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Sol 1. Let $(x_1, x_2 \dots x_n)$ be a given normal sample with mean $= \theta_1$ & var $= \theta_2$

p.d.f of normal dist. is :-

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Density of x_i is

$$\frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\frac{(x_i-\theta_1)^2}{\theta_2}}$$

Jt. density $L(x_1, x_2 \dots x_n) =$

$$\prod_{i=1}^{n}\left(\frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\frac{(x_i-\theta_1)^2}{\theta_2}}\right)$$

$$= \left(\frac{1}{\sqrt{2\pi\theta_2}}\right)^n e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}$$

Taking log on both sides

$$\ln(L(x_1, x_2, \dots x_n)) = n\ln\left(\frac{1}{\sqrt{2\pi\theta_2}}\right) + \ln\left(e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}\right)$$

Let $Z = -\frac{n}{2}(\ln(2\pi\theta_2)) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$

For M.L.E of mean, put $\frac{dZ}{d\theta_1} = 0$

$$\Rightarrow \frac{d}{d\theta_1}\left(-\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2\right) = 0$$

$$\Rightarrow -\frac{1}{2\theta_2}\sum_{i=1}^{n}\frac{d}{d\theta_1}(x_i-\theta_1)^2 = 0$$

$$\Rightarrow \frac{1}{\cancel{2}\theta_2}\sum_{i=1}^{n}\cancel{2}(x_i-\theta_1) = 0$$

$$\Rightarrow \sum_{i=1}^{n}(x_i-\theta_1) = 0$$

$$\Rightarrow \sum_{i=1}^{n}x_i - n\theta_1 = 0$$

$$\Rightarrow \boxed{\theta_1 = \frac{\sum_{i=1}^{n}x_i}{n}}$$

For M.L.E of var, put $\frac{dZ}{d\theta_2} = 0$

$$\Rightarrow \frac{d}{d\theta_2}\left(-\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2\right) = 0$$

$$\Rightarrow -\frac{n}{2}\left(\frac{\cancel{2\pi}}{\cancel{2\pi}\theta_2}\right) + \frac{1}{2\theta_2^2}\left(\sum_{i=1}^{n}(x_i-\theta_1)^2\right) = 0$$

$$\Rightarrow \frac{1}{\cancel{2}\theta_2^{\cancel{2}}}\left(\sum_{i=1}^{n}(x_i-\theta_1)^2\right) = \frac{n}{\cancel{2}\cancel{\theta_2}}$$

$$\Rightarrow \boxed{\theta_2 = \frac{\sum_{i=1}^{n} (x_i - \theta_1)^2}{n}}$$

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Sol 2. Let $B(m, \theta)$ be the given distribution of the sample $(x_1, x_2, \ldots x_n)$

m = no. trials ; θ = prob. of success $\in (0,1)$

p.d.f of binomial dist.

$$f(x) = {}^mC_x \, p^x (1-p)^{m-x}$$

Jt. density of x_i =

$$L(x_1, x_2 \ldots x_n) \quad \prod_{i=1}^{n} {}^mC_{x_i} \, \theta^{x_i} (1-\theta)^{m-x_i}$$

taking log on both sides.

$$\ln(L(x_1, x_2 \ldots x_n)) = \ln\left(\prod_{i=1}^{n} {}^mC_{x_i} \, \theta^{x_i} (1-\theta)^{m-x_i}\right)$$

Let $$Z = \sum_{i=1}^{n}\left(\ln({}^mC_{x_i}) + x_i \ln(\theta) + m-x_i (\ln(1-\theta))\right)$$

Put $\frac{dZ}{d\theta} = 0$

$$\Rightarrow \sum_{i=1}^{n}\left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\Rightarrow (1-\theta)\sum_{i=1}^{n} x_i = \theta\left(\sum_{i=1}^{n}(m-x_i)\right)$$

$$\Rightarrow (1-\theta)\sum x_i = \theta(nm - \sum x_i)$$

$$\Rightarrow \sum x_i - \theta \sum x_i = \theta n m - \theta \sum x_i$$

$$\Rightarrow \sum x_i = \theta n m$$

$$\Rightarrow \boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{n m}}$$