



北京大学

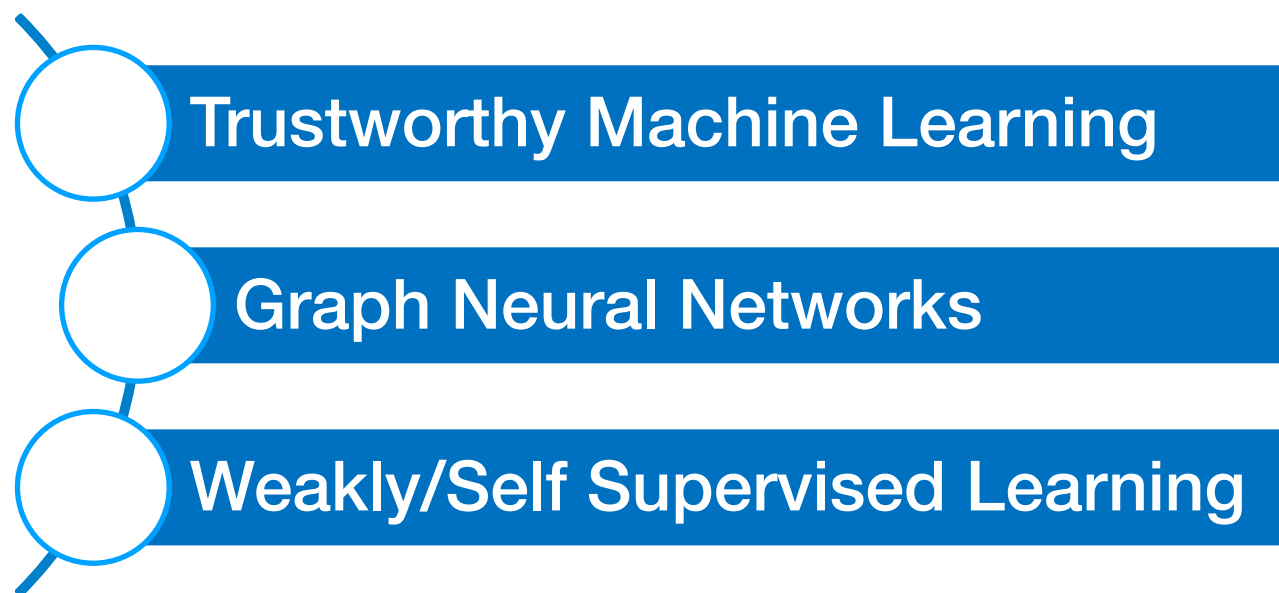
# 机器学习介绍

# Introduction of Machine Learning

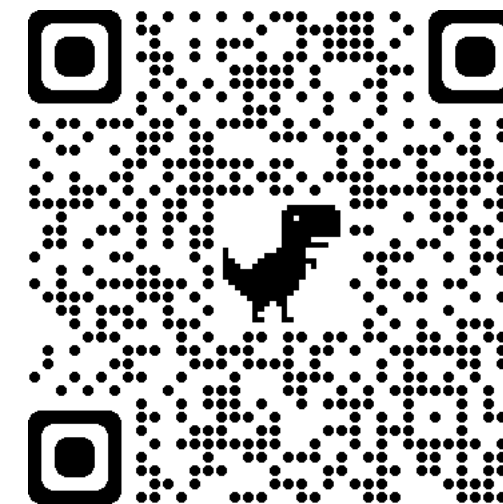


主讲人：王奕森

- Assistant Professor, Ph.D. Advisor
- Research Interests: Machine Learning



Homepage



<https://yisenwang.github.io/>

- Welcome interns!

中国计算机学会推荐国际学术会议

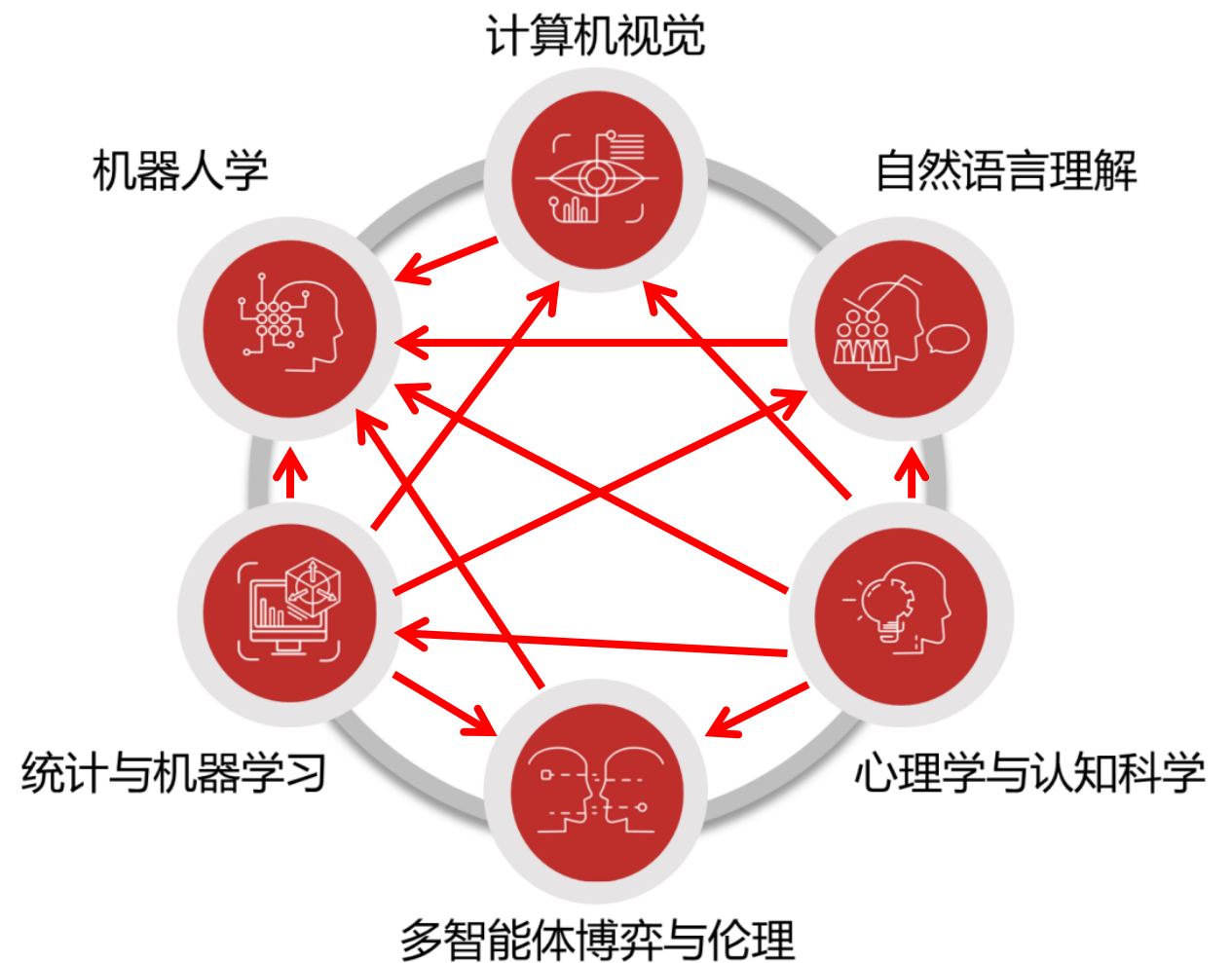
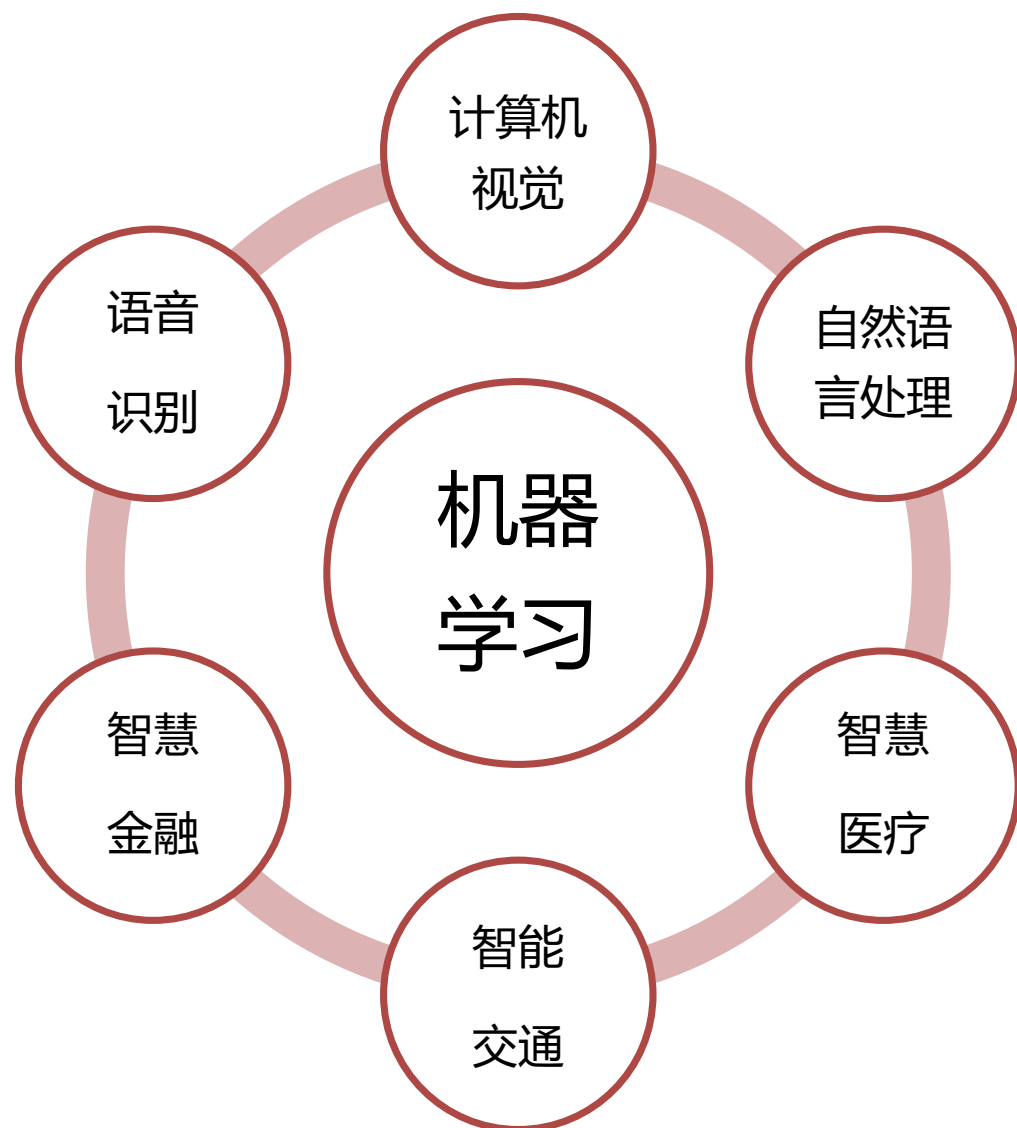
(人工智能)



## 一、A 类

序号	会议简称	会议全称	出版社	网址
1	AAAI	AAAI Conference on Artificial Intelligence	AAAI	<a href="http://dblp.uni-trier.de/db/conf/aaai/">http://dblp.uni-trier.de/db/conf/aaai/</a>
2	NeurIPS	Annual Conference on Neural Information Processing Systems	MIT Press	<a href="http://dblp.uni-trier.de/db/conf/nips/">http://dblp.uni-trier.de/db/conf/nips/</a>
3	ACL	Annual Meeting of the Association for Computational Linguistics	ACL	<a href="http://dblp.uni-trier.de/db/conf/acl/">http://dblp.uni-trier.de/db/conf/acl/</a>
4	CVPR	IEEE Conference on Computer Vision and Pattern Recognition	IEEE	<a href="http://dblp.uni-trier.de/db/conf/cvpr/">http://dblp.uni-trier.de/db/conf/cvpr/</a>
5	ICCV	International Conference on Computer Vision	IEEE	<a href="http://dblp.uni-trier.de/db/conf/iccv/">http://dblp.uni-trier.de/db/conf/iccv/</a>
6	ICML	International Conference on Machine Learning	ACM	<a href="http://dblp.uni-trier.de/db/conf/icml/">http://dblp.uni-trier.de/db/conf/icml/</a>
7	IJCAI	International Joint Conference on Artificial Intelligence	Morgan Kaufmann	<a href="http://dblp.uni-trier.de/db/conf/ijcai/">http://dblp.uni-trier.de/db/conf/ijcai/</a>

ICLR, International Conference on Learning Representations



# What is Machine Learning?

- Machine Learning is for designing **algorithms** that can **learn and build computable models** from **training data** and then perform desired tasks **on new data**.

A computer program is said to **learn from experience E** with respect to **some task T** and some **performance measure P**, if its performance on T, as measure by P, **improves with experience E**.

So ML is not rule based and in principle it can enable machines to **evolve**.



Tom Mitchell  
(Member of NAE & AAAS,  
Prof. @ CMU)



## Machine Learning $\approx$ Looking for Function

- Speech Recognition

$$f(\text{audio waveform}) = \text{"How are you"}$$

- Image Recognition

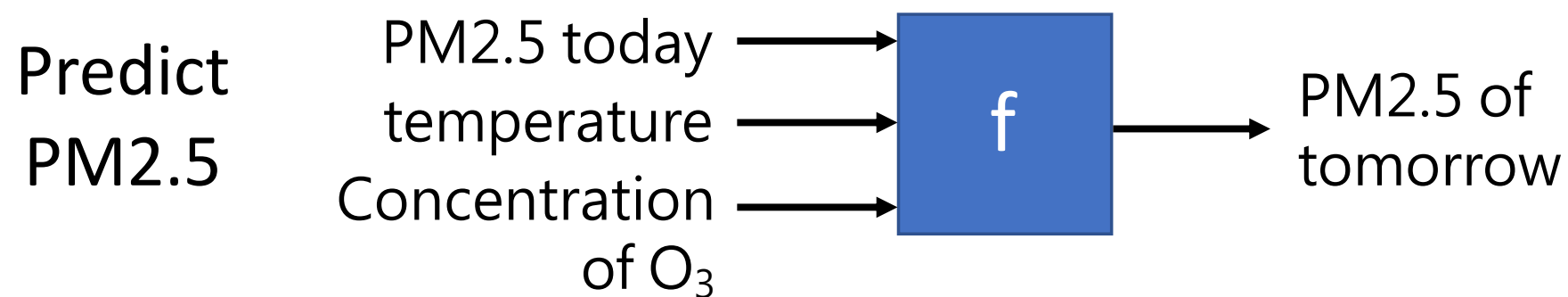
$$f(\text{cat image}) = \text{"Cat"}$$

- Playing Go

$$f(\text{Go board state}) = \text{"5-5" (next move)}$$

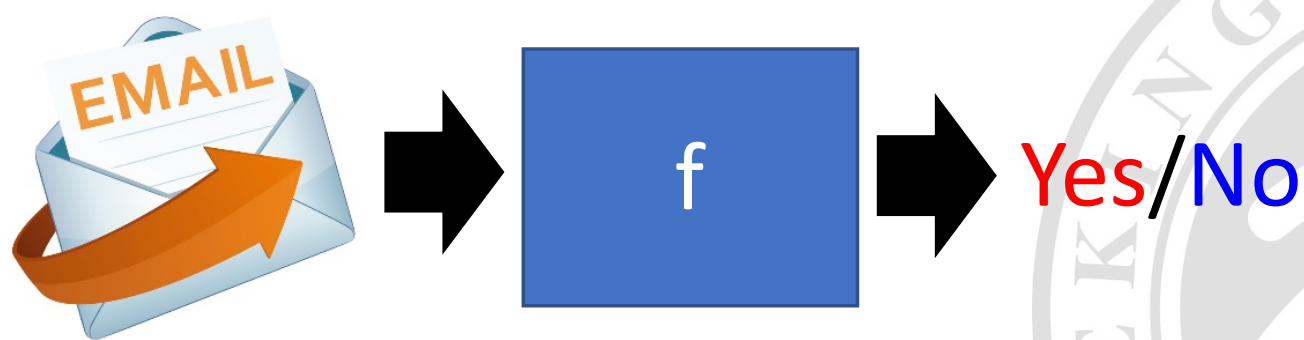


**Regression:** The function outputs a scalar.

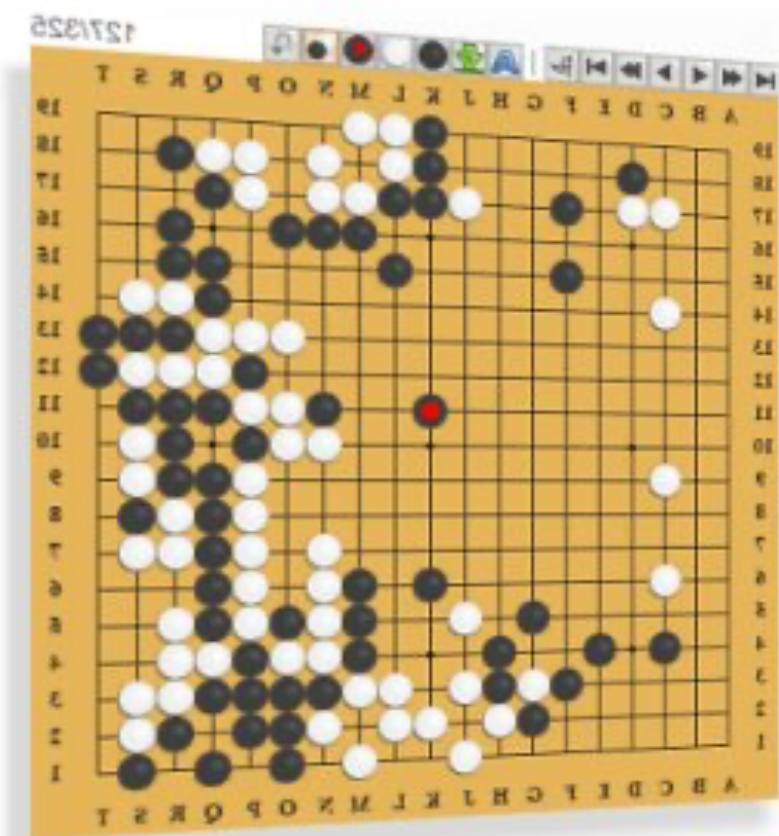


**Classification:** Given options (**classes**), the function outputs the correct one.

Spam filtering



**Classification:** Given options (**classes**), the function outputs the correct one.



**Playing GO**



Each position  
is a class  
(19 x 19 classes)



a position on  
the board

**Next move**

Taking examples (some slides) from Prof. Hungyi Lee@NTU

$y = f(\text{no. of views on 2/26})$

Hung-yi Lee									
篩選器									
影片	流量來源	地理位置	觀眾年齡	觀眾性別	日期	訂閱狀態	訂閱來源	播放清單	
日期 ↓					+	喜歡的人數		訂閱人數	
2021年1月26日						54	4.9%	69	5.5%
2021年1月27日						60	5.4%	71	5.6%
2021年1月28日						36	3.2%	63	5.0%
2021年1月29日						27	2.4%	40	3.2%
2021年1月30日						40	3.6%	40	3.2%
2021年1月31日						47	4.2%	51	4.0%
2021年2月1日						61	5.5%	29	2.3%
2021年2月2日						49	4.4%	43	3.4%
2021年2月3日						26	2.3%	44	3.5%
2021年2月4日						43	3.9%	33	2.6%
2021年2月5日						45	4.0%	49	3.9%
2021年2月6日						29	2.6%	42	3.3%
2021年2月7日						26	2.3%	46	3.6%
2021年2月8日						38	3.4%	26	2.1%
2021年2月9日						29	2.6%	25	2.0%
2021年2月10日						31	2.8%	35	2.8%



