LPP using graphical method minimize $Z = x_1 + x_2$ subject to

$$2x_1 + x_2 \geq 4$$

$$x_1 + x_2 \geq 7$$

$$x_1, x_2 \geq 0$$

Ans :-

$2x_1 + x_2 = 4$ ——— (1)

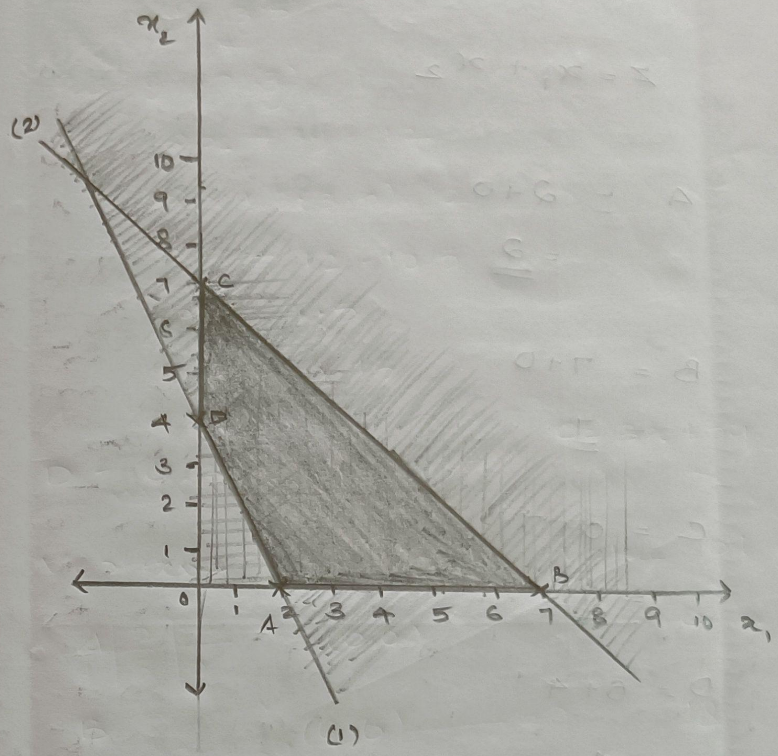
x_1	0	2
x_2	4	0

$x_1 + x_2 = 7$ ——— (2)

x_1	0	7
x_2	7	0

$x_1 = x_2 = 0$ ——— (3)

$Z = x_1 + x_2$



A - (2, 0)

B - (7, 0)

C - (0, 7)

D - (0, 4)

$Z = x_1 + x_2$

$A = 2+0$
$= \underline{\underline{2}}$

$B = 7+0$
$= \underline{\underline{7}}$

$C = 0+7$
$= \underline{\underline{7}}$

$D = 0+4$
$= \underline{\underline{4}}$

Vertices	Coordinate	Value
A	(2,0)	2 Min Value
B	(7,0)	7
C	(0,7)	7
D	(0,4)	4

The solution of given LPP is $x = 2$, $y = 0$

* Solve the following using graphical method. Maximize $P = 2x + 3y$ subject to $x + 2y \leq 2$, $4x + 3y \geq 12$, $x, y \geq 0$.

Ans:-

$x + 2y = 2$ —— (1)

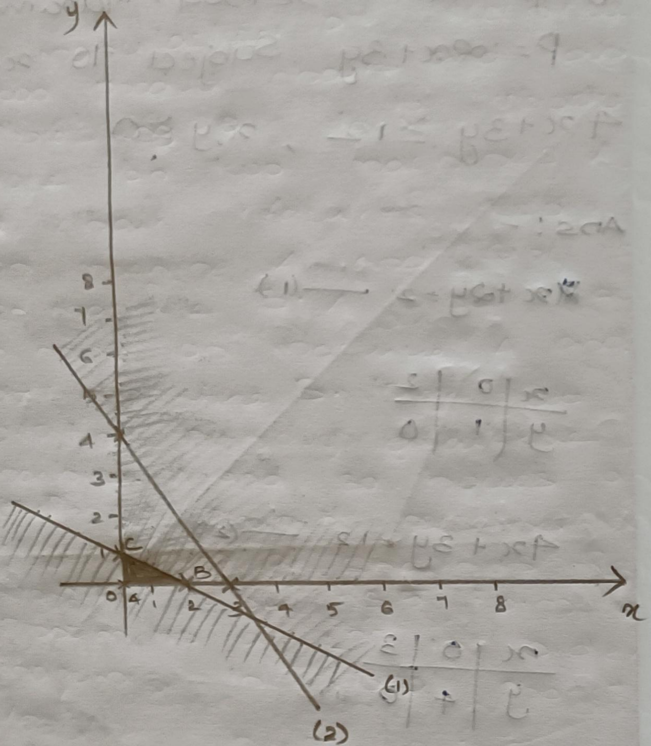
x	0	2
y	1	0

$4x + 3y = 12$ —— (2)

x	0	3
y	4	0

$x = y = 0$ —— (3)

$P = 2x + 3y$



A - (0,0)

B - (2,0)

C - (0,1)

$P = 2x + 3y$

$A = 2\times0 + 3\times0$

$= \underline{\underline{0}}$

$B = 2\times2 + 3\times0$

$= \underline{\underline{4}}$

$C = 2\times0 + 3\times1$

$= \underline{\underline{3}}$

Vertex	coordinate	value of P
A	(0,0)	0
B	(2,0)	4
C	(0,1)	3

The solution of given LPP is $y = 0$, $x = 0$

LPP example

Q: A manufacturer of furniture make two product chairs and tables. Processing of these product is done on two machines A and B. A chair requires two hours on machine A and six hours on machine B. A table requires 5 hours on machine A and no time on machine B. There are 16 hours of time per day available on machine A and 30 hours on machine B. Profit gained by the manufacturer from a chair is Rs 1 and from a table Rs 5. Formulate the problem into a LPP in order to maximize the total profit?

Ans:-

x_1 - No. of chairs

x_2 - No. of tables.

Profit from chair $= 1 \times x_1$

Profit from table $= 5 \times x_2$

$\therefore$ Total profit $Z = x_1 + x_2$

i) Machine A

Time required for chair $= 2 \times x_1$

$= 2x_1$

Time required for tables $= 5 \times x_2$

$= 5x_2$

$\therefore$ Total time required $= 2x_1 + 5x_2$

Available time $= 16$ hrs

$\therefore 2x_1 + 5x_2 \leq 16$

ii) Machine B

Time required for chairs $= 6 \times x_1$

$= 6x_1$

Time required for tables $= 0 \times x_2$

$= 0$

$\therefore$ Total time required $= 6x_1 + 0$

Available time $= 30$ hrs

$\therefore 6x_1 + 0 \leq 30$

LPP to find x_1 & x_2 which

maximize $Z = x_1 + \cancel{x_2}\ 5x_2$

Subject to

$2x_1 + 5x_2 \leq 16$

$6x_1 \leq 30$

$x_1 \geq 0,\ x_2 \geq 0$

The standard form of LPP
wp

1) converting an LPP to std form.

