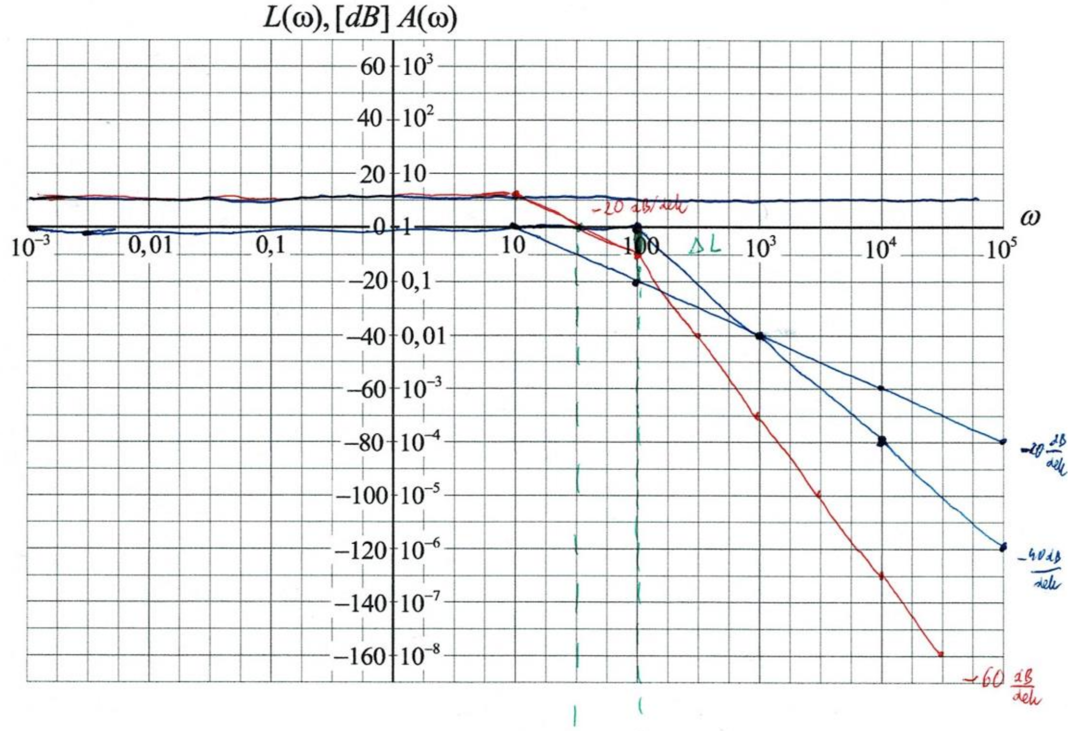
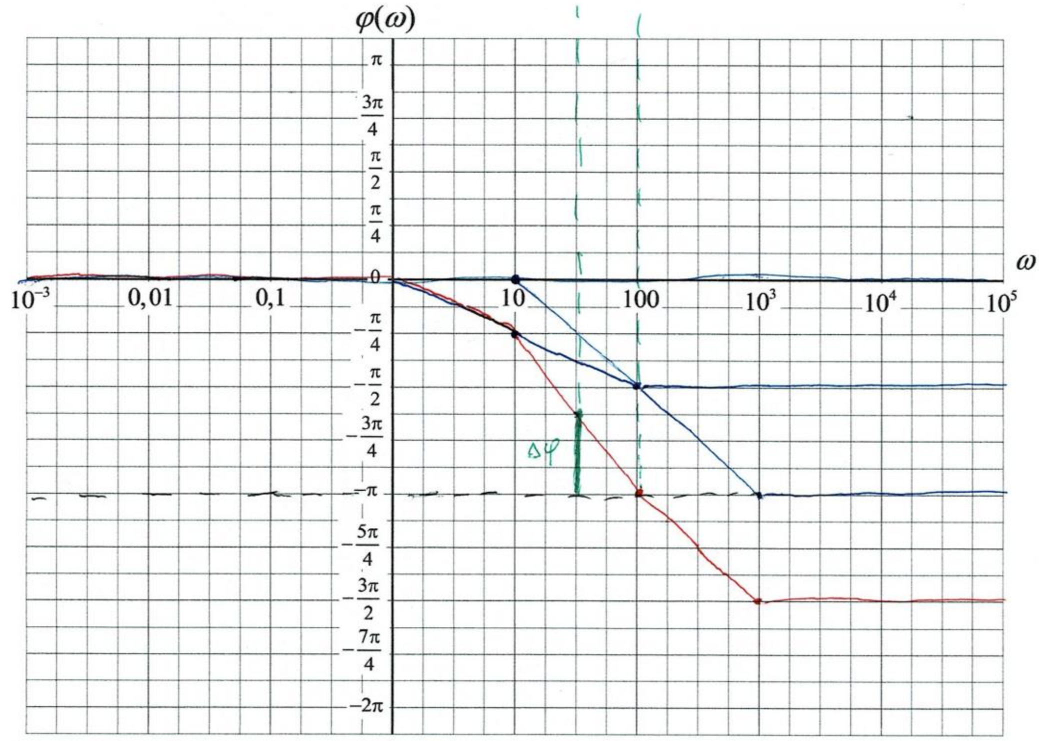
zad. 1 – wariant 8





