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Data Visualization in ProB

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Erklärung

Hiermit versichere ich, dass ich diese Bachelorarbeit selbstständig verfasst habe. Ich habe dazu keine anderen als die angegebenen Quellen und Hilfsmittel verwendet.

Düsseldorf, den 14. Juni 2013

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Abstract

The ProB tool performs the animation and verification of models written in several different formal specification languages. Here we propose a method for the generation of dynamic visualizations to help users better understand the results that are produced from ProB. In order to do this, we used the D3 JavaScript library to generate visualizations based on the data produced by ProB. In this work, we will present three new visualizations that are available for the user. We will also describe the new visualization framework that has been implemented to allow the user to interact with the visualizations and to allow the visualizations to be updated when changes in ProB take place. In the future, it should be relatively easy for developers to extend the existing framework to make new visualizations.

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1 Introduction

ProB is the name given to a group of software tools dealing with the verification of models written in formal specification languages. The main specification language that ProB deals with is the B-Method. The B-Method is a method of specifying and designing software systems that was originally created by J.R. Abrial [1]. Central to the B-Method are the concepts of *abstract machines* that specify how a system should function [2]. However, ProB also provides support for the Event-B, CSP-M, TLA+, and Z specification languages.

In order to verify a formal model, ProB simulates the state space that corresponds to that model. A *state space* is a labeled transition system that is represented as a directed multigraph. The vertices in the graph represent every possible state in the system. The edges in the graph represent all of the transitions between any given states. ProB verifies a model primarily by performing *consistency checking*. This is the systematic check of all states that are accessible from the initial state. A model is found to be correct if its state space contains no deadlocks (i.e. all vertices in the graph have at least one outgoing edge) and if no state violates the *invariant* for the model. The *invariant* is a predicate that must evaluate to true for every state in the model. The user can also use ProB to perform animation of a particular model, i.e. to interactively create a trace of states throughout the state space.

During the course of model verification, ProB produces a great deal of data. The purpose of this work is to dynamically generate visualizations based on this data. The approach for data visualization here differs from that applied in B-Motion studio [3] which allows users to build visualization that then are updated when the current state in an animation changes.

1.1 Visualization of the State Space

Since the concept of the state space is so central to ProB, the focus of this work will be on generating a visualization of the state space. Algorithms for drawing graphs are not trivial; hence this visualization problem is not trivial. However, the focus of this work is in applying existing graph algorithms to create visualizations as opposed to writing a new algorithm or modifying an existing algorithm. Therefore, it is necessary to find a suitable library to take care of the drawing of the state space.

ProB already includes support for the creation of a graph description of the state space in the DOT graph description language. These description files can then be visualized with GraphViz¹. However, the graph algorithms available in GraphViz are relatively inefficient. During verification, the state space often grows exponentially. For state spaces that have a very large number of vertices, the GraphViz algorithms take too long to be useful. The graph generation in GraphViz is also inherently offline. If any thing changes in the state space, the whole graph must be rerendered. During consistency checking or animation, states are constantly being added to the graph. We therefore want a graph engine that can easily add vertices and that can handle state spaces with a large number of vertices.

¹<http://www.graphviz.org>

We also want the user to be able to manipulate and interact with the graph. For instance, ProB supports the ability to create smaller graphs that are derived from the original state space [4]. Because state spaces become so large so quickly, a derived graph can be much more meaningful and useful for a user. One of the features that we want to implement for the visualization of the state space is to enable the user to apply these algorithms to their state space to simplify its representation. The user should be able to seamlessly transfer between the representation of the whole state space and derived state spaces.

1.2 Other Visualizations

Although the visualization of the state space is the focus of this work, there are visualizations that can be generated which will be useful for the user. For instance, ProB supports the generation of a DOT file which, when rendered, shows how a given formula is broken down into subformulas [5]. The formula and its subformulas are also evaluated for a given state, and the resulting nodes are colored so as to specify the value of the formula (e.g. if a given predicate evaluates to true at the specified state, the predicate would be colored green). During the course of this work, we will also recreate this visualization using the new graph engine.

One of the features that is currently being integrated into ProB is the simulation of multiple models concurrently. In this case, the user is less interested in viewing the state space and more interested in being able to see what value a particular expression takes on over the course of a given animation. We will therefore also create a chart to visualize this information.

1.3 Integration of the Visualizations into ProB

The graph engine that we have chosen is D3. Instead of a graph engine in the traditional sense, D3 provides a domain specific language to enable the user to generate elements based on data that is provided. D3 is written in JavaScript, and the visualizations that can be produced are pure HTML and SVG documents.

The visualizations that are produced using D3 will be integrated into the ProB 2.0 Java application using the Jetty server that is already present. The visualizations will also make use of the existing listener framework that is triggered when changes in the state space or the current animation take place. It should also be possible to create and view multiple visualizations at any given time.

2 Background

2.1 ProB 2.0

ProB is written in primarily in SICStus prolog [6]. This core is packaged as a binary executable command line interface, and several other tools have been built on top of it to provide the user with a functioning user interface. The user interface for the current stan-

alone ProB application is written in Tcl/Tk. It is in this application that the GraphViz rendering engine has been integrated.

ProB 2.0 is another tool that is built on top of the ProB command line interface. It is the successor of an Eclipse RCP plugin that has been in development since 2005 [7]. This application was created so that ProB could be integrated with the Rodin software, which is an easy to use and extensible tool platform for editing specifications written in the Event-B specification language.

In 2011, development for ProB 2.0 began. The main goal of ProB 2.0 was to adapt and optimize the existing Java application to produce a programmatic API. One of the main improvements made available in this tool was the introduction of a programmatic abstraction of the state space. It also provides a programmatic abstraction for the representation of animations. This abstraction consists of the trace of transitions that have been executed during the course of an animation. This also provides the user with the concept of a current state. ProB 2.0 also contains a listener framework that is triggered whenever changes take place within a state space or within an animation.

The core harnesses the power of the Groovy programming language. It integrates integrates a fully functioning groovy console into the final product. It is now possible for users and developers to write Groovy scripts that carry out desired functionality. There is also support for creating web applications that communicate with ProB. The console is actually a user interface that makes use of the jetty server that is integrated in the ProB 2.0. There are no eclipse dependencies present in the core, so it can be deployed as a jar file and integrated into any Java based application.

ProB 2.0 also contains an Eclipse RCP Plugin that builds on top of the programmatic core and provides the interface with which the application can communicate with the Rodin platform. The graphical user interface is now built on top of the new programmatic abstractions that are available from the core, and changes that take place within the graphical interface are triggered by the listener framework.

The visualizations that we have created in the course of this work are integrated into ProB 2.0. They are pure JavaScript applications that communicate with Java servlets responsible for generating the data that needs to be visualized. Within the Eclipse user interface, they are simply loaded into a browser.

2.2 D3 and JavaScript

Since a jetty server was already available in the ProB 2.0 Plugin, it was plausible to create visualizations using JavaScript and HTML. Because the ProB 2.0 Plugin is an Eclipse application, it also would have been possible to create visualizations using a native Java or Eclipse library. We carried out an experiment at the beginning of this work to determine the feasibility of the different graph libraries. JUNG was considered because it is the software framework that is the ProB 2.0 API currently uses. It would have been relatively simple to embed the visualizations into the existing ProB 2.0 Plugin, but customizing JUNG graphs is extremely difficult and it would not have been possible to update the graph visualization dynamically. The ZEST graph library was also considered, but in the end we chose to use the D3 library.

D3 (Data-Driven Documents) is “an embedded domain-specific language for transforming the document object model based on data” which is written in JavaScript [8]. Developers can embed the library into a JavaScript application and use the D3 functions to create a pure SVG and HTML document object model (DOM). The focus of D3 is not on creating data visualizations. It is on providing the user the capability of defining exactly which elements the DOM should contain based on the data that the user has provided. Because the objects that are being manipulated are pure SVG and HTML, the user can use D3 to create objects that can be styled using CSS or by dynamically manipulating the style tags of the elements.

