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Beliefs, Actions, and Rationality in Strategical Decisions

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Abstract

A puzzling finding from research on strategical decision making concerns the effect that predictions have on future actions. Simply stating a prediction about an opponent changes the total probability (pooled over predictions) of a player taking a future action compared to not stating any prediction. This is called an interference effect. We first review five different findings of interference effects from past empirical work using the prisoner's dilemma game. Then we report interference effects obtained from a new experiment in which 493 participants played a six-stage centipede game against a computer agent. During the first stage of the game, the total probability following prediction for cooperation was higher than making a decision alone; during later stages, the total probability following prediction for cooperation was lower than making a decision alone. These interference effects are difficult to explain using traditional economic models, and instead these results suggest turning to a quantum cognition approach to strategic decision making. Toward this end, we develop a belief-action entanglement model that provides a good account of the empirical results.

Keywords: Rationality; Centipede game; Prisoner's dilemma game; Interference effects; Quantum cognition; Belief assessment; Strategical decisions; Disjunction effect

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1. Introduction

It seems difficult to define what is rational for strategical decisions that involve interactions among intelligent actors. The definitions eventually run directly counter to how human decision makers actually behave. For example, definitions formulated by economists, including the Nash and subgame perfect equilibrium formulations, are inconsistent with findings from experimental economics using mainstream games such as the prisoner's dilemma (PD) game and the centipede game. These findings call for new ideas about strategical decision making (Camerer, 1997), and one new approach comes from quantum computing, information, and probability theory. On the one hand, quantum physicists, anticipating the development of quantum computers, have developed quantum strategies for games, which provide new and better equilibrium solutions (Alonso-Sanz, 2019; Eisert, Wilkens, & Lewenstein, 1999; Meyer, 1999; Piotrowski & Sladkowski, 2002; Santos, 2020). On the other hand, quantum cognition researchers have used quantum theory to better describe how people actually make strategical decisions (Asano, Ohya, & Khrennikov, 2011; Denolf et al., 2016; Lambert-Mogiliansky & Busemeyer, 2012; Martínez-Martínez & Sánchez-Burillo, 2016; Pothos & Busemeyer, 2009; Tesar, 2020; Yukalov & Sornette, 2014). In this article, (a) we review puzzling empirical findings using the PD game that motivates our consideration of a quantum approach to strategical decision making, (b) we describe a new experiment using a centipede game, and (c) we extend a quantum model, called the belief-action entanglement (BAE) model, to account for the new empirical findings.

2. Five puzzling findings of the PD game

In a pair of seminal articles (Shafir & Tversky, 1992; Tversky & Shafir, 1992), Tversky and Shafir called into question a rational principle of decision making proposed by Savage (1954), called the sure-thing principle. According to this principle, if under the state of the world s, a person prefers action A over B, and if under the complementary state of the world, $\sim s$, the person also prefers action A over B, then even if the state of the world is unknown, the person should also prefer action A over B. They tested this principle using both a two-stage gambling paradigm (Tversky & Shafir, 1992) and a one-shot PD game (Shafir & Tversky, 1992). The present article is concerned primarily with strategical decisions like the PD game; see Broekaert et al. (2020) for a recent review of the two-stage gambling game.

In a PD game, there are two players (say, Bill and Angela) and each can choose to defect or cooperate. In a one-shot game, they play each other only once. The payoffs for the PD game are illustrated in Table 1. According to the Nash equilibrium, both players should defect (and both get punished with P), even though both would be better off cooperating (and receive rewards $R > P$). Of course, this is not what players normally do, and instead they frequently cooperate (Rapoport, 1988), producing better outcomes than mutual defection. Cooperation in the PD game seems to go against the Nash equilibrium, but it can be explained by additional principles such as fairness (Fehr & Schmidt, 1999), which are not necessarily considered irrational by many economic and political science researchers.

Table 1
Payoffs for Angela and Bill produced by each pair of strategies in a PD game

	Bill Defects	Bill Coop's
Angela defects	B gets P, A gets P	B gets S, A gets G
Angela Coop's	B gets G, A gets S	B gets R, A gets R
	$G > R > P > S$	

Shafir and Tversky (1992) reported findings more puzzling than cooperation behavior in the PD game, which they called a disjunction effect. Usually in a PD game, both players move simultaneously and remain uncertain about the choice of the opponent before making a decision. Shafir and Tversky (1992) included this condition, but also added two new conditions to the design: On different trials, the player was informed that the opponent already made a move to (a) cooperate in one condition, or (b) defect in another, or (c) the player was given no information. They found that the percentage of cooperation in the unknown condition (37%) was significantly greater than the percentage of cooperation when the opponent was known to defect (3%) and when the opponent was known to cooperate (16%). Shafir and Tversky (1992) claimed that this disjunction effect reflected a violation of the sure-thing principle because 25% of the participants chose to defect when the opponent defected and also chose to defect when the opponent cooperated, but then switched and chose to cooperate when the opponent's decision was unknown. Busemeyer, Matthews, and Wang (2006) later replicated these findings.