# 1) Function

$$y = f($$



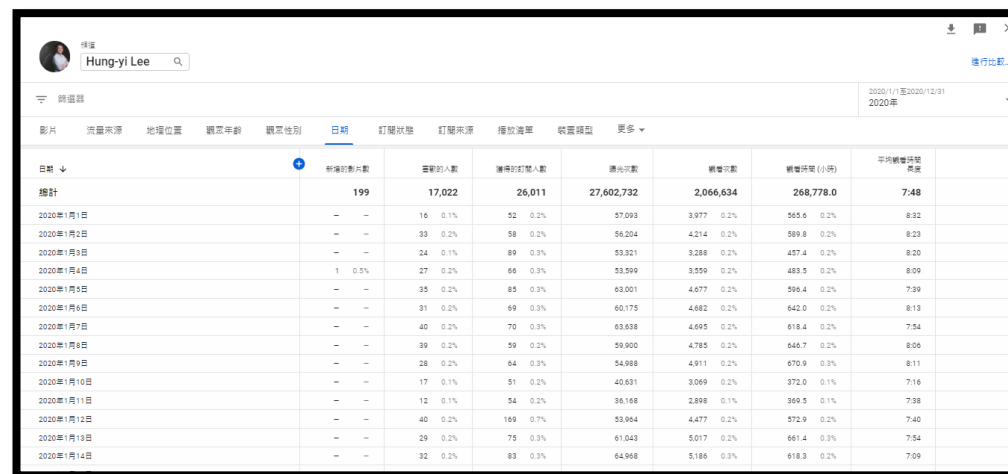
**Model**  $y = b + wx_1$  based on domain knowledge

**feature**

$y$ : no. of views on 2/26,  $x_1$ : no. of views on 2/25

$w$  and  $b$  are unknown parameters (learned from data)

**weight**    **bias**



日期	新增影片數	觀看的人數	獲得的訂閱人數	總播放數	觀看次數	觀看時長 (小時)	平均觀看時長 (分鐘)
總計	199	17,022	26,011	27,602,732	2,066,634	268,778.0	7:48
2020年1月1日	-	16	52	57,093	3,977	565.6	8:32
2020年1月2日	-	33	58	56,204	4,214	589.8	8:23
2020年1月3日	-	24	89	53,321	3,288	457.4	8:20
2020年1月4日	1	27	66	53,599	3,559	483.5	8:09
2020年1月5日	-	35	85	63,001	4,577	596.4	7:39
2020年1月6日	-	31	69	60,175	4,682	642.0	8:13
2020年1月7日	-	40	70	63,838	4,695	616.4	7:54
2020年1月8日	-	39	59	59,900	4,785	645.7	8:06
2020年1月9日	-	28	64	54,988	4,911	670.9	8:11
2020年1月10日	-	17	51	40,631	3,069	372.0	7:16
2020年1月11日	-	12	54	36,168	2,898	369.5	7:38
2020年1月12日	-	40	169	53,964	4,477	572.9	7:40
2020年1月13日	-	29	75	61,043	5,017	661.4	7:54
2020年1月14日	-	32	83	64,968	5,186	618.3	7:09

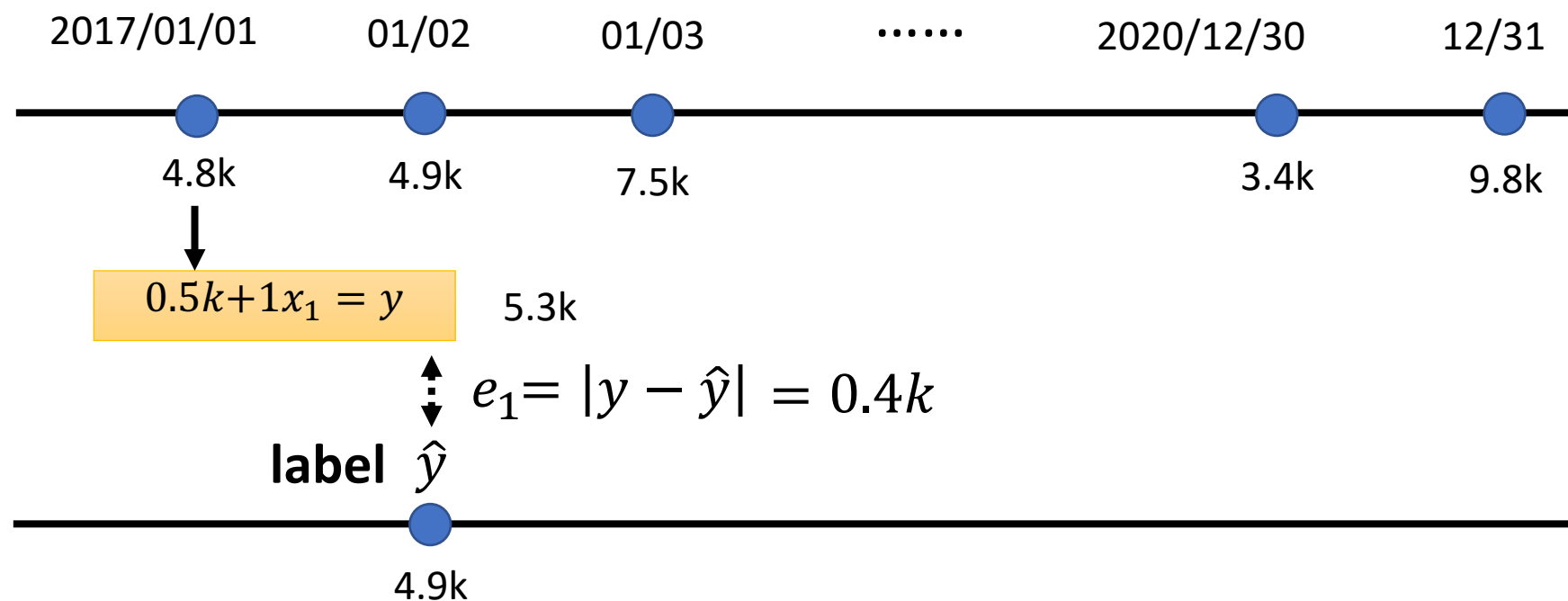


### Defining Loss from Training Data

- Loss is a function of parameters  $L(b, w)$
- Loss: how good a set of values is.

$$L(0.5k, 1) \quad y = b + wx_1 \quad \longrightarrow \quad y = 0.5k + 1x_1 \quad \text{How good it is?}$$

Data from 2017/01/01 – 2020/12/31

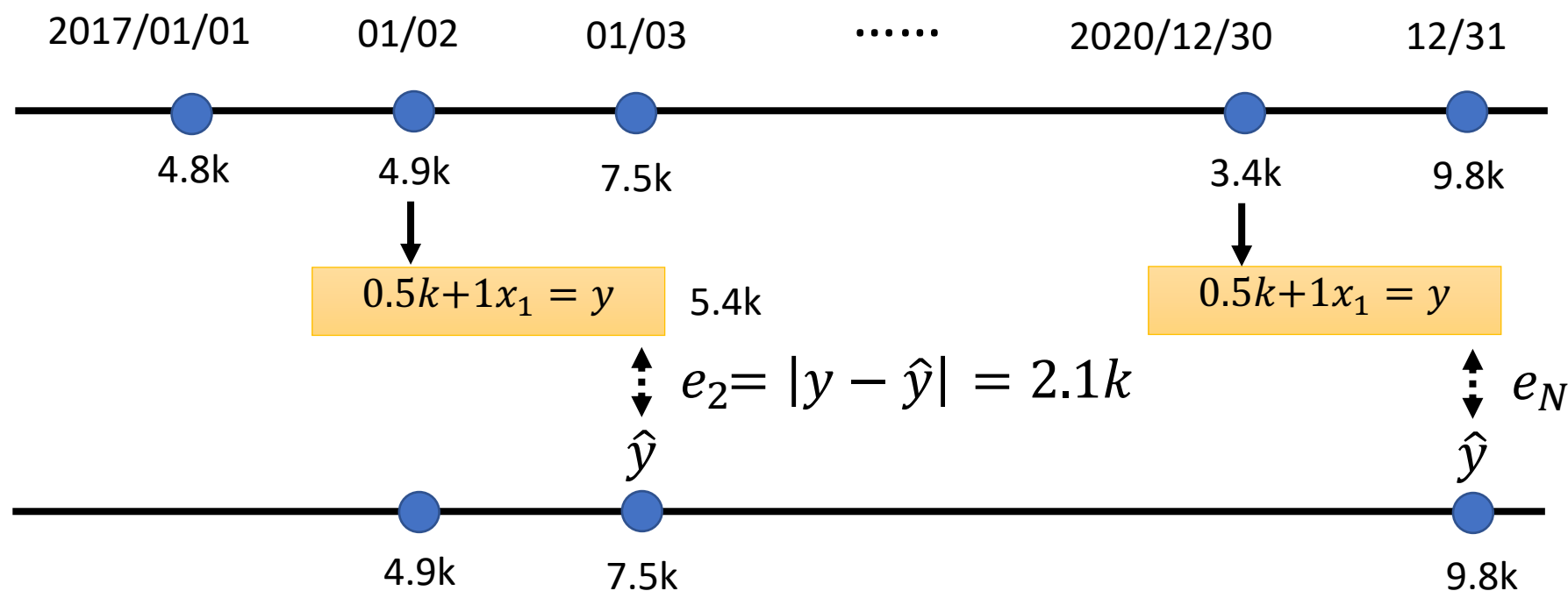


## Defining Loss from Training Data

- Loss is a function of parameters  $L(b, w)$
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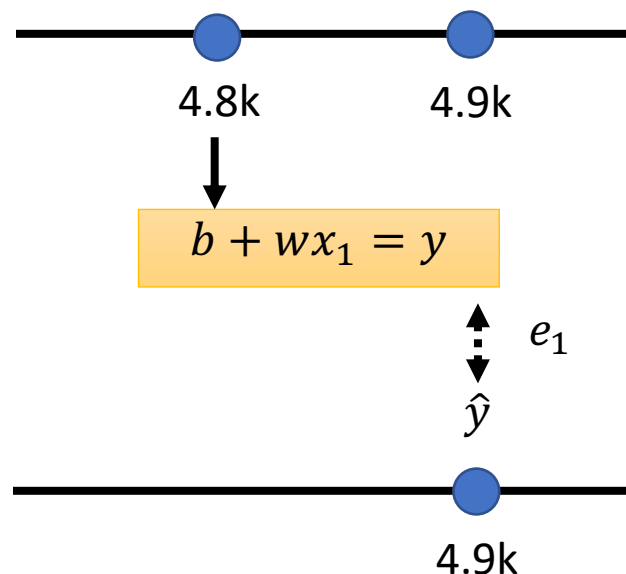
$$L(0.5k, 1) \quad y = b + wx_1 \quad \longrightarrow \quad y = 0.5k + 1x_1 \quad \text{How good it is?}$$

Data from 2017/01/01 – 2020/12/31



### Defining Loss from Training Data

- Loss is a function of parameters  $L(b, w)$
- Loss: how good a set of values is.



Loss: 
$$L = \frac{1}{N} \sum_n e_n$$

$e = |y - \hat{y}|$        $L$  is mean absolute error (MAE)

$e = (y - \hat{y})^2$        $L$  is mean square error (MSE)

If  $y$  and  $\hat{y}$  are both probability distributions ➡ Cross-entropy

$$e = y \log \hat{y}$$

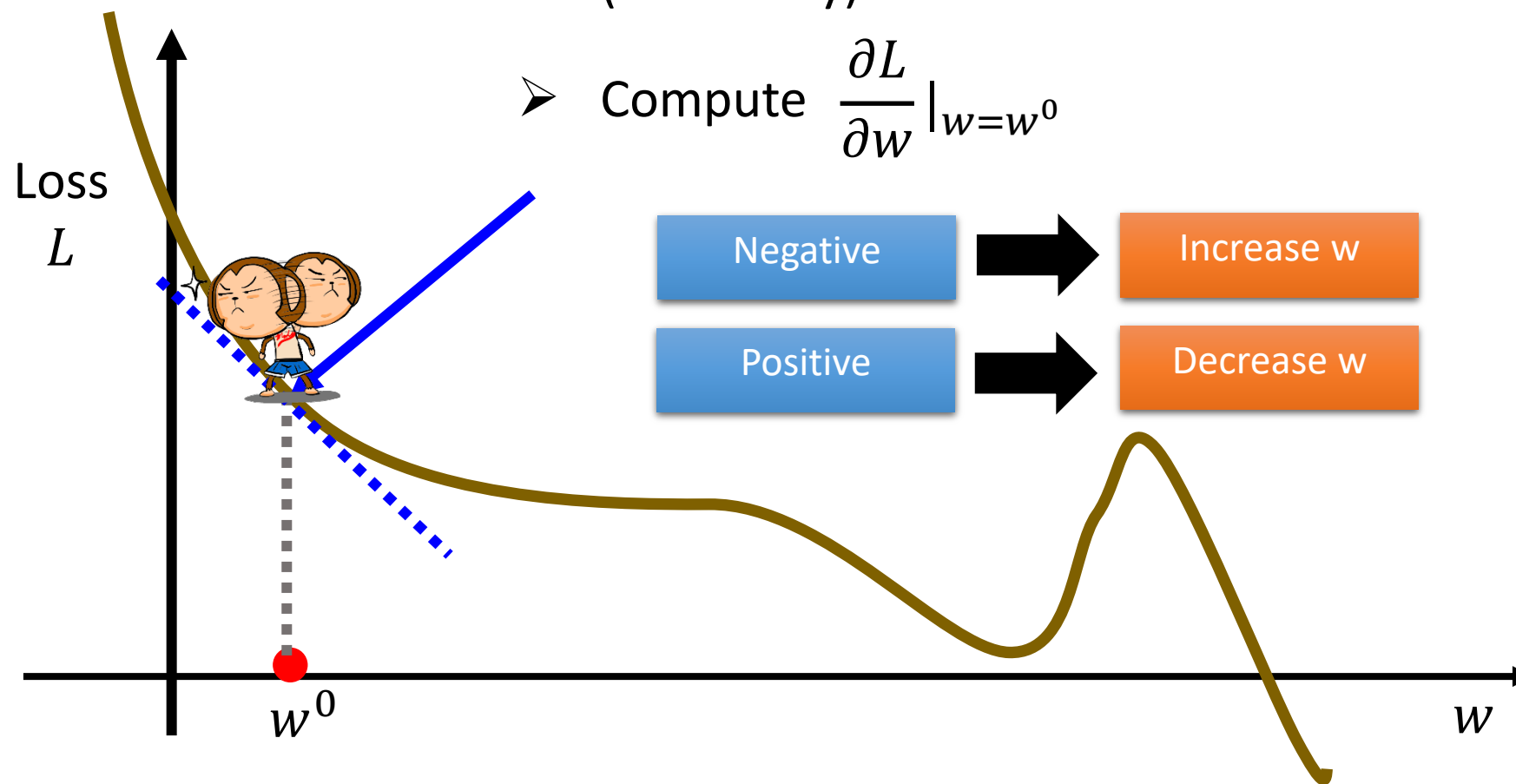


### 3) Optimization

$$w^* = \arg \min_w L$$

#### Gradient Descent

- (Randomly) Pick an initial value  $w^0$
- Compute  $\frac{\partial L}{\partial w} \big|_{w=w^0}$





