Dynamic Stabilisation of a Reaction-Wheel Actuated Wheel-Robot

Joydeep Biswas
Department of Physics
Bhartendu Seth*
Department of Mechanical Engineering
Indian Institute of Technology, Bombay
Powai, Mumbai - 400076.
INDIA
*seth@me.iitb.ac.in http://www.me.iitb.ac.in/~seth/

Abstract: **Reactobot is an enclosed single wheel robot that was developed at the Indian Institute of Technology Bombay. The robot is balanced by actuating an internal reaction wheel suspended from the central axis. By pitching the suspended mass within the wheel forward or backwards, the robot can be made to accelerate forward and backward, respectively. Turns are executed by tilting the robot to the left or to the right while it is in motion. In this paper, we describe the dynamics of the system relevant to its balancing about the roll axis, and propose a control scheme to balance it. Experimental results demonstrate the effectiveness of this scheme.**

Key-Words: **roll, balance, gyro, reaction wheel, LQR, mobile robot**

1. Introduction

The invention of the wheel was, without a doubt, a revolutionary step in the development of transport. Today, the application of microcontrollers is another such step leading to entirely new mechatronic systems capable of autonomous operation. The wheel has not escaped this transformation. Several teams around the world have successfully demonstrated automatic balancing of wheels in the upright position, thereby liberating them from the vehicles to which they have been traditionally attached to, thus giving birth to, what could be called the *Wheel Robots.*

Considerable work has been done in the area of dynamic balancing of a wheel in the upright position including Zenkov's work on stabilisation of the unicycle with rider [1], "JOE" (a mobile inverted pendulum) [2], CMU's "Gyrover" [3], and NUS's "Gyrobot" [4].

Zenkov et. al. [1] Model the case of a unicycle with a rider as a two link inverted pendulum, and demonstrate that the system can be stabilised by controlling the motion of the second link in the plane perpendicular to that containing the wheel. "JOE" is a two wheel robot that is stabilized by accelerating the system forward and backward, much like a pendulum on a cart. The "Gyrover [3] and the "Gyrobot" [4] are both single wheel robots that are stabilised using mechanical gyroscopes. In the paper "Electric Vehicle on one Wheel" [5], K. Hofer describes an electric unicycle capable of carrying a person, that is roll stabilized by the use of a wide wheel base. Pitch stabilization is then achieved by driving the wheel. More recently, the development of a reaction wheel stabilised unicycle robot, "MURATA GIRL" was announced by Murata Manufacturing Co., Ltd [6].

In this paper, we present the construction, dynamics and control of our unique configuration for a single wheel robot. The system is inherently stable along the pitch axis since the centre of mass of the system is lower than the wheel axle. In this respect, it is similar to "Gyrover" [3] and "Gyrobot" [4], but the mechanism used to stabilise it along the roll axis is quite different. Whereas the former robots use the gyroscopic effect of an internal flywheel to stabilize themselves, Reactobot is balanced by the actuation of an internal reaction wheel which has its axis parallel to the roll axis. In construction, the system is completely enclosed, with the reaction wheel, electronics, batteries and drive motor suspended from the wheel axle that is connected to an outer shell enclosure, unlike the unicycle constructions of [1], [5] and [6].

Resolved along the roll axis, the system dynamics are similar to Spong's Inertia Wheel Inverted Pendulum [7] and Sumiko Majima and Takashi Kasai's reaction wheel stabilized unicycle [8]. Unlike the case of the single wheel robot stabilized by a mechanical gyroscope (where the flywheel must be kept spinning at a large and constant angular velocity), the reaction wheel of the Reactobot can be brought to rest once it achieves the dynamically stable position, as shown in section 4.

