

Assessment -3 [Lucky Number]

Editorial By – Joydeep Biswas

Difficulty:- Easy

Prerequisites:- Fibonacci sequence including its pattern & behaviour.

Problem Understanding:- An integer N is given, we need to develop a Fibonacci series upto N-1 (i.e:- N numbers of number should be there). Then we need to select even place number from the series of number and have to repeat this until there is only one number left & then we have to print the (number%10) unit place's number.

Solution Approach:-

[N.B:- details about subtasks or sub test cases are not mentioned in question (intentionally)]

For 20 points:-

Basic approach that we can follow is, simply calculate the Fibonacci series and then select the even place's number until it become a single number then just print the unit digit of number. But this approach will not work for 100 point because as we can see $N \leq 10^{18}$. For a moment let's forget about variable to store, just think that if you can calculate max 10^8 operation per sec. So, with this approach we are solving it with minimum $O(n \log n)$ time complexity which will not fit with in time limit.

For 100 points:- [with $O(1)$ time complexity]

You can follow two approach one you can follow a obvious pattern and hardcode it or follow the well known fact [Fibonacci Numbers are Periodic](#) , & its unit digit repeat after every 60 intervals.

Explanation:-

Solution using pattern:-

[N.B:- more informal explanation]

More informally say, when we select the even place number until there exists one number we run this operation $\log_2(N)$ times. Because we select from half of these number each time.

Now, the main pattern that you can see here, if the count of operation is less than 4 then the unit digit must be the count itself. If the number greater or equal to 4 then we have to see if the remainder of $\text{count} \% 4 == 1$ the unit digit must be 1, if $\text{count} \% 4 == 2$ unit digit must be 2, if the remainder is 3 then unit digit must be 3 and if the remainder is 0 then unit digit must be 0. [**Based on Intuition**]

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Now, this pattern happens only because the repeating nature of Fibonacci series. Unit digit repeats for every 60. Link of main source is given below (**all about Fibonacci number**).

<http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibmaths.html>

Formal Explanation:-

More formally say, when we select from half of number each time, we can see a pattern that the last number we are getting is the 2^{th} , 4^{th} , 8^{th} , 16^{th} , number from the actual series. Now if you carefully observe that on that position the number is repeating like this 0, 9, 2, 3 [unit place number]. So when we calculate the count we actually calculate the power of 2 [2^x where $x \geq 0$] is the position of the last element we can get. Now as we know the pattern on that position is repeating with 4 elements [0, 9, 2, 3] so, we can simply check the remainder of $\text{count} \% 4$, **[where count is the number of repetition we have done in order to select even place number each time until there exists exactly one element ($\log_2(N)$ which is nothing but the power of 2)]** to select the element from the repeating element set. Done.

Complexity:- $O(1)$

This editorial is given below is slightly different form mine, but this is also based on repetition of Fibonacci series. To under stand this editorial you have to know about bit manipulation additionally.

Codechef's editorial link (from original problem FIBEASY) :-

<https://discuss.codechef.com/t/fibeasy-editorial/37931/1>

N is given 11 N fibonacci numbers are { 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 }

$S = \{ 0, 1, 1, 2, 3, 5, 8, 3, 1, 4, 5 \}$

$Y = \{ 1, 2, 5, 3, 4 \} \text{ ----> } S = \{ 1, 2, 5, 3, 4 \}$

$Y = \{ 2, 3 \} \text{ -----> } S = \{ 2, 3 \}$

$Y = \{ 3 \} \text{ -----> } S = \{ 3 \}$

Output is 3

N is given 4:- N fibonacci numbers are { 0, 1, 1, 2 }

$S = \{ 0, 1, 1, 2 \}$

$Y = \{ 1, 2 \} \text{ ----> } S = \{ 1, 2 \}$

$Y = \{ 2 \} \text{ -----> } S = \{ 2 \}$

Output is 2

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Code:-

For 20 point:-

```
//  
// By Joydeep Biswas - 20/04/2020  
//  
//solution for 20 points only [note:- here we take n as integer input]  
//  
import java.io.BufferedReader;  
import java.io.InputStreamReader;  
import java.util.*;  
class TestClass {  
    public static void main(String args[] ) throws Exception {  
        BufferedReader br = new BufferedReader(new InputStreamReader(System.in));  
        int t = Integer.parseInt(br.readLine());  
        while(t-->0){  
            int n = Integer.parseInt(br.readLine());  
            if(n==1){  
                System.out.println(0); //if n==1  
            }  
            else{  
                int[] arr = new int[n]; // interger arr of size n  
                arr[0] = 0; // first element  
                arr[1] = 1; // second elemnt  
                for(int i=2;i<n;i++){  
                    arr[i] = arr[i-1]+arr[i-2]; // fn = fn-1 + fn-2  
                }  
                int number = arr[1];  
                int l=arr.length; //length of initial array  
                while(l>3){  
                    int[] temp = new int[l/2]; //arr size of half of previous  
array  
                    int c=0;  
                    for(int i=1;i<l;i+=2){  
                        temp[c] = arr[i];  
                        c++;  
                    }  
                    arr = temp; // now arr change to temp(arr is now temp)  
                    number = temp[1];  
                    l=l/2;  
                }  
                System.out.println(number%10); //unit digit  
            }  
        }  
    }  
}
```

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For 100 points:-

```
//  
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//  
//solution for 100 points  
//  
  
#include <stdio.h>  
#include <math.h>  
int main(void) {  
    // your code goes here  
    int t;  
    scanf("%d",&t);  
    while(t--)  
    {  
        long long int n;  
        scanf("%lld",&n);  
        long long int c=0;  
        c=log(n)/log(2);  
        if(c<4)  
            printf("%d\n",c);  
        else  
        {  
            if(c%4==0)  
                printf("0\n");  
            else if(c%4==1)  
                printf("9\n");  
            else if(c%4==2)  
                printf("2\n");  
            else if(c%4==3)  
                printf("3\n");  
        }  
    }  
    return 0;  
}
```