Breadth-first search

CSC263 Tutorial 8

BFS algorithm

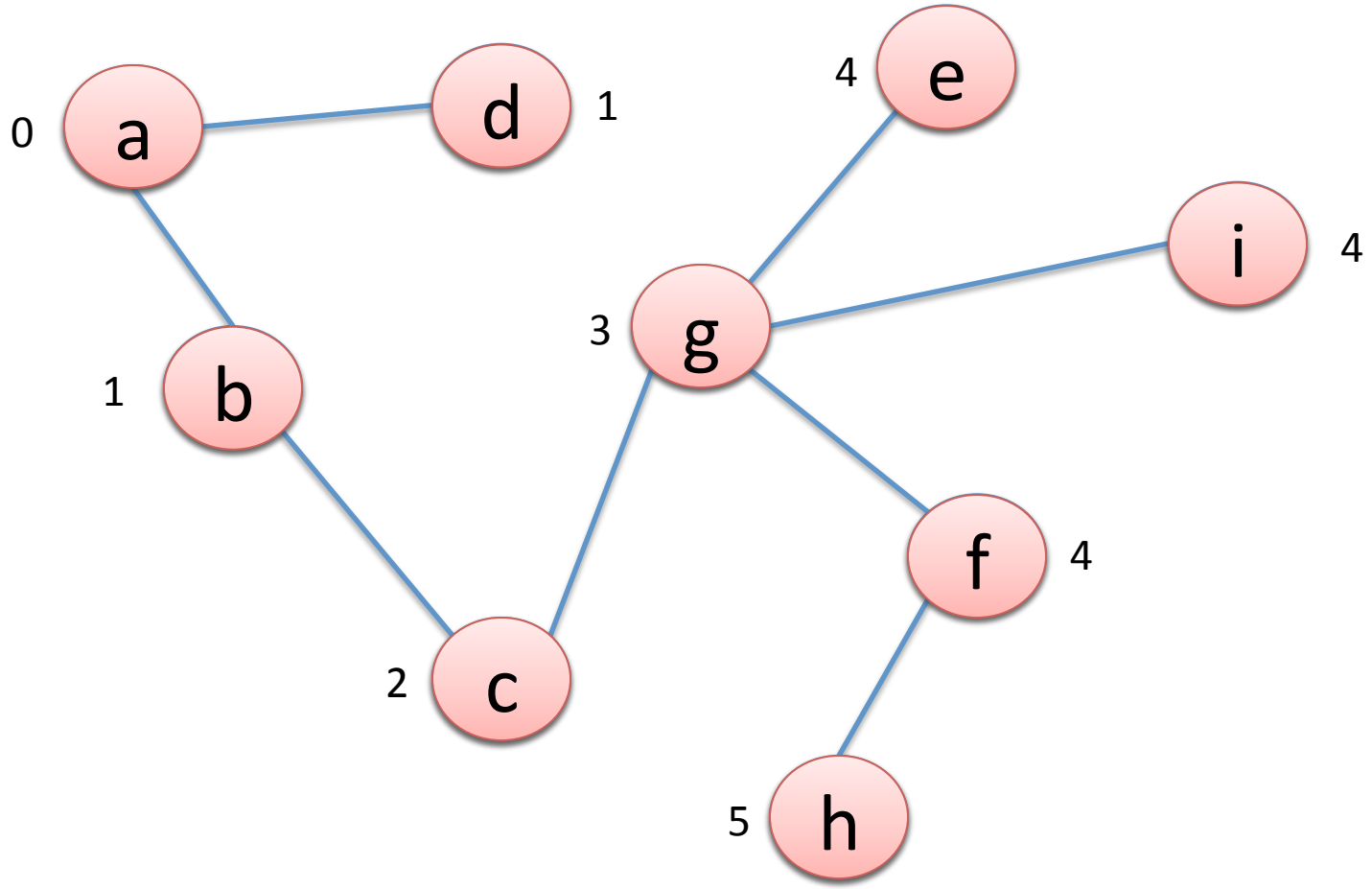
```
BFS(G, s)      /* BFS of G starting from node s */
   colour[1..n] = {white, …, white}
   d[1..n] = {∞, …, ∞}
   P[1..n] = {Nil, Nil, …, Nil}
   colour[s] = grey
   d[s] = 0
   Create new queue Q
   Q.enqueue(s)

   while Q is not empty
        u = Q.dequeue()
        for each node v adjacent to u
             if colour[v] = white then
                  colour[v] = grey
                  d[v] = d[u] + 1
                  P[v] = u
                  Q.enqueue(v)
        colour[u] = black
```

Suppose we have a graph **G = (V,E)** containing **|V| = n** nodes and **|E| = m** edges.

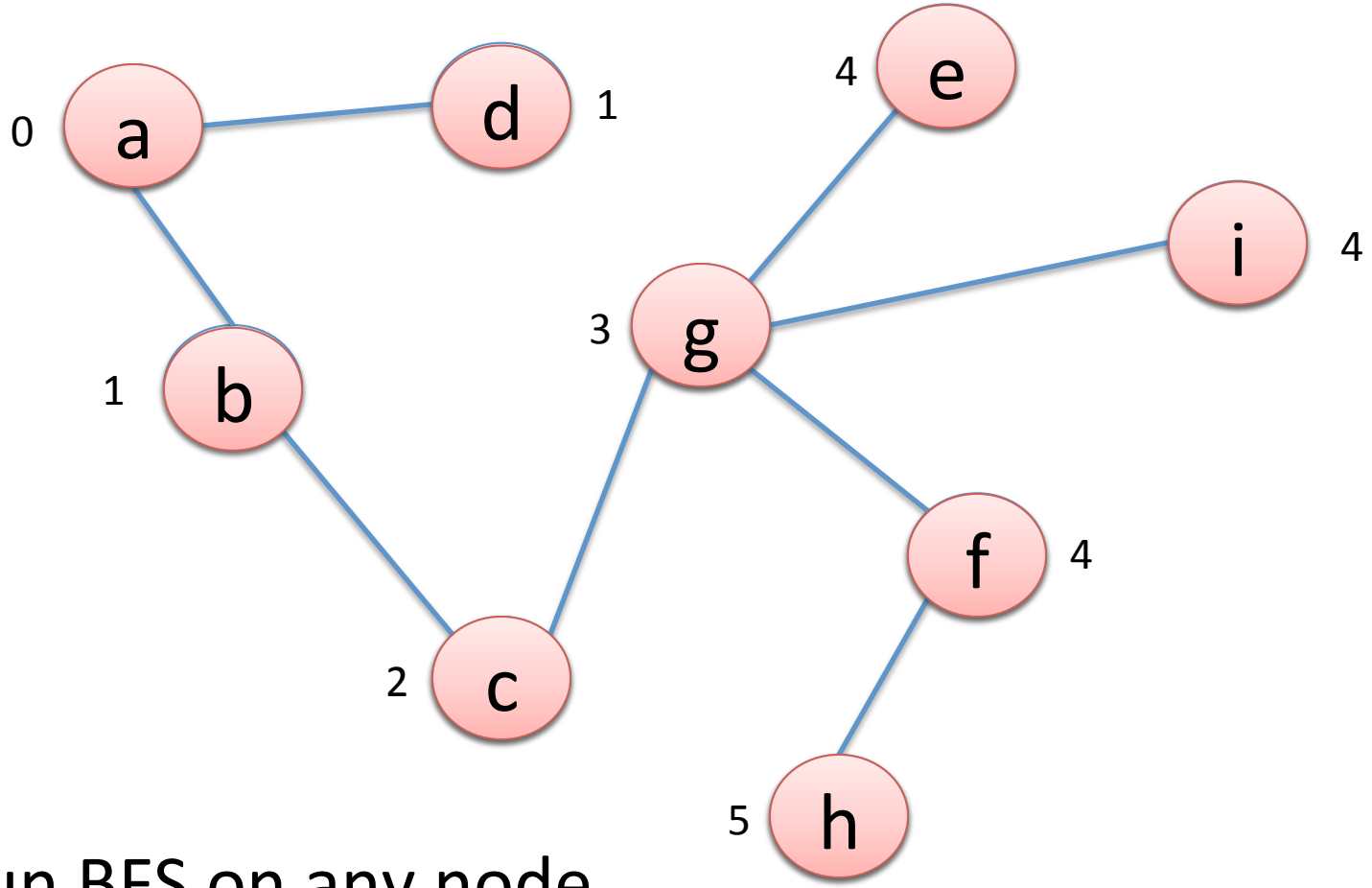
BFS takes **O(n+m)** time and space.

Example: BFS starting at node **a**

- All weights initially set to infinity.

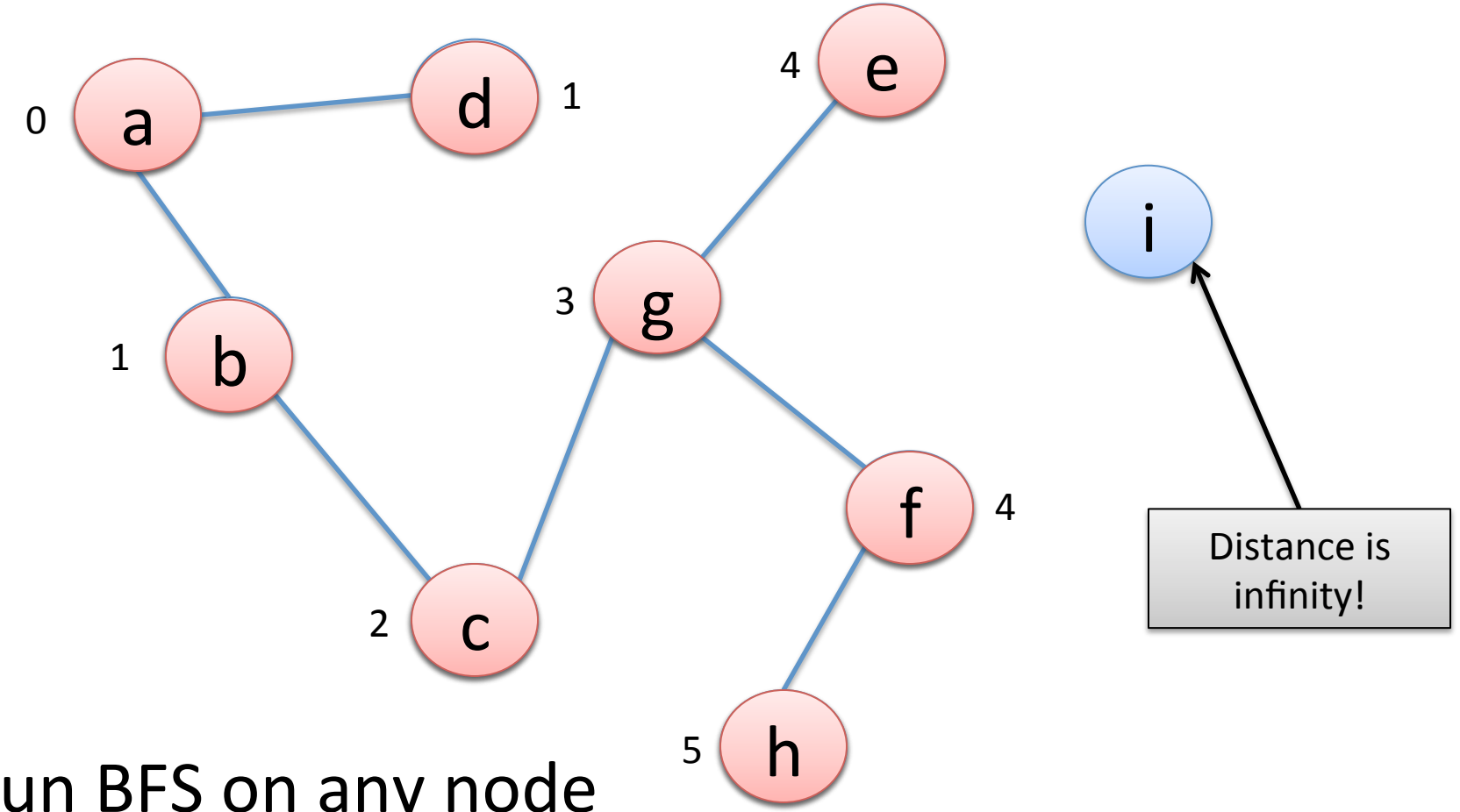


Application 1: checking connectedness



- Run BFS on any node
- If no node has distance infinity, it's **connected**!

Application 1: checking connectedness



- Run BFS on any node
- If a node has distance infinity, **disconnected**!

Application 2: finding connected components

Algorithm:

initially label each node 0

c := 1

for each node u = 1..n do

    if u is still labeled 0 then

        do a BFS starting at u

        give the label c to each node reached by the BFS

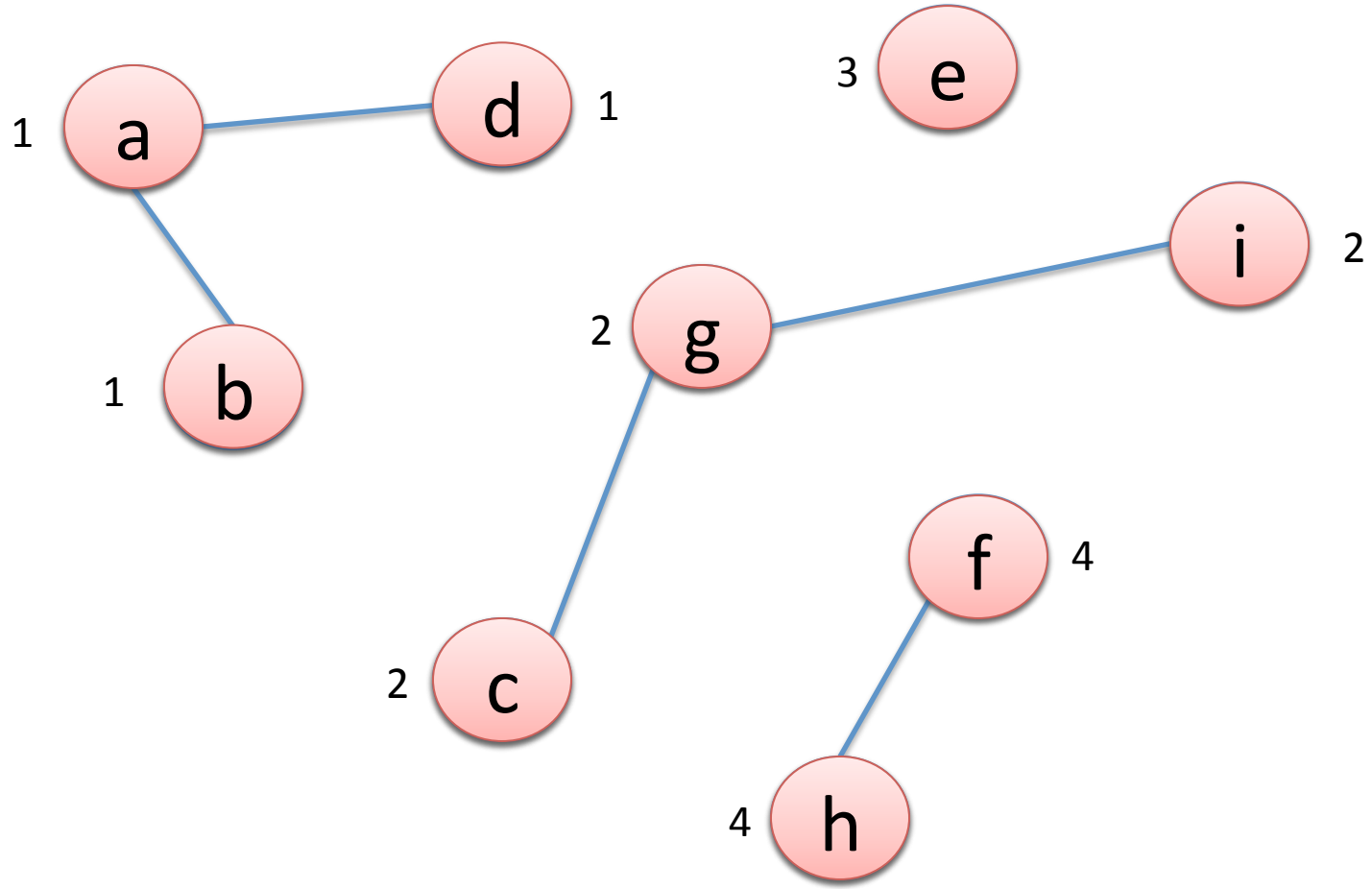
        c := c+1

    end if

end for

Application 2: finding connected components

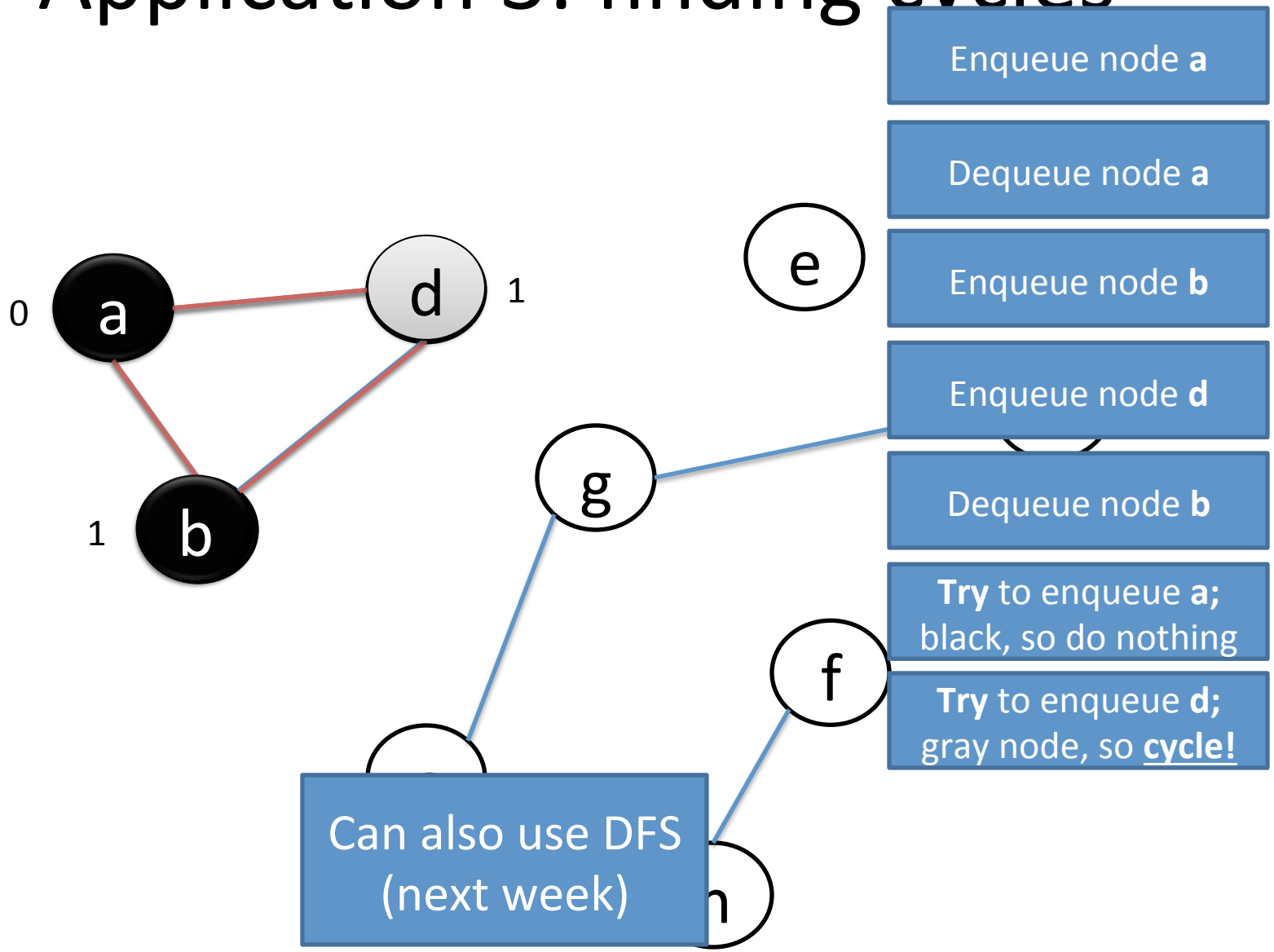
- Initially, each node has label 0



Application 3: finding cycles

- Repeatedly do BFS from any unvisited node, until all nodes are visited.
- In any of these BFSs, if we see an edge that points to a **gray** node, then **there is a cycle**!

