Change of Variables

In this section we are going to show a technique to find distribution of interconnected variables. Let be Y and X random variables correlated by $Y = f(x)$ where f is invertible and monotonic. With the distribution of X is well known, we want to find the distribution of Y.

$$P(X \cap Y) = P(X) \cdot P(Y \mid X)$$

If X is given, then Y can only be $f(x)$ so $P(Y \mid X) = \delta(f(x) - y)$, hence:

$$P(Y) = \int_{-\infty}^{\infty} P(X \cap Y) \cdot dx = \int_{-\infty}^{\infty} P(X) \cdot \delta(f(x) - y) \cdot dx$$

Lets be $U = f(x) \implies \frac{dU}{dx} = f'(x)$, so we have:

$$P(Y) = \int_{-\infty}^{\infty} P(X) \cdot \delta(f(x) - y) \cdot dx = \int_{-\infty}^{\infty} P(X) \cdot \frac{1}{f'(x)} \cdot \delta(U - y) \cdot dU$$

Using the filtering property:

$$P(Y) = P(X) \cdot \frac{1}{f'(x)}\Big|_{x=f^{-1}(y)}$$

Where $P(X)$ is evaluated at $x = f^{-1}(y)$

This is a very important result, but notice that this expression is true only for monotonic functions. If you are dealing with non-monotonic functions, you can divide the function in monotonic intervals and then apply the Change of Variables technique.