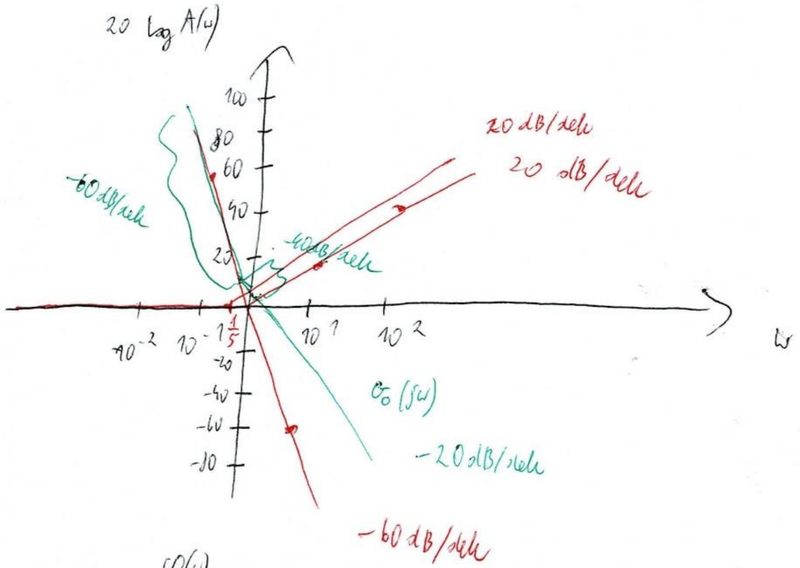
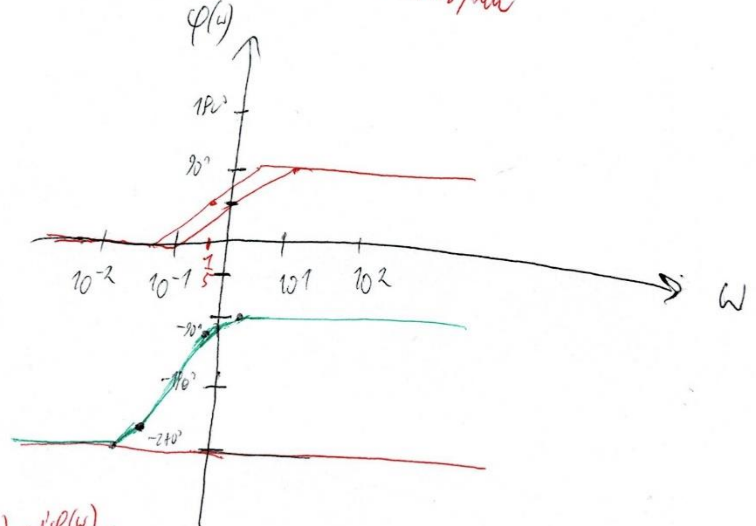
Zad. 6

1) $G_0(s) = \frac{5(s+1)(s+0,2)}{s^3} = \frac{1}{s^3}(s+1)\left(\frac{s}{\frac{1}{5}}+1\right)$





$G_0(j\omega) = A(\omega)e^{j\varphi(\omega)}$;

$$G_0(j\omega) = \frac{1}{\omega^3}\sqrt{(\omega^2+1)}\sqrt{(25\omega^2+1)}\; e^{j\left(-\frac{3}{2}\pi + \mathrm{arctg}\,\omega + \mathrm{arctg}\,5\omega\right)}$$

$G_0(j\omega) = P(\omega) + jQ(\omega)$:

$$G_0(j\omega) = \frac{(j\omega+1)(5j\omega+1)}{(j\omega)^3} = \frac{-5\omega^2+6j\omega+1}{-j\omega^3} = \frac{-6\omega + j(-5\omega^2+1)}{\omega^3}$$

