8-3-24
Monday

Module: V

→ Network Analysis:-

Three phases on project management

i) Planning
ii) Scheduling
iii) Controlling

Activity:- An activity is a task associated with a project. it is a physically identifiable part of a project, which consumes time and resources.

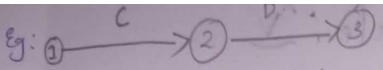
Eg: Lay pipeline is an activity

It is represented by an arrow, tail of which represents its start and head represent its finish

Eg: these A or 1-2 denote the activity. by '1' represent start of the activity and '2' represents finish of the activity.

→ Preceding Activity:-

Activity which must be completed before a given activity. An activity 'C' is called the preceding activity of 'D' if 'D' follows 'C'.

Eg: 

C → preceding activity of D. if D follows C.

Here 'D' is called successor of C.

~> Start and Terminal Activity:-

Activity which have no ~~previous~~ predecessor

or.

OR.

Activity which have no preceding activity.

Terminal activity: Activity which have no successor or no succeeding activity.

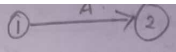
~> Event

Event represent instance in time, when certain activity have been started or completed. In other words an event describes, start or complete of a task.

Event is a point in time, and doesn't consume any resources.

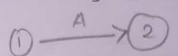
Eg: Pipeline layed.

* Tail Event: Event which marks the beginning of an activity.



'1' is the tail event.

* Head Event:- Event which maks the end of an activity.



'2' is the ~~tail~~ Head Event.

~> Network Diagram:-

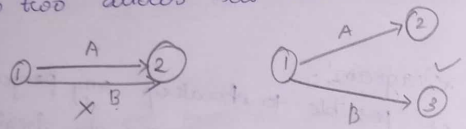
It is possible to ~~break~~ breakup any project into a number of 'jobs' called activity. The beginning or end of each activity constitute the event of the project.

A ~~graph~~ graph drawn connecting various activities and event of a project is called network diagram. In a network diagram activities represented by arrow & event is represented by numbered cycle.

~> Rules for Network Construction:-

1) Use straight arrows
2) Try to avoid arrows, that cross each other
3. No event can occur until every activity preceding it has been completed

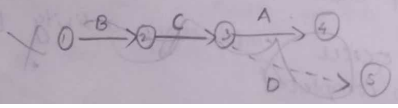
4. Use arrows from left to right.
5. Dummy Activity should be introduce if extremely necessary.
6. The network has only one entry point and one exit point.
7. No two arrows has same heads and tail event.

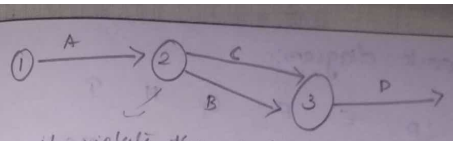


⇝ Dummy Activity:-

Usually a job requires time and cost but there are certain activities which dont take time or resources, they are known as dummy activity. They are used to maintain a proper precedence relation between two events. and is denoted by doted arrows.

Eg: A is the first activity followed by 'B' and 'C'.
'D' starts after 'B' and 'C'

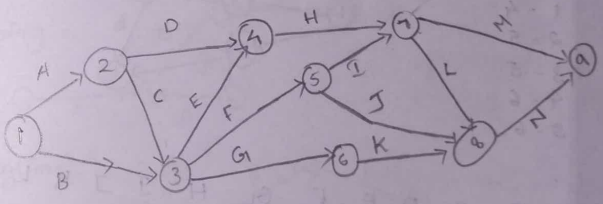




21-B-24

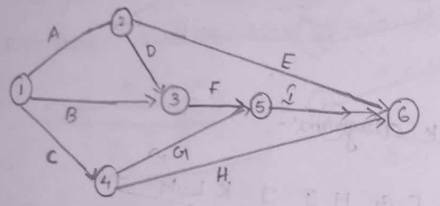
⇝ Draw Network diagram:-

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Activity Δ-				B,C	B,C	DE	F	G	H,I	HI	JKL			
Prec activity	–	–	A	A	B,C	B,C	DE	F	G	H,I	HI	JKL		



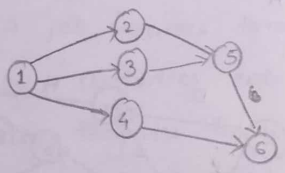
Q. Draw network diagram.

