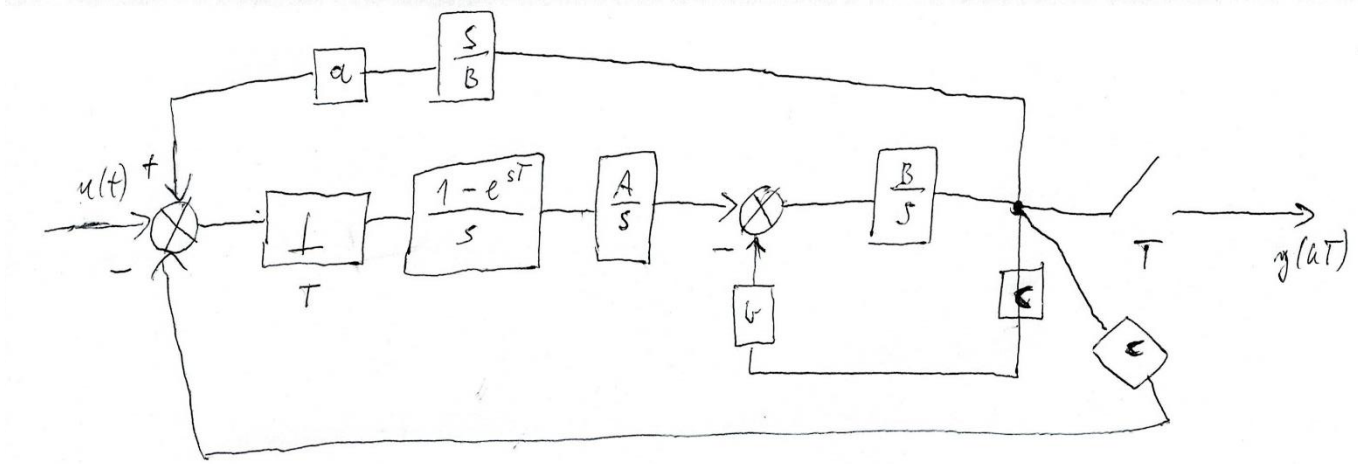
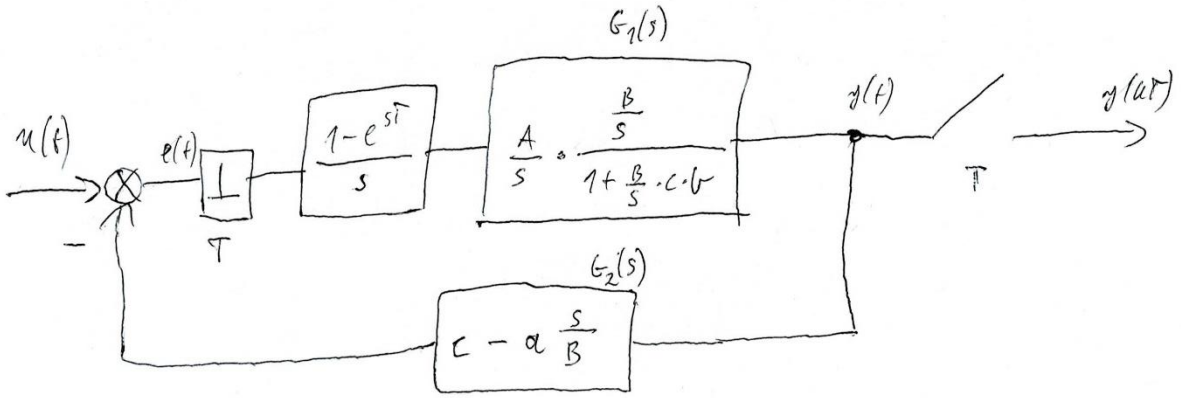
Zad. 1 – Wariant 1

1.





2.

$$G_1(s) = \frac{AB}{s^2 \cdot \left(1 + \frac{Bcb}{s}\right)} = \frac{2s}{s^3 + 3s^2}$$

$$G_2(s) = c - \frac{as}{B} = -2s + 1$$

3. Wzory z których policzyłem transmitancje:

3.

a) Transmitancja Toru Głównego

$$G_{TG}(s) = G_1(s) \Rightarrow H_{TG}(s) = \frac{G_1(s)}{s} \Rightarrow H_{TG}(z) = \mathcal{Z}\left\{\mathcal{L}^{-1}\left\{H_{TG}(s)\right\}\Big|_{t=kT}\right\}$$

$$G_{TG}(z) = \frac{z-1}{z} H_{TG}(z)$$

b) Transmitancja Układu Otwartego

$$G_0(s) = G_1(s)G_2(s) \Rightarrow H_0(s) = \frac{G_1(s)G_2(s)}{s} \Rightarrow H_0(z) = \mathcal{Z}\left\{\mathcal{L}^{-1}\left\{H_0(s)\right\}\Big|_{t=kT}\right\}$$

$$G_0(z) = \frac{z-1}{z} H_0(z)$$

c) Transmitancja Układu Zamkniętego

$$G(z) = \frac{G_{TG}(z)}{1 + G_0(z)}$$

3. cd Transmitancje obliczone przy pomocy Matlaba:

```
G_ToruGlownego =

  0.0090707 (z+0.9049)
  --------------------
    (z-1) (z-0.7408)


G0 =

   -0.3365 (z-1.051)
   -----------------
   (z-1) (z-0.7408)
```

```
G =

   0.0090707 (z+0.9049)
  ----------------------
  (z^2 - 2.077z + 1.095)
```