Application 3: finding cycles



Application 4: computing distance

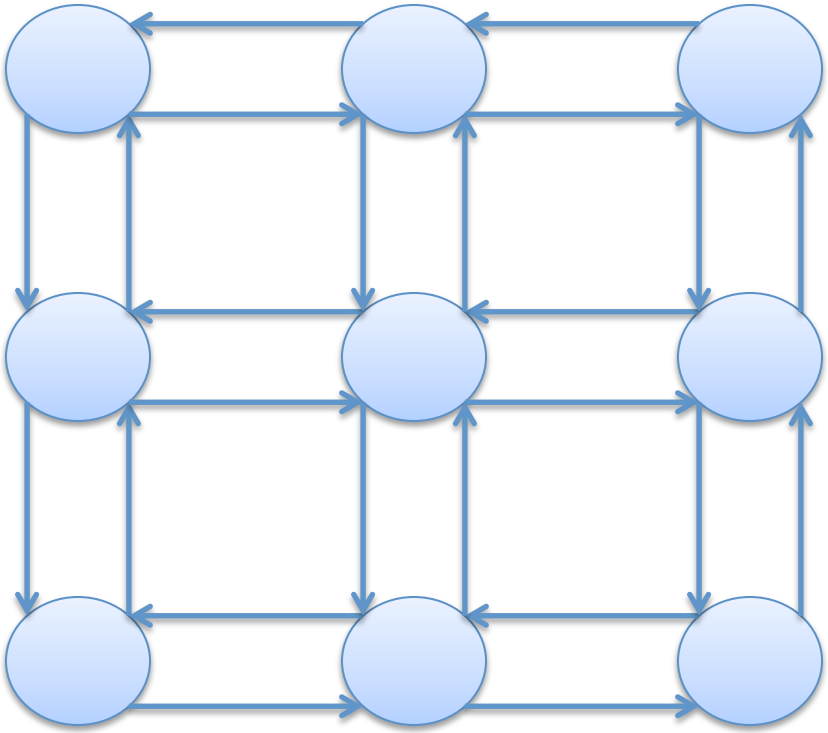
- Suppose we have an **m** x **n** grid of farms.
- Initially, one of the farms is on fire!
- At each hour, the fire spreads from each burning farm to each farm (going up, down, left and right).
- How long before all farms are on fire?

Application 4: computing distance

- How can we represent this problem as a graph problem?
  - Nodes are farms.
  - There is an edge between two farms if the farms are adjacent (next to one another).

Application 4: computing distance

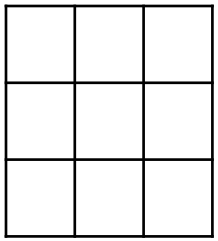
- How should we store this graph in memory?
- One possibility (for a 3x3 grid of farms):



Wastes lots of memory!

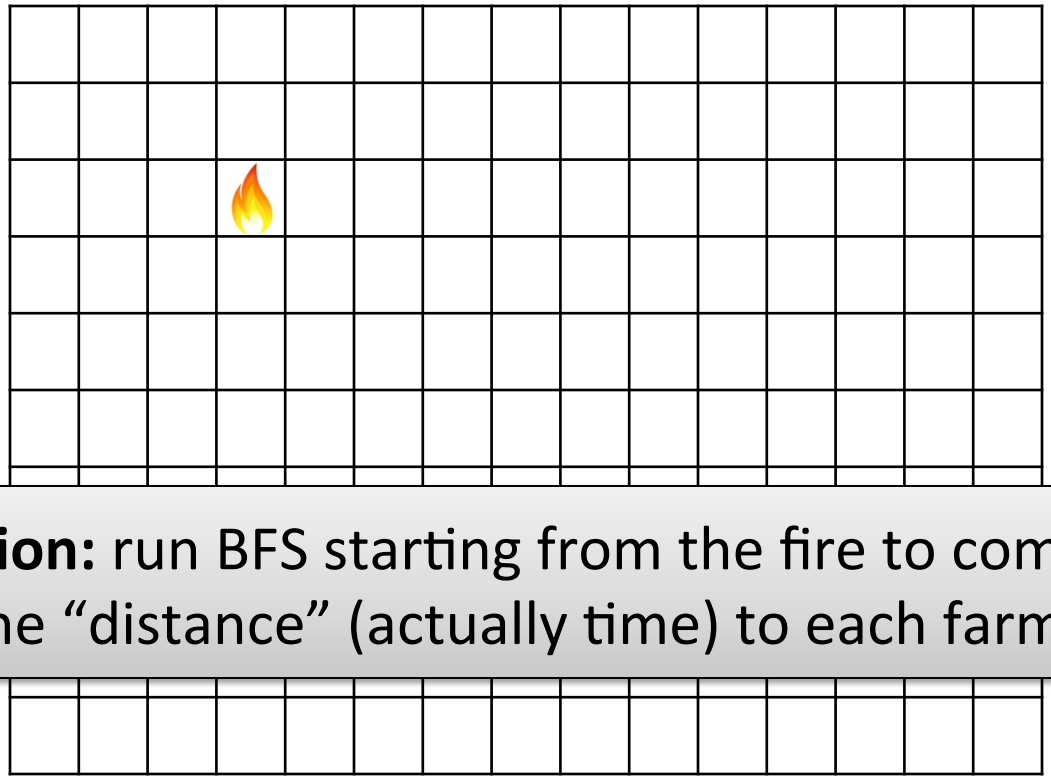
Application 4: computing distance

- How should we store this graph in memory?
- A more memory efficient way:
  - Store any data associated with the farms in a 3x3 array (one element for each farm)



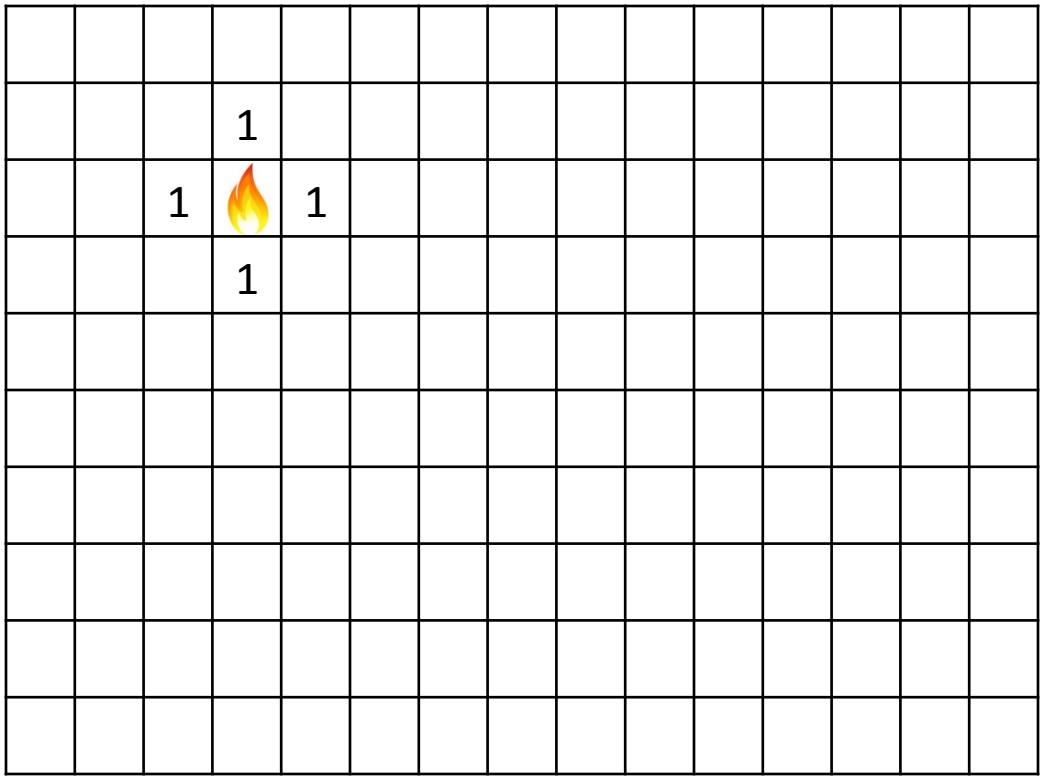
  - Two farms are adjacent if their array elements are adjacent in the 3x3 array.

Application 4: computing distance (for a 15 x 10 grid of farms)

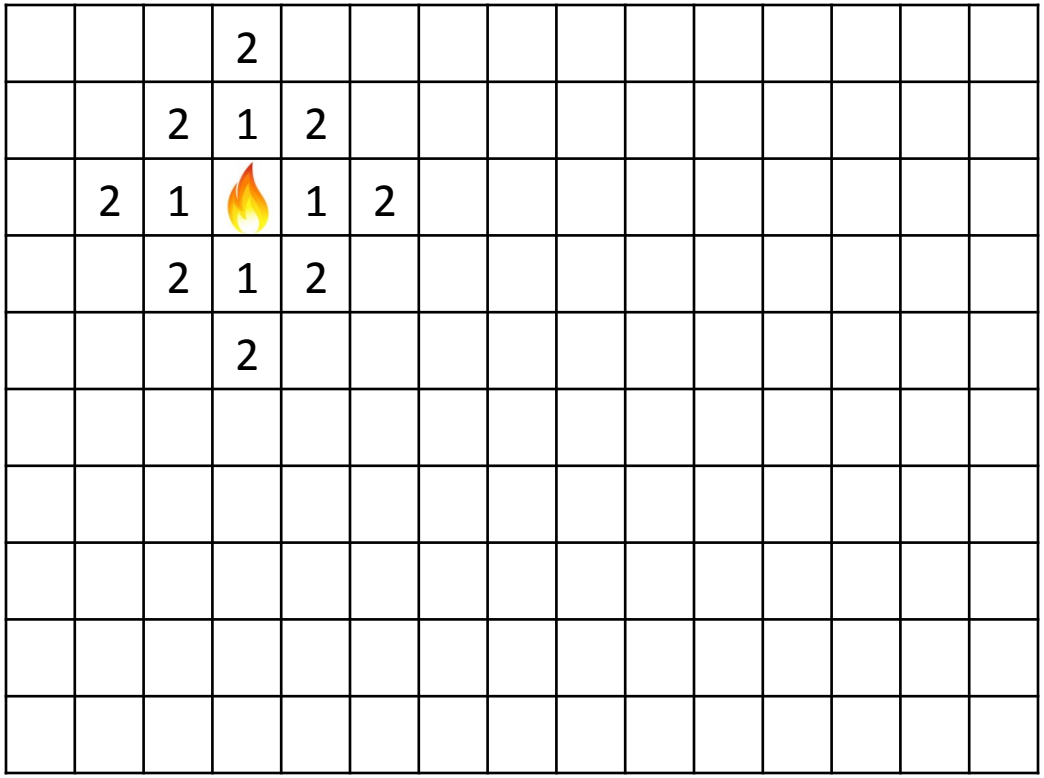


Solution: run BFS starting from the fire to compute the "distance" (actually time) to each farm.

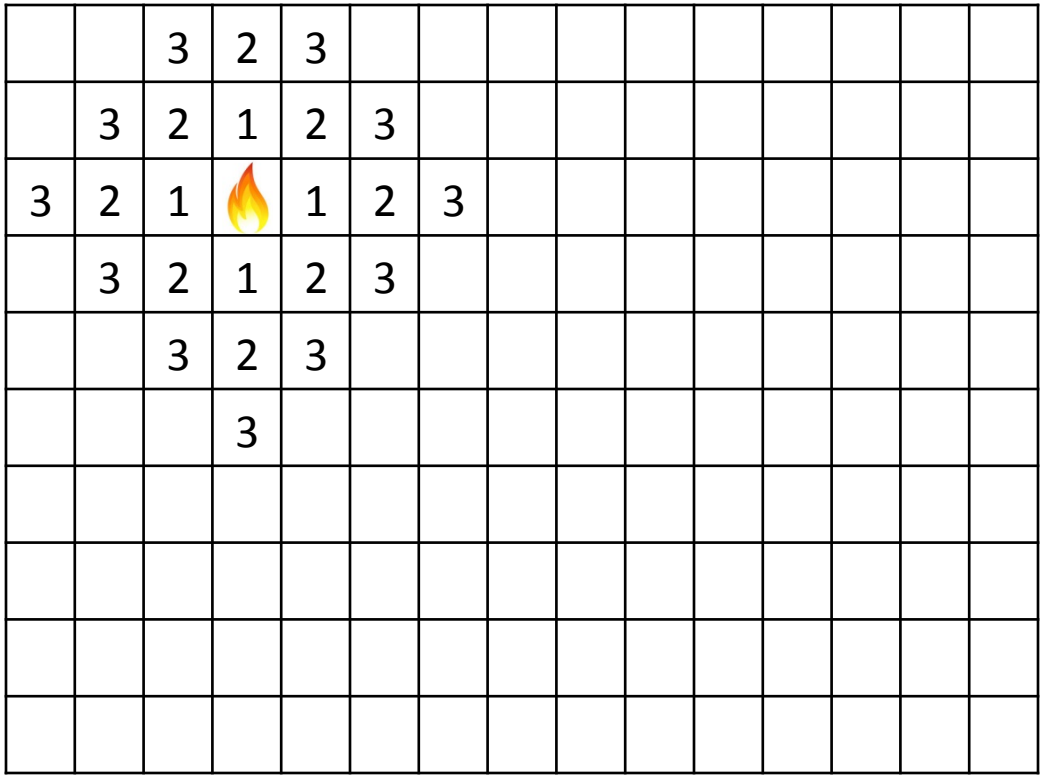
Application 4: computing distance



Application 4: computing distance



Application 4: computing distance



Application 4: computing distance

	4	3	2	3	4									
4	3	2	1	2	3	4								
3	2	1	🔥	1	2	3	4							
4	3	2	1	2	3	4								
	4	3	2	3	4									
		4	3	4										
			4											

Application 4: computing distance

5	4	3	2	3	4	5								
4	3	2	1	2	3	4	5							
3	2	1		1	2	3	4	5						
4	3	2	1	2	3	4	5							
5	4	3	2	3	4	5								
	5	4	3	4	5									
		5	4	5										
			5											

Application 4: computing distance

5	4	3	2	3	4	5	6							
4	3	2	1	2	3	4	5	6						
3	2	1		1	2	3	4	5	6					
4	3	2	1	2	3	4	5	6						
5	4	3	2	3	4	5	6							
6	5	4	3	4	5	6								
	6	5	4	5	6									
		6	5	6										
			6											

Application 4: computing distance

5	4	3	2	3	4	5	6	7						
4	3	2	1	2	3	4	5	6	7					
3	2	1	🔥	1	2	3	4	5	6	7				
4	3	2	1	2	3	4	5	6	7					
5	4	3	2	3	4	5	6	7						
6	5	4	3	4	5	6	7							
7	6	5	4	5	6	7								
	7	6	5	6	7									
		7	6	7										
			7											

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8					
4	3	2	1	2	3	4	5	6	7	8				
3	2	1		1	2	3	4	5	6	7	8			
4	3	2	1	2	3	4	5	6	7	8				
5	4	3	2	3	4	5	6	7	8					
6	5	4	3	4	5	6	7	8						
7	6	5	4	5	6	7	8							
8	7	6	5	6	7	8								
	8	7	6	7	8									
		8	7	8										

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9				
4	3	2	1	2	3	4	5	6	7	8	9			
3	2	1		1	2	3	4	5	6	7	8	9		
4	3	2	1	2	3	4	5	6	7	8	9			
5	4	3	2	3	4	5	6	7	8	9				
6	5	4	3	4	5	6	7	8	9					
7	6	5	4	5	6	7	8	9						
8	7	6	5	6	7	8	9							
9	8	7	6	7	8	9								
	9	8	7	8	9									

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10			
4	3	2	1	2	3	4	5	6	7	8	9	10		
3	2	1	🔥	1	2	3	4	5	6	7	8	9	10	
4	3	2	1	2	3	4	5	6	7	8	9	10		
5	4	3	2	3	4	5	6	7	8	9	10			
6	5	4	3	4	5	6	7	8	9	10				
7	6	5	4	5	6	7	8	9	10					
8	7	6	5	6	7	8	9	10						
9	8	7	6	7	8	9	10							
10	9	8	7	8	9	10								

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11		
4	3	2	1	2	3	4	5	6	7	8	9	10	11	
3	2	1	🔥	1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	
5	4	3	2	3	4	5	6	7	8	9	10	11		
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9	8	7	6	7	8	9	10	11						
10	9	8	7	8	9	10	11							

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1	🔥	1	2	3	4	5	6	7	8	9	10	11
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7	6	5	4	5	6	7	8	9	10	11	12			
8	7	6	5	6	7	8	9	10	11	12				
9	8	7	6	7	8	9	10	11	12					
10	9	8	7	8	9	10	11	12						

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1	🔥	1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
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7	6	5	4	5	6	7	8	9	10	11	12	13		
8	7	6	5	6	7	8	9	10	11	12	13			
9	8	7	6	7	8	9	10	11	12	13				
10	9	8	7	8	9	10	11	12	13					

