Exercise

- Find the entropy a normal distribution with mean μ and deviation σ^2.

Solution:

$$H[P] = -\int_{-\infty}^{\infty} P(x)\log P(x)dx = -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} log[\ \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\ ]\ dx$$

$$H[P] = \frac{1}{2\sigma^2}\int_{-\infty}^{\infty} \frac{(x-\mu)^2}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx + \frac{log 2\pi\sigma^2}{2}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

The first integral is just the definition of deviation so is equal to σ^2 and the second integral is 1 by definition of normalization. So we have:

$$H[P] = \frac{1}{2}(1 + \log 2\pi\sigma^2)$$