The disjunction effect is puzzling also because it violates a prediction based on the law of total probability applied to a triad. According to this law (where C_A symbolizes A cooperates, C_B symbolizes B cooperates, D_B symbolizes B defects)

$$\begin{aligned} p((C_B \cup D_B) \cap C_A) &= p(C_B \cap C_A) + p(D_B \cap C_A) \\ &= p(C_B) \cdot p(C_A|C_B) + p(D_B) \cdot p(C_A|D_B). \end{aligned} \tag{1}$$

If it is assumed that the event C_A^* for the unknown condition is the same as the event $(C_B \cup D_B) \cap C_A$ from the known conditions, then we should observe that $p(C_A^*)$ lies in between the two conditional probabilities, $p(C_A|C_B) > p(C_A|D_B)$. The disjunction effect, however, violates this prediction based on the law of total probability. Shafir and Tversky (1992) explained the results by arguing that the advantages of defection are clear when the opponent's action is known, but these advantages become unclear when the opponent's action is unknown. Somehow the two thoughts about the opponent's action (to defect or cooperate) interfere with each other, to produce no clear thought when the opponent's action is unknown.

Croson (1999) conducted a different but related study of the disjunction effect using a traditional simultaneous PD game. Instead of informing a player of an opponent's action, one group of participants were asked to predict the opponent's move before making their own decision, and a second group made a decision without expressing any prediction. This experiment provided another type of test of the law of total probability. The total probability that Angela cooperates (pooled across predictions about Bill) is expected to be equal to the probability that Angela cooperates without predictions. Once again a disjunction effect occurred, violating the prediction from the law of total probability: In a first experiment, 77.5% of the participants in the no-prediction group chose to cooperate, but this dropped to 55% for the group with prediction; in a second experiment, 62.5% cooperated in the no-prediction group, which dropped to 42.5% in the prediction group. Simply asking participants to report their prediction significantly reduced the overall percentage of cooperation by 20% in the simultaneous PD game. Croson (2000) replicated these findings.

Tesar (2020) replicated and extended the work by Shafir and Tversky (1992) and Croson (1999). This new experiment included (a) a manipulation, such as Shafir and Tversky (1992), that did or did not provide the information about the other opponent's (e.g., Bill's) move before the player (e.g., Angela) made her decision; (b) a second manipulation, such as Croson (1999), that did or did not request a guess about the opponent (e.g., Bill) before the player (e.g., Angela) made her decision; and (c) a third manipulation that changed the order of guessing and decision. Tesar (2020) replicated both findings by Shafir and Tversky (1992) and Croson (1999), and also found that there was an order effect: The percentage of cooperation was 65% when the player (e.g., Angela) made a decision before guessing, but this percentage was significantly reduced to 42% when the player (e.g., Angela) guessed the opponent (Bill) first (pooled across guesses).

Blanco et al. (2014) examined the effects of prediction on a sequential PD game in which one player moves first and the second player responds to the first player's move. The experiment included three groups of participants: In the prediction group, each player was asked to predict how many players would cooperate as second movers; in the feedback group, each player was told the number of others who cooperated as second movers; in the baseline group, no prediction was made nor any feedback provided. The results for the first move indicated an interference effect of prediction on action: The percentage of cooperation in the first move was 55% for the prediction group, which was significantly greater than the 28% for the baseline group; the feedback group produced a percentage of cooperation 57% similar to the prediction group. In sum, once again, simply asking for a prediction changes the total probability of an action.

Kvam et al. (2014) examined a different type of measurement effect using a version of the PD game that allowed for "cheap talk." Each player played several repetitions with computer agents and each repetition involve a sequence of four stages. During Stage 1, they were asked to choose whether they were willing to promise agent A that they would cooperate in a future PD game with agent A; during Stage 2, they played a standard PD game with a different agent B; during Stage 3, they played a standard PD game against agent A that they faced in the first stage; and during Stage 4, they played several games with other agents without any promises. Half of the computer agents had a reputation for cooperation, and half had a reputation for defection. The key comparison was the within-participant rate of cooperation to agent B in the second stage (immediately following a promise) compared to the fourth stage (which did not include promises). Kvam et al. (2014) found that there was little difference between cooperation rates when playing an agent with a defecting reputation: In both cases the rate was very low at 16%. However, for agents with a cooperative reputation, the rate of cooperation was significantly higher in Stage 2 (47%) than in Stage 4 (40%). Here we see