2.2.1 Core Functionality

D3 provides a selector API based on CSS3 that is similar to jQuery². The user creates visualizations by selecting sections of the document and binding them to user provided data in the form of an array of arbitrary values [8]. D3 provides support for parsing JSON, XML, HTML, CSV, and TSV files. Once the data is bound to the desired section of the document, D3 can append an HTML or SVG element onto the section for each element of data. This is where the real power of D3 lies because the user can define the attributes of the element dynamically based on the values of the datum in question. By changing these attributes (e.g. size, radius, color) the resulting document already presents the data in a way that the viewer visually understands. The core also provides support for working with arrays and for defining transitions that can be used to animate the document. In order to better understand how D3 works, we have provided a simple example of a how a developer can use D3 to create an HTML dropdown menu (see Listing 1). The generated HTML snippet is also provided (see Listing 2). A more complicated example using the force layout is available in the appendix (see Appendix A).

2.2.2 Further Functionality

D3 also provides further functionality for manipulating the DOM. Developers can define a scale based on the domain and range of values that are defined in the data provided by the user. The placement of elements within the document can then be placed according to the desired scale. D3 provides support for many different types of scales including linear scales, power scales, logarithmic scales, and temporal scales. Axes can also be created to correspond to the defined scale.

The user has the ability to change the DOM as needed. However, D3 also supports a large number of visualization layouts so that the user does not have to define the positions for the elements in a given visualization. The two layouts that are of relevance for this work are the tree layout and the spring layout.

The tree layout uses the Rheingold-Tilford algorithm for drawing tidy trees [9]. The force layout uses an algorithm created by Dwyer [10] to create a scalable and constrained graph layout. The physical simulations are based on the work by Jakobsen [11]. The implementation “uses a quadtree to accelerate charge interaction using the Barnes–Hut approxi-

²<http://jquery.com>

mation. In addition to the repulsive charge force, a pseudo-gravity force keeps nodes centered in the visible area and avoids expulsion of disconnected subgraphs, while links are fixed-distance geometric constraints. Additional custom forces and constraints may be applied on the “tick” event, simply by updating the x and y attributes of nodes” [12].

To help the viewer interact with the visualization, D3 provides support for the zoom and drag behaviors. This listens to the mouse clicks commonly associated with zooming (i.e. scrolling, double clicking) and enlarges the image as would be expected. With this same mechanism, the developer can enable the user to grab hold of the canvas and pan through the image to inspect it closer.

It is very easy to begin developing with D3. The API is described in detail on the D3 Wiki [12], and the D3 website³ includes an extensive array of examples that new developers can use as a starting off point. The D3 developer community is very large, so it is easy to find answers to almost every question online.

Listing 1: Dynamically create a dropdown menu.

```
// Select element with id "body" and append a select tag onto it. When it
// changes, the defined function will be triggered.
var dropdown = d3.select("#body")
    .append("select")
    .on("change", function() {
        var choice = this.options[this.selectedIndex].__data__;
        // handle choice
    });

var options = [{id: 1, name: "Option 1"},
    {id: 2, name: "Option 2"},
    {id: 3, name: "Option 3"}];

// Create an option tag with id and text elements for each option that is
// defined in options
dropdown.selectAll("option")
    .data(options)
    .enter()
    .append("option")
    .attr("id", function(d) { return "op" + d.id; })
    .text(function(d) { return d.name; });

// Select option 3 by default
dropdown.select("#op3")
    .attr("selected", true);
```

Listing 2: Html generated from Listing 1

```
<div id=body>
  <select>
    <option id="op1">Option 1</option>
    <option id="op2">Option 2</option>
    <option id="op3" selected=true>Option 3</option>
  </select>
</div>
```

³<http://d3js.org>

2.3 GraphViz integration with emscripten

The current visualizations available in ProB application are powered using the GraphViz⁴ graph visualization software. There is support for generating graphs for the state space in the DOT graph language. The problem with this, and the reason that we are researching other alternatives, is that drawing GraphViz graphs is rather inefficient. However, for the state spaces that are derived using the signature merge algorithm, or for the transition diagrams that can be created from a state space, these graphs are quite pleasing to the eye. The derived graphs are also usually small enough that they can be drawn efficiently.

For this reason, we wanted to somehow be able to visualize small graphs written in the DOT language. In order to do this, we took advantage of the emscripten compiler [13]. This is a compiler that compiles LLVM bitcode to JavaScript so that it can be run in any browser. C programs can be compiled to LLVM. We used the Viz.js JavaScript library⁵ developed by Mike Daines which has compiled GraphViz from C to JavaScript and provided a wrapper function to produce svg visualizations. For example, the code shown in Listing 3 will produce Figure 1.

Listing 3: Create a visualization with viz.js and insert it into an html page.

```
svg = Viz("digraph { a -> b; a -> c; }", "svg");  
$("#elementId").replaceWith(svg);
```

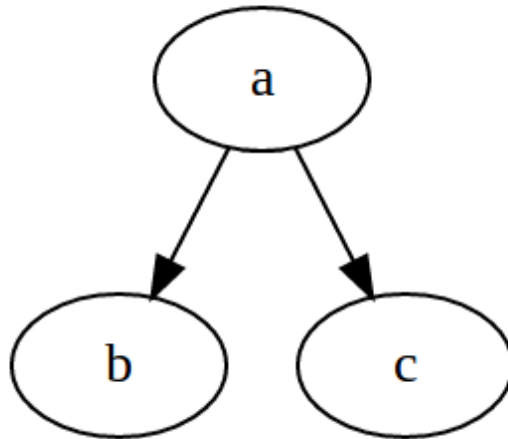


Figure 1: Generated graph image from Listing 3.

⁴<http://www.graphviz.org>

⁵<https://github.com/mdaines/viz.js>

3 Contribution

3.1 Visualization framework

3.1.1 Structure

One of the main issues that we had to be deal with at the beginning of the development process was the issue of how to integrate the visualizations into ProB 2.0. A functioning jetty server was already available, so we had to integrate our JavaScript visualizations into the existing framework. We ran into problems at the beginning, because a Java servlet is a singleton object. We needed to create a way to let the servlet know for which visualization it should be calculating. In order to do this, we created a session based servlet. When the user wants to open a visualization, the servlet is contacted. The servlet then creates a servlet responsible for the visualization and a unique session id. When a visualization sends a GET request, it includes its session id. The session based servlet then forwards the request to the the servlet that is responsible for the data calculations.

The visualization communicates with the servlet by setting up a polling interval. It tells the state space if it needs the complete data set or just the changes since the last polling interval. The servlet then responds by sending the correct data (see Figure 2).

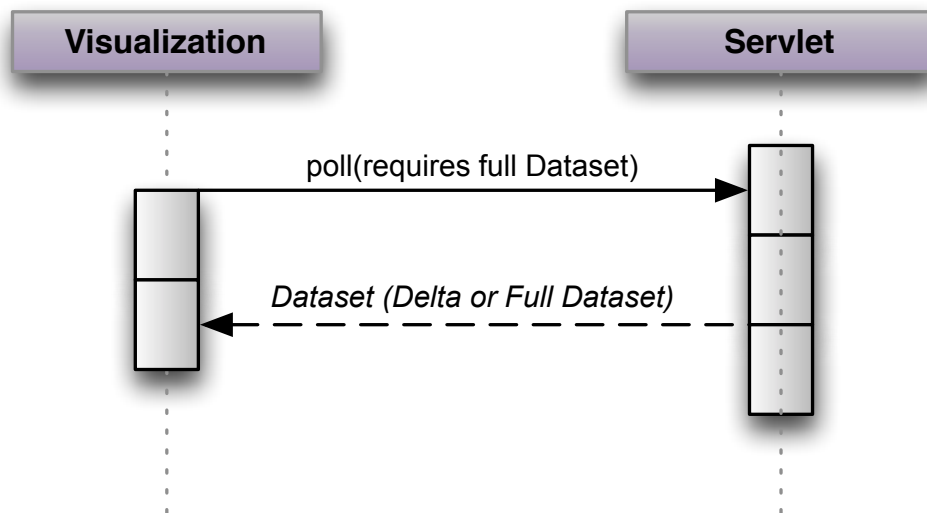


Figure 2: How visualizations communicate with the servlet that is responsible.