$$P(\omega) = \frac{-6}{\omega^2}$$

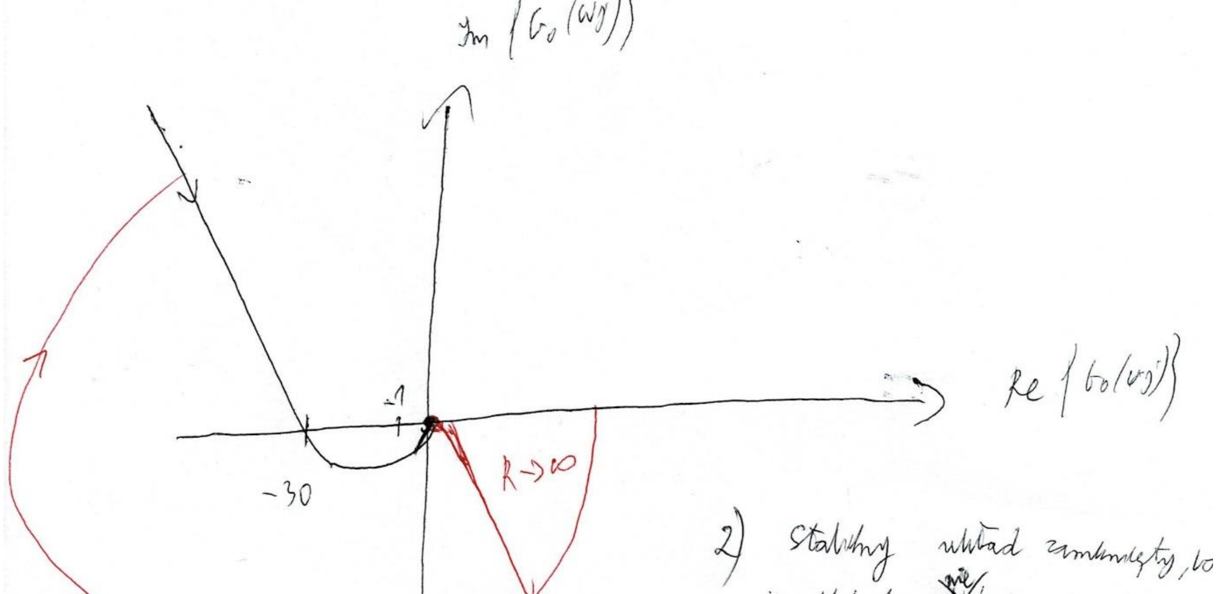
$$Q(\omega) = \frac{-5\omega^2+1}{\omega^3}$$

Wykres Nyquista

ω	0^+	$\frac{\sqrt{5}}{5}$	$\nearrow$	∞
$P(\omega)$	$-\infty$	-30	$-$	0
$Q(\omega)$	∞	0	$-$	0

Bieguny:

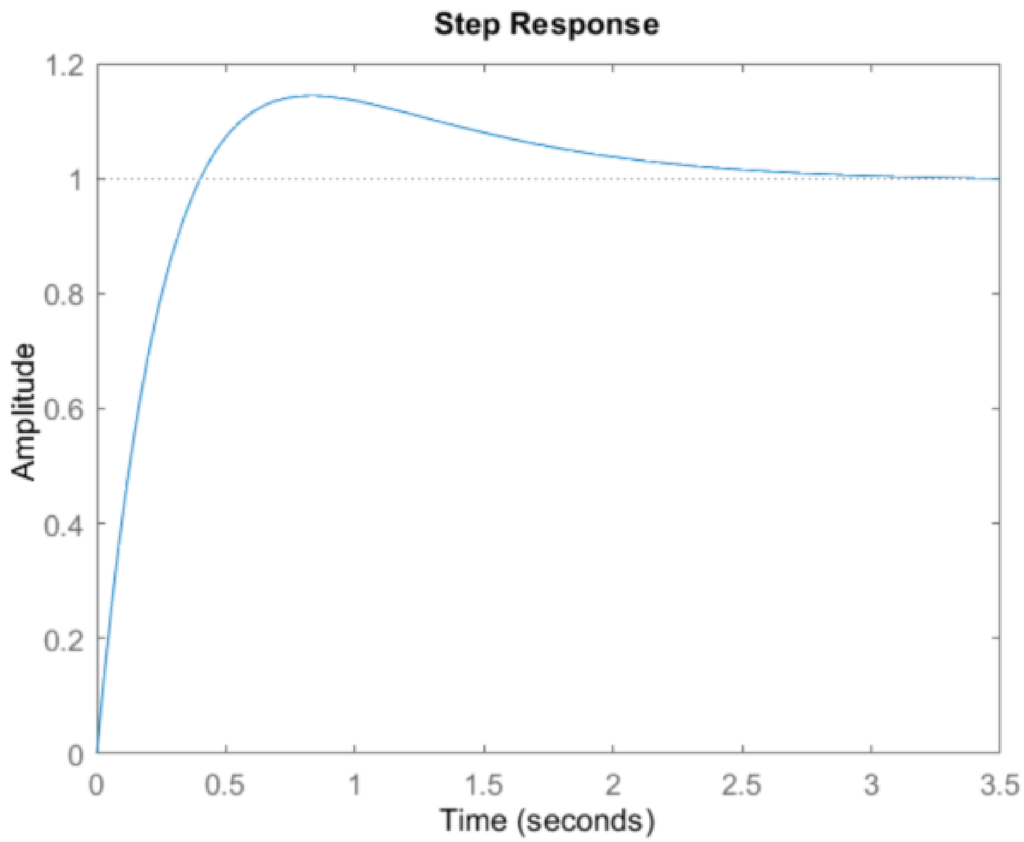
$s = 0$ (krytyczny)