	A	B	C	D	E	F	G	H	I
Preceding act.	-	-	A	A	BD	C	C	FG	H

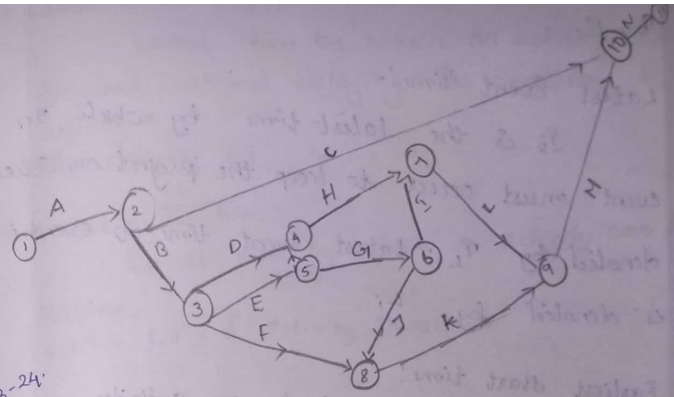


Q. Activity:

1 - 2
1 - 3
1 - 4
2 - 5
3 - 5
4 - 6
5 - 6



Q.	A	B	C	D	E	F	G	H	I	J	K	L	M	N
	-	A	A	B	B	B	E	DE	G	G	FJ	HI	LK	ME

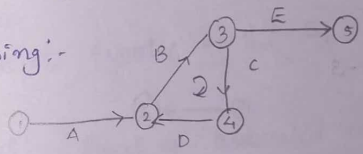


23-3-24

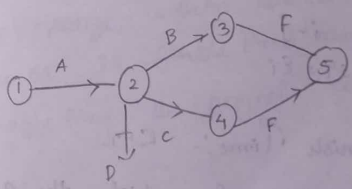
⇒ Errors in Network Constructions:-

1. Redundancy:- using dummy activity unnecessarily.

2. Looping:-



3. Dangling:



Earliest Event Time (T_E)

It is the earliest at which an event can occur. Earliest event time of event i is denoted

by E_i.

Latest Event Time:-

It is the latest time by which an event must occur to keep the project on schedule. denoted by T_L. Latest event time of event i is denoted by L_i

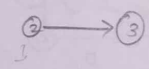
~> Earliest start time:

Earliest time by which an activity can commence.

EST = Earliest occurence of tailevent of the activity.

for activity 2-3

EST = E_2.



for activity i-j

EST = E_i

~> Earliest finish Time:- EFT.

Earliest time by which the activity can be finished

Interms EFT = EST + Activity duration.

~> Latest finish time:-

Latest time by which an activity can be finished, without delaying the completion of an activity.

for activity i-j LFT = L_j

~> Latest start Time:-

Latest time by which an activity can be started,

LST = LFT - Activity Duration.

~> Slack

It is associated with event, difference btw earliest event time and latest event time.

Slack of Event i = $L_i - E_i$

~> Float:- Float is associated with activity times it denotes range within which activities start time or its finish time may fluctuate without affecting the project completion.

~> Total Float:-

It is the time spend by which starting (or finishing) of an activity can be delayed without delaying the project completion

Total float = LFT - EFT

or

LST - EST.

→ Critical Activity

Activity cohose total float is zero. So any delay in the start of critical activity will cause a further delay in the completion of the project.

The path connecting the critical activity is called the critical path. In a network there may be a number of path from initial event to terminal event connecting the critical activity among these critical path is longest on the basis of finial duration.

Two Basic Planing and Controlling Technique.

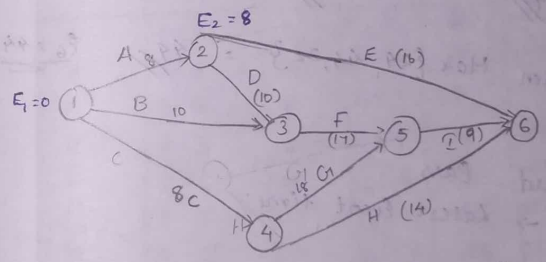
1) CPM: Critical Path method.

2) PERT: Programme Evaluation and Review Techniques.

19-4-24

Find critical path and duration of project?

Activity	Exceeding Activity	Time.
A	–	8
B	–	10
C	–	8
D	A	10
E	A	16
F	B D	17
G	C	18
H	C	14
I	F, G	9



(Forward pass)

T_E → Easliest time

$\mathcal{E}_1 = 0$

$\therefore \mathcal{E}_2 = 0 + 8 = 8$

$\mathcal{E}_3$ through D = 8 + 10 = 18//

through B 0 + 10 = 10

then Max {18, 10} = 18// $\therefore \mathcal{E}_3 = 18$//

• $\xi_4 = 0+8 = 8$ //

• ξ_5 through F: $\xi_3 + 17 = 18+17 = 35$; through G: $\xi_4 + 18 = 8+18 = 26$

then Max {35, 26} = 35 //

$\therefore \xi_5 = 35$

• ξ_6 through E: $\xi_2 + 16 = 8+16 = 24$ //

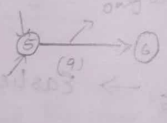
through I: $= \xi_5 + 9 = 35+9 = 44$ //

through H: $= \xi_4 + 14 = 8+14 = 22$

then Max {24, 44, 22} = 44 // $\xi_6 = 44$

⇒ Backward pass

T_L → Latest Event time.



$L_6 = \xi_6 = 44$.