All LP solvers

first convert, the given program to std form which means

* All variables involve are restricted to be non negative

* All constraints are equalities with constant non negative right hand sides.

converting may require new variables and rearranging constraints.

* An inequality can be multiplied by negative one to get non negative RHS

* inequalities can be converted to equalities by adding or subtracting non negative variables.

* unrestricted variables can be dealt with by writing the variables as the difference of two new non negative variables.

Q. Solve the following LPP using graphical method. Write the given LPP in STD form. Minimize $z = x + 4y$ subject to $x - 3y \geq 10$, $2x + y = 6$, $x \geq 0$?

Let y' & y'' be two non negative variable such that $y' \geq y''$

Now put $y = y' - y''$

Minimize $z = x + 4(y' - y'')$

subject to,

$$x - 3(y' - y'') - S1 = 10$$

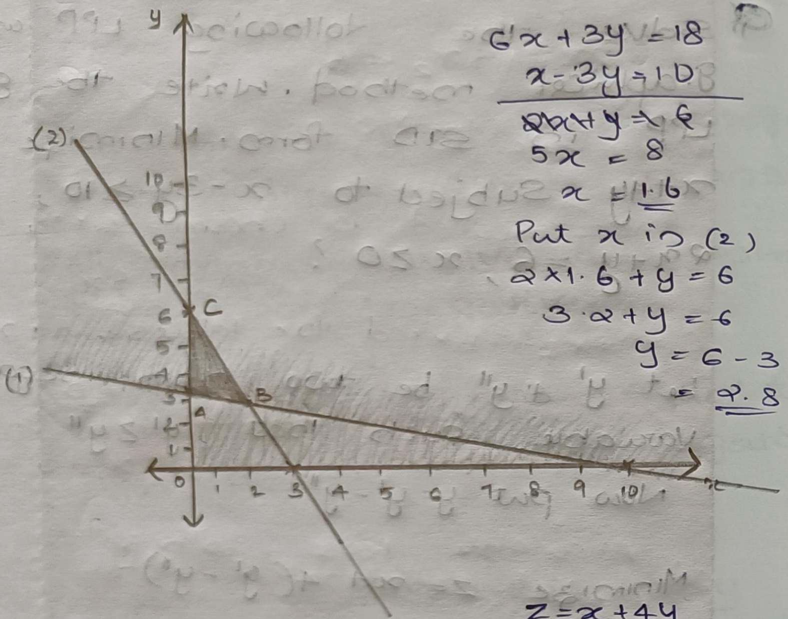
$$2x + (y' - y'') = 6$$

$$x, y', y'' \geq 0$$

(1) $x-3y=10$ (2) $2x+y=6$ (3) $x=0$

x	0	10
y	3.3	0

x	0	3
y	6	0



$$6x+3y=18$$
$$x-3y=10$$
$$5x=8$$
$$x=1.6$$

Put x in (2)

$$2\times1.6+y=6$$
$$3.2+y=6$$
$$y=6-3.2$$
$$=2.8$$

$Z=x+4y$

Vertex	Coordinate	Value of Z
A	(0,3)	12 Min value
B	(1.6, 2.8)	12.8
C	(0,6)	24 max value

The Solution of the given LPP is $x=0$, $y=3$

* A furniture company produces tables and chairs. the Production process for each is similar in that both require a certain number of hours of carpentry work and a certain number of labour hour in the painting department. Each table takes 4hrs of carpentry and 2hrs in painting. Each chair requires 3hrs of carpentry and 1 hr in the Painting department. During the current production period 240 hrs of carpentry time are available and 100hrs in Painting is available. Each table sold yields a profit of Rs 7, each chair produced is sold for a profit of Rs 5. Find the best combination of table & chair to manufacture inorder to reach maximum Profit?

Let x & y be the No. of tables & chairs

Profit gained when x No. of tables sold $= 7x$.

Profit gained when y No. of chair sold $= 5y$.

Total profit $= 7x + 5y$

maximize:-

$\therefore Z = 7x + 5y$

Subject to:-

$4x + 3y \leq 240$

Since carpentry work is maximum 240 hrs. (table 4hr chair 3hr)

$2x + y \leq 100$

Since Painting work is maximum 100 hrs. (table 2hr chair 1hr)

$x, y \geq 0$

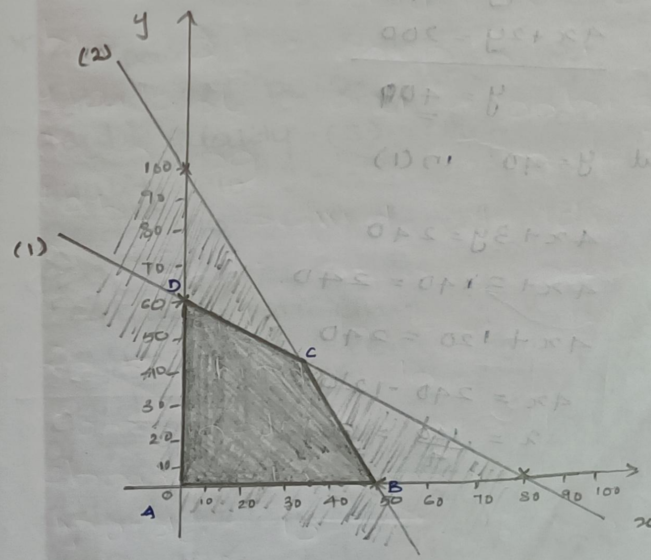
$4x + 3y = 240$ —— (1)

x	0	60
y	80	0

$2x + y = 100$ —— (2)

x	0	50
y	100	0

$x = y = 0$ —— (3)



A - (0,0)

B - (50,0)

C - (30,40)

D - (0,60)

$4x + 3y = 240$ —— (1)

$2x + y = 100$ —— (2)

(2) × 2

$4x + 2y = 200$

(1) - (2)

$$\begin{array}{r} 4x + 3y = 240 \\ 4x + 2y = 200 \\ \hline y = 40 \end{array}$$

Put $y = 40$ in (1)

$4x + 3y = 240$

$4x + 3 * 40 = 240$

$4x + 120 = 240$

$4x = 240 - 120$

$x = \frac{120}{4} = 30$

$z = 7x + 5y$

Vertex	Coordinate	Value of z
A	(0,0)	0
B	(50,0)	350
C	(30,40)	410 Max Value
D	(0,60)	300

Solution of given LPP is $x = 30$ and $y = 40$

* A paint manufacturer produce 2 types of paint one type of standard quality (S) other of top quality. To make these paint be pigment needs 2 ingredient the pigment and the resin. S quality paint requires 2 unit pigment and 8 unit of resin for each unit paid and is sold at a profit of Rs 1. Top quality paint require 4 unit of pigment and 2 unit of resin for each unit and sold at a profit of Rs 1.5 per unit

He has Stocks for 12 unit of Pigment and 10 unit of Resin?