1.

A.

a) $u(t) = \sin t$

$$y(t) = \sqrt{10}\,\sin(t + 0) = 3{,}16\,\sin t$$

b) $u(t) = \sin 10t$

$$y(t) = \sqrt{10}\,\sin\left(10t - \frac{\pi}{4}\right) = 3{,}16\,\sin(10t - 0{,}79)$$

B.

$$\omega_0 = \sqrt{10 \cdot 100} = 31{,}62\ \frac{rad}{s}$$

$$\omega_{-\pi} = 100\ \frac{rad}{s}$$

$$\Delta L = 0 - (-10) = 10\ dB$$

$$\Delta\varphi = -\frac{5}{8}\pi - (-\pi) = \frac{3}{8}\pi = 1{,}18\ rad = 67{,}5^\circ$$

C.

$$T_{osc} = \frac{2\pi}{\omega_{-\pi}} = \frac{2\pi}{100} = 0{,}063\ s$$

D.

$$T_0 = \frac{\frac{1}{2}\Delta\varphi}{\omega_0} = \frac{\frac{1}{2}\cdot 1{,}18}{31{,}62} = 0{,}019\,s$$

E.

$$20\log k = -\Delta L$$

$$k = 10^{\frac{-\Delta L}{20}} = 10^{-\frac{10}{20}} = 0{,}32$$

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2. A.

a) ~~$u(t) = \sin(10t)$~~ $u(t) = \sin t$

$$G_0(j\omega) = A(\omega)\, e^{j\varphi(\omega)}$$

$$A(\omega) = \sqrt{10} \cdot \frac{1}{\left(\sqrt{0.01\omega^2+1}\right)\left(\sqrt{0.0001\omega^2+1}\right)^2}$$

$$\varphi(\omega) = -\operatorname{arctg}\frac{\omega}{10} - 2\operatorname{arctg}\frac{\omega}{100}$$

~~$A(10)$~~

$$A(1) = \sqrt{10} \cdot \frac{1}{\left(\sqrt{0.01+1}\right)\left(0.0001+1\right)} = 3,15$$

$$\varphi(1) = \cancel{0,12} - \operatorname{arctg}\frac{1}{10} - 2\operatorname{arctg}\frac{1}{100} = -0,12 \text{ rad}$$

$$y(t) = 3,15 \sin\left(t - 0,12\right)$$

b) $u(t) = \sin 10t$

$$A(10) = \sqrt{10} \cdot \frac{1}{\left(\sqrt{0.01 \cdot 10^2+1}\right)\left(0.0001 \cdot 10^2+1\right)} = \cancel{3,01} = 2,21$$

$$\varphi(10) = -\operatorname{arctg}\frac{10}{10} - 2\operatorname{arctg}\frac{10}{100} = -0,98 \text{ rad}$$

~~$y(t) = 3,01 \sin(10t - 0,98)$~~

$$y(t) = 2,21 \sin\left(10t - 0,98\right)$$

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B. $\omega_0 \in (0,\infty)$