2) Stabilny układ zamknięty, bo punkt krytyczny nie leży wewnątrz charakterystyki uzupełnionej o fragment okręgu

$$\underset{0<\omega<\infty}{\Delta \arg} \{G_0(j\omega)+1\} = 0^\circ$$

3


Step Response
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5
3
3.5

4) Zapas modułu i zapas fazy

Najpierw muszę znaleźć ω_0 i $\omega_{-\pi}$

ω_0:

$$A(\omega_0) = 1$$

$$\frac{\sqrt{25\omega_0^4 + 26\omega_0^2 + 1}}{\omega_0^3} = 1$$

⋮

$$\boxed{\omega_0 \approx 5{,}1 \ \frac{rad}{s}}$$

$\omega_{-\pi}$: $\boxed{\omega_{-\pi} = \frac{\sqrt{5}}{5} \ \frac{rad}{s}}$ (wiemy to z tabelki nad wykresem Nyquista)

$\Delta\varphi$:

$$\Delta\varphi = \pi + \arg\{G_0(j\omega_0)\} = \pi + \left(-\frac{3}{2}\pi + \operatorname{arctg} 5{,}1 + \operatorname{arctg} 25{,}5\right) = \ldots \text{ rad} = 76{,}7^\circ$$

ΔL:

$$\Delta L = 20\log\frac{1}{|G_0(j\omega_{-\pi})|} = 20\log\frac{1}{\frac{\sqrt{25\left(\frac{\sqrt{5}}{5}\right)^4 + 26\left(\frac{\sqrt{5}}{5}\right)^2 + 1}}{\left(\frac{\sqrt{5}}{5}\right)^3}} =$$

$$= -29{,}5 \text{ dB}$$

~~5) dla k: 0 < k < $\frac{1}{|G_0(j\omega_{-\pi})|}$ = 22 dB = ΔL układ nie zmieni stabilności~~

~~6) $\tau_{kr} = \frac{\Delta\varphi}{\omega_0} = \frac{5{,}1}{5{,}1} = 1$, czyli $\tau < \frac{\Delta\varphi}{\omega_0} = 1$~~