Let x & y be the quantity of Paints in the quality standard & top respectively.

	S	T
Pigment - P	2	4
Resin - R	3	2
Profit - ₹	1	1.5

Maximize $z = x + \frac{3}{2}y$

Subject to,

$2x + 4y \leq 12$

$3x + 2y \leq 10$

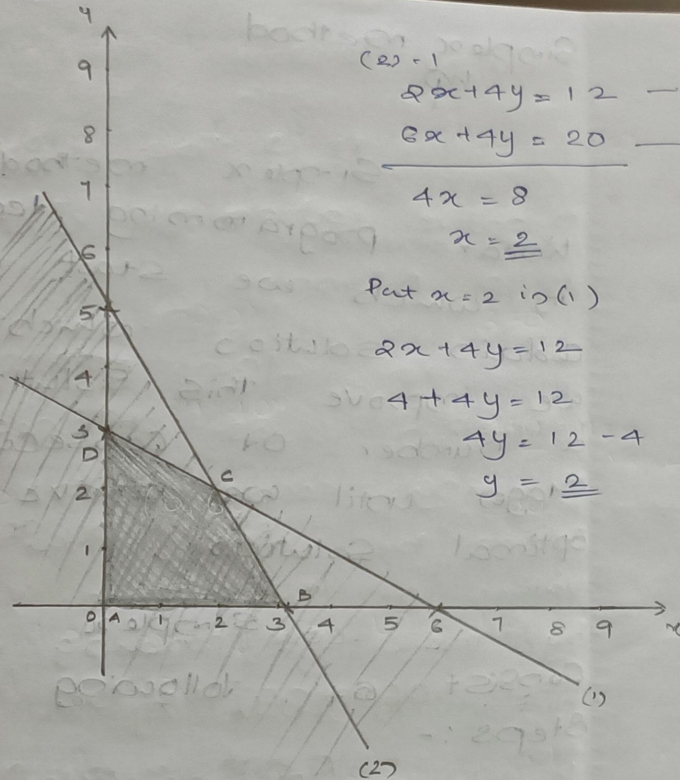
$x, y \geq 0$

$2x + 4y = 12$ ——(1) $3x + 2y = 10$ ——(2)

x	0	6
y	3	0

x	0	3.3
y	5	0

$x = y = 0$ ——(3)



(2) - 1

$2x + 4y = 12$ —(1)

$6x + 4y = 20$ —(2)

$4x = 8$

$x = \underline{\underline{2}}$

Put $x = 2$ in (1)

$2x + 4y = 12$

$4 + 4y = 12$

$4y = 12 - 4$

$y = \underline{\underline{2}}$

Vertex	cordinate	Value of z
A	(0,0)	0
B	(3.3,0)	3.3
C	(2,2)	5 Max Value
D	(0,3)	4.5

Solution of given LPP is $x = 2$
$y = 2$

make it we have to subtract ~~some variable from its LHS~~
~~The variables which are subtracted from LHS convert them into equality called surplus variable.~~

$$\cancel{eg: 2x_1 + 4x_2 \geq 12}$$

$$\cancel{2x_1 + 4x_2 - S_2 = 12}$$

S_2 - surplus variable

---

* Solve the LPP using simplex method, maximize $Z = 7x_1 + 5x_2$
subject to

$$x_1 + 2x_2 \leq 6$$

$$4x_1 + 3x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

Introducing slag variable the inequality constraint become equality and we have

$$x_1 + 2x_2 + S_1 = 6$$

$$4x_1 + 3x_2 + S_2 = 12$$

We can write the equation in matrix form.

$$\begin{array}{cccc} x_1 & x_2 & S_1 & S_2 \end{array} \qquad x_B$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 4 & 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

The objective function is

$$Z = 7x_1 + 5x_2 + 0S_1 + 0S_2$$

B	C_B	X_B	x_1	x_2	S_1	S_2	Ratio (X_B/key column)	
S_1	0	6	1	2	1	0	$6/1 = 6$	
S_2	0	12	$\boxed{4}$	3	0	1	$12/4 = 3$	Key Row (least value)
$S_1(C_B \cdot x_1) + S_2(C_B \cdot x_2)$		Z_j	0	0	0	0		
Value of Z		C_j	7	5	0	0		
		$Z_j - C_j$	-7	-5	0	0		

Key column (most -ve value)

key row

key column

Here 4 is our key element

x_1 is the incoming vector and S_2 is the outgoing vector

Here our key element is 4 we have to multiply 1/4 to each of the element in the key row except C_B for transforming 4 to 1

Now make the transformation

$S_1 \rightarrow S_1 - x_1$

B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	3	0	5/4	1	-1/4
x_1	7	3	1	3/4	0	1/4
		Z_j	7	21/4	0	7/4
		C_j	7	5	0	0
		$Z_j - C_j$	0	1/4	0	7/4

For the given LPP the solution is

$x_1 = 3 \quad x_2 = 0 \quad S_1 = 3 \quad S_2 = 0$

$Z = 7x_1 + 5x_2 + 0S_1 + 0S_2$

$= (7 \times 3) + (5 \times 0) + (0 \times 3) + (0 \times 0)$

$= 21 + 0 + 0 + 0$

$= \underline{\underline{21}}$

* Solve the following LPP using simplex method

Maximize $Z = 5x_1 + 3x_2$ subject to:-

$x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0$

Introducing slag variable & converting the constraint into equations we have

$x_1 + x_2 + S_1 = 2$

$$5x_1 + 2x_2 + S_2 = 10$$

$$3x_1 + 8x_2 + S_3 = 12$$

These equations can be expressed as

$$\begin{array}{ccccc} x_1 & x_2 & S_1 & S_2 & S_3 \end{array} \qquad X_B$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 5 & 2 & 0 & 1 & 0 \\ 3 & 8 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 12 \end{bmatrix}$$

