Decision Variable

X_{ij}^{k} = Binary Variable; 1 if lorry k travels directly from location i to location j .

S_i^k =Service start time at location i by lorry k

$deliv_i^k$= Integer variable, representing the number of new batteries delivered by lorry k up to, in location i.

$Pick_i^k$= Integer variable, representing the number of old batteries picked by lorry k up to, in location i.

$LowSOC_i =$ Binary variable, 1 if the SOC at location i is below 20%, and 0 otherwise.

Parameters

N= all nodes

K= no of lorries

d_{ij}= Distance between location i to j.

Q= capacity

e_i, , l_i, = earliest and last time of time windows.

p_i=Service time at location i.

D_i= Demand of delivery of charged ESS at location i.

P_i= Original demand of pickup discharged ESS at location i.

SOC_i = State of Charge at location i as a percentage.

T_{ij}= Travel time from location i to j.

FC= Fuel cost per kilometer.

MC= Manpower cost per hour.

RT= Road toll.

VMC= Vehicle maintenance cost per kilometer.

Objective Function:

The objective is to minimize the travel distance and operational costs:

$$\text{Min Z} = \sum_{k\in K}\sum_{i\in N}\sum_{j\in N, i\neq j}(d_{ij} \cdot (FC + VMC) \cdot X_{i,j}^{k}) + \sum_{k\in K}\sum_{i\in N}\sum_{j\in N, i\neq j} MC \cdot (S_i^k + p_i + T_{ij}) + \sum_{k\in K} RT \cdot \sum_{j\in N, j\neq 0} X_{0j}^{k}$$

Constraints:

1. **Delivery and pickup demand fulfillment:**

$deliv_i^k = D_i$ $\forall\ i\epsilon\ N\backslash\{0\}$
$Pick_i^k = P_i$ $\forall\ i\epsilon\ N\backslash\{0\}$

2. **Vehicle routing constraint:** Sum all lorries k and all possible destination nodes j to calculate the total number of trips allocated towards serving each customer. This constraint is to satisfy the total demand for MESS delivery and pickup each customer sites.

$\sum_{k\epsilon K}\sum_{j\epsilon N, j\neq i} X_{i,j}^k \geq\ D_i + P_i$ $\forall\ i\epsilon\ N\backslash\{0\}$

3. **Vehicle Capacity Constraint**

$\sum_{k\epsilon K}\ deliv_i^k\ \leq$ Q $\forall\ i\epsilon\ N\backslash\{0\}$
$\sum_{k\epsilon K}\ pick_i^k\ \leq$ Q $\forall\ i\epsilon\ N\backslash\{0\}$

4. SOC Pickup Requirement

$\sum_{k\epsilon K}\ pick_i^k\ \geq\ LowSOC_i \cdot\ (1 - \frac{SOC_i}{100})\ \times\ P_i$ $\forall\ i\epsilon\ N\backslash\{0\}$

5. Service and travel time window constraint:

$e_i\ \leq\ S_i^k \leq l_i$ $\forall\ k\epsilon K\ , \forall\ i\epsilon N$

$S_i^k + p_i + T_{ij} \leq S_j^k$ $\forall\ k\epsilon K\ , \forall\ i, j\epsilon N, i \neq 0$

6. Depot start and end constraint:

$\sum_{j\in N, j\neq 0} X_{0j}^k = 1$ $\forall\ k\epsilon K$

$\sum_{i\in N, i\neq 0} X_{i0}^k = 1$ $\forall\ k\epsilon K$

7. Subtour elimination
$X_{i,j}^k +\ X_{j,i}^k \leq 1$ $\forall\ k\epsilon K\ , \forall\ i, j\epsilon N, i \neq 0$

8. No loop between nodes

$\sum_{i\in \mathrm{N}} X_{ii}^k$ == 0 $\forall\ k\epsilon K$