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1	🔥	1	2	3	4	5	6	7	8	9	10	11
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8	7	6	5	6	7	8	9	10	11	12	13	14		
9	8	7	6	7	8	9	10	11	12	13	14			
10	9	8	7	8	9	10	11	12	13	14				

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
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6	5	4	3	4	5	6	7	8	9	10	11	12	13	14
7	6	5	4	5	6	7	8	9	10	11	12	13	14	15
8	7	6	5	6	7	8	9	10	11	12	13	14	15	
9	8	7	6	7	8	9	10	11	12	13	14	15		
10	9	8	7	8	9	10	11	12	13	14	15			

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
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7	6	5	4	5	6	7	8	9	10	11	12	13	14	15
8	7	6	5	6	7	8	9	10	11	12	13	14	15	16
9	8	7	6	7	8	9	10	11	12	13	14	15	16	
10	9	8	7	8	9	10	11	12	13	14	15	16		

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
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8	7	6	5	6	7	8	9	10	11	12	13	14	15	16
9	8	7	6	7	8	9	10	11	12	13	14	15	16	17
10	9	8	7	8	9	10	11	12	13	14	15	16	17	

Application 4: computing distance

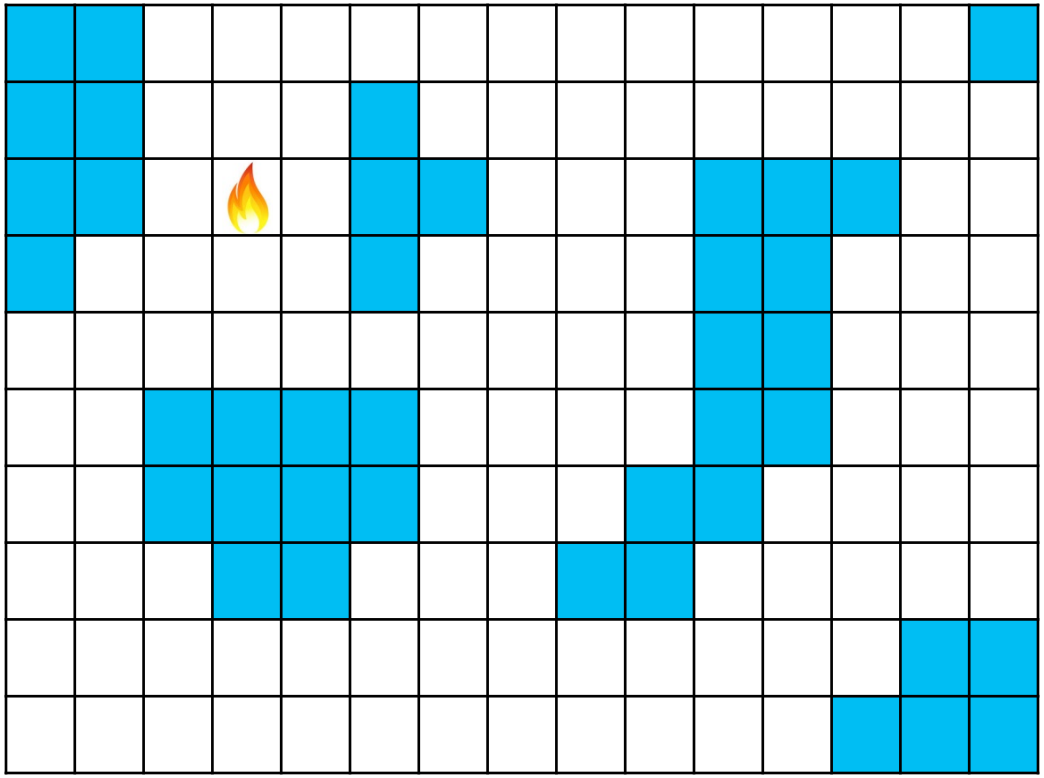
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
6	5	4	3	4	5	6	7	8	9	10	11	12	13	14
7	6	5	4	5	6	7	8	9	10	11	12	13	14	15
8	7	6	5	6	7	8	9	10	11	12	13	14	15	16
9	8	7	6	7	8	9	10	11	12	13	14	15	16	17
10	9	8	7	8	9	10	11	12	13	14	15	16	17	18

18 hours until all farms are on fire!

Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

Just don't let BFS visit the lakes!



Application 4: **modification 1**

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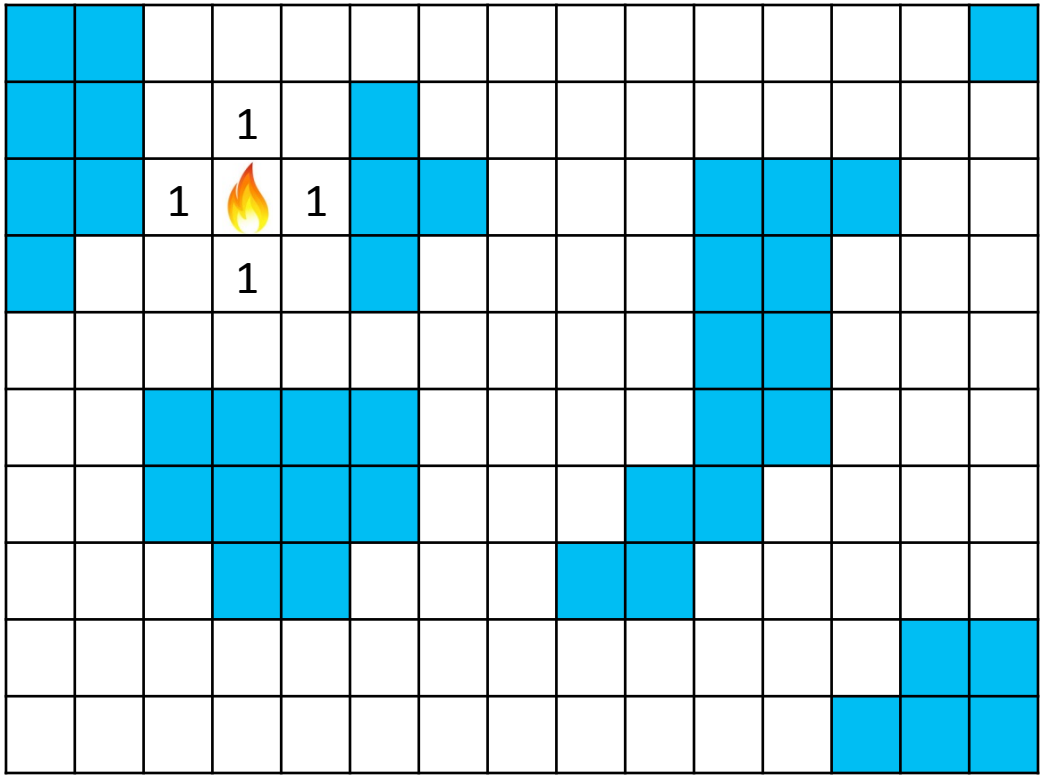

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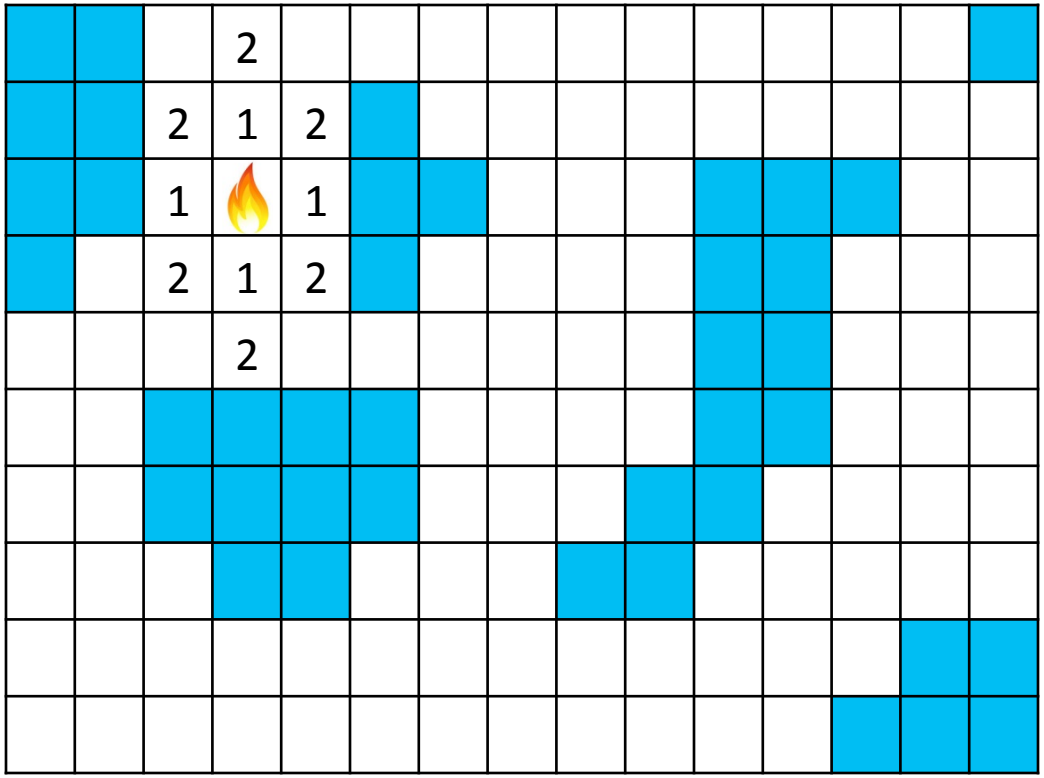

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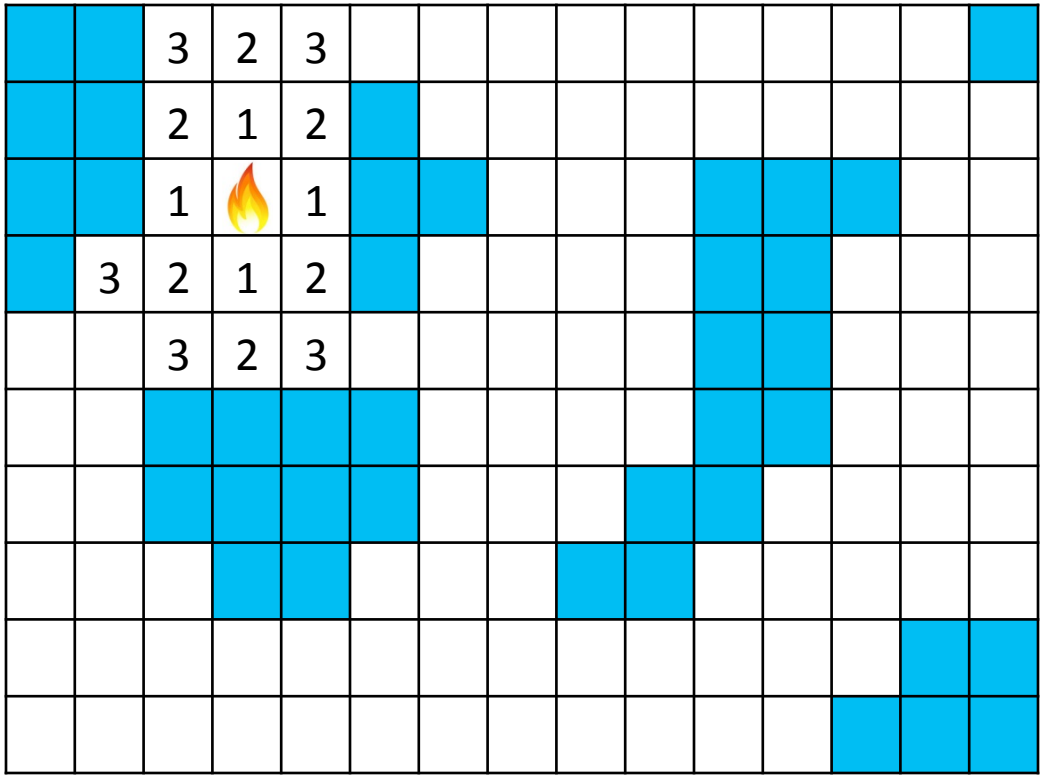

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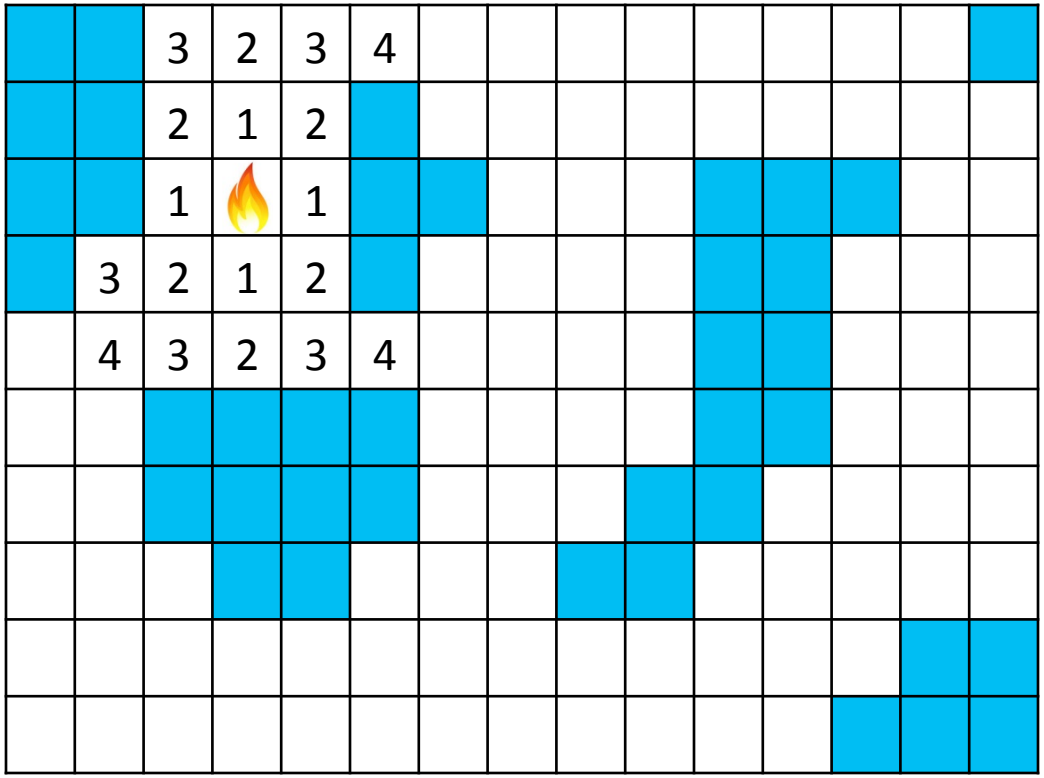

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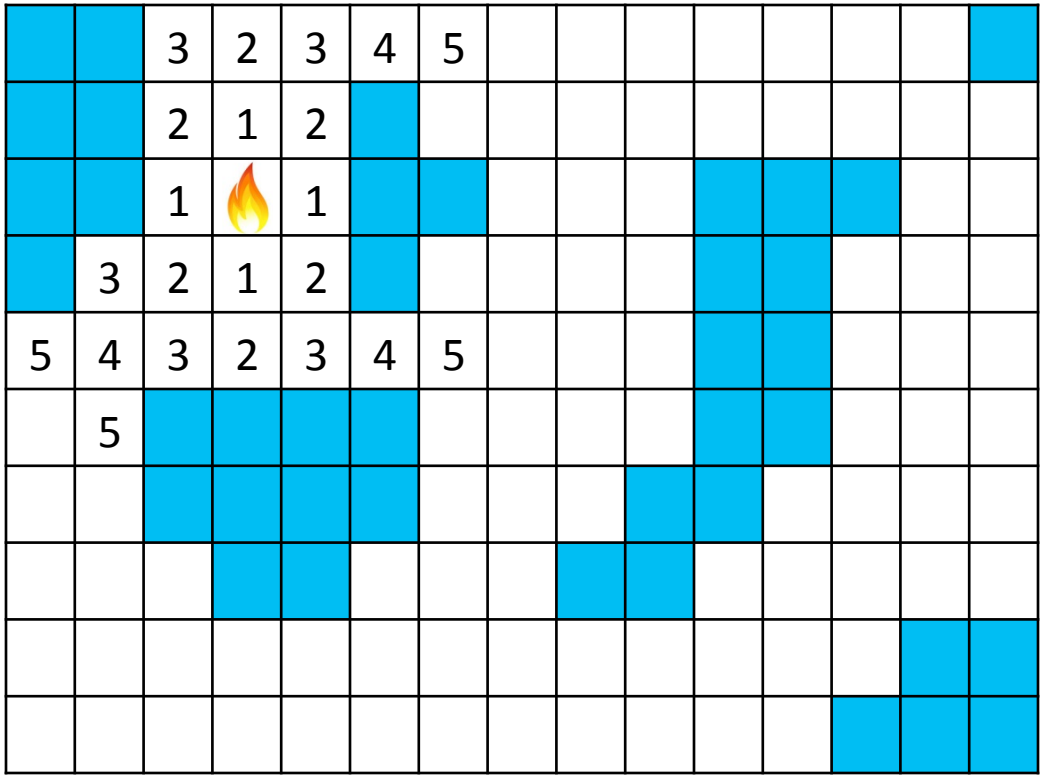

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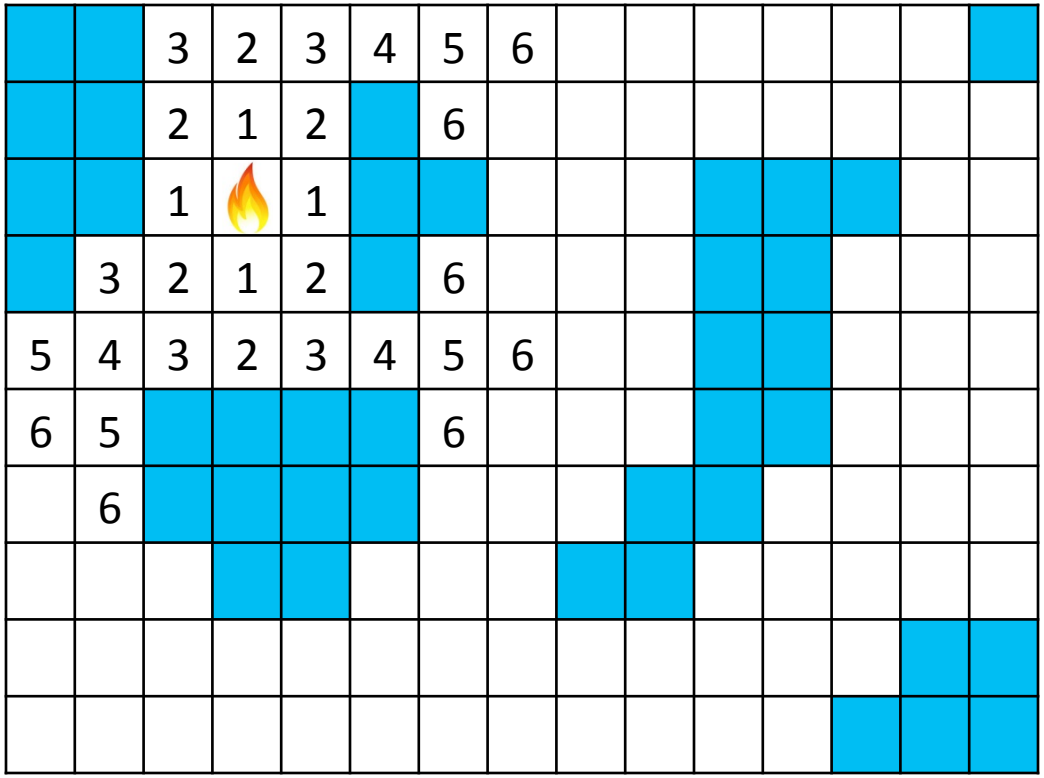