The objective function is

$$Z = 5x_1 + 3x_2 + 0S_1 + 0S_2 + 0S_3$$

B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	Ratio
S_1	0	2	$\boxed{1}$	1	1	0	0	2/2 = 1 ← Key Row
S_2	0	10	5	2	0	1	0	10/5 = 2
S_3	0	12	3	8	0	0	1	12/3 = 4
		z_j	0	0	0	0	0	
		c_j	5	3	0	0	0	
		$z_j - c_j$	-5	-3	0	0	0	

↑ Key column

B	C_B	X_B	x_1	x_2	S_1	S_2	S_3
x_1	5	2	1	1	1	0	0
S_2	0	0	0	-3	-5	1	0
S_3	0	6	0	5	-3	0	1
		z_j	5	5	5	0	0
		c_j	5	3	0	0	0
		$z_j - c_j$	0	2	5	0	0

$S_2 \rightarrow S_2 - 5x_1$

$S_3 \rightarrow S_3 - 3x_1$

So the solution of given LPP is

$x_1 = 2 \quad x_2 = 0$

$S_1 = 0 \quad S_2 = 0 \quad S_3 = 6$

$$Z = 5x_1 + 3x_2 + 0S_1 + 0S_2 + 0S_3$$

$$= (5\times2) + (3\times0) + (0\times0) + (0\times0) + (0\times6)$$

$$= \underline{10}$$

Que

1 * Solve the following problem by Simplex method Maximize subject to :-

$Z = 3x + 5y + 7z$

$3x + 2y + 4z \le 100$

$x+4y+2z \le 100$

$x+y+3z \le 100$

$x, y, z \ge 0$

2.* Solve the following LPP by Simplex method maximise $z = 2x - 3y + 4z$ subject to

$4x - 3y + z \le 3$

$x + y + z \le 10$

$2x + y - z \le 10$

$x, y, z \ge 0$

Ans

1*

Introducing slag variable and converting the constraints into equations we have

$3x + 2y + 4z + S_1 = 100$

$x + 4y + 2z + S_2 = 100$

$x + y + 3z + S_3 = 100$

This equation can be expressed as:-

$$\begin{array}{cccccc} x & y & z & S_1 & S_2 & S_3 \end{array} \qquad x_B$$

$$\begin{bmatrix} 3 & 2 & 4 & 1 & 0 & 0 \\ 1 & 4 & 2 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}$$

The objective function is

$3x + 5y + 7z + 0S_1 + 0S_2 + 0S_3$.

B	C_B	X_B	x	y	z	S_1	S_2	S_3	Ratio
S_1	0	100	3	2	$\boxed{4}$	1	0	0	$100/4 = 25$ key Row
S_2	0	100	1	4	2	0	1	0	$100/2 = 50$
S_3	0	100	1	1	3	0	0	1	$100/3 = 33.33$
		z_j	0	0	0	0	0	0	
		c_j	3	5	7	0	0	0	
		$z_j - c_j$	-3	-5	-7	0	0	0	

Key Colum

S outgoing Vector

Z incoming Vector

B	C_B	X_B	x	y	z	S_1	S_2	S_3	Ratio
z	7	25	3/4	2/4	1	1/4	0	0	50
S_2	0	50	-1/2	$\boxed{3}$	0	-1/2	1	0	50/3 Key Row
S_3	0	25	-5/4	-1/2	0	-3/4	0	1	-50
		Z_j	21/4	7/2	7	7/4	0	0	
		C_j	3	5	7	0	0	0	
		$Z_j - C_j$	9/4	-3/2	0	7/4	0	0	

Key column

$S_2 \to S_2 - 2Z$

$S_3 \to S_3 - 3Z$

S_2 - outgoing

Y - incoming vector

B	C_B	X_B	x	y	z	S_1	S_2	S_3
z	7	50/3	5/6	0	1	1/3	-1/6	0
y	5	50/3	-1/6	1	0	-1/6	1/3	0
S_3	0	100/3	-4/3	0	0	-1/4	1/6	1
		Z_j	5	5	7	3/2	1/2	0
		C_j	3	5	7	0	0	0
		$Z_j - C_j$	2	0	0	3/2	1/2	0

The solution of the given LPP is

$z = 50/3$

$y = 50/3$

$S_3 = 100/3$

$x = 0$

$z = 0$

$S_1 = 0$

$S_2 = 0$

$P = 3x + 5y + 7z + 0S_1 + 0S_2 + 0S_3$

$= (3 \times 0) + 5(50/3) + 7(50/3) + (0 \times 0) + (0 \times 0) + (0 \times 100/3)$

$= \frac{250}{3} + \frac{350}{3}$

$= \frac{600}{3}$

$= 200$

2* Introducing slag variable and converting the constraints into equation we have

$$4x - 3y + z + S_1 = 3$$

$$x + y + z + S_2 = 10$$

$$2x + y - z + S_3 = 10$$

this equation can be expressed as

$$\begin{array}{cccccc c c} x & y & z & S_1 & S_2 & S_3 & & X_B \\ \end{array}$$

$$\begin{bmatrix} 4 & -3 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 10 \\ 10 \end{bmatrix}$$

The objective function is
Z =

$$2x - 3y + 4z + 0s_1 + 0s_2 + 0S_3$$

B	C_B	X_B	x	y	z	S_1	S_2	S_3	Ratio
S_1	0	3	4	-3	$\boxed{1}$	1	0	0	$3/1 = 3$ Key Row
S_2	0	10	1	1	1	0	1	0	$10/1 = 10$
S_3	0	10	2	1	-1	0	0	1	$10/-1 = -10$
		Z_j	0	0	0	0	0	0	
		C_j	2	-3	4	0	0	0	
		$Z_j - C_j$	-2	3	-4	0	0	0	

key column

S_1 - outgoing vector

Z - incoming vector

B	C_B	X_B	x	y	z	S_1	S_2	S_3	Ratio
Z	4	3	4	-3	1	1	1	0	-1
S_2	0	7	-3	$\boxed{4}$	0	-1	0	0	1.75 key Row
S_3	0	13	6	-2	0	1	1	1	-6.25
		Z_j	16	-12	4	4	0	0	
		C_j	2	-3	4	0	0	0	
		$Z_j - C_j$	14	-9	0	4	0	0	

key column

Y - incoming vector

S_2 - outgoing vector

B	C_B	X_B	x	y	z	S_1	S_2	S_3
z	4	33/4	7/4	0	1	1/4	3/4	0
y	3	7/4	-3/4	1	0	-1/4	1/4	0
S_3	0	33/2	9/2	0	0	1/2	1/2	1
		Z_j	19/4	7/4	4	1/4	15/4	0
		C_j	2	-3	4	0	0	0
		$Z_j - C_j$	11/4	19/4	0	1/4	15/4	0