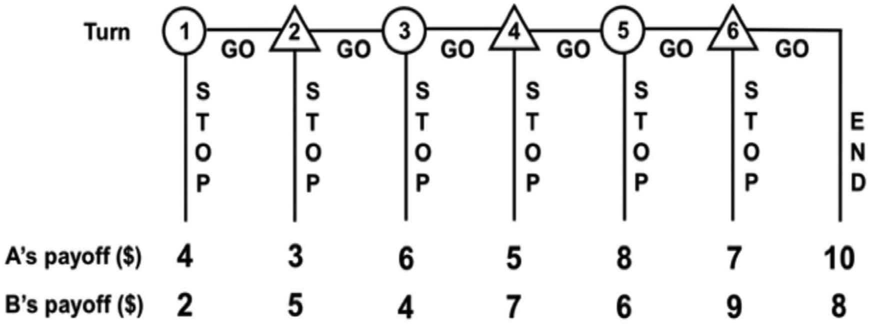

Fig. 1. Six-stage centipede game with linear increasing payoffs.

that making a promise (predicting one's own behavior) on a preceding game carries over to change the probability of cooperation in what is supposed to be an independent game.

In general, we define an interference effect as the difference between (a) the total probability of an action (e.g., cooperate), pooled across all possible predictions, and (b) the probability of the same action (e.g., cooperate) when the action was made directly. Besides the PD game, interference effects have also been reported using gambling tasks (Broekaert et al., 2020; Tversky & Shafir, 1992) and categorization-decision tasks (Wang & Busemeyer, 2016).

3. A new centipede game experiment

So far, our review of interference effects in strategical games has been limited to experiments using the PD game. A broader review of the effect that belief assessment has on strategical game playing indicates that these effects depend on the type of strategic game (Schotter & Trevino, 2014). The centipede game is a social dilemma game similar to the PD game, but it involves a sequence of stages rather than a single stage. Beliefs about an opponent's behavior in future stages of the game play a critical role in the centipede game (Krockow et al., 2016). However, the effect of belief measurement on later actions has never been investigated using the centipede game. The purpose of the new experiment was to test for interference effects in the centipede game. The centipede game provides a new dynamic examination of interference effects across the sequential stages of the game.

The centipede game (Rosenthal, 1981) has been extensively studied in the experimental economics literature (Krockow et al., 2016). The game in an extensive form is shown in Fig. 1: Two players, Angela and Bill, decide sequentially whether to allocate a pot of rewards in a predetermined way or whether to pass the allocation choice to the other player. Fig. 1 illustrates player A (Angela) going first and B (Bill) going next. If Angela continues and passes the decision to Bill, the total pot to allocate increases in size; and if Bill continues passing, then the pot grows even more. Passing can be done only a finite number of times (6 in this example). Once a player stops and splits the pot, the game is over with that player gaining the higher share of the pot and the other player obtaining the smaller share. All standard game-theoretic equilibrium concepts predict that the play stops and the pot is split at the first decision node of player A (see, e.g., Aumann 1998).

3.1. Previous findings for the centipede game

The subgame perfect and Nash equilibrium for the centipede game is to defect on the very first decision stage (Aumann, 1998). Empirically, players most frequently play for several stages, starting from a relatively high rate of cooperation during the first stage, and decreasing across stages, but some players even continue to play during the last stage (Krockow et al., 2016). The tendency to cooperate increases with the number of stages of the game, decreases with large compared to small incentives, decreases with experience and learning, increases with geometric compared to linear increases in the pot, decreases with group compared to individual decisions, and increases with symmetric compared to asymmetric payoffs (Krockow et al., 2016).

3.2. New experiment

In the present study, $N = 493$ (43% males, 54% females, remainder unknown), participants were recruited from undergraduate students at a large Midwestern U.S. university. They played the game shown in Fig. 1 against computer agents programmed to behave like a human. The experiment consisted of two types of games: Half were predict-act (P-then-A) games in which the human expressed a prediction about the agent's action on the next round before taking an action; and half were act-only (A-only) games in which the human simply chose an action without expressing any prediction about the agent. These types of games were presented in different blocks of trials, with blocks randomized in order across participants. Each participant played 15 games with the human player going first (position A), and 15 games with the computer agent going first (position B). More details about the methods and procedures are presented in the Supporting Information.

3.3. Results of new experiment

Table 2 shows the joint relative frequency of predictions about the agent by the human player and actions taken by the human player for the first stage (when the human goes first) and the second stage (when the agent goes first). Most frequently, the human player predicts continuation and then decides to continue. However, there is an asymmetry when the predictions and action disagree: The player predicts that the agent will continue and then decides to stop more often than the player predicts the agent will stop and then decides to continue.

