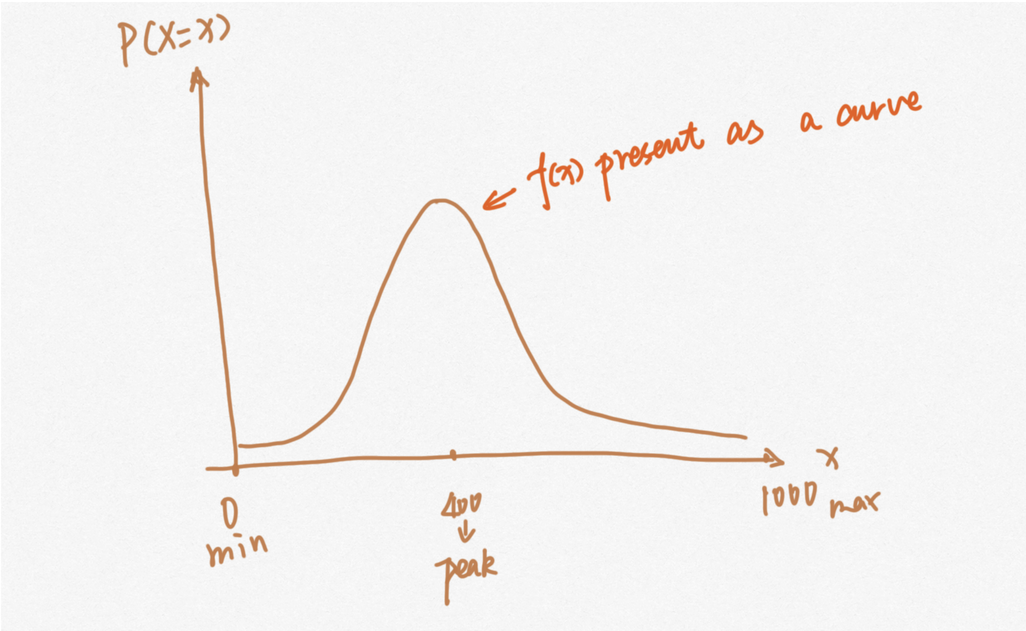
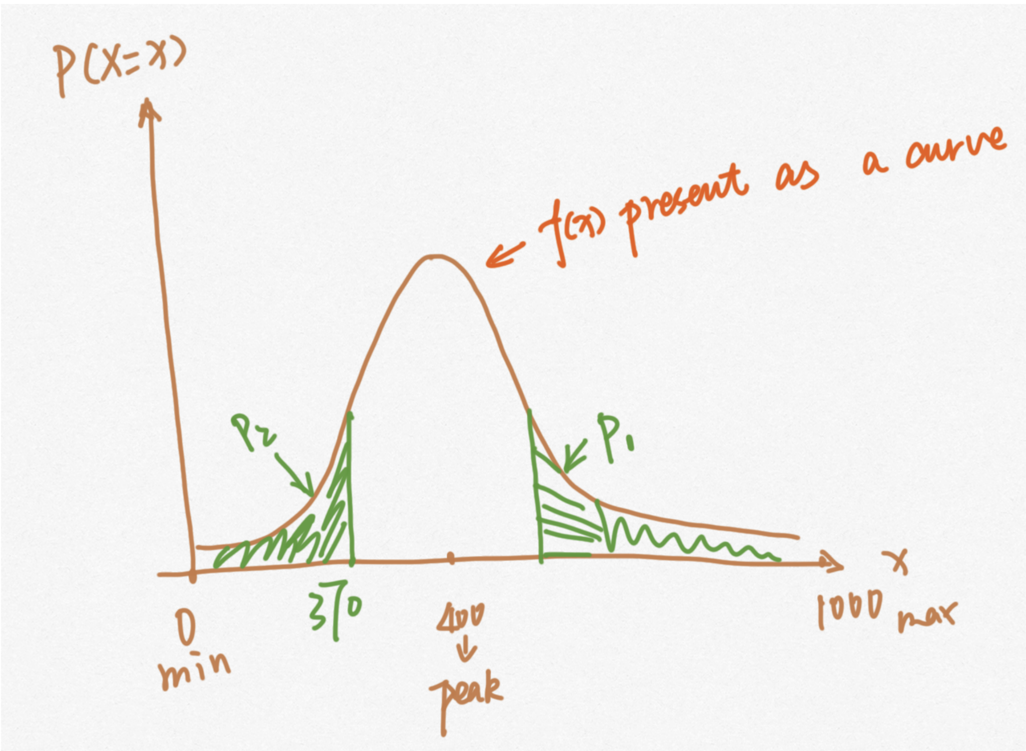
ID: 107664618
Joy HU
UPI: mhu138

