

- Para una línea recta,  $\chi^2$  tiene la siguiente forma

$$\chi^2 = \sum_{i=1}^N \frac{(y_i - a_0 - a_1 x_i)^2}{\sigma_i^2}$$

Minimizando

$$\frac{\partial \chi^2}{\partial a_0} = -2 \sum \frac{(y_i - a_0 - a_1 x_i)}{\sigma_i^2} = 0$$

$$\frac{\partial \chi^2}{\partial a_1} = -2 \sum \frac{x_i (y_i - a_0 - a_1 x_i)}{\sigma_i^2} = 0$$

$$\sum \frac{y_i}{\sigma_i^2} = \left( \sum \frac{1}{\sigma_i^2} \right) a_0 + \left( \sum \frac{x_i}{\sigma_i^2} \right) a_1.$$

$$\sum \frac{x_i y_i}{\sigma_i^2} = \left( \sum \frac{x_i}{\sigma_i^2} \right) a_0 + \left( \sum \frac{x_i^2}{\sigma_i^2} \right) a_1.$$

- tenemos un sistema de dos ecuaciones

$$\begin{bmatrix} \sum \frac{y_i}{\sigma_i^2} \\ \sum \frac{x_i y_i}{\sigma_i^2} \end{bmatrix} = \begin{bmatrix} \sum \frac{1}{\sigma_i^2} & \sum \frac{x_i}{\sigma_i^2} \\ \sum \frac{x_i}{\sigma_i^2} & \sum \frac{x_i^2}{\sigma_i^2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

- Hallando la inversa.

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \sum \frac{x_i^2}{\sigma_i^2} & - \sum \frac{x_i}{\sigma_i^2} \\ - \sum \frac{x_i}{\sigma_i^2} & \sum \frac{1}{\sigma_i^2} \end{bmatrix} \begin{bmatrix} \sum \frac{y_i}{\sigma_i^2} \\ \sum \frac{x_i y_i}{\sigma_i^2} \end{bmatrix}$$

$$\Delta = \left( \sum \frac{1}{\sigma_i^2} \right) \left( \sum \frac{x_i^2}{\sigma_i^2} \right) - \left( \sum \frac{x_i}{\sigma_i^2} \right)^2$$

tomando  $\sigma_i = \sigma$ , encontramos las soluciones para mínimos cuadrados

$$\Delta = N \sum x^2 - (\sum x)^2 \text{ encontramos}$$

$$a_0 = \frac{\sum y \sum x^2 - \sum xy \sum x}{\Delta}$$

$$a_1 = \frac{N \sum xy - \sum x \sum y}{\Delta}$$