Table 3 shows the statistics obtained from the predictions by the humans about the agent and the actions by the human for the first stage when the human goes first, and for the second stage when the agent goes first. As can be seen in the column under the label Int, there is a negative interference effect indicating a higher probability to defect and stop when no

Table 2
Joint relative frequency of predictions about agent and actions by human player for the first stage (when human player goes first) and the second stage (when computer agent goes first)

Human Player Goes First		
	Act Stop	Act Continue
Predict stop	0.1389	0.0522
Predict continue	0.1141	0.6947
Computer Agent Goes First		
	Act Stop	Act Continue
Predict stop	0.1756	0.0635
Predict continue	0.1174	0.6436

Table 3
First-stage (when human goes first) and second-stage (when agent goes first) predictions about agent and actions by human player

$p(S_A)$	$p(S_H \mid S_A)$	$p(C_A)$	$p(S_H \mid C_A)$	TP	$p(S_H)$	LL	Int	UL
Human player goes first								
0.1912	0.7268	0.8088	0.1411	0.2530	0.2848	−0.01	−0.03	−0.06
Computer agent goes first								
0.2391	0.7345	0.7609	0.1542	0.2930	0.3234	−0.01	−0.03	−0.05

Note: $p(S_A)$, proportion predicted agent will stop; $p(C_A)$, proportion predicted agent will continue; $p(S_H|S_A)$, proportion human player stops given agent predicted to stop; $p(S_H|S_C)$, proportion human player stops given agent predicted to continue; TP, total probability to stop after a prediction; $p(S_H)$, proportion human stops without prediction; $Int = TP - p(S_H)$; LL, lower limit of 95% confidence interval; UL, upper limit of 95% confidence interval. The confidence interval was computed using the interference effect scores computed for the 493 participants.

prediction is made compared to when a prediction is made beforehand. These first-stage results are in the same direction as those reported by Blanco et al. (2014).

Table 4 shows the distribution that the human player stops across the six stages of the game separately for the predict-act condition and the act-alone condition. The columns add to 1.00. While the human players stopped on some early games, they most frequently stopped at Stage 4 (when going second) and Stage 5 (when going first). The largest difference (10%) between P-A versus A-alone occurred at stop 3 when the human played first. The distributions for human players to stop differed between the P-then-A and A-alone conditions, and a chi-square test of distribution difference indicated that the difference is significant: $\chi^2 = 111.29$, $df = 3$, $p < .001$ when the player went first, and $\chi^2 = 28.41$, $df = 3$, $p < .001$ when the computer agent went first.

Table 5 shows the proportion of predictions and actions conditioned on reaching a stage. The rows labeled with D under the column "Source" display the observed proportions (the rows labeled with M are modeling results discussed later). The proportion to predict continue,

Table 4
Marginal stopping distributions for actions following predictions, and for actions to stop without predictions

Stage	Action After Prediction	Action Without Prediction
Human player goes first		
Stop 1	0.2530	0.2848
Stop 3	0.2503	0.1527
Stop 5	0.3519	0.3868
Continue 5	0.1448	0.1756
Computer agent goes first		
Stop 2	0.2930	0.3234
Stop 4	0.4228	0.4002
Stop 6	0.2475	0.2194
Continue 6	0.0368	0.0571

Table 5
Observed proportions for the prediction to continue and observed proportion for the action to continue across stages for conditions when human went first and when agent went first

Stage	Source	Predict	Act_P	Act_W
Human Went First				
1	D	0.81	0.75	0.72
1	M	0.80	0.77	0.76
3	D	0.72	0.76	0.79
3	M	0.74	0.72	0.74
5	D	0.36	0.41	0.44
5	M	0.34	0.48	0.49
Agent Went First				
2	D	0.76	0.71	0.68
2	M	0.71	0.71	0.70
4	D	0.50	0.56	0.58
4	M	0.60	0.60	0.61
6	D	0.43	0.34	0.35
6	M	0.31	0.37	0.36

Note: D, observed data; M, quantum model results. Act_P refers to action following prediction and Act_W refers to action without prediction.

conditioned on reaching each stage, is shown under the column "Predict"; the proportion for the human player to take the action to continue, in the P-then-A condition and following a prediction, is shown under column Act_P; and the proportion for the human player to take the action to continue, in the A-alone condition (without any prediction), is shown under column Act_W.

As can be seen in the table, the proportion of predictions to continue, conditioned on reaching a stage of the game, decreased across the stages. Also, as seen in the table, the proportion of games that the human player decided to continue, conditioned on reaching a stage, also decreased across stages. Finally, there is an interference effect, $Int = Act_W - Act_P$, between the column for $\mathrm{Act_P}$ (from the P-then-A condition) and the column for the $\mathrm{Act_W}$(from the A-alone condition). The P-then-A condition produced higher continuation (lower stopping rates) than A-alone at the early stages ($t(492) = -2.59$, $p = .01$, Stage 1; $t(492) = -2.51$, $p = .0125$, Stage 2), but A-alone condition produced higher continuation (lower stopping rates) at the later stages ($t(492) = +3.04$, $p = .0025$, Stages 3 and 5; $t(492) = +0.28$, $p = .78$, Stages 4 and 6).

