Breadth-first search

CSC263 Tutorial 8

BFS algorithm

```
BFS(G, s) /* BFS of G starting from node s */
  colour[1..n] = {white, ..., white}
  d[1..n] = {\infty, ..., \infty}
  P[1..n] = {Nil, Nil, ..., Nil}
  colour[s] = grey
  d[s] = 0
  Create new queue Q
  Q.enqueue(s)
  while Q is not empty
     u = Q.dequeue()
     for each node v adjacent to u
         if colour[v] = white then
           colour[v] = grey
           d[v] = d[u] + 1
           P[v] = u
           Q.enqueue(v)
     colour[u] = black
```

```
Suppose we have a graph

G = (V,E) containing

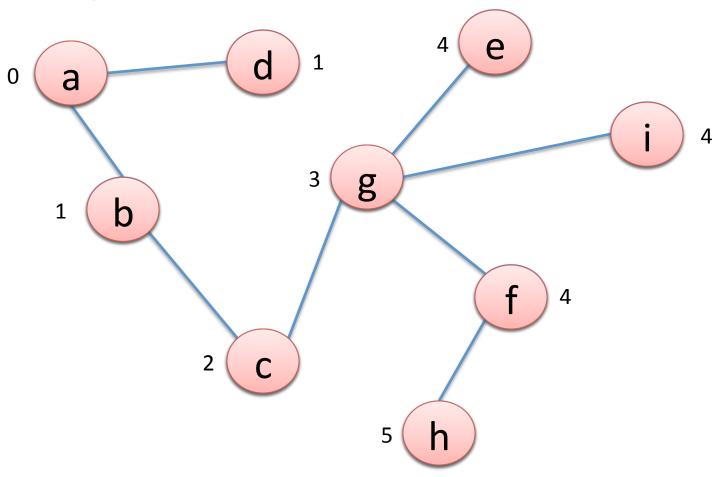
|V| = n nodes and

|E| = m edges.
```

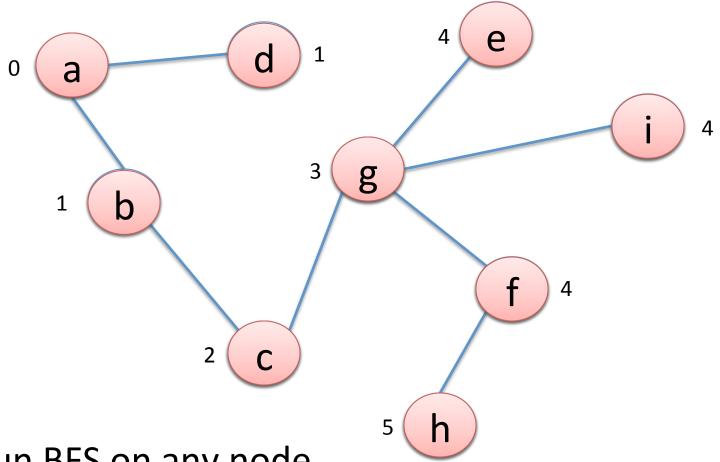
BFS takes **O(n+m)** time and space.

Example: BFS starting at node a

• All weights initially set to infinity.

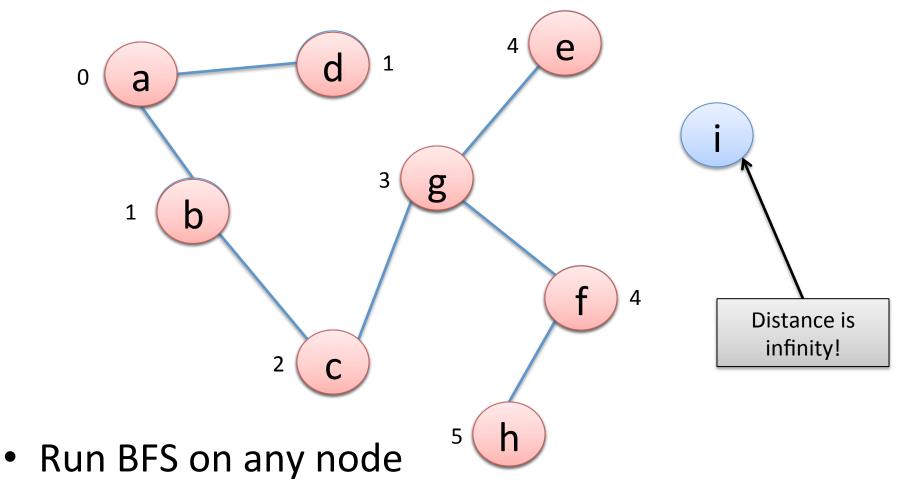


Application 1: checking connectedness



- Run BFS on any node
- If no node has distance infinity, it's connected!