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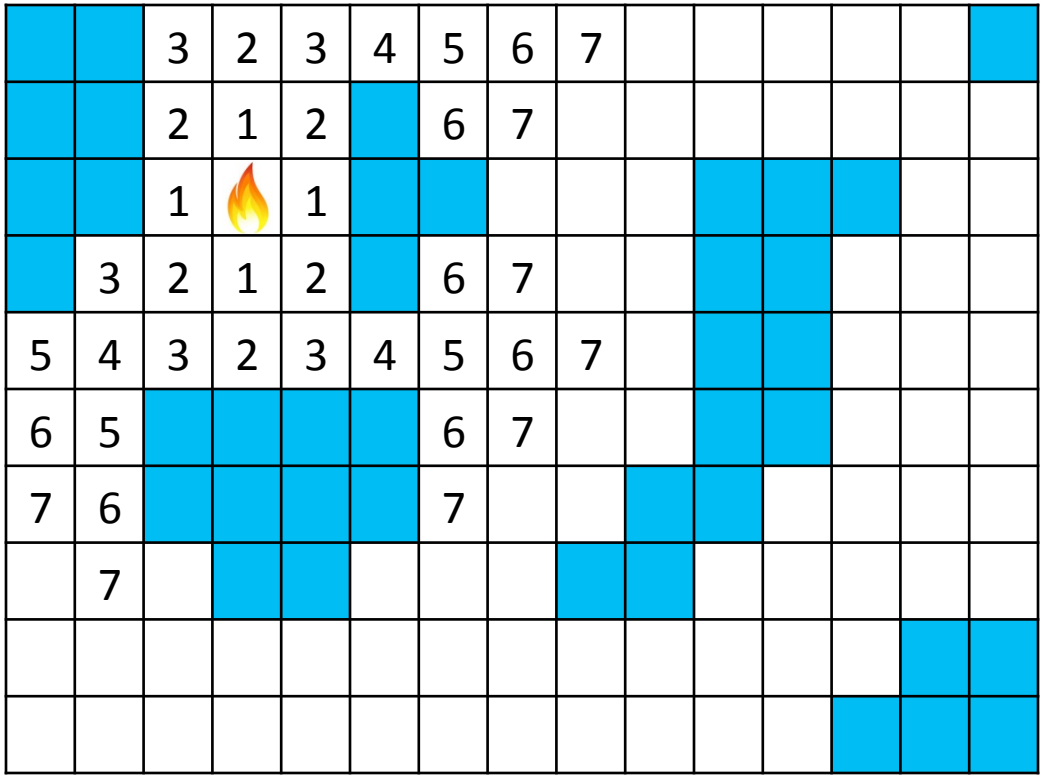

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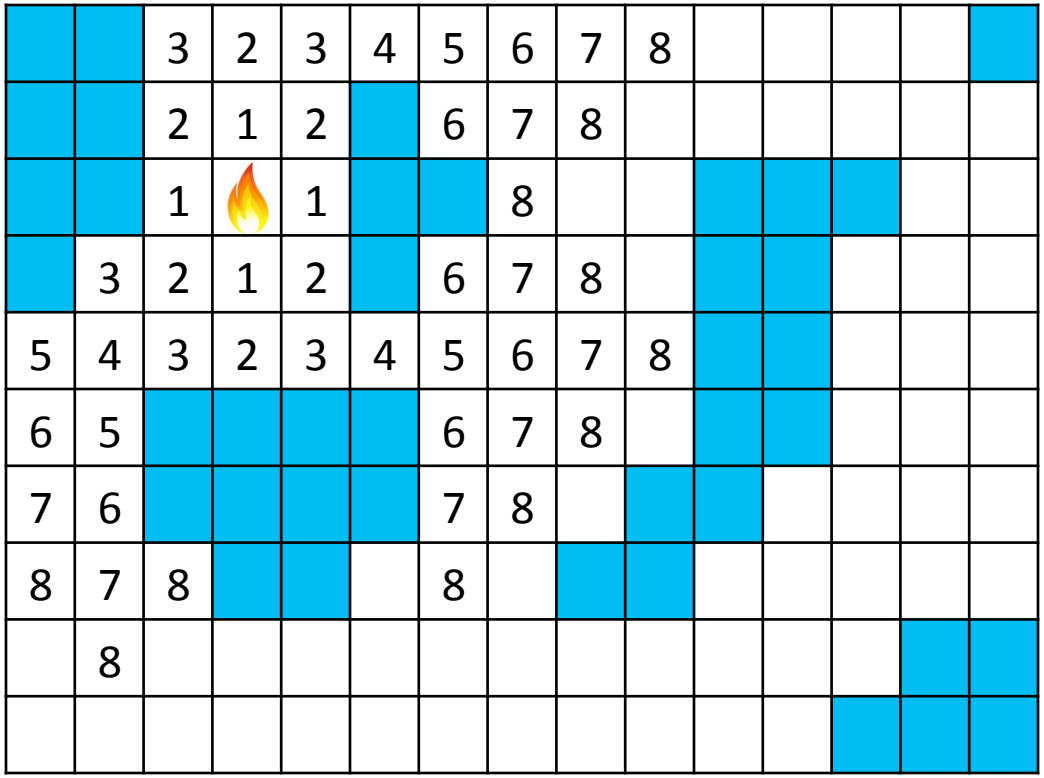

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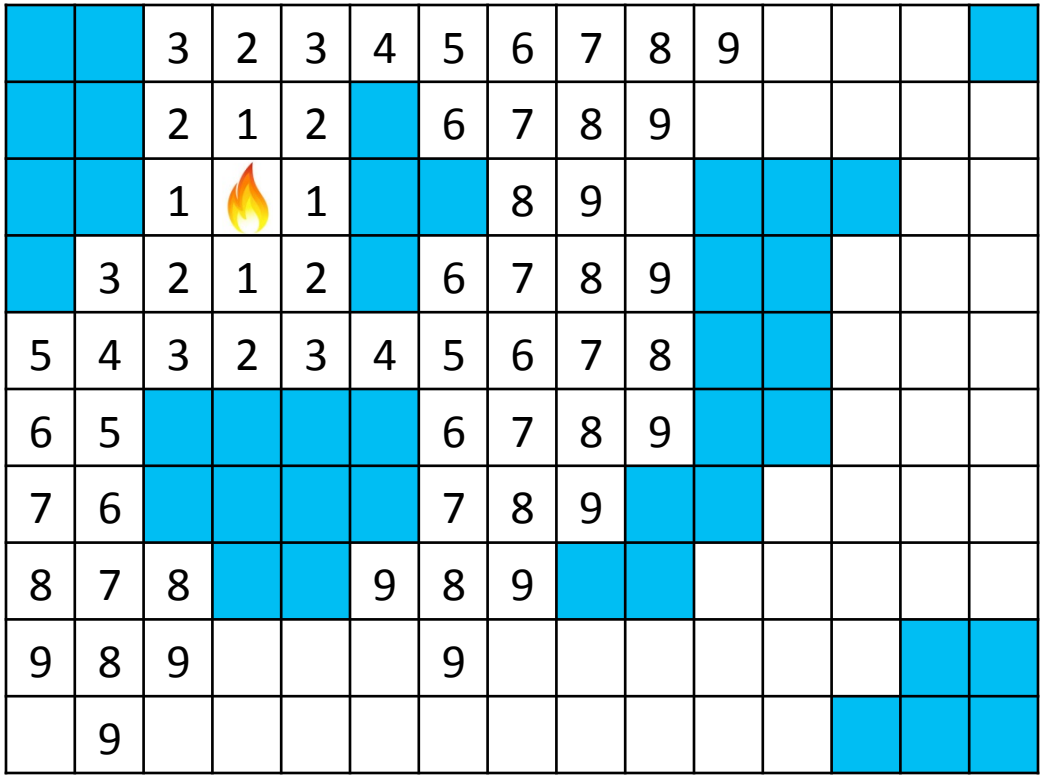

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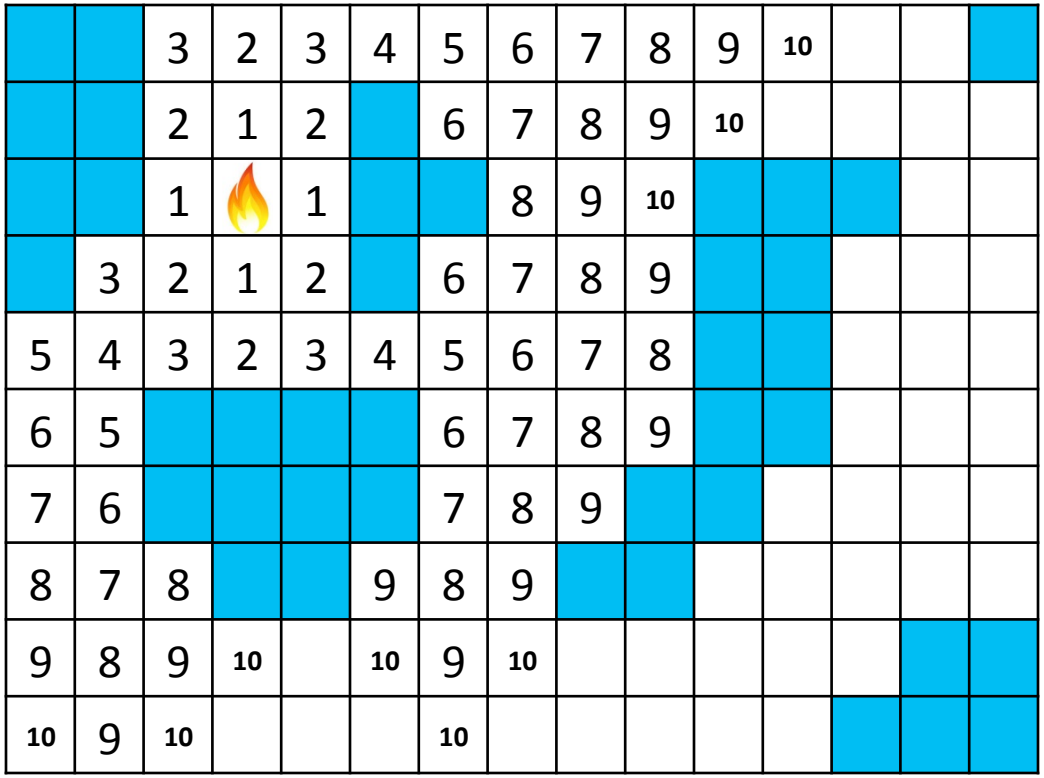

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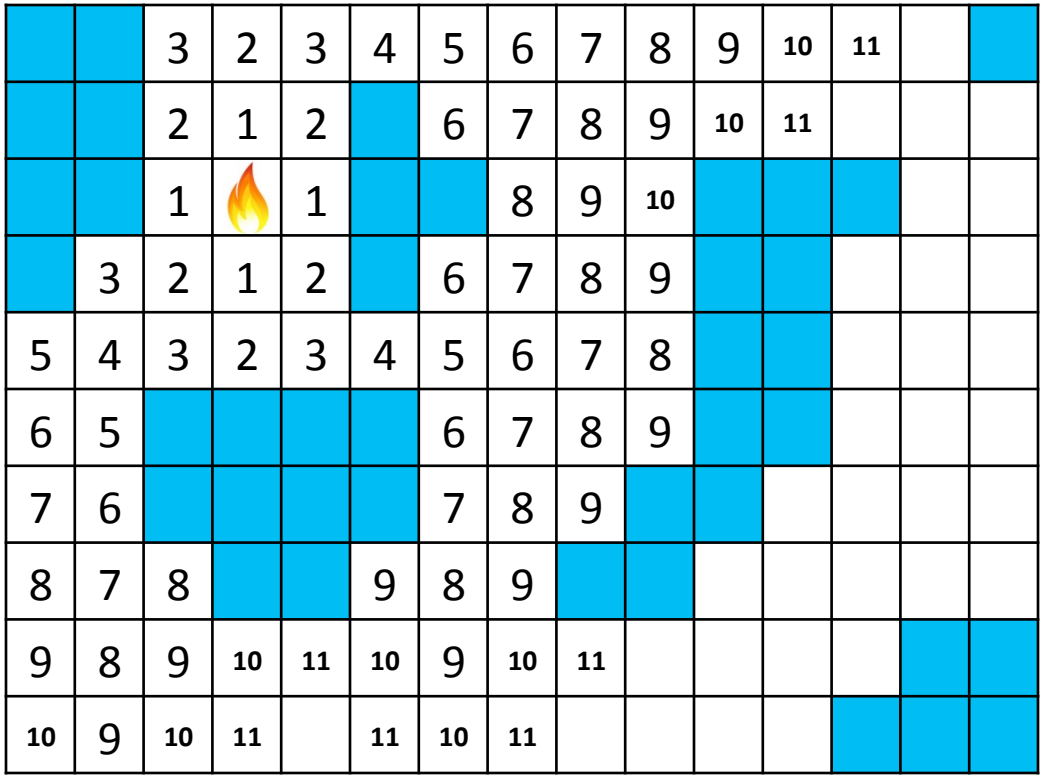

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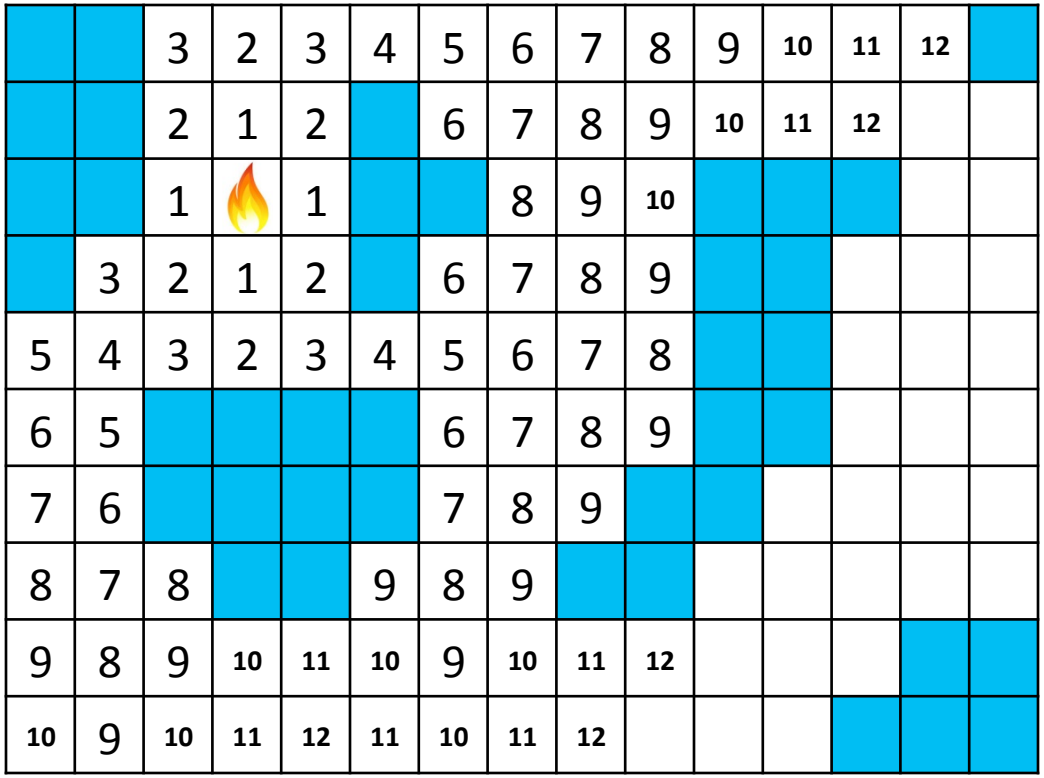

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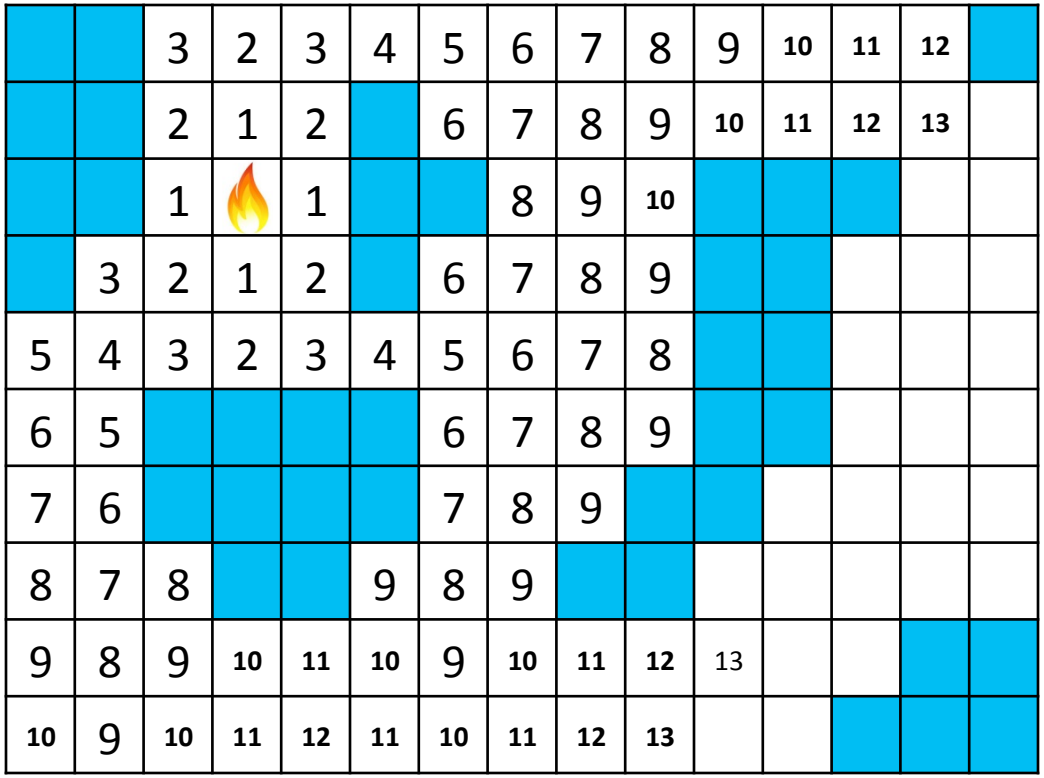