The servlet is also connected to ProB 2.0 through the listener framework. Depending on the data that is needed for a particular visualization, the servlet registers either to receive notifications about changes in the current animation or about changes in the state space (see Figure 3). Every time the servlet receives a new notification, the data needed for the visualization is recalculated.

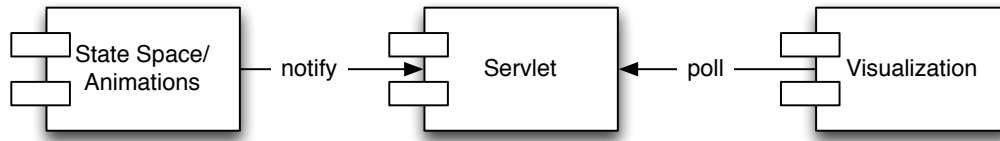


Figure 3: Model of the program flow.

3.1.2 User interaction with the visualizations

Once we implemented a way to integrate the visualization servlets into the ProB 2.0 application, it was still necessary to implement an easy way for the user to interact with the visualizations. We wanted the user to be able to directly manipulate how the visualization appears. One of the main advantages of the D3 visualization framework is the flexibility that it provides for the developer. Using D3 selectors, it is possible for a developer to select and change the attributes of any of the elements of the visualization. We wanted the user to also be able to select and manipulate the visualization from within ProB.

In order to allow the user to directly manipulate the DOM of the visualization, we decided to lift the functionality of the D3 selectors from the JavaScript level into the Java application. For this purpose, we defined a `Transformer` object. A `Transformer` consists mainly of a selector string and the `set` method. The `Transformer` is created by defining a selector based on the W3C Selectors API and calling the `set` method for every attribute that the user wants to manipulate. Because the `set` method returns the instance of the `Transformer`, it is possible chain the method calls together (see Listing 4).

Listing 4: Define a `Transformer` in Java

```
// Select elements with ids "sroot" and "s1" and set their fill and stroke
// attributes
Transformer t = new Transformer("#sroot,#s1").set("fill","red").set("stroke",
    "gray");
```

Every visualization servlet contains a list of `Transformers` that is sent to the visualization every time that it is polled. The visualization can then apply the `Transformers` to the DOM. We now needed a way for the user to create `Transformers` and add them to the visualization. It is possible to add a `Transformer` to a visualization servlet by calling the `apply` method present in all of the visualization servlets. When we create a new visualization servlet, we automatically create a variable in the Groovy console. The user can also use the `transform` closure available in the Groovy console to create `Transformers` which can then be applied to the visualization (see Listing 5).

Listing 5: Define rules for the transformation of visualization elements

```
// Select elements with ids "sroot" and "s1" and set their fill and stroke
// attributes
```



```

x = transform("#sroot,#s1") {
    set "fill", "red"
    set "stroke", "gray"
}

// Apply to visualization
viz0.apply(x)

```

It is also possible to harness the power of the Groovy closure in order to create a Transformer that can be parameterized (see Listing 6).

Listing 6: Use Groovy closures to generate Transformers

```

// Create a closure that can be parameterized
colorize = { selection, color ->
    transform(selection) {
        set "fill", color
    }
}

// Color elements "sroot" and "s1" green
viz0.apply(colorize("#sroot,#s1", "green"))

```

The downside to this mechanism is that the user needs to have a good idea about the internal representation of the DOM in order to manipulate the visualizations. This mechanism is especially interesting for the state space visualization, so we spent more time defining the ids for the DOM elements. The SVG objects that are created using D3 are generated using the state ids and transition ids that are assigned by ProB. These element ids can be seen in Table 1.

Table 1: Generated Element Ids for Elements in State Space Visualization

Element	Generated Element Id
transition	t + <i>transition id</i>
text on transition	tt + <i>transition id</i>
state	s + <i>state id</i>
text on state	st + <i>state id</i>

It is also possible in ProB to filter a state space based on a predicate. This means that it is possible to give ProB a predicate and receive a list of state ids for which the given predicate holds. Ideally, however, we would want a Transformer based on a given predicate to be updated in the case that the state space changes.. For this purpose, we have introduced a Transformer that will be updated in the case that new states are added to the state space. The user can create such a transformer by specifying a predicate and a state space object (see Listing 7). Currently, this will only update the state objects in the DOM, but in the future, the same concept can be applied for all of the different elements.

Listing 7: Create a Transformer based on the states that match a given predicate

```
// Define your formula
predicate = "active\\waiting={}" as ClassicalB

// Create a Transformer that will filter the given state space (saved in
// space0) according to the given predicate
x = transform(predicate, space_0 ) {
  set "fill", "blue"
  set "stroke", "white"
}

// Apply the Transformer to the visualization
viz0.apply(x)
```

3.1.3 Extensibility

In addition to creating visualizations that are useful to the user, we also wanted to make it easy for other developers to create similar visualizations. In order to help with this, we encapsulated certain elements common to all of the visualizations into a separate script. In order to have access to these elements, a developer simply needs to include the script before that of his visualization. Currently, there are two main elements included in this script. Firstly, the user can use the `createCanvas` function to create a D3 selection that includes support for zooming (see Listing 8). Secondly, if the user wants to be able to apply the `Transformers` that are described in the last section, there is a built in function to apply a list of `Transformers` to the visualization (see Listing 9). In the future, it will also be possible to add more functions to this script to make it even easier to create visualizations for ProB.

Listing 8: Append a D3 selection to an element that includes support for zooming

```
var width = 600, height = 400;
var svg = createCanvas("#elementId", width, height);

// Append all further elements to this selection
svg.append(...);
```

Listing 9: Apply list of `Transformers` received from servlet

```
// The servlet has been polled, and a response has been received
var styling = response.styling;

// ... Render the visualization
// ... Then apply the user defined styling
applyStyling(styling);
```

3.2 Visualization of the State Space

3.2.1 Main visualization

When a state space visualization is opened, the visualization servlet responsible for the state space visualization takes the state space associated with the current animation and extracts the information about the nodes and edges contained within the graph. This information is then processed by D3 using the force layout and rendered to create a visualization (see Figure 4). As a basis for the visualization, we used the Force Directed Graph example created by Michael Bostock (see Appendix A).

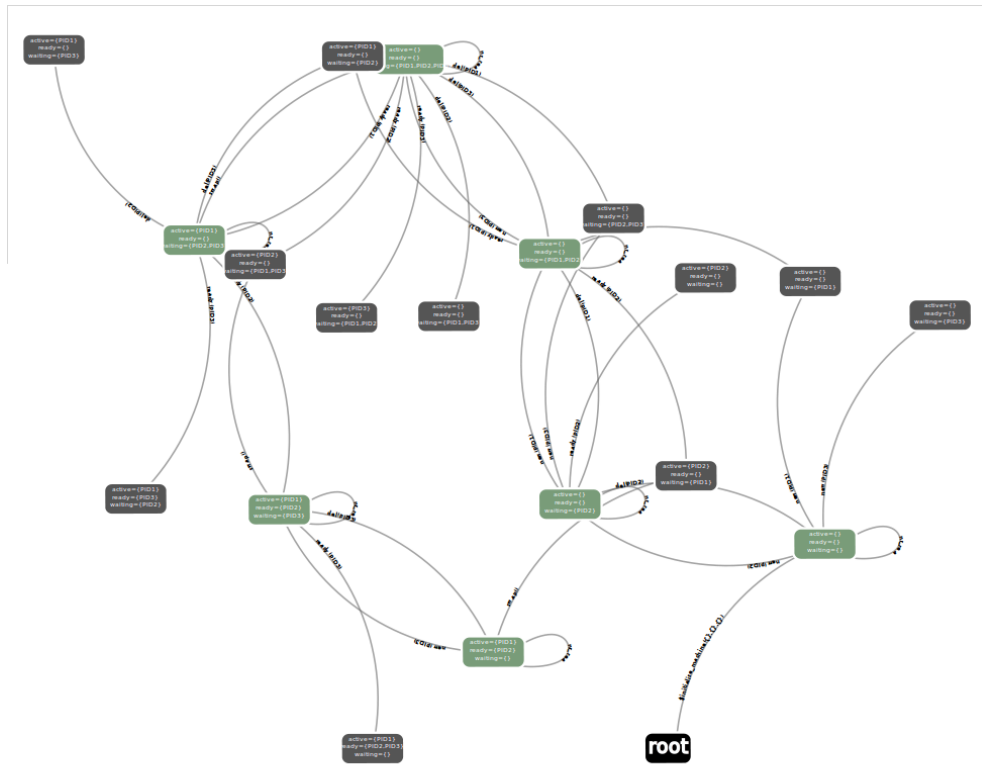


Figure 4: Visualization of a partially explored state space for the Scheduler example

