CSC236 tutorial exercises, Week #12
best before Thursday evening

These exercises are intended to give you some practice manipulating regular expressions.

1. Let $\Sigma = \{0, 1\}$. For each language below provide a regular expression over Σ

   (a) $L_1 = \{x \in \Sigma \mid x$ has an even number of 1s or an odd number of 0s$\}$.

   sample solution: I design the regex for each part, and then combine them. Two 1s can be denoted by a prefix of 0 or more 0s, followed by a pair of 1s separated by 0 or more 0s, followed by 0 or more 0s. Repeat the pairs of 1s as needed: $0 * (10 * 10*)*$ for an even number of 1s. An odd number of 0s can be denoted by an expression for the first 0, concatenated with the expression for an even number of 0s, $1 * 01 * (01 * 01*)*$. Combine the two with union:

   $$0 * (10 * 10*) * +1 * 01 * (01 * 01*)*$$

   (b) $L_2 = \{x \in \Sigma \mid x$ has at least one 1 and at least one 0$\}$.

   sample solution: One expression where the required 1 precedes the required 0, one expression where it follows, then take the union:

   $$(0 + 1) * 1(0 + 1) * 0(0 + 1) * +(0 + 1) * 0(0 + 1) * 1(0 + 1)*$$

   (c) $L_3 = \{x \in \Sigma \mid$ every 1 in x is immediately preceded and followed by a 0$\}$

   sample solution: If $x \neq \varepsilon$ then it must end in at least one zero, preceded by repeats of the $0 * 01$ pattern. Otherwise $x = \varepsilon$:

   $$(0 * 01) * 00 * +\varepsilon$$

2. Let $\Sigma = \{0, 1\}$, let $\mathcal{RE}$ be the regular expressions over Σ, and let $r_1, r_2, r_3 \in \mathcal{RE}$. Say whether each of the following is true or false, and justify your claim:

   (a) If $r_1 r_2 \equiv r_2 r_1$ and $r_1 \not\equiv \varepsilon \not\equiv r_2$ and $r_1 \not\equiv \emptyset \not\equiv r_2$, then $r_1 \equiv r_2$.

   sample solution: False. As a counterexample $r_1 = 0, r_2 = 0*$.

   (b) If $r_1 r_2 \equiv r_1 r_3$ and $r_1 \not\equiv \emptyset$, then $r_2 \equiv r_3$.

   sample solution: False. As a counterexample $r_1 = (0 + 1)*, r_2 = \varepsilon, r_3 = (0 + 1)*$.