### 3) Optimization

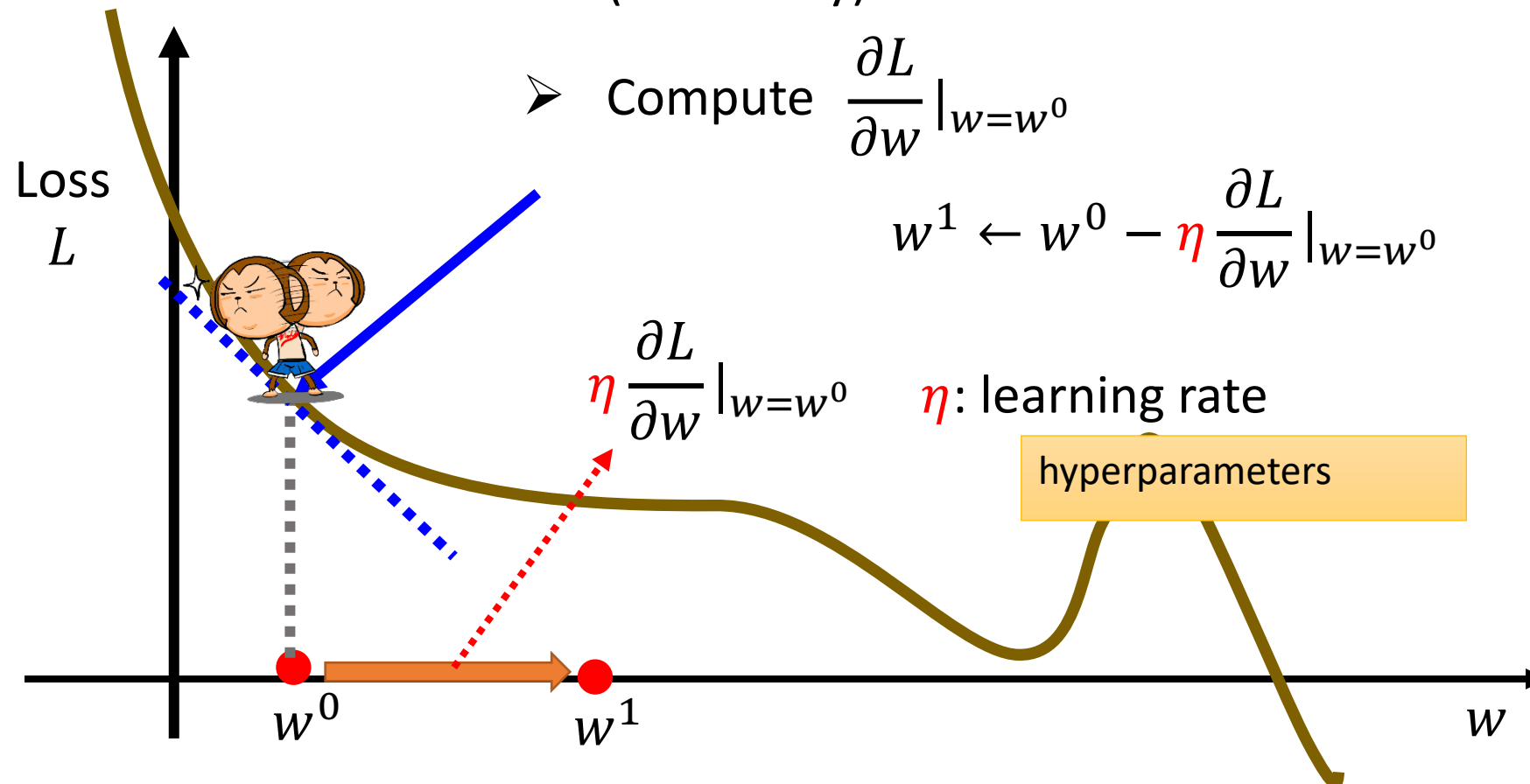
$$w^* = \arg \min_w L$$

#### Gradient Descent

➤ (Randomly) Pick an initial value  $w^0$

➤ Compute  $\frac{\partial L}{\partial w} \big|_{w=w^0}$

$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w} \big|_{w=w^0}$$



### 3) Optimization

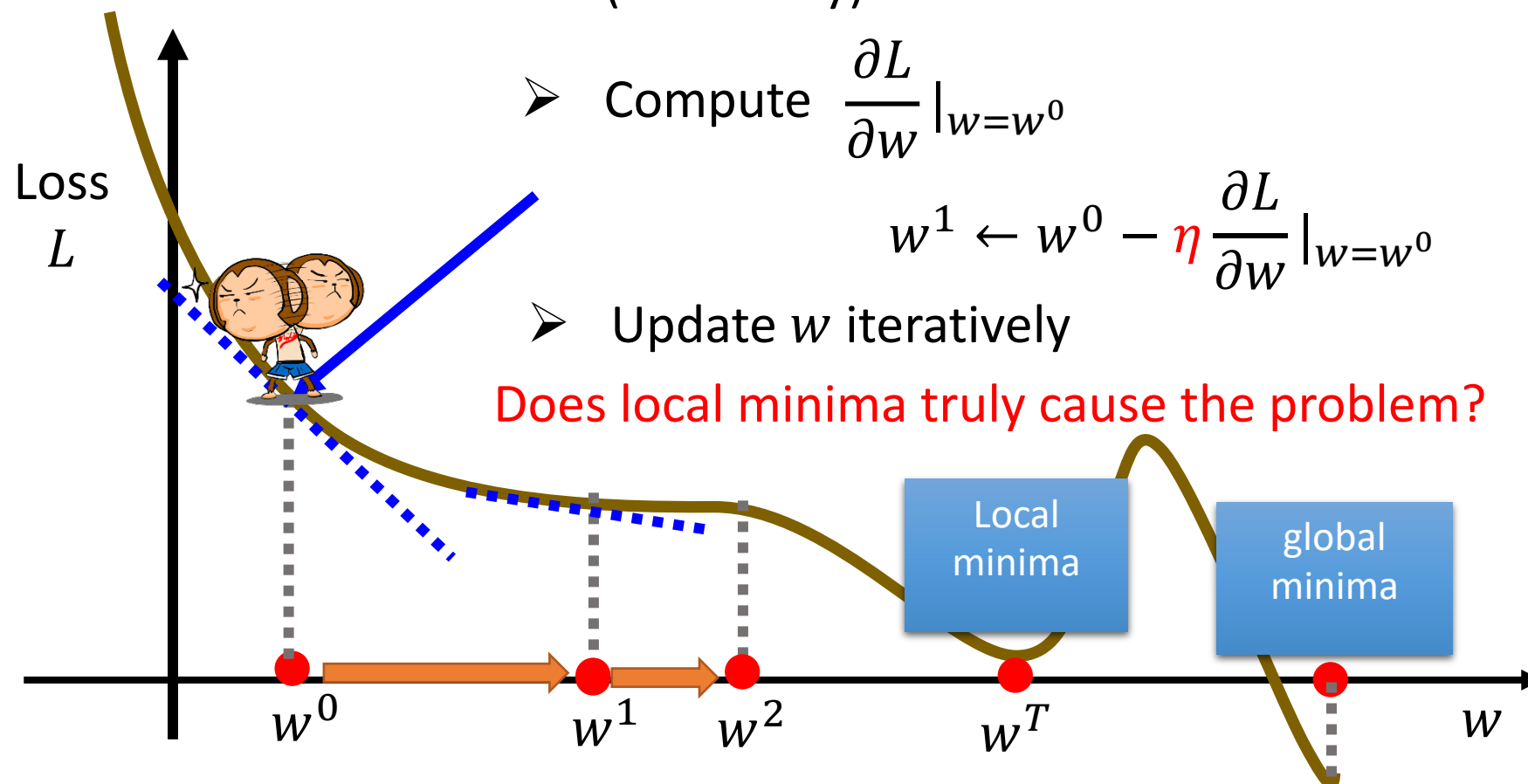
$$w^* = \arg \min_w L$$

#### Gradient Descent

- (Randomly) Pick an initial value  $w^0$
- Compute  $\frac{\partial L}{\partial w} \big|_{w=w^0}$
- Update  $w$  iteratively

$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w} \big|_{w=w^0}$$

Does local minima truly cause the problem?



### 3) Optimization

$$w^*, b^* = \arg \min_{w, b} L$$

➤ (Randomly) Pick initial values  $w^0, b^0$

➤ Compute

$$\frac{\partial L}{\partial w} \Big|_{w=w^0, b=b^0}$$

$$\frac{\partial L}{\partial b} \Big|_{w=w^0, b=b^0}$$

$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w} \Big|_{w=w^0, b=b^0}$$

$$b^1 \leftarrow b^0 - \eta \frac{\partial L}{\partial b} \Big|_{w=w^0, b=b^0}$$

Can be done in one line in most deep learning frameworks

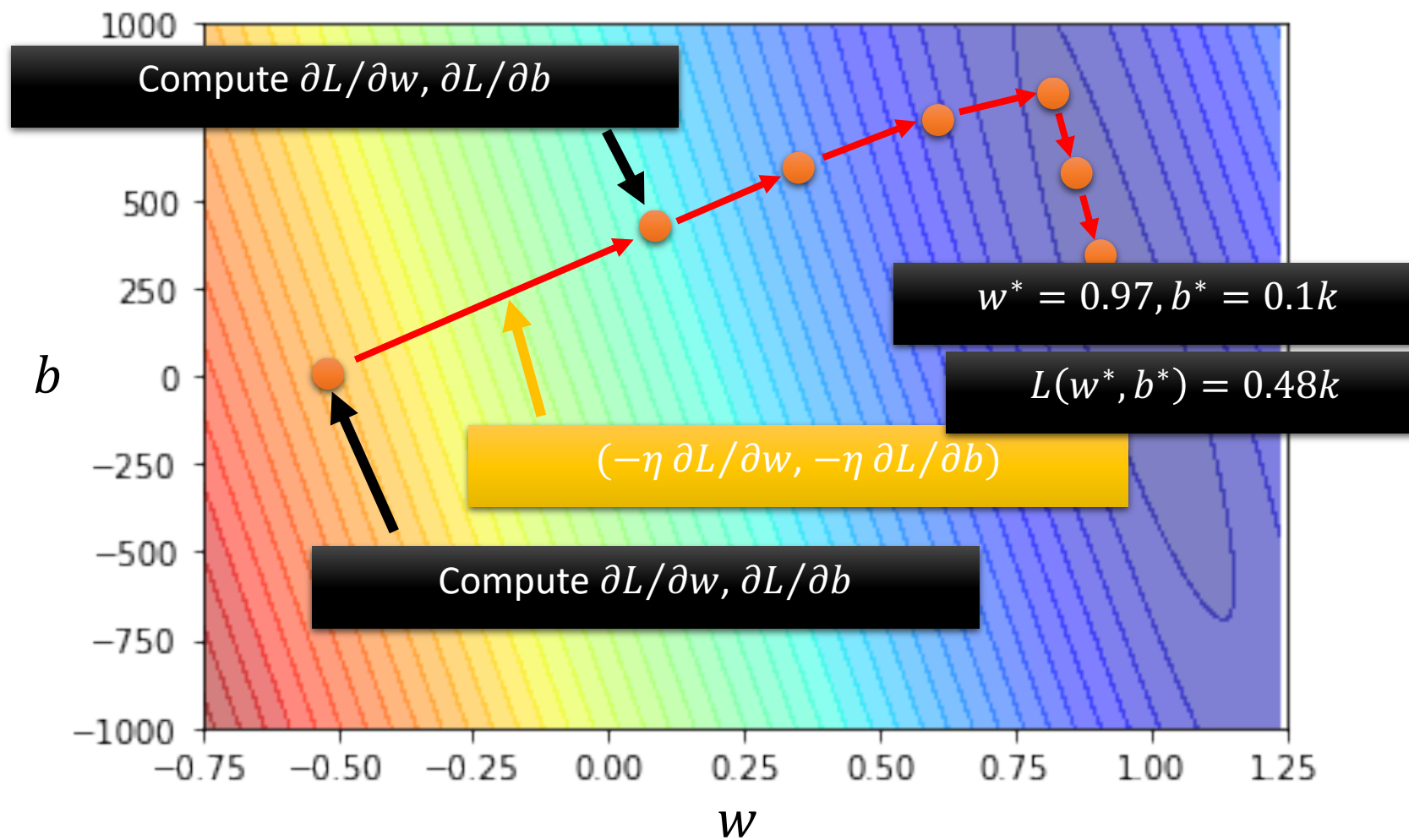
➤ Update  $w$  and  $b$  iteratively



### 3) Optimization

Model  $y = b + wx_1$

$$w^*, b^* = \arg \min_{w, b} L$$



$$y = b + wx_1$$

Step 1:  
function with  
unknown



Step 2: define  
loss from  
training data



Step 3:  
optimization

$$w^* = 0.97, b^* = 0.1k$$
$$L(w^*, b^*) = 0.48k$$







$y = 0.1k + 0.97x_1$  achieves the smallest loss  $L = 0.48k$  on data of 2017 – 2020 (**training data**)

How about data of 2021 (**unseen during training**)?

$$L' = 0.58k$$



$$y = b + wx_1$$

2017 - 2020	2021
$L = 0.48k$	$L' = 0.58k$

$$y = b + \sum_{j=1}^7 w_j x_j$$

2017 - 2020	2021
$L = 0.38k$	$L' = 0.49k$

$b$	$w_1^*$	$w_2^*$	$w_3^*$	$w_4^*$	$w_5^*$	$w_6^*$	$w_7^*$
0.05k	0.79	-0.31	0.12	-0.01	-0.10	0.30	0.18

$$y = b + \sum_{j=1}^{28} w_j x_j$$

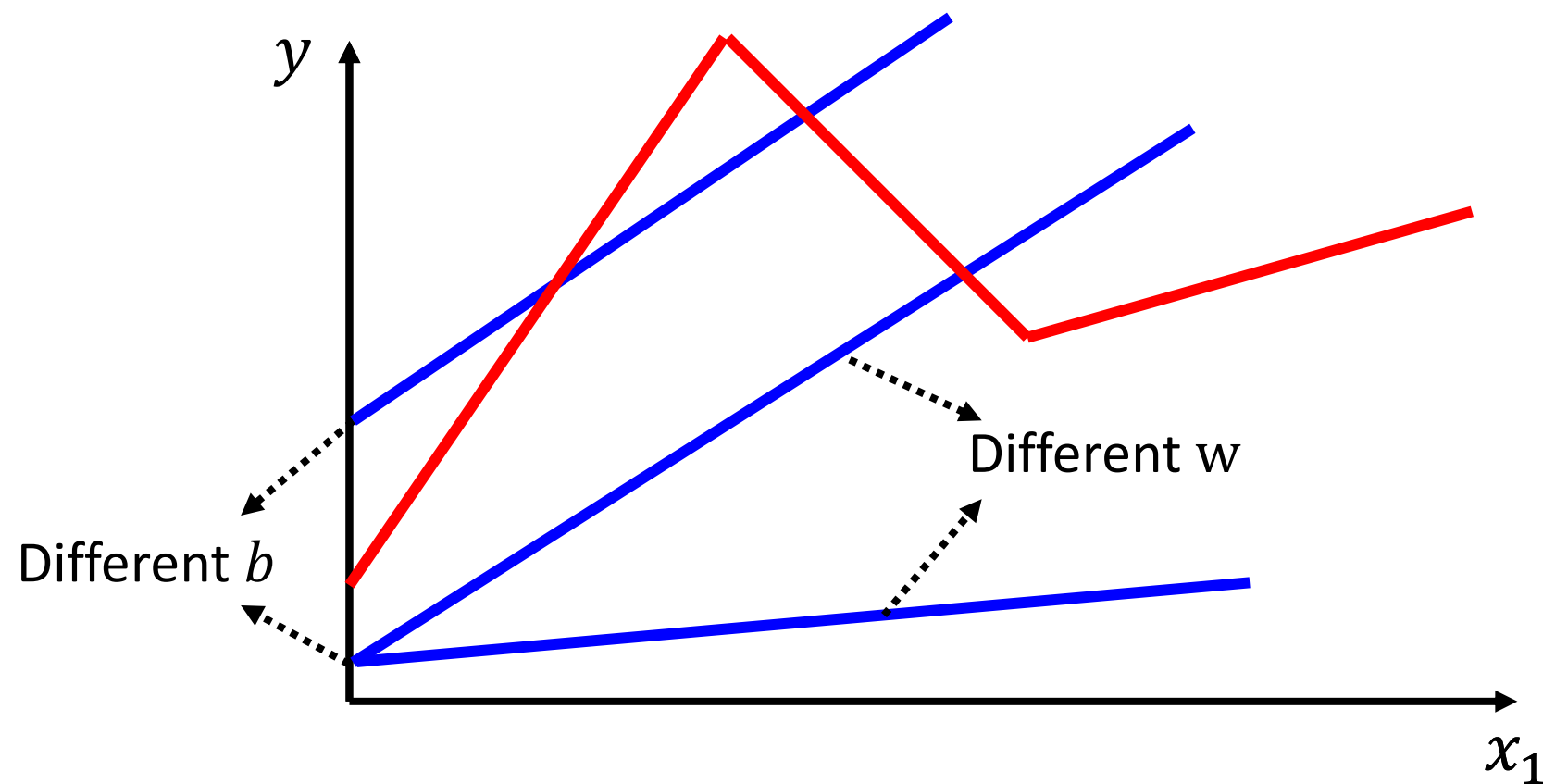
2017 - 2020	2021
$L = 0.33k$	$L' = 0.46k$

$$y = b + \sum_{j=1}^{56} w_j x_j$$

2017 - 2020	2021
$L = 0.32k$	$L' = 0.46k$

Linear models





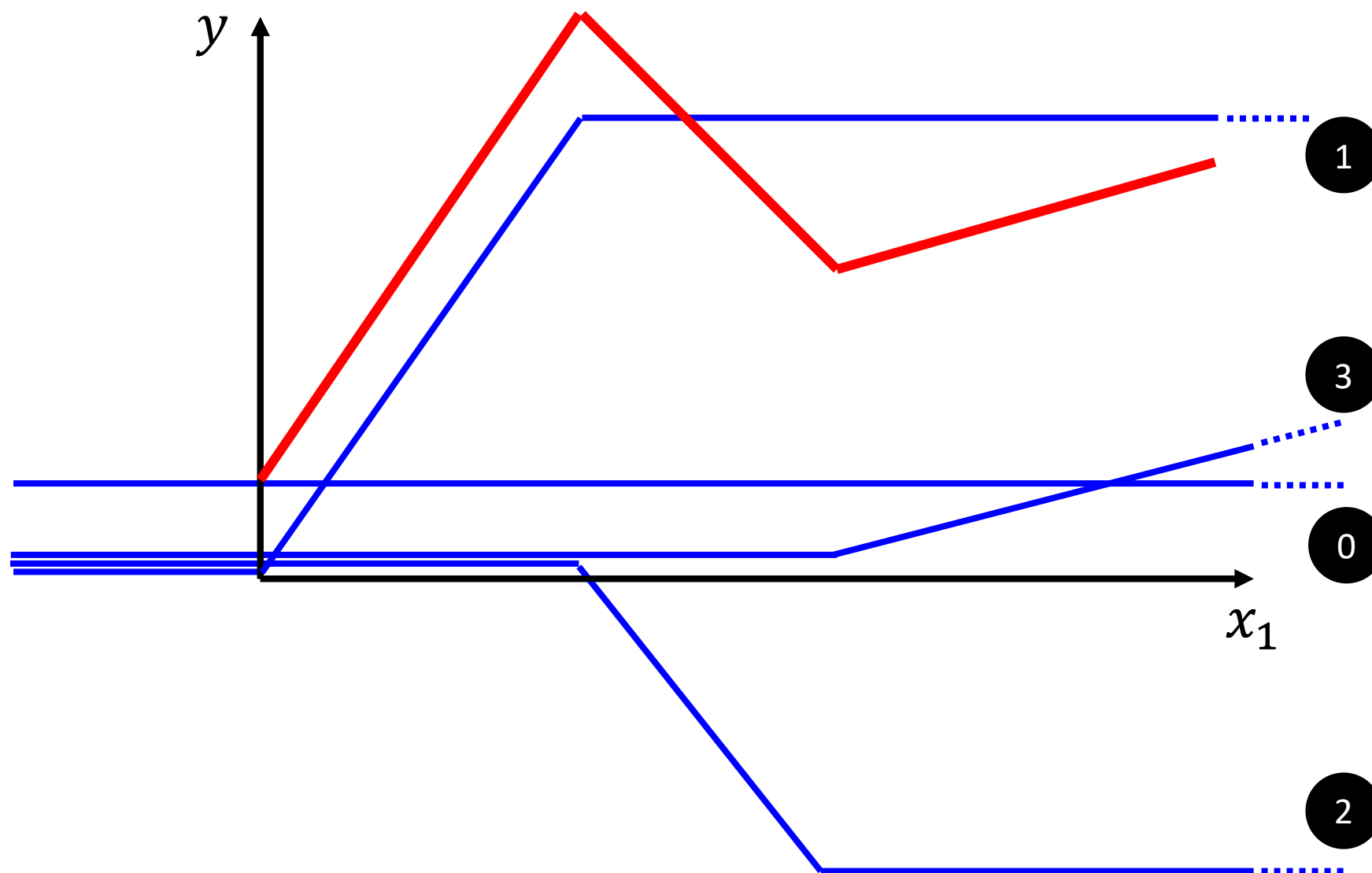
Linear models have severe limitation. **Model Bias**

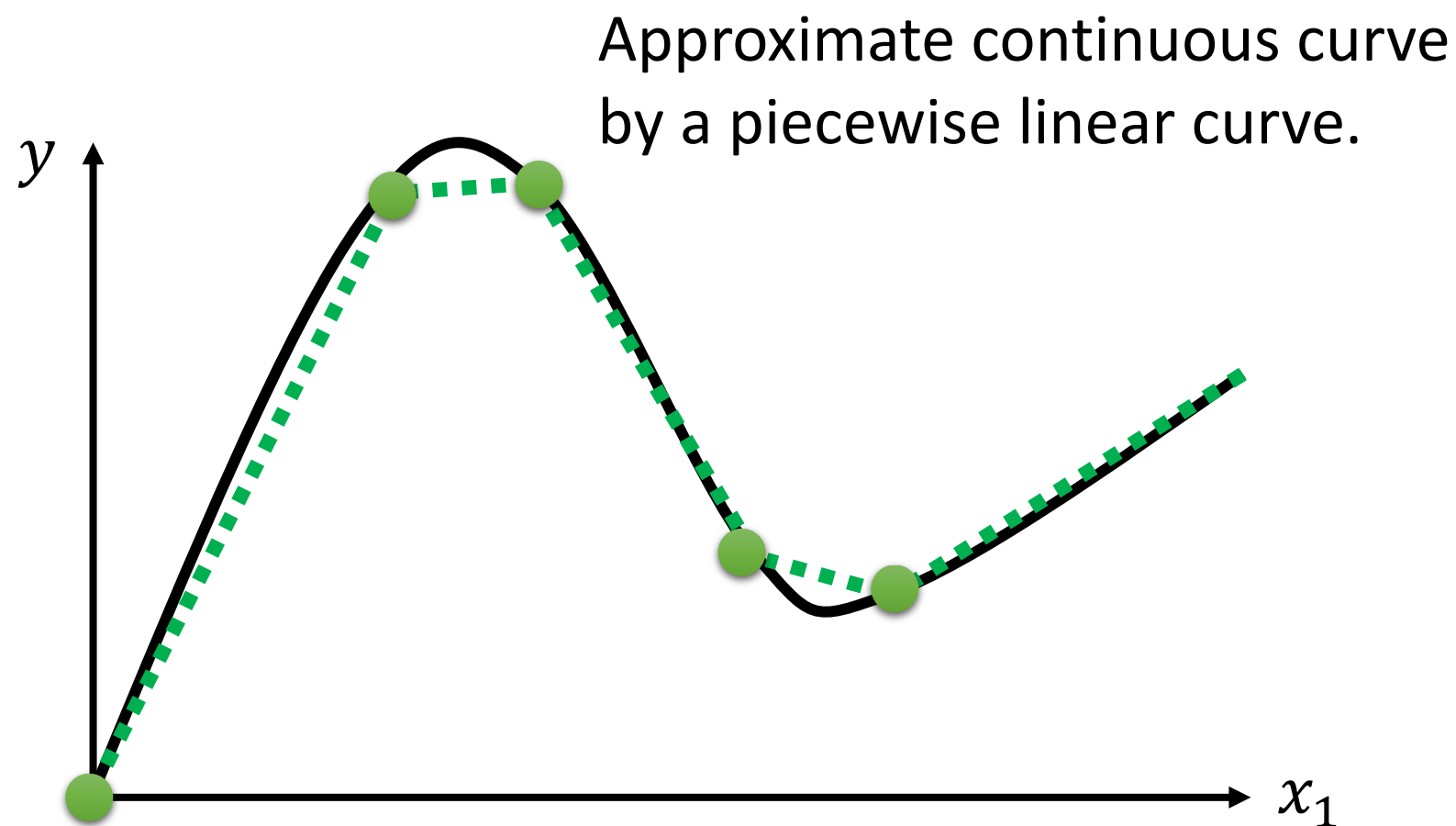
We need a more flexible model!



