**Solution**

1. Sample Solution:

   Let $x \in \mathbb{R}$.

   # Proof by contrapositive

   Assume $x \leqslant -1$.

   Then $x < 0$.

   Also $x + 1 \leqslant 0$.

   Then $x - 1 < 0$. #subtract 2 from both sides and use the fact $-2 < 0$.

   Then $(x+1)(x-1) \geqslant 0$ # multiply $x + 1 \leqslant 0$ by $x - 1$.

   Then $x^2 - 1 \geqslant 0$.

   Then $x(x^2 - 1) \leqslant 0$ # multiply by $x < 0$.

   Then $x^2 - x \leqslant 0$.

   Then $x \leqslant -1 \Rightarrow x^3 - x \leqslant 0$.

   Then $x^3 - x > 0 \Rightarrow x > -1$.

   Therefore $\forall x \in \mathbb{R} : x^3 - x > 0 \Rightarrow x > -1$.

2. Sample Solution:
   Let $n \in \mathbb{Z}$.
   Assume $\exists a, b \in \mathbb{Z} : n + 5 = 5a + 6b$.
   Let $a_0, b_0 \in \mathbb{Z}$ such that $n + 5 = 5a_0 + 6b_0$.
   Then, $n + 2 = n + 5 - 3$.
   Then, $n + 2 = 5a_0 + 6b_0 - 3$.
   Then, $n + 2 = 5a_0 + 6b_0 - 15 + 15 - 3$.
   Then, $n + 2 = 5(a_0 + 3) + 6(b_0 - 3)$.
   Let $a_1 = a_0 + 3$.
   Then, $a_1 \in \mathbb{Z}$.
   Let $b_1 = 2(b_0 - 3)$.
   Then, $b_1 \in \mathbb{Z}$.
   Then, $n + 2 = 5a_1 + 3b_1$.
   Then, $\exists a, b \in \mathbb{Z} : n + 2 = 5a + 3b$.
   Then, $[\exists a, b \in \mathbb{Z} : n + 5 = 5a + 6b] \Rightarrow [\exists a, b \in \mathbb{Z} : n + 2 = 5a + 3b]$.
   Therefore $\forall n \in \mathbb{Z} : [\exists a, b \in \mathbb{Z} : n + 5 = 5a + 6b] \Rightarrow [\exists a, b \in \mathbb{Z} : n + 2 = 5a + 3b]$.