1. Sample Solution:
Let $x \in \mathbb{R}^+$.
We'll prove that $\forall n \in \mathbb{N} : (1+x)^n \geqslant 1 + nx$ by induction.

Base step: Prove $P(0)$
Let $n = 0$.
Then $(1+x)^n = 1$ and $1 + nx = 1$.
Then $(1+x)^0 \geq 1 + (0)x$.
Then $P(0)$.

Induction step: Prove $\forall k \in \mathbb{N} : ((1+x)^k \geqslant 1 + kx) \Rightarrow ((1+x)^{k+1} \geqslant 1 + (k+1)x)$
Let $k \in \mathbb{N}$.
Assume $(1+x)^k \geqslant 1 + kx$.
We want to prove that $(1+x)^{k+1} \geqslant 1 + (k+1)x$.
We'll start with the quantity on the left, and show that it's $\geqslant$ the quantity on the right.

$$\begin{aligned}
\text{Then } (1+x)^{k+1} &= (1+x)^k(1+x) \\
&\geqslant (1+kx)(1+x) && \text{(by our assumption)} \\
&= 1 + kx + x + kx^2 \\
&\geqslant 1 + kx + x && \text{(since } kx^2 \geqslant 0\text{)} \\
&= 1 + (k+1)x
\end{aligned}$$

Then $(1+x)^{k+1} \geqslant 1 + (k+1)x$.
Then $((1+x)^k \geqslant 1 + kx) \Rightarrow ((1+x)^{k+1} \geqslant 1 + (k+1)x)$.
Then $\forall k \in \mathbb{N} : ((1+x)^k \geqslant 1 + kx) \Rightarrow ((1+x)^{k+1} \geqslant 1 + (k+1)x)$.
Then $\forall n \in \mathbb{N} : (1+x)^n \geqslant 1 + nx$.
Proof by induction is complete

Therefore, $\forall x \in \mathbb{R}^+ : \ \forall n \in \mathbb{N} : \ (1+x)^n \geqslant 1 + nx$.