1. (a) Let X be the number of Ardent-voters out of 1000 respondents.

$$
\begin{aligned}
&X \sim \text{Binomial}(1000, p_A) \\
&H_0 : p_A = 0.4 \\
&H_1 : p_A \neq 0.4 && \text{(two-sided test)}
\end{aligned}
$$

(b) Under H_0, we have $X \sim \text{Binomial}(1000, 0.4)$, The probability function of X peaks at about $1000 \times 0.4 = 400$





(c)

ID: 107664618
Joy HU
UPI: mhu138

(d) For the p-value,

$$\mathbb{P}(X \leq 370) = F_X(370)$$

The R code for the p-value is `2 * pbinom(370,1000,0.4)`

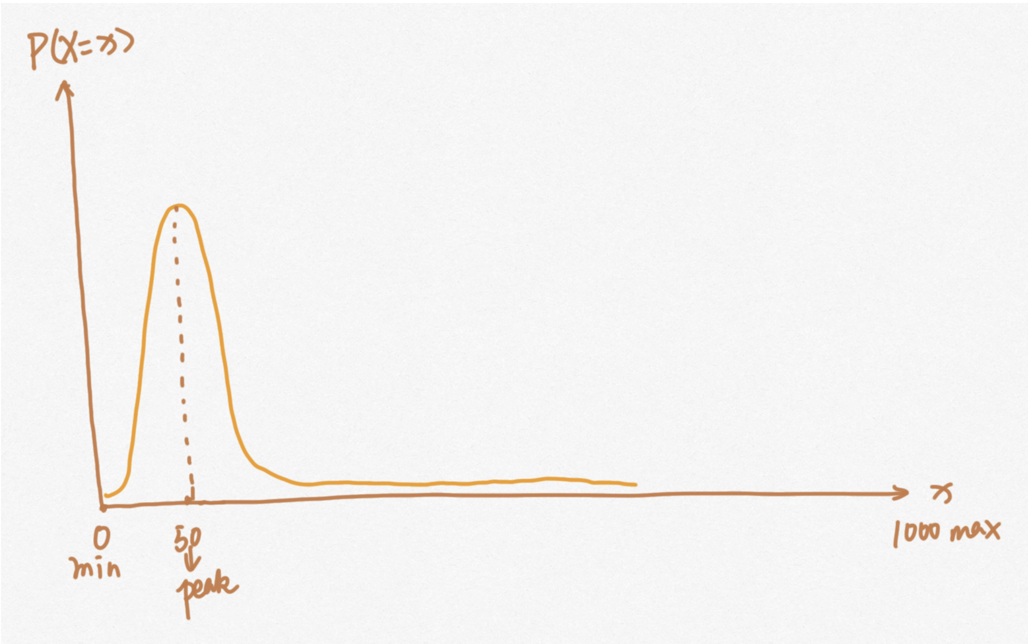
(e) If the true value of p is 0.4, just around 5.6% of the time the polling of 1000 people will give an answer as extreme as $x = 370$. This p-value is slightly greater than 0.05 and it's non-significant, therefore we have no evidence against the null hypothesis that $p = 0.4$ at 5% level of significance. The observed polling is **compatible** with possibility that the true support for Ardent is 40%.