Unfortunately, the state space contains an extremely large amount of information that has to be processed. This includes the values of the variables and the invariant for every state in the graph and the names and parameters of the operations that correspond to every edge in the graph. Because of this, it is rather difficult to create a useful visualization of the whole state space because the user not only wants to inspect how the state space appears as a whole but also the individual states within the operation. As a proposed solution of this problem, we used the zoom functionality that is available in D3. The main problem was that if the visualization of the nodes was large enough for the user

to read the values of the variable at the given state, it would no longer be possible to see the state space as a whole. Instead of trying to meet both requirements at once, we simply made the text that is printed on the node and edge objects very small. When the visualization is created, the user can inspect the graph as a whole how the graph appears as a whole. The text for the given nodes, however, is virtually indiscernable. If the user wants to inspect a particular node, they can do so by zooming into the visualization. The text is then larger, and the user can see the values of the variables for the given state and the out going transitions (see Figure 5). Then user can also click on the background of the visualization in order to pan through the visualization and inspect other nodes and edges. The visualization is also interactive. The user can grab a node and move it around to a desired position.

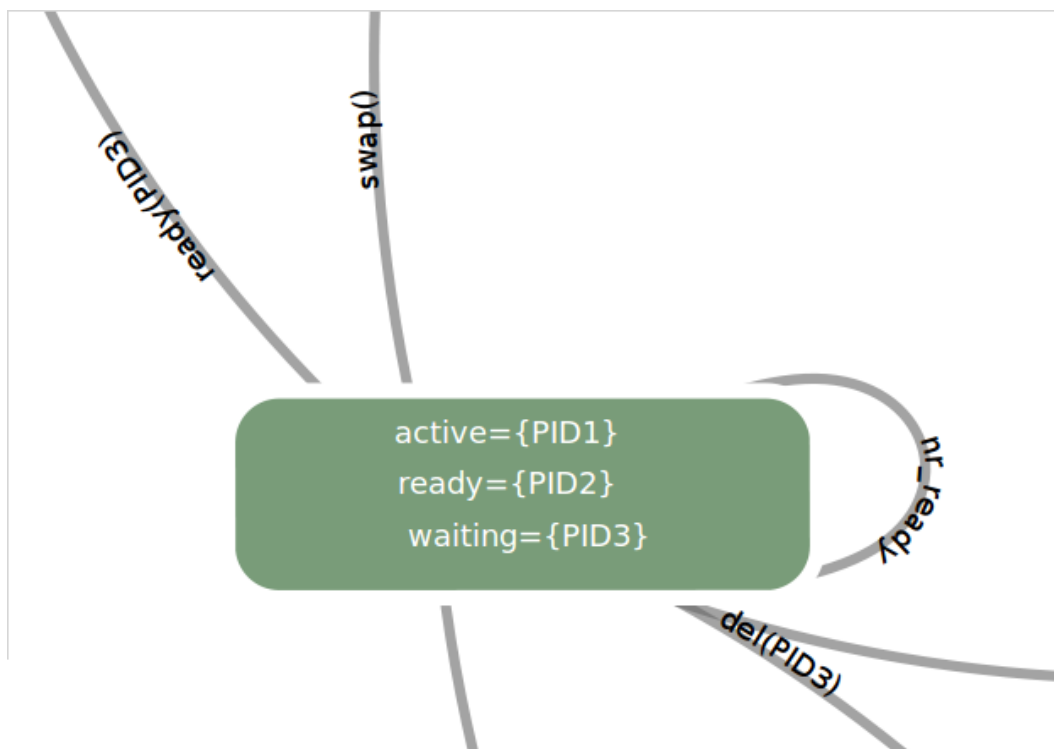


Figure 5: By zooming in, the user can inspect individual nodes.

We had some performance issues that were associated with very large state spaces. The force layout keeps adjusting the graph until it reaches a fix point. The problem was that as the state space grew, there were more and more objects that had to be accounted for. The force layout just kept calculating and moving the the nodes. This didn't only affect the appearance of the visualization. It cost enough resources that the whole eclipse plugin would become unresponsive. A quick fix to this problem was adding a play/pause button to the upper left hand corner of the visualization (see Figure 6). When the user is satisfied with the visualization and how it is laid out, he can press the pause button, and the graph will stop being rendered. When he presses the play button, the rendering

will begin again. The iterative force layout keeps running in the background, so when the rendering begins again, the visualization has had time to stabilize.

When new states are added to the state space, these states are added to the graph when the visualization receives them in the next poll. The graph is then updated. This results in a nice animation.

Status about the invariant is available in the graph based on the color of the nodes. If an invariant violation is present, the node is colored red. If the invariant is ok, the node is colored green. Otherwise, if the invariant has not yet been calculated for the given node, the node is colored gray.

The state spaces often grow exponentially. This is known as the state space explosion problem. For this reason, a visualization of the entire state space can often be a great deal too much information for a user to process at any given time. For this reason, we have also made smaller graphs available that are derived from the original state space and can be more useful for the user to visualize. The user can choose from these visualizations in the dropdown menu in the upper left hand corner (see Figure 6).

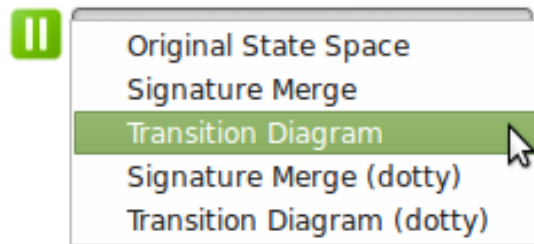


Figure 6: The user can select the desired visualization and play and pause the rendering.

3.2.2 Signature Merged State Spaces

The signature merge algorithm is the first of two algorithms for reducing the state space that have been implemented in this work. The algorithm works by merging all of the states which have the same outgoing transitions. This creates a state space that is considerably smaller but that still preserves information about the operations that are enabled for a given state [4] (see Figure 7).

The graph that is generated with the signature merge algorithm differs based on the transitions that are of the interest of to the user. By default, all transitions are selected when the algorithm is run. However, it is also possible to remove or readd transitions into the calculation of the algorithm. In order to allow the user to select the transitions of interest, we have implemented a small user interface that pops up when the user clicks on the settings icon (see Figure 8). The settings icon only becomes available when the user is in a mode dealing with the signature merge algorithm. The label for the nodes in

this graph consists of a list of the outgoing transitions from the node and the number of states that have been merged into the node.