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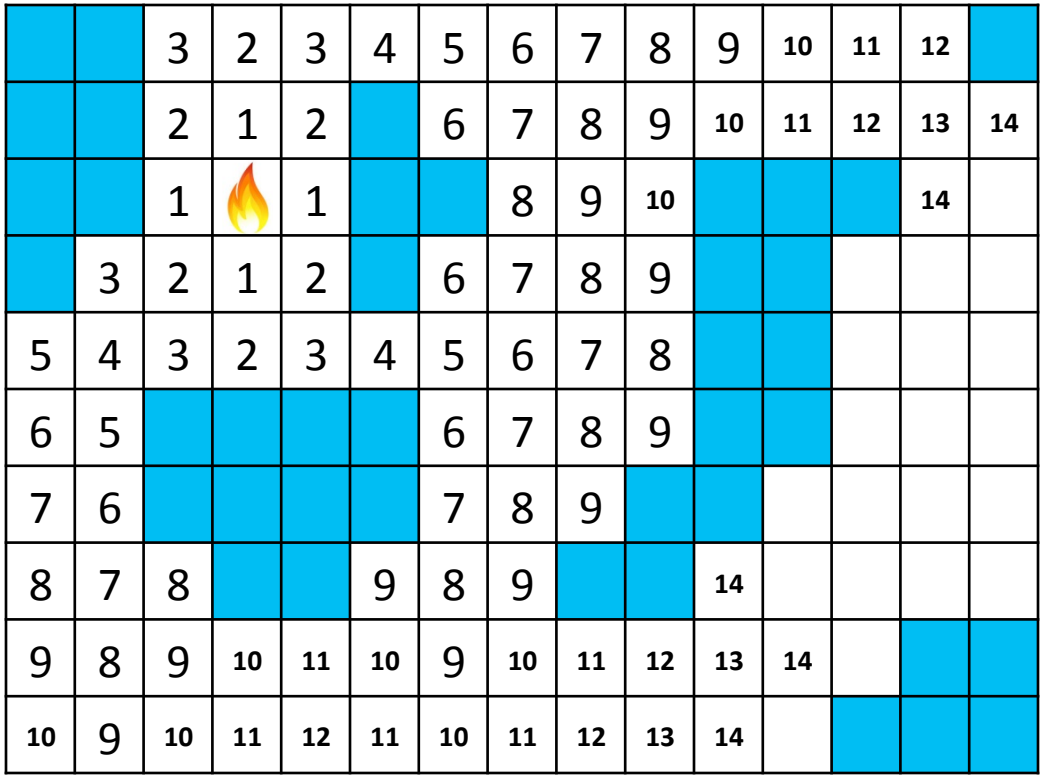

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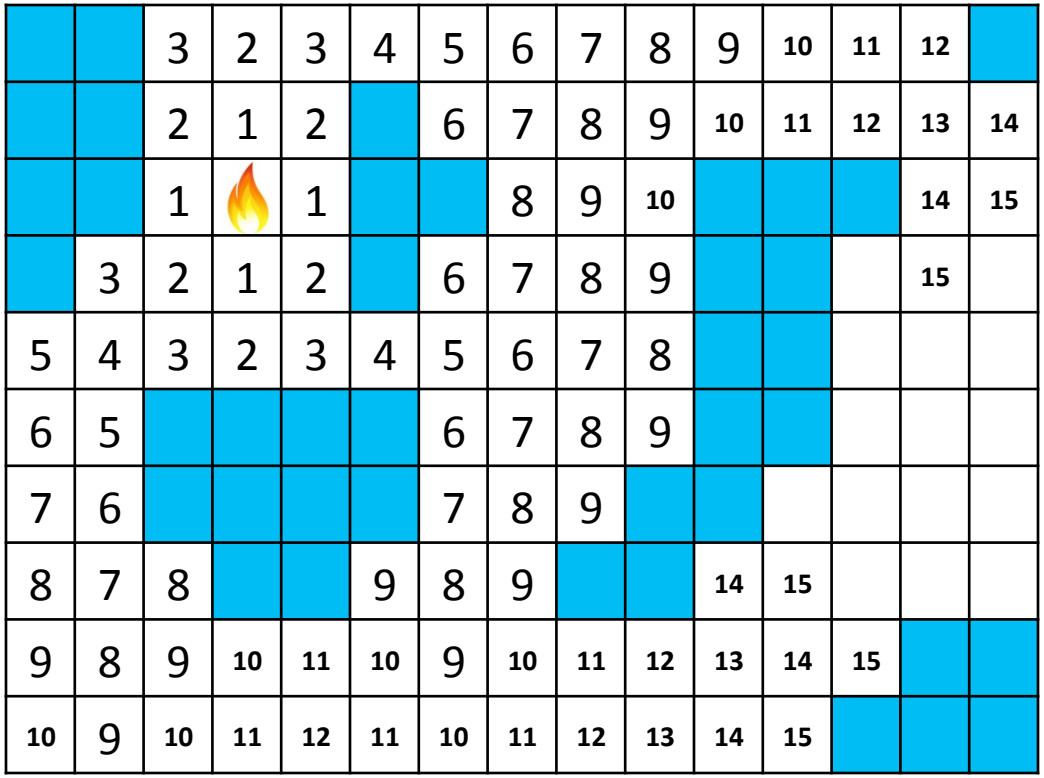

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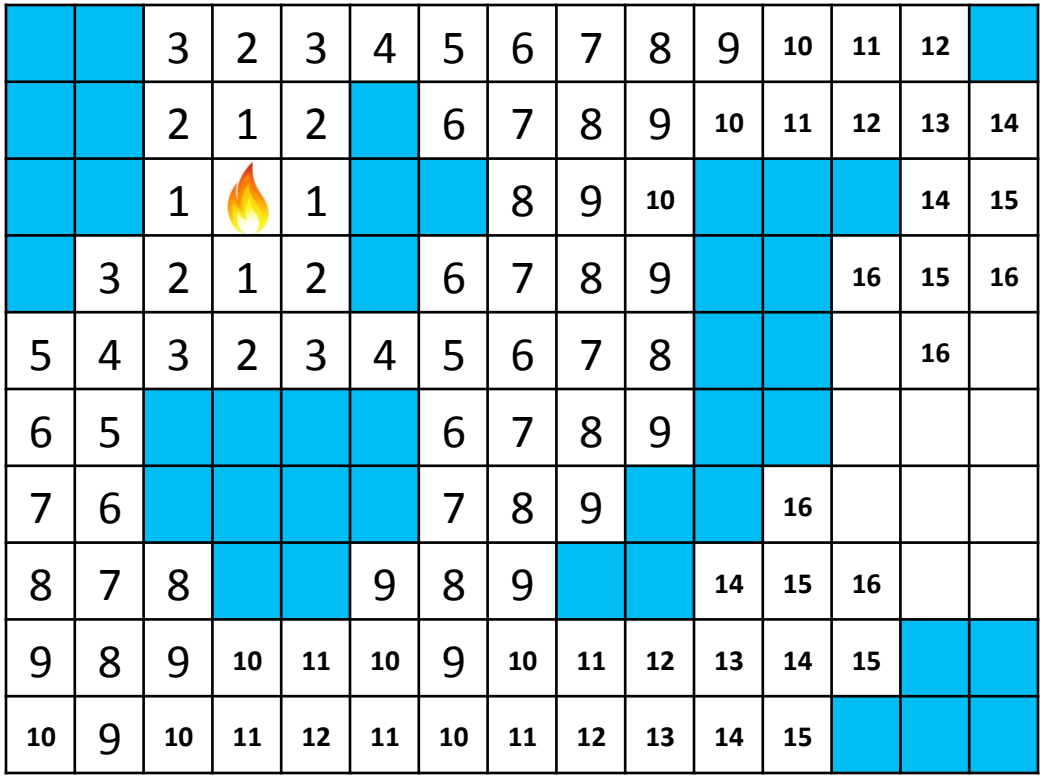

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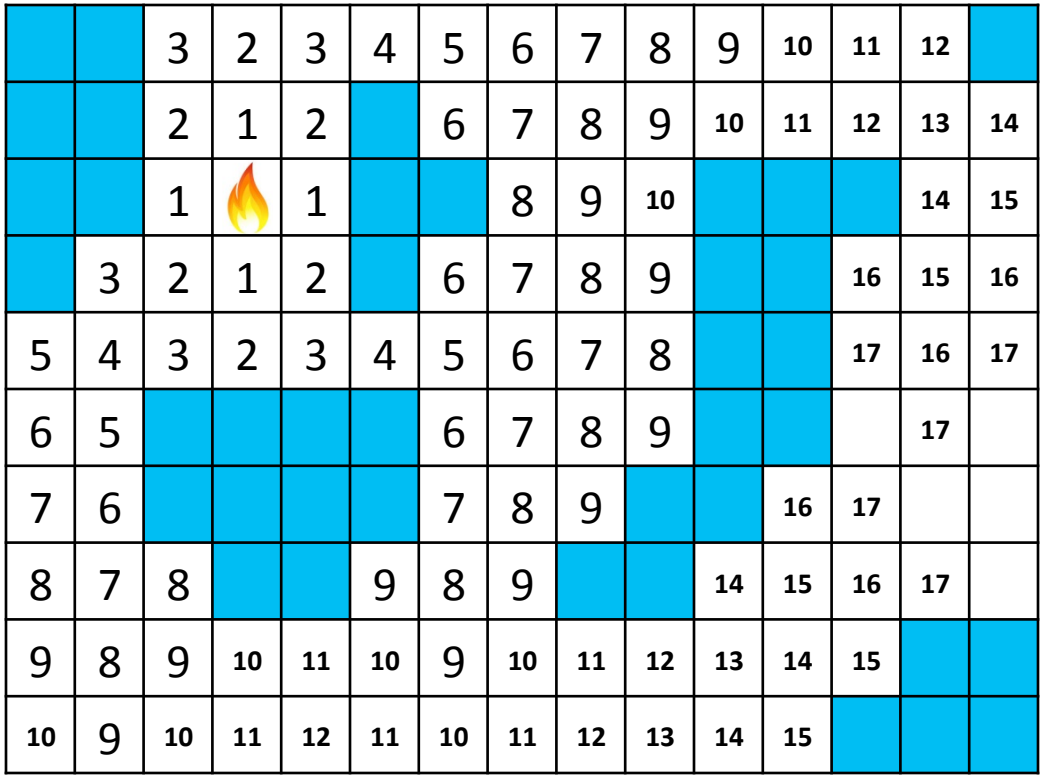

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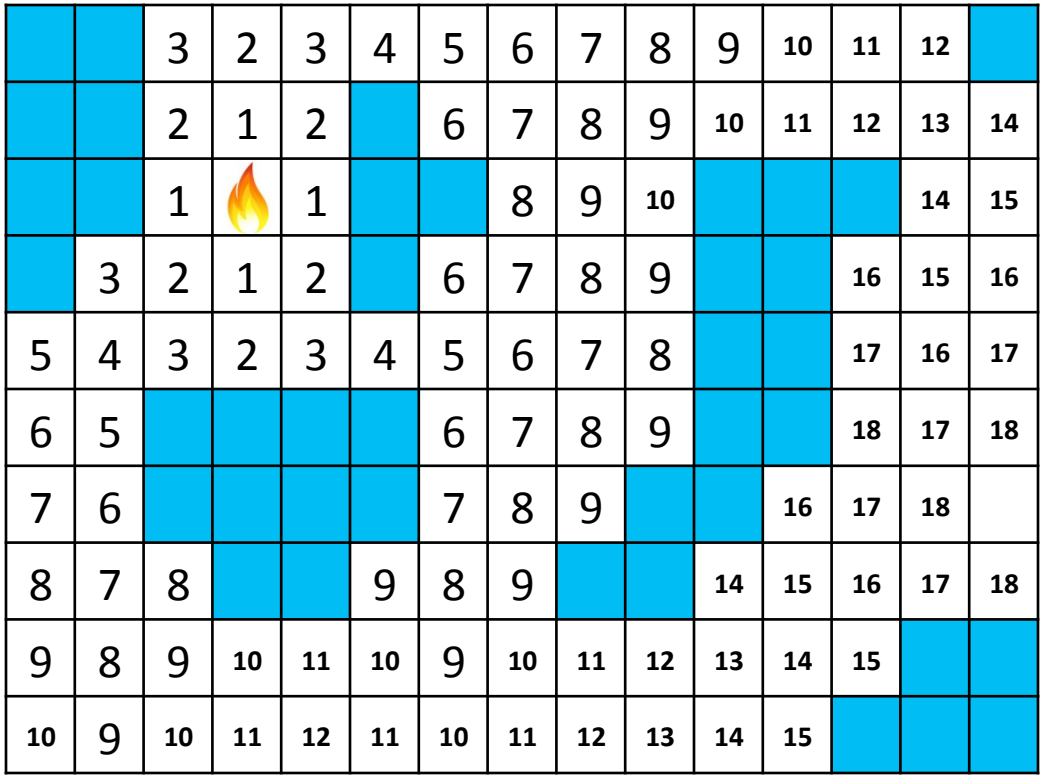

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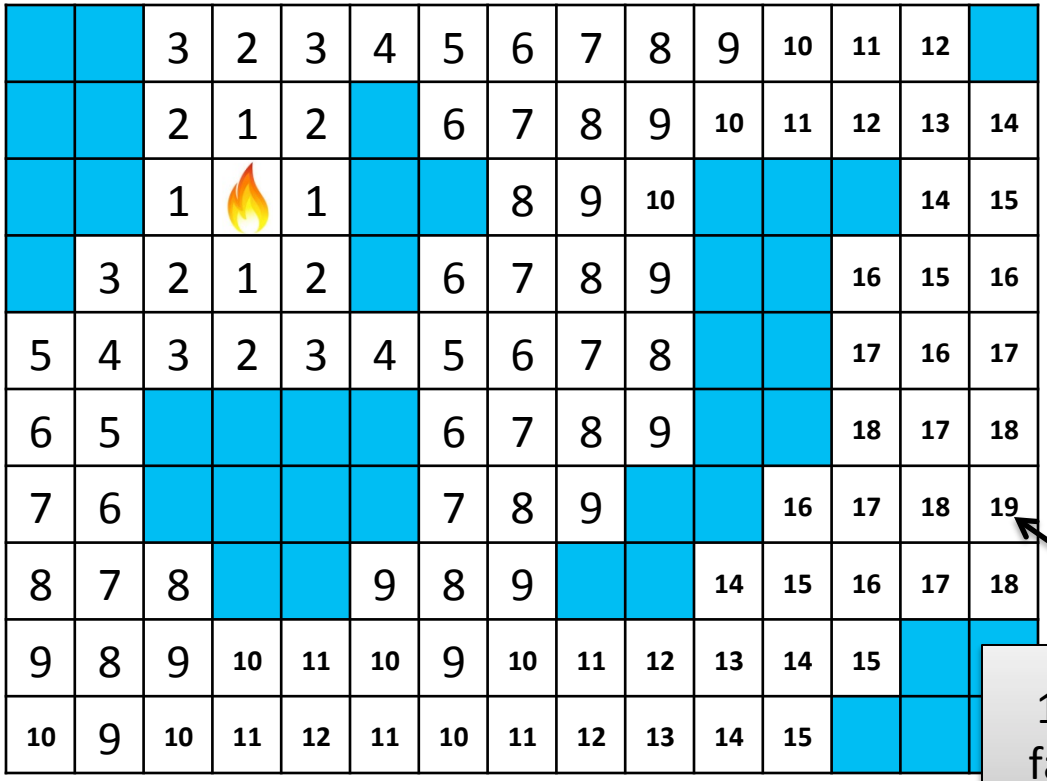

Application 4: **modification 1**

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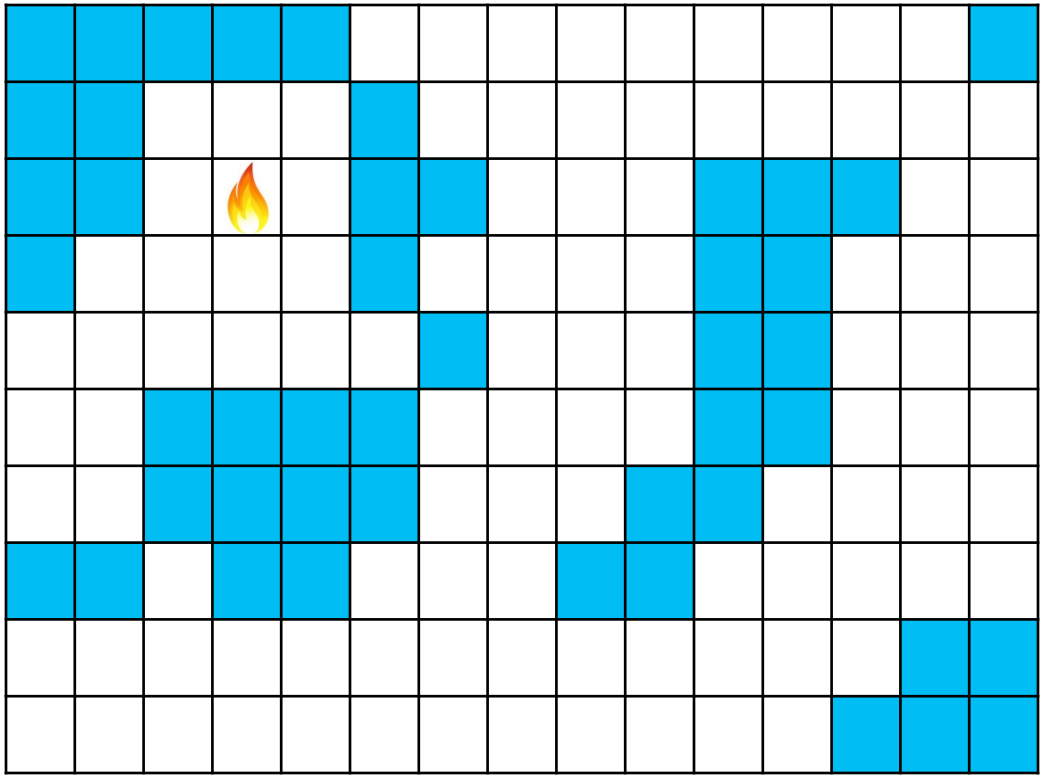

