SCIENCE

STATS 210: Statistical Theory

Assignment Tracking Sheet

Student Information

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Assignment Information

Assignment Name:	Tutorial 4	Due:	11:00 p.m. - 31 Aug, 2020 (NZ Time)
Department:	statistics		
Lab / Tutorial Day:		Time:	
Lab / Tutorial Group:		Tutor:	
Notes:		Word Count:	

Declaration: (please read and sign)

By submitting this assignment, I confirm that I am aware of The University expectation that all students complete coursework with integrity and honesty as stated in the Student Academic Conduct Statute.
http://www.auckland.ac.nz/uoa/home/about/teaching-learning/honesty/tl-uni-regs-statutes-guidelines

- I understand that the University of Auckland will not tolerate cheating or assisting other to cheat, and views cheating in coursework as a serious academic offence.
- I declare that where work from other sources (including sources on the world-wide web) has been used, it has been properly acknowledged and referenced.
- I confirm that this work represents my individual/ our team's effort and does not contain plagiarised material.
- I have checked the above details and verify them to be correct for the assignment I am submitting.
- I understand that the University of Auckland takes no responsibility for lost assignments and that I agree to provide a duplicate copy if requested.
- I understand that uncollected assignments will be retained in secure storage until the end of the examination period and thereafter destroyed.
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Signed: ______________________ Date: ______________________

Note:

1. Assignments are not accessible after they have been handed in. No additions/removals will be permitted.
2. Marks may be withheld for students who have not submitted their work to Turnitin.com if required in the course outline.
3. The University of Auckland views cheating in coursework as a serious academic offence. Accordingly it may require submitted work to be reviewed against electronic source material using computerised detection mechanisms.

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1. (a) X is the number of boys out of Garfield's seven children, it's has binomial distribution. Therefore, the distribution of X is
$X \sim \text{Binimoal}(7, p)$

   (b) X is the number of boys out of Garfield's seven children and he has 5 sons, therefore $x = 5$

   (c) The likelihood function:

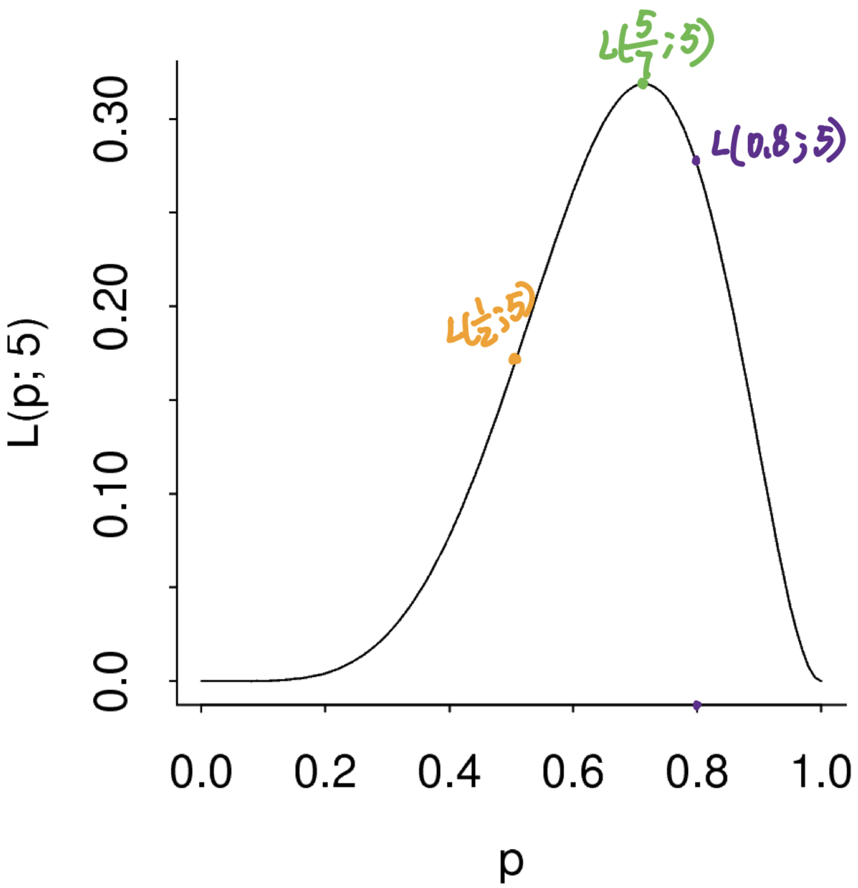
$$\begin{aligned} L(p;5) &= \mathbb{P}(X = 5) \text{ when } X \sim \text{Binimoal}(7, p), \\ &= \binom{7}{5} p^5 (1-p)^2 \\ &= 21 p^5 (1-p)^2 \quad \text{for } 0 < p < 1 \end{aligned}$$

   (d) $L(\frac{1}{2}; 5) = 21 \times (\frac{1}{2})^5 (1 - \frac{1}{2})^2 = 0.164$

   (e) $L(\frac{5}{7}; 5) = 21 \times (\frac{5}{7})^5 (1 - \frac{5}{7})^2 = 0.319$

   (f) $L(0.8; 5) = 21 \times 0.8^5 (1 - 0.8)^2 = 0.275$

   (g) The graph is attached below



(h)

$$\begin{aligned}\frac{dL}{dp} &= 21 \times \left(5 \times p^4 \times (1-p)^2 + p^5 \times 2 \times (1-p) \times (-1)\right) \quad \text{Product rule} \\ &= 21 \times p^4 \times (1-p) \times (5 \times (1-p) - 2p) \\ &= 21p^4(1-p)(5-7p)\end{aligned}$$

(i) The maximising value of p occurs when $\left.\frac{dL}{dp}\right|_{p=\hat{p}} = 0$, this gives:

$$\begin{aligned}\left.\frac{dL}{dp}\right|_{p=\hat{p}} &= 21\hat{p}^4(1-\hat{p})(5-7\hat{p}) = 0 \\ &\implies 5 - 7\hat{p} = 0 \\ &\implies \hat{p} = \frac{5}{7}\end{aligned}$$

(j) We have decided that a sensible parameter estimate for p is the maximum likelihood estimate ($\hat{p} = \frac{5}{7}$): the value of p at which the observation $X = 5$ is more likely than at any other value of p.

2. (a) The likelihood function:

$$\begin{aligned}&L(p;3) = \mathbb{P}(X = 3) \text{ when } X \sim \text{Geometric}(p) \\ &L(p;3) = (1-p)^3 p \text{ for } 0 < p < 1\end{aligned}$$

(b)

$$\begin{aligned}\frac{dL}{dp} &= (1-p)^3 + p \times 3 \times (1-p)^2 \times (-1) \quad \text{(by product rule)} \\ &= (1-p)^3 - 3p(1-p)^2 \\ &= (1-p)^2(1-p-3p) \\ &= (1-p)^2(1-4p)\end{aligned}$$

(c) The miximising value of p occurs when $\left.\frac{dL}{dp}\right|_{p=\hat{p}} = 0$, this gives

$$\begin{aligned}\left.\frac{dL}{dp}\right|_{p=\hat{p}} &= (1-\hat{p})^2(1-4\hat{p}) = 0 \\ \implies 1 - 4\hat{p} &= 0 \\ \hat{p} &= \frac{1}{4}\end{aligned}$$

(d) In common-sense, Sammie first succeeded on his fourth jump, we would think that the probability of success would be $\frac{1}{4}$. And our MLE indeed same as our common-sense.