1. Let $P(n)$ be a predicate "Positive integer n can be written in base 2". For example, $5 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$, so $P(5)$ is true. $P(n)$ can be written formally as follows:

$$(\exists k \in \mathbb{N}, \exists c_0, c_1, \ldots, c_k \in \{0,1\}) : n = \sum_{i=0}^{k} c_i 2^i$$

   For instance, for $n = 5$, we let $k = 2, c_0 = 1, c_1 = 0, c_2 = 2$, so $5 = \sum_{i=0}^{2} c_i 2^i$.

   Prove the following statement :

$$\forall n \in \mathbb{Z}^+ : P(n).$$

2. Use the definition of big-Ω to prove or disprove:

$$6n^3 - 4n^2 + 3n + 2 \in \Omega(5n^3 - n^2 + n + 1).$$

3. Use the definition of big-O to prove that:

$$\frac{n^3 + 1}{2n + 1} \in O(n^2).$$

4. Use the definition of big-Ω to prove that:

$$\frac{1}{5}n^2 - 42n - 8 \in \Omega(n^2).$$

5. Use the definition of big-Ω to prove or disprove:

$$9n^2 + 3n - 1 \in \Omega(n^3).$$