Application 1: checking connectedness



If a node has distance infinity, disconnected!

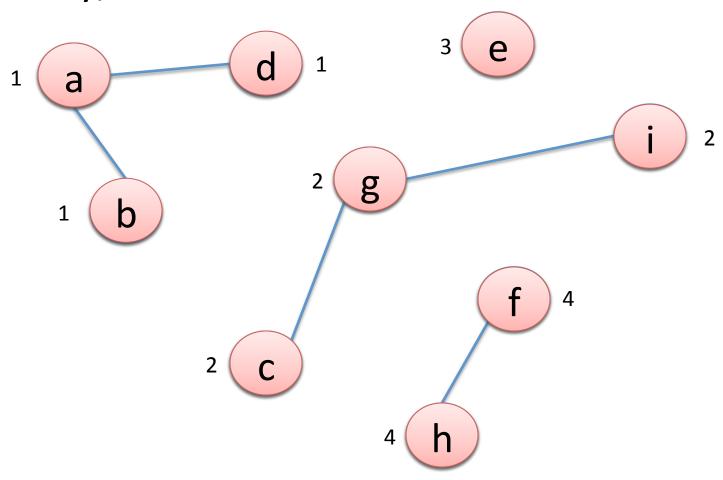
Application 2: finding connected components

Algorithm:

```
initially label each node 0
c := 1
for each node u = 1..n do
  if u is still labeled 0 then
    do a BFS starting at u
    give the label c to each node reached by the BFS
    c := c+1
  end if
end for
```

Application 2: finding connected components

Initially, each node has label 0



Application 3: finding cycles

- Repeatedly do BFS from any unvisited node, until all nodes are visited.
- In any of these BFSs, if we see an edge that points to a gray node, then there is a cycle!

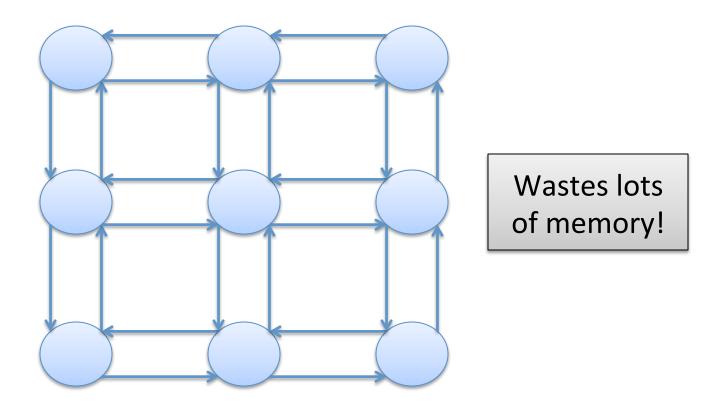
Application 3: finding cycles

Enqueue node a Dequeue node a e Enqueue node **b** 1 0 Enqueue node d g Dequeue node **b** 1 Try to enqueue a; black, so do nothing Try to enqueue d; gray node, so cycle! Can also use DFS (next week)

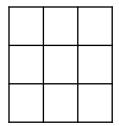
- Suppose we have an m x n grid of farms.
- Initially, one of the farms is on fire!
- At each hour, the fire spreads from each burning farm to each farm (going up, down, left and right).
- How long before all farms are on fire?

- How can we represent this problem as a graph problem?
 - Nodes are farms.
 - There is an edge between two farms if the farms are adjacent (next to one another).