As we mentioned earlier, the first-stage results of this experiment are in the same direction as those found by Blanco et al. (2014), but at the later stage the results reversed and are in the same direction as those found by Croson (1999). The interference effects are smaller for this centipede game compared to the previous results obtained with the PD game at the first stage. This may be produced by the difference between the games (single stage vs. six stage). Alternatively, it may be produced by differences in procedures, including the use of a within-subject design and the use of a large number of games for each person.

4. Models of the centipede game

4.1. Previous economic models

McKelvey and Palfrey (1992) initially proposed a model based on the assumption that a small proportion of players in the game are altruisitic. However, a later study by Fey, McKelvey, and Palfrey (1996), using a constant sum game in which there is no social benefit to cooperation, also showed sizable cooperation, and so they proposed an alternative model based on the assumption that players make mistakes. Based on findings showing learning and sequential effects, Rapoport et al. (2003) proposed a learning model that adjusted the probability to play based on whether a defection occurred on the first trial during the previous game. Finally, Kawagoe and Takizawa (2012) proposed a *k*-level theory in which a player makes a best response to an opponent that plans *k*-steps ahead. All players range from L_1 to L_4. The L_1 player assumes the opponent is an L_0 (random) player, and makes the best response to this player (with some error). An L_k player assumes the opponent is an L_{k-1} player and makes the best response (with some error). The distribution of L_k types is estimated from the data. However, these models do not provide any account of the relations between predictions and actions and their interference, which is the main issue in the present experiment. Quantum cognition models are designed to account for these relations and interactions. We eventually propose a quantum model that incorporates some of the basic ideas in the works by Kawagoe and Takizawa (2012) and Rapoport et al. (2003).

4.2. Belief-action entanglement model

There are now at least seven different quantum cognition models that have been developed for the PD game (Asano, Ohya, & Khrennikov, 2011; Denolf et al., 2016; Lambert-

Mogiliansky & Busemeyer, 2012; Martínez-Martínez & Sánchez-Burillo, 2016; Pothos & Busemeyer, 2009; Tesar, 2020; Yukalov & Sornette, 2014). However, none have been extended for application to dynamic games, such as the centipede game. The previous BAE (Pothos & Busemeyer, 2009) PD model is more readily extended to the centipede game than the other six models. The BAE model can be viewed as a quantum version of a player that makes best responses to her predictions about the opponents move with error prone responses.

Suppose the player goes first and the agent goes second (a parallel model is developed for the case when the computer agent goes first). At each stage, the player's (e.g., Angela's) cognitive state is assumed to be a unit length vector formed by a superposition $\psi = [\psi_{SS} \quad \psi_{SC} \quad \psi_{CS} \quad \psi_{CC}]^\dagger$ over a set of four basis cognitive states $S = \{|SS\rangle, |SC\rangle, |CS\rangle|CC\rangle\}$, where, for example, $|CS\rangle$ represents the state in which Angela predicts that the computer agent will continue but Angela decides to stop. Her predictions and actions for each stage are based on this state, which is updated after each stage of the centipede game. The projector for predicting the computer agent continues equals $P_C = diag[0 \quad 0 \quad 1 \quad 1]$, and to stop equals $P_S = diag[1 \quad 1 \quad 0 \quad 0]$. The projector for Angela deciding to take the action to continue equals $A_C = diag[0 \quad 1 \quad 0 \quad 1]$, and to stop equals $A_S = diag[1 \quad 0 \quad 1 \quad 0]$.

4.2.1. Stage 1

First consider the P-then-A condition. At the beginning of Stage 1, Angela is in a superposition state $\psi_1 = \frac{1}{\sqrt{2}}[b_S(2) \quad b_S(2) \quad b_C(2) \quad b_C(2)]$. In this state, she has some probability $p_1(C_{pred}) = \|P_C \cdot \psi_1\|^2 = |b_C(2)|^2$ to predict that the agent will continue at Stage 2, and a probability $1 - |b_C(2)|^2$ to predict that the agent will stop. If she predicts that the agent will continue, then she updates to the conditional state $\psi_{1C} = \frac{P_C \cdot \psi_1}{\sqrt{p_1(C_{pred})}} = [0 \quad 0 \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}]^\dagger$, and if she predicts that the agent will stop, then she updates to the conditional state $\psi_{1S} = [\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad 0 \quad 0]^\dagger$. Before taking an action, Angela evaluates the payoffs at Stage 1 for stopping and continuing using a unitary transformation, denoted as U_1, of the current state $\psi_{1j}, j = C, S$ to generate an evaluation state $(U_1 \cdot \psi_{1j})$. (We describe the unitary matrices, U_k for stage k, later after describing all the stages of the game.) The probability that Angela chooses the action to continue, conditioned on the current state ψ_{1j}, then equals $p_1(C_{act}|j) = \|A_C \cdot U_1 \cdot \psi_{1j}\|^2$, and the probability to stop equals $1 - p_1(C_{act}|j)$, for $j = C, S$. Next consider the A-alone condition. Starting from the same superposition state ψ_1, Angela evaluates the payoffs for stage 1 using the same unitary transformation U_1. Then the probability that Angela chooses the action to continue equals $p_1(C_{act}) = \|A_C \cdot U_1 \cdot \psi_1\|^2$ and the probability to stop equals $p_1(S_{act}) = 1 - p_1(C_{act})$.