The solution of given LPP is

$x = 0$ $\quad S_1 = 0$

$y = 7/4$ $\quad S_2 = 0$

$z = 33/4$ $\quad S_3 = 33/2$

$$Z = 2x - 3y + 4z + 0S_1 + 0S_2 + 0S_3$$

$$= (2\times 0) - (3\times 7/4) + (4 \times 33/4) + (0\times 0) + (0\times 0) + (0 \times 33/2)$$

$$= -21/4 + 132/4$$

$$= \underline{27.75}$$

Artificial variable technique

wp

Artificial variable:-

~~These are fictitious variable. They are incorporated only for computational purpose. They have no physical meaning. Artificial variables are introduced when the constraints are of the type $\geq$ or $=$~~

~~e.g.~~

~~$2x_1 + 3x_2 + x_3 \geq 10$~~

$\Rightarrow 2x_1 + 3x_2 + x_3 - S_1 + A_1 = 0$

Artificial variable technique:-

~~constraints of some linear programming problems may have $\geq$ or $=$ sign. In such problem even after introducing surplus variable the simplex table may not contain identity~~ matrix.

To get identity matrix we in the simplex table

Q:- Min $z = 5x_1 + 6x_1$

Subject to $2x_1 + 5x_2 \geq 1500$

$3x_1 + x_2 \geq 1200$

$x_1 \geq 0, \ x_2 \geq 0$

converting the constraints into equations by introducing surplus and artificial variables

$$2x_1 + 5x_2 - S_1 + A_1 = 1500$$

$$3x_1 + x_2 - S_2 + A_2 = 1200$$

This is a minimization problem so we convert it into a maximization problem by changing the signs of c values in Z

$$Z' = -5x_1 - 6x_2$$

$$\therefore Z' = -5x_1 - 6x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$$

$$\begin{array}{cccccc} x_1 & x_2 & S_1 & S_2 & A_1 & A_2 \\ \end{array} \qquad x_B$$

$$\begin{bmatrix} 2 & 5 & -1 & 0 & 1 & 0 \\ 3 & 1 & 0 & -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1500 \\ 1200 \end{bmatrix}$$

B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	θ
A_1	$-M$	1500	2	$\boxed{5}$	-1	0	1	0	300
A_2	$-M$	1500	3	1	0	-1	0	1	1200
		z_j	$-5M$	$-6M$	M	M	$-M$	$-M$	
		c_j	-5	-6	0	0	$-M$	$-M$	
		$z_j - c_j$	$-5M+5$	$-6M+6$	M	M	0	0	

Highest negative Δj is $-6M+6$ and is selected. so x_2 is the incoming vector and A_1 is the outgoing vector. key element is 5. converting the element of x_2 as $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ we get the Second Simplex table

B	C_B	X_B	x_1	x_2	S_1	S_2	A_2	θ
x_2	-6	300	$2/5$	1	$-1/5$	0	0	750
A_2	$-M$	900	$\boxed{13/5}$	0	$1/5$	-1	1	$4500/13$
		z_j	$-12/5 - 13M/5$	-6	$6/5 - M/5$	M	$-M$	
		c_j	-5	-6	0	0	$-M$	
		$z_j - c_j$	$-13M/5 + 13/5$	0	$-M/5 + 6/5$	M	0	

x_1 is the incoming vector
Since it has highest negative Δ_j
minimum ratio show A_2 is the
the outgoing vector.

$\therefore$ Key element is 13/5

Converting vector x_1 into $\begin{bmatrix}0\\1\end{bmatrix}$

The third simplex table is :-

B	C_B	X_B	x_1	x_2	S_1	S_2
x_2	-6	2100/13	0	1	-3/13	2/13
x_1	-5	4500/13	1	0	1/13	-5/13
		z_j	-5	-6	1	1
		c_j	-5	-6	0	0
		$z_j - c_j$	0	0	1	1

Since no Δ_j is negative the solution is optimum so the solution is

$x_1 = 4500/13$

$x_2 = 2100/13$

$Z = 5 \times (4500/13) + 6 \times (2100/13)$

$= \underline{2100}$

*

	X	Y	Z	operating cost(hr)
Plant A	2	4	3	9
Plant B	4	3	2	10
Order on hand	50	24	60	

You are required to use the simplex method to find the No. of production hours needed to fulfill the order on hand at minimum cost.

Ans:- Let x_1 be the No. of production hours used in plant A, x_2 be the No. of production hours used in plant B.

Then the LPP is

Minimize $Z = 9x_1 + 10x_2$

Subject to $2x_1 + 4x_2 \geq 50$ / $x_1 + 2x_2 \geq 25$

$4x_1 + 3x_2 \geq 24$

$3x_1 + 2x_2 \geq 60$

$x_1 \geq 0,\ x_2 \geq 0$

converting the inequality into equation by introducing surplus and artificial variable

$$x_1 + 2x_2 - S_1 + A_1 = 25$$

$$4x_1 + 3x_2 - S_2 + A_2 = 24$$

$$3x_1 + 2x_2 - S_3 + A_3 = 60$$

For converting the ~~opt~~ minimization into maximization read the objective function as

$$z' = -9x_1 - 10x_2$$

ie,

$$z' = -9x_1 - 10x_2 + 0S_1 + 0S_2 + 0S_3 - MA_1 - MA_2 - MA_3$$

B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	A_1	A_2	A_3	θ
A_1	$-M$	25	1	2	-1	0	0	1	0	0		25
A_2	$-M$	24	$\boxed{4}$	3	0	-1	0	0	1	0		6 ←
A_3	$-M$	60	3	2	0	0	-1	0	0	1		20
		z_j	$-8M$	$-7M$	M	M	M	$-M$	$-M$	$-M$		
		c_j	-9	-10	0	0	0	$-M$	$-M$	$-M$		
		$z_j - c_j$	$-8M+9$ ↑	$-7M+10$	M	M	M	0	0	0		

Highest negative Δ_j is $-8M+9$

So x_1 is the incoming vector and A_2 is the outgoing vector

Key element is 4

Convert x_1 column into $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	A_1	A_3	θ
A_1	$-M$	19	0	5/4	-1	1/4	0	1	0	76
x_1	-9	6	1	3/4	0	-1/4	0	0	0	-24
A_3	$-M$	42	0	-1/4	0	$\boxed{3/4}$	-1	0	1	56 ←
		z_j	-9	$-M-27/4$	M	$-M+9/4$	M	$-M$	$-M$	
		c_j	9	-10	0	0	0	$-M$	$-M$	
		$z_j - c_j$	0	$-M+13/4$	M	$-M+9/4$ ↑	M	0	0	

