ID: 107664618
Joy HU
UPI: mhu138

1. (a) Yes, $X \sim$ Binomial $(n = 500, p = 0.14)$

   (b) Yes, $X \sim$ Binomial$(n = 8, p = 0.7)$

   (c) No, not a fixed number of trials.

   (d) Yes, $X \sim$ Binomial$(n = 12, p = 0.02)$

   (e) No, the number of coffee surved is not fixed.

2. (a) $X \sim$ Binomial$(n = 10, p = 0.2)$

   (b) $p_2 = \mathbb{P}(X = 2) = \binom{10}{2} \times 0.2^2 \times 0.8^8 = 0.30$
   $p_3 = \mathbb{P}(X = 3) = \binom{10}{3} \times 0.2^3 \times 0.8^7 = 0.20$

   (c) $c_0 = \mathbb{P}(X \leq 0) = 0.11$
   $c_4 = \mathbb{P}(X \leq 4) = F_X(3) + \mathbb{P}(X = 4) = 0.88 + 0.09 = 0.97$
   $c_5 = \mathbb{P}(X \leq 5) = F_X(4) + \mathbb{P}(X = 5) = 0.97 + 0.02 = 0.99$

   (d)

$$f_X(x) = \begin{cases} 0.11 & \text{when } x = 0 \\ 0.27 & \text{when } x = 1 \\ 0.30 & \text{when } x = 2 \\ 0.20 & \text{when } x = 3 \\ 0.09 & \text{when } x = 4 \\ 0.02 & \text{when } x = 5 \\ 0.01 & \text{when } x = 6 \\ 0 & \text{otherwise} \end{cases}$$

$$F_X(x) = \begin{cases} 0.11 & \text{when } x = 0 \\ 0.38 & \text{when } x = 1 \\ 0.68 & \text{when } x = 2 \\ 0.88 & \text{when } x = 3 \\ 0.97 & \text{when } x = 4 \\ 0.99 & \text{when } x = 5 \\ 1 & \text{when } x \geq 6 \end{cases}$$

   The probability of Tom will get at least 6 out of 10 will be:

$$\mathbb{P}(x \geq 6) = 1 - F_X(5) = 1 - 0.99 = 0.01$$

   (e) $T \sim$ Binomial$(n = 40, p = 0.2)$

   (f) $\mathbb{P}(T = 10) = \binom{40}{10} \times 0.2^{10} \times 0.8^{30} = 0.107$

   (g) $\mathbb{P}(t \geq 20) = 1 - F_T(t \leq 19) = 1 - (F_T(18) + \mathbb{P}(X = 19)) = 1 - (0.9999148 + 6.348 \times 10^{-5}) = 1 - 0.9999783 = 2.171 \times 10^{-5}$ I would never recommend this strategy as it has close to 0 probability. Study the course before the test is recommended for sure!