1 Demonstrating Improved Accuracy

We have gone to considerable trouble to develop our skew-normal approximation for the binomial; now comes the time to justify our efforts by demonstrating its improved accuracy over the regular normal one.

1.1 Visual Comparison

The first, and most obvious, way of judging accuracy is by visual inspection. Figures 1, 2, and 3 compare the binomial, normal, and skew-normal at small values of p for $n = 25$, $n = 50$, and $n = 100$, respectively. It is not hard to see that, especially at very small n and p, our skew-normal curve follows the shape of the binomial much more accurately.

1.2 Maximal Absolute Error

Another more numerical method would be to compare the maximal absolute errors of our two approximations, defined by Schader and Schmid (1989) as

$$\text{MABS}(n, p) \;=\; \max_{k \in \{0,1,\ldots,n\}} \left| F_{B(n,p)}(k) - F_{\text{appr}(n,p)}(k + 0.5) \right| \tag{1}$$

where $F_{B(n,p)}$ is the cdf of the binomial and $F_{\text{appr}(n,p)}$ is the cdf of either the normal or skew-normal approximation; the 0.5 is a continuity correction.

Figure 4 shows the MABS as a function of p, for $n = 25$ and $n = 100$. Figure 5, on the other hand, shows the MABS as a function of n, for $p = 0.05$ and $p = 0.1$. Again, the skew-normal outperforms the normal considerably in the extreme ranges, with the two approximations converging as $n \to \infty$ or $p \to 0.5$.

References

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Martin Schader and Friedrich Schmid. Two rules of thumb for the approximation of the binomial distribution by the normal distribution. *American Statistical Association*, 43(1):23–24, February 1989. MABS of the normal approximation.