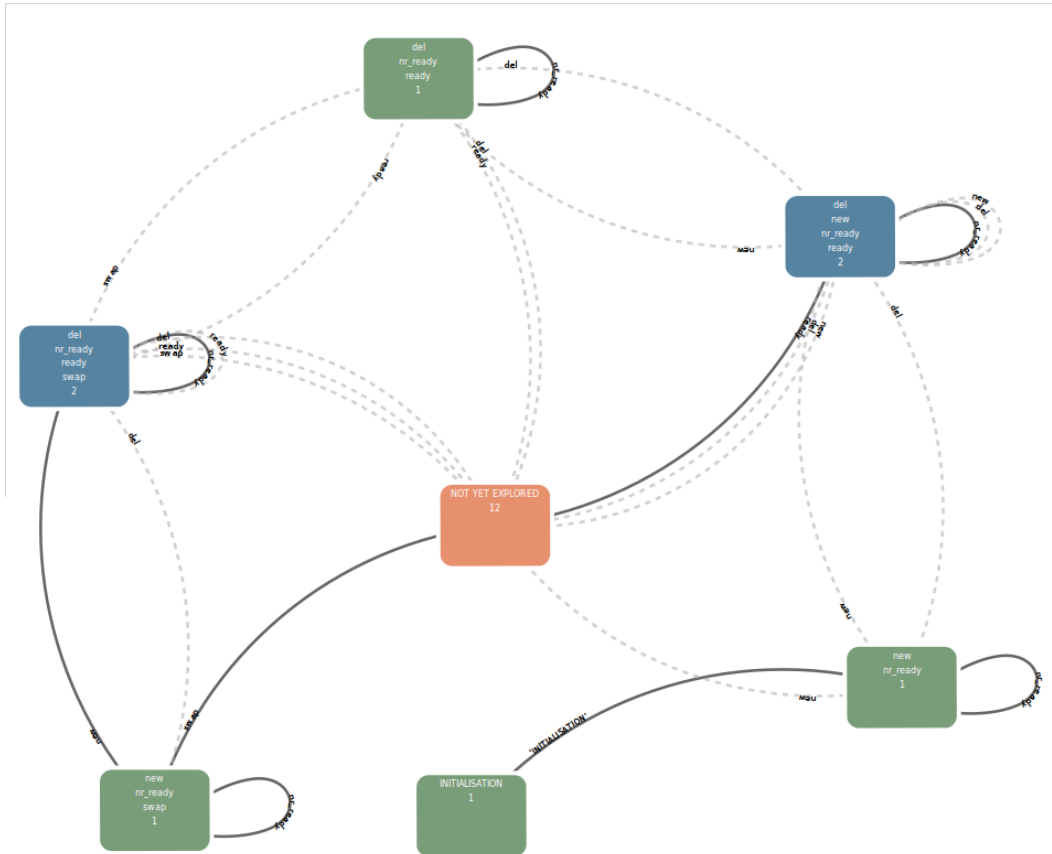


Figure 7: D3 Visualization of signature merge for Scheduler example

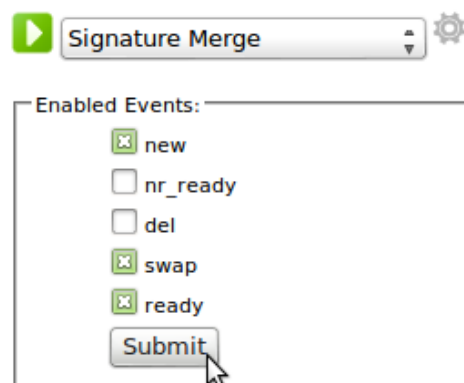


Figure 8: User interface to chose events for signature merge.

If the user wants to use the GraphViz algorithms and rendering engine, there is also support for visualizing the DOT representation of the signature merged state space that is generated from ProB (see Figure 9). This is done using the Viz.js JavaScript library. The same user interface is available for the GraphViz generated graph, and the visualization supports zooming and panning in the same way as the D3 powered visualizations do.

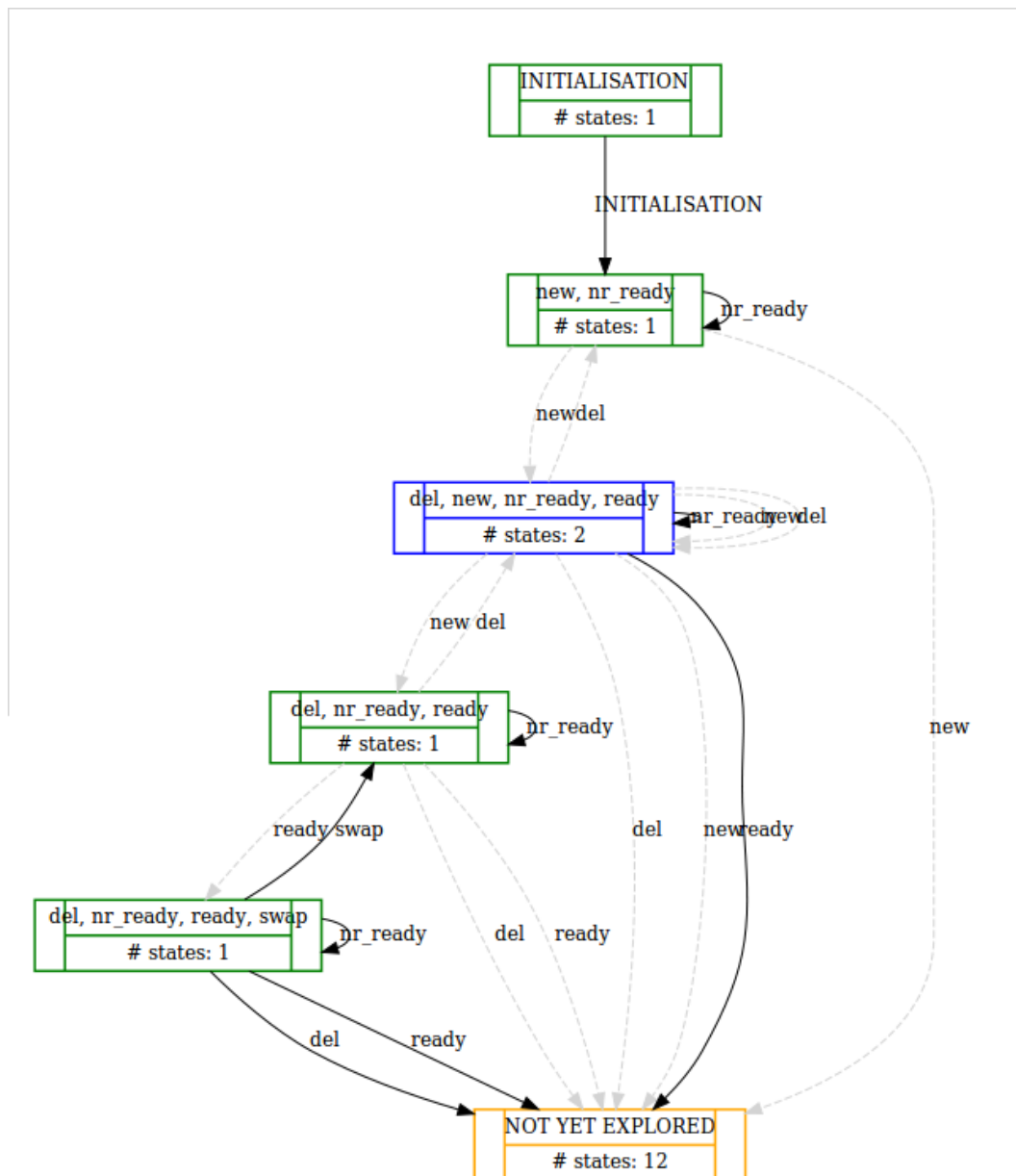


Figure 9: GraphViz powered Visualization of the signature merge algorithm for the Scheduler example

3.2.3 Transition Diagrams

The other reduction algorithm that is supported in this implementation is the creation of transition diagrams. In order to perform this algorithm, ProB receives an expression from the user. Then ProB calculates all of the possible solutions to the expression. These become the vertices in the graph. The edges in the graph show the transitions that change the value of the given expression to that of the value shown in the target state (see Figure 10).

