CS450: hw4

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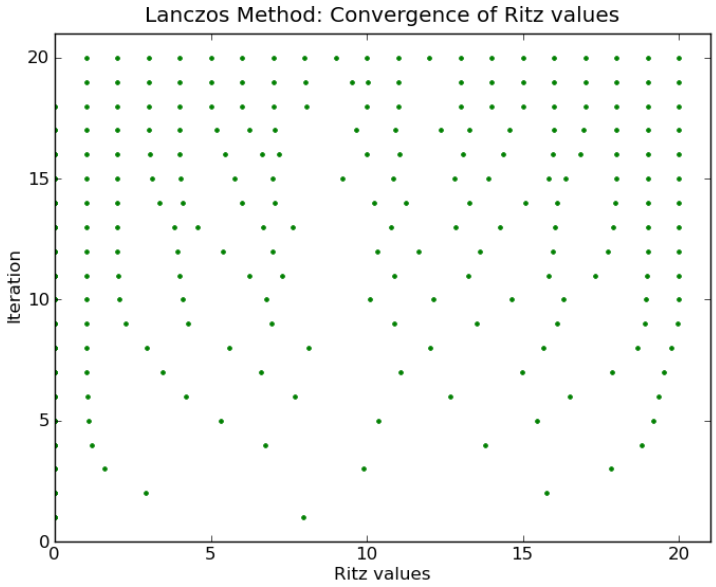
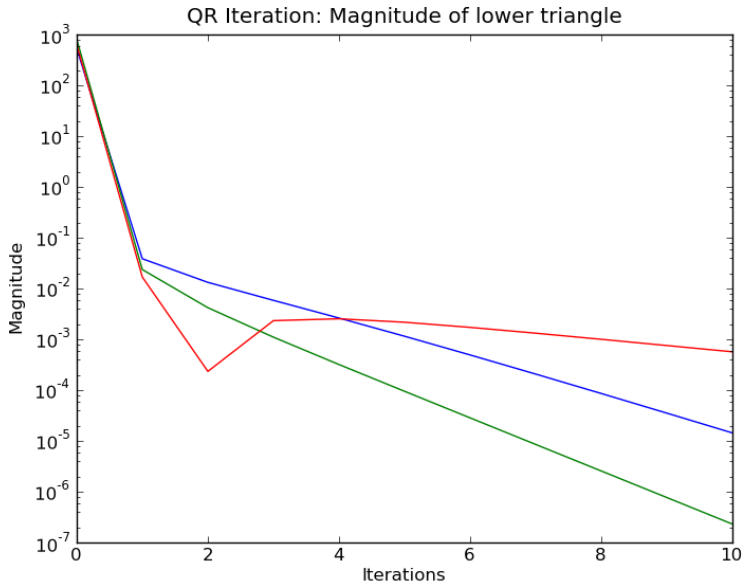
1.)

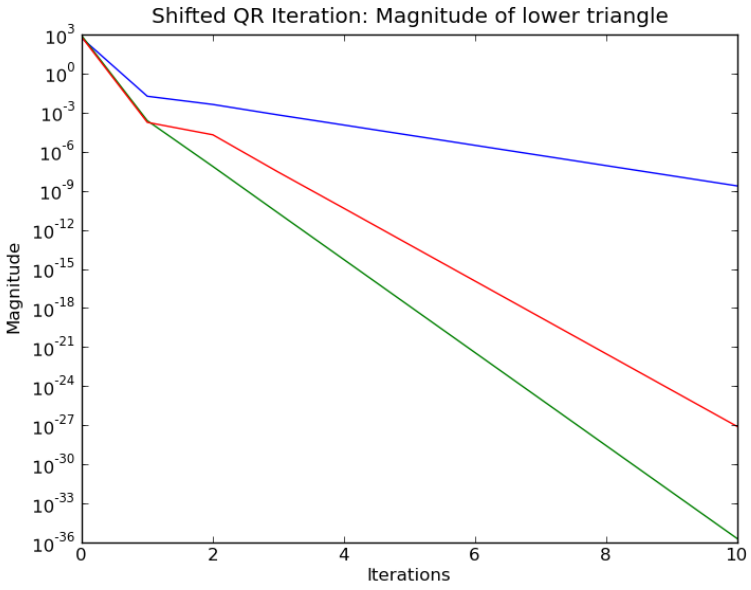


2.)

a.)


QR Iteration: Magnitude of lower triangle
Magnitude
Iterations
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
2
4
6
8
10

b.)


Shifted QR Iteration: Magnitude of lower triangle
Magnitude
Iterations
10^{3}
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
10^{-24}
10^{-27}
10^{-30}
10^{-33}
10^{-36}
0
2
4
6
8
10

c.)

