

Exercise

- Let be X a random variable linearly distributed in the $[0, 1]$ interval, if Y is another random variable such that $y = \sqrt{x}$, find the probability distribution of Y .

Solution:

Since X is linearly distributed we know that:

$$P(X) = Ax$$

$$\int_0^1 P(x)dx = 1 \implies \left. \frac{Ax^2}{2} \right|_0^1 = 1 \implies A = 2$$

$$P(X) = 2x$$

In the section: Change of Variables, we proof that:

$$P(Y) = \frac{1}{y'(x)} \cdot P(x)|_{x=y^2}$$

We know that:

$$y'(x)|_{x=y^2} = \frac{1}{2\sqrt{x}} = \frac{1}{2y}$$

$$P(x)|_{x=y^2} = 2y^2$$

So we have:

$$P(Y) = 4y^3$$