# Piecewise Linear Curves

red curve = constant + sum of a set of



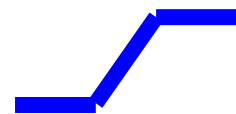


To have good approximation, we need sufficient pieces.



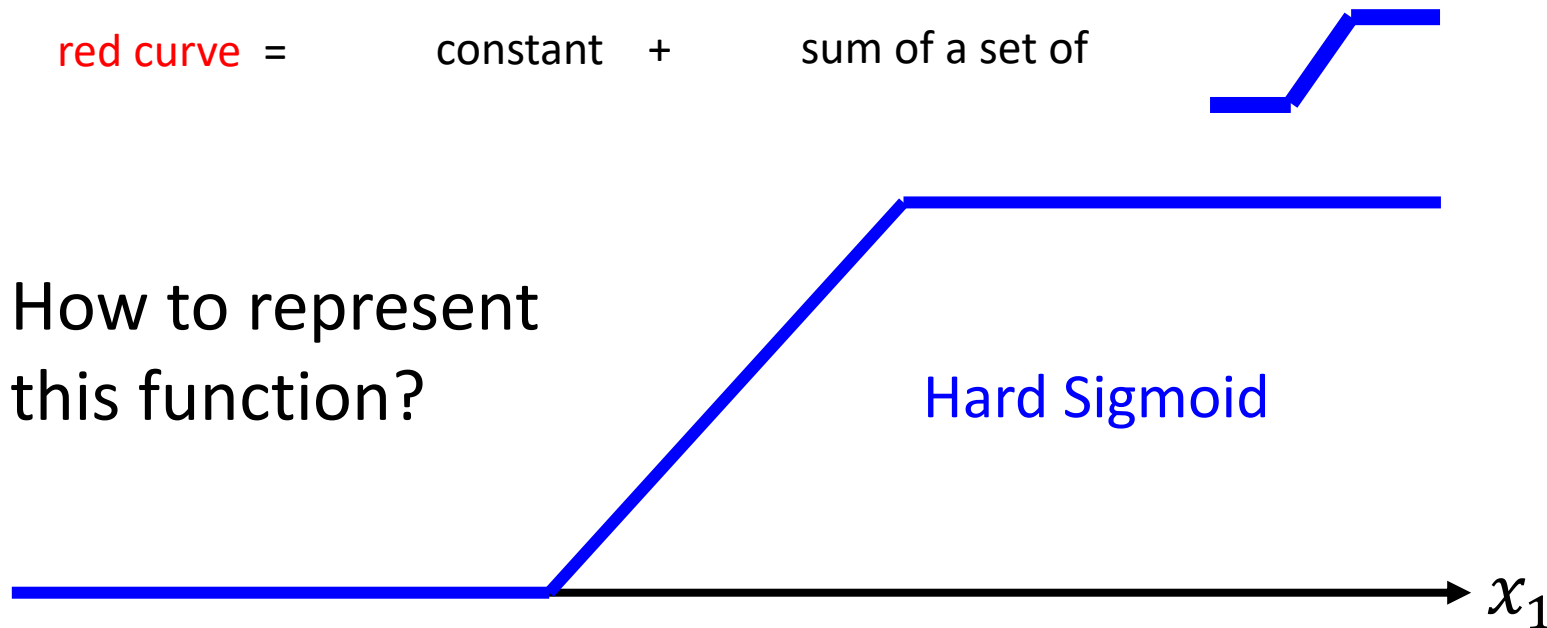


red curve = constant + sum of a set of



How to represent  
this function?

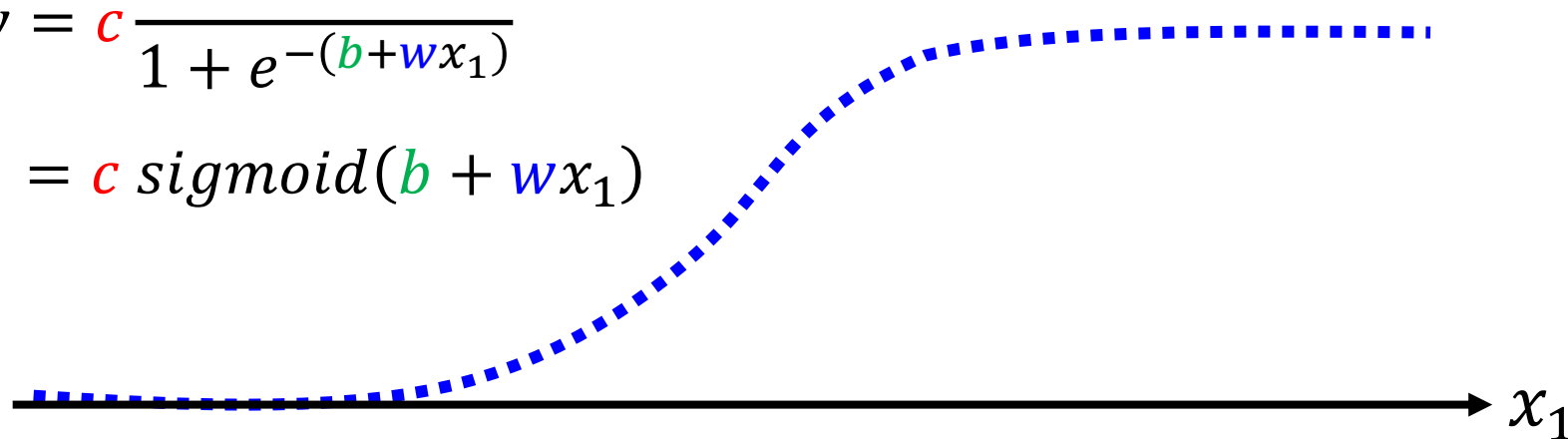
Hard Sigmoid



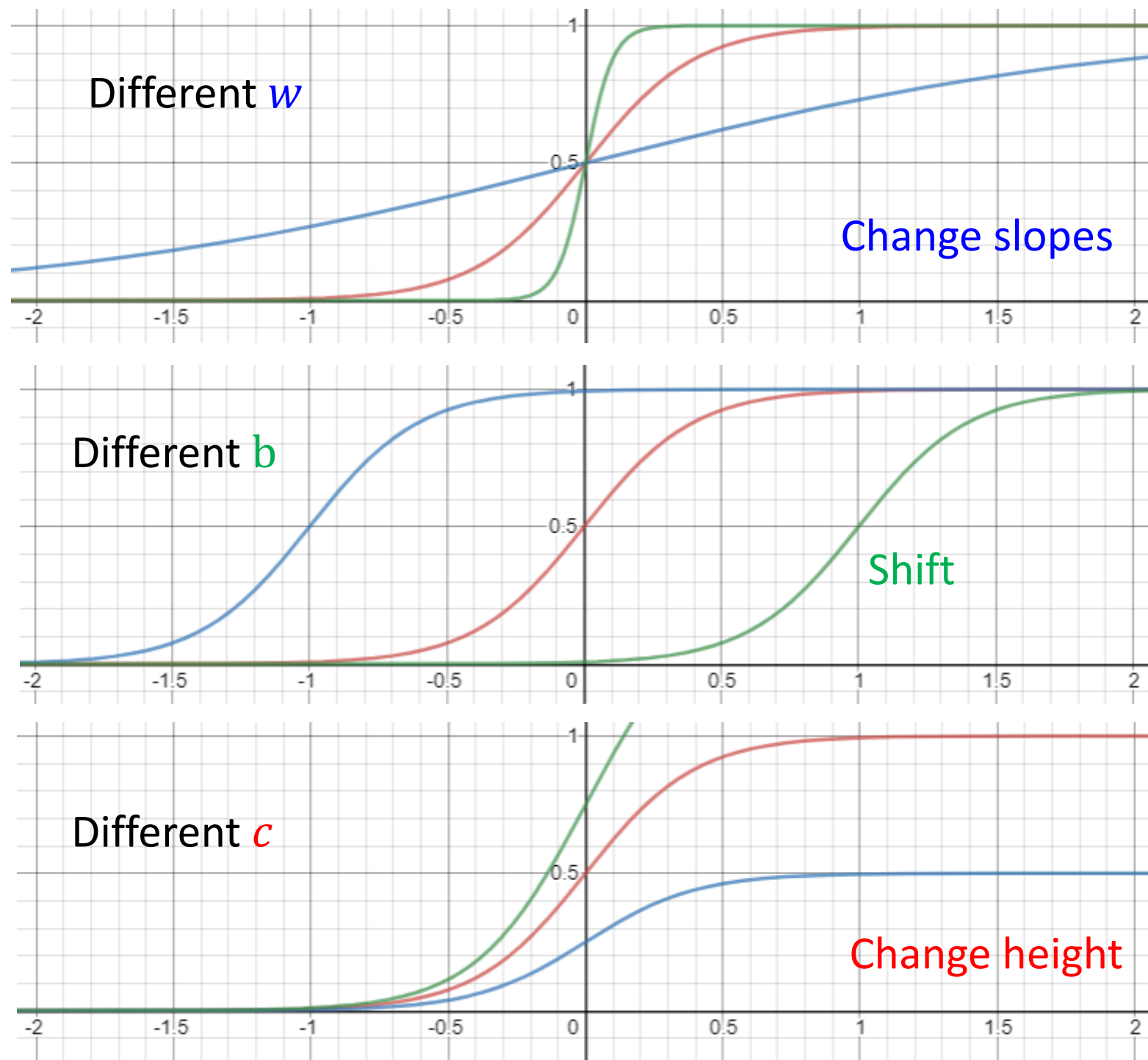
Sigmoid Function

$$y = c \frac{1}{1 + e^{-(b+wx_1)}}$$

$$= c \operatorname{sigmoid}(b + wx_1)$$

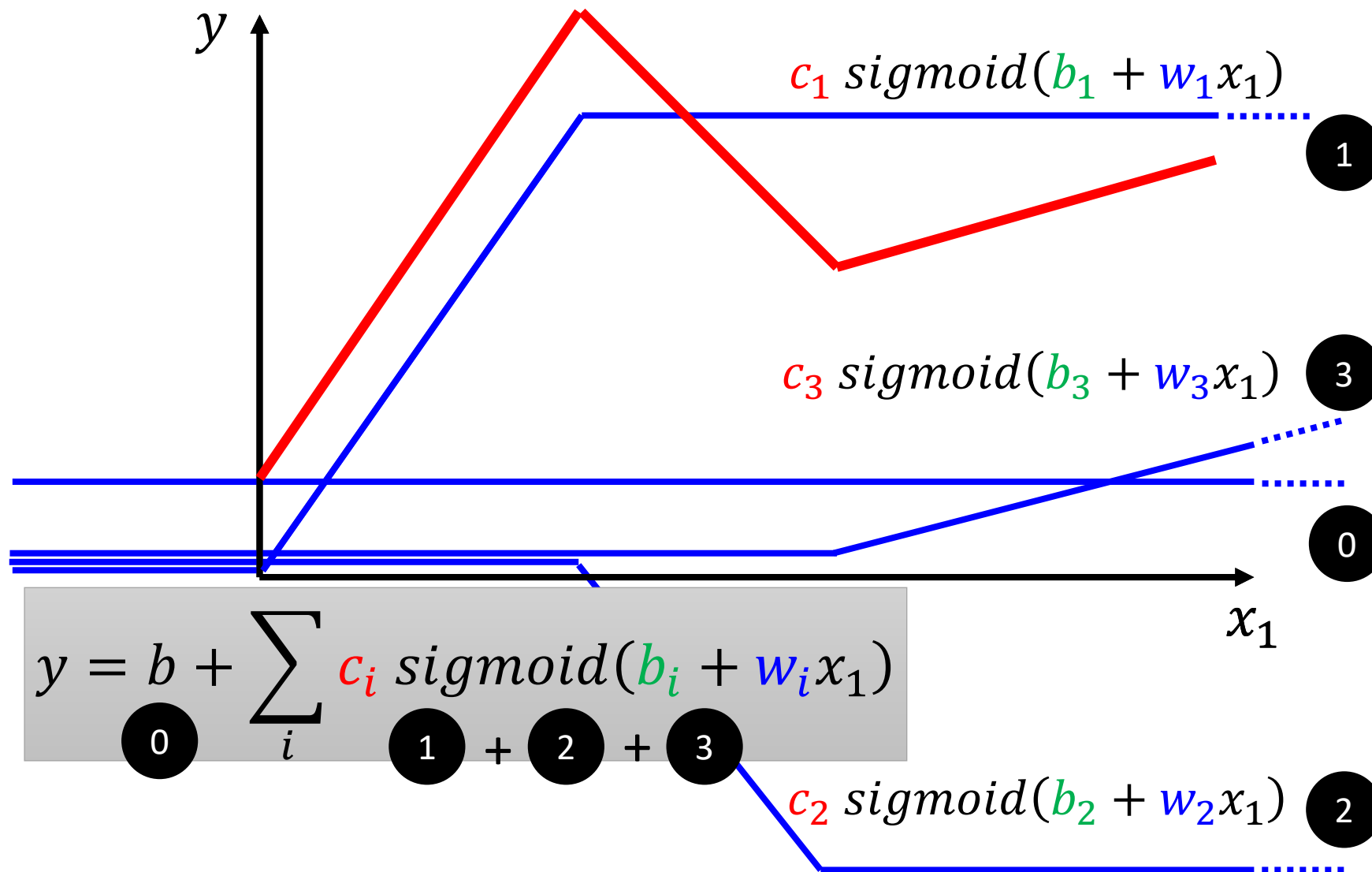


# Property of sigmoid



# The nonlinear becomes...

red curve = sum of a set of  + constant



$$y = \underline{b + wx_1}$$

$$y = b + \sum_i c_i \operatorname{sigmoid}(\underline{b_i + w_i x_1})$$

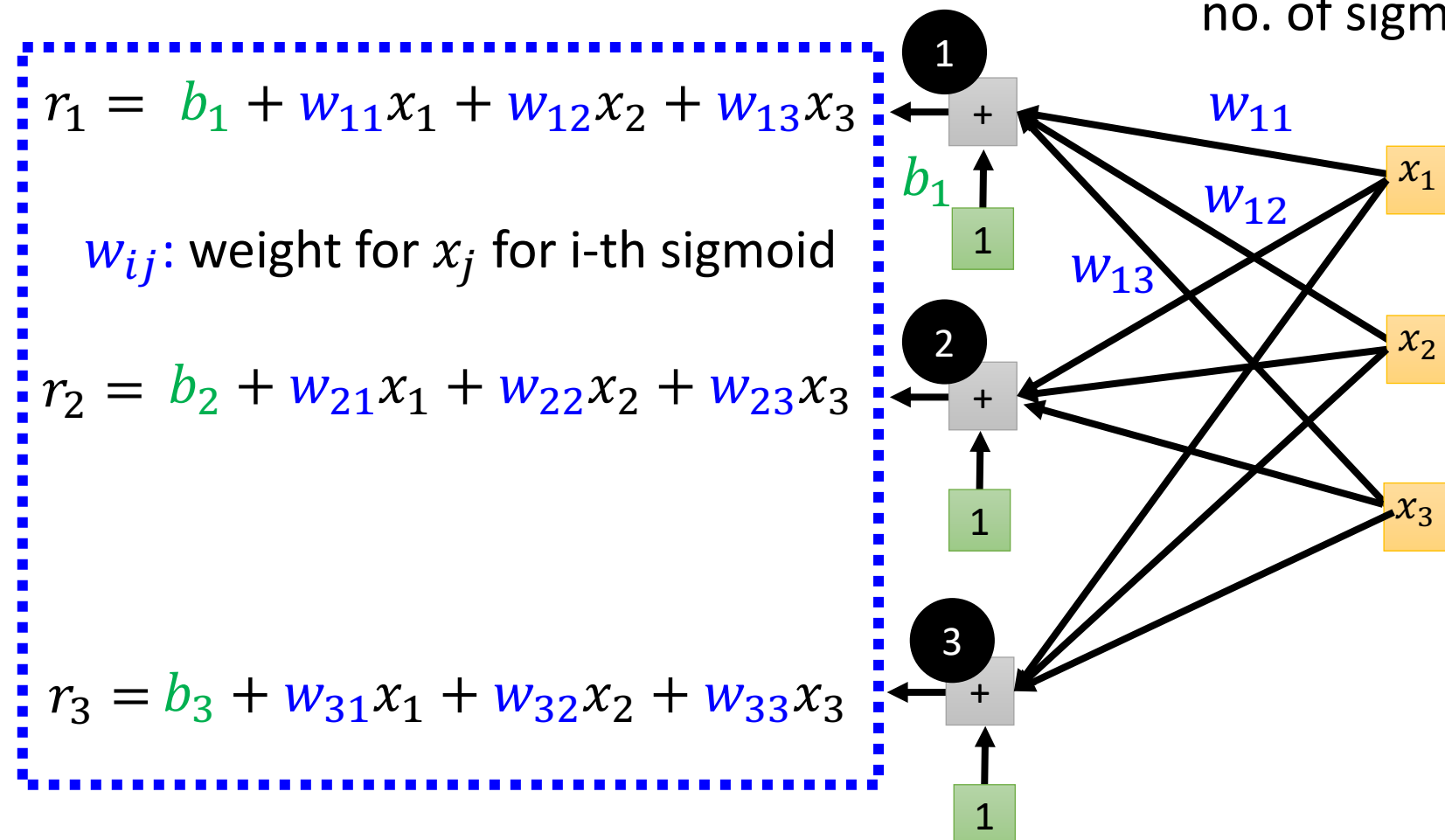
$$y = \underline{b + \sum_j w_j x_j}$$

$$y = b + \sum_i c_i \operatorname{sigmoid}\left(\underline{b_i + \sum_j w_{ij} x_j}\right)$$



$$y = b + \sum_i c_i \operatorname{sigmoid} \left( b_i + \sum_j w_{ij} x_j \right)$$

$j: 1, 2, 3$   
 no. of features  
 $i: 1, 2, 3$   
 no. of sigmoid





$$y = b + \sum_i c_i \operatorname{sigmoid} \left( b_i + \sum_j w_{ij} x_j \right) \quad \begin{matrix} i: 1,2,3 \\ j: 1,2,3 \end{matrix}$$

$$r_1 = b_1 + w_{11}x_1 + w_{12}x_2 + w_{13}x_3$$

$$r_2 = b_2 + w_{21}x_1 + w_{22}x_2 + w_{23}x_3$$

$$r_3 = b_3 + w_{31}x_1 + w_{32}x_2 + w_{33}x_3$$

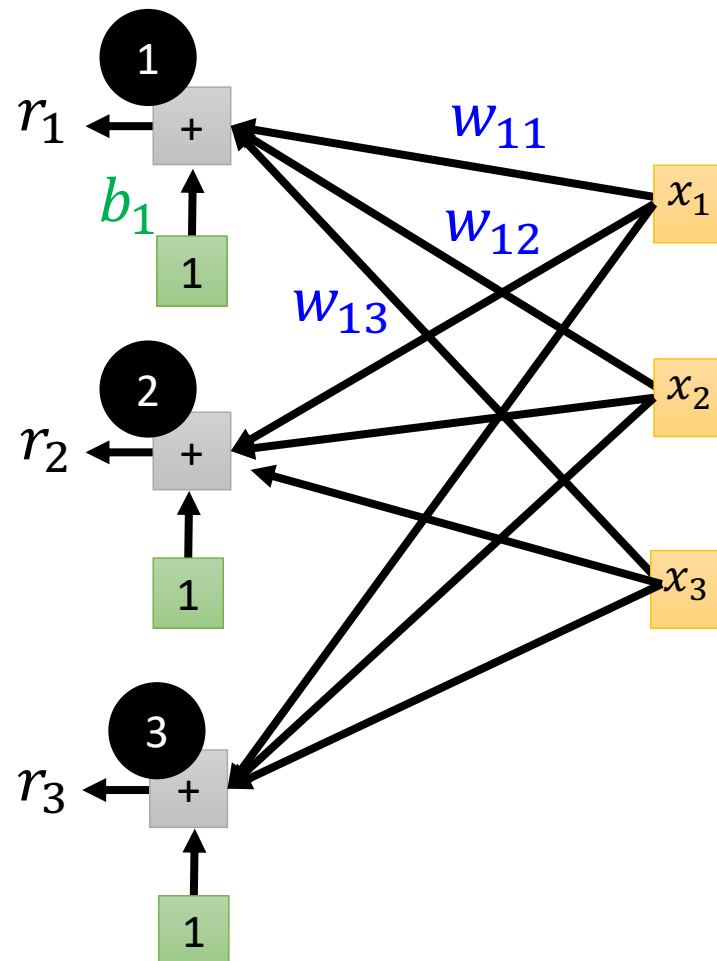
$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\mathbf{r} = \mathbf{b} + \mathbf{W} \mathbf{x}$$

