

THE SKEW-NORMAL APPROXIMATION OF THE BINOMIAL DISTRIBUTION

Joyce Tipping

Truman State University

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INTRODUCTION

$X \sim \text{Bin}(n, p)$ where $0 < p < 1$ and $n = 1, 2, 3, \dots$

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$$F_X(x) = P(X \leq x) = \sum_{k=0}^x f_X(k)$$

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For example ...

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When $n = 3$,

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When $n = 25$,

$$\begin{aligned} F(12) = & \binom{25}{12} p^{12} q^{13} + \binom{25}{11} p^{11} q^{14} + \binom{25}{10} p^{10} q^{15} + \binom{25}{9} p^9 q^{16} \\ & + \binom{25}{8} p^8 q^{17} + \binom{25}{7} p^7 q^{18} + \binom{25}{6} p^6 q^{19} + \binom{25}{5} p^5 q^{20} \\ & + \binom{25}{4} p^4 q^{21} + \binom{25}{3} p^3 q^{22} + \binom{25}{2} p^2 q^{23} + \binom{25}{1} p^1 q^{24} \\ & + \binom{25}{0} p^0 q^{25} \end{aligned}$$

INTRODUCTION

Normal Approximation of the Binomial:

$$F_X(x) \approx \Phi\left(\frac{x + 0.5 - \mu}{\sigma}\right),$$

where $\mu = np$, $\sigma = \sqrt{np(1-p)}$, and Φ is the standard normal cdf.

When does this work well? ...

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Normal Approximation of the Binomial:

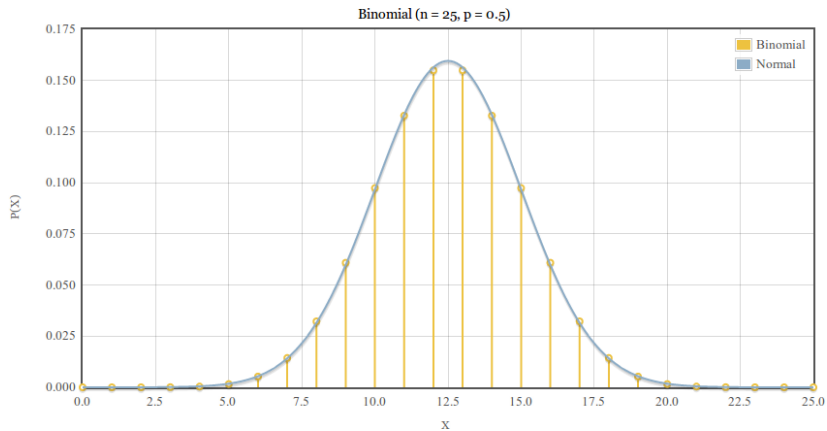
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When does this work well? ... In a nutshell, when the binomial is symmetric.

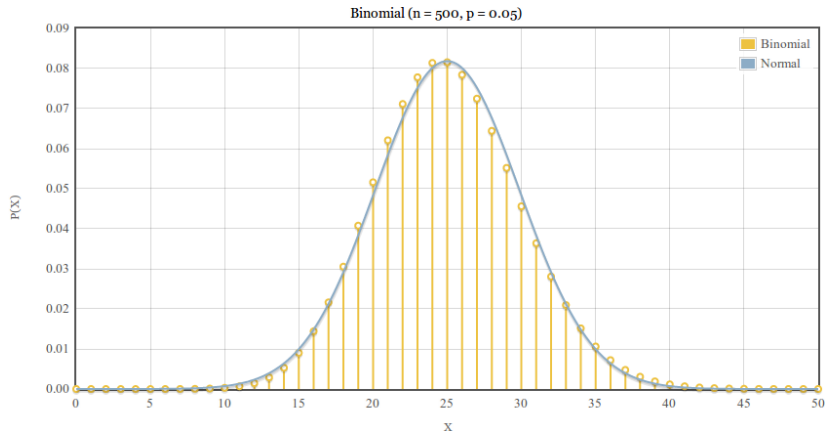
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The binomial is symmetric when $p = 0.5$



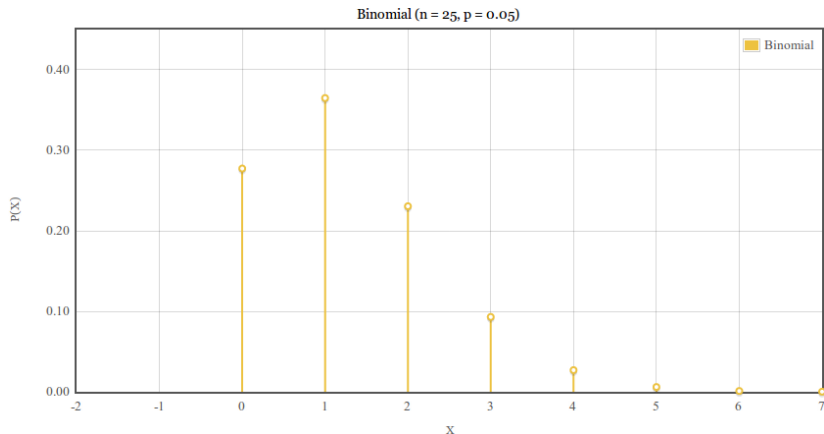
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The binomial is symmetric when $p = 0.5$ or n is very large.



