

1. Let $f : \mathbb{N} \rightarrow \mathbb{R}^+$ that satisfies the following condition:

$$\forall n \in \mathbb{N} : f(n) \geq 1.$$

Prove the following:

$$2^{\sqrt{f(n)}} \in O(2^{f(n)}).$$

2. Prove that the function `meaning_of_life` below is not computable:

```
def meaning_of_life(f, I) :
    """
    Return True if f(I) returns 42, False otherwise.
    """
```

Emulate the technique from the course notes to reduce `halt` to `meaning_of_life`

3. Consider the following Python function:

```
def has_duplicate(L):
    """
    :param L: List
    :return: True if the list has at least a pair of duplicates,
             False otherwise
    """
    if L == []:
        return False
    i = 0
    while i < len(L):
        j = 0
        while j < len(L):
            if (i != j) and (L[i] == L[j]):
                return True
            j = j + 1
        i = i + 1
    return False
```

Denote `TDup(n)` the function that gives the running time complexity of the function `has_duplicate(L)` where $n = \text{len}(L)$. Compute `TDup(n)`. Justify every step.

4. Consider the following Python function:

```
def prime(n):
    i = 2
    while i*i <= n:
        if n%i == 0:
            return False
        else:
            i = i + 1
    return True
```

Denote $TP(n)$ the function that gives the running time complexity of the function `prime(n)`. Compute $TP(n)$. Justify every step.

5. **Prove or disprove:**

$$\forall e \in \mathbb{R}^+ : \exists d \in \mathbb{R}^+ : \forall x, y \in \mathbb{R}^+ : |x - y| > d \Rightarrow |x + y| > e$$