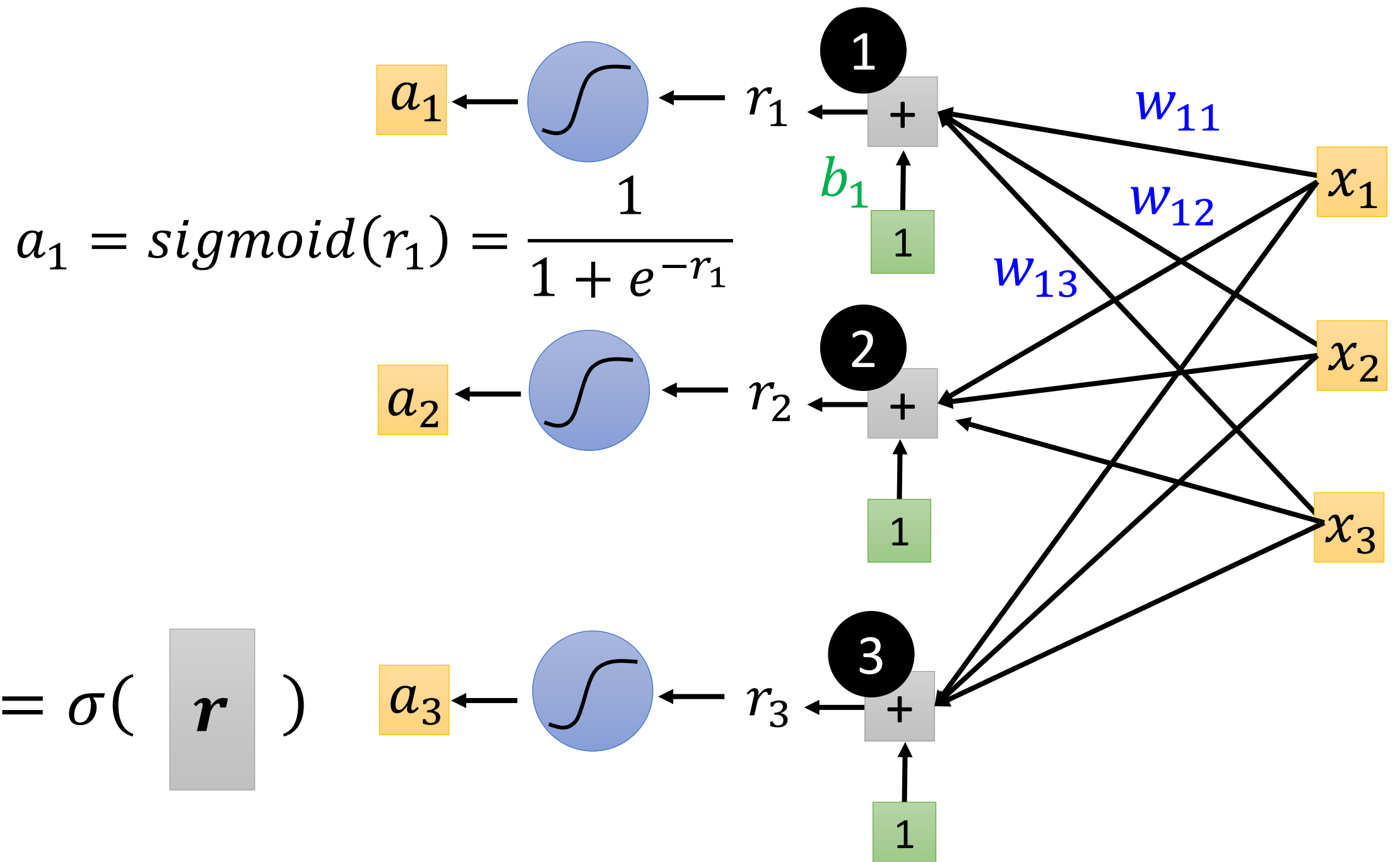


$$y = b + \sum_i c_i \operatorname{sigmoid} \left( b_i + \sum_j w_{ij} x_j \right) \quad \begin{matrix} i: 1,2,3 \\ j: 1,2,3 \end{matrix}$$

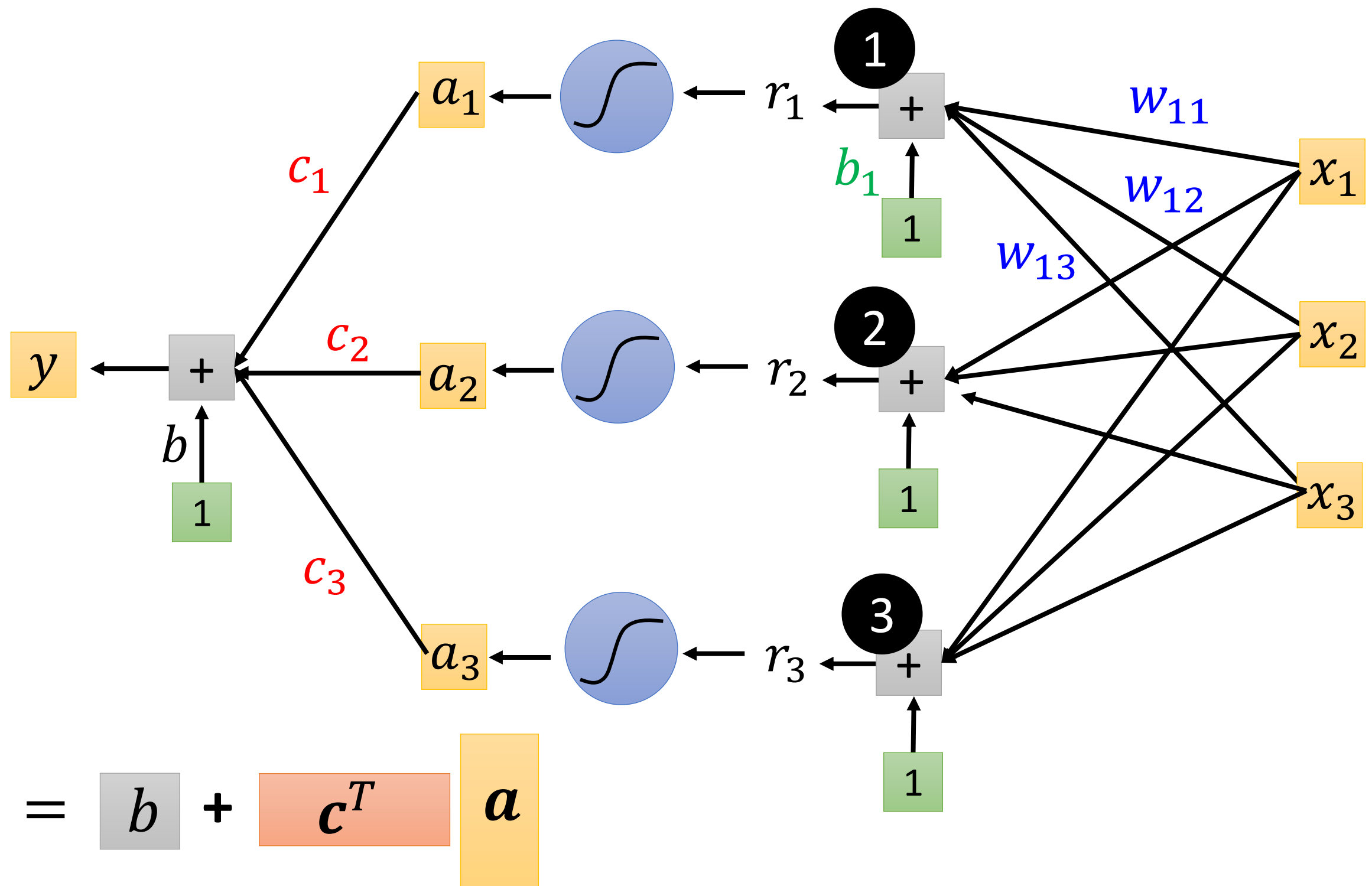
$$\mathbf{r} = \mathbf{b} + \mathbf{W} \mathbf{x}$$

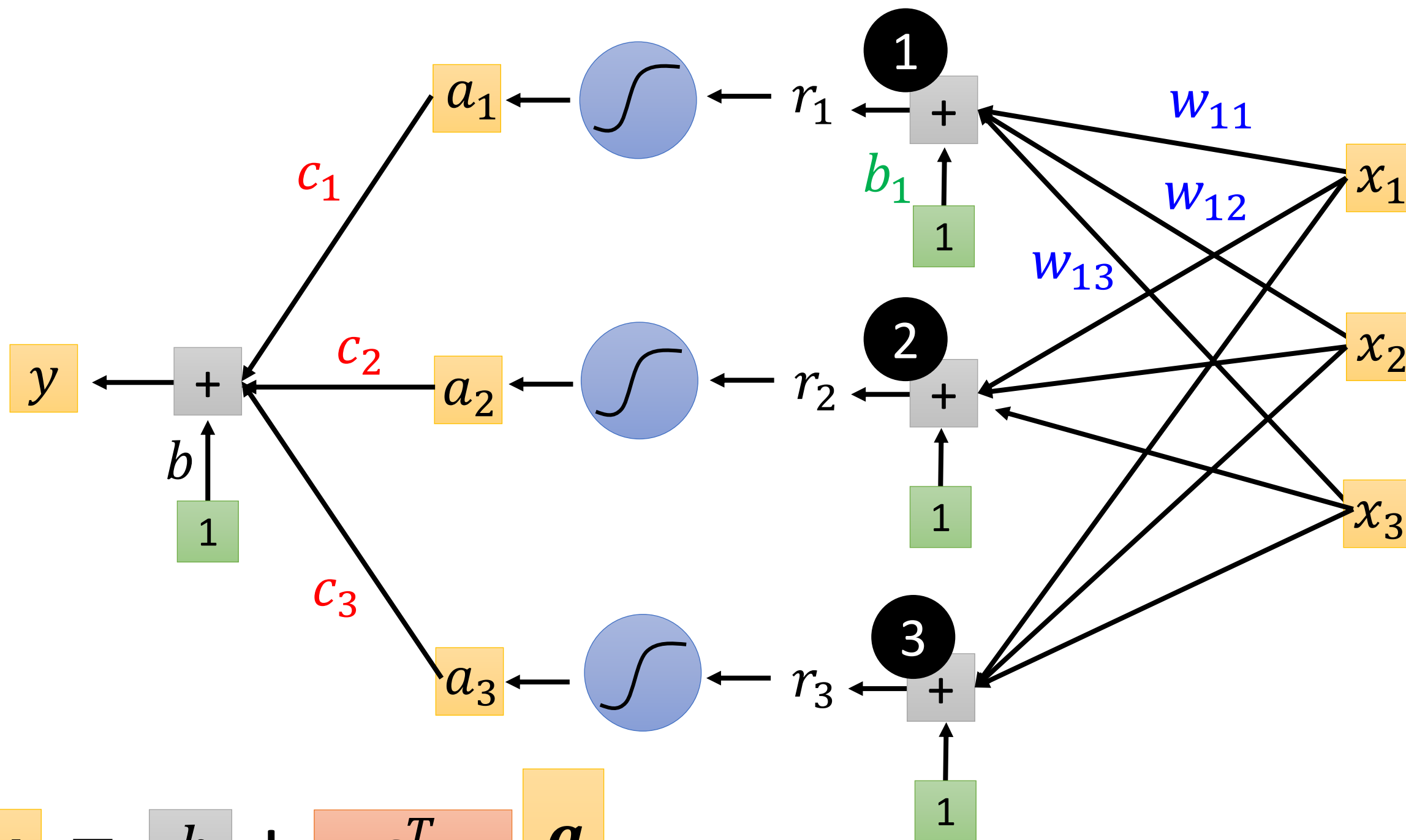


$$y = b + \sum_i c_i \text{sigmoid} \left( b_i + \sum_j w_{ij} x_j \right) \quad \begin{array}{l} i: 1,2,3 \\ j: 1,2,3 \end{array}$$



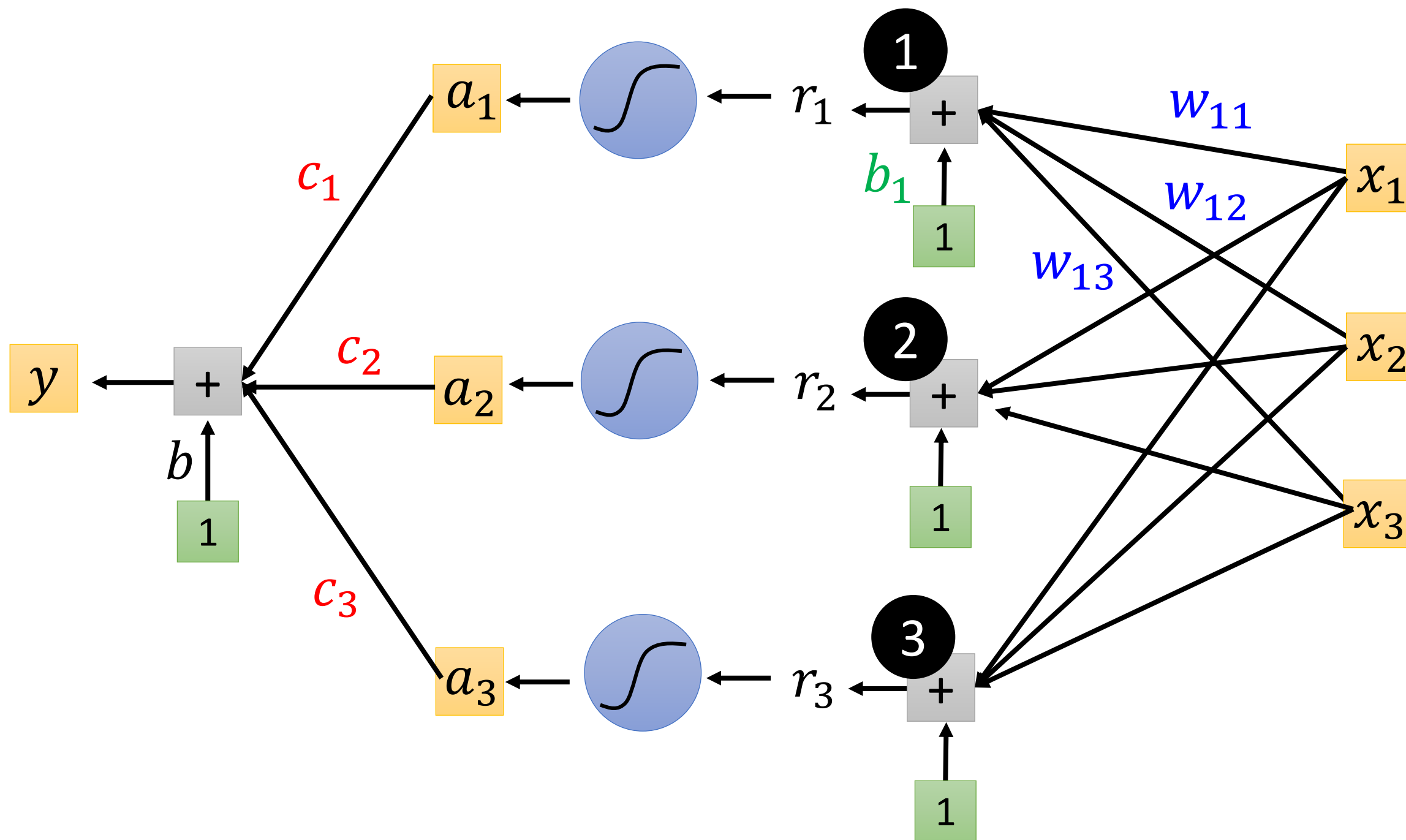
$$y = b + \sum_i \mathbf{c}_i \operatorname{sigmoid} \left( \mathbf{b}_i + \sum_j \mathbf{w}_{ij} x_j \right) \quad \begin{array}{l} i: 1,2,3 \\ j: 1,2,3 \end{array}$$





$$y = b + c^T a$$

$$a = \sigma(r) \quad r = b + Wx$$



$$y = b + c^T \sigma(b + Wx)$$

# Function with unknown parameters

$$y = b + c^T \sigma(b + Wx)$$

$x$  feature

Unknown parameters

$W$

$b$

$c^T$

$b$

Rows of  $W$

$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \end{bmatrix}$



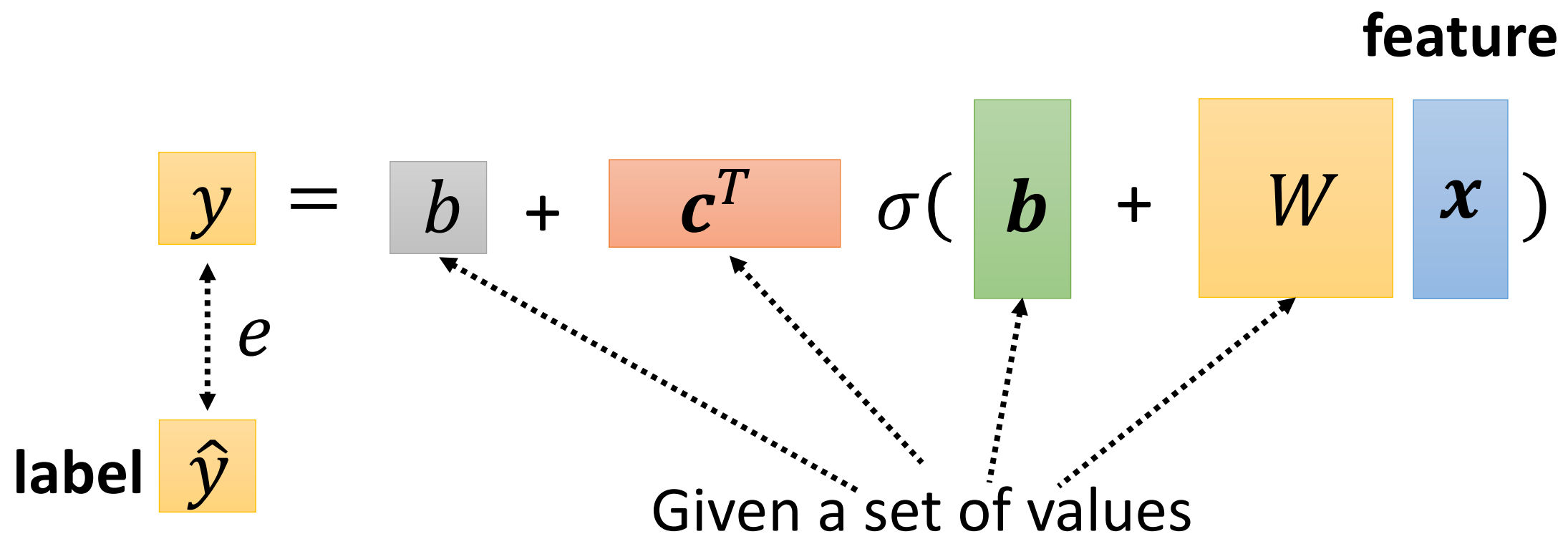


$$y = b + c^T \sigma( b + W x )$$

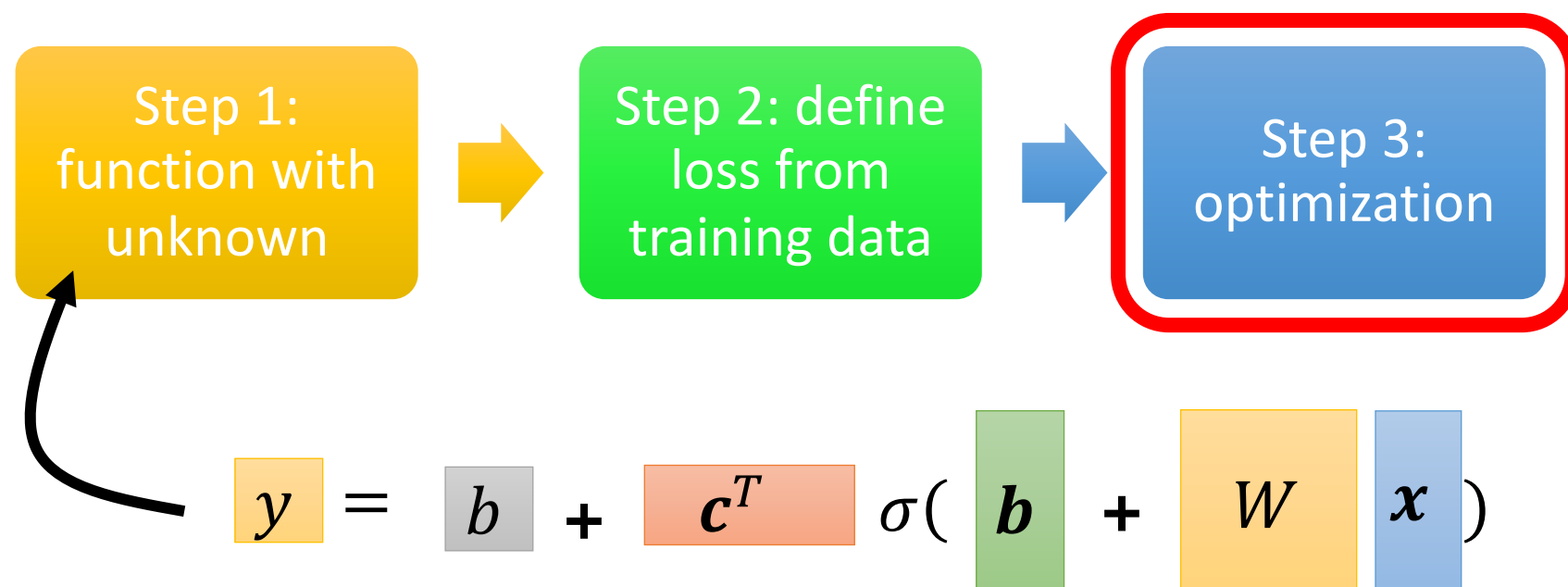


# LOSS

- Loss is a function of parameters  $L(\theta)$
- Loss means how good a set of values is.



