Probalibility

0.1 The language of probability

Kolmogorov definitions:

- We define ω as the set of all possible elementary events. ω is called the sample space of the experiment
- We define Σ as a set of subsets of ω , Σ is a set of random events (combination of elementary events) with the following properties:

  if $A, B \in \Sigma$ then $A \cap B$ and $A \cup B \in \Sigma$

  $\omega \in \Sigma$

  $\varnothing \in \Sigma$
- Let $A \in \Sigma$, we define the Probability of A as a non negative real number, that can be assigned for all $A \in \Sigma$. $\exists\ P(A) \geq 0$ with the following properties:

  $P(\omega) = 1$

  let $A, B \in \Sigma$, if $A \cap B = \varnothing \implies P(A \cup B) = P(A) + P(B)$

Alternative definitions:

- We define ω as the set of all possible elementary events
- We define Σ as a σ-algebra of random events (combination of elementary events) with the following properties:

  Σ contains at least one subset of ω

  if $A \in \Sigma \implies \omega / A \in \Sigma$

  if $A_1, A_2 \dots A_n \in \Sigma \Longrightarrow \bigcup_{i=1}^{n} A_i \in \Sigma$
- An arbitrary element of Σ is called a random event
- Let $A \in \Sigma$, we define the Probability of A as a non negative real number, that can be assigned for all $A \in \Sigma$. $\exists\ P(A) \geq 0$ with the following properties:

$P(\omega)$=1

let $A, B \in \Sigma$, if $A \cap B = \varnothing \Longrightarrow P(A \cup B) = P(A) + P(B)$