$L_5 = 44 - 9 = 35$ //

$L_4 = \min \{35-18, 44-14\}$

$= \{17, 30\}$

$= 17$ //

• $L_3 = L_5 - 17 = 35 - 17 = 18$ //

• $L_2 = \min \{L_6 - 16, L_3 - 10\}$

$= \{44-16, 18-10\}$

$= 28, 8 = 8$ //

• $L_1 = \min \{L_2 - 8, L_3 - 10, L_4 - 8\}$

$= \{8-8, 18-10, 17-8\}$

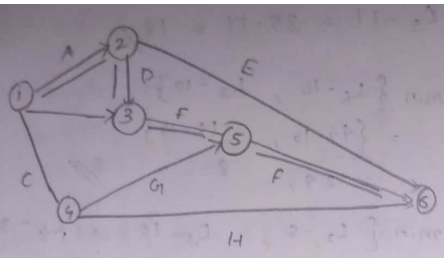
$= \{0, 8, 9\} = 0$ //

Activity	i-j	time	EST ξ_i	EFT $\xi_i + t$	LST $L_j - t$	LFT L_j	Total float LST - EST
A	1-2	8	$\xi_1 = 0$	8	8-8=0	$L_2 = 8$	0-0=0
B	1-3	10	$\xi_1 = 0$	10	18-10=8	$L_3 = 18$	8-0=8
C	1-4	8	$\xi_1 = 0$	8	17-8=9	$L_4 = 17$	9-0=9
D	2-3	10	$\xi_2 = 8$	18	18-10=8	$L_3 = 18$	8-8=0
E	2-6	16	$\xi_2 = 8$	24	44-16=28	$L_6 = 44$	28-8=20
F	3-5	17	$\xi_3 = 18$	35	35-17=18	$L_5 = 35$	18-18=0
G	4-5	18	$\xi_4 = 8$	26	35-18=17	$L_5 = 35$	17-8=9
H	4-6	14	$\xi_4 = 8$	22	44-14=30	$L_6 = 44$	30-8=22
I	5-6	9	$\xi_5 = 35$	44	44-9=35	$L_6 = 44$	35-35=0

Critical activities are A, D F and I

ie, total float = 0.

24-4-2024
Wednesday



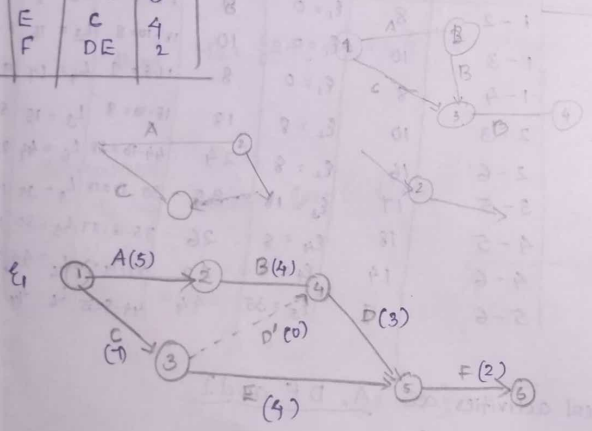
Critical path $A \rightarrow D \rightarrow F \rightarrow I$

Project duration = $8 + 10 + 17 + 9 = 44$ //

Q

		Time
A	-	5
B	A	4
C	-	7
D	BC	3
E	C	4
F	DE	2

find project duration.



T_E → Earliest time

$\xi_1 = 0$

$\xi_2 = 0 + 5 = 5$ //

$\xi_3 = 0 + 7 = 7$ //

$\xi_4 = \max\{5+4, 7+0\}$

$= \{9, 7\} = 9$ //

$\xi_5 = \max\{5+4+3, 7+4\}$

$= \{12, 11\} = 12$ //

$\xi_6 = \max\{5+4+3+2, 7+4+2\}$

$= \{14, 13\} = 14$ //

- T_L → Latest event time Backward pass:-

$L_6 = \xi_6 = 14$

$L_5 = \xi_6 - 2 = 14 - 2 = 12$ //

$L_4 = \xi_5 - 3 = 12 - 3 = 9$ //

$L_3 = \min\{\xi_4 - 0, \xi_5 - 4\}$

$= \{9 - 0, 12 - 4\} = 8$ //
(a, 8)

$L_2 = \xi_4 - 4 = 9 - 4 = 5$ //

$L_1 = \min\{\xi_2 - 5, \xi_3 - 7\}$

$= 5 - 5, 7 - 7 = 0$ //

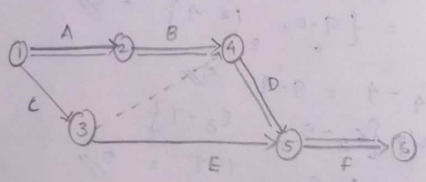
Activity	i-j	time	EST ξ_i	EFT $\xi_i + t$	$L_j - t$	LFT L_j	Total float $L_{6?} - E_{ST}$
A	1-2	5	$\xi_1 = 0$	5	0	$L_2 = 5$	0-0 = 0
B	2-4	4	$\xi_2 = 5$	4	5	$L_4 = 9$	5-5 = 0
C	1-3	7	$\xi_1 = 0$	7	1	$L_3 = 8$	1-0 = 1
D	4-5	3	$\xi_4 = 9$	12	9	$L_5 = 12$	9-9 = 0
E	3-5	4	$\xi_3 = 7$	11	8	$L_5 = 12$	8-7 = 1
F	5-6	2	$\xi_5 = 12$	14	12	$L_6 = 14$	12-12 = 0
D'	3-4	0	$\xi_3 = 7$	7	9	$L_4 = 9$	9-7 = 2

∴ critical activities are A, B, D and F

∴ critical path = $A \rightarrow B \rightarrow D + F$

= 5 + 4 + 3 + 2

= 14 //

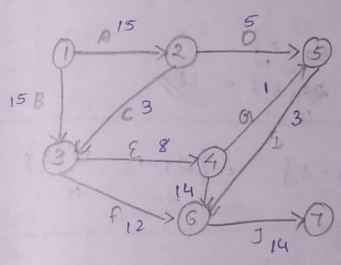


Assignment.