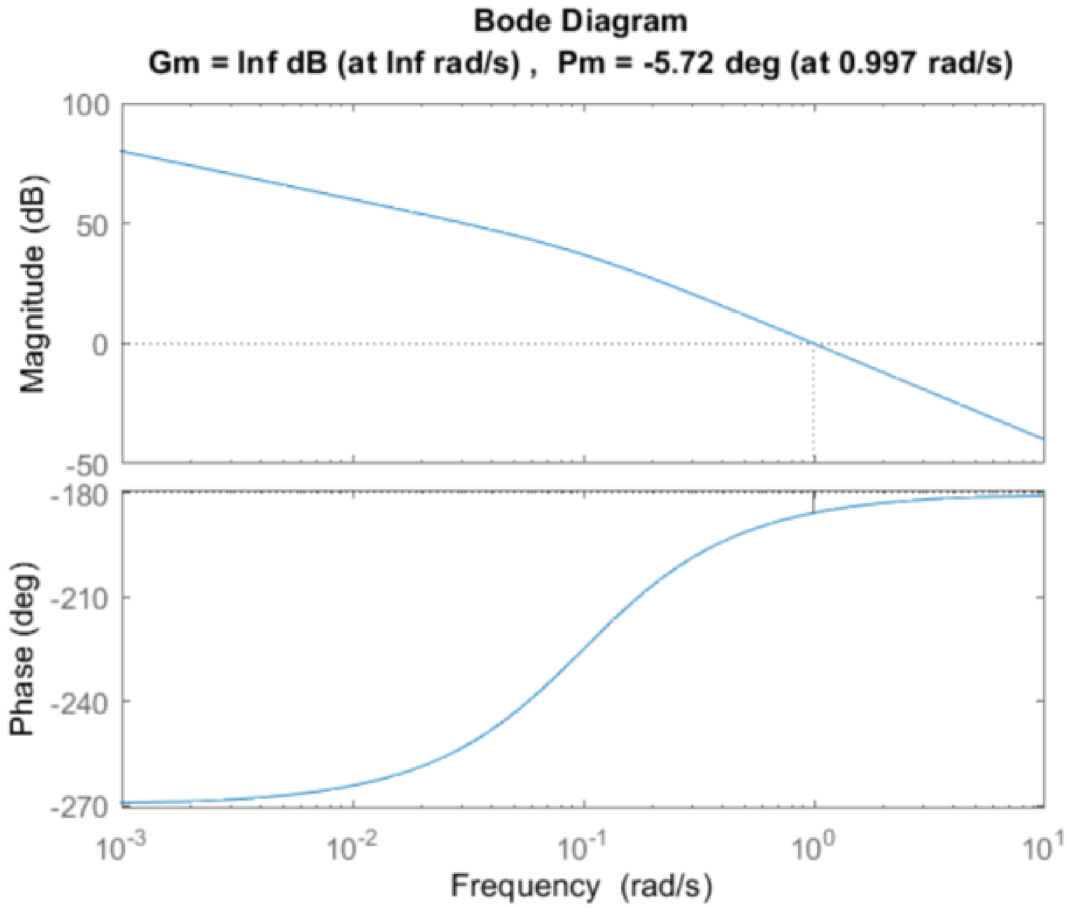
4

5) $$0 < k < \frac{1}{|G_0(j\omega_{-\pi})|} = 0{,}033$$

6) $$\tau < \tau_{kr} = \frac{\Delta\varphi}{\omega_0} = 0{,}26\ s$$

5

Zad. 13


Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = -5.72 deg (at 0.997 rad/s)
Magnitude (dB)
100
50
0
-50
Phase (deg)
-180
-210
-240
-270
10^-3
10^-2
10^-1
10^0
10^1
Frequency (rad/s)

$$G_0(s) = \frac{1}{s(s-0{,}1)}$$

$$G_0(j\omega) = \frac{1}{j\omega(j\omega - 0{,}1)} = \frac{1}{-\omega^2 - j\omega \cdot 0{,}1} = \frac{-1}{\omega^2 + 0{,}1 j\omega}$$