ω_0:

$$\left|G_0(j\omega_0)\right| = 1$$

$$\sqrt{10}\,\frac{1}{\left(\sqrt{0{,}01\omega_0^2+1}\right)\left(\sqrt{0{,}0001\,\omega_0^2+1}\right)^2} = 1$$

$$10 = \left(0{,}01\,\omega_0^2+1\right)\left(0{,}0001\,\omega_0^2+1\right)^2$$

$$10^{-10}\omega_0^6 + 2{,}01\cdot 10^{-6}\,\omega_0^4 + 0{,}0102\,\omega_0^2 - 9 = 0$$

$$\underset{\omega_0 \notin D}{\omega_0 = -27{,}63\ \tfrac{rad}{s}} \quad \vee \quad \underset{\omega_0 \in D}{\underbrace{\omega_0 = 27{,}63}}\ \tfrac{rad}{s}$$

$\omega_{-\pi} \in (0,\infty)$

$\omega_{-\pi}$:

$$\arg\left\{G_0(j\omega_{-\pi})\right\} = -\pi$$

$$-\operatorname{arctg}\left(\frac{\omega_{-\pi}}{10}\right) - 2\operatorname{arctg}\left(\frac{\omega_{-\pi}}{100}\right) = -\pi$$

$$\omega_{-\pi} = 109{,}54\ \frac{rad}{s}$$

$$A(\omega_{-\pi}) = 0{,}13$$

ΔL:

$$\Delta L = 20\lg\frac{1}{A(\omega_{-\pi})} = 20\lg\frac{1}{0{,}13} = 17{,}68\ dB$$

$\Delta\varphi$:

$$\Delta\varphi = \pi + \arg\left\{G_0(j\omega_0)\right\} = \pi - 1{,}76 = 1{,}38\ rad = 79^\circ$$

C. $$T_{osc} = \frac{2\pi}{\omega_{-\pi}} = \frac{2\pi}{109{,}54} = 0{,}057\ s$$

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D.

$$T_0 = \frac{\frac{1}{2}\Delta\varphi}{\omega_0} = \frac{\frac{1}{2}\cdot 1{,}38}{27{,}63} = 0{,}025\,s$$

E.

$$20\lg k = -\Delta L$$

$$k = 10^{\frac{-\Delta L}{20}} = 10^{\frac{-17{,}68}{20}} = 0{,}13$$

	Wartości dokładne	wartości na podstawie charakterystyk
A.		
a) $y(t)$; $A=1$; $\omega=1$	$3{,}15 \sin(t-0{,}12)$	$3{,}16 \sin t$
b) $y(t)$; $A=1$; $\omega=10$	$2{,}21 \sin(10t-0{,}98)$	$3{,}16 \sin(10t-0{,}79)$
B.		
ω_0	$27{,}63 \frac{rad}{s}$	$31{,}67 \frac{rad}{s}$
$\omega_{-\pi}$	$109{,}54 \frac{rad}{s}$	$100 \frac{rad}{s}$
ΔL	$17{,}68$ dB	10 dB
$\Delta\varphi$	$79°$	$67{,}5°$
C.		
$T_{osc.}$	$0{,}057$ s	$0{,}063$ s
D.		
T_0	$0{,}025$ s	$0{,}019$ s
E.		
k	~~3,85~~ 0,13	~~4,58~~ 0,32

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3. $G(s) = \frac{G_0(s)}{1+G_0(s)} = \frac{3{,}16 \cdot 10^5}{(s+147{,}9)(s^2+62{,}11s+2814)}$

$$K = G(0) = \frac{3{,}16 \cdot 10^5}{147{,}9 \cdot 2814} = 0{,}76$$

4. $u(t) = 1(t)$

$$e_{ust} = \frac{1}{1+G_0(0)} = \frac{1}{1+\sqrt{10}} = 0{,}24$$

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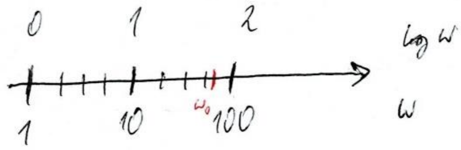
5) $C(s) = k_p$, więc charakterystyka fazowa nie ulega zmianie, zatem z wykresu:

$$\Delta\varphi = \frac{\pi}{8} \text{ dla } \omega_0 = 10^{\frac{11}{6}} = 68{,}13 \ \frac{rad}{s}$$

