STATS 210 **Semester 2, 2020** **Assignment 1**

Due: 11:00pm, Thursday 20$^{\text{th}}$ August, 2020

1. *Some STATS 125 Revision:* For two events, A and B, we have the following:

$$\mathbb{P}(A) = 0.8; \qquad \mathbb{P}(B \,|\, A) = 0.5; \qquad A \text{ and } B \text{ are independent}$$

For parts (a) to (e) below, state whether the statement is *TRUE*, *FALSE*, or *NOT POSSIBLE TO TELL*. Justify your answers.

(a) A and B are mutually exclusive.
(b) A and $A \cup B$ are independent.
(c) $\mathbb{P}(B) = \mathbb{P}(A \,|\, B)$.
(d) $\mathbb{P}(A \,|\, C) > \mathbb{P}(B \,|\, C)$, where C is a third event with $\mathbb{P}(C) = 0.9$.
(e) $\mathbb{P}(B) \leq \mathbb{P}(A)$. (10)

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2. Each match played by the *Bampots* field hockey team is won by them with probability $\frac{1}{2}$, drawn with probability $\frac{1}{6}$, or lost with probability $\frac{1}{3}$. The result of each match is independent of all other results. For each of the situations described below in parts (a) to (e), when asked for the distribution of the specified random variable, take your answer from the following list:

Binomial   Negative Binomial   Geometric   Hypergeometric   Poisson   Other

State your choice with reasons, and give all appropriate parameters. If you choose 'Other', explain why the distribution described is different from those in the list. Give your answer to part (i) in the format of the sample answer below where appropriate:

Sample Question: Let X be the number of drawn matches in a season lasting 20 matches. State the distribution of X from the list above.

Sample Answer: Let 'Trial'= hockey match
Let 'Success'= a draw, so $\mathbb{P}(\text{Success})=\frac{1}{6}$
$X \sim \text{Binomial}(n = 20, p = \frac{1}{6})$, because X is the number of successes out of a fixed number n of independent trials, each with constant probability p of success.

(a) Let X be the number of matches before their first win.
 (i) State the distribution of X from the list above. (2)
 (ii) Find $\mathbb{E}(X)$. (2)
 (iii) Find $\mathbb{P}(X = 3)$. (2)

(b) Let X be the number of matches between their first win and their next loss.
 (i) State the distribution of X from the list above. (2)
 (ii) Find $\mathbb{E}(X)$. (2)
 (iii) Find $\mathbb{P}(X = 0)$. (2)

(c) Let X be the number of losses in the 5 matches following their first win.
 (i) State the distribution of X from the list above. (2)
 (ii) Find $\mathbb{E}(X)$. (2)
 (iii) Find $\mathbb{P}(X \geq 2 \,|\, X \geq 1)$. (4)

(d) Let X be the number of draws or losses before their third win.

   (i) State the distribution of X from the list above. (2)

   (ii) Find $\mathbb{E}(X)$. (2)

   (iii) Find $\mathbb{P}(X = 1)$. (2)

(e) Let X be the number of draws from match i, where $i = 1, \ldots, 5$, and let Y be the number of losses from match j, where $j = 6, \ldots, 10$. Let $Z = X + Y$.

   (i) State the distribution of Z from the list above. (2)

   (ii) Find $\mathbb{E}(Z)$. (2)

   (iii) Find $\mathbb{P}(Z = 2)$. (2)

Following a successful season, the *Bampots* field hockey team progressed to the final stage of their regional tournament. As is customary in such situations, if the final game results in a draw, a penalty shootout is used to decide which team wins the tournament. One method of penalty shootout[1] is the **Penalty Stroke Competition** which is the best-of-five penalty strokes. The **Penalty Stroke Competition** works by players from each team taking alternate strokes until a team has won or until 10 strokes have been completed. If the result is a tie after 10 strokes, the same players will continue to take strokes in a **Sudden Death** format until a winner is established.

(f) Assume players P_1, P_2, P_3, P_4 and P_5 are selected to take the five penalty strokes for the *Bampots* team. For each player P_i, the probability of a successful penalty stroke, is 0.8^{i-1} independently of all other players. Let Y be the total number of successful penalty strokes. Find $\mathbb{E}(Y)$ and $\text{Var}(Y)$ justifying all steps of your working. (5)

   Hint: It may be useful to think of each penalty stroke as a Bernoulli trial.

[1] Since 2011 it has been more common to use a series of one-on-ones between an attacking player and a goalkeeper to determine the winning team.

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3. The number of exam scripts that Stephanie marks per hour, denoted X, follows a Poisson process with parameter λ_1, while the number of exam scripts Marie marks per hour, denoted Y, follows a Poisson process with parameter λ_2. This means that $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$. Assume that X and Y are independent. Furthermore, assume that $\text{Var}(X) + \text{Var}(Y) = 8$.

   (a) Calculate $\mathbb{P}(X + Y \geq 3)$. Show your working. (3)

   After collecting some information over a few semesters, and making use of maximum likelihood procedures that will be introduced in Chapter 3, we have estimated λ_2. Assume that our estimate of λ_2 is $\widehat{\lambda}_2 = 5$.

   (b) Given that a total of 12 exam scripts were marked in one hour, find the probability that 6 of them were marked by Stephanie. That is, find $\mathbb{P}(X = 6 \,|\, X + Y = 12)$. (5)

Total: (55)