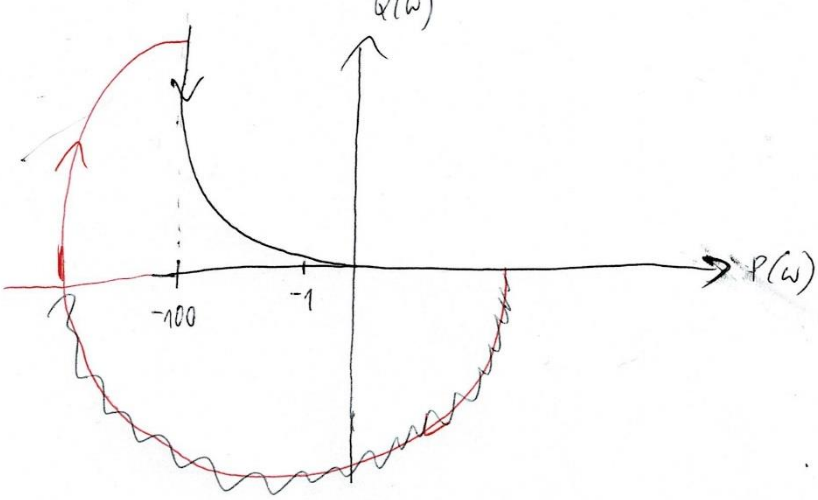
$$G_0(j\omega) = A(\omega)\, e^{j\varphi(\omega)} = \frac{1}{\omega} \cdot \frac{1}{\sqrt{\omega^2 + 0{,}01}}\, e^{j\left(-\frac{\pi}{2} - \operatorname{arctg}(-10\omega)\right)}$$

$$G_0(j\omega) = P(\omega) + Q(\omega) = \frac{-1(\omega^2 + j\omega\, 0{,}1)}{\omega^4 + 0{,}01\omega^2} = \begin{cases} P(\omega) = \dfrac{-\omega^2}{\omega^4 + 0{,}01\omega^2} = \dfrac{-1}{\omega^2 + 0{,}01} \\[2ex] Q(\omega) = \dfrac{+0{,}1\omega}{\omega^4 + 0{,}01\omega^2} = \dfrac{+0{,}1}{\omega^3 + 0{,}01\omega} \end{cases}$$

Bieguny: $s_1 = 0$ – zakładam, że należy do lewej półpłaszczyzny
$s_2 = 0{,}1$ – niestabilny

$\left.\begin{matrix} s_1 \\ s_2 \end{matrix}\right\} k = 1$

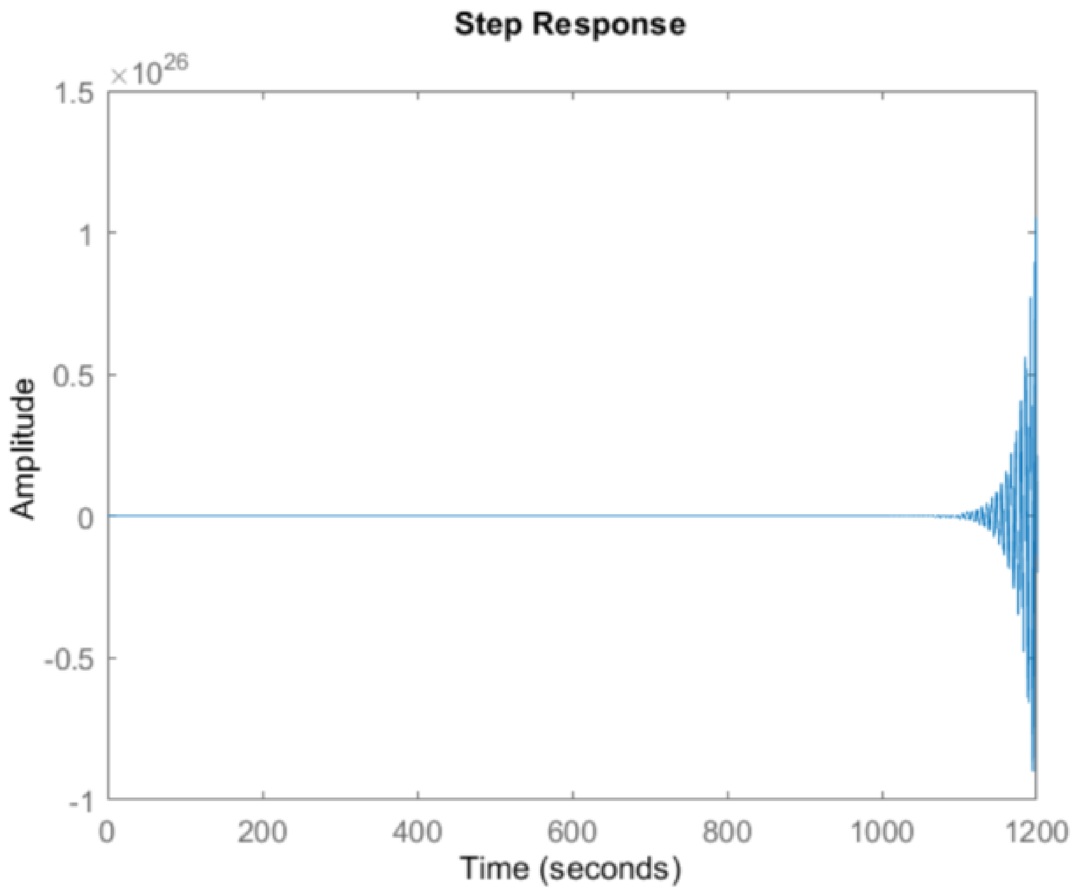
ω	0	∞
$P(\omega)$	-100	0
$Q(\omega)$	∞	0