Q)

Job	A 1-2	B 1-3	C 2-3	D 2-5	E 3-4	F 3-6	G 4-5	H 4-6	I 5-6	J 6-7
Duration (days)	15	15	3	5	8	12	1	14	3	14

- Draw an arrow diagram representing the project
- find the total float for each activity.
- find the critical path and the total project duration.



ξ_i → Earliest time

$E_1 = 0$

$E_2 = 0 + 15 = 15$ //

$\xi_3 = \text{Max}\{0+15 = 15, 15+3\} = 18$ //

$\xi_4 = \text{Max}\{\xi_3 + 8, 8\ 15 + 8\} = \{18+8, 15+8\} = 26$ //
(26, 23)

$\xi_5 = \text{Max}\{\xi_2 + 5, 15+8+1, \xi_3 + 8 + 1\} = \{15+5, 15+9, 18+9\} = 27$ //
(20, 24, 27)

$\xi_6 = \text{Max}\{\xi_3 + 12, \xi_4 + 14, \xi_5 + 3\} = \{18+12, 26+14, 27+3\} = 40$ //
(30, 40)

$\xi_7 = \xi_6 + 14 = 40 + 14 = \underline{54}$

- $T_L \rightarrow$ Latest Event

$L_7 = \xi_7 = 54$//

$L_6 = \xi_7 - 14 = \underline{40}$

$L_5 = \xi_6 - 3 = 40 - 3 = 37$//

$L_4 = \min\{\xi_6 - 14, \xi_5 - 1\}$

$= \{40 - 14, 27 - 1\} = 26$//

$L_3 = \min\{\xi_6 - 12, \xi_4 - 8\}$

$\{40 - 12, 26 - 8\} = 18$//

$L_2 = \min\{\xi_5 - 1, \xi_3 - 3\} = \{27 - 1, 18 - 3\}$

$= 15$//

$L_1 = \min\{\xi_2 - 15, \xi_3 - 15\}$

$\{15 - 15, 18 - 15\} = 0$//

∴ A, C, E, H, J are the critical activity

critical path: $A \rightarrow C \rightarrow E \rightarrow H \rightarrow J$

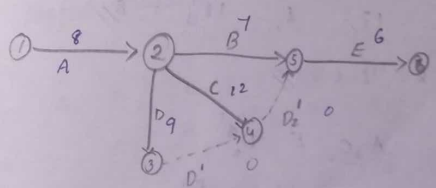
$15 + 3 + 8 + 14 + 4 = 54$//

∴ project duration = 54//

	i-j	Time	EST ξ_i	EFT $\xi_i + t$	LST $L_j - t$	LFT L_j	Total float LST-EST
A	1-2	15	0	15	0	15	0
B	1-3	15	0	15	3	18	3
C	2-3	3	15	18	15	18	0
D	2-5	5	15	20	32	37	17
E	3-4	8	18	26	18	26	0
F	3-6	12	18	30	28	40	10
G	4-5	1	26	27	36	37	10
H	4-6	14	26	40	26	40	0
I	5-6	3	27	30	37	40	10
J	6-7	14	40	54	40	54	0

Q.

Job	Predecessor	Time
A	-	8
B	A	7
C	A	12
D	A	9
E	B C D	6



- T_E → Earliest time

$\varepsilon_1 = 0$

$\varepsilon_2 = 0+8 = 8$//

$\varepsilon_3 = \varepsilon_2 + 9 = 8+9 = 17$//

$\varepsilon_4 = \text{Max}\{\varepsilon_3+0,\ \varepsilon_2+12\}$

$= \text{Max}\{17+0,\ 8+12\} = 20$//

$\varepsilon_5 = \text{Max}\{\varepsilon_2+7,\ \varepsilon_4+0\}$

$= \text{Max}\{8+7,\ 20+0\} = 20$//

$\varepsilon_6 = \text{Max}\{\varepsilon_5+6\} = 20+6 = 26$//

T_L → Latest Event:-

$L_6 = \varepsilon_6 = 26$//

$L_5 = 26-6 = 20$//

$L_4 = \varepsilon_5 - 0 = 20-0 = 20$//

$L_3 = \varepsilon_4 - 0 = 20-0 = 20$//

$L_2 = \text{Min}\{\varepsilon_5-7,\ \varepsilon_4-12,\ \varepsilon_3-9\}$

$= \text{Min}\{20-7,\ 20-12,\ 17-9\}$

$= \text{Min}\{13,\ 8,\ 18\} = 8$//

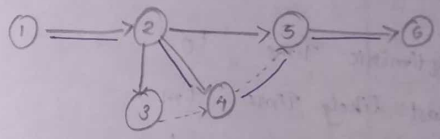
$L_1 = \varepsilon_2 - 8 = 8-8 = 0$//

* A, C, E, D_2'

A → C → D_2' → E.

