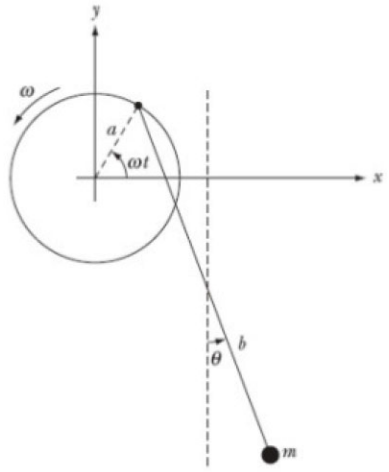
1. A point particle of mass m moves without friction down a wedge of mass M that is free to slide on a frictionless table. The wedge is inclined at the angle α to the horizontal.

   How many degrees of freedom does the particle have here? Identify the generalized coordinates here.

   (In the next homework the students will be asked to solve this problem using Lagrange's formalism.)

2. A simple pendulum of length b and mass m moves attached to a massless rim of radius a rotating with constant angular velocity ω. How many degrees of freedom do we have here? Find the Lagrangian.

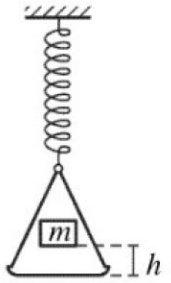


3. Find the equations of motion of a particle of mass m constrained to move on the surface of a sphere, acted upon a conservative force $\mathbf{F} = F_0 \hat{n_\theta}$, with F_0 a constant.

   Hint. To find the potential energy find the scalar product $\mathbf{F} \cdot d\mathbf{r}$ for the infinitesimal displacement on the sphere and use the fact that it is equal to $-dU$ (the force is conservative).

4. Double pendulum: (1) identify the generalized coordinates; (2) find the Lagrangian; (3) write down the Euler–Lagrange equations of motion (you may skip this part); (4) please look up some animations of trajectories of such a pendulum.

6. A block with mass m falls down from height h on a horizontal plane suspended on a spring with spring constant k, and remains on the plane. Find the amplitude of resulting oscillations.



8. Find the center of mass of a non-uniform cylinder with the z axis as the axis of symmetry and $\rho(\mathbf{r}) = \alpha z^2$ (or another example that can be simply calculated by reducing the triple integral to a single one, based on the symmetry).