$$\arg_{0<\omega<\infty}\left\{G_0(j\omega)+1\right\} = -\pi$$

Układ niestabilny

Byłby stabilny, gdyby $\arg_{0<\omega<\infty}\left\{G_0(j\omega)+1\right\} = \pi$



Step Response
×10^26
1.5
1
0.5
0
-0.5
-1
Amplitude
0
200
400
600
800
1000
1200
Time (seconds)

~~ω_0~~

ω_0:

$$A(\omega_0) = 1$$

$$\frac{1}{\omega_0\left(\omega_0^2 + 0{,}01\right)} = 1$$

$\vdots$

$$\omega_0 \approx 0{,}997 \ \frac{rad}{s}$$

$\omega_{-\pi}$:

$$\omega_{-\pi} = \infty$$

$$\Delta\varphi = \pi + \arg\left\{G_0(j\omega_0)\right\} = -5{,}72^\circ$$

$$\Delta L = 20\lg \frac{1}{\left|G_0(j\omega_{-\pi})\right|} = \infty \ dB$$

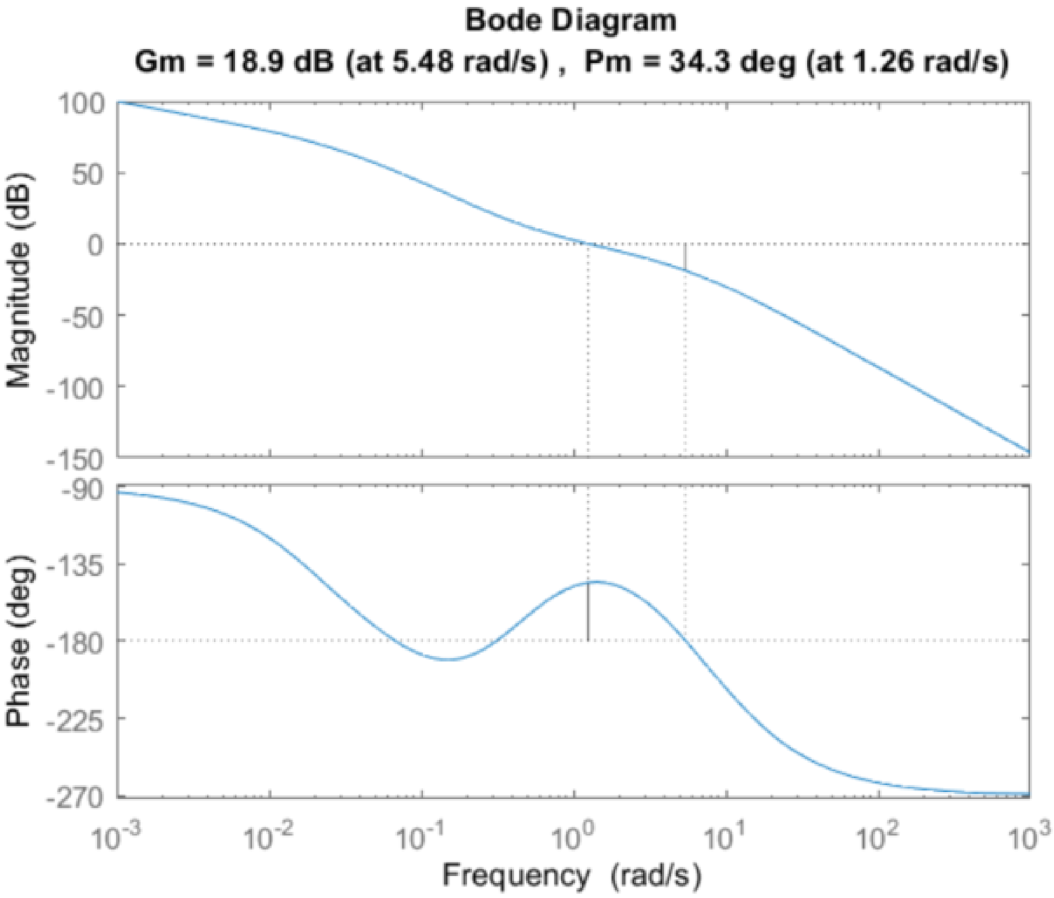
$$\tau < t_{kr} = \frac{\Delta\varphi}{\omega_0} = \frac{0{,}0998}{0{,}997} = 0{,}1001 \ s$$

