1. Prove the following claim:

$$\forall u, v, w, z \in \mathbb{R}^+ : \frac{u}{v} < \frac{z}{w} \Rightarrow \frac{u+z}{v+w} < \frac{z}{w}$$

2. Prove the following claim:

$$\forall x, y \in R : (y^3 + yx^2 \leqslant x^3 + xy^2) \Rightarrow (y \leqslant x)$$

3. Recall the predicates $E(n)$: "n is even" and $O(n)$: "n is odd". Prove the following claim:

$$\forall n \in \mathbb{N} : E(n^2 + 3) \Rightarrow O(n)$$

4. Prove or disprove the following statement

$$\forall n \in \mathbb{N} : [\exists a, b \in \mathbb{Z} : n = 5a + 7b] \Rightarrow [\exists a, b \in \mathbb{Z} : n + 1 = 5a + 7b]$$

5. Prove or disprove the following statements

   (a) For all real numbers r, s, if r and s are both positive, then $\sqrt{r} + \sqrt{s} = \sqrt{r+s}$

   (b) For all real numbers r, s, if r and s are both positive, then $\sqrt{r} + \sqrt{s} \neq \sqrt{r+s}$

6. Let $\mathbb{I}$ be the set of irrational numbers and $\mathbb{Q}$ the set of rational numbers. Prove the following:

$$\forall x \in \mathbb{Q} : x \neq 0 \Rightarrow [\exists a, b \in \mathbb{I} : x = a \cdot b].$$