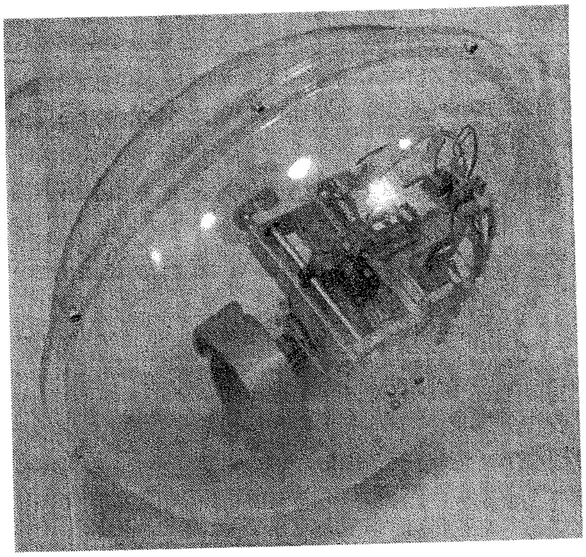

Fig. 2.1: Photograph of the Reactobot

2. System Configuration

As shown in Figure 2.1, the Reactobot consists of a hollow plastic disc containing the actuators, sensors, microprocessors batteries and other electronics necessary for its operation. These are suspended from the main axle of the wheel like a pendulum. This arrangement ensures that all wiring remains on the pendulum only and there is no problem of twisting wires as the wheel rotates. The user interface is through a remote control using RF communication.

Control is achieved through two actuators in the Reactobot. The pitch motor is coupled to the axle by means of bevel gears for forward and reverse motion of the robot. The balancing about the roll direction is achieved by the reaction wheel actuator, which is suspended on from the main axle so that the axis of the reaction wheel is parallel to the ground and along the roll axis when the robot is moving at a constant velocity and the suspended mass is in its equilibrium position. Fig 2.2 schematically shows the front and side views of the robot.

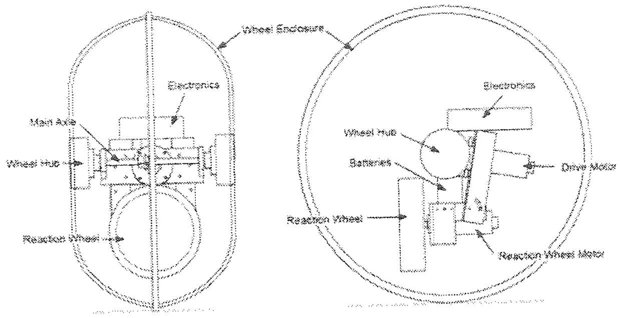

Fig. 2.2: Front and Side Views of the Reactobot

The challenge in balancing this system is that the system should be able to estimate and attain its equilibrium position before the reaction wheel motor reaches its power output limit. Additionally, the reaction wheel should come to rest after the robot reaches its equilibrium position, so that the robot can deal with further external perturbations.

Since the equilibrium position itself is dependent on the mass distribution of the system, the surface characteristics of the ground and external wind effects, the robot should be able to dynamically estimate the equilibrium position. We have used an inertial measurement unit (IMU) comprising of three, one-axis gyroscopes and two, two-axis accelerometers/inclinometers. A complementary filter is used to estimate the angles as the angle estimates from the gyroscopes alone are prone to drift due to noise in the angular rate measurements and the angle estimates from the inclinometers alone are inaccurate in the presence of accelerations.

The parameters of the complementary filter are tuned to obtain satisfactory step response with respect to a constant gyroscope bias error (step response) of 1°/s, and a step response with respect to the inclinometer of a static inclination of 1°.

For tuning the Reactobot and to collect data, a telemetry program written in Visual Basic was used. This could then be used with a USB Zigbee module to setup a wireless link with the Reactobot for telemetry and control as shown in Fig. 2.3.

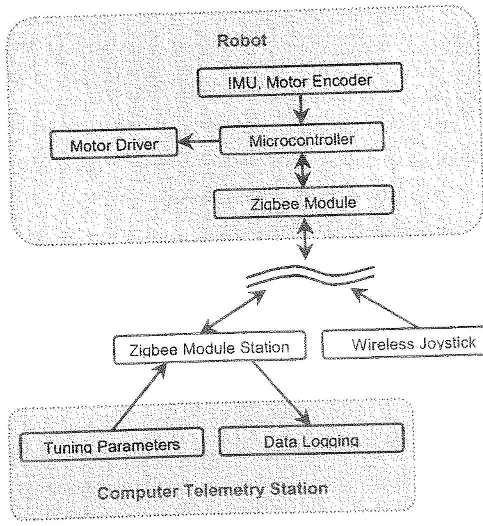

Fig. 2.3: Schematic of RF Communication

The telemetry software is capable of graphically displaying the variation of up to six channels in real time, and saving the graphs to bitmap files. Alternatively, a comma-separated value (CSV) listing can be saved for analysis with other programs. This was indispensable while developing the IMU complimentary filters and while tuning the control loop.

