Calculating a Skew-Normal Approximation

Although easier with a computer program, calculating estimates for μ, σ, and λ by hand is perfectly possible.

Calculating a Skew-Normal Approximation: Finding λ

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Our goal is to find λ such that $f(\lambda)$ is within a certain margin of error (e) of $k_{n,p}$.

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This convenient fact allows us to find lower and upper bounds for λ and repeatedly bisect our interval until we are within e of $k_{n,p}$.

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Since we're doing this by hand, we'll take our error margin e to be a modest 0.1.

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Our values: $a = 1$, $b = 3$.

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Repeat this step until $f(c)$ is within e of $k_{n,p}$, or more precisely $k_{n,p} - 0.01 < f(c) < k_{n,p} + 0.01$.

Calculating a Skew-Normal Approximation: Finding λ

(Step 3)

The following table shows our iterations:

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We take the last value of c: 2.3125.

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Our final answer: $\lambda = 2.3125$.

Calculating a Skew-Normal Approximation: Finding σ

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$$\sigma = \sqrt{\frac{np(1-p)}{1 - \frac{2}{\pi} \cdot \frac{\lambda^2}{1+\lambda^2}}} = \sqrt{\frac{25 \cdot 0.1 \cdot 0.9}{1 - \frac{2}{\pi} \cdot \frac{2.3125^2}{1+2.3125^2}}} = 2.2029.$$

CALCULATING A SKEW-NORMAL APPROXIMATION: FINDING μ

And with λ and σ, we can also find μ:

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$$\begin{aligned}
\mu &= np - \sigma \cdot \sqrt{\frac{2}{\pi}} \cdot \frac{\lambda}{\sqrt{1+\lambda^2}} \\
&= 25 \cdot 0.1 - 2.2029 \cdot \sqrt{\frac{2}{\pi}} \cdot \frac{2.3125}{\sqrt{1+2.3125^2}} \\
&= 0.8867.
\end{aligned}$$

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