- How should we store this graph in memory?
- One possibility (for a 3x3 grid of farms):

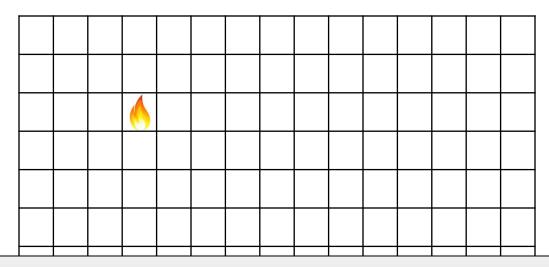


- How should we store this graph in memory?
- A more memory efficient way:
 - Store any data associated with the farms in a 3x3 array (one element for each farm)

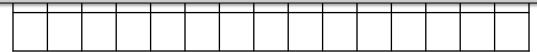


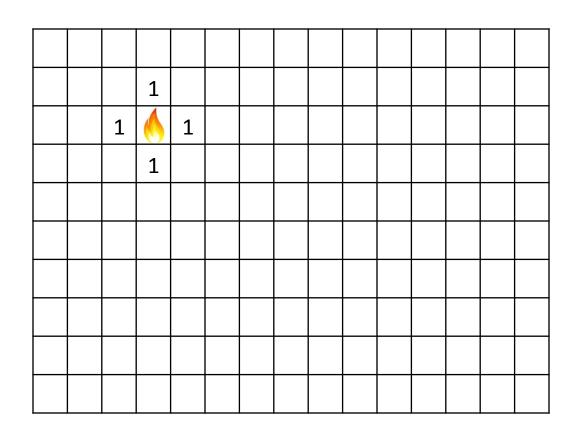
 Two farms are adjacent if their array elements are adjacent in the 3x3 array.

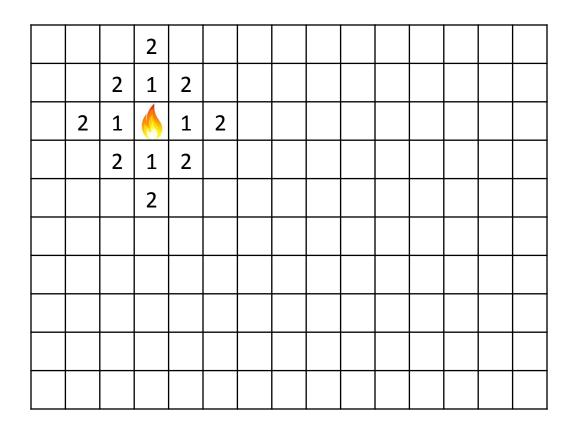
Application 4: computing distance (for a 15 x 10 grid of farms)



Solution: run BFS starting from the fire to compute the "distance" (actually time) to each farm.







		3	2	3						
	3	2	1	2	3					
3	2	1		1	2	3				
	3	2	1	2	3					
		თ	2	თ						
			3							
				_						_

	4	3	2	3	4						
4	3	2	1	2	3	4					
3	2	1		1	2	3	4				
4	3	2	1	2	3	4					
	4	3	2	3	4						
		4	3	4							
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4	3	2	1	2	3	4	5				
5	4	3	2	3	4	5					
	5	4	3	4	5						
		5	4	5							
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5	4	3	2	3	4	5	6					
4	3	2	1	2	3	4	5	6				
3	2	1		1	2	3	4	5	6			
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5	4	3	2	თ	4	5	6	7				
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	7	6	5	6	7							
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		8	7	8									

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8	7	6	5	6	7	8	9						
9	8	7	6	7	8	9							
	9	8	7	8	9								