∴ 8 + 12 + 0 + 6 = 26//

Activity	i-j	time t	EST ε_i	EFT ε_{i+t}	LST L_{j-t}	LFT L_j	Total float LST − EST
A	1-2	8	0	8	0	8	0
B	2-5	7	8	15	13	20	5
C	2-4	12	8	20	8	20 20	0
D	2-3	9	8	17	11	20	3
E	5-6	6	20	26	20	26	0
D'	3-4	0	17	17	20	20	3
D_2'	4-5	0	20	20	20	20	0



Q. Activity A B C D E F G H I J K

Preceding activity – A B C B E D,f E H G, I J

Time 13 8 10 9 11 10 8 6 7 14 18.

find project duration.

Q. Activity A B C D E F G H I

Preceding activity – – – A A B,D C C F,G.

Time 8 10 8 10 16 17 18 14 9

Q. Activity A B C D E f G H

Ps - A A B B DE D cfG

2 4 8 3 2 3 4 8.

→ PERT

Three time estimate for each activity say pert.

i, Optimistic Time t_o

ii, Most likely Time t_m

iii, Pessimistic Time t_p.

Expected Time $= \frac{t_o + 4t_m + t_p}{6}$

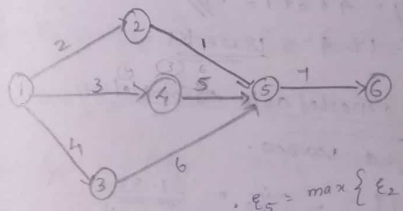
variance of activity $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

1. Find project duration and project variance
2) Probability of completing project 4 weeks earlier than expected.

Activity	t_o	t_m	t_p
1-2	1	1	1
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-5	2	5	8
5-6	3	6	15

Ans:

Activity	t_o	t_m	t_p	$t_e = \frac{t_o+4t_m+t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	1	1	1	$\frac{1+4+1}{6} = 2$ //	$\left(\frac{1-1}{6}\right)^2 = 1$
1-3	1	4	7	$\frac{1+16+7}{6} = 4$ //	$\left(\frac{7-1}{6}\right)^2 = 1$
1-4	2	2	8	$\frac{2+8+8}{6} = 3$ //	$\left(\frac{8-2}{6}\right)^2 = 1$
2-5	1	1	1	$\frac{1+4+1}{6} = 1$	$\left(\frac{1-1}{6}\right)^2 = 0$
3-5	2	5	14	$\frac{2+20+14}{6} = 6$	$\left(\frac{14-2}{6}\right)^2 = 4$
4-5	2	5	8	$\frac{2+20+8}{6} = 5$	$\left(\frac{8-2}{6}\right)^2 = 1$
5-6	3	6	15	$\frac{3+24+15}{6} = 7$	$\left(\frac{15-3}{6}\right)^2 = 4$



$\therefore E_1 = 0$

$E_2 = 0+2 = 2$ //

$E_3 = 0+4 = 4$ //

$E_4 = 0+3 = 3$

$E_5 = \max\{E_2+1, E_4+5, E_3+6\}$

$= \{3, 8, 10\} = 10$ //

$E_6 = E_5 + 7 = 10 + 7 = 17$ //

$L_6 = E_6 = 17$.

$L_5 = 10$

$L_3 = 9$

$L_2 = 9$

$L_1 = 0$

Activity	time	ESij Ei	EFij Ei+t	ESij Lj-t	LFij Lj	Total float LFij - EFij
1-2	2	0	2	7	9	7
1-3	4	0	4	0	4	0 ✓
1-4	3	0	3	2	5	2
2-5	1	2	3	9	10	7
3-5	6	4	10	4	10	0 ✓
4-5	5	3	8	5	10	2
5-6	7	10	17	10	17	0 ✓

variance of project = Σ variance of critical activity

$= \because$ 1-3, 3-5, 5-6.

$\therefore 1 + 4 + 4 = 9$

∴ critical activity = $4 + 6 + 7 = 17$ // week.

∴ due date = $17 - 4 = 13$ weeks

Where D = due date − expected date of completion / $\sqrt{\text{project variance}}$.

$\therefore \frac{13 - 17}{\sqrt{9}} = -4/3 = -1.33$

required probability = $P(Z \leq D)$

$P(Z \leq -1.33)$

$= .5 - .4082$

$= .0918$

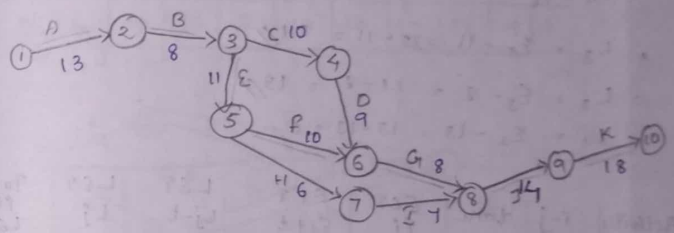
$= 9.18\%$

Assignment.