~~z wykresu~~: skąd się wzięło $\omega_0 = 10^{\frac{11}{6}}$?



$$\log \omega_0 = 1\tfrac{3}{4} + \tfrac{1}{3} \cdot \tfrac{1}{4} = 1\tfrac{10}{12} = 1\tfrac{5}{6} = \tfrac{11}{6} \iff \omega_0 = 10^{\frac{11}{6}}$$

$k_p = ?$

Trzeba podnieść charakterystykę modułową o $20 \log k_p$ dB tak żeby $\omega_0 = 10^{\frac{11}{6}}$

$$20 \log k_p = \frac{2 \cdot 10}{3} \iff k_p = 10^{\frac{2 \cdot 10}{3 \cdot 20}} = 2{,}15$$

$$\Delta L = 10 - \frac{2}{3} \cdot 10 = 3{,}33 \text{ dB}$$

5)

$$C(s) = k_p \quad , \quad G_0(s) = C(s)\,P(s) = \frac{k_p\sqrt{10}}{(0{,}1s+1)(0{,}01s+1)^2}$$

$$\Delta\varphi = \pi + \arg\{G_0(j\omega_0)\} = \frac{\pi}{8}$$

$$\arg\{G_0(j\omega_0)\} = -\frac{7}{8}\pi$$

$$-\operatorname{arctg}\frac{\omega_0}{10} - 2\operatorname{arctg}\frac{\omega_0}{100} = -\frac{7}{8}\pi$$

$$\omega_0 = 76{,}64\ \frac{rad}{s}$$

$$|G_0(j\omega_0)| = 1$$

$$k_p\sqrt{10}\cdot\frac{1}{\left(\sqrt{0{,}01\omega_0^2+1}\right)\left(\sqrt{0{,}0001\omega_0^2+1}\right)^2} = 1$$

$$k_p = \frac{\sqrt{0{,}01\cdot 76{,}64^2+1}\cdot\left(\sqrt{0{,}0001\cdot 76{,}64^2+1}\right)^2}{\sqrt{10}} = 3{,}88$$

$$\Delta L = 20\log\frac{1}{|A(\omega_{-\pi})|\,k_p} = 20\lg\frac{1}{0{,}13\cdot 3{,}88} = 5{,}94\ dB$$

zad. 2 wariant 0

1. $G_0(s) = \sqrt{10} \cdot \frac{1}{s}(s+1)\left(\frac{1}{\frac{s}{\sqrt{10}}+1}\right)\left(\frac{1}{\frac{s}{10}+1}\right)^2 =$

$$= \frac{\sqrt{10}\,(s+1)}{s\left(\frac{s}{\sqrt{10}}+1\right)\left(\frac{s}{10}+1\right)^2}$$

2.