Highest negative Δ_j is $-M+9/4$

∴ S_2 is the incoming vector. Minimum ratio is 56 so A_3 is the outgoing vector

Key element = 3/4

Column S_2 into $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	A_1	θ
A_1	$-M$	5	0	$\boxed{4/3}$	-1	0	$1/3$	1	$15/4$ ←
x_1	-9	20	1	$2/3$	0	0	$-1/3$	0	30
S_2	0	56	0	$-1/3$	0	1	$-4/3$	0	-
		z_j	-9	$-4M/3-6$	M	0	$-M/3+3$	$-M$	
		c_j	-9	-10	0	0	0	$-M$	
		z_j-c_j	0	$-4M/3+4$ ↑	M	0	$-M/3+3$	0	

Highest negative Δ_j is $-4M/3+4$
minimum ratio is A1 Key element is $4/3$
$\therefore x_2$ is the incoming vector and A_1 is the outgoing vector.

B	C_B	X_B	x_1	x_2	S_1	S_2	S_3
x_2	-10	$14/5$	0	1	$-3/4$	0	$1/4$
x_1	-9	$35/2$	1	0	$1/2$	0	$-1/2$
S_2	0	$229/4$	0	0	$-1/4$	1	$-5/4$
		z_j	-9	-10	3	0	2
		c_j	-9	-10	0	0	0
		z_j-c_j	0	0	3	0	2

$x_1 = \frac{35}{2}$

$x_2 = \frac{15}{4}$

$Z = (9 \times \frac{35}{2}) + (10 \times \frac{15}{4})$

$= 195$

Duality

Dual of a given primal

i) If primal is maximization, the dual is minimization and viceversa

ii) constants in the RHS of the constraints of primal become the coefficience in the objective function of the dual and viceversa.

iii) The transpose of the coefficience in the constraints of primal form the coefficient of the dual

iv) The inequalities in the

Constraints ~~are~~ reverse.

* Maximize $z = 3x_1 + x_2 + 2x_3$ subject to $x_1 + x_2 + x_3 \le 5$

$2x_1 + x_3 \le 10$

$x_2 + 3x_3 \le 15$

$x_1, x_2, x_3 \ge 0$

Find the dual of the above Primal ?

The coefficient matrix of the subject to constraints is :-

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

Transpose of above matrix is :-

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

≤ 5
≤ 10
≤ 15

The dual of given primal

Minimize $z' = 5y_1 + 10y_2 + 5y_3$

Subject to,

$y_1 + 2y_2 \ge 3$ $\quad 3x_1 + x_2 + 2x_3$

$y_1 + y_3 \ge 1$

$y_1 + y_2 + 3y_3 \ge 2$

$y_1, y_2, y_3 \ge 0$

* Write the dual of the following LPP

i) Maximize $z = x - y + 3z$

subject to, $x + y + z \le 10$

$2x - y - z \le 2$

$2x - 2y - 3z \le 6$

$x, y, z \ge 0$

The coefficient matrix of the subject to constraints is :-

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 2 & -2 & -3 \end{bmatrix}$$

Transpose of given matrix is

$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & -1 & -2 \\ 1 & -1 & 3 \end{bmatrix}$$

The dual of given primal

Minimize $z' = 10x' + 2y' + 6z'$

Subject to,

$x' + 2y' + 2z' \geq 1$

$x' - y' - 2z' \geq -1$

$x' - y' + 3z' \geq 3$

$x', y', z' \geq 0$

* Minimize $z = 3x + y + 4z$ subject to $x + 2y \geq 3$

$x + 2z \geq 2$

$2x + 3y + z \geq 4$

$x, y, z \geq 0$

The coefficient matrix of the subject to constraint is:-

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

The transpose of above matrix is

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix}$$

The dual of given primal

mi.

subject to

$$x' + y' + 2z' \quad 3$$

$$2x' + 3z' \quad 1$$

$$2y' + z' \quad 4$$

$$x', y', z' \quad 0$$

Transportation Problem

Transportation problems are particular class of allocation problem. The objective in these problem is to transport various amount of a single homogeneous commodity that are stored at several origin to a number of destinations.

Transportation technique can be applied not only to the cost minimising problem but also to time minimising problem, distance minimising problem, profit maximising problem etc.

Transportation Table

Denotes the origin as $O_1, O_2, \ldots O_m$ and destination as $D_1, D_2, \ldots D_n$. Let the quantity produced at the origin be respectively $a_1, a_2, \ldots a_m$. Let the requirements in various destination be respectively $b_1, b_2, \ldots b_n$. The total quantity produced and total quantity required must be equal

ie,

$$a_1 + a_2 + \ldots a_m = b_1 + b_2 + \ldots b_n$$

or

$$\Sigma a = \Sigma b$$

Let C_{ij} be the cost of transportation of one unit from the i^{th} origin to j^{th} destination

These information

~~Total transportation cost~~
~~$= (2\times5) + (3\times2) + (3\times6) +$~~
~~$(4\times3) + (1\times4) + (2\times1)$~~

$= \underline{\underline{102}}$

---

* obtain an initial basic feasible solution for given transportation problem using North-west corner rule.

	Destination				
Origins	P	Q	R	S	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

Req total $= 200 + 225 + 275 + 250$
$= \underline{\underline{950}}$

Available total $= 250 + 300 + 400$
$= \underline{\underline{950}}$

$\therefore$ The problem is balanced.

$\boxed{200}$ 11	13	17	14	~~250~~ 50
16	18	14	10	300
21	24	13	10	400
~~200~~ 0	225	275	250	

$\boxed{50}$ 13	17	14	~~50~~ 0
18	14	10	300
24	13	10	400
~~225~~ 175	275	250	

$\boxed{175}$ 18	14	10	~~300~~ 125
24	13	10	400
~~175~~ 0	275	250	

$\boxed{125}$ 14	10	~~125~~ 0
13	10	400
275 150	250	

$\boxed{150}$ 13	$\boxed{250}$ 10	~~400~~ 250
~~150~~ 0	250	

$\boxed{200}$ 11	$\boxed{50}$ 13	17	14
16	$\boxed{175}$ 18	$\boxed{125}$ 14	10
21	24	$\boxed{150}$ 13	$\boxed{250}$ 10

Total cost $= (11 \times 200) + (13 \times 50) + (18 \times 175) + (14 \times 125) + (13 \times 150) + (10 \times 250)$

$= 2200 + 650 + 3150 + 1750 + 1950 + 2500$

$= \underline{\underline{12200}}$

Lowest cost entry method

~~choose the cell having the lowest cost in the matrix. Allocate them as much as possible which is in the minimum of the row total and column total. Thus either a row total or a column total is exhausted. Cross off the corresponding row or column. From the reduced matrix locate the lowest cost. Allocate to that~~

686

* Find an initial basic feasible solution for the following transportation Problem ?