①

Activity	A	B	C	D	E	f	G	H	I	J	K
Preceding Activity	-	A	B	C	B	E	Df	E	H	G, I	J
Time	13	8	10	9	11	10	8	6	7	14	18

find project duration?



T_E → Earliest Time

(•) $E_1 = 0$ (•) $E_2 = 0 + 13 = 13$ // (•) $E_3 = 13 + 8 = 21$ //

• $E_4 = E_3 + 10 = 31$ // (•) $E_5 = E_3 + 11 = 32$ //

• $E_6 = \text{Max}\{E_5 + 10, E_4 + 9\} = \{42, 40\} = 42$ //

• $E_7 = E_5 + 6 = 38$ // (•) $E_8 = \text{Max}\{E_6 + 8, E_4 + 7\}$

$= \{50, 45\} = 50$ //

• $E_9 = E_8 + 14 = 64$ // (•) $E_{10} = E_9 + 18 = 82$ //

T_L → Latest ...

- $L_{10} = \xi_{10} = 82$ //
- $L_8 = \xi_9 - 14 = 50$ //
- $L_7 = \xi_8 - 8 = 50 - 8 = 42$ //
- $L_6 = \xi_8 - 8 = 50 - 8 = 42$ //
- $L_4 = \xi_6 - 9 = 42 - 9 = 33$ //
- $L_3 = \xi_5 - 11 = 32 - 11 = 21$ //
- $L_2 = \xi_3 - 8 = 21 - 8 = 13$ //

$L_1 = \xi_2 - 13 = 13 - 13 = 0$ //

(•) $L_9 = \xi_{10} - 18 = 64$ //

$L_4 = \xi_8 - 7 = 50 - 7 = 43$

- $L_5 = \text{Min}\{\xi_6 - 10, \xi_4 - 6\}$
  $= \{42 - 10, 38 - 6\}$
  $= \{32, 32\} = 32$ //

$L_{10} = \xi_{10} = 82$

$L_9 = L_{10} - 18 = 64$

$L_8 = L_9 - 14 = 50$

Activity	i-j	time	EST ξ_i	EFT $\xi_i + t$	LST $L_j - t$	LFT L_j	Total float LST - EST
A	1-2	13	0	13	0	13	0
B	2-3	8	13	21	13	21	0
C	3-4	10	21	31	23	33	2
D	4-6	9	31	40	33	42	2
E	3-5	11	21	32	21	32	0
f	5-6	10	32	42	32	42	0
G	6-8	8	42	50	42	50	0
H	5-7	6	32	38	37	43	5
I	7-8	7	38	45	43	50	5
J	8-9	14	50	64	50	64	0
K	9-10	18	64	82	64	82	0

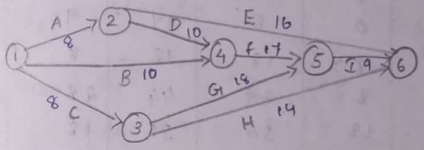
∴ A, B, E, f G J K are the critical activity.

∴ critical path : A → B → E → f → G → J → K

= 13 + 8 + 11 + 10 + 8 + 14 + 18 = 82 //

2.

Activity	A	B	C	D	E	f	G	H	I
Preceeding Activity	-	-	-	A	A	BD	C	C	fG
Time	8	10	8	10	16	14	18	14	9



T_E → Earliest Time :-

(•) $\xi_1 = 0$ // (•) $\xi_2 = 8$ // (•) $\xi_3 = 8$ //

- $\xi_4 = \text{Max}\{10, \xi_2 + 10\} = \{10, 18\} = 18$ //
- $\xi_5 = \text{Max}\{\xi_3 + 18, \xi_4 + 17\} = \{26, 35\} = 35$ //
- $\xi_6 = \text{Max}\{\xi_2 + 16, \xi_5 + 9, \xi_3 + 14\}$
  $= \{24, 44, 22\} = 44$

T_L → Latest time:-

(•) $L_6 = \varepsilon_6 = 44$ //

(•) $L_5 = 44 - 9 = 35$ //

• $L_4 = 35 - 17 = 18$ //

(•) $L_3 = \min\{35-18,\ 44-14\}$
$= \{17,\ 30\} = 17$ //

• $L_2 = \min\{44-16,\ 18-10\}$
$= \{28,\ 8\} = 8$ //

$L_1 = \min\{8-8,\ 8-8,\ 18-10\}$
$= 0$ //

Activity	i-j	time	ES↑ ε_i	Eft ε_{i+t}	LF↑ L_j	LS↑ L_{j-t}	Total float LS↑ − ES↑
A	1-2	8	0	8	8	0	0
B	1-4	10	0	10	18	8	8
C	1-3	8	0	8	17	9	9
D	2-4	10	8	18	18	8	0
E	2-6	16	8	24	44	28	20
F	4-5	17	18	35	35	18	0
G	3-5	18	8	26	35	17	9
H	3-6	14	8	22	44	30	22
I	5-6	9	35	44	44	35	0

