

PC 5. Exercise 5-4

i). Critical region: $H_0: \mu \leq 100$. $\alpha = 0.05$.

Assume normal distribution: $Z_{0.05} = 1.64$.

$$\text{critical value: } \mu_0 + Z_{\alpha} \frac{\sigma}{\sqrt{n}} = 100 + 1.64 \cdot \frac{4}{\sqrt{8}} = 102.32$$

$$[102.3, +\infty)$$

Since $102.2 < 102.3$, we fail to reject H_0 .

$$\text{ii) } P[\bar{x} \geq 102.2 | \mu \leq 100, \sigma = 4]$$

$$= P\left[Z \geq \frac{102.2 - 100}{\frac{4}{\sqrt{8}}} \right]$$

$$= P[Z \geq 1.556] = 0.059$$

$$\text{iii) power: } \beta = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{2 - \frac{\sqrt{8}}{4}} e^{-\frac{t^2}{2}} dt.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-1.89} e^{-\frac{t^2}{2}} dt.$$

$$= 0.029$$

$$1 - \beta = 0.971$$

$$\text{iv) } \beta \leq 0.15, \alpha = 0.05, \delta = 5, \sigma = 4.$$

$$-Z_{\beta} \approx Z_{\alpha} - \delta\sqrt{n}/\sigma$$

$$\Rightarrow n \geq 463$$

at least 5 samples