STATS 210 **Tutorial 4**

1. President James Abram Garfield (1831-1881) was an American president with lots of children. He had 5 sons and 2 daughters. He also gave his name to the cartoon character Garfield, although the resemblence is not entirely convincing.

Suppose that each of President Garfield's seven children were boys with probability p and girls with probability $1-p$, all independently of each other. Define the random variable X to be the number of boys out of Garfield's seven children.

(a) Write down the distribution of X, in terms of the unknown parameter p. (1)

(b) Write down x, the observed value of X, from the information above. (1)

(c) Write down the likelihood function, $L(p\,; x)$, substituting the correct value of x. Remember to state the range of values of p for which your expression applies. In your answer, replace any expression of the form $\binom{a}{b}$ (i.e. ${}^aC_b$) with its actual value. (1)

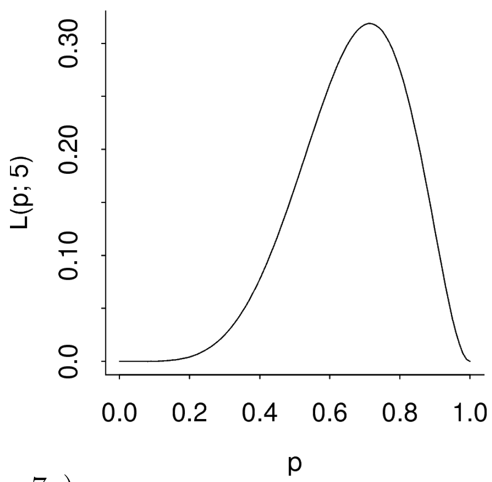
(d) Find $L\left(\frac{1}{2}\,; 5\right)$. (1)

(e) Find $L\left(\frac{5}{7}\,; 5\right)$. (1)

(f) Find $L\,(0.8\,; 5)$. (1)

(g) The graph of $L(p\,; 5)$ for $0 < p < 1$ is here:

Redraw the graph as a sketch, and mark on it the answers from parts (d), (e), and (f). (3)



(h) Show that

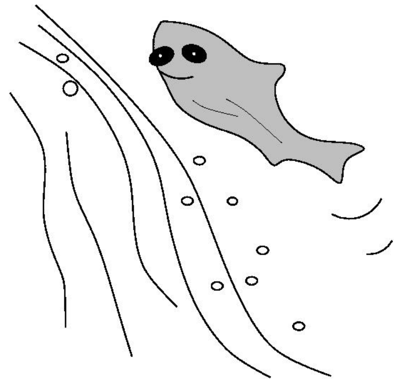
$$\frac{dL}{dp} = 21p^4(1-p)(5-7p).$$

(3)

(i) The sketch in part (g) shows that the value of p that maximizes $L(p\,; 5)$ is *not* $p = 0$ or $p = 1$. Use the expression in (h) to find the value of p that *does* maximize the likelihood. (1)

(j) Write a sentence in English to explain what the maximum likelihood estimate from (i) represents. [Hint: see page 63 of the course book - the comment in blue below "Maximizing the likelihood".] (1)

2. Have you ever seen a salmon leap? Salmon leaps are waterfalls where you can see salmon repeatedly leaping out of the water in their attempts to jump to the top of the waterfall, so they can continue upstream to breed.



Sammie the Salmon is trying to jump up a waterfall. Each time he jumps, he has probability p of making it to the top. All Sammie's jumps are independent. Let X be the total number of jumps that Sammie makes until he has finally made it to the top.

The probability function of X is:

$$\mathbb{P}(X = x) = (1 - p)^{x-1} p,$$

because a total of x jumps requires $x - 1$ failures to begin with (probability $(1 - p)^{x-1}$), followed by a single success (probability p).

Suppose that Sammie manages to jump to the top of the waterfall after $X = 4$ jumps.

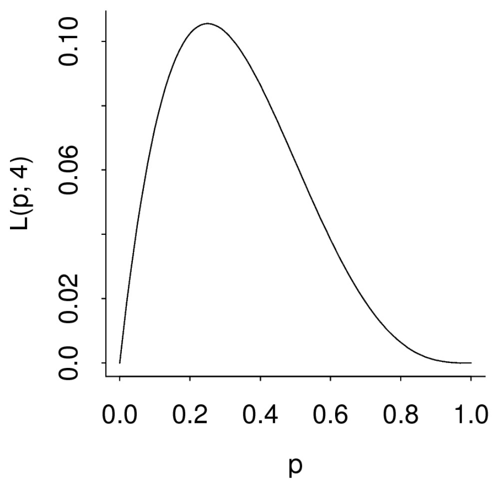
(a) Write down the likelihood function, $L(p\,; 4)$. State the range of values of p for which the function is defined. (1)

(b) Show that

$$\frac{dL}{dp} = (1 - p)^2 (1 - 4p).$$

(3)

(c) The likelihood function is plotted below. Using the expression in (b), find the maximum likelihood estimate of p. (1)



(d) Explain why the maximum likelihood estimate is also a common-sense estimate of p, remembering that Sammie first succeeded on his fourth jump. (1)

Total: 20