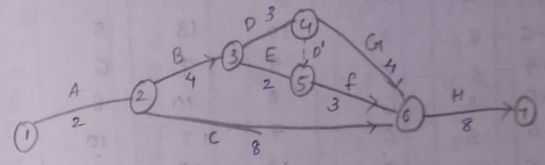
Critical Activity = A, D, F, I

path A → D → F → I

$= 8 + 10 + 17 + 9 = 44$ //

Q. (3)

Activity	A	B	C	D	E	f	G	H
preceding activity	-	A	A	B	B	DE	D	CfG
Time	2	4	8	3	2	3	4	8



Earliest Time: T_E

$\varepsilon_1 = 0$ $\varepsilon_2 = 2$

$\varepsilon_3 = \varepsilon_2 + 4 = 2+4 = 6$ //

$\varepsilon_4 = \varepsilon_3 + 3 = 6+3 = 9$ //

$\varepsilon_5 = \text{Max}\{\varepsilon_3 + 2,\ \varepsilon_4 + 0\}$
$= 6+2,\ 9$
$= 8, 9 = 9$ //

$\varepsilon_6 = \text{Max}\{\varepsilon_5 + 3,\ \varepsilon_4 + 4,\ \varepsilon_2 + 8\}$
$= \{9+3,\ 9+4,\ 2+8\}$
$= 13$ //

$\varepsilon_7 = \varepsilon_6 + 8 = 13 + 8 = 21$ //

T_L → Latest time:-

• $L_7 = \varepsilon_7 = 21$ //

• $L_6 = 21 - 8 = 13$ //

• $L_5 = \varepsilon_6 - 3 = 13 - 3 = 10$ //

• $L_4 = \min\{\varepsilon_6 - 4,\ \varepsilon_5 - 0\} = \{13-4,\ 9-0\} = 9$ //

• $L_3 = \min\{\varepsilon_4 - 3,\ \varepsilon_5 - 2\}$
$= \{9-3,\ 9-2\} = 6$ //

$L_2 = \min\{\varepsilon_3 - 4,\ \varepsilon_6 - 6\} = 6-4,\ 21-8$
$= 2$ //

$L_1 = E_2 - 2 = 2 - 2 = 0$ //

Activity	i-j	time	EST E_i	EFT $E_i + t$	LFT L_j	LST $L_j - t$	Total float LST - EST
A	1-2	2	0	2	2	0	0
B	2-3	4	2	6	6	2	0
C	2-6	8	2	10	13	5	3
D	3-4	3	6	9	9	6	0
E	3-5	2	6	8	10	8	2
F	3-6	3	6	9	13	10	4
G	4-6	4	9	13	13	9	0
H	6-7	8	13	21	21	13	0
D'	4-5	0	9	9	10	10	1

A B D, G H

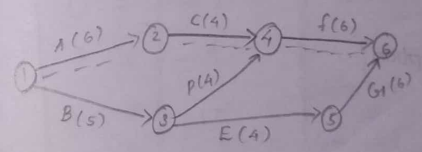
2 + 4 + 3 + 4 + 8 = 21 //

Q. Find S.D of project and probability of completing project in 18 week s.

Activity	Predecessor	to	tm	tp
A	–	3	6	9
B	–	2	5	8
C	A	2	4	6
D	B	2	3	10
E	B	1	3	11
F	CD	4	6	8
G	E	1	5	15

Ans:

Activity	to	tm	tp	$te = \frac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A	3	6	9	$\frac{3+24+9}{6} = 6$ //	$(6/6)^2 = 1$ //
B	2	5	8	$\frac{2+20+8}{6} = 5$ //	$(6/6)^2 = 1$
C	2	4	6	$\frac{2+16+6}{6} = 4$ //	$(4/6)^2 = .44$
D	2	3	10	$\frac{2+12+10}{6} = 4$ //	$(8/6)^2 = 1.77$
E	1	3	11	$\frac{1+12+11}{6} = \frac{24}{6} = 4$ //	$(10/6)^2 = 2.77$
f	4	6	8	$\frac{4+24+8}{6} = 6$ //	$(4/6)^2 = .44$
G	1	5	15	$\frac{1+20+15}{6} = 6$ //	$(14/6)^2 = 5.44$



$E_1 = 0$

$E_2 = 6$

$E_3 = 5$

$E_4 = 10$

Critical path - A → C - F

project length = 6+4+6 = 16

Due date :

expected = 16

$\mathcal{D} = \frac{18-16}{\sqrt{\text{project variance}}}$

variance of project = 1.88

$SD = \sqrt{1.88} = 1.374$

- Total Elapsed Time.
- Idle time.

  Amount of time for which a machine doesn't work or remain at rest.