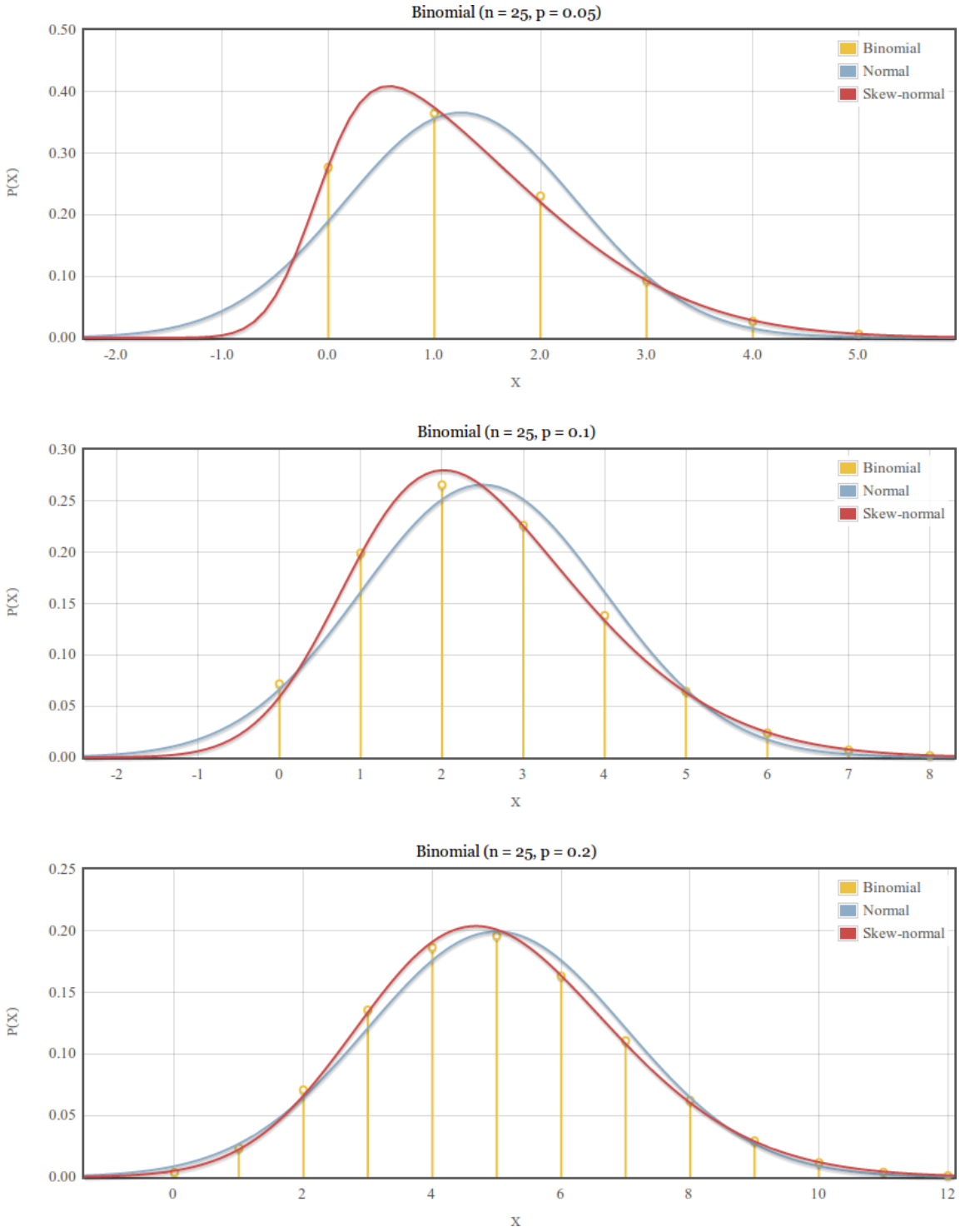

Figure 1: Binomial, normal, and skew-normal, $n = 25$

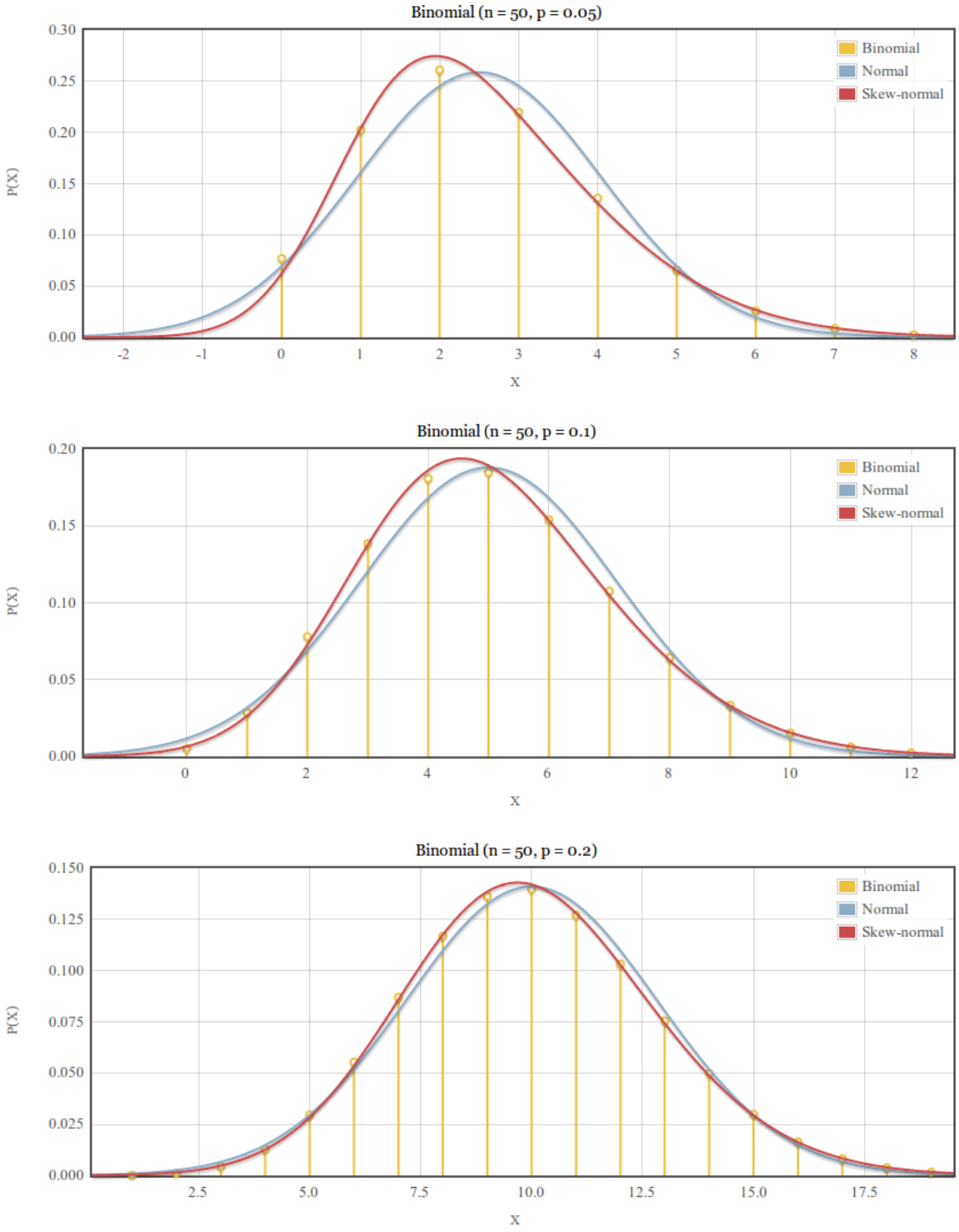