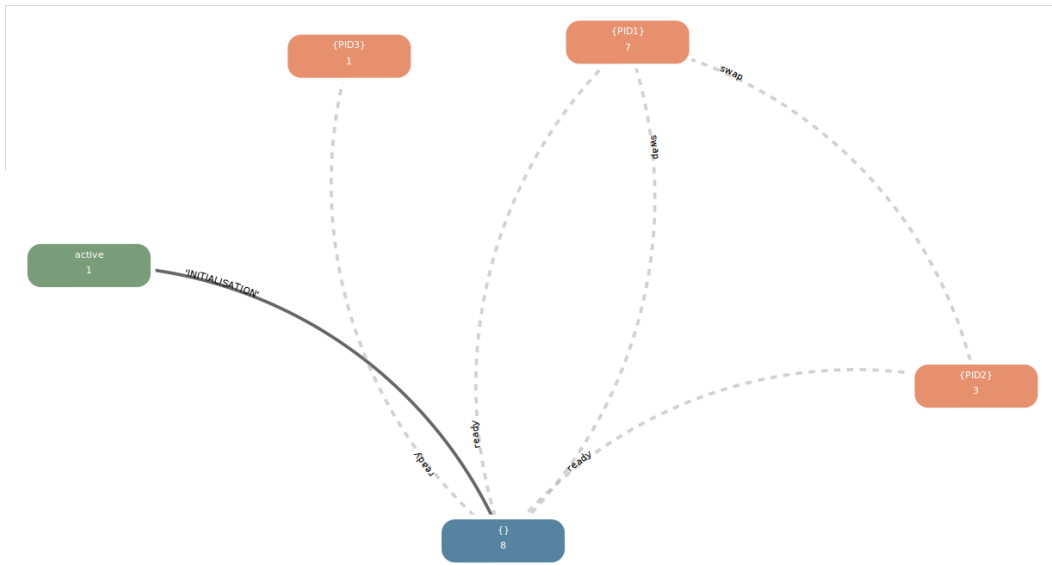


Figure 10: D3 Visualization of the transition diagram of `active` in Scheduler example

When the user chooses to create a transition diagram from the menu, a prompt appears. This is how the user can specify the initial expression for the calculation of the transition diagram. Once the graph is calculated and rendered, a text field appears next to the drop down menu (see Figure 11). If the user wants to change the expression that is being visualized, they can input a new expression here and submit it. The algorithm will then be recalculated and the graph will be rerendered.

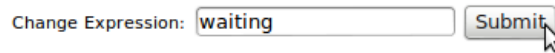


Figure 11: The user can input a new expression to recalculate the transition diagram.

If the user prefers the GraphViz representation over the D3 powered visualization, it is also possible to generate a GraphViz based visualization for the transition diagram (see

Figure 12). As with the signature merged state space, the UI for the GraphViz powered transition diagram is the same as that for the D3 powered visualization, and the visualization supports zooming and panning. Although the GraphViz algorithms are inherently offline, these visualizations make use of the visualization framework and are recalculated and rerendered every time that a change takes place within the state space.

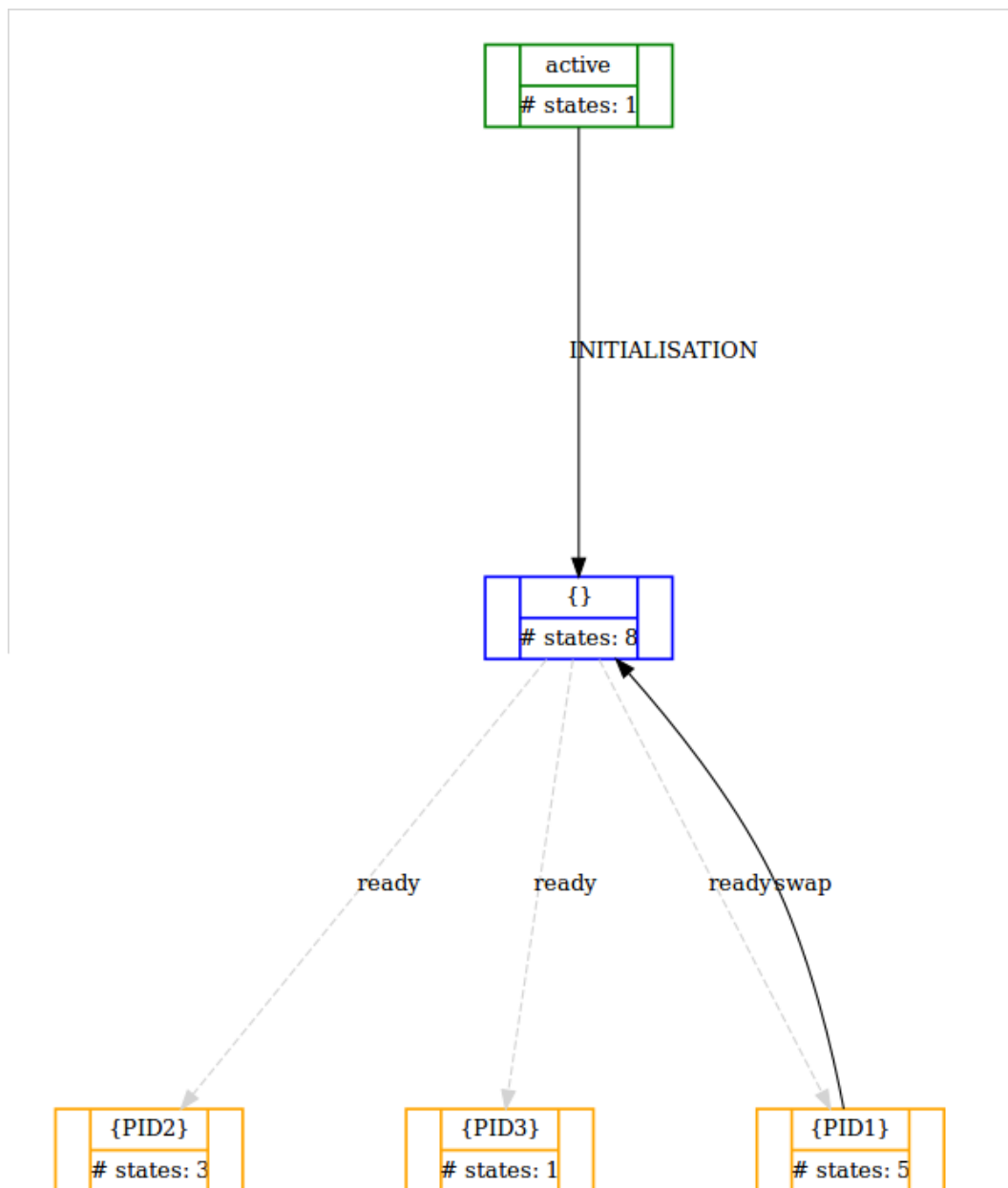


Figure 12: GraphViz powered visualization of the transition diagram of `active` in Scheduler model

3.3 Visualization of a B-type Formula

ProB already supported the functionality of expanding a formula into its subformulas and finding its value at a given state. However, for any given formula, only the subformulas directly under the desired formula would be calculated. This algorithm was adapted so that all the subformulas are calculated automatically and a tree structure is built automatically. This structure is then cached, and can then be evaluated for any given state.

The final visualization is interactive (see Figure 13). If a formula has subformulas, the user can select it from within the visualization to expand or to retract the subformulas. The subformulas are always either predicates or expressions. If they are expressions, they are colored white or light grey depending on whether they have subformulas or not. If the formula is a predicate that has evaluated to true for the given formula, the node is colored green. If the formula is a predicate that has evaluated to false, node is colored in red. The value of the given formula is also printed beneath the formula. This allows the user to visually identify the parts of the formulas and their given values.