Loss: 
$$L = \frac{1}{N} \sum_n e_n$$



# Optimization of New Model

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} L$$

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \end{bmatrix}$$

➤ (Randomly) Pick initial values  $\boldsymbol{\theta}^0$

$$\text{gradient } \mathbf{g} = \begin{bmatrix} \frac{\partial L}{\partial \theta_1} |_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \\ \frac{\partial L}{\partial \theta_2} |_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \\ \vdots \end{bmatrix} \quad \begin{bmatrix} \theta_1^1 \\ \theta_2^1 \\ \vdots \end{bmatrix} \leftarrow \begin{bmatrix} \theta_1^0 \\ \theta_2^0 \\ \vdots \end{bmatrix} - \begin{bmatrix} \eta \frac{\partial L}{\partial \theta_1} |_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \\ \eta \frac{\partial L}{\partial \theta_2} |_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \\ \vdots \end{bmatrix}$$

$$\mathbf{g} = \nabla L(\boldsymbol{\theta}^0)$$

$$\boldsymbol{\theta}^1 \leftarrow \boldsymbol{\theta}^0 - \eta \mathbf{g}$$

# Optimization of New Model

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} L$$

➤ (Randomly) Pick initial values  $\boldsymbol{\theta}^0$

➤ Compute gradient  $\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^0)$

$$\boldsymbol{\theta}^1 \leftarrow \boldsymbol{\theta}^0 - \eta \boldsymbol{g}$$

➤ Compute gradient  $\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^1)$

$$\boldsymbol{\theta}^2 \leftarrow \boldsymbol{\theta}^1 - \eta \boldsymbol{g}$$

➤ Compute gradient  $\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^2)$

$$\boldsymbol{\theta}^3 \leftarrow \boldsymbol{\theta}^2 - \eta \boldsymbol{g}$$

# Optimization of New Model

$$\theta^* = \arg \min_{\theta} L$$

➤ (Randomly) Pick initial values  $\theta^0$

➤ Compute gradient  $\mathbf{g} = \nabla L^1(\theta^0)$

$$\text{update } \theta^1 \leftarrow \theta^0 - \eta \mathbf{g}$$

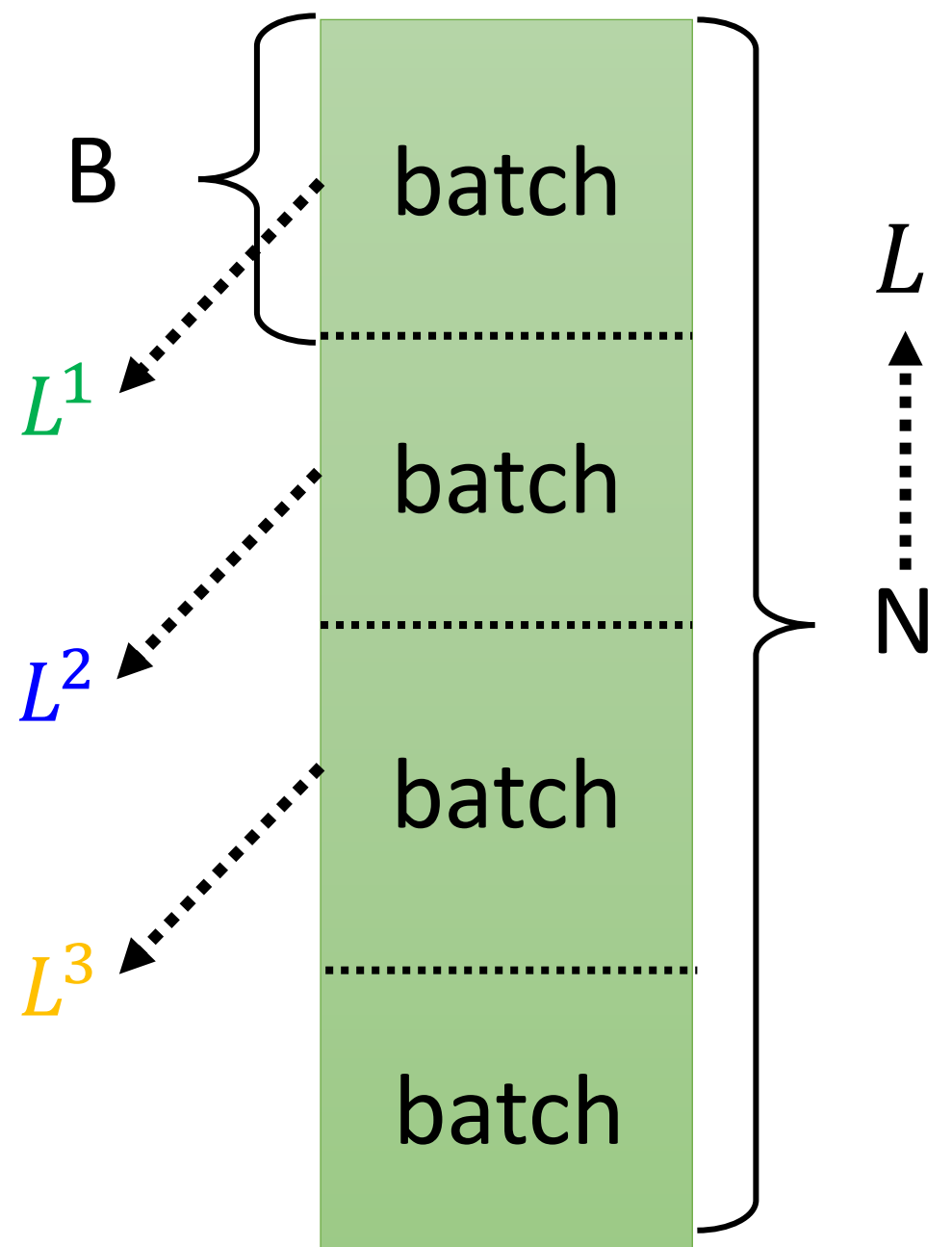
➤ Compute gradient  $\mathbf{g} = \nabla L^2(\theta^1)$

$$\text{update } \theta^2 \leftarrow \theta^1 - \eta \mathbf{g}$$

➤ Compute gradient  $\mathbf{g} = \nabla L^3(\theta^2)$

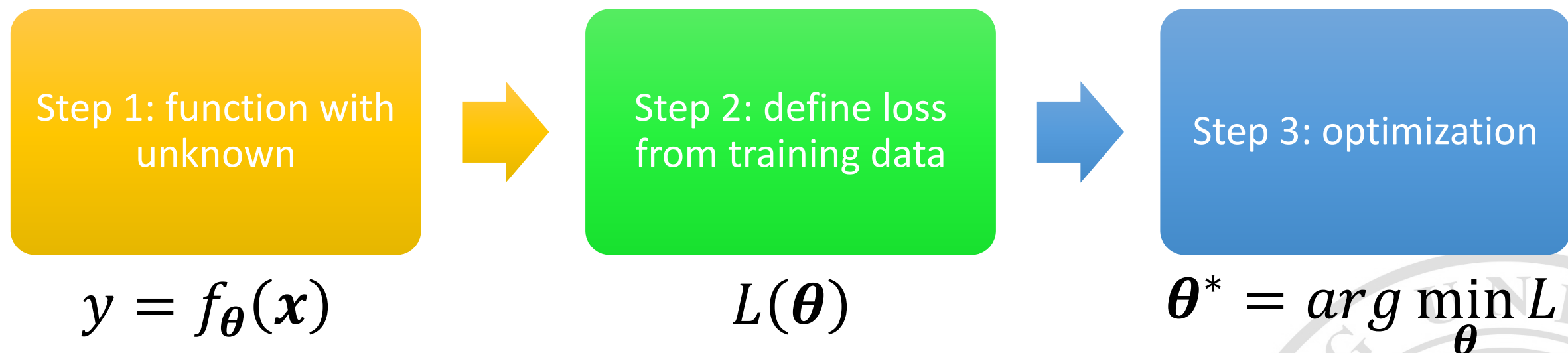
$$\text{update } \theta^3 \leftarrow \theta^2 - \eta \mathbf{g}$$

1 **epoch** = see all the batches once



Training data:  $\{(\mathbf{x}^1, \hat{y}^1), (\mathbf{x}^2, \hat{y}^2), \dots, (\mathbf{x}^N, \hat{y}^N)\}$

Training:



Testing data:  $\{\mathbf{x}^{N+1}, \mathbf{x}^{N+2}, \dots, \mathbf{x}^{N+M}\}$

Use  $y = f_{\theta^*}(\mathbf{x})$  to label the testing data

$\{y^{N+1}, y^{N+2}, \dots, y^{N+M}\}$



- Small loss on training data, large loss on testing data.

## An extreme example

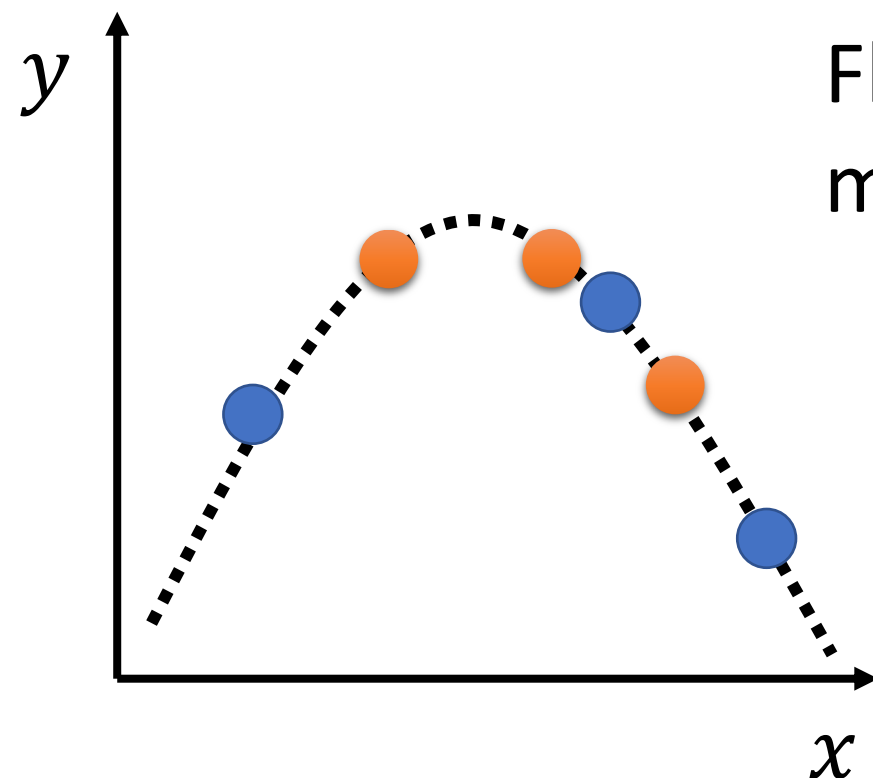
Training data:  $\{(\mathbf{x}^1, \hat{y}^1), (\mathbf{x}^2, \hat{y}^2), \dots, (\mathbf{x}^N, \hat{y}^N)\}$

$$f(\mathbf{x}) = \begin{cases} \hat{y}^i & \exists \mathbf{x}^i = \mathbf{x} \\ \text{random} & \text{otherwise} \end{cases}$$

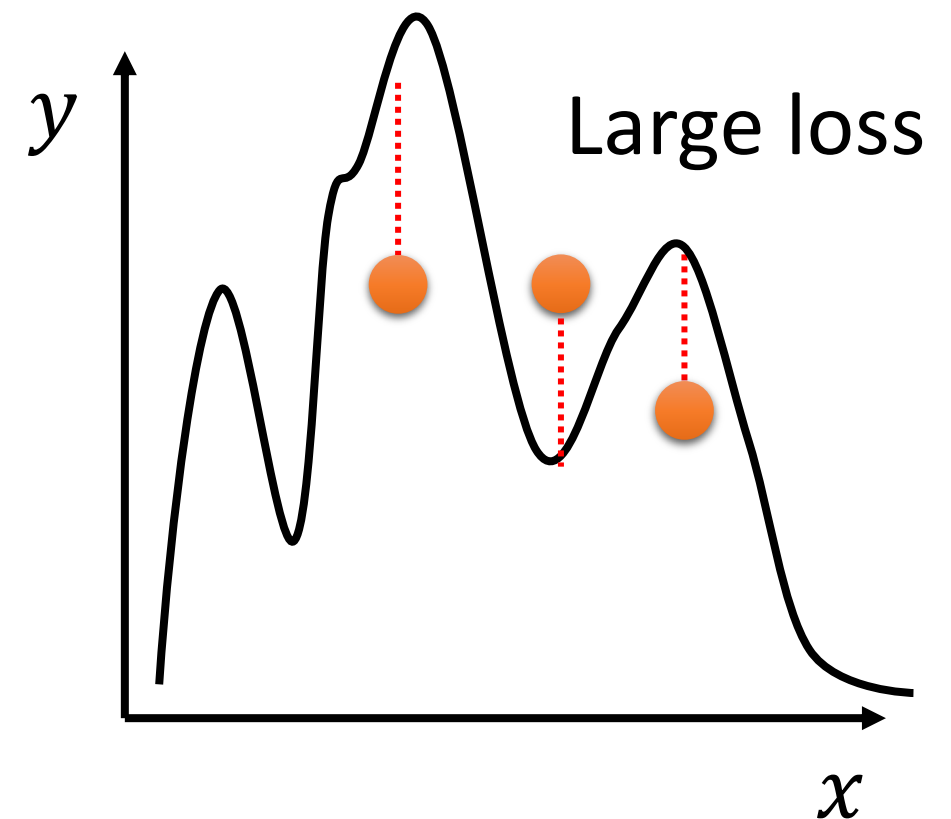
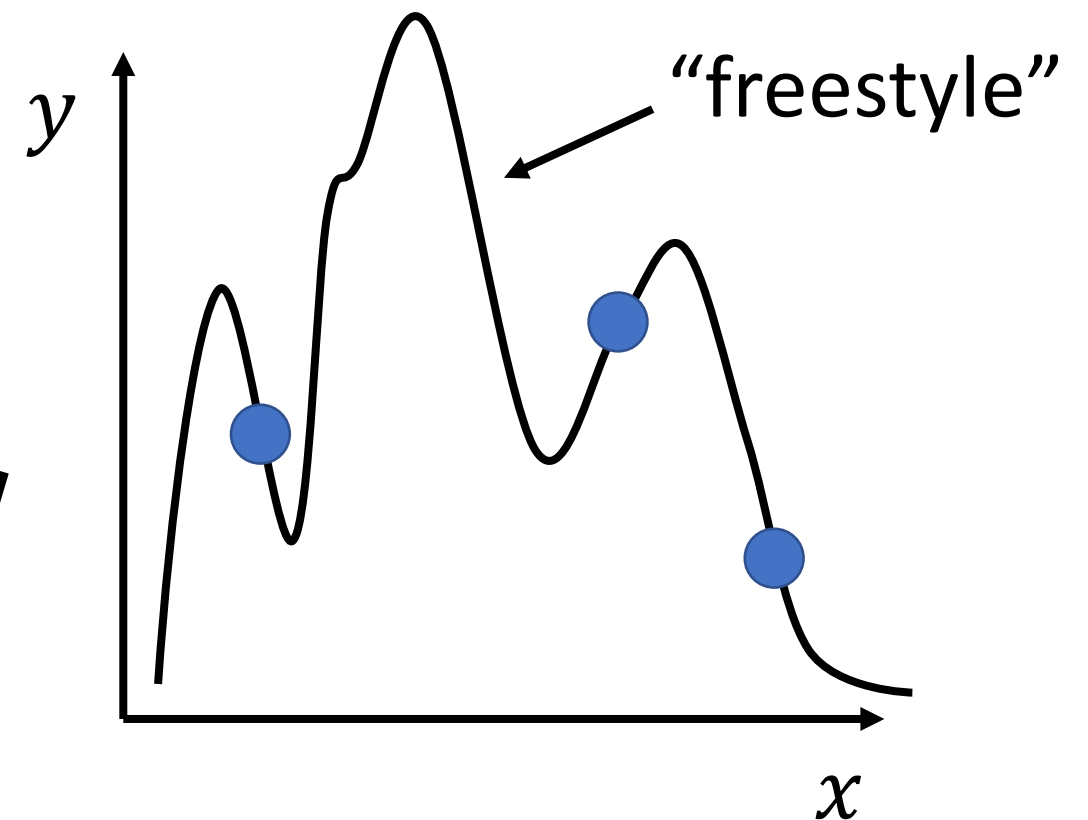
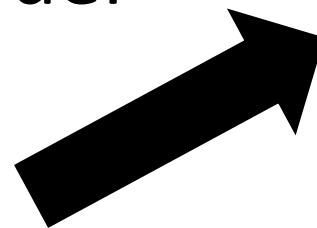
Less than useless ...

This function obtains **zero training loss**, but **large testing loss**.