3. Simplified System Dynamics

Table 3.1: System Variables and Parameters

x_1	The lean angle of the robot from the vertical	
x_2	The angular velocity of the robot about the roll axis	
x_3	The angular velocity of the reaction wheel	
x_4	The reaction wheel angle	
$\dot{x}_i$	The rate of change of x_i with time (i=1,2,3,4)	
$\bar{x}_1$	The equilibrium value of the lean angle of the robot	
$\hat{\bar{x}}_1$	The system's estimate of the equilibrium value of lean angle of the robot	
k_i	The torque constant of the reaction wheel motor	37.7 mNm/A
k_ω	The back-emf constant of the reaction wheel motor	0.0377 V/rad/s
R_C	The armature resistance of the reaction wheel motor	1.63 Ω
k_f	The coefficient of viscous friction at the reaction wheel bearing	1 Nms
M	The mass of the entire robot	4.9 kg
g	Acceleration due to gravity	9.8 m/s^2
R	The distance of the centre of mass of the robot from the lowest point of the wheel rim, where the robot touches the ground	0.18 m
I_D	The moment of inertia of the reaction wheel about its own axis	0.013 kgm^2
I_B	The moment of inertia of the robot along with the reaction wheel considered as a point mass, about the roll axis	0.4093 kgm^2
n	The gear reduction between the reaction wheel motor and the reaction wheel	3.7
v	The voltage applied to the reaction wheel motor	

Although the system has five degrees of freedom, we consider only the problem of roll stabilisation here. While deriving the dynamic equations, governing the balancing of the system, it is assumed that the axis of the reaction wheel is parallel to the ground, thus its actuation only affects the "roll" motion of the robot. This would be true while the robot is at rest, but during linear acceleration and deceleration, the reaction wheel's axis would no longer be parallel to the ground, and would also affect the "yaw" motion of the robot. The system variables and parameters are tabulated in Table 1, and the state variables x_1, x_2, x_3, x_4 are graphically depicted in Fig. 3.1.

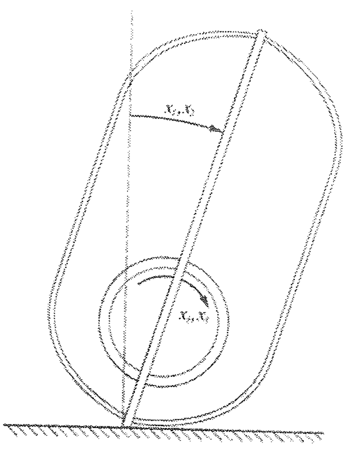

Fig. 3.1: System State Variables

The system dynamics are governed by the following equations:

$$\dot{x}_1 = x_2 \tag{3.1}$$

$$\dot{x}_2 = \frac{mgR}{I_B}\sin(x_1 - \bar{x}_1) - \frac{nk_i I_D}{I_B R_C (n^2 I_r + I_D)} v + \left(\frac{n^2 k_i k_\omega I_D}{I_B R_C (n^2 I_r + I_D)} + \frac{nk_f I_D}{I_B (n^2 I_r + I_D)} \right) x_3 \tag{3.2}$$

$$\dot{x}_3 = -\left(\frac{n^2 k_i k_\omega}{R_C (n^2 I_r + I_D)} + \frac{nk_f}{n^2 I_r + I_D} \right) x_3 + \frac{nk_i}{R_C (n^2 I_r + I_D)} v \tag{3.3}$$

$$\dot{x}_4 = x_3 \tag{3.4}$$

4. Stabilization of the Robot

For small lean angles about the equilibrium position, the system dynamics can be linearised by small angle approximation of x_1 about $\bar{x}_1$. The system can then be expressed in the standard form shown below,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{4.1}$$

