1. (a) $F_X(x)$ is cumulative distribution function of X, and X is a random variable represents the number shown when we roll the D_1 which has 9 sides, so the distribution of X is discrete. When $X < 1$, the probability will be 0, the cumulative distribution $F_X(x) = 0$ when $x \in (-\infty, 1)$, and $F_X(x) = 1$ when $x \in (9, \infty)$
When $x \in [1, 9], \mathbb{P}(X = x) = \log_{10}(\dfrac{x+1}{x})$ and $F_X(x) = \log_{10}(\frac{1+1}{1}) + \log_{10}(\frac{1+2}{2}) \ldots \log_{10}(\frac{x+1}{x})$
$F_X(x) = \log_{10}(\dfrac{2 \times 3 \times \cdots \times (x+1)}{1 \times 2 \times \cdots \times x}) = \log_{10}(x+1)$. Because x can only be an integer and within the range $[1, 9]$, therefore $\lfloor x \rfloor = x$. So, $F_X(x) = \log_{10}(\lfloor x \rfloor + 1)$ when $x \in [1, 9]$

   (b) To calculate $\mathbb{P}(X + Y = 10)$, there are 9 combinations. $X = 1, Y = 9$ or $X = 2, Y = 8$ or $\ldots X = 8, Y = 2$ or $X = 9, Y = 1$, therefore,
$\mathbb{P}(X+Y = 10) = \mathbb{P}(X = 1) \times \mathbb{P}(Y = 9) + \mathbb{P}(X = 2) \times \mathbb{P}(Y = 8) \ldots \mathbb{P}(X = 9) \times \mathbb{P}(Y = 1) = (\mathbb{P}(X = 1) + \mathbb{P}(X = 2) \ldots \mathbb{P}(X = 9)) \times \dfrac{1}{9} = F_X(9) \times \frac{1}{9} = \frac{1}{9}$

2. (a) From the truth table, we noticed that Z only has two outcome 0 or 1, therefore it's a Bernoulli distribution. When $Z = 1$, $\mathbb{P}(Z = 1) = \mathbb{P}(X = 1) \times \mathbb{P}(Y = 0) + \mathbb{P}(X = 0) \times \mathbb{P}(Y = 1) = \frac{1}{10} \times \frac{1}{2} + \frac{9}{10} \times \frac{1}{2} = \frac{1}{2}$
So $Z \sim$ Bernoulli $(\frac{1}{2})$

   (b) $\mathbb{P}(Y = 0, Z = 0) = \mathbb{P}(Y = 0 \mid Z = 0) \times \mathbb{P}(Z = 0)$
$= \dfrac{\mathbb{P}(X = 0, Y = 0)}{\mathbb{P}(Z = 0)} \times \mathbb{P}(Z = 0) = \dfrac{9}{10} \times \dfrac{1}{2} = \dfrac{9}{20}$

   (c) no, variable Y, Z are not independent. As $\mathbb{P}(Y = 0, Z = 0) \neq \mathbb{P}(Y = 0) \times \mathbb{P}(Z = 0)$