# Overfitting



Flexible  
model

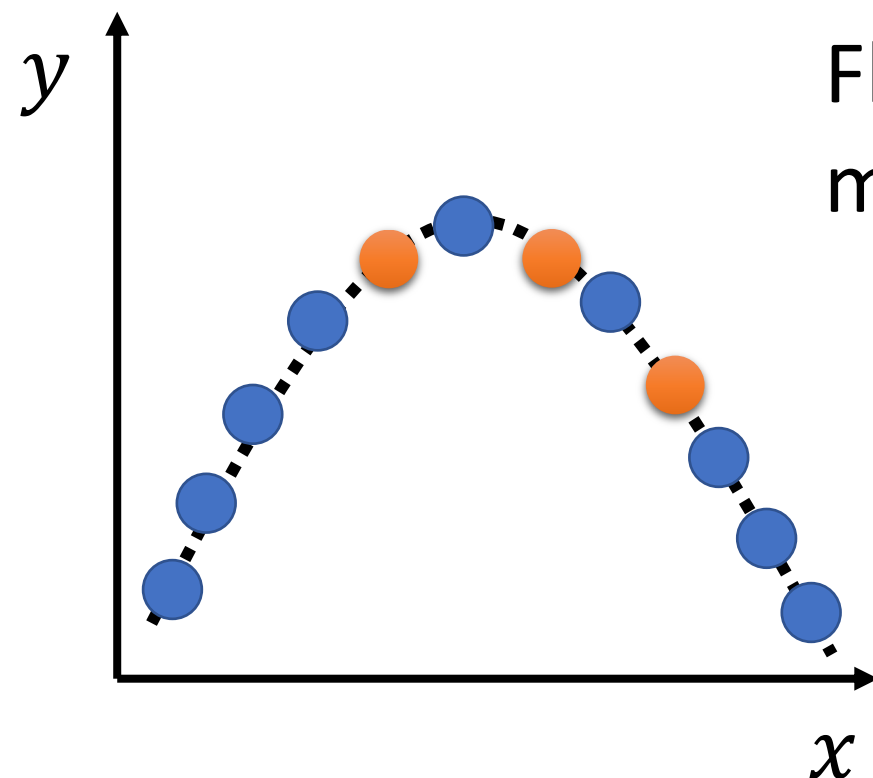


..... Real data distribution  
(not observable)

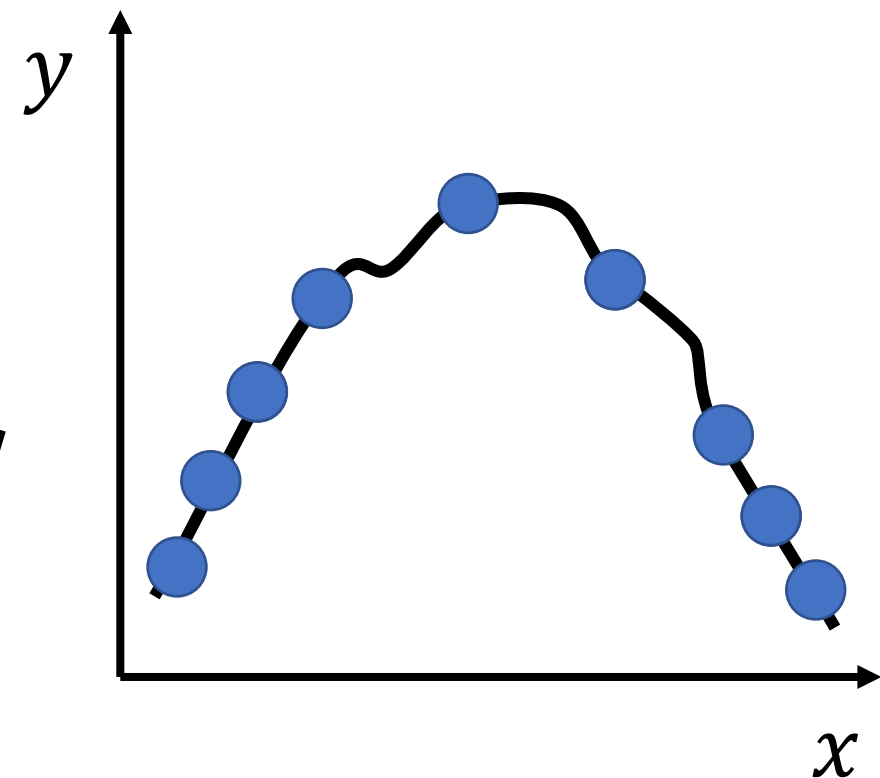
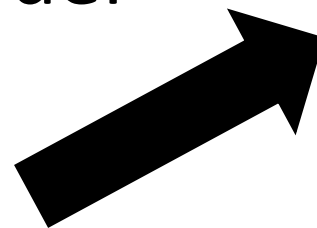
● Training data

● Testing data

# Overfitting



Flexible  
model

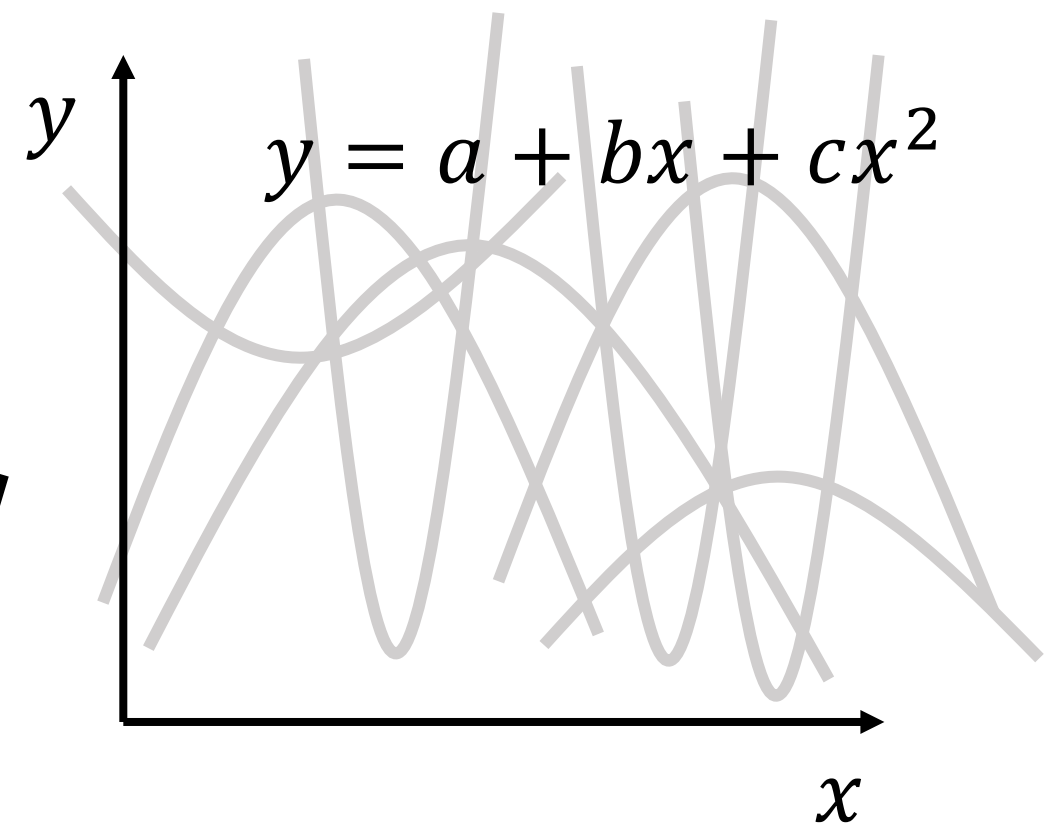
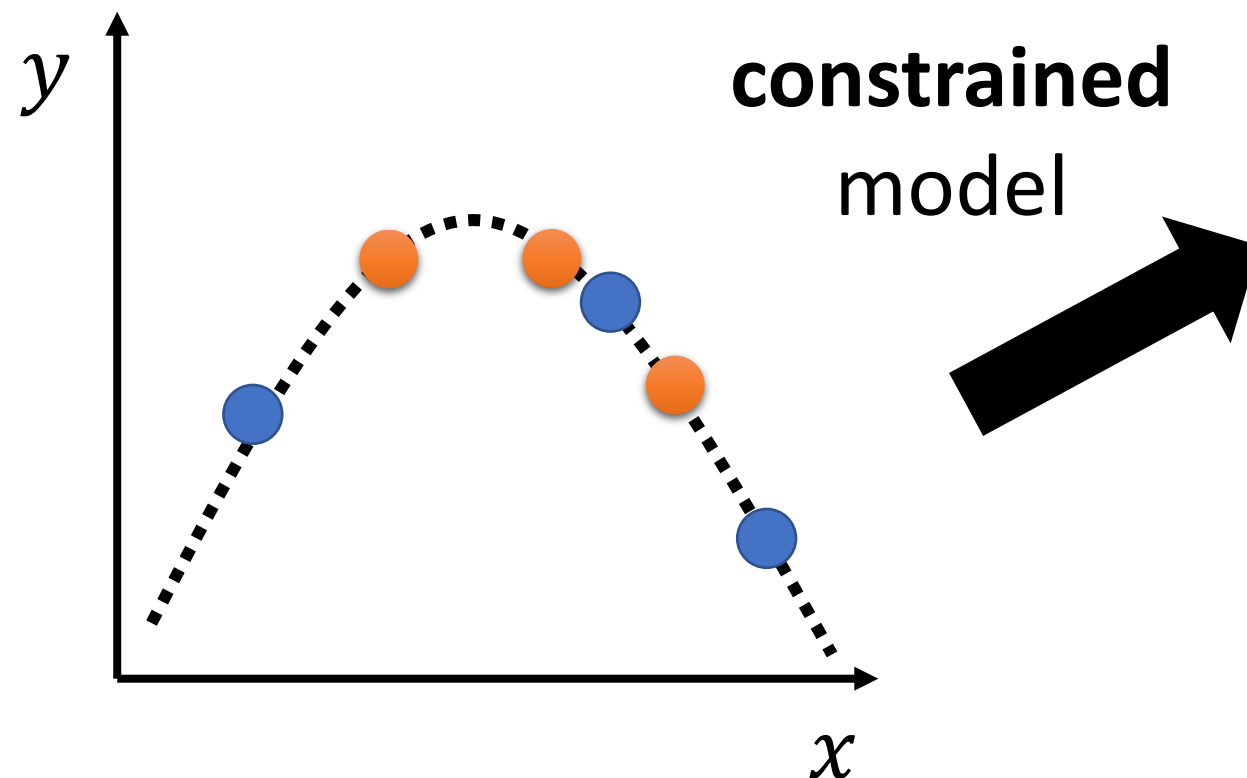


**More training data**

## Data augmentation



# Overfitting

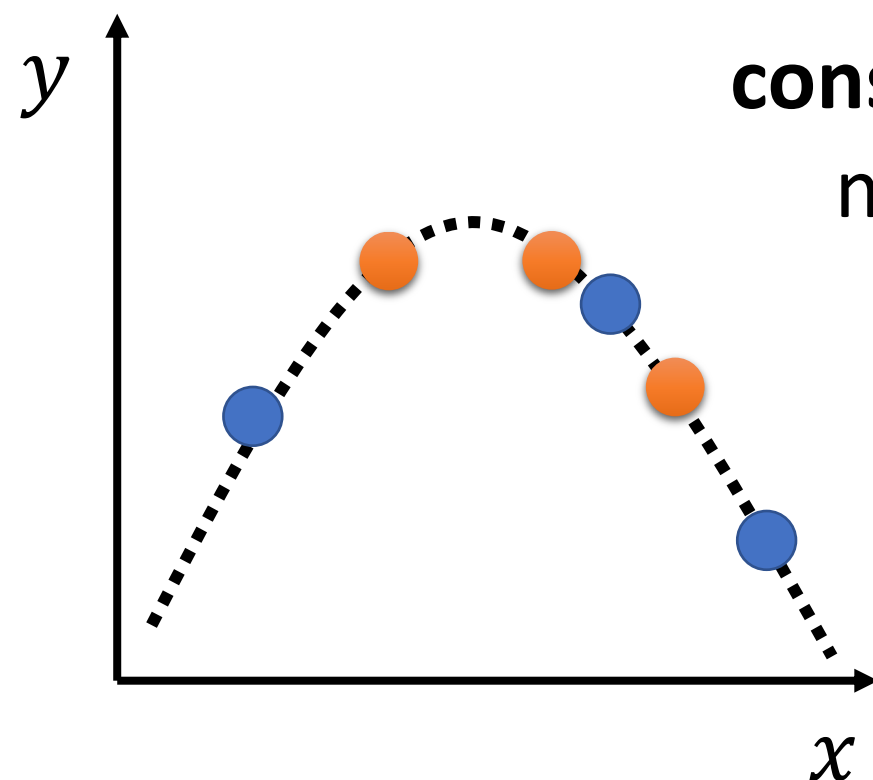


..... Real data distribution  
(not observable)

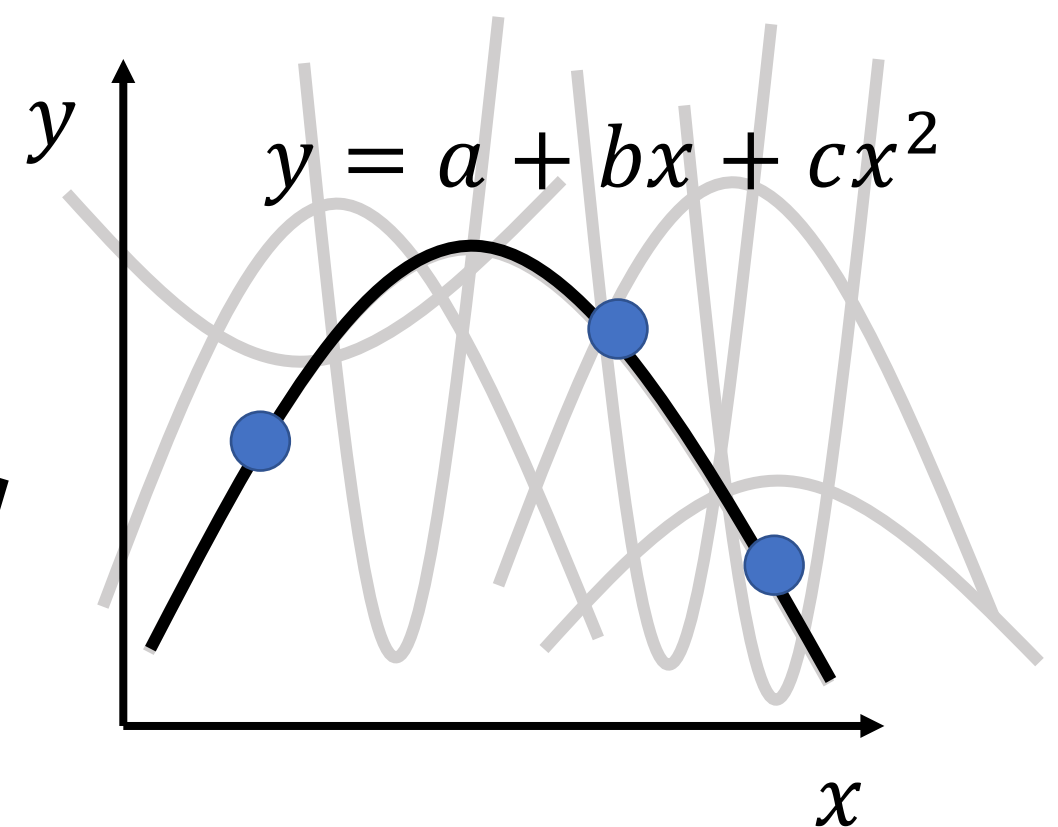
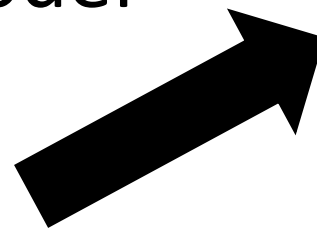
● Training data

● Testing data

# Overfitting



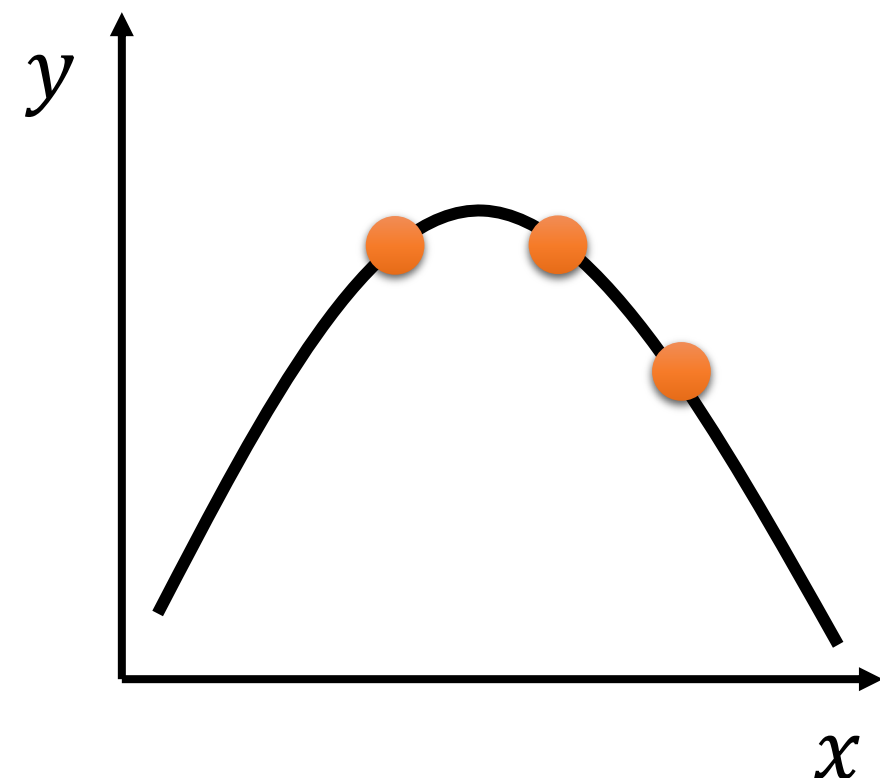
**constrained  
model**



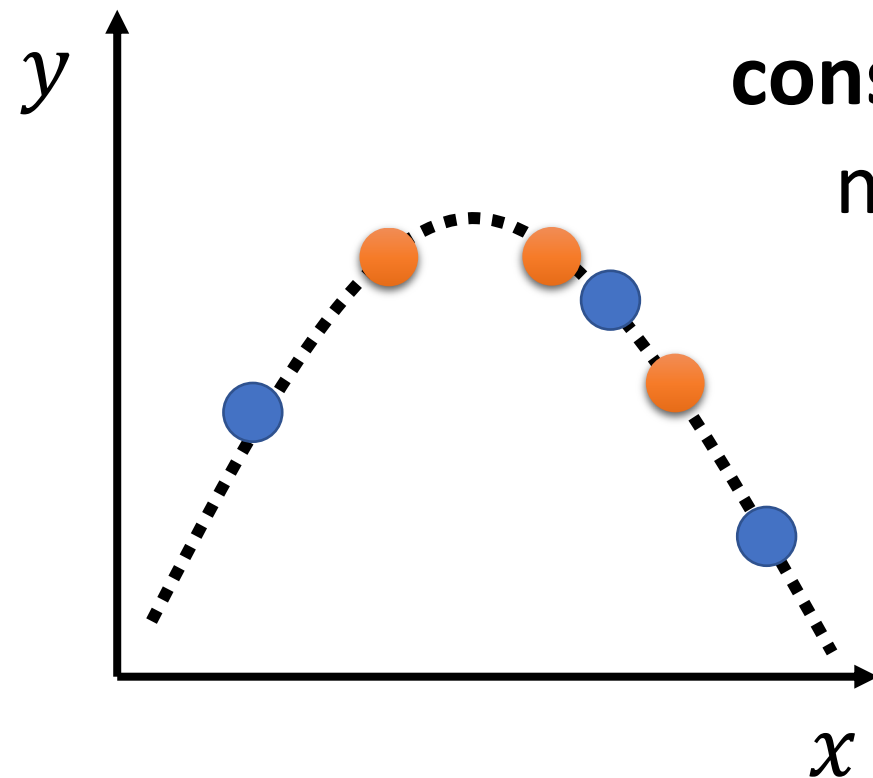
..... Real data distribution  
(not observable)

