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1.)a.)

$x < \epsilon_{mach}$

b.)

$$\begin{aligned}
\frac{1}{1-x} - \frac{1}{1+x} &= \frac{1+x}{(1-x)(1+x)} - \frac{1-x}{(1-x)(1+x)} \\
&= \frac{(1+x)-(1-x)}{(1-x)(1+x)} \\
&= \frac{2x}{(1-x^2)} = \frac{2x}{(1-x^2)} * \frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \frac{\frac{2}{x}}{\frac{1}{x}-x}
\end{aligned}$$

c.)

In this equation $\frac{1}{1-x} - \frac{1}{1+x}$ for values less than ϵ_{mach} each term contributes a value loss of ϵ_{mach} totaling $2\epsilon_{mach}$.
Where as in this equation $\frac{\frac{2}{x}}{\frac{1}{x}-x}$ only a single term equal to ϵ_{mach} is lost.

2.)

Given a singular matrix, by definition there exists at least one linear dependency. Meaning that through Gaussian elimination at least one of the rows will become all zeros. Given an all zero row, if the corresponding answer column is not equal to zero, no solution exists. If the answer column does contain a zero then when solving for x_{n-1} becomes a free variable. Therefore for $x_{n-1} = s$ for all s existing in R, there exists a unique solution.

3.)a.)

$$\begin{bmatrix} e & 1 & 1 & 1 \\ 0 & -8^{20} & -8^{20} & -8^{20} \\ 0 & -8^{20} & -8^{20} & -8^{20} \end{bmatrix}$$

b.)

The small value for e has created a fraudulent lenear dependence. Do to the fact its scale differed from the rest of the matrix by more than ϵ_{mach}

c.)

$$\begin{bmatrix} 4 & 5 & 6 & 3 \\ 0 & \frac{-1}{4} & \frac{3}{2} & \frac{5}{4} \\ 0 & -\frac{5*8^{20}}{4} & -\frac{3*8^{20}}{2} & -\frac{3*8^{20}}{4} \end{bmatrix}$$

4.)

$$(A - uv^T)^{-1} = A^{-1} + A^{-1}u(1 - v^T A^{-1}u)^{-1}v^T A^{-1}$$

$$(A - uv^T)^{-1}(A - uv^T) = (A^{-1} + A^{-1}u(1 - v^T A^{-1}u)^{-1}v^T A^{-1})(A - uv^T)$$

$$I = (A^{-1} + \frac{A^{-1}uv^T A^{-1}}{(1 - v^T A^{-1}u)})(A - uv^T)$$

$$I = A^{-1}A + A^{-1}uv^T A^{-1}A - \frac{A^{-1} * uv^T - A^{-1}uv^T A^{-1}uv^T}{(1 - v^T A^{-1}u)}$$

$$I = I + A^{-1}uv^T A^{-1}A - \frac{A^{-1}uv^T - A^{-1}uv^T A^{-1} * uv^T}{(1 - v^T A^{-1}u)}$$

$$I = I + A^{-1}uv^T - \frac{A^{-1}uv^T - A^{-1}u(v^T A^{-1}u)v^T}{(1 - v^T A^{-1}u)}$$

Since $(v^T A^{-1}u)$ is a scalar, it can commute

$$I = I + A^{-1}uv^T - \frac{A^{-1}uv^T - (v^T A^{-1}u)A^{-1}uv^T}{(1 - v^T A^{-1}u)}$$

$$I = I + A^{-1}uv^T - \frac{1 - (v^T A^{-1}u)}{(1 - v^T A^{-1}u)}A^{-1}uv^T$$

$$I = I + A^{-1}uv^T - A^{-1}uv^T$$

$$I = I$$

5.)a.)

