1. Prove that no natural number between two consecutive squares can be a square. This formally can be expressed as:

$$\forall n \in \mathbb{N} : \forall k \in \mathbb{N} : (n > 0 \land k^2 < n < (k+1)^2) \Rightarrow (\forall m \in \mathbb{N} : n \neq m^2).$$

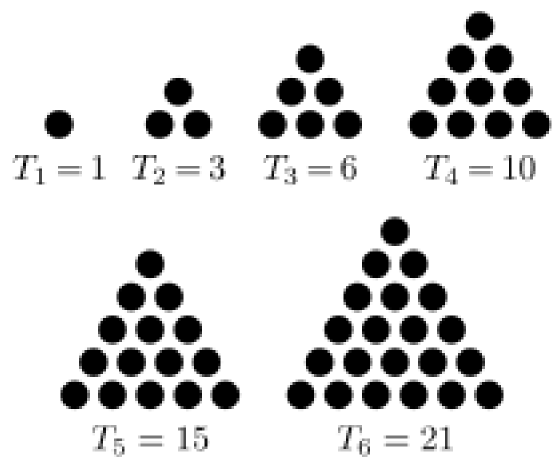
2. Prove that for all positive natural numbers n, the following expression cannot be a complete square:

$$n + \left\lfloor \sqrt{n} + \frac{1}{2} \right\rfloor$$

3. Let a be the sequence: $0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3, 0, \ldots$. prove or disprove the following claim:

$$\exists i \in \mathbb{N} : [\forall j \in \mathbb{N} : [a_j \neq a_i \Rightarrow [\exists k \in \mathbb{N} : a_k = 1 + a_j]]]$$

4. If marbles are arranged to form an equilateral triangle shape, with n marbles on each side, a total of $\sum_{i=1}^{n} i$ marbles will be required.



In lecture, we proved that $\sum_{i=1}^{n} i = n(n+1)/2$. For each $n \in \mathbb{N}$, let $T_n = n(n+1)/2$; these numbers are usually called the *triangular numbers*. Prove the following:

$$\forall n \in \mathbb{N} : \sum_{j=0}^{n} T_j = \frac{n(n+1)(n+2)}{6}$$