4. Czy układ $G(z)$ jest stabilny?

Mianownik $G(z)$:

$$M(z) = z^2 - 2{,}077z + 1{,}095$$

$$\left(z - (1{,}039 + i\,0{,}126)\right)\left(z - (1{,}039 - i\,0{,}126)\right) = 0$$

$$\Downarrow$$

$$|z_{1,2}| = 1{,}046 > 1,$$ czyli układ niestabilny

5.

$$h_{ust} = \lim_{z \to 1} \frac{z-1}{z} G(z) \frac{z}{z-1} = G(1) = 1$$

6.

$$e(t) = u(t) - y(t)$$

$$e_{ust} = 1 - h_{ust} = 1 - 1 = 0$$

7. Z Metody Bezpośredniej:

$$A = \begin{bmatrix} 0 & 1 \\ -1,095 & 2,077 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

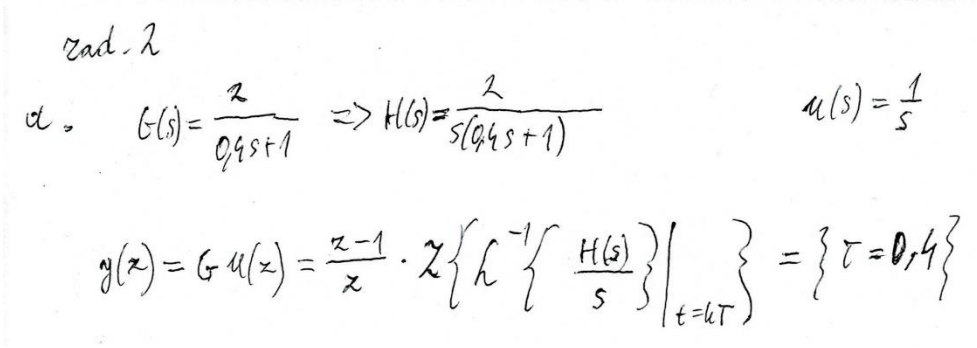
$$C = \begin{bmatrix} 0,008 & 0,009 \end{bmatrix}$$

$$D = [0]$$

$$x((k+1)T) = \begin{bmatrix} 0 & 1 \\ -1,095 & 2,077 \end{bmatrix} x(kT) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(kT)$$

$$y(kT) = \begin{bmatrix} 0,008 & 0,009 \end{bmatrix} x(kT)$$

Zad. 2 – Wariant 1



$$L^{-1}\{H(s)U(s)\}=L^{-1}\left\{\frac{2}{s^2(0.4s+1)}\right\}=2(t+0.4e^{-2.5t}-0.4)$$

$$y(z)=\frac{z-1}{z}Z\{2(kT+0.4e^{-2.5kT}-0.4)\}=\frac{z-1}{z}\left(\frac{2\mathrm{z}}{(\mathrm{z}-1)^2}+\frac{0.8\mathrm{z}}{\mathrm{z}-\mathrm{e}^{-2.5\mathrm{T}}}-\frac{0.8\mathrm{z}}{\mathrm{z}-1}\right)=$$
$$=\frac{z-1}{z}\left(\frac{2\mathrm{z}}{(\mathrm{z}-1)^2}+\frac{0.8\mathrm{z}}{\mathrm{z}-0.2}-\frac{0.8\mathrm{z}}{\mathrm{z}-1}\right)=\frac{1.36z+0.24}{(z-1)(z-0.2)}$$

$$y(kT)=Z^{-1}\{y(z)\}=\left(2-16\cdot 5^{-k-1}\right)\cdot u(kT)$$

Odpowiedź:

$$y(kT)=Z^{-1}\{y(z)\}=\left(2-16\cdot 5^{-k-1}\right)\cdot u(kT)$$

6.

$$G(s) = \frac{2}{0{,}4s+1} \Rightarrow H(s) = \frac{2}{s(0{,}4s+1)} \qquad u(z) = \frac{z}{z-1}$$

$$y(z) = \frac{z-1}{z} \mathcal{Z}\left\{ L^{-1}\left\{ H(s) \right\}\Big|_{t=kT} \right\} \cdot \frac{z}{z-1} = \left\{ T = 0.4 \right\} =$$

$$= \frac{2z}{z-1} - \frac{2z}{z - e^{-\frac{T}{\tau}}} = \left\{ e^{-\frac{T}{\tau}} = 0.2 \right\} = \frac{2z}{z-1} - \frac{2z}{z-0.2}$$

$$y(kT) = 2 - 2 \cdot (0.2)^k$$

Odpowiedź:

$$y(kT) = \left(2 - 2 \cdot 0.2^k\right) \cdot u(kT)$$