Theorem 1. *let* $A \in \Sigma$ *so* $P(A) + P(A^c) = 1$, $\forall\ A \in \Sigma$ *where* A^c *is the logical negation of* A

Proof. Since ω is our sample space A^c is defined as ω / A we have:

$$P(A) + P(A^c) = P(A) + P(\omega / A) = P(\omega)$$

By the definition of probability $P(\omega) = 1$, so we have $P(A) + P(A^c) = 1$ ■

Theorem 2. *The probability of the empty set is 0,* $P(\varnothing) = 0$

Proof. Let $A \in \Sigma$, since $A \cup \varnothing = A$ we have:

$$P(A \cup \varnothing) = P(A) + P(\varnothing) = P(A)$$
$$P(\varnothing) = P(A) - P(A) = 0$$

■

Theorem 3. *Let* $A_1, A_2 \ldots A_N \in \Sigma$, *if* $A_i \cap A_j = \varnothing\ \forall i, j$ *1* $\leq$ *i,j* $\leq N$ *so we have* $P(\bigcup_{i=1}^{N} A_i) = \sum_{i=1}^{N} P(A_i)$

Proof. The equation is true for N=2, since this reduce to the case in the definition of probability.

Assume that $P(\bigcup_{i=1}^{N} A_i) = \sum_{i=1}^{N} P(A_i)$ is true for N. Since $A_{N+1} \cap \bigcup_{i=1}^{N} A_i = \varnothing$ we have:

$$P(\bigcup_{i=1}^{N} A_i) + P(A_{N+1}) = \sum_{i=1}^{N} P(A_i) + P(A_{N+1}) = \sum_{i=1}^{N+1} P(A_i)$$
$$P(\bigcup_{i=1}^{N} A_i) + P(A_{N+1}) = P([\bigcup_{i=1}^{N} A_i] \cup A_{N+1}) = P(\bigcup_{i=1}^{N+1} A_i) \text{ hence:}$$
$$P(\bigcup_{i=1}^{N+1} A_i) = \sum_{i=1}^{N+1} P(A_i)\ \forall N \in \mathbb{N}$$