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10	9	8	7	8	9	10								

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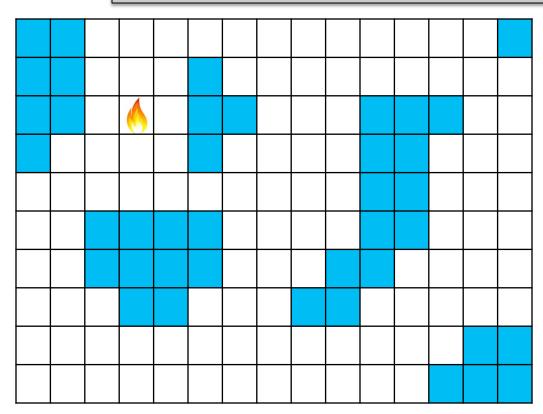
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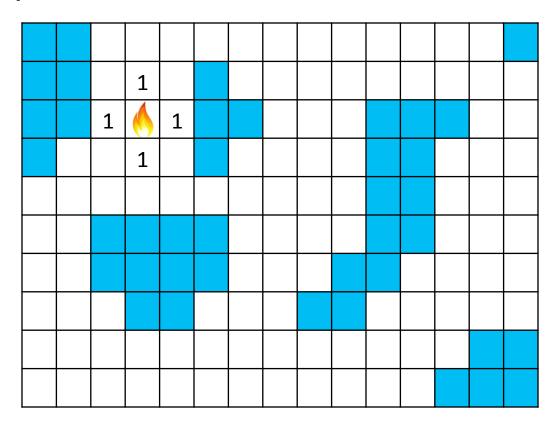
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9	8	7	6	7	8	9	10	11	12	13	14	15	16	17
10	9	8	7	8	9	10	11	12	13	14	15	16	17	18

18 hours until all farms are on fire!

• What if there are lakes in the grid, where fire cannot pass? Just don't let BFS visit the lakes!



 What if there are lakes in the grid, where fire cannot pass?



 What if there are lakes in the grid, where fire cannot pass?

		2						
	2	1	2					
	1	6	1					
	2	1	2					
		2						

 What if there are lakes in the grid, where fire cannot pass?

	3	2	3					
	2	1	2					
	1	6	1					
3	2	1	2					
	3	2	3					

	3	2	3	4					
	2	1	2						
	1	6	1						
3	2	1	2						
4	3	2	თ	4					

		3	2	3	4	5				
		2	1	2						
		1	6	1						
	3	2	1	2						
5	4	3	2	3	4	5				
	5									

		3	2	3	4	5	6				
		2	1	2		6					
		1		1							
	3	2	1	2		6					
5	4	3	2	3	4	5	6				
6	5					6					
	6										

		3	2	3	4	5	6	7			
		2	1	2		6	7				
		1	6	1							
	3	2	1	2		6	7				
5	4	3	2	თ	4	5	6	7			
6	5					6	7				
7	6					7					
	7										

