CS 374 Spring 2016
Home Work Number: 3
Problem number: 1
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Problem 1

$$h(w) \begin{cases} \varepsilon & if \;\; w = \varepsilon \\ 01 & if \;\; w = a \\ 10 & if \;\; w = b \\ h(a)h(u) & if \;\; w = au, where \;\; a\varepsilon\Sigma \;\; and \;\; u\varepsilon\Sigma^* \end{cases}$$

1.)

a.)

$$h^{-1}(\{0101\}) = \{aa\}$$
$$h^{-1}(\{00\}) = \{\}$$
$$h^{-1}(\{001\}) = \{\}$$
$$h^{-1}(\{1001\}) = \{ba\}$$

b.)

$$L = \mathbb{L}((00+1)^*)$$
$$h^{-1}(L) = \{ba\}^*$$
$$h(h^{-1}(L)) = \{1001\}^*$$

2.)

Definition DFA_M **&** DFA_N (I)

$$DFA_M = (Q, \Delta, \delta, s, A)$$
$$DFA_N = (Q', \Sigma, \delta', s', A')$$

where,

$$Q' = Q$$
$$s' = s$$
$$A' = A$$
$$\delta'(q, a) = \delta^*_M(q, h(a)$$

a.)

$$\forall w \in \Sigma^*, \;\; \delta^*_N(s', w) = \delta^*_M(s, h(w)) \qquad \text{(II)}$$

b.) Inductive proof of statment (I):

BaseCases:

$w = \varepsilon$

$$
\begin{aligned}
\delta_N^*(s', w) &= \delta_N^*(s', \varepsilon) \\
&= \delta_N^*(s, \varepsilon) && \text{by definition (I)} \\
&= \delta_M^*(s, h(\varepsilon)) && \text{by definition (I)} \\
&= \delta_M^*(s, h(w))
\end{aligned} \tag{1}
$$

$w = a,$ where $a \in \Sigma$

$$
\begin{aligned}
\delta_N^*(s', w) &= \delta_N^*(s', a) \\
&= \delta_N^*(s, a) && \text{by definition (I)} \\
&= \delta_M^*(s, h(a)) && \text{by definition (I)} \\
&= \delta_M^*(s, h(w))
\end{aligned} \tag{2}
$$

Suppose that for all w, where $|w| < n$ statement (I) holds.
Let w be a string such that $|w| = n$ and $w = au$, where $a \in \Sigma^{n-1}$ and $u \in \Sigma$
Then...

$$
\begin{aligned}
\delta_N^*(s', w) &= \delta_N^*(s', au) \\
&= \delta_N^*(\delta_N^*(s', a), u) && \text{by definition (I)} \\
&= \delta_N^*(\delta_M^*(s, h(a)), u) && \text{by the Inductive Hypothesis} \\
&= \delta_M^*(\delta_M^*(s, a), h(u)) && \text{by definition (I)} \\
&= \delta_M^*(s, h(au)) \\
&= \delta_M^*(s, h(w))
\end{aligned} \tag{3}
$$

c.)
Note that $L_M = L$

$$
\delta_N^*(s', w) = \delta_M^*(s, h(w)) \qquad \text{As proven by induction}
$$

Then...

$$
\begin{aligned}
&\Rightarrow L_M \supseteq h(L_N) \\
&\Rightarrow h^{-1}(L_M) = \{w \in \Sigma^* \mid h(w) \in L_M\} = L_N
\end{aligned} \tag{4}
$$