● Training data

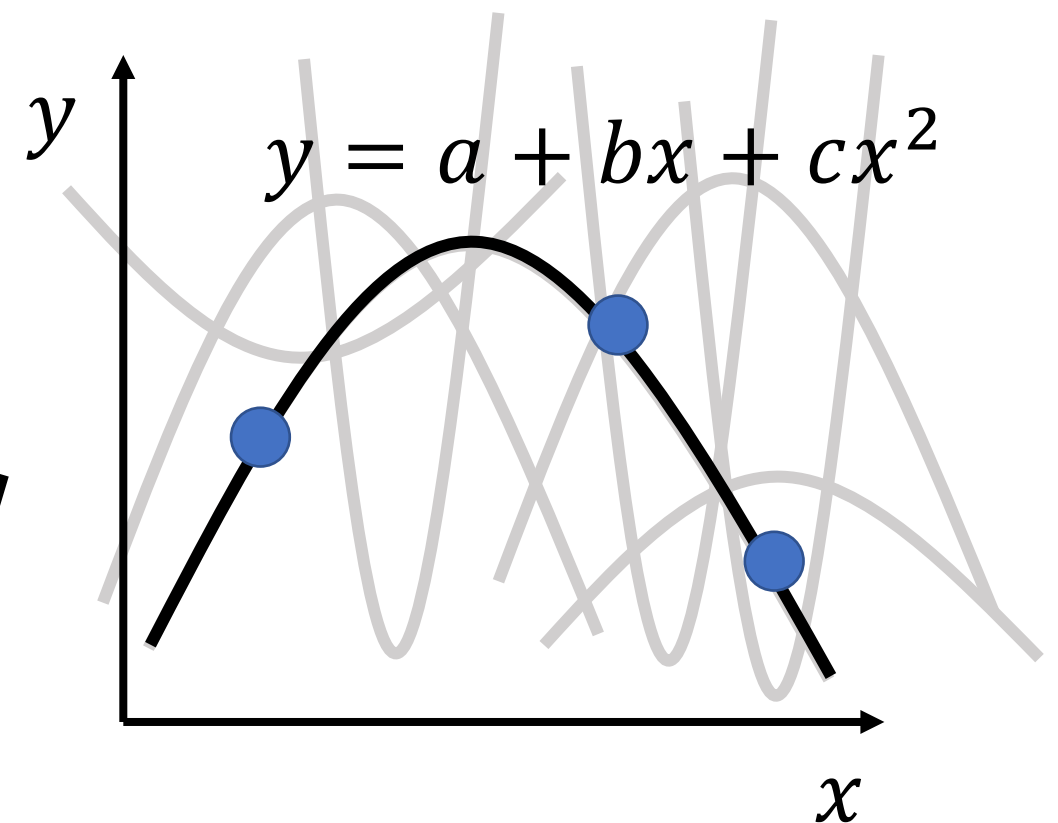
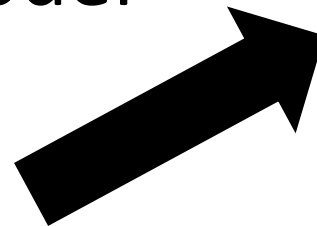
● Testing data



# Overfitting

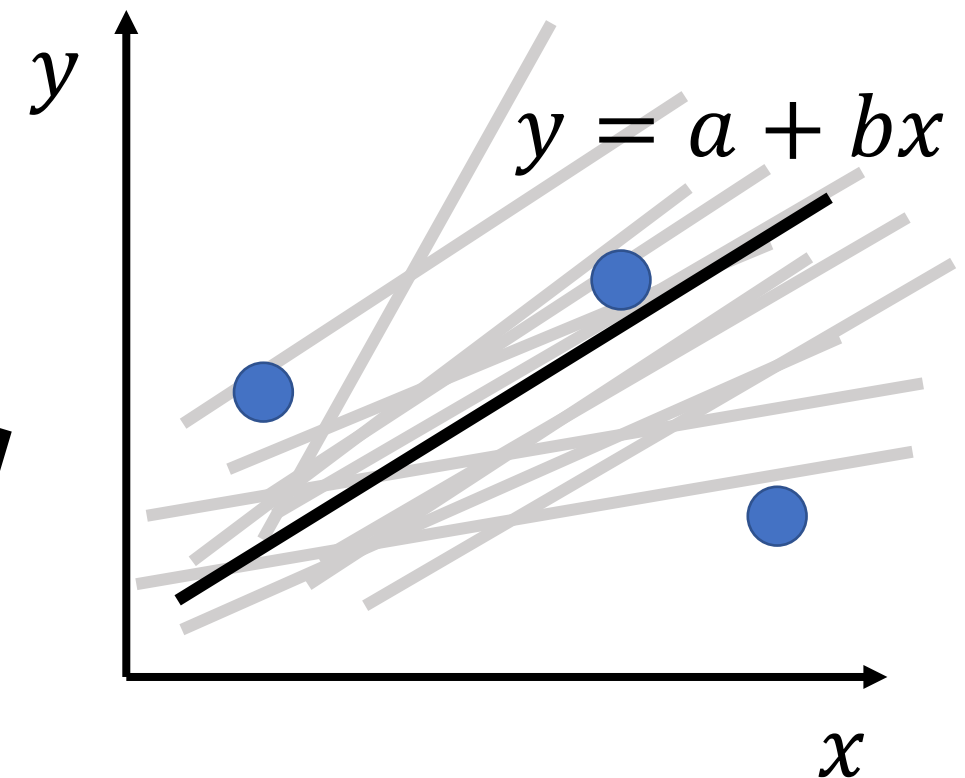
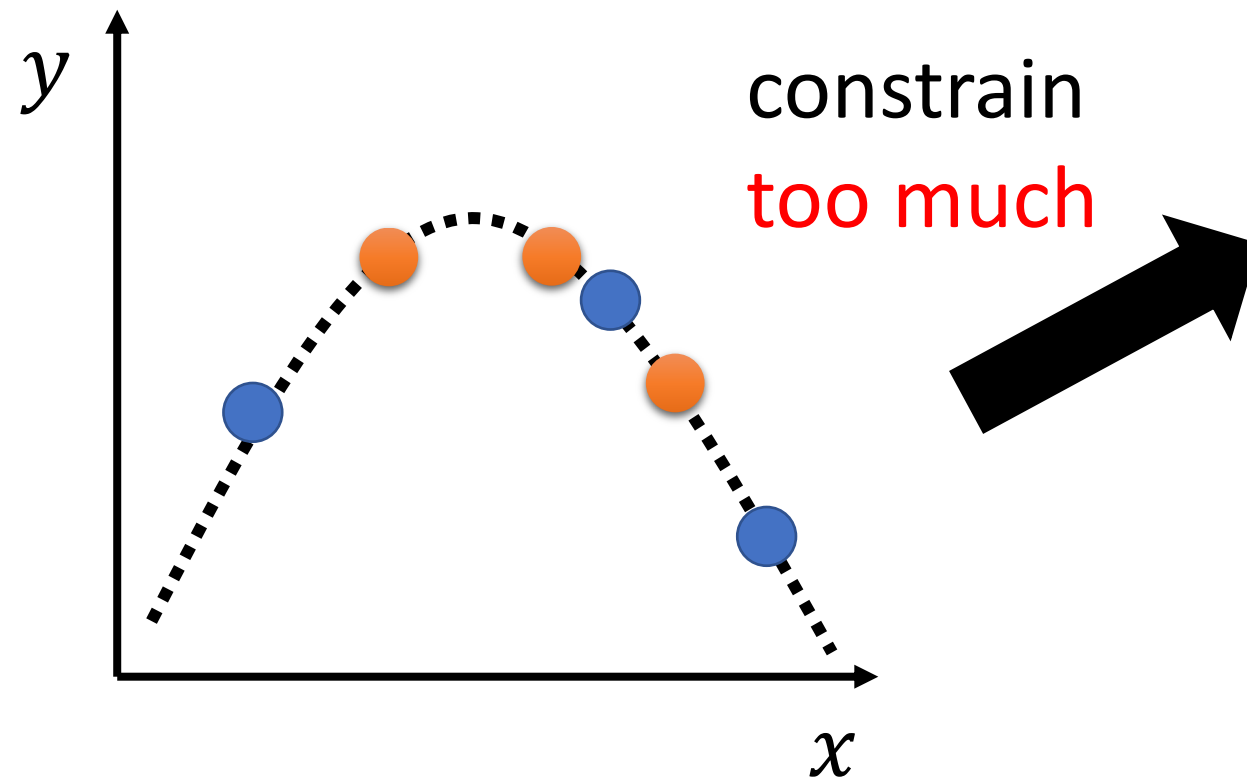


**constrained  
model**



- Less parameters, sharing parameters
- Less features
- Early stopping
- Regularization
- Dropout

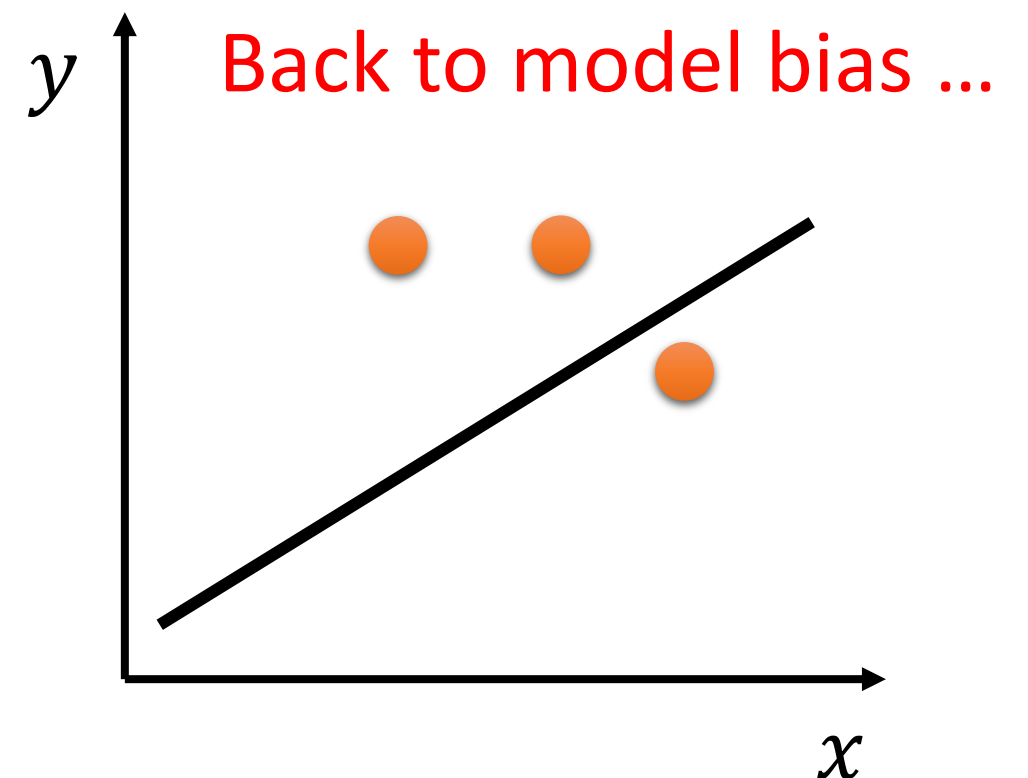
# Overfitting



..... Real data distribution  
(not observable)

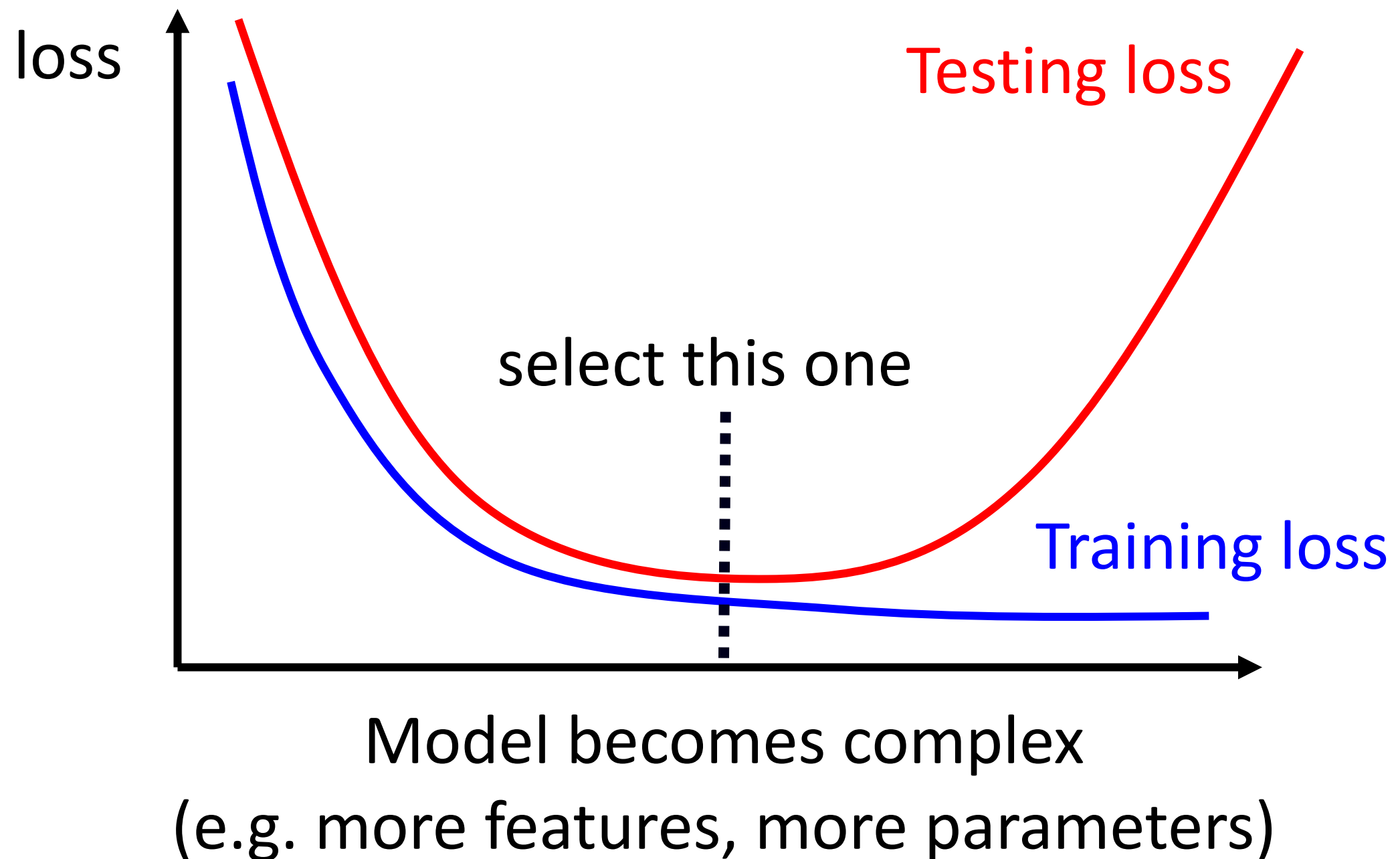
● Training data

● Testing data

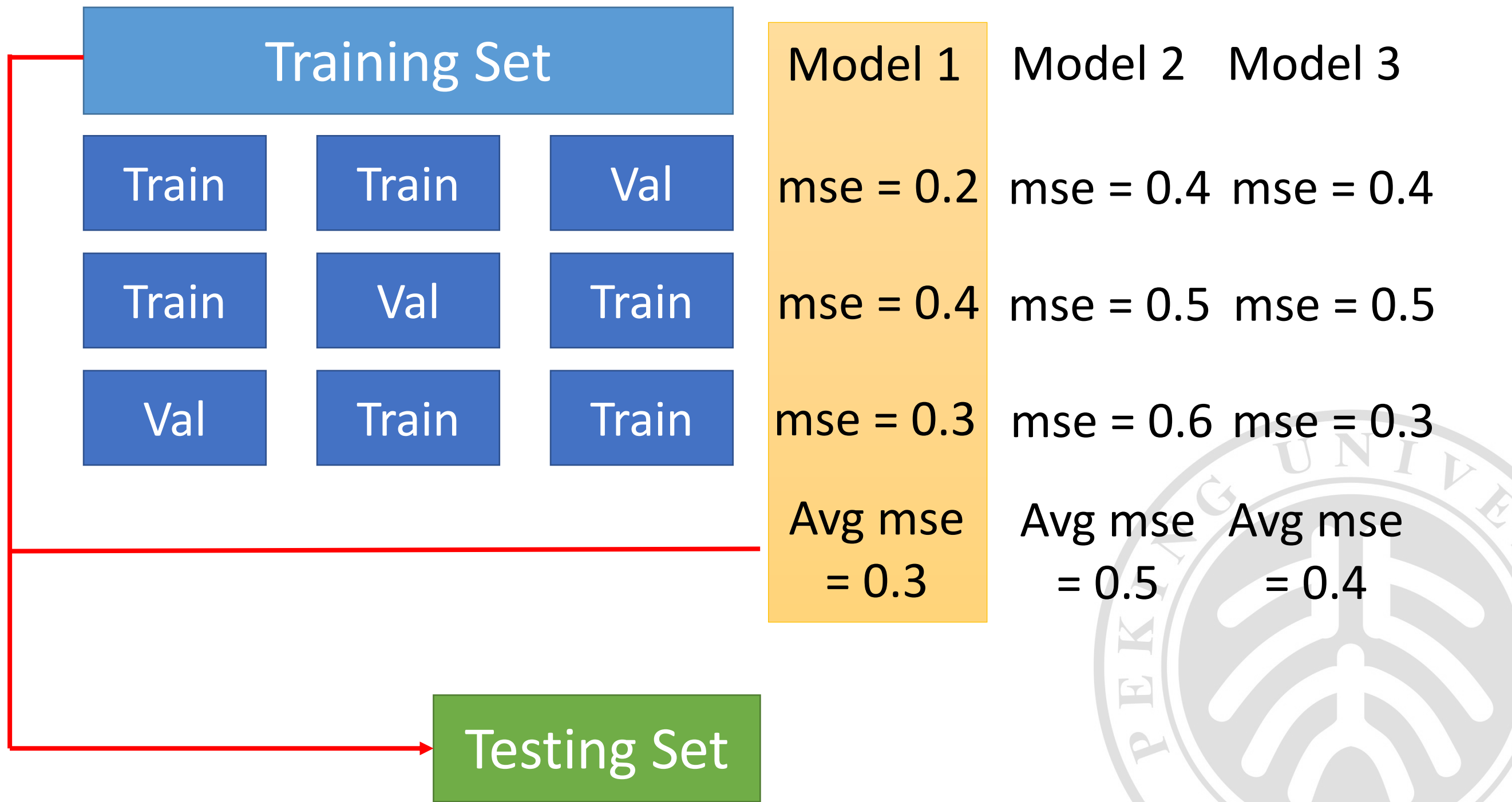




# Bias-Complexity Trade-off



## N-Fold Cross Validation



- What information is available for learning?
  - What does the data look like?
  - How is it annotated?
- What output is desired?
  - What should the algorithm produce?
  - How will it be used?



- Learning with a teacher
  - Explicit feedback in the form of labeled examples
  - Goal: make prediction
  - Pros: Good performance
  - Cons: Labeled data is difficult to find
  - “Classical” (labeled data simply there; do your best)
  - Query-based (can ask for labeled examples)
- Examples
  - Classification
    - Is an email spam or not?
  - Regression
    - What is the expected rate of return on a specific investment?



- Learning without labels
  - Only observed unlabeled examples
  - Goal: uncover structure in data
  - Pros: Easy to find lots of data
  - Cons: what are we looking for?
- Examples
  - Clustering
    - Group emails by topic
  - Manifold learning
    - Find a low dimensional data representation



- Learn a behavior policy by interacting with the world
  - How to navigate in a world
  - Success measured by rewards received for actions taken
  - Maximize sum of rewards
- Examples
  - Chess (and checkers)
  - Robot control
  - Piloting an airplane