4.2.2. Stage 3

For the P-then-A condition, at the beginning of Stage 3, Angela is transformed back to a superposition state $\psi_3 = \frac{1}{\sqrt{2}}[b_S(4) \quad b_S(4) \quad b_C(4) \quad b_C(4)]$.[1] In this state, she has some probability $p_3(C_{pred}) = |b_C(4)|^2$ to predict that the agent will continue at stage 4, and a probability $|b_S(4)|^2 = 1 - |b_C(4)|^2$ to predict that the agent will stop. If she predicts that the agent will take action $j \in \{C, S\}$, then she updates to the conditional state $\psi_{3j} = \frac{1}{\sqrt{2}}[\delta_{jS} \quad \delta_{jS} \quad \delta_{jC} \quad \delta_{jC}]^{\dagger}$, where $\delta_{ij} = 1$ if $i = j$, and zero otherwise. Before taking an action, Angela evaluates Stage 3 payoffs using a unitary transformation, U_3, to generate an evaluation state $(U_3 \cdot \psi_{3j})$. The probability that Angela chooses the action to continue then equals $p_3(C_{act}|j) = \|A_C \cdot U_3 \cdot \psi_{3j}\|^2$, and the probability to stop equals $p_3(S_{act}|j) = 1 - p_3(C_{act}|j)$, for $j = C, S$. For the A-alone condition, at the beginning of Stage 3, Angela is in the superposition state ψ_3, and evaluates the payoffs using the same unitary transformation U_3. The probability that Angela chooses the action to continue equals $p_3(C_{act}) = \|A_C \cdot U_3 \cdot \psi_3\|^2$ and the probability to stop equals $p_3(S_{act}) = 1 - p_3(C_{act})$.

4.2.3. Stage 5

This stage follows the same rules as Stage 3 using a superposition state ψ_5 and a unitary transformation U_5.

4.2.4. Beliefs

Similar to Rapoport et al. (2003), we use a learning model to change beliefs across games based on previous actions by the computer agent. This learning process allows beliefs to vary across participants depending on their personal experience with the game. Consider the condition when the human player goes first, and define the initial vector $B(0) = [1 \quad 1 \quad \beta \quad 1]/(3 + \beta)$, which is used on the first game. The parameter β is used to represent the near end game effect regarding predictions for stage 6. This vector is used to define $b_S(2 \cdot j) = \sqrt{B_j(0)}$, for $j = 1, 2, 3$, on the first game. After the first game, the vector is updated according to the learning rule (for $t = 1, \ldots, 30$ games by a participant):

$$B'(t) = \delta(t) \cdot ((1 - \alpha) \cdot B(t - 1) + \alpha \cdot T(t)) + (1 - \delta(t)) \cdot B(t),$$

$$B(t) = \frac{B'(t)}{\sum_{j=1}^{4} B'(t)},$$

where $\delta(t) = 1$ if the agent stopped at some stage during the game, and zero otherwise, and $T(t)$ is a vector with all zeros except a one located at a stage (2,4,6) that the computer agent stopped. Then for game t, we set $b_S(2 \cdot j) = \sqrt{B_j(t)}$, for $j = 1, 2, 3$. Then $b_C(2 \cdot j) = \sqrt{1 - B_j(t)}$.