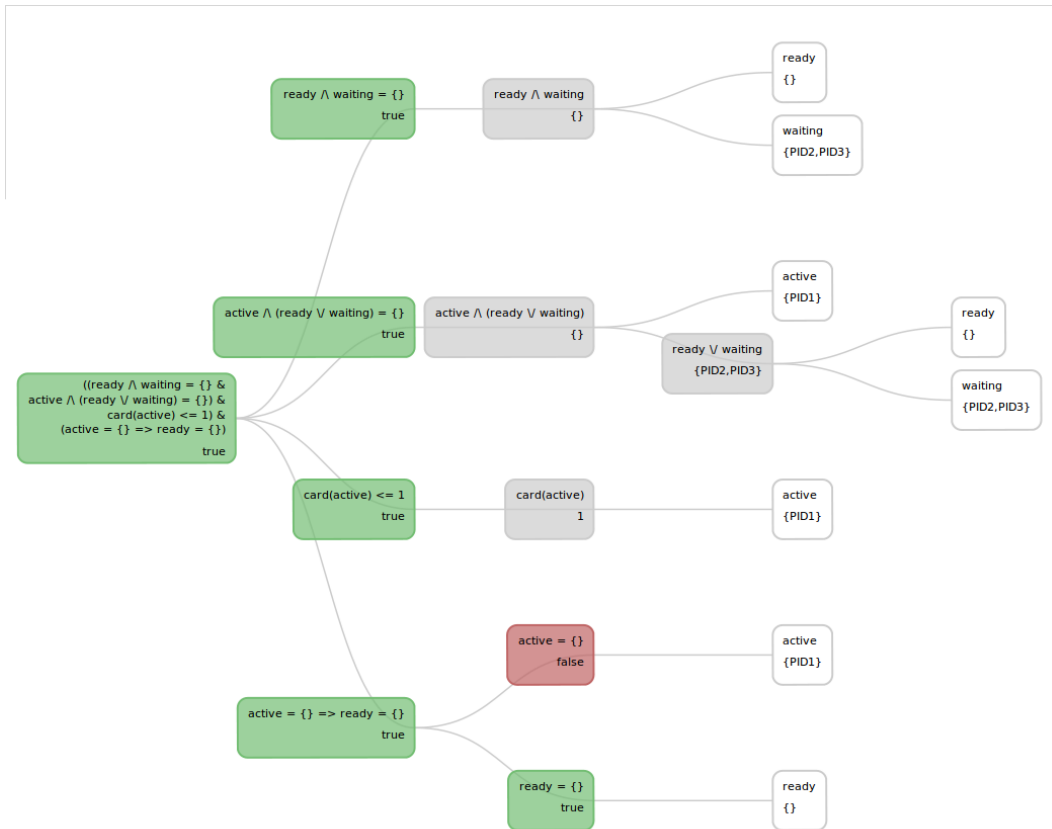


Figure 13: Visualization of the invariant of the Scheduler model

In the implementation of the formula, the D3 tree layout is used. The expanding and

collapsing of the nodes takes place with a simple JavaScript function. This visualization is based on the Collapsible Tree Layout from the D3 website (see Appendix B). By harnessing the power of the D3 zoom behavior, it is also possible to zoom in and out of the visualization and to pan the image to inspect it closer. The servlet responsible for the visualization implements a listener to identify if any changes in the animation occur. If they do, the formula is recalculated for the new current state, and the visualization is redrawn.

3.4 Visualization of the Value of a Formula Over Time

The last visualization that we have created in the course of this work is the visualization of the value of a given formula over the course of an animation. Because a state is defined by the values that the variables take on when dealing with B type specification languages, it can be interesting to be able to examine the value of a variable over the course of a trace. This is particularly interesting when the concept of time is present in the model being animated. In this case, the user is often only interested in inspecting a particular variable to see how it changes over time.

When the visualization is created, the user specifies the formula that is to be visualized. The user can also specify a second expression which will serve as the value for the time axis. Then it contacts ProB and extracts the value that the formula takes on for each state in the list. This information is then processed by D3 to produce a simple line plot (see Figure 14). Expressions that take on boolean values and integer values can be visualized. If the current state changes, the formula is recalculated and a new plot is produced.

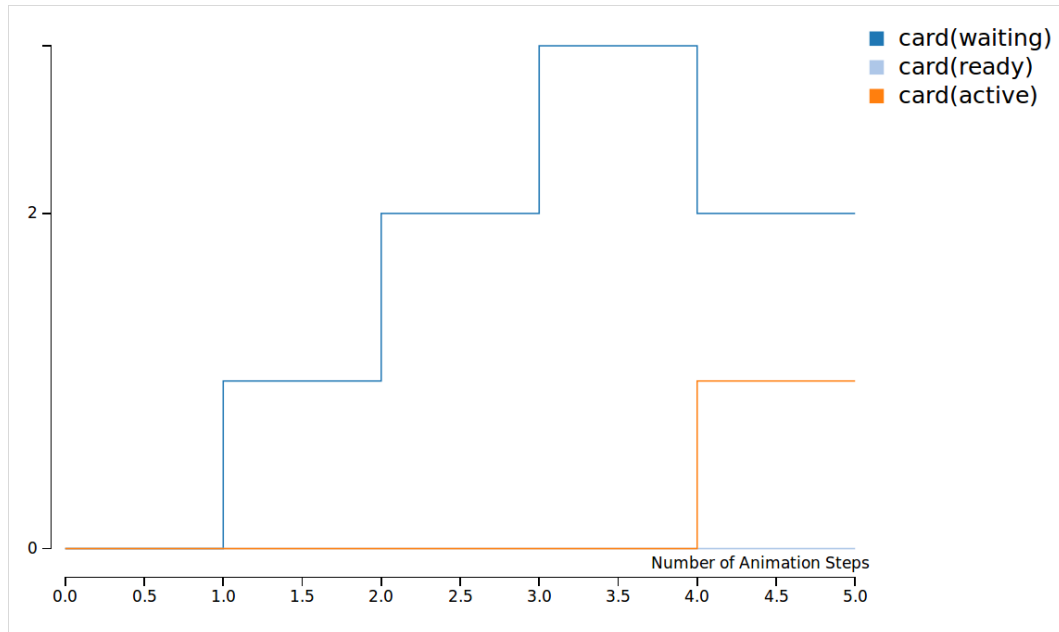


Figure 14: Visualization of value of the cardinality of the variables from the Scheduler model over the course of an animation.

As seen in Figure 14, it is possible to visualize multiple formulas at the same time. In order to add a formula to the visualization, the user can input a new formula in the text box in the upper left hand corner (see Figure 15).

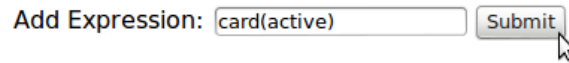


Figure 15: The user can add an expression to the visualization.

4 Related Work

The focus of this work was to apply existing graph algorithms to the problem of visualizing a state space. However, a great deal of work has been done in developing algorithms to simplify the state space so that it can be better visualized. Two algorithms have already been integrated into ProB [4]. One of these, the signature merge algorithm, has already been presented in this work. However, the DFA-Abstraction algorithm presented there has not yet been integrated into the new tool.

Several tools exist specifically for the visualization of directed graphs. Walrus⁶ and GraphViz⁷, which has already been introduced in this paper, are two tools that can be used for graph visualization. Some tools which require the visualization of labeled transition systems use the graph visualization features that are provided by these tools. As we have seen, ProB is one of these tools. CSPM⁸ is another tool which supports the generation of DOT files in order to visualize the state space associated with CSP specifications.

However, there are also several tools developed for the specific purpose of visualizing state spaces. Van Ham et al. [14] have considered the problem of visualizing labeled transition systems and developed the LTSView⁹ tool for visualizing the structure of state spaces. This tool produces a 3D representation of the state spaces. With the LTSView tool in view, work has been done to develop a method of converting 3D models of labeled transition systems to 2D [15]. The StateVis tool is the result of this research¹⁰. These tools can help users to get a rough idea of the feel of the whole state space. However, they do not allow for a close inspection of interesting parts of the state space. In order to provide a solution to this problem, Pretorius and van Wijk have proposed a method for interacting with the visualization of state transition graphs [16]. The tools NoodleView¹¹ and its successor DiaGraphica¹² attempt to apply this method.

⁶<http://www.caida.org/tools/visualization/walrus/>

⁷<http://www.graphviz.org>

⁸<http://hackage.haskell.org/package/CSPM-cspm>

⁹http://www.mcr2.org/release/user_manual/tools/ltsview.html

¹⁰<http://www.win.tue.nl/vis1/home/apretori/statevis/>

¹¹<http://www.comp.leeds.ac.uk/scsajp/applications/noodleview/>

¹²<http://www.comp.leeds.ac.uk/scsajp/applications/diagraphica/>

5 Future Work

One of the main features we implemented in this work was the creation of a visualization framework. Each visualization has its own unique elements, but some elements have been encapsulated so that they can be reused in other visualizations. This is true both on the server and the client side. The basic elements of a ProB visualization have been extracted into a JavaScript library. For instance, developers can now use this library to create a basic canvas which can be zoomed and panned. We have also implemented a basic servlet that can handle the communication between the visualization and the servlet responsible for the visualization. This can easily be extended to create new servlets.