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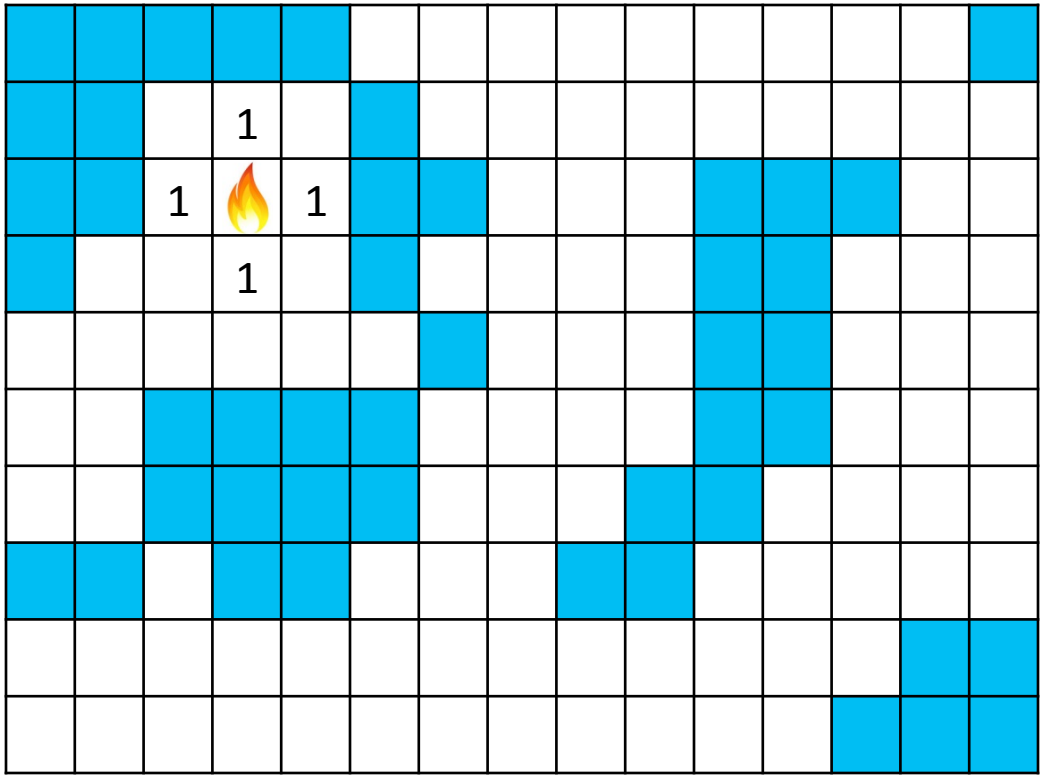

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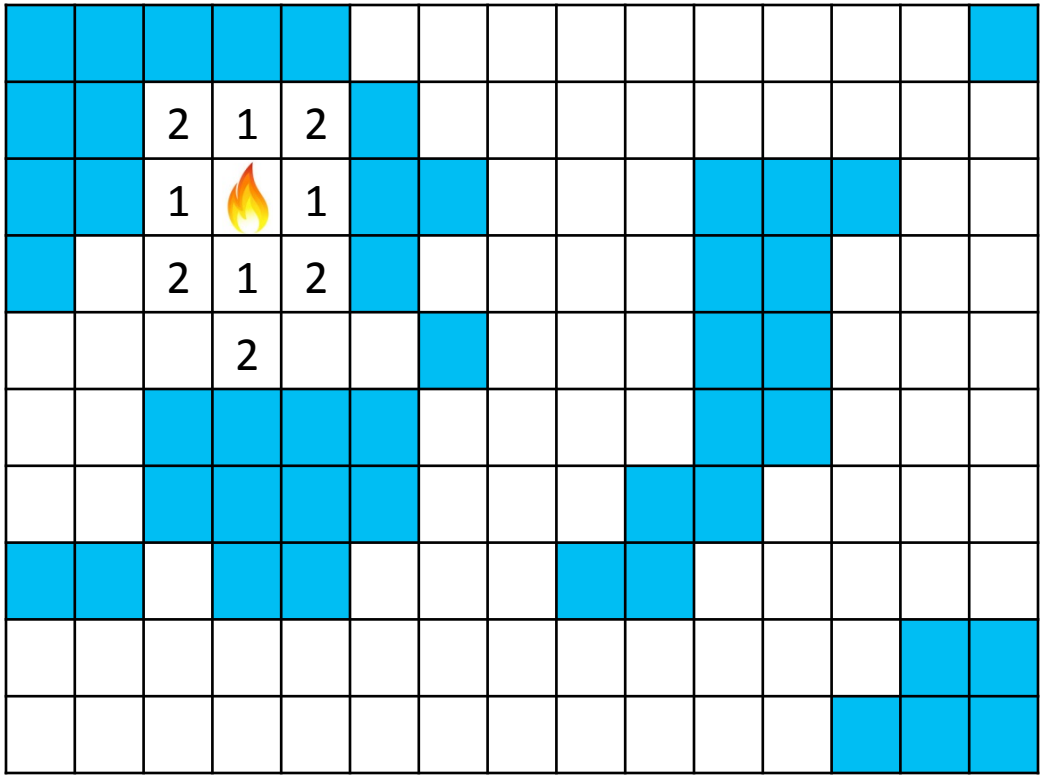

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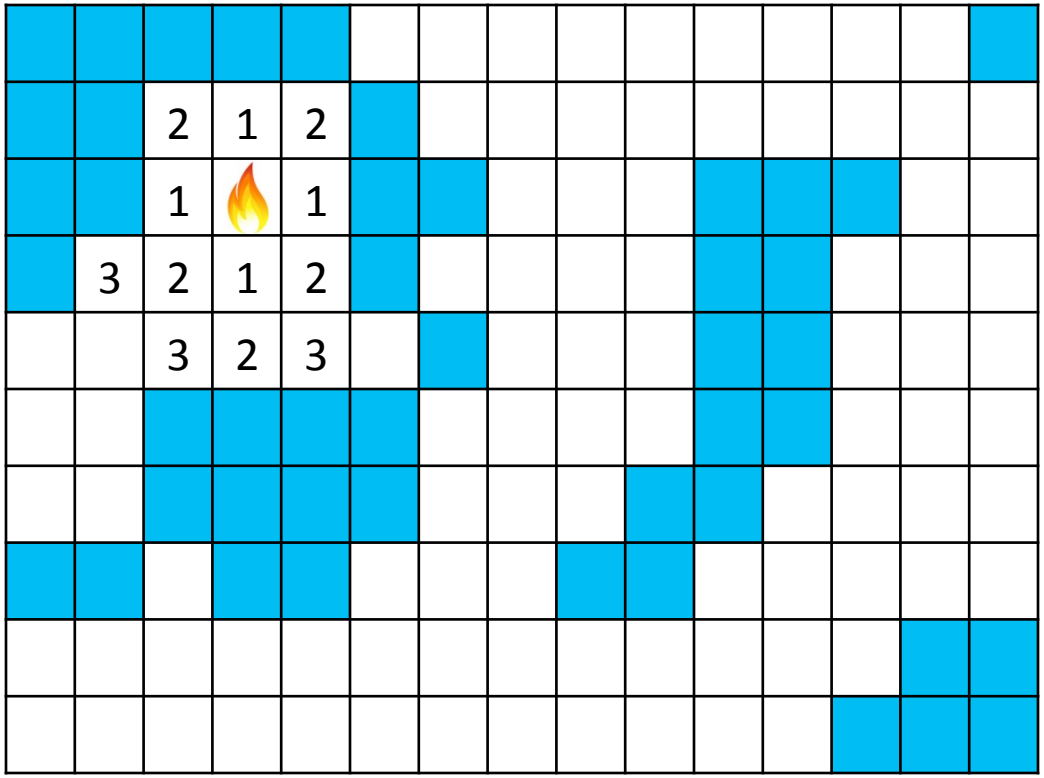

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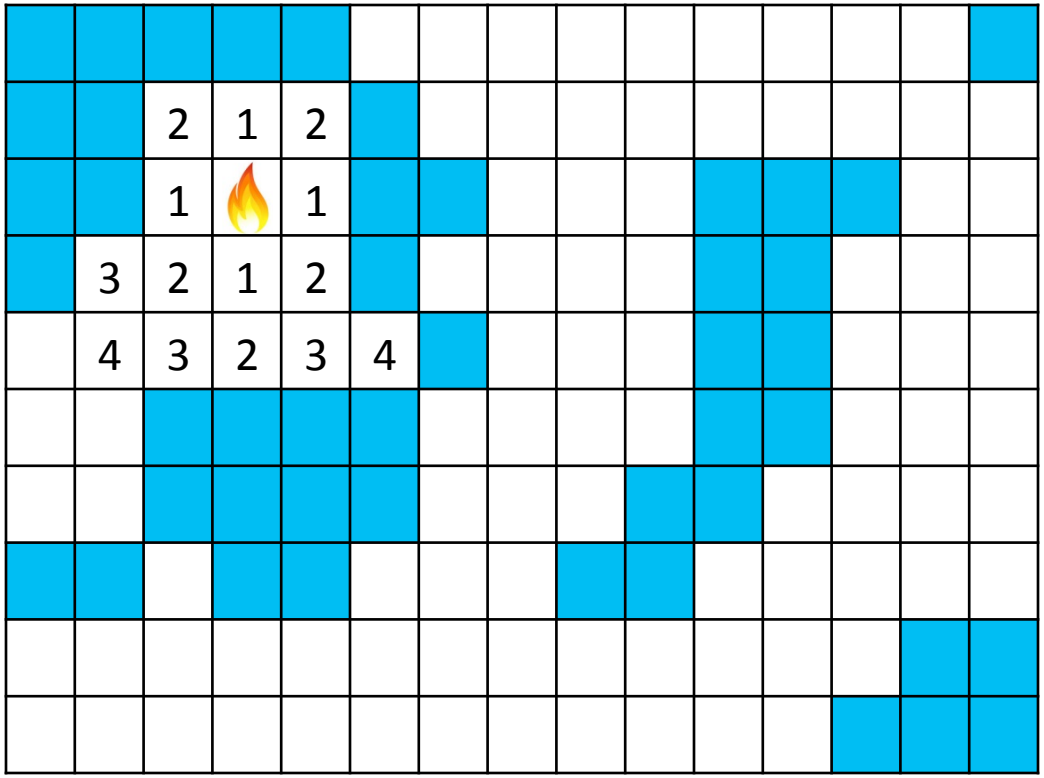

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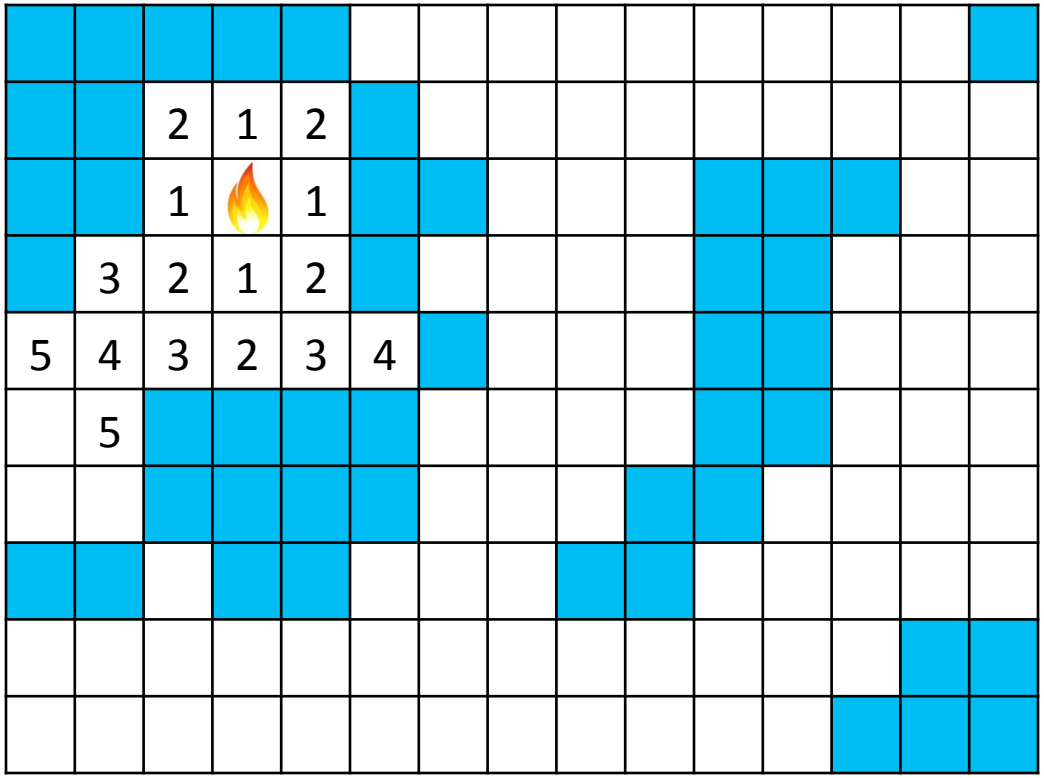

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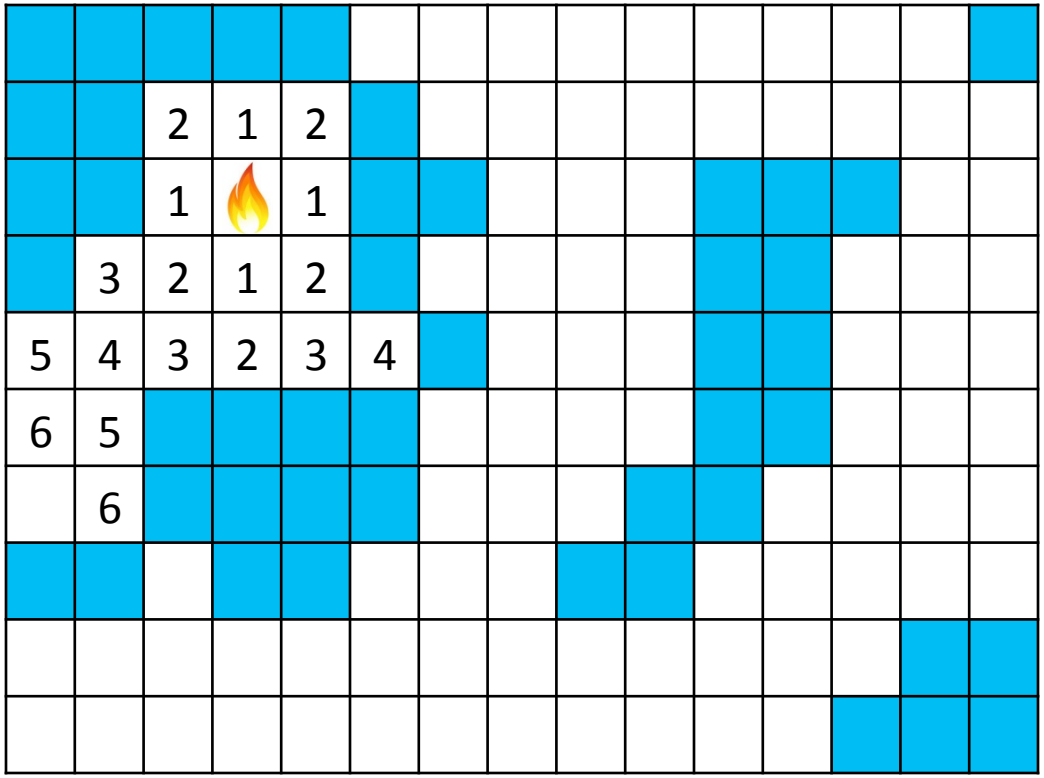

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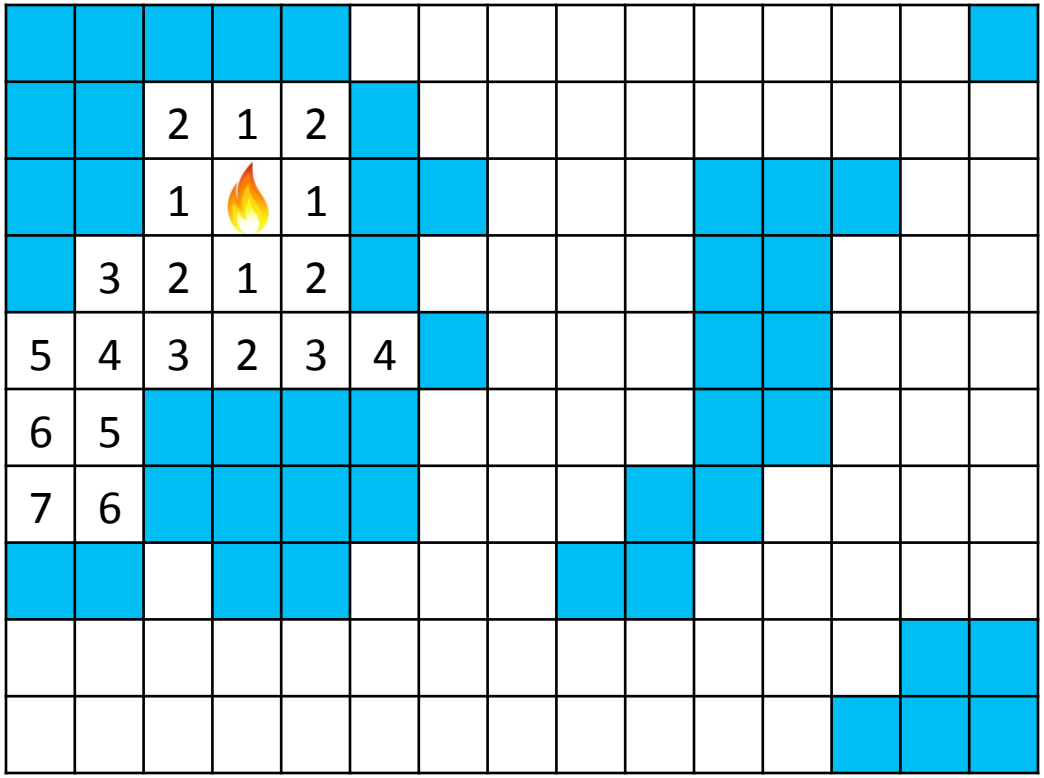