- Principal Assumption :-

Q. Find total elapsed time, from order AB

Job	Machine A	Machine B
1	5	2
2	1	6
3	9	7
4	3	8
5	10	4

Find the order that of that 5 job.

least time is 1. It occurs for Job 2 in machine A.
So place it first.

2 4 3 5 1

then draw a table with the same order

Machine A Machine B.

Job	Intime	Outtime	Intime	Outtime
2	0	1	1	7
4	1	4	7	15
3	4	13	15	22
5	13	23	23	27
1	23	28	28	30

∴ total elapsed time = 30hrs.
idel time of machine A = 30 - 28 = 2hrs

idle time of Machine B =

0 → 2 hr, 22 → 23 hr, 27 → 28 hr

1 + 1 + 1

= 3 hr

Q. Find total elapsed time, order PQ?

Job	A	B	C	D	E	F	G	H	I
Machine P	2	5	4	9	6	8	7	5	4
Machine Q.	6	8	7	4	3	9	3	8	11

A I C B H F D E G

Job	Machine P Intime	Machine P Outtime	Machine Q. Intime	Machine Q. Outtime
A	0	2	2	8
I	2	6	8	19
C	6	10	19	26
B	10	15	26	34
H	15	20	34	42
F	20	28	42	51
D	28	37	51	55
E	37	43	55	58
G	43	50	58	61

∴ total elapsed time = 61 hrs

idle time of machine P = 61 − 50 = 11 hr

" Machine Q = 0 → 2 = 2 hrs

14-5-24

Q. Find total elapsed Time.

Job	1	2	3	4	5	6	7	8	9	10
A	7	8	10	3	7	4	5	8	5	6
B	4	2	6	6	5	7	2	6	7	6

4 6 9 10 8 3 5 1 7 2

Job	Machine A Intime	Machine A Outtime	Machine B. Intime	Machine B. Outtime
4	0	3	3	9
6	3	7	9	16
9	7	12	16	23
10	12	18	23	29
8	18	26	29	35
3	26	36	36	42
5	36	43	43	48
1	43	50	50	54
7	50	55	55	57
2	55	63	63	65

∴ total elapsed time = 65 hrs

idle time of machine A = 65 − 63 = 2 hrs

idle time of machine B

= 0 → 3 , 35 → 36 , 42 → 43 , 48 → 50 , 54 → 55 , 56 → 57 → 63.

= 3 + 1 + 1 + 2 + 1 + 6 = 14 hrs

→ Simulation:

It is the representation of reality through the use of a model which react in the same manner as reality under given set of condition.

Eg: planitorium, testing of a aircraft model in a wind tunnel

* Monte - Carlo simulation:

Q. A tourist car operator find that, during the past two months the car usage has varied so much the demand for car fluctuated during the past 200 wk is given below.

Trips per week	0	1	2	3	4	5
No. of week	16	24	30	60	40	30

Simulate demand for 10 weeks.

Case Random no: (82, 96, 18, 96, 20, 84, 56, 11, 52, 3)

Trips	Frequency	Probability	Cumulative Probability	Random No Range
0	16	16/200 = .08	.08	0 − [.08 × 100 − 1] = 7
1	24	24/200 = .12	.08 + .12 = .2	8 − [.2 × 100 − 1] = 19
2	30	30/200 = .15	.2 + .15 = .35	20 − [.35 × 100 − 1] = 34
3	60	60/200 = .3	.35 + .3 = .65	35 − [.65 × 100 − 1] = 64
4	40	40/200 = .2	.65 + .2 = .85	65 − [.85 × 100 − 1] = 84
5	30	30/200 = .15	.85 + .15 = 1	85 − [1 × 100 −] = 99

Weeks	Random no	demand/Trip
1	82 → 65 − 84 − 50	4
2	96	5
3	18	1
4	96	5
5	20	2
6	84	4
7	56	3
8	11	1
9	52	3
10	3	0
		Σ = 28.

Average demand = 28/10 = 2.8

Q. A

17-5-24 friday

9 54 94 1, 80 73 20 26 90 79

Trip per day	0	1	2	3	4	5
Number of days.	5	15	40	20	15	5

A tourist car operator finds that during the past 100 days the demand for the car had been varied as shown below.

Using random number simulate the demand for a 10 days.

Trips	frequency	Probability	Cumulative Probability	Random no range.
0	5	$5/100 = .05$	$.05$	$0 \to (.05 \times 100 - 1) = 4$
1	15	$15/100 = .15$	$.05 + .15 = .2$	$5 \to (.02 \times 100 - 1)$ $5 \to 19$
2	40	$40/100 = .4$	$.2 + .4 = .6$	$20 \to 59$
3	20	$20/100 = .2$	$.6 + .2 = .8$	$60 \to 79$
4	15	$15/100 = .15$	$.8 + .15 = .95$	$80 \to 94$
5	5	$5/100 = .05$	$.95 + .05 = 1$	$95 \to 99$.

.8 + .15 / .95 / .05 / 1.00 / 0

days.	Random no.	demand
1	9	1
2	54	2
3	94	4
4	1	0
5	80	4
6	73	3
7	20	2
8	26	2
9	90	4
10	79	2

Σ demand = 25