where, $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$, $\mathbf{u} = \begin{bmatrix} v & \bar{x}_1 \end{bmatrix}^T$,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ mgR/I_B & 0 & A_{23} & 0 \\ 0 & 0 & A_{33} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_{23} = \frac{n^2 k_i k_\omega I_D}{I_B R_C (n^2 I_r + I_D)} + \frac{nk_f I_D}{I_B (n^2 I_r + I_D)},$$

$$A_{33} = -\left(\frac{n^2 k_i k_\omega}{R_C(n^2 I_r + I_D)} + \frac{n k_f}{n^2 I_r + I_D} \right),$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ -\dfrac{n k_i I_D}{I_B R_C(n^2 I_r + I_D)} & -\dfrac{mgR}{I_B} \\ \dfrac{n k_i}{R_C(n^2 I_r + I_D)} & 0 \\ 0 & 0 \end{bmatrix}$$

$\mathbf{B}$ can be split up into $\mathbf{b}_\mathbf{v}$ and $\mathbf{b}_{\bar{\mathbf{x}}_1}$ as:

$$\mathbf{b}_\mathbf{v} = \begin{bmatrix} 0 \\ -\dfrac{n k_i I_D}{I_B R_C(n^2 I_r + I_D)} \\ \dfrac{n k_i}{R_C(n^2 I_r + I_D)} \\ 0 \end{bmatrix}, \text{ and}$$

$$\mathbf{b}_{\bar{\mathbf{x}}_1} = \begin{bmatrix} 0 \\ -\dfrac{mgR}{I_B} \\ 0 \\ 0 \end{bmatrix}$$

where $\mathbf{b}_\mathbf{v}$ denotes the input matrix of the system when the reaction wheel motor is driven, and $\mathbf{b}_{\bar{\mathbf{x}}_1}$ denotes the input matrix of the system when $\bar{x}_1 \neq 0$. The controllability matrix using $\mathbf{A}$ and $\mathbf{b}_\mathbf{v}$ is seen to be full rank, indicating that the system is controllable by driving the reaction wheel motor.

A full state feedback LQR controller with state feedback law $v = -\mathbf{Kx}$ for the model $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{b}_\mathbf{v} v$ is then designed to stabilize the system. The matrices in the cost function

$$J = \int_0^\infty (\mathbf{x}^\mathrm{T}\mathbf{Qx} + Rv^2)dt,$$

are

$$R = 600000 \text{ and } \mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30 \end{bmatrix}$$

This particular form of $\mathbf{Q}$ ensures that at the steady state, $x_3 = 0$ and $x_1 = \bar{x}_1$. This is because from the equation of the system dynamics (Eq. 4.1),

$$\dot{x}_4 = 0 \Rightarrow x_3 = 0$$

$$\therefore \dot{x}_3 = 0 \Rightarrow v = 0$$

$$\therefore \dot{x}_2 = 0 \Rightarrow x_1 = \bar{x}_1$$

Also,

$$\therefore \dot{x}_1 = 0 \Rightarrow x_2 = 0$$

The magnitudes of the components of $\mathbf{Q}$ and R are chosen such that the system is capable of starting up from $x_1=0$ and coming to a steady state of $|\bar{x}_1| = 5$ degrees without the control, v exceeding 24 volts (the rated voltage of the reaction wheel motor). Also, $\mathbf{Q}$ and R are tuned so that the first time that $x_1 = \bar{x}_1$ is at $t \leq 1s$.

For our robot, the actual equilibrium orientation is unknown a priori. Hence, the robot is started up in close vicinity of the equilibrium point and the magnitude of rotation of the reaction wheel dynamically determines the equilibrium point. Thus the optimal controller stabilizes the robot and at that point, the magnitude of rotation of the reaction wheel lets us determine the estimate of the equilibrium point.