Output

$$\begin{bmatrix} 5.0 & 4.0 & 3.0 & 8.0 & 9.0 \\ 1.0 & 2.0 & 3.0 & 4.0 & 5.0 \\ 2.0 & 1.0 & 2.0 & 1.0 & 2.0 \\ 2.0 & 1.0 & 1.0 & 1.0 & 2.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} 5.0 & 4.0 & 3.0 & 8.0 & 9.0 \\ 0.0 & 1.2 & 2.4 & 2.4 & 3.2 \\ 0.0 & \text{-}0.6 & 0.8 & \text{-}2.2 & -1.6 \\ 0.0 & \text{-}0.6 & -0.2 & \text{-}2.2 & -1.6 \\ 0.0 & 0.2 & 0.4 & \text{-}0.6 & -0.8 \end{bmatrix} \begin{bmatrix} 5.0 & 4.0 & 3.0 & 8.0 & 9.0 \\ 0.0 & 1.2 & 2.4 & 2.4 & 3.2 \\ 0.0 & 0.0 & 2.0 & \text{-}1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & \text{-}1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & \text{-}1.0 & -1.3 \end{bmatrix} \begin{bmatrix} 5.0 & 4.0 & 3.0 & 8.0 & 9.0 \\ 0.0 & 1.2 & 2.4 & 2.4 & 3.2 \\ 0.0 & 0.0 & 2.0 & \text{-}1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & \text{-}0.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & \text{-}1.0 & -1.3 \end{bmatrix}$$

$$\begin{bmatrix} 5.0 & 4.0 & 3.0 & 8.0 & 9.0 \\ 0.0 & 1.2 & 2.4 & 2.4 & 3.2 \\ 0.0 & 0.0 & 2.0 & \text{-}1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & \text{-}0.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -1.3 \end{bmatrix}$$

b.)

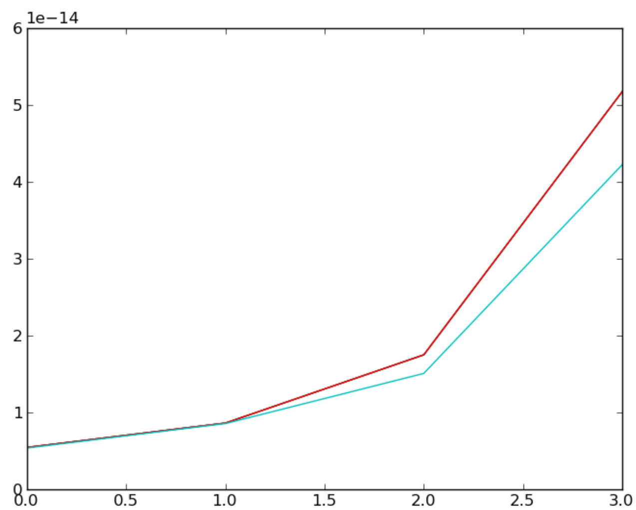

Figure 1: Accuacy and Residual- My implementation vs. Standard lib.

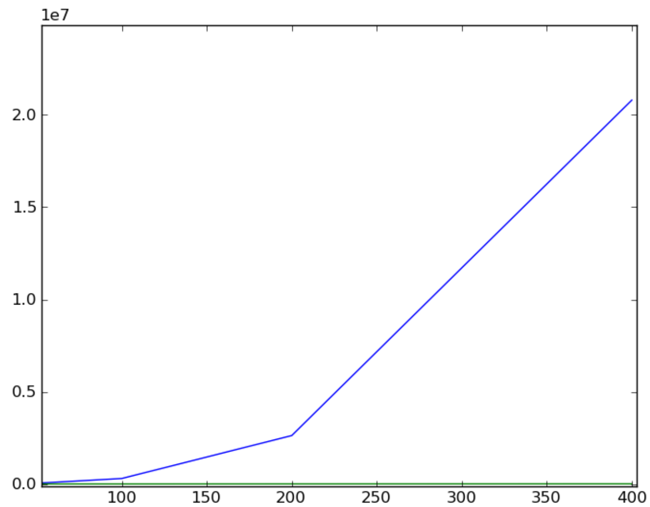

Figure 2: Computation Time: My implemtation vs. standard lib.

c.)

6.)a.)b.)c.)d.)