Figure 2: Binomial, normal, and skew-normal, $n = 50$

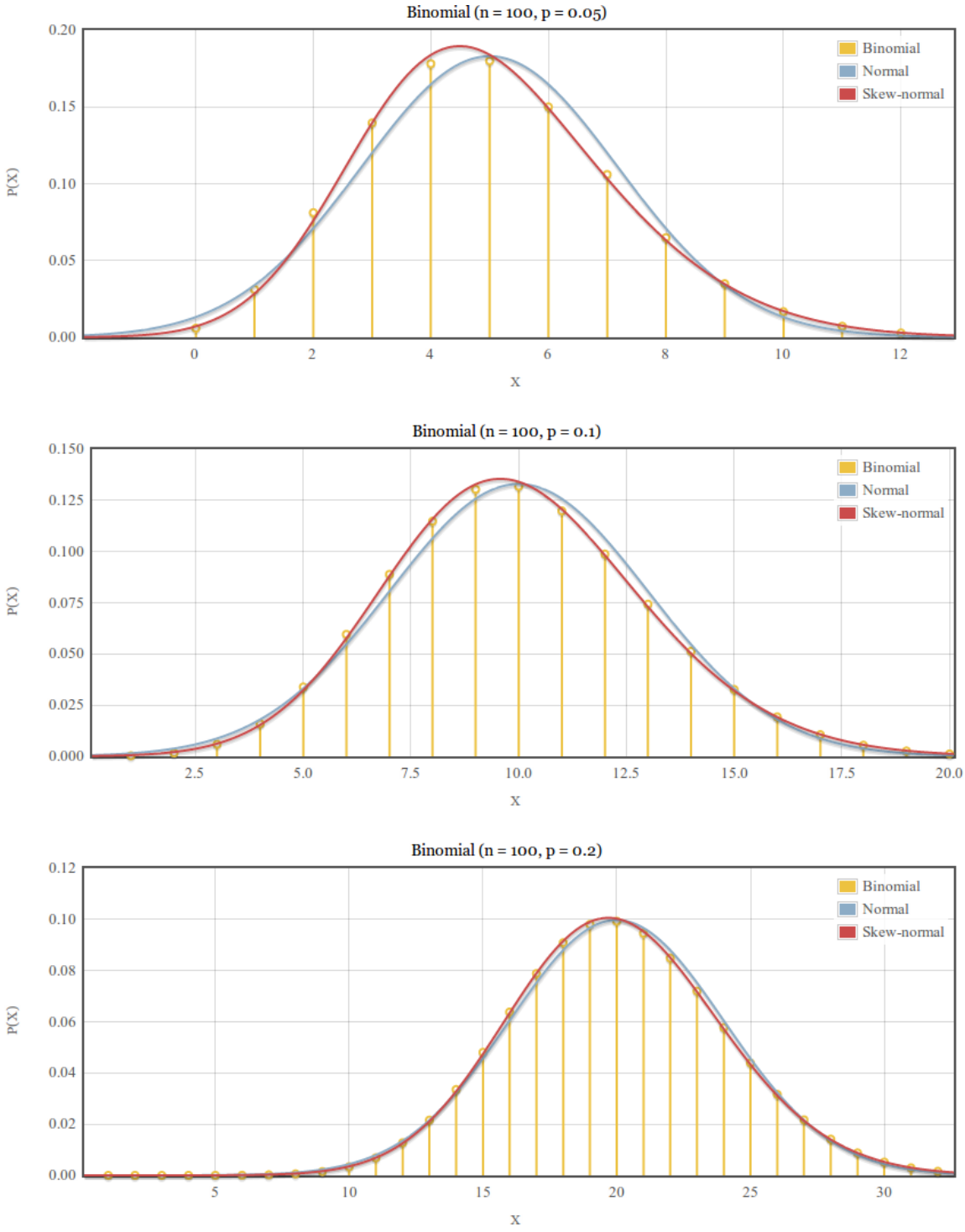

Figure 3: Binomial, normal, and skew-normal, $n = 100$