At steady state, only x_1 and x_4 can be non-zero. Further,

$$K_1 x_1 = -K_4 x_4$$

and the acquired estimate of $\bar{x}_1$ is given by,

$$\hat{\bar{x}}_1 = -\frac{K_4}{K_1} x_4$$

5. Experimental And Simulation Results

The optimal gain matrix obtained from the system model is given by,

$$\mathbf{K} = \begin{bmatrix} -2105 & -0.595 & -8.717 & -0.007 \end{bmatrix}$$

After some experimental tuning, the robot is seen to behave in a more robust manner with a slightly modified gain matrix of,

$$\mathbf{K} = \begin{bmatrix} -1952 & -0.624 & -8.717 & -0.006 \end{bmatrix}$$

During the experiment, the robot is started up from an initial angle of $x_1=0$ and once it has reached the steady state, the value of x_1 is noted. The system model response is then simulated with $\bar{x}_1$ set to this value. The simulation results are plotted in Fig. 5.1. The first plot in the Figure shows the lean angle (about roll axis) of the wheel along with the estimated equilibrium position of the robot. The rise time is well under 1 second and the robot settles to the equilibrium position within 2 seconds at which time the correct value of the equilibrium angle is also estimated. The second graph in the simulation results shows the time history of the roll velocity and is simply the derivative of the lean angle. The