Destination

Orgin	A	B	C	D	Supply
1	21	16	15	3	11
2	17	18	14	23	13
3	32	27	18	41	19
Demand	6	10	12	15	

Supply = 11 + 13 + 19

= 43

Demand = 6+10+12+15

= 43

Penalty = least value - next least value.

21	16	15	[11] 3	~~11~~ 0	(12)
17	18	14	23	13	(3)
32	27	18	4	19	(9)
6	10	12	~~15~~ 4		
(4)	(2)	(1)	(20)		

17	18	14	[4] 23	~~13~~ 9	(3)
32	27	18	4	19	(9)
6	10	12	~~4~~ 0		
(15)	(9)	(4)	(18)		

[6] 17	18	14	~~9~~ 3	(3)
32	27	18	19	(9)
0 ~~6~~	10	12		
(15)	(9)	(4)		

[3] 18	14	~~3~~ 0	(4)
27	18	19	(9)
7 ~~10~~	~~12~~		
(9)	(4)		

12
27 18 19
7 ~~1~~20

7
27 ~~7~~0
~~7~~0

Total $= (11 \times 3) + (23 \times 4) + (6 \times 17) + (3 \times 18) + (12 \times 18) + (7 \times 27)$

$= 686$

Q1 * obtain an initial feasible solution to the following transportation problem using least cost method?

Origin	Destination D_1	D_2	D_3	D_4	Capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

Q2 * obtain an initial feasible solution using vogel's approximation method.

Origin	D	E	F	G	Supply
A	20	25	28	31	200
B	32	28	32	41	180
C	18	35	24	32	110
Demand	150	40	180	170	

Q1*

Demand = 4 + 6 + 8 + 6

= 24

Capacity = 6 + 8 + 10

= 24

1	2	3	4	6
4	3	2	$\boxed{6}$ 0	~~8~~ 2
0	2	2	1	10
4	6	8	~~6~~ 0	

1	2	3	6
4	3	2	2
$\boxed{4}$ 0	2	2	~~10~~ 6
~~4~~ 0	6	8	

2	3	6
3	$\boxed{2}$ 2	~~2~~ 0
2	2	6
6	~~8~~ 6	

2	3	6
2	$\boxed{6}$ 2	~~6~~ 0
6	~~8~~ 6	

$\boxed{6}$ 2	~~6~~ 0
~~6~~ 0	

Total = (6×0) + (4×0) + (2×2) + (6×2) + (6×2)

= 4 + 12 + 12

= 28

Q2*

Demand = 150 + 40 + 180 + 170

= 540

Supply = 200 + 180 + 110

= 490

20	25	28	31	200 (5)
32	28	32	41	180 (4)
18	35	24	32	110 (6)
0	0	0	$\boxed{50}$ 0	~~50~~ 0 (0)
150	40	180	~~170~~ 120	
(18)	(25)	(24)	(31)	

Demand total ≠ supply total

∴ Problem is unbalanced so we add an extra row with supply (50)

ie, 540 − 490 = $\underline{\underline{50}}$
D S

20	25	28	31	200 (5)
32	28	32	41	180 (4)
$\boxed{110}$ 18	35	24	32	~~110~~ 0 (6)
40 ~~150~~	40	180	120	
(2)	(3)	(4)	(1)	

$\boxed{40}$ 20	25	28	31	~~200~~ 160 (5)
32	28	32	41	180 (4)
0 ~~40~~	40	180	120	
(12)	(3)	(4)	(10)	

25	28	$\boxed{120}$ 31	~~160~~ 40 (3)
28	32	41	180 (4)
40	180	~~120~~ 0	
(3)	(4)	(10)	

25	$\boxed{40}$ 28	~~40~~ 0 (3)
28	32	180 (4)
40	~~180~~ 140	
(3)	(4)	

$\boxed{40}$ 28	32	~~180~~ 140 (4)
~~40~~ 0	140	

$\boxed{140}$ 32	~~140~~ 0
~~140~~ 0	

$(50\times0)+(110\times18)+(40\times20)+(120\times31)+(40\times28)+(40\times28)+(140\times32)$

$= \underline{\underline{13220}}$

MODI Method

Step 1: When the initial basic feasible solution is obtained some cells are occupied and others are unoccupied. No. of occupied cells is m+n−1. Let c_{ij} be the cost of the cell (i,j)

Then we determine m+n numbers called u_i and v_j values by forming m+n-1 equations of the form $u_i+v_j = c_{ij}$ corresponding to each occupied cell is (2,3) then the equation is $u_2+v_3 = c_{23}$ where c_{23} is the cost in the cell (2,3) for solving the equations we take one of u_i or v_j values are zero

Step 2: Then we calculate cell evaluation or d_{ij} values for unoccupied cell, by the formula $d_{ij} = c_{ij} - (u_i + v_j)$

Step 3: If all d_{ij} values are positive the solution is optimal but alternative solution exist. If atleast one d_{ij} is negative the solution is not optimal

Step 4: If solution is not optimal make reallocation. Then repeat 1-4

* Solve the following transportation problem

origin	Destination I	II	III	Supply
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	

Supply = 5+8+7+14

= 34

Demand = 7+9+18

= 34

Demand total = supply total so the problem is balanced.

Now we have to find an initial basic feasible solution for the transportation problem. We are using Vogel's approximation method for the same.