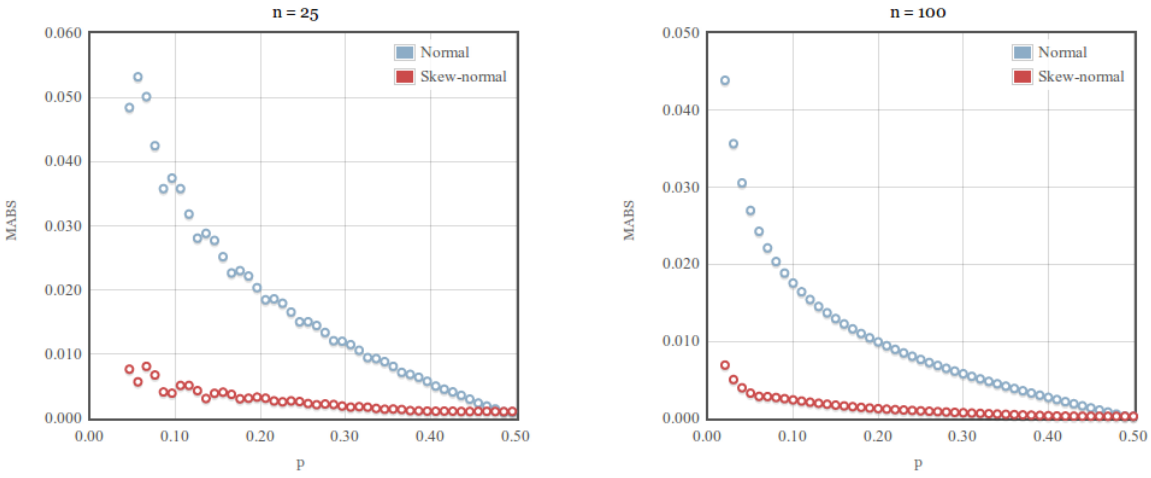

Figure 4: MABS as a function of p

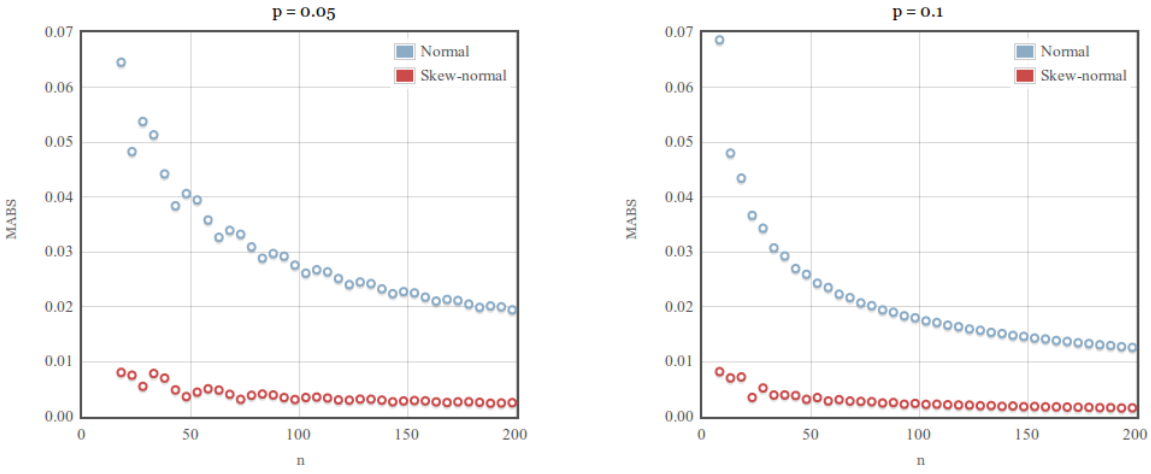

Figure 5: MABS as a function of n