		3	2	3	4	5	6	7	8			
		2	1	2		6	7	8				
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	3	2	1	2		6	7	8				
5	4	3	2	თ	4	5	6	7	8			
6	5					6	7	8				
7	6					7	8					
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		1		1			8	9				
	3	2	1	2		6	7	8	9			
5	4	3	2	თ	4	5	6	7	8			
6	5					6	7	8	9			
7	6					7	8	9				
8	7	8			9	8	9					
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		3	2	3	4	5	6	7	8	9	10		
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		1	6	1			8	9	10				
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		2	1	2		6	7	8	9	10	11		
		1	6	1			8	9	10				
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		1	6	1			8	9	10					
	თ	2	1	2		6	7	8	9					
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		1		1			8	9	10				14	
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7	6					7	8	9						
8	7	8			9	8	9			14				
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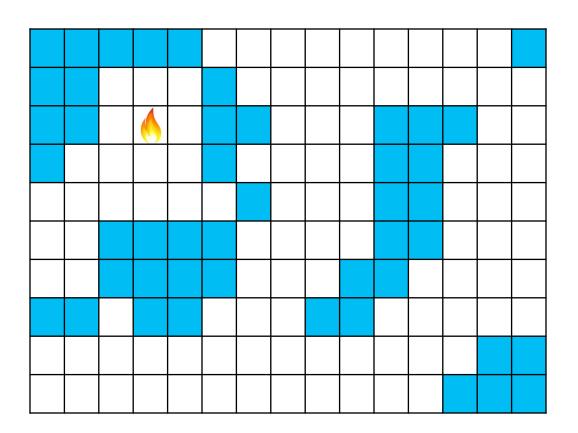
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	თ	2	1	2		6	7	8	9				15	
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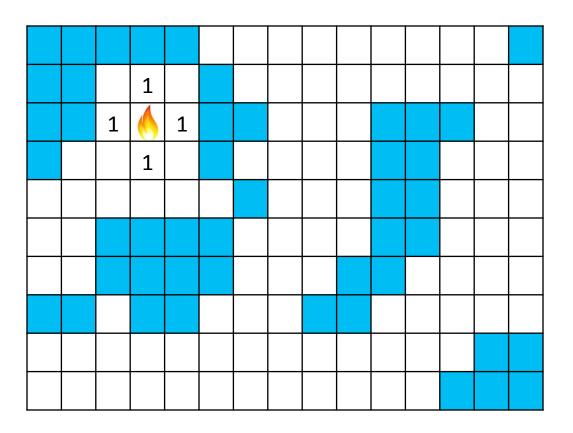
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	3	2	1	2		6	7	8	9			16	15	16
5	4	3	2	თ	4	5	6	7	8				16	
6	5					6	7	8	9					
7	6					7	8	9			16			
8	7	8			9	8	9			14	15	16		
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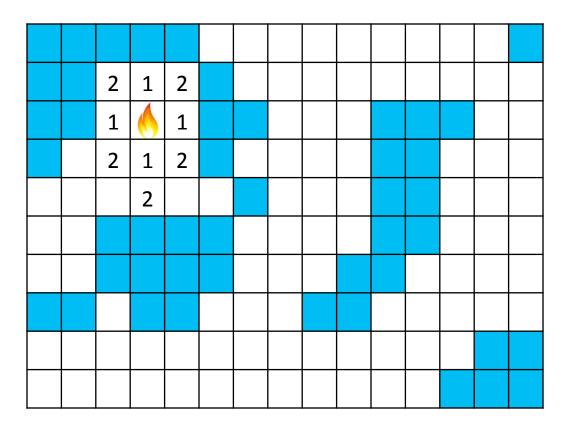
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7	6					7	8	9			16	17		
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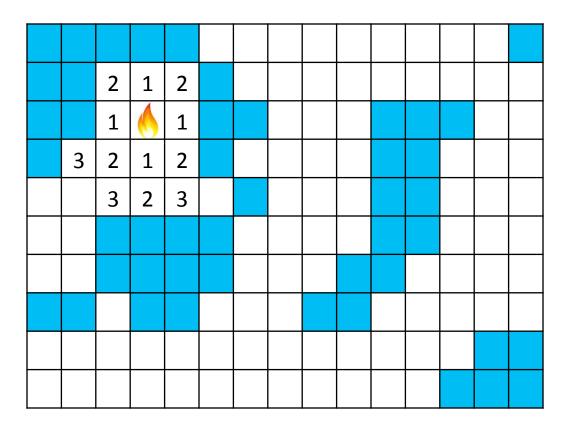
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		1		1			8	9	10				14	15
	თ	2	1	2		6	7	8	9			16	15	16
5	4	3	2	თ	4	5	6	7	8			17	16	17
6	5					6	7	8	9			18	17	18
7	6					7	8	9			16	17	18	
8	7	8			9	8	9			14	15	16	17	18
9	8	9	10	11	10	9	10	11	12	13	14	15		
10	9	10	11	12	11	10	11	12	13	14	15			

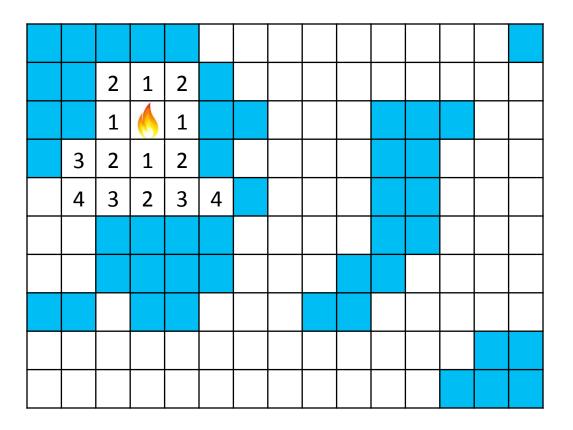
		3	2	3	4	5	6	7	8	9	10	11	12		
		2	1	2		6	7	8	9	10	11	12	13	14	
		1		1			8	9	10				14	15	
	3	2	1	2		6	7	8	9			16	15	16	
5	4	3	2	3	4	5	6	7	8			17	16	17	
6	5					6	7	8	9			18	17	18	
7	6					7	8	9			16	17	18	19	
8	7	8			9	8	9			14	15	16	17	18	
9	8	9	10	11	10	9	10	11	12	13	14	15			
10	9	10	11	12	11	10	11	12	13	14	15				.9 hours until all arms are on fire!
														10	arris are on me:

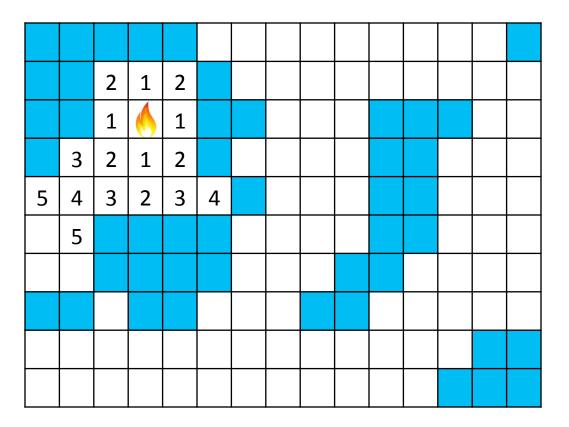


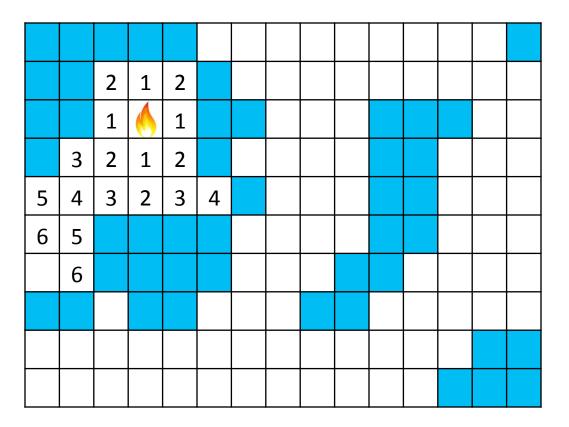


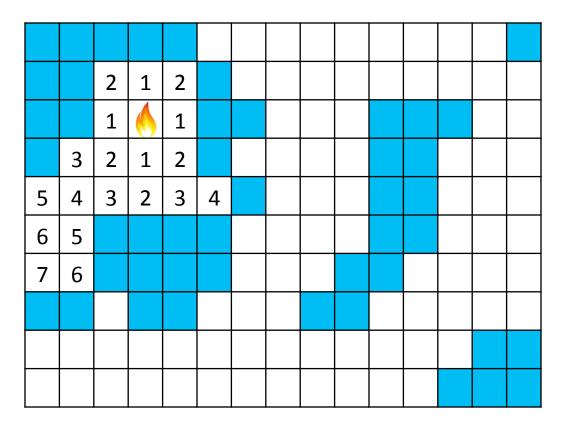


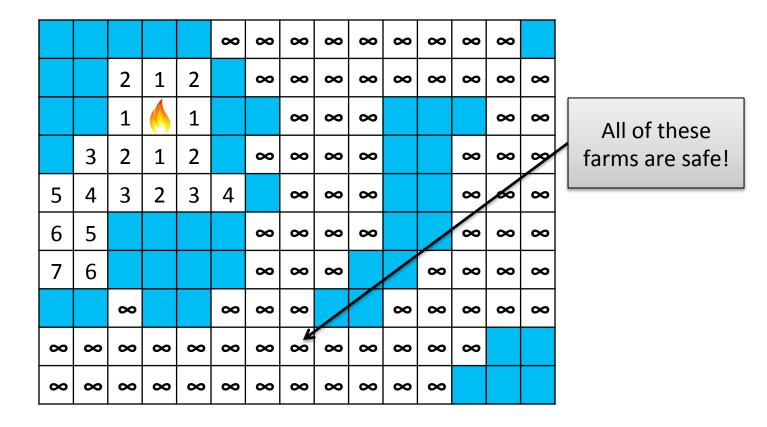




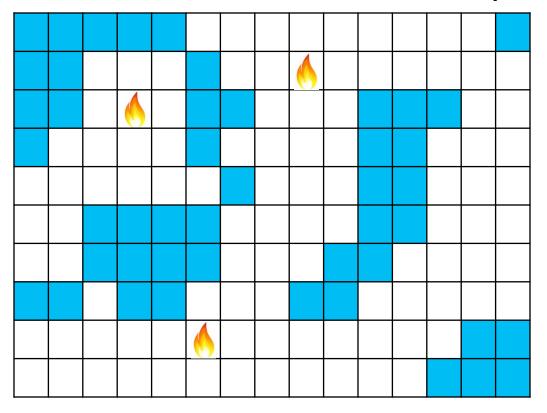




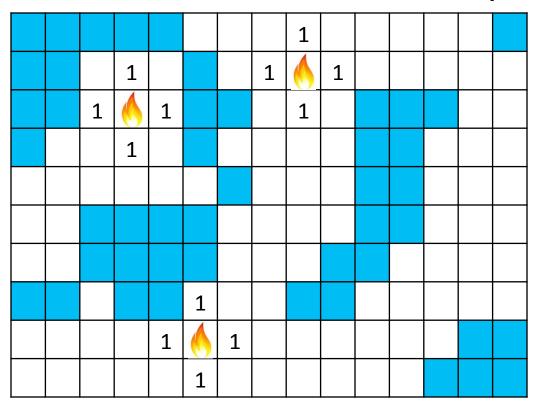




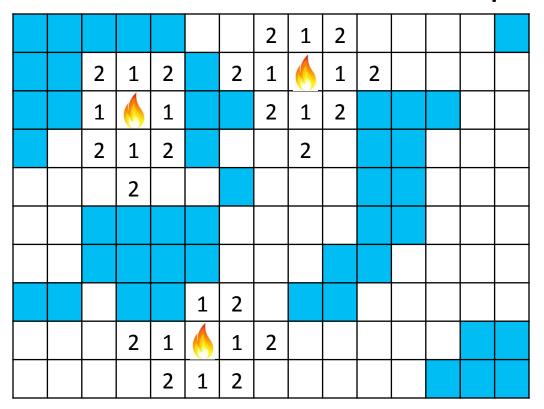
- What if multiple fires start at the same time?
- Just place all fires in the initial BFS queue!



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					3	2	1	2	3			
	2	1	2		2	1	6	1	2	3		
	1	6	1			2	1	2				
3	2	1	2			3	2	თ				
	3	2	თ				3					
					3							
				1	2	3						
	3	2	1	6	1	2	3					
		3	2	1	2	3						

- What if multiple fires start at the same time?
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				4	3	2	1	2	3	4		
	2	1	2		2	1	6	1	2	3	4	
	1	6	1			2	1	2				
3	2	1	2		4	3	2	3				
4	3	2	თ	4		4	3	4				
					4		4					
					3	4						
	4			1	2	3						
4	3	2	1		1	2	3	4				
	4	3	2	1	2	3	4					

- What if multiple fires start at the same time?
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					4	3	2	1	2	3	4	5		
		2	1	2		2	1	6	1	2	3	4	5	
		1	6	1			2	1	2					
	3	2	1	2		4	3	2	თ					
5	4	3	2	თ	4		4	3	4					
	5					4	5	4	5					
						3	4	5						
		4			1	2	3							
5	4	3	2	1	6	1	2	3	4	5				
	5	4	3	2	1	2	3	4	5					

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					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1		1	2	3	4	5	6
		1		1			2	1	2				6	
	3	2	1	2		4	3	2	3					
5	4	3	2	თ	4		4	3	4					
6	5					4	5	4	5					
	6					3	4	5						
		4			1	2	3			6				
5	4	3	2	1		1	2	3	4	5	6			
6	5	4	3	2	1	2	3	4	5	6				

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					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1	6	1	2	3	4	5	6
		1		1			2	1	2				6	7
	თ	2	1	2		4	3	2	თ				7	
5	4	3	2	თ	4		4	3	4					
6	5					4	5	4	5					
7	6					3	4	5						
		4			1	2	3			6	7			
5	4	3	2	1	6	1	2	3	4	5	6	7		
6	5	4	3	2	1	2	3	4	5	6	7			