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Error of non-shifted QR eigenvalues after 2 iterations is:
First:  0.00628408349861  Second:  0.0607178308072  Third:  0.0207289826417
Total Error:  0.0877308969475

Error of shifted QR eigenvalues after 2 iterations is:
First:  0.0100642325499  Second:  2.51639116211e-08  Third:  0.00301927983055
Total Error:  0.0130835375443

The Error for the non-shifted QR is about 6.7 times greater then the error
of the shifted QR.
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3.)

a.)

Function $g_1(x) = \frac{x^2+2}{3} = \frac{1}{3} * x^2 + \frac{2}{3}$

$$g_1'(x) = \frac{2}{3} * x$$

$$g_1'(2) = \frac{4}{3}$$

Function $g_2(x) = \sqrt{3x-2} = (3x-2)^{\frac{1}{2}}$

$$g_2'(x) = 3 * \frac{1}{2} * (3x-2)^{\frac{-1}{2}} = \frac{3}{2(3x-2)^{\frac{1}{2}}}$$

$$g_2'(2) = \frac{2}{2(6-2)^{\frac{1}{2}}} = \frac{3}{4} = 0.75$$

Function $g_3()x = 3 - \frac{2}{x}$

$$g_3'(x) = \frac{2}{x^2}$$

$$g_3'(2) = \frac{2}{4} = 0.5$$

Function $g_4(x) = \frac{x^2-2}{2x-3} = (x^2-2) * (2x-3)^{-1}$

$$g_4'(x) = (x^2-2) * 2 * -1 * (2x-3) - 2 + 2x(1)(2x-3)^{-1}$$

$$g_4'(x) = \frac{-2(x^2-2)}{(2x-3)^2} + \frac{2x}{2x-3}$$

$$g_4'(2) = \frac{-4}{1} + \frac{4}{1} = 0$$

b.)

	Function g1	Function g2	Function g3	Function g4
0	4.0000000000E+00	4.1143782777E-01	1.0000000000E-01	5.8823529412E-02
1	1.2000000000E+01	5.2403119310E-01	3.5714285714E-01	1.1724137931E-01
2	1.2933333333E+02	5.9654576299E-01	4.3750000000E-01	2.3130336868E-01
3	1.6556000000E+04	6.4348477843E-01	4.7058823529E-01	4.3911361099E-01
4	2.7407906267E+08	6.7465748929E-01	4.8571428571E-01	7.3626084401E-01
5	7.5119332227E+16	6.9591582951E-01	4.9295774648E-01	9.5489316960E-01
6	5.6429140742E+33	7.1074124724E-01	4.9650349650E-01	9.9893576370E-01
7	3.1842479249E+67	7.2126478595E-01	4.9825783972E-01	9.9492015302E-01
8	1.0139434847E+135	7.2883684806E-01	4.9913043478E-01	
9	INF	7.3434163775E-01	4.9956559513E-01	
10		7.3837476155E-01	4.9978289188E-01	
11		7.4134697292E-01	4.9989146950E-01	
12		7.4354697472E-01	4.9994574064E-01	
13		7.4518076365E-01	4.9997287179E-01	
14		7.4639706492E-01	4.9998643627E-01	
15		7.4730423888E-01	4.9999321822E-01	
16		7.4798179159E-01	4.9999660914E-01	
17		7.4848837140E-01	4.9999830458E-01	
18		7.4886741733E-01	4.9999915225E-01	
19		7.4915120280E-01	4.9999957619E-01	

g_1 is the only function with a $g'(2)$ greater than one and as expected g_1 is confirmed to diverge experimentally. g_1, g_2, and g_3 all converge to the predicted value of $g'(2)$. The quadratic convergence of g_4 causes some undefined behavior, most likely due to an inaccurate r value.

4.)

Tolerance: 1.000000E-10 Initial guess for x: 8.1 Initial guess for x_(-1): -8

	Bisection	Secant	Newtons
Function A:			
Iterations:	38	14	9
Final value:	2.094551	2.094551	2.094551
Error Convergence:	0.500000	6.903444E-01	0.562997
Function B:			
Iterations:	38	9	6
Final value:	0.567143	0.567143	0.567143
Error Convergence:	0.500000	3.293669E-01	0.186175

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Function C:
         Iterations:                38                10                 5
        Final value:         -6.439117          2.772605          9.317243
  Error Convergence:          0.499997      6.841148E-01          0.160413

Function D:
         Iterations:                38                47                36
        Final value:          1.000006          1.000006          1.000007
  Error Convergence:          0.500001      0.000000E+00          0.000000
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Bisection error converges to .5 as expected however secant and newtons errors have undefined behavior. This is due to the convergence rates being dependent on initial guesses proximity to solution.