Note that the notation $\sum_{i=j}^{k} f(i)$ gives us a short form for expressing the sum $f(j) + f(j+1) + \cdots + f(k-1) + f(k)$. Also, note that $\prod_{i=j}^{k} f(i)$ gives us a short form for expressing the product $f(j) \times f(j+1) \times \cdots \times f(k-1) \times f(k)$.

1. Expand the following expressions to get the long sum/product they represent. Do not simplify.

   (a) $\sum_{k=1}^{3}(k+1)$

   (b) $\sum_{m=0}^{1} \dfrac{1}{2^m}$

   (c) $\sum_{k=-1}^{2}(k^2+3)$

   (d) $\sum_{j=0}^{4}(-1)^j \dfrac{j}{j+1}$

   (e) $\sum_{k=1}^{5}(2k)$

   (f) $\prod_{i=2}^{4} \dfrac{i(i+2)}{(i-1)(i+1)}$

2. Simplify each of the following expressions by using $\sum$ or $\prod$ notation.

   (a) $3+6+12+24+48+96$

   (b) $\dfrac{1}{3}+\dfrac{4}{9}+\dfrac{9}{27}+\dfrac{16}{81}+\dfrac{25}{243}+\dfrac{36}{729}$

   (c) $0+1-2+3-4+5$

   (d) $\left(\dfrac{1}{1+1}\right) \times \left(\dfrac{2}{2+1}\right) \times \left(\dfrac{3}{3+1}\right) \times \cdots \times \left(\dfrac{k}{k+1}\right)$

   (e) $\left(\dfrac{1\cdot 2}{3\cdot 4}\right) \times \left(\dfrac{2\cdot 3}{4\cdot 5}\right) \times \left(\dfrac{3\cdot 4}{5\cdot 6}\right)$

3. It is not to hard to prove manipulation results like the following that can be used to help us manipulate sums and products. If $a_m, a_{m+1}, a_{m+2}, \ldots$ and $b_m, b_{m+1}, b_{m+2}, \ldots$ are sequences of real numbers and c is any real number, then the following equations hold for any integer $n \geqslant m$:

$$\sum_{k=m}^{n}(a_k+b_k) = \sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k$$

$$\sum_{k=m}^{n} c\cdot a_k = c\cdot \sum_{k=m}^{n} a_k$$

$$\prod_{k=m}^{n}(a_k\cdot b_k) = \left(\prod_{k=m}^{n} a_k\right)\left(\prod_{k=m}^{n} b_k\right)$$

   Using these laws, rewrite each of the following as a single sum or product, but do not simplify your final sum/product.

   (a) $3\cdot\sum_{k=1}^{n}(2k-3)+\sum_{k=1}^{n}(4-5k)$

   (b) $\left(\prod_{k=1}^{n}\dfrac{k}{k+1}\right)\left(\prod_{k=1}^{n}\dfrac{k+1}{k+2}\right)$

4. Let our universe be seven different programs: $U = \{A, B, E, G, L, K, M\}$, each meant to carry out the same task, written in different languages. Denote $P = \{B, G, K, M\}$ the set of programs written in Python and $J = \{A, E, L\}$ the set of programs written in Java. Suppose that the set $C = \{B, A, E, L, M\}$ contains correct programs and $I = \{G, K\}$ contains incorrect programs, but you have not yet verified this by testing the programs.

   For the following two statements, say whether the statement is true or false, give the *smallest* number of programs that must be tested to verify your claim, and justify each answer:

   (a) All Python programs are correct.

   (b) All Java programs are correct.

Solution