4.2.5. Evaluations

Unitary matrices are used to represent a probabilistic best response to the predicted move by the computer agent. Recall that U_j is the unitary transformation used to evaluate the payoffs for stage $j = 1, 3, 5$ when the human player goes first. The Supporting Information describes the details for constructing the unitary matrices, and here we briefly summarize the main ideas. Each unitary matrix is constructed from a matrix exponential of a Hermitian matrix H_j. To build this matrix, we used exactly the same assumptions as used in the previous work (Busemeyer, Wang, & Shiffrin, 2015; Pothos & Busemeyer, 2009; Wang & Busemeyer, 2016). Each Hermitian matrix $H_j = F_j + G_j$ is built from the sum two matrices: The matrix F_j is designed to rotate the action in the direction appropriate for a prediction. If the agent is predicted to stop, then F_j rotates the action toward stop; if the agent is predicted to continue, then F_j rotates the action toward continue. The matrix G_j is designed to *entangle* beliefs and actions in such a way as to coordinate beliefs with actions. If Angela decides to stop, then her beliefs rotate toward believing that the agent will stop, too; if Angela decides to continue, then her beliefs rotate toward believing that the agent will continue, too. Psychologically, this is related to idea of cognitive consistency to reduce dissonance (Festinger, 1957). It is also related to the idea of social projection, that is, the idea that naïve observers often believe that others will behave as they do (Krueger, DiDonato, & Freestone, 2012). The matrix G_j is *essential* for producing the interference between predictions and actions.

Altogether the BAE model requires estimating four free parameters from the data: an initial belief parameter β, a learning rate parameter α, a payoff rate parameter λ used in F_j, and an entanglement parameter γ used in G_j. The latter parameter is a critical parameter for the model, which is used to capture the interference effects. If it is set to zero, then no interference effects can occur. The individual data and code used to fit the individual data are located at https://osf.io/9hupw/.

4.2.6. Application of BAE to centipede experiment

We estimated the parameters and evaluated the accuracy of the model in two ways. The aggregate method was to estimate the same fixed parameters across all participants, but note that the beliefs $b_S(j)$ varied across individuals because of the learning process that depended on each person's unique experience during the 30 games. For the second way, we allow for individual differences in the key entanglement parameter by estimating γ separately for each person, and we kept the remaining parameters fixed across participants (again, the beliefs in $b_S(j)$ varied across individuals because of learning).

Using the aggregate method, we compared the four-parameter model with $\{\alpha, \beta, \gamma, \lambda\}$ all free to a three-parameter model with $\gamma = 0$ using a *BIC* method. The $G^2 = -2 \cdot$ log-likelihood was computed for each model, producing $G^2(4) = 4.4661 \times 10^4$ for the four-parameter model, and $G^2(3) = 4.5180 \times 10^4$ for the three-parameter model. The *BIC* penalty for one extra parameter is $\ln(N) = \ln(493 \cdot 30) = 9.6$, and so the *BIC* difference equals $BIC(3) - BIC(4) = 519$ favoring the four-parameter model over the three-parameter model. This provides evidence for the contribution of γ to improve the accuracy of the model predictions. The best-fit parameters are $\alpha = 0.12$, $\beta = 1.85$,$\lambda = 1.367$, and $\gamma = 0.4258$.

When using the method allowing for individual differences in γ, we allowed the parameter γ to change sign from the first stage to the last two stages to allow more flexibility across stages. The predictions produced by averaging across the BAE predictions for each person are shown in the rows labeled M in Table 5. As can be seen, the model provides a reasonably accurate account of the pattern of results. The model reproduces the direction of the interference effects; however, it underestimates the size of the interference. The largest model errors occur under the Pred column in the right panel of Table 5 (when the computer agent went first) during Stages 4 and 6. This could reflect some problems with the learning model when the computer agent goes first.

A histogram of the distribution of γ estimated from the 493 participants is shown in Fig. 2. The mean of this distribution equals 0.52 with a 95% confidence interval [0.41, 0.63] for the

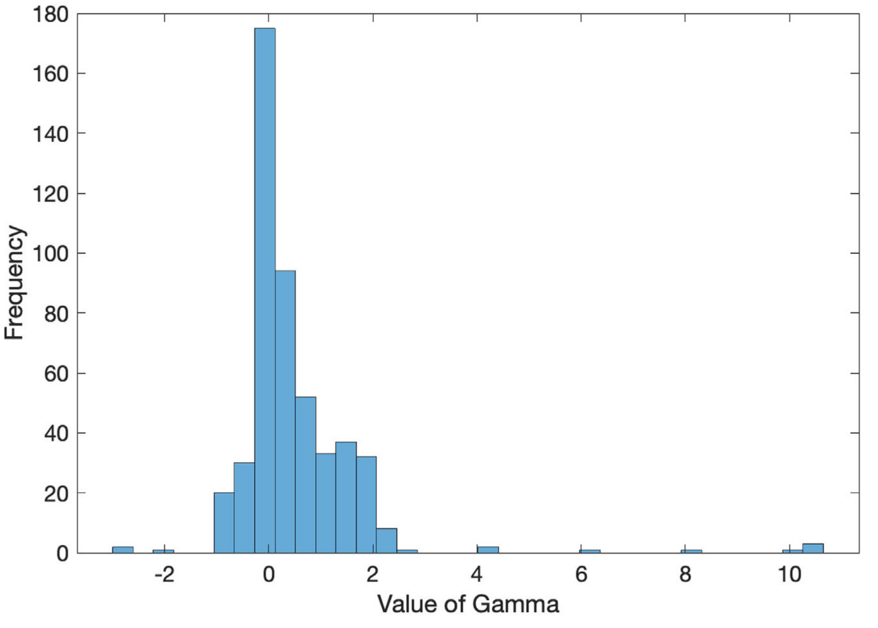

Fig. 2. Frequency distribution across 493 participants for values of γ.

