Let $\mathbb{Z}$ be the standard normal distribution. Show that $\mathbb{X} = \mu + \mathbb{Z}\sigma$ has a normal distribution with mean μ and standard deviation σ.

Proof.

$$\begin{aligned} M_{\mathbb{X}}(t) &= e^{\mu t} \cdot M_{\mathbb{Z}}(\sigma t) \\ &= e^{\mu t} \cdot e^{\frac{(\sigma t)^2}{2}} \\ &= e^{\mu t + \frac{1}{2}\sigma^2 t^2} \\ &= M_{\mathbb{N}}(t) \end{aligned}$$

$\mathcal{Q}.\mathcal{E}.\mathcal{D}.$