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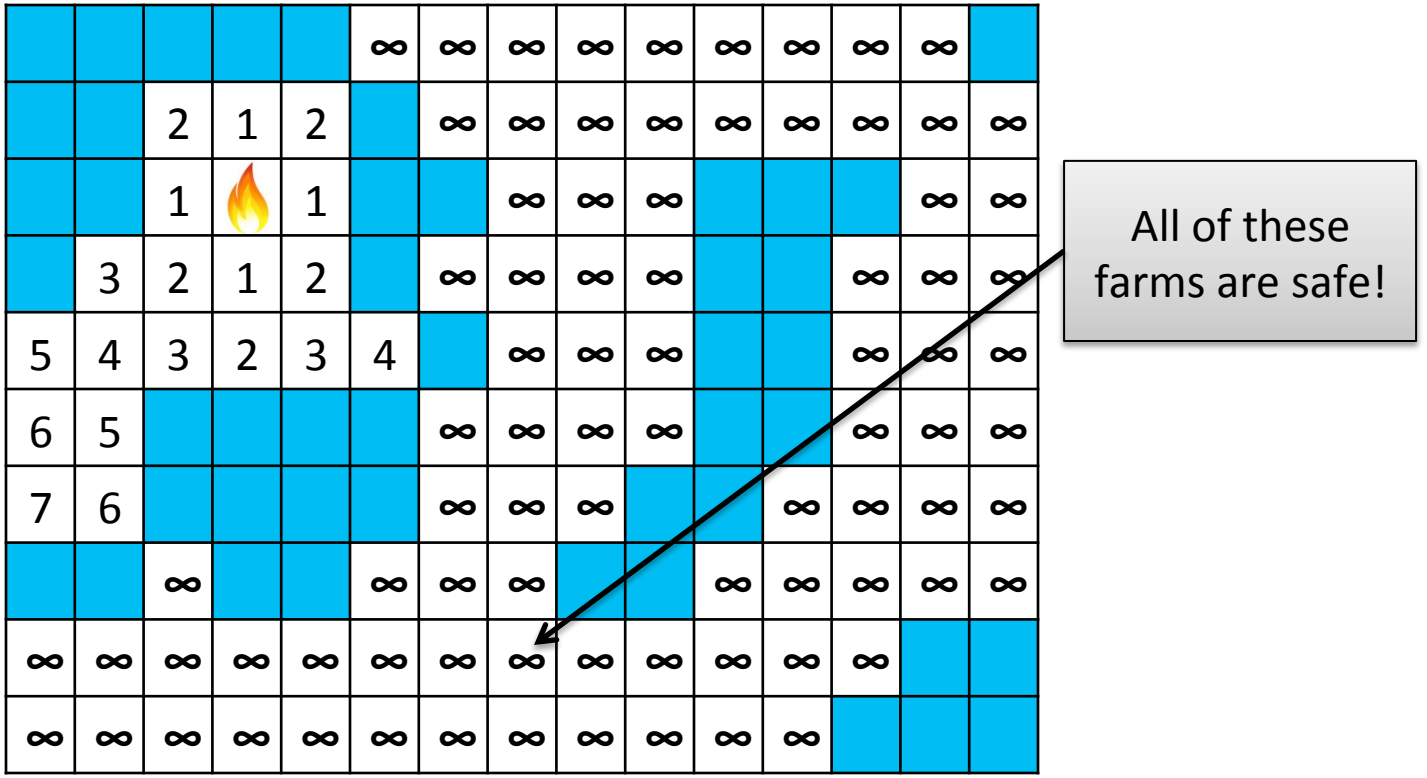

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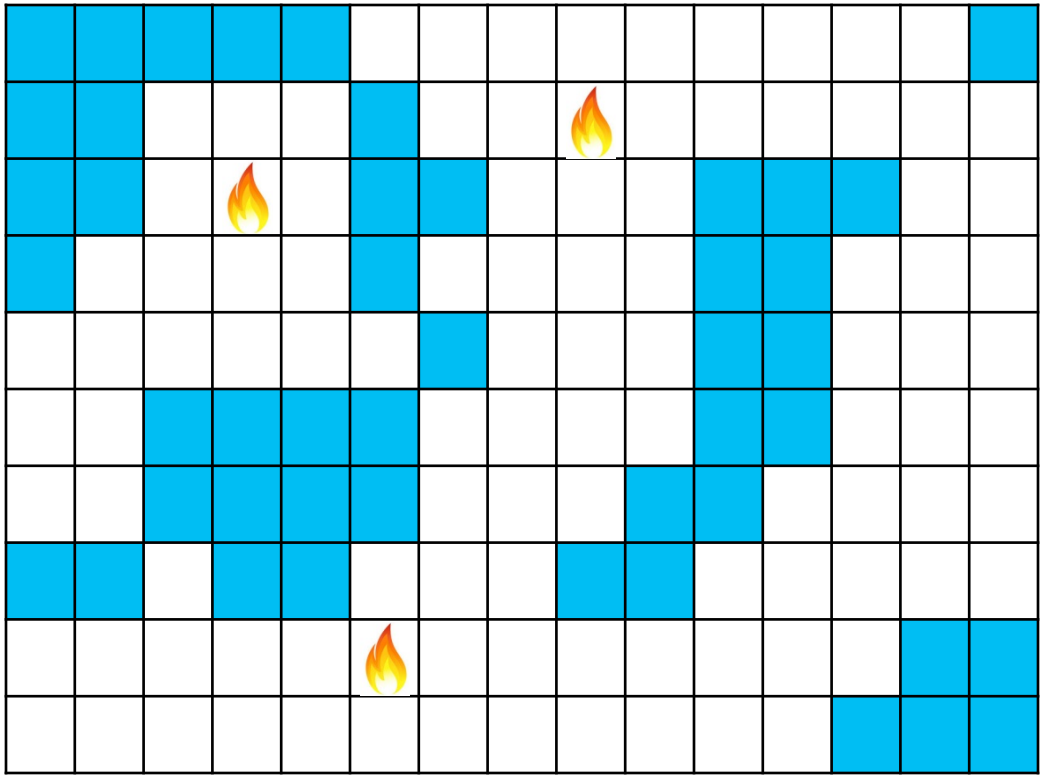
- What if multiple fires start at the same time?
- Just place **all** fires in the initial BFS queue!



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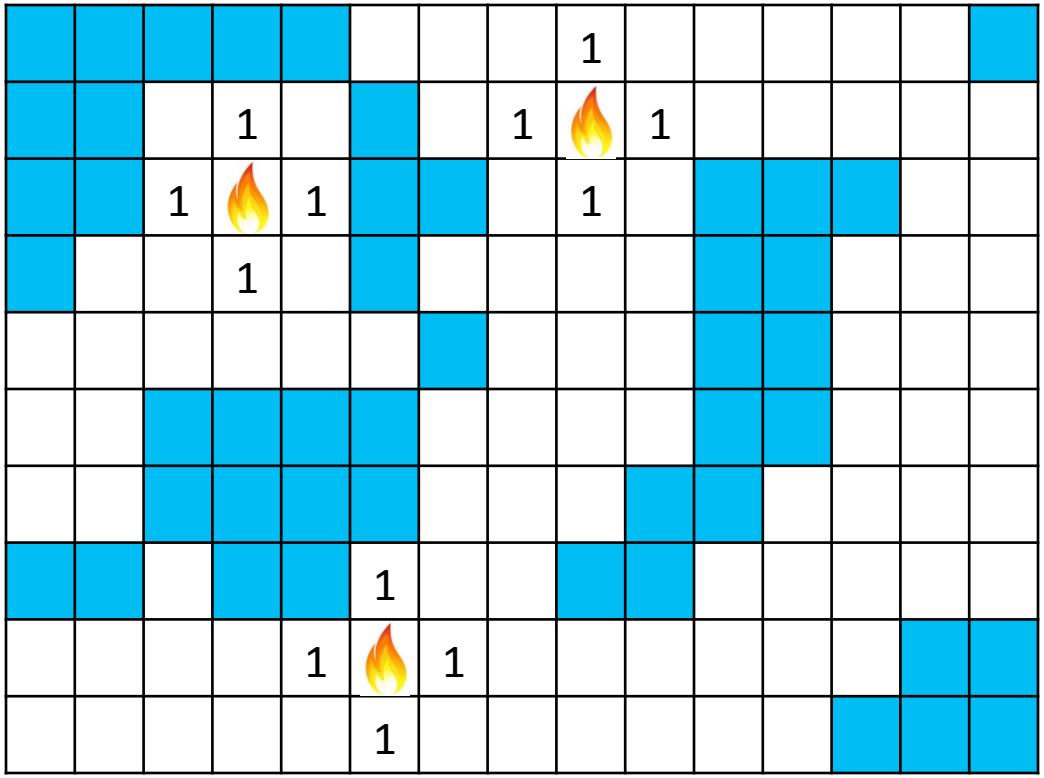

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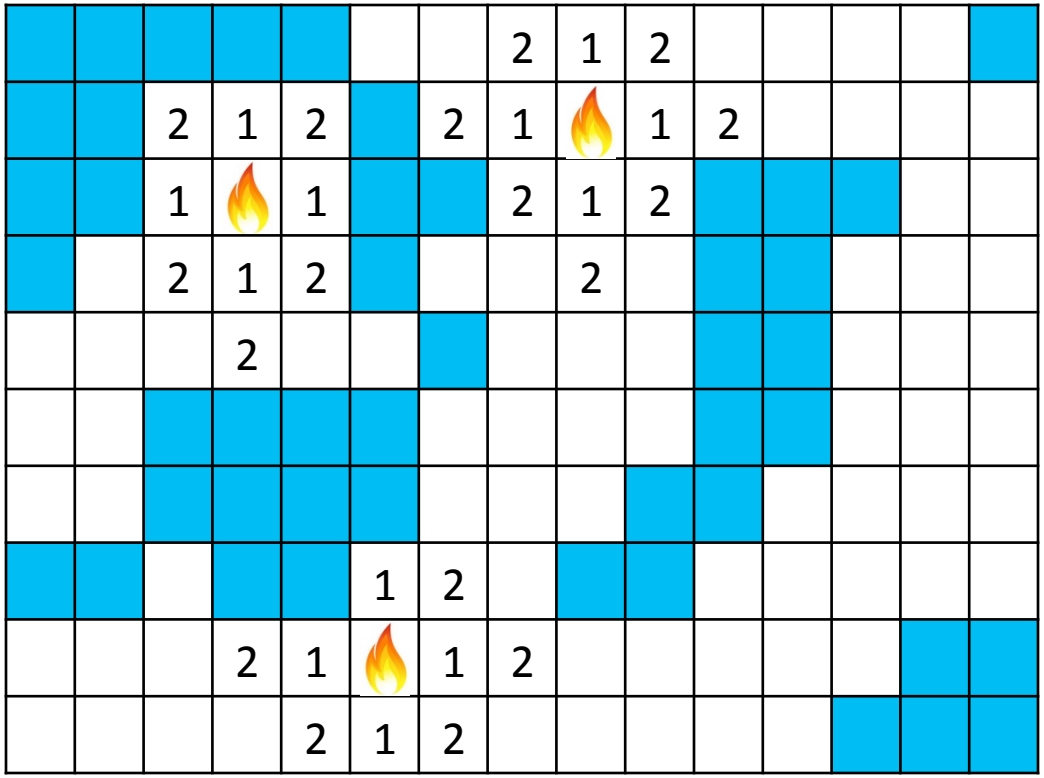

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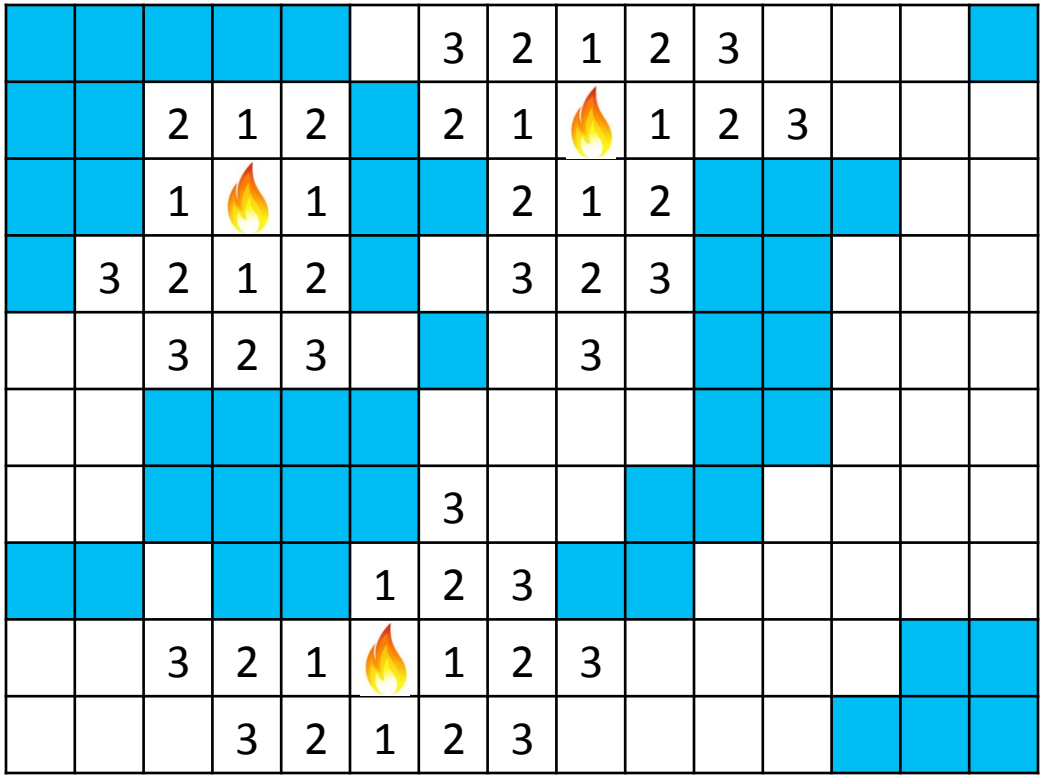

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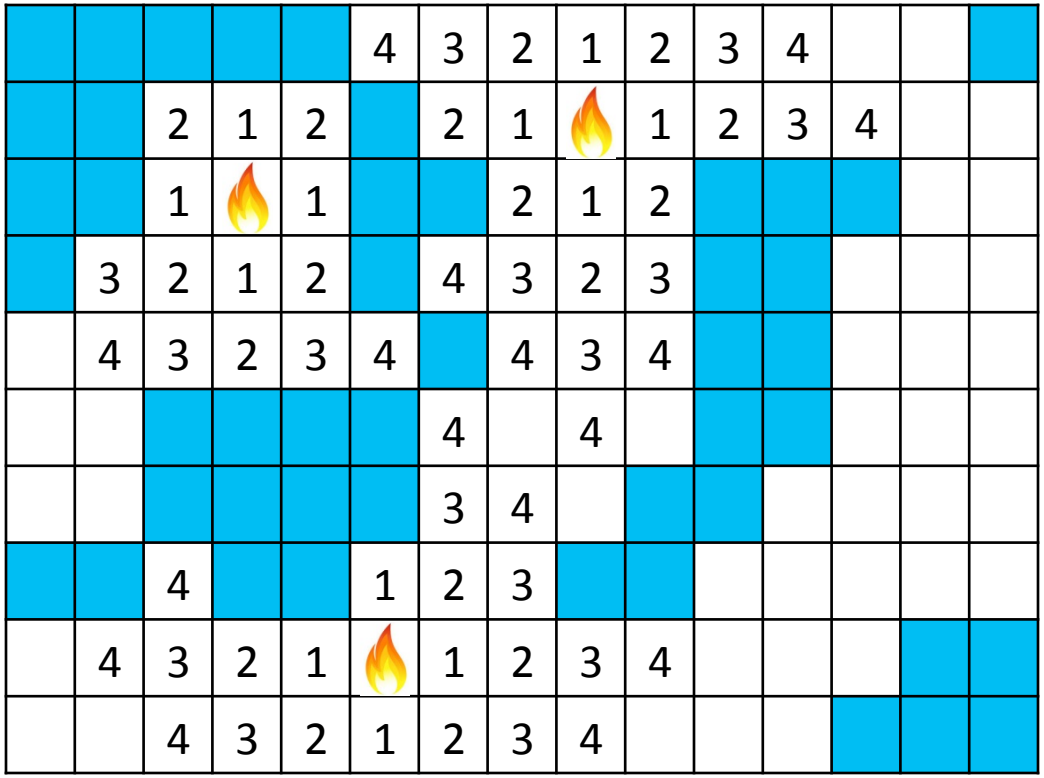

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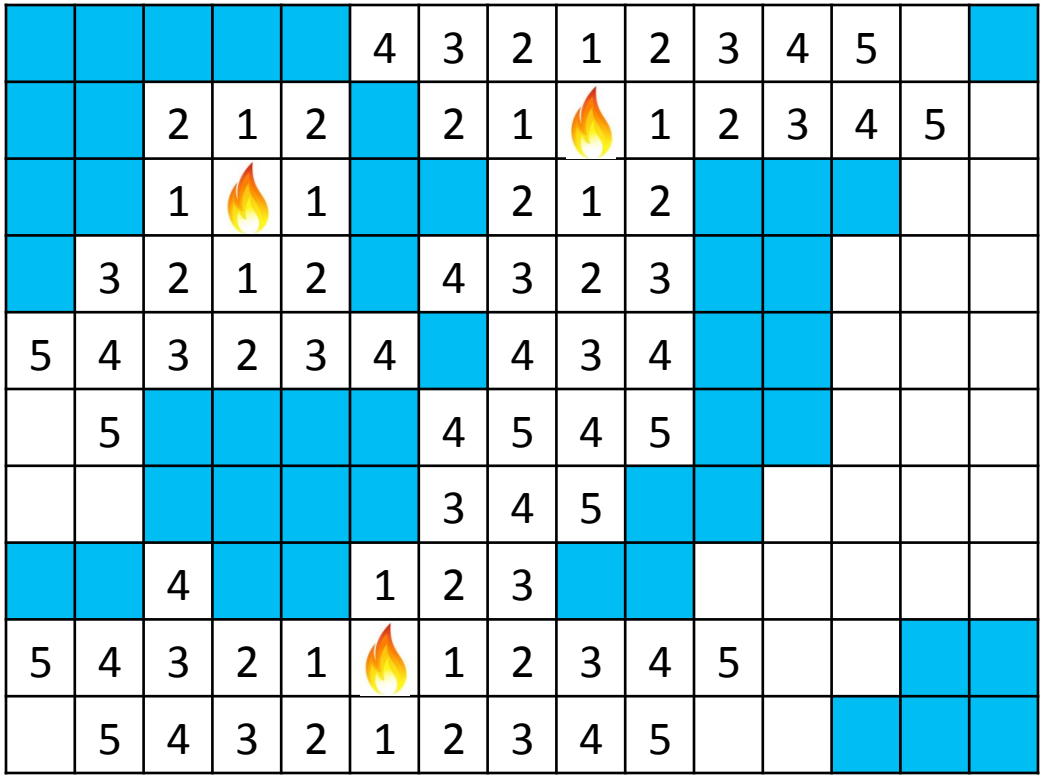