[5] 2	7	4	~~5~~ 0 (2)
3	3	1	8 (2)
5	4	7	7 (1)
1	6	2	14 (1)
2 ~~7~~ (1)	9 (1)	18 (1)	

3	3	[8] 1	~~8~~ 0 (2)
5	4	7	7 (1)
1	6	2	14 (1)
(2) 2	(1) 9	(1) ~~18~~ 10	

5	4	7	7(1)
1	6	$\boxed{10}$ 2	4 ~~14~~ (1)
2 (4)	9 (2)	~~10~~ (5)	

5	4	7(1)
$\boxed{2}$ 1	6	2 ~~4~~ (5)
~~2~~0 (4)	9 (2)	

4	7 (4)
$\boxed{2}$ 6	0 ~~2~~ (6)
~~9~~ 7 (2)	

$\boxed{7}$ 4	~~7~~ 0
~~7~~ 0	

Occupied cells are

$\boxed{5}$ 2	7	4
3	3	$\boxed{8}$ 1
5	$\boxed{7}$ 4	7
$\boxed{2}$ 1	$\boxed{2}$ 6	$\boxed{10}$ 2

(1,1) (2,3) (3,2), (4,1) (4,2) (4,3)

The cost in these cells are

2, 1, 4, 1, 6, 2

$\therefore$ The equations are

$U_1 + V_1 = 2$

$U_2 + V_3 = 1$

$U_3 + V_2 = 4$

$U_4 + V_1 = 1$

$U_4 + V_2 = 6$

$U_4 + V_3 = 2$

Put $U_4 = 0$

$V_1 = 1$

$V_2 = 6$

$V_3 = 2$

$\Rightarrow U_1 + 1 = 2$

$U_1 = \underline{\underline{1}}$

$U_2 + 2 = 1$

$U_2 = \underline{\underline{-1}}$

$U_3 + 6 = 4$

$U_3 = -2$

$U_1 = 1$ $V_1 = 1$

$U_2 = -1$ $V_2 = 6$

$U_3 = -2$ $V_3 = 2$

$U_4 = 0$

Now we have to find dij for all unoccupied cells.

Here the unoccupied cells are

(1,2) (1,3) (2,1) (2,2) (3,1) (3,3)

$d_{ij} = c_{ij} - (U_i + V_j)$

$d_{12} = C_{12} - (U_1 + V_2)$

$= 7 - (1+6)$

$= 0$

$d_{13} = C_{13} - (U_1 + V_3)$

$= 4 - (1+2)$

$= 1$

$d_{21} = C_{21} - (U_2 + V_1)$

$= 3 - (-1+1)$

$= 3$

$d_{22} = C_{22} - (U_2 + V_2)$

$= 3 - (-1+6)$

$= -2$

$d_{31} = C_{31} - (U_3 + V_1)$

$= 5 - (-2+1)$

$= 6$

$d_{33} = C_{33} - (U_3 + V_3)$

$= 7 - (-2+2)$

$= 7$

$\boxed{5}$ 2	0 7	1 4
3 3	-2 + 3	$\boxed{8}$ − 1
6 5	$\boxed{7}$ 4	7 7
$\boxed{2}$ 1	$\boxed{2}$ − 6	$\boxed{10}$ + 2

$\boxed{5}$ 2	7	4
3	$\boxed{2}$ 3	$\boxed{6}$ 1
5	$\boxed{7}$ 4	7
$\boxed{2}$ 1	6	$\boxed{12}$ 2

$0+2=2$
$8-2=6$
$10+2=12$
$2-2=0$

$U_1+V_1=2$

$U_2+V_2=3$

$U_2+V_3=1$

$U_3+V_2=4$

$U_4+V_1=1$

$U_4+V_3=2$

Put $U_4=0$

$U_4+V_1=1$

$0+V_1=1$

$V_1=1$

$U_4+V_3=2$

$0+V_3=2$

$V_3=2$

$U_1+V_1=2$

$U_1+1=2$

$U_1=2-1$

$U_1=1$

$U_2+V_3=1$

$U_2+2=1$

$U_2=1-2$

$U_2 = -1$

$U_2 + V_2 = 3$

$-1 + V_2 = 3$

$V_2 = 3 + 1$

$V_2 = 4$

$U_3 + V_2 = 4$

$U_3 + 4 = 4$

$U_3 = 4 - 4$

$U_3 = 0$

$U_1 = 1 \quad V_1 = 1$

$U_2 = -1 \quad V_2 = 4$

$U_3 = 0 \quad V_3 = 2$

$U_4 = 0$

Here the unoccupied cells are

(1,2) (1,3) (2,1) (3,1) (3,3) (4,2)

$d_{ij} = c_{ij} - (u_i + v_j)$

$d_{12} = 7 - (U_1 + V_2)$

$= 7 - (1+4)$

$= 2$

$d_{13} = C_{13} - (U_1 + V_3)$

$= 4 \ (1+2)$

$= 4 - 3$

$= 1$

$d_{21} = C_{21} - (U_2 + V_1)$

$= 3 - (-1+1)$

$= 3 - 0$

$= 3$

$d_{31} = C_{31} - (U_3 + V_1)$

$= 5 - (0+1)$

$= \underline{4}$

$d_{33} = C_{33} - (U_3 + V_3)$

$= 7 - (0+2)$

$= \underline{5}$

$d_{42} = C_{42} - (U_4 - V_2)$

$= 6 - (0-4)$

$= 6 - (-4)$

$= \underline{10}$ ∴ The problem is optimal

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unbalanced transportation problem
up

* A steel company has 3 open hearth furnaces and 4 Rolling mills. Transportation cost for shipping steel from furnaces to rolling mills are shown in the following table.

	M_1	M_2	M_3	M_4	capacities
F_1	6	1	9	3	70
F_2	11	5	2	8	55
F_3	10	12	4	7	70
Requirement	85	35	50	45	

Here total capacity $= 70+55+70$
$= \underline{195}$

total requirement $= 85+35+50+45$
$= \underline{215}$

Here the problem is not balanced

total capacity is < total requirement

$215 - 195 = \underline{20}$

~~we add added a element again~~

so we add a row with all elements 0.

6	1	9	3	70	(2)
11	5	2	8	55	(3)
10	12	4	7	70	(3)
$\boxed{20}$ 0	0	0	0	~~20~~ 0	(0)
65 ~~85~~	35	50	45		
(6)	(1)	(2)	(3)		

6	1	9	$\boxed{45}$ 3	25 ~~70~~	(2)
11	5	2	8	55	(3)
10	12	4	7	70	(3)
65	35	50	~~45~~ 0		
(4)	(4)	(2)	(4)		

6	$\boxed{25}$ 1	9	0 ~~25~~	(5)
11	5	2	55	(3)
10	12	4	70	(6)
65	10 ~~35~~	50		
(4)	(4)	(2)		

11	$\boxed{10}$ 5	2	45 ~~55~~	(3)
10	12	4	70	(6)
65	0 ~~10~~	50		
(1)	(7)	(2)		

11	$\boxed{45}$ 2	0 ~~45~~	(9)
10	4	70	(6)
65	5 ~~50~~		
(1)	(2)		

$\boxed{65}$ 10	4	5 ~~70~~	(6)
0 ~~65~~	5		

$\boxed{5}$ 4	0 ~~5~~
0 ~~5~~	

$$= (20 \times 0) + (45 \times 3) + (25 \times 1) + (10 \times 5) +$$

$$(45 \times 2) + (10 \times 65) + (5 \times 4)$$

$$= \underline{\underline{970}}$$