(f) If the opinion poll has National polling at 41% and Labour polling at 37%, this does not mean that National is definitely ahead of Labour. As we have shown in previous questions, given the sample with 41% and 37% polling respectively, we do not have evidence to against the null hypothesis which both Parties have 40% supporters from the sample population.
We could not tell whether one party's supporter is more than the other from the testing we did above.

2. (a) Let X be the number of Peters-voter out of 1000 respondents.

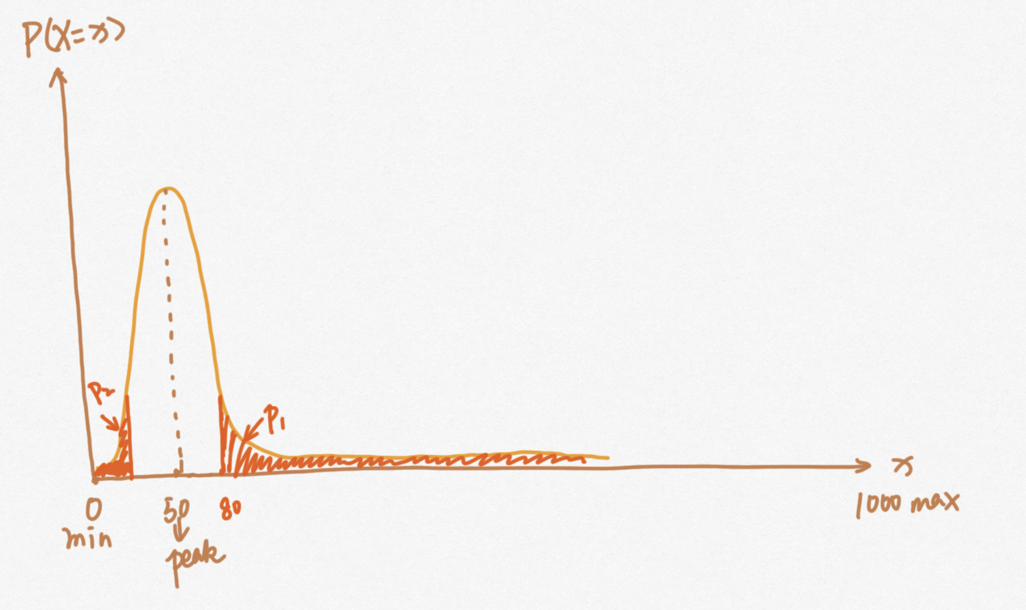
$$X \sim \text{Binomial}(1000, P_p)$$
$$H_0 : P_p = 0.05$$
$$H_1 : P_p \neq 0.05 \qquad \text{(two-sided test)}$$

(b) Under H_0, we have $X \sim \text{Binomial}(1000, 0.05)$. The probability function of X peaks at about $1000 \times 0.05 = 50$



(c)

ID: 107664618
Joy HU
UPI: mhu138



(d) For the p-value,

$$\begin{aligned}\mathbb{P}(X \geq 80) &= 1 - \mathbb{P}(X < 80) \\ &= 1 - \mathbb{P}(X \leq 79) \\ &= 1 - F_X(79)\end{aligned}$$

The R code for the p-value is `2 * (1 - pbinom(79,1000,0.05))`

(e) If the true value of p is 0.05, we have close to 0% of the chance to see that out of 1000 people will vote for Peters as extreme as $x = 80$. This p-value is very small and therefore we have strong evidence against the null hypothesis that $p = 0.05$. The observed polling is **not compatible** with the possibility that the true support for Peters is 5%.

(f) The conclusion is not the same. For 1(e), we are testing the true p is a right shift of 3 percentage points away from the observed polling, we have no evidence to against our H_0. While 2(e) we are testing the true p is a left shift of 3 percentage points away from the observed polling and we have strong evidence to against our H_0.
I think it's our hypothesis is different. In 1(e) we are testing whether the true supporter is about 40%, but 2(e) we are testing against the worst-scenario.