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					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1		1	2	3	4	5	6
		1		1			2	1	2				6	7
	3	2	1	2		4	3	2	3			8	7	8
5	4	3	2	თ	4		4	3	4				8	
6	5					4	5	4	5					
7	6					3	4	5			8			
		4			1	2	3			6	7	8		
5	4	3	2	1		1	2	3	4	5	6	7		
6	5	4	3	2	1	2	3	4	5	6	7			

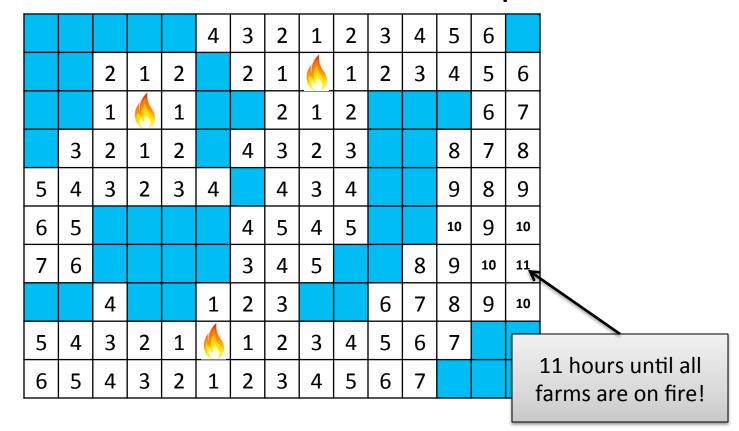
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					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1	6	1	2	3	4	5	6
		1		1			2	1	2				6	7
	3	2	1	2		4	3	2	3			8	7	8
5	4	3	2	3	4		4	3	4			9	8	9
6	5					4	5	4	5				9	
7	6					3	4	5			8	9		
		4			1	2	3			6	7	8	9	
5	4	3	2	1	6	1	2	3	4	5	6	7		
6	5	4	3	2	1	2	3	4	5	6	7			

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					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1		1	2	3	4	5	6
		1		1			2	1	2				6	7
	3	2	1	2		4	3	2	3			8	7	8
5	4	3	2	თ	4		4	3	4			9	8	9
6	5					4	5	4	5			10	9	10
7	6					3	4	5			8	9	10	
		4			1	2	3			6	7	8	9	10
5	4	3	2	1		1	2	3	4	5	6	7		
6	5	4	3	2	1	2	3	4	5	6	7			

- What if multiple fires start at the same time?
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Application 4: other modifications to think about

- What if fires start at different times?
- What if fires spread at different speeds?

A neat problem to think about

• It's the year 2241. You're in a cave and it's collapsing! A device tells you when each section of ceiling will cave in. Can you escape?

					19	16	14	12	16	13	20	21	19	
		27	24	20		20	10	9	18	20	17	16	17	18
		23	18	16			9	12	14				14	20
	19	20	16	13		7	7	9	11			16	24	20
6	7	Ė	12	14	20	5	8	12	9			25	22	15
5	6					14	9	14	10			25	23	21
1	1					15	16	11			19	26	19	25
		2			12	18	24			24	13	27	30	5]
7	8	13	17	17	13	19	22	20	21	23	20	12		
10	12	15	18	16	15	20	18	16	19	18	15			

A neat problem to think about

 It's the year 2241. You're in a cave and it's collapsing! A device tells you section of ceiling will cave in

Here's a sample solution. How would you solve it in general?

						1								
					19	16	14	12	16	13	20	21	19	
		27	24	20		20	10	7	18	20	17	16	17	18
		23	18	16			9	12	14				14	20
	19	20	16	13		7	7	9	11			16	24	20
6	7	•==	12	14	20	5	8	12	9			25	22	15
5	6					14	9	14	10			25	23	21
1	1					15	16	11			19	26	19	25
		2			12	18	24			24	13	27	30	5
7	8	13	17	17	13	19	22	20	21	23	20	12		
10	12	15	18	16	15	20	18	16	19	18	15			