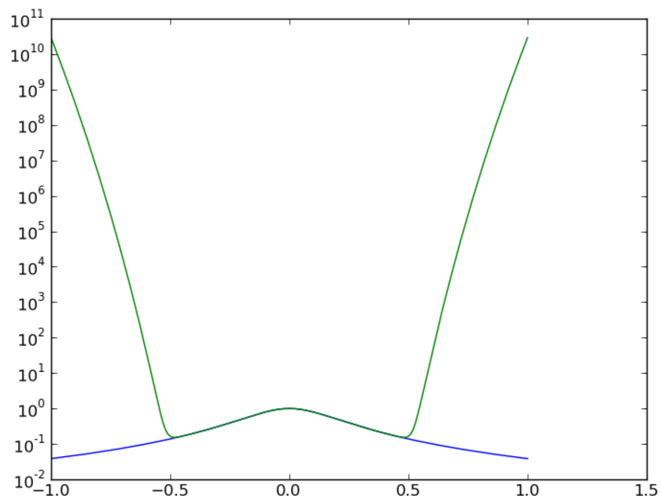

Figure 3: Uniformily Distributed Approximation

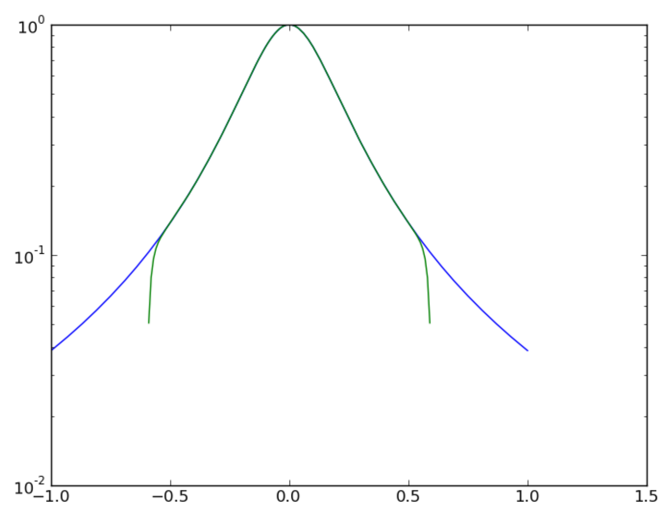

Figure 4: Chebyshev Distributed Approximation

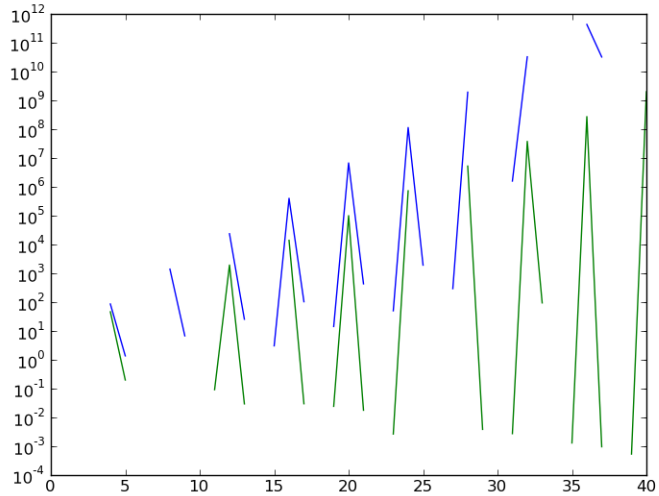

Figure 5: Maximum error- My function vs. Standard lib.

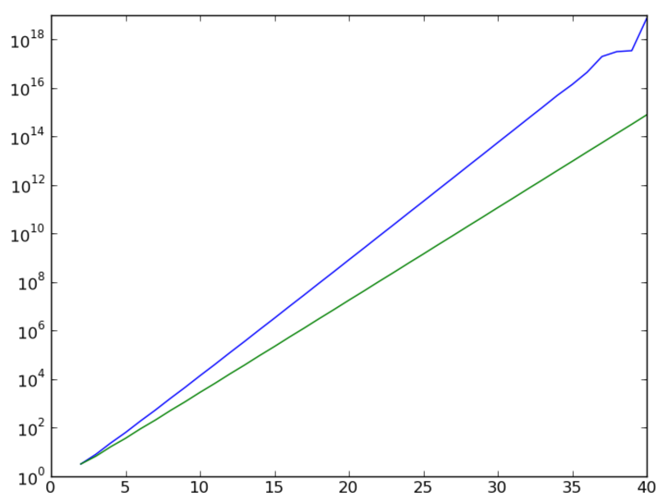

Figure 6: Condition Number- My function vs. Standard lib.