Quantifiers

What is that?

- A quantifier is an expression that indicates the scope of a term to which it is attached.
  - all cars are red
  - some cars are green
- Quantifiers connect properties of elements to properties of sets.

Universal quantification: $\forall$

Employee	Gender	Salary
Al	male	60,000
Betty	female	500
Carlos	male	40,000
Doug	male	30,000
Ellen	female	50,000
Flo	female	20,000

- Every employee makes less than \$70,000
- Each employee makes less than \$70,000.
- All employees make less than \$70,000.
- Employees make less than \$70,000

Universal quantification: $\forall$

Employee	Gender	Salary
Al	male	60,000
Betty	female	500
Carlos	male	40,000
Doug	male	30,000
Ellen	female	50,000
Flo	female	20,000

- All female employees earn less than \$65,000. (T or F?)
- Every employee earning less than \$55,000 is female. (T or F?)
- Male employees earn less than \$55,000. (T or F?)

Prove / Disprove Universal Quantification

Every P is a Q

- To prove, verify that every element of the domain is an example that satisfies the quantification.
- To disprove, give a single counter-example that does not satisfy the quantification

Existential quantification: $\exists$

Employee	**Gender**	**Salary**
Al	male	60,000
Betty	female	500
Carlos	male	40,000
Doug	male	30,000
Ellen	female	50,000
Flo	female	20,000

- Some employee earns over \$57,000.
- There is an employee who earns over \$57,000.

Existential quantification: $\exists$

Employee	Gender	Salary
Al	male	60,000
Betty	female	500
Carlos	male	40,000
Doug	male	30,000
Ellen	female	50,000
Flo	female	20,000

- Some employee earns over \$80,000. (T or F?)
- There exists a male employee who earns less than \$27,000. (T or F?)
- At least one female employee earns over \$42,000. (T or F?)

Prove / Disprove Existential Quantification

Some P is a Q

- To prove, give a single example that satisfies the quantification.
- To disprove, verify that every element of the domain is a counter-example that does not satisfy the quantification.

Prove/disprove with “no”, “one”, “example”, “counter-example”

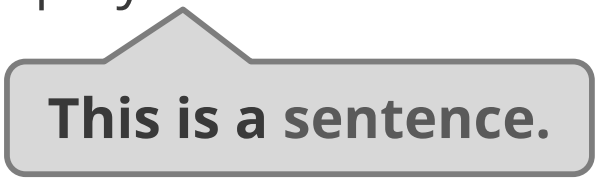
	Universal	Existential
Verify (prove)	no counter-example	one example
Falsify (disprove)	one counter-example	no example

“Duality”, “anti-symmetric”

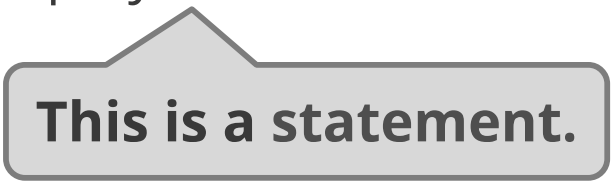
Sentences & Statements

Sentence and Statement

The employee earns less than $55,000.



Every employee earns less than $55,000.



Sentence vs Statement

- ➔ A **statement** is always a **sentence.**
- ➔ A **sentence** is **NOT** always a **statement**.
- ➔ A **statement** is a **sentence** that is NOT "**open**".
- ➔ The object in an **open sentence** is **unspecified (unquantified)**, thus the sentence cannot be evaluated.
- ➔ The object in a **statement** is **quantified**, thus a **statement** can be evaluated **true** or **false**.

Predicate

Predicates

L: the employees who earn less than $55,000.

To say: employee ***x*** earns less than $55,000.

We can write: $x \in L$ — ***x* is in the set *L***

or we can write: *L(x)* — ***x* has property *L***

***L(x)* is a boolean function returning True or False (whether *x* earns less than $55,000)**

***L(x)* is called a predicate.**

Predicates

$L(x) : x$ earns less than \$55,000

$L(\text{Carlos})$ Carlos earns less than $55,000.

$\neg L(\text{Carlos})$

Carlos earns no less than $55,000

"¬" means "not": negation

Predicates

$F(x) : x$ is a female employee

$F(\text{Ellen})$ Ellen is a female employee.

$\neg F(\text{Carlos})$

Carlos is not a female employee

Predicates

L(x) without specifying ***x*** is an **open sentence**.

Turn it into a **statement** by **universally** quantifying it.

For each employee, the employee earns less than $55,000.

∀ employee ***x***: ***x*** earns less than $55,000.

$\forall x \in E: L(x)$

Predicates

Similarly, turn it into a **statement** by **existentially** quantifying it.

There exists an employee, the employee earns less than $55,000.

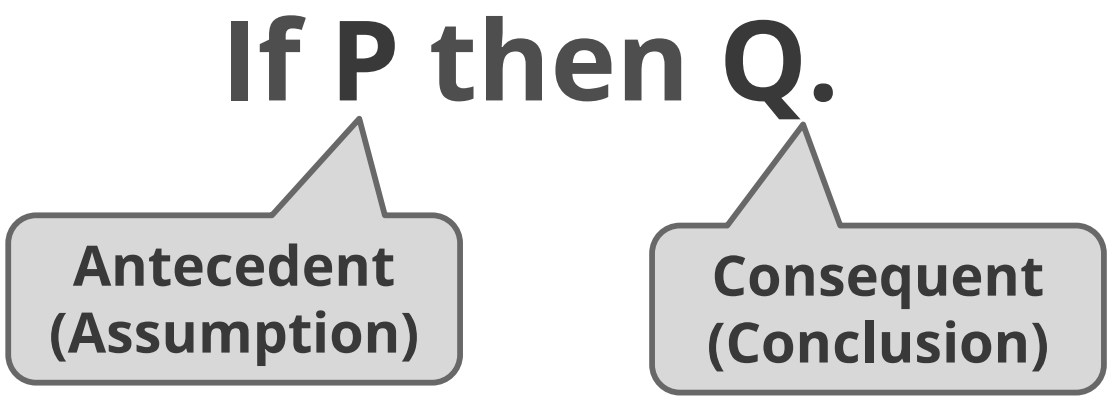
∃ employee ***x***: ***x*** earns less than $55,000.

$\exists x \in E: L(x)$

Implication

Implication

If an employee is male, **then** he makes less than $55,000.



$$P \Rightarrow Q$$

"P implies Q"

Verify / falsify implication

If an employee is male, **then** he makes less than $55,000.

$\forall x \in M : L(x)$

Employee	Gender	Salary
Al	male	60,000
Betty	female	500
Carlos	male	40,000
Doug	male	30,000
Ellen	female	50,000
Flo	female	20,000

Same as verifying / falsifying a universal quantification.

Converse of implication

E: set of all employees
F(x): ***x*** is female
L(x): ***x*** earns less than $55,000

Original: $\forall x \in E: F(x) \Rightarrow L(x)$

Converse: $\forall x \in E: L(x) \Rightarrow F(x)$

What's the relation between the two?

Contrapositive

E: set of all employees
F(x): ***x*** is female
L(x): ***x*** earns less than $55,000

Original: $\forall x \in E: F(x) \Rightarrow L(x)$

Contrapositive: $\forall x \in E: \neg L(x) \Rightarrow \neg F(x)$

What's the relation between these two?