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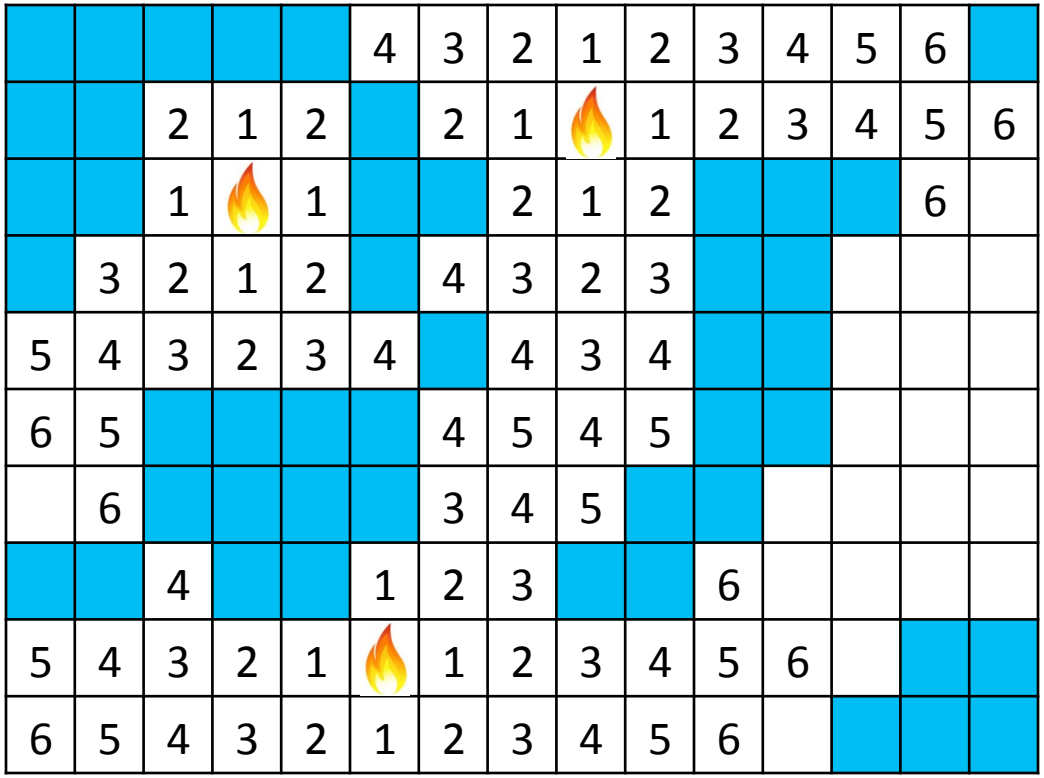

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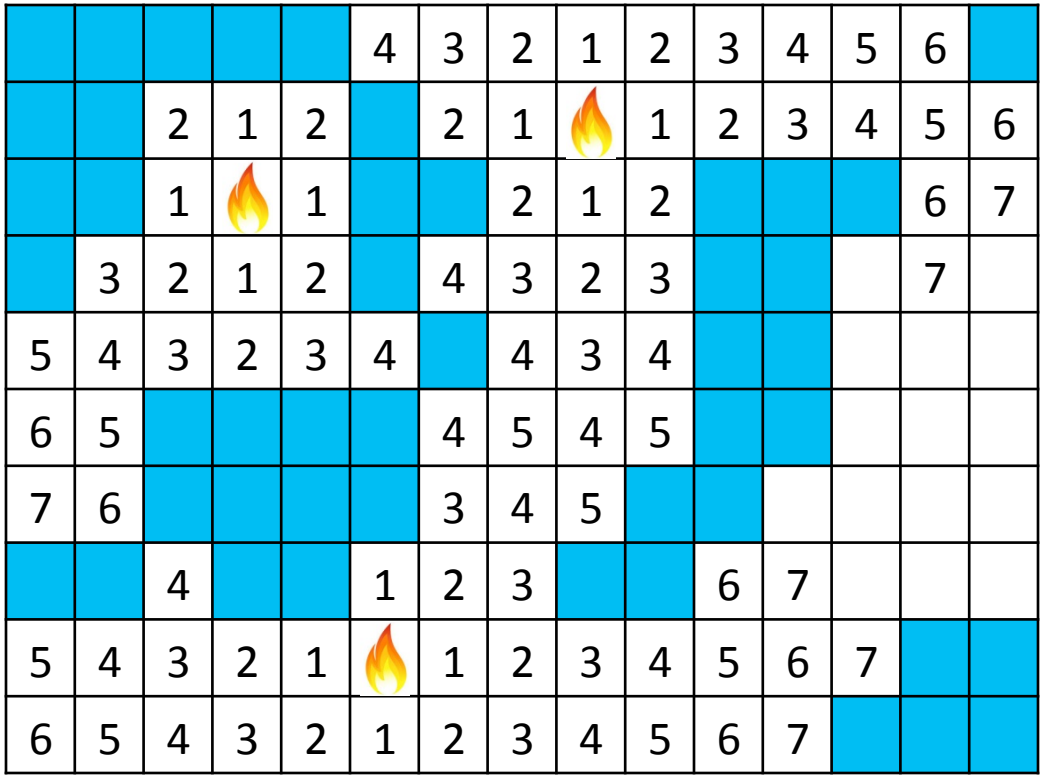

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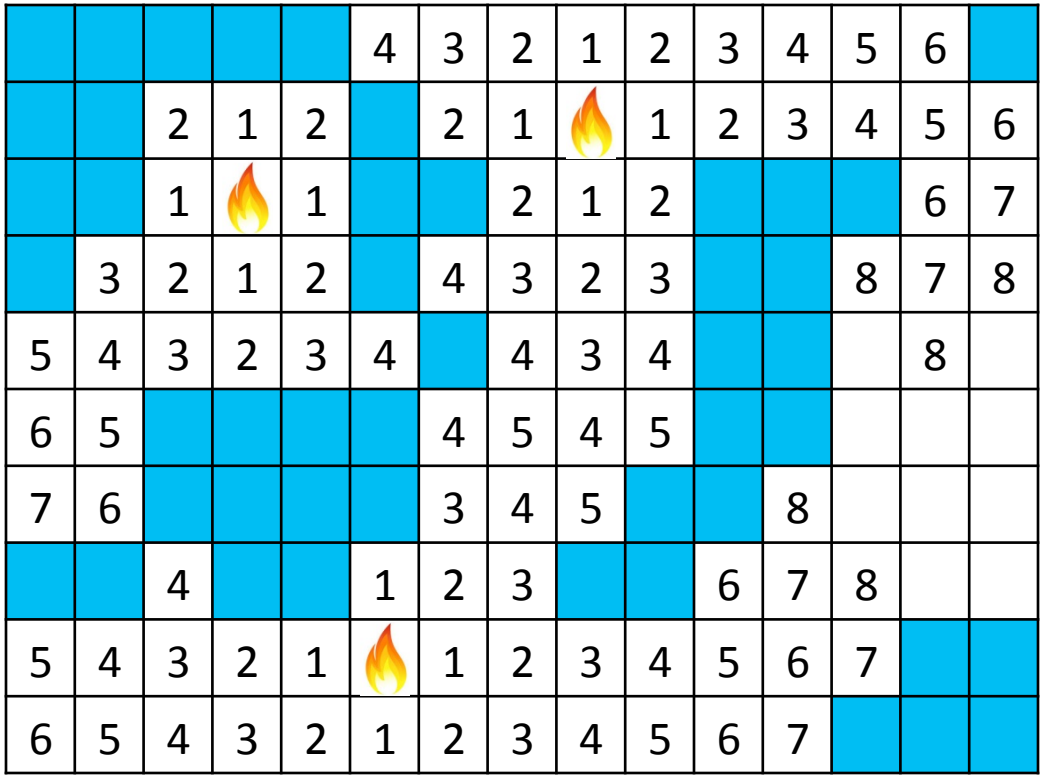

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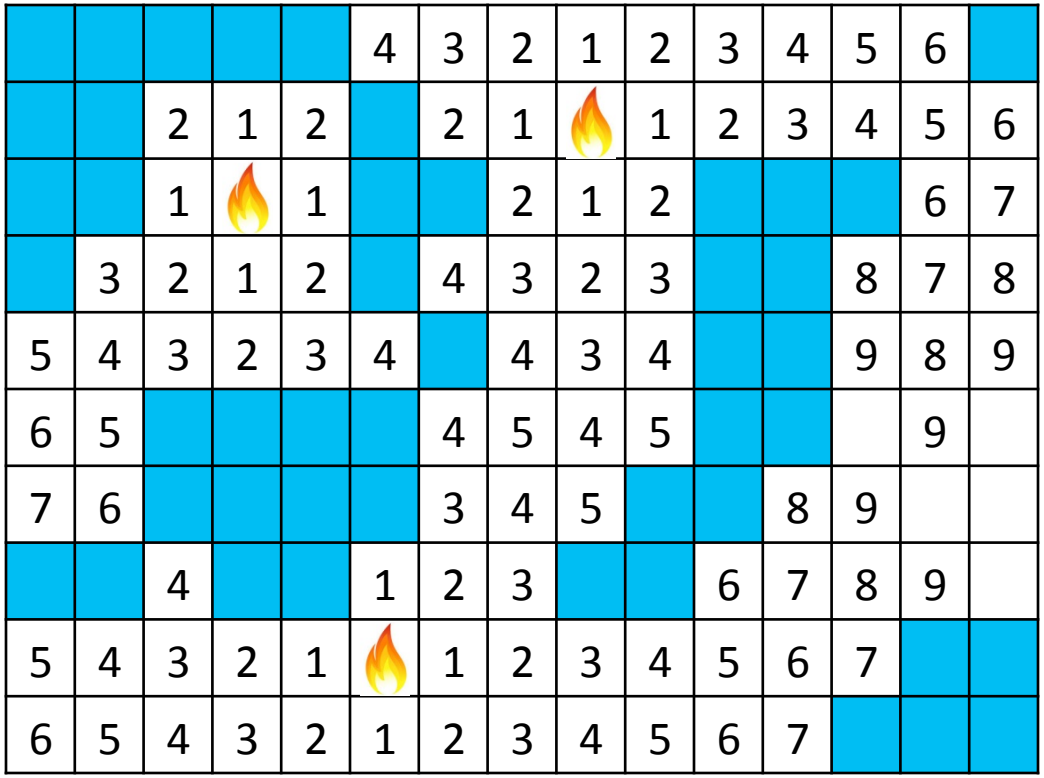

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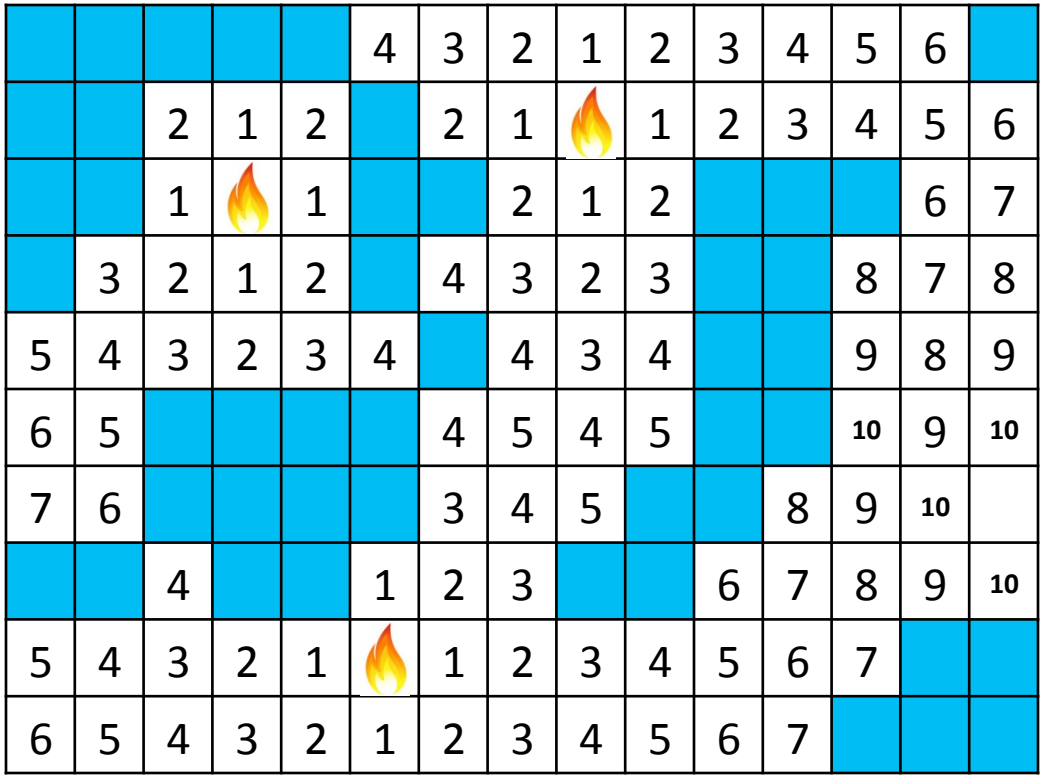

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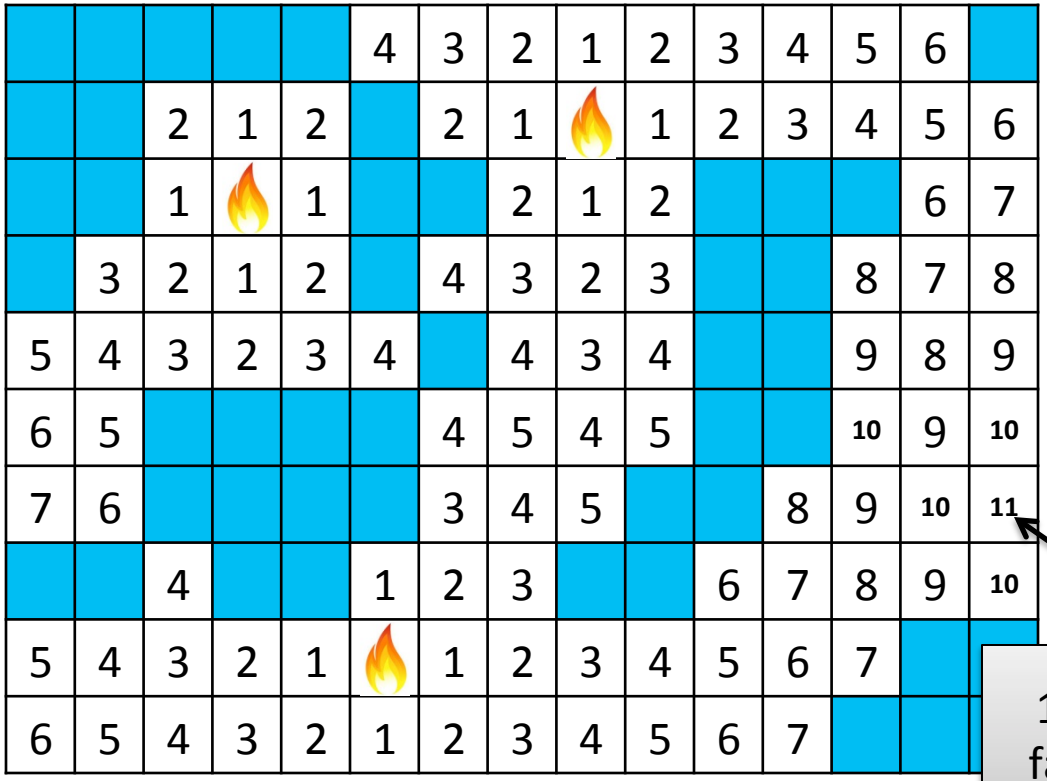

Application 4:
other modifications to think about

- What if fires start at different times?
- What if fires spread at different speeds?

A neat problem to think about

- It's the year 2241. You're in a cave and it's collapsing! A device tells you when each section of ceiling will cave in. Can you escape?

					19	16	14	12	16	13	20	21	19	
		27	24	20		20	10	9	18	20	17	16	17	18
		23	18	16			9	12	14				14	20
	19	20	16	13		7	7	9	11			16	24	20
6	7		12	14	20	5	8	12	9			25	22	15
5	6					14	9	14	10			25	23	21
1	1					15	16	11			19	26	19	25
		2			12	18	24			24	13	27	30	
7	8	13	17	17	13	19	22	20	21	23	20	12		
10	12	15	18	16	15	20	18	16	19	18	15			

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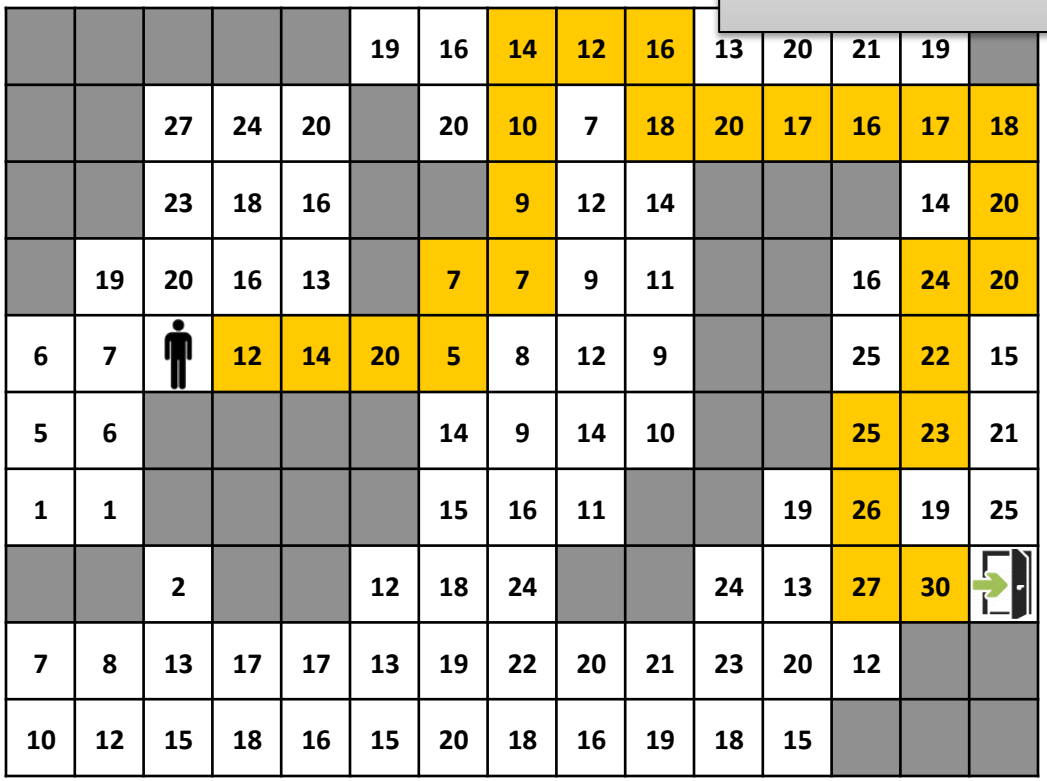
Here's a sample solution. How would you solve it in general?