Derivation Rules of SD+

1 Rules of Inference

All the Derivation Rules of SD

AND

Modus Tollens (MT)

$$\begin{array}{l|l} & P \supset Q \\ & \sim Q \\ \rhd & \sim P \end{array}$$

Hypothetical Syllogism (HS)

$$\begin{array}{l|l} & P \supset Q \\ & Q \supset R \\ \rhd & P \supset R \end{array}$$

Disjunctive Syllogism (DS)

$$\begin{array}{l|l} & P \vee Q \\ & \sim P \\ \rhd & Q \end{array} \quad \text{OR} \quad \begin{array}{l|l} & P \vee Q \\ & \sim Q \\ \rhd & P \end{array}$$

Paradoxes of Material Implication (PMI)

$$\begin{array}{l|l} & \sim P \\ \rhd & P \supset Q \end{array} \quad \text{OR} \quad \begin{array}{l|l} & Q \\ \rhd & P \supset Q \end{array}$$

2 Rules of Replacement

Commutation (Com)

$P \,\&\, Q :: Q \,\&\, P$

$P \vee Q :: Q \vee P$

Association (Assoc)

$P \,\&\, (Q \,\&\, R) :: (P \,\&\, Q) \,\&\, R$

$P \vee (Q \vee R) :: (P \vee Q) \vee R$

Implication (Impl)

$P \supset Q :: \sim P \vee Q$

Double Negation (DN)

$P :: \sim\sim P$

DeMorgan (DeM)

$\sim (P \,\&\, Q) :: \sim P \vee \sim Q$

$\sim (P \vee Q) :: \sim P \,\&\, \sim Q$

Idempotence (Idem)

$P :: P \,\&\, P$

$P :: P \vee P$

Transposition (Trans)

$P \supset Q :: \sim Q \supset \sim P$

Exportation (Exp)

$P \supset (Q \supset R) :: (P \,\&\, Q) \supset R$

Distribution (Dist)

$P \,\&\, (Q \vee R) :: (P \,\&\, Q) \vee (P \,\&\, R)$

$P \vee (Q \,\&\, R) :: (P \vee Q) \,\&\, (P \vee R)$

Equivalence (Equiv)

$P \equiv Q :: (P \supset Q) \,\&\, (Q \supset P)$

$P \equiv Q :: (P \,\&\, Q) \vee (\sim P \,\&\, \sim Q)$

Conditional Negation (CN)

$\sim (P \supset Q) :: P \,\&\, \sim Q$