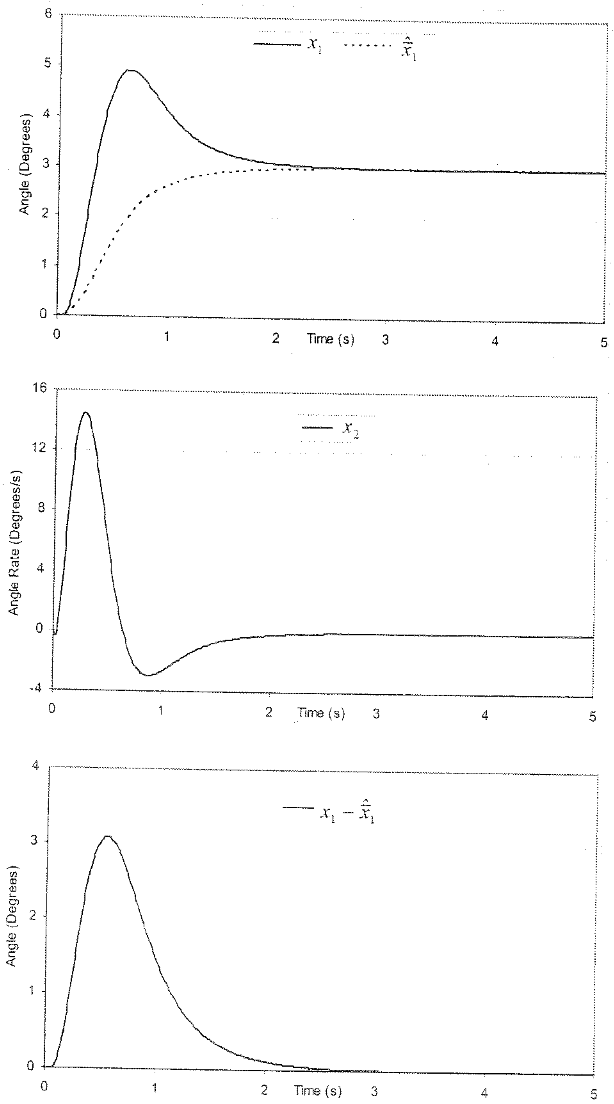

Figure 5.1: Simulation Results

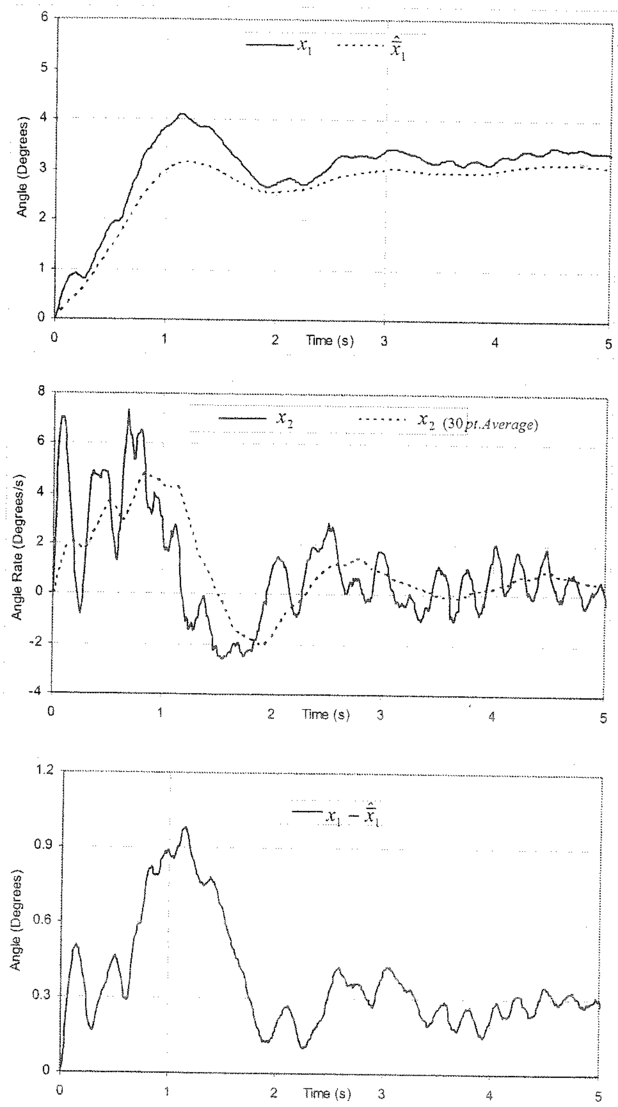

Figure 5.2: Experimental Results

last graph shows the difference between the lean angle and estimate of the equilibrium position.

The experimental results are plotted in Fig. 5.2. For obtaining these results, the robot was held approximately vertically and was let go and simultaneously the servo control was initiated. The robot was stabilized with the action of the reaction wheel. The data for this time period was saved using the telemetry link and software described in Section 2. The experimental results are generally in line with the simulation results. The major difference appears to be due to uncertainty in the system parameters. The reaction-wheel, for example, had a noticeable asymmetry. To reduce the effect of this error, a running average of 30 points is shown as dotted line in the second graph. Despite the error, the rise time remains less than 1 second but the robot continues to shows some oscillations with amplitude of less than 0.2° beyond the settling time obtained in the simulations. The last graph shows that the estimated equilibrium point remains off by approximately 0.3° even after 5 seconds.

6. Conclusion

A novel configuration of a single wheel robot, Reactobot has been reported here, which is stabilized by a reaction wheel actuator. An LQR optimal controller based on a simplified linearised model is able to stabilize the system.

The simplified model is, however, insufficient for achieving complex manoeuvres of the Reactobot. Work is currently going on to model the complete dynamics of this wheel robot and design a controller that takes into account the coupling between the different pitch, roll and yaw motions of the wheel.

References

[1] D.V. Zenkov, A.M. Bloch, and J.E. Marsden. Stabilization of the unicycle with rider. In *Decision and Control, 1999. Proceedings of the*

38th IEEE Conference on, volume 4, pages 3470–3471 vol.4, 1999.

[2] F. Grasser, A. D'Arrigo, S. Colombi, and A.C. Rufer. Joe: a mobile, inverted pendulum. *Industrial Electronics, IEEE Transactions on*, 49(1):107–114, 2002.

[3] Shu-Jen Tsai, E.D. Ferreira, and C.J.J. Paredis. Control of the gyrover. a single-wheel gyroscopically stabilized. In *Intelligent Robots and Systems, 1999. IROS '99. Proceedings. 1999 IEEE/RSJ International Conference on*, volume 1, pages 179–184 vol.1, 1999.

[4] A. Al Mamun, Z. Zhen, P. Vadakkepat, and T.H. Lee, "Tracking control of the gyrobot-a gyroscopically stabilized single-wheeled robot" in *Industrial Electronics Society, 2005. IECON 2005. 31st Annual Conference of IEEE*, pages 6 pp.–, 2005.

[5] Hofer, K., "Electric vehicle on one wheel," in *Vehicle Power and Propulsion, 2005 IEEE Conference* , pp. 5 pp.-, 7-9 Sept. 2005

[6] http://www.murata.com/new/news_release/2008/0923.html

[7] Mark W. Spong, Peter Corke, and Rogelio Lozano. Nonlinear control of the reaction wheel pendulum. *Automatica*, 37(11):1845–1851, Nov 2001.

[8] S. Majima and T. Kasai. A controller for changing the yaw direction of an underactuated unicycle robot.