We are currently developing a worksheet element for ProB 2.0. This will serve as documentation and a means to run groovy scripts within the application. In the future, it will be possible to embed the visualizations created in the scope of this work within the worksheet.

It will also be necessary to implement other data visualizations in ProB 2.0. For instance, there is currently no graphical representation of a state. This visualization is available in the ProB Tcl/Tk standalone application, but it still needs to be implemented within ProB 2.0.

The state space visualization and the visualization of a formula over time are not language specific. It is possible to use the visualizations for any specification language that ProB supports. For the visualization of the breakdown of a formula, however, this is not the case. The current implementation supports only Classical B and Event-B formulas. In the future, work could be done to support formulas from other formalisms.

Current visualizations will also need to be maintained and updated to add functionality. For instance, it might be possible to integrate the proposed state visualization with the existing state space visualization. Then, when a user would select a state, a window would pop up displaying the state visualization for that particular state.

6 Conclusion

Over the course of this work, we have shown the feasibility of using D3 to create data visualizations within ProB. It is not only possible to create the desired visualizations, it is also easy to adapt the servlets so that the visualizations are updated to reflect changes made in the course of model animation. The styling for these visualizations can be easily defined by the user. This gives the user a large amount of control over the visualization.

The visualization of the state space was the focus of this work. The visualization that was created is interactive and is automatically updated as soon as states are calculated and cached in the state space. It also can handle relatively large state spaces (TEST THIS OUT TO DETERMINE HOW LARGE!!!!).

The decision to support GraphViz visualizations in the state space visualization in addition to D3 visualizations took place relatively late in the development process. Using the existing visualization framework, it was possible to implement this feature in a relatively short period of time. This shows that the visualization framework is relatively flexible

and can be easily extended.

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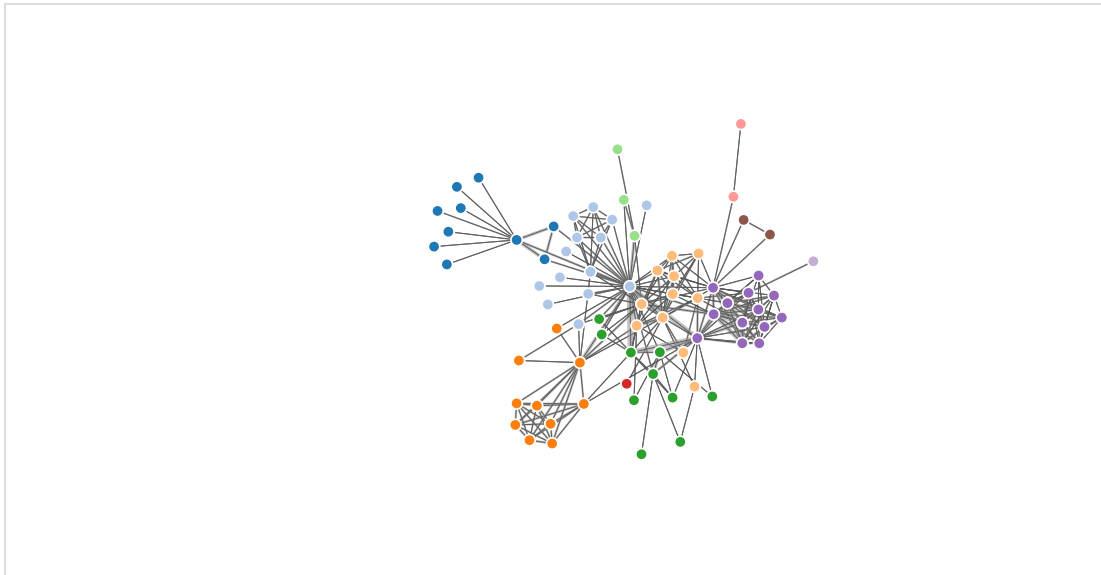
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A Force Directed Graph

mbostock's block #4062045

Force-Directed Graph

May 8, 2013



This simple force-directed graph shows character co-occurrence in *Les Misérables*. A physical simulation of charged particles and springs places related characters in closer proximity, while unrelated characters are farther apart. Layout algorithm inspired by [Tim Dwyer](#) and [Thomas Jakobsen](#). Data based on character coappearance in Victor Hugo's *Les Misérables*, compiled by [Donald Knuth](#).

[Open in a new window.](#)

index.html

```

<!DOCTYPE html>
<meta charset="utf-8">
<style>

.node {
  stroke: #fff;
  stroke-width: 1.5px;
}

.link {
  stroke: #999;
  stroke-opacity: .6;
}

</style>
<body>
<script src="http://d3js.org/d3.v3.min.js"></script>
<script>

var width = 960,
    height = 500;
  
```

bl.ocks.org/mbostock/4062045

1/7

```

var color = d3.scale.category20();

var force = d3.layout.force()
    .charge(-120)
    .linkDistance(30)
    .size([width, height]);

var svg = d3.select("body").append("svg")
    .attr("width", width)
    .attr("height", height);

d3.json("miserables.json", function(error, graph) {
    force
        .nodes(graph.nodes)
        .links(graph.links)
        .start();

    var link = svg.selectAll(".link")
        .data(graph.links)
        .enter().append("line")
        .attr("class", "link")
        .style("stroke-width", function(d) { return Math.sqrt(d.value); });

    var node = svg.selectAll(".node")
        .data(graph.nodes)
        .enter().append("circle")
        .attr("class", "node")
        .attr("r", 5)
        .style("fill", function(d) { return color(d.group); })
        .call(force.drag);

    node.append("title")
        .text(function(d) { return d.name; });

    force.on("tick", function() {
        link.attr("x1", function(d) { return d.source.x; })
            .attr("y1", function(d) { return d.source.y; })
            .attr("x2", function(d) { return d.target.x; })
            .attr("y2", function(d) { return d.target.y; });

        node.attr("cx", function(d) { return d.x; })
            .attr("cy", function(d) { return d.y; });
    });
});
</script>

```

miserables.json

```

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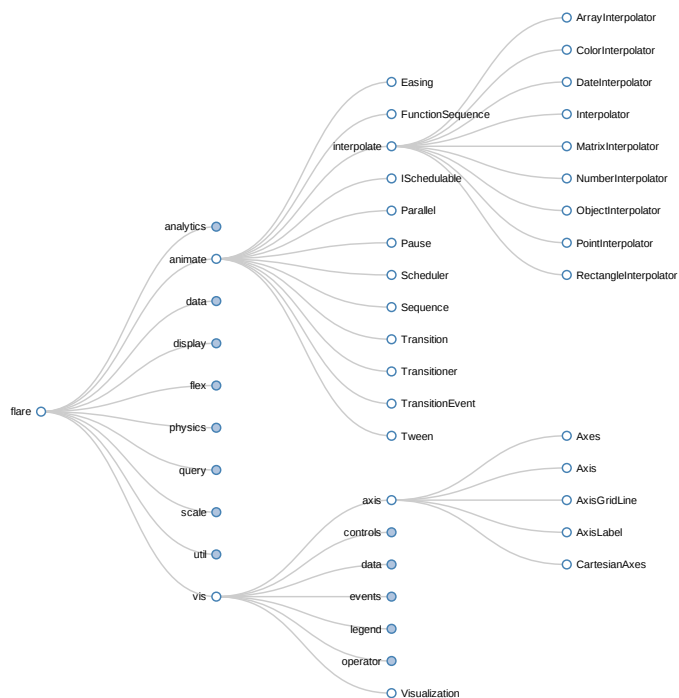


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B Collapsible Tree Layout



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click or option-click to expand or collapse