STATS 210 **Tutorial 2**

1. Answer the following questions. Show your working for each part.

   (a) Suppose that X is a random variable that represents the number shown when we roll the die D_1 which has 9 sides. The probability function of X is given by

   $$\mathbb{P}(X = x) = \log_{10}\left(\frac{x+1}{x}\right), \text{ for } x \in \{1, 2, \ldots, 9\},$$

   where $\log_{10}(\cdot)$ denotes the logarithm base 10. Show that the cumulative distribution function of X is:

   $$F_X(x) = \begin{cases} 0, & \text{if } x \in (-\infty, 1), \\ \log_{10}\left(\lfloor x \rfloor + 1\right), & \text{if } x \in [1, 9], \\ 1, & \text{if } x \in (9, \infty), \end{cases}$$

   where $\lfloor \cdot \rfloor$ is the greatest integer less than or equal to the real number in the argument. For example, $\lfloor 2.0 \rfloor = 2$, $\lfloor 2.5 \rfloor = 2$, $\lfloor 2.999999 \rfloor = 2$. (6)

   (b) Let Y be a random variable that represents the number shown when we roll the 9-sided die D_2 which is known to be fair. Hence, we have:

   $$\mathbb{P}(Y = y) = \frac{1}{9}, \text{ for } y \in \{1, 2, \ldots, 9\}.$$

   Calculate $\mathbb{P}(X + Y = 10)$ for the case when the random variables X and Y are assumed to be independent. (4)

2. Suppose that $X \sim$ Bernoulli(1/10), $Y \sim$ Bernoulli(1/2), and the random variables X and Y are independent. The random variable Z is given by $Z = X \oplus Y$, where the operator $\oplus$ is defined in Table 1. The operation that corresponds to the symbol $\oplus$ is called *exclusive or* (often abbreviated to XOR), or *addition mod 2*.

   Note that for any $x, y \in \{0, 1\}$, $x \oplus y = 0$ if $x = y$ and $x \oplus y = 1$ if $x \neq y$.

x	y	$z = x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

Table 1: Definition of the operator $\oplus$.

   Answer the following questions:

   (a) Show that $Z \sim$ Bernoulli(1/2). (3)

   (b) Calculate $\mathbb{P}(Y = 0, Z = 0)$. (4)

   (c) Decide if the random variables Y and Z are independent (or not). Explain your answer. (3)

Total: 20