Zad. 20

```
G0 =

       48 (s+0.5) (s+0.3333)
  -------------------------------
  s (s+10) (s+4) (s+0.1) (s+0.02)

Continuous-time zero/pole/gain model.
```

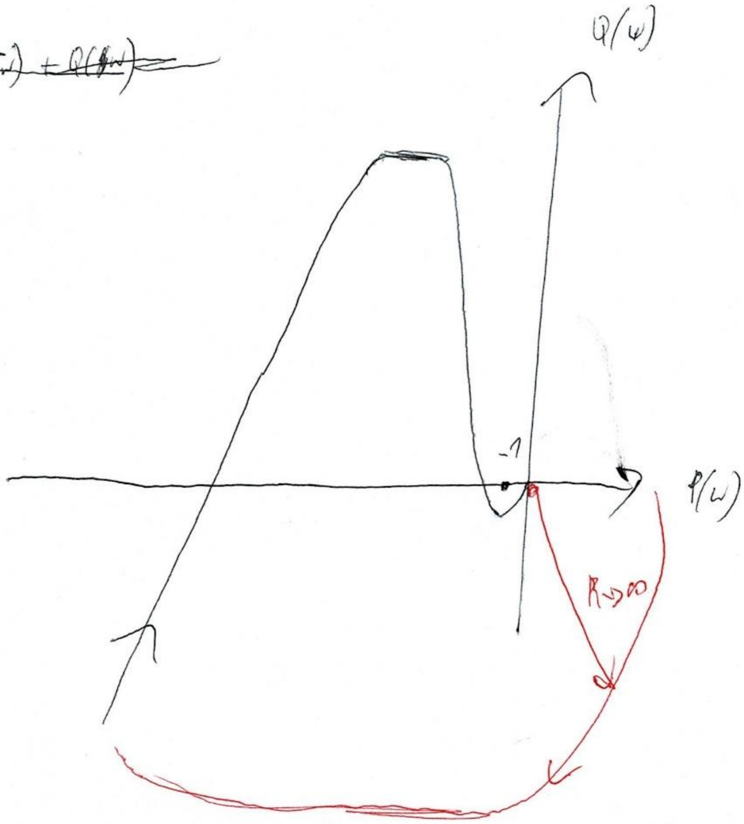


$$G_0(s) = \frac{48(s+0,5)(s+0,33)}{s(s+10)(s+4)(s+0,1)(s+0,02)} \; ;$$

$s=0$ - przyjmuję, że stabilny, zatem $k=0$

~~$G_0(j\omega) = P(j\omega) + Q(j\omega)$~~

~~$P(\omega)$~~ =



Układ jest stabilny, bo punkt krytyczny nie leży wewnątrz charakterystyki uzupełnionej o fragment półokręgu

