1. Let $I(x)$ be the predicate "x is an integer". Prove or disprove the following claims:
   (a) $\forall x \in \mathbb{R} : I(x) \Rightarrow I(\frac{x}{3})$.
   (b) $\forall x, y \in \mathbb{R} : I(x) \wedge I(y) \Rightarrow I(x \cdot y)$. # this dot is for multiplication, not a decimal dot :)
   (c) $\forall x \in \mathbb{R} : I(x) \Rightarrow [\exists y \in \mathbb{R} : I(y) \wedge x + y = 0]$
   (d) Repeat all that with the converse.

2. In addition to $I(x)$ defined above, consider the predicate "x is rational", denoted by $Q(x)$, and defined as

$$\forall r \in \mathbb{R} : Q(x) \Rightarrow [\exists p, q \in \mathbb{R} : I(p) \wedge I(q) \wedge q \neq 0 \wedge x = \frac{p}{q}]$$

   Prove or disprove the following:
   (a) $\forall x \in \mathbb{R} : Q(x) \Rightarrow Q(-x)$.
   (b) $\forall x \in \mathbb{R} : Q(x) \Rightarrow Q(x + 5)$.
   (c) $\forall x \in \mathbb{R} : Q(x) \wedge Q(y) \Rightarrow Q(x + y)$.
   (d) $\forall x, y \in \mathbb{R} : Q(x) \wedge I(y) \Rightarrow I(x \cdot y)$. # this dot is for multiplication

3. Call a number of the form $r + s\sqrt{2}$, where r, s are rational numbers, a quadratic number. Let $R(x)$ be the predicate "x is a rational number", and let $Q(x)$ be the predicate "x is a quadratic number". Prove or disprove the following claim:

$$\forall x \in \mathbb{R} : R(x) \Rightarrow Q(x).$$

   Note: Predicate Q here is different from the one defined in the previous problem.