mean; the median of γ equals 0.16, which is significantly different from zero according to a Wilcoxon signed-rank test ($z = 10.57$, $p \approx 0$). A total of 64% of the participants had $\gamma > 0.05$. These results indicate that the key entanglement γ parameter made a useful contribution to the predictions from the model.

5. Discussion

A growing body of empirical evidence demonstrates interference effects in strategical decision making, which refers to a violation of a prediction derived from the law of total probability. It is expected that the *total probability* of a player taking an action, following a request for an explicit prediction about the opponent's action, should equal the probability of the player taking the same action when no prediction is requested. On the contrary, prior research using the PD game has demonstrated systematic interference effects that violate this expectation. The present experiment is the first to demonstrate that interference effects also occur in a centipede game, which is a more dynamic game consisting of a sequence of stages (six in our case).

Quantum models of cognition provide a natural account of interference effects. In fact, these effects are a psychological analogue of the interference effects observed in Mach–Zehnder interferometer experiments that inspired quantum physics. Seven quantum models have been developed in the past work to account for these interference effects in a PD game. However, extending these models to the more complex centipede game remains a challenge.

We identified one model, the BAE model, as the most promising of the seven for extension to the centipede game. The BAE model was reasonably successful to capture the complex pattern of predictions and actions across stages, and the changes of interference effects across stages. We examined the distribution of individual estimates of the key entanglement parameter of the BAE model, which is needed to produce interference effects. We found that the distribution was significantly different from zero, indicating that this parameter makes a significant contribution to the predictive accuracy of the model. The BAE model provides one of many ways that quantum models can be used to account for interference effects in strategical games. In particular, the BAE model assumed a compatibility relation between predictions and actions, and interference was produced by a unitary transformation that entangled beliefs with actions. An alternative way to model interference is to assume incompatibility between prediction and action measurements (e.g., Lambert-Mogiliansky & Busemeyer, 2012). These alternatives can be compared to the BAE model more systematically in future work.

Shafir and Tversky (1992) argued that the disjunction effect results from a failure of consequential reasoning, which seems to indicate a flaw in human rational thinking. Quantum cognition models provide a different view concerning the rationality of the disjunction effect in particular, and interference effects in general. Quantum probability theory is an axiomatic theory, however it is based on different axioms than classical (Bayesian) probability theory (Gudder, 1988). Classical probability theory represents events as subsets of a sample space, whereas quantum theory represents events as subspaces of a vector space (each subspace corresponds to a projector). The algebra of subsets of a sample space is Boolean, which satisfies the axiom of distributivity, and the latter is responsible for the law of total probability. The algebra of subspaces of a vector space is more general than Boolean because it allows but does not require the distributive axiom (Hughes, 1989). Therefore, the single event "Angela chooses to cooperate" is not necessarily the same as the disjunctive event "Bill is predicted to cooperate and then Angela chooses to cooperate, or Bill is predicted to defect and then Angela chooses to cooperate." In this way, quantum probability can be considered to use a quantum logic that is a relaxation of an overly restrictive Boolean logic to model human reasoning and decision making.

Quantum models certainly do not exhaust all the possible explanations for interference effects. One could try using resource rationality and probabilistic language of thought principles to understand these effects by interpreting them as some type of commitment effect and use a "conditioned Bayesian observer" model to account for these effects (Qiu, Luu, & Stocker, 2020). Alternatively, one could try using the adaptive toolbox heuristic approach to construct an ecologically rational heuristic that strategically connects predictions with actions (Spiliopoulos & Hertwig, 2020). However, we can only speculate at this point because these approaches have not yet been applied to the interference effects reported here. The main advantage of the BAE model is that it provides not only a principled and coherent theoretical account of interference effects found with the centipede game but also for findings from the PD game (Pothos & Busemeyer, 2009), the two-stage gambling game (Broekaert et al., 2020), and the categorization-decision task (Wang & Busemeyer, 2016). Future work will examine applications to other types of dynamic games.

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Note

1. The reduced belief and action state after taking an action on the first stage is a tensor product, $\phi_B \otimes \phi_A$, which can be transformed by a tensor product of unitary transformations to produce a desired initial state for Stage 3, $\psi = (U_B \otimes U_A) \cdot (\phi_B \otimes \phi_A) = (U_B \cdot \phi_B) \otimes (U_A \cdot \phi_A)$.

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