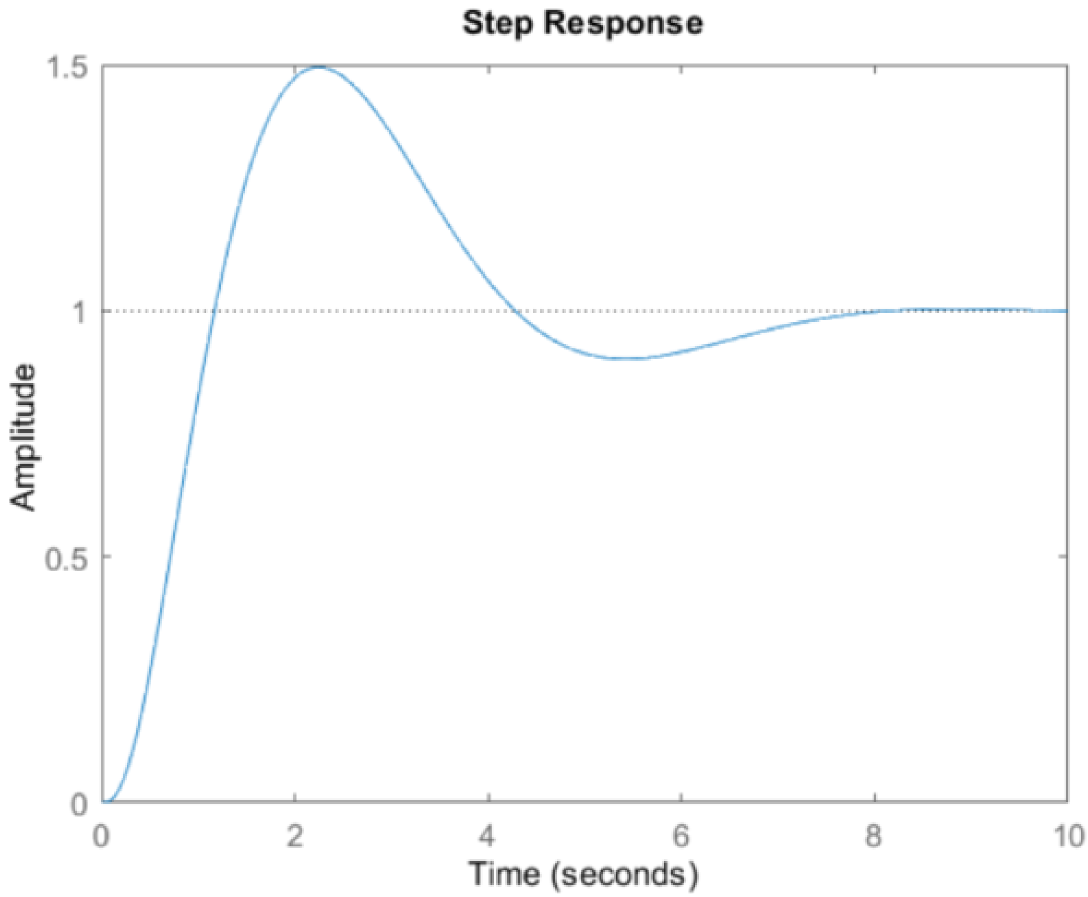
$$\Delta \arg_{0<\omega<\infty} \{G_0(j\omega)+1\} = 0^\circ = 0 \cdot \pi$$

$\omega_0 = 1,26 \frac{rad}{s}$; $\omega_{-\pi} = 5,48 \frac{rad}{s}$

$\Delta\phi = 34,3^\circ$; $\Delta L = 18,9\ dB$

$$0 < k < \frac{1}{|G_0(j\omega_{-\pi})|} = \ldots$$

$$T < T_k = \frac{\Delta\phi}{\omega_0} = \ldots$$


Step Response
Amplitude
Time (seconds)
1.5
1
0.5
0
0
2
4
6
8
10