Exercise

- Solve the difference equation $y_{n+1} = ay_n + b$ for initial value equal y_0.

Solution:

Lets now analyse the behavior of the equation for 1, 2 ,3 ... n:

$$y_1 = ay_0 + b$$
$$y_2 = a^2 y_o + ab + b$$
$$y_3 = a^3 y_0 + a^2 b + ab + b$$

$$y_n = a^n y_0 + \sum_{i=0}^{n-1} a^i b$$

Since the sum: $\sum_{i=0}^{n-1} a^i b$ is a geometrical progression sum, we know that:

$$\sum_{i=0}^{n-1} a^i b = \frac{1-a^n}{1-a} b$$

So the final solution is:

$$y_n = a^n y_0 + \frac{1-a^n}{1-a} b$$