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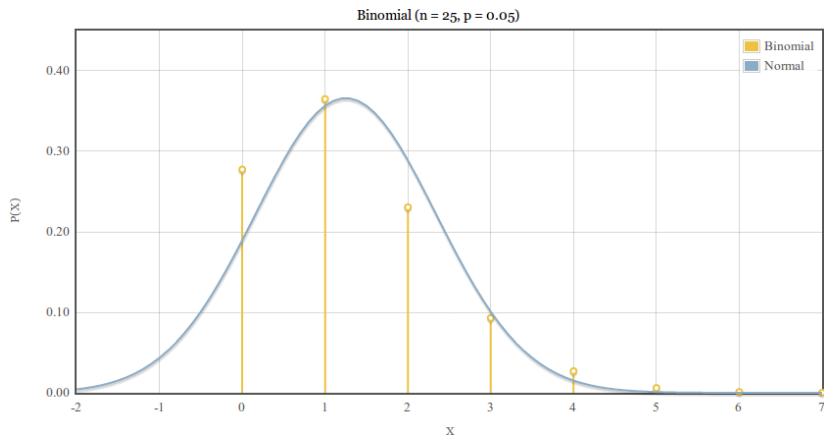
However, when n is medium and p is extreme ...



the binomial is very skewed.

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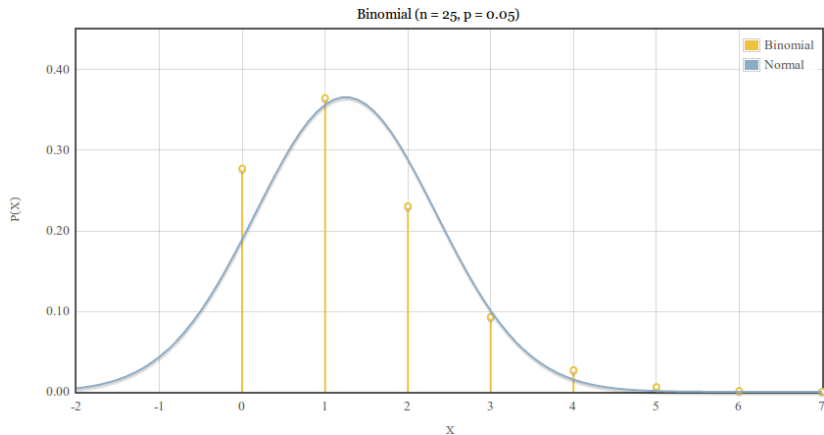
However, when n is medium and p is extreme ...



the normal approximation doesn't work very well.

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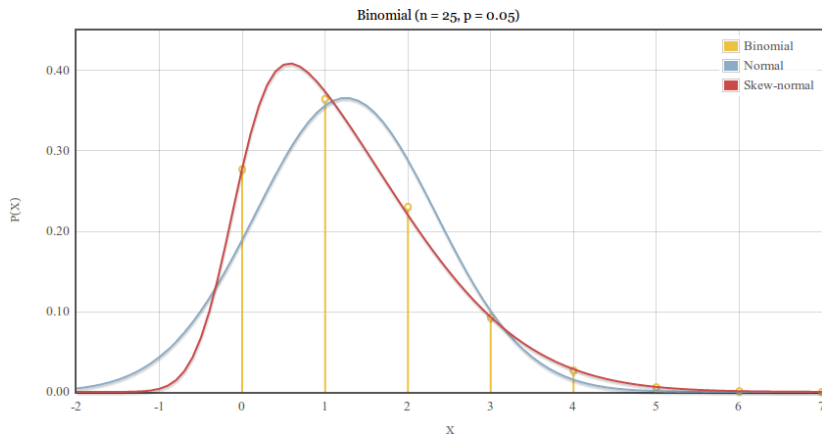
However, when n is medium and p is extreme ...



Can we do better?

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However, when n is medium and p is extreme ...



Can we do better? Introducing ... the skew-normal distribution.

OUTLINE

What we're going to cover:

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1. Skew-Normal distribution – basic properties

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2. Method of Moments – derive an approximation

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What we're going to cover:

1. Skew-Normal distribution – basic properties
2. Method of Moments – derive an approximation
3. Accuracy – compare to the normal approximation

THE SKEW-NORMAL DISTRIBUTION

DEFINITION (SKEW-NORMAL)

Let Y be a skew-normal distribution, with location parameter $\mu \in \mathbb{R}$, scale parameter $\sigma > 0$, and shape parameter $\lambda \in \mathbb{R}$. Then Y has pdf

$$f(x|\mu, \sigma, \lambda) = \frac{2}{\sigma} \cdot \phi\left(\frac{x - \mu}{\sigma}\right) \cdot \Phi\left(\frac{\lambda(x - \mu)}{\sigma}\right), \quad x \in \mathbb{R},$$

where ϕ is the standard normal pdf and Φ is the standard normal cdf.

We write $Y \sim SN(\mu, \sigma, \lambda)$.

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A few notes:

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