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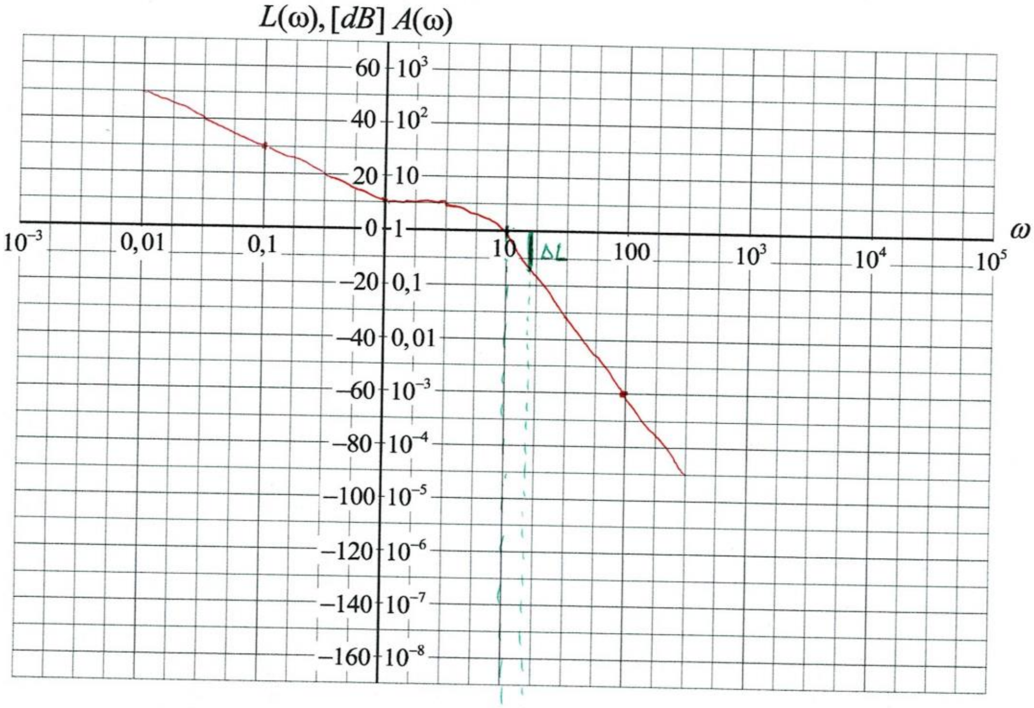
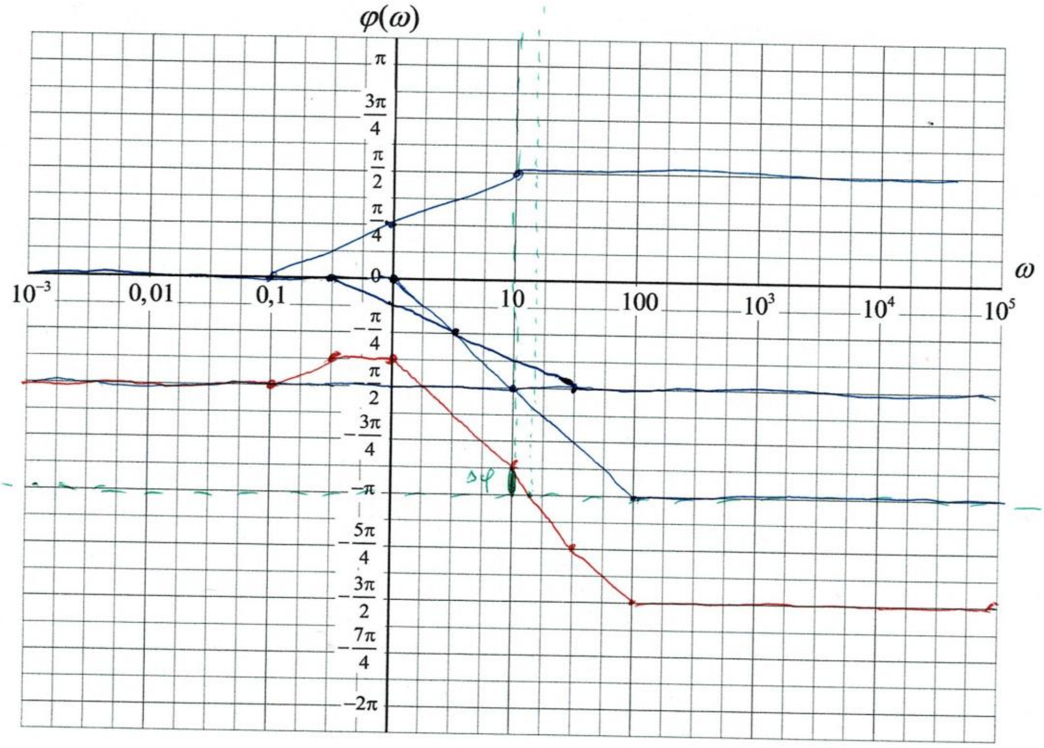
2.



L(ω), [dB] A(ω)
60
40
20
0
−20
−40
−60
−80
−100
−120
−140
−160
10^3
10^2
10
1
0,1
0,01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
ω
10^{-3}
0,01
0,1
10
100
10^3
10^4
10^5
ΔL



φ(ω)
π
3π/4
π/2
π/4
0
−π/4
−π/2
−3π/4
−π
−5π/4
−3π/2
−7π/4
−2π
ω
10^{-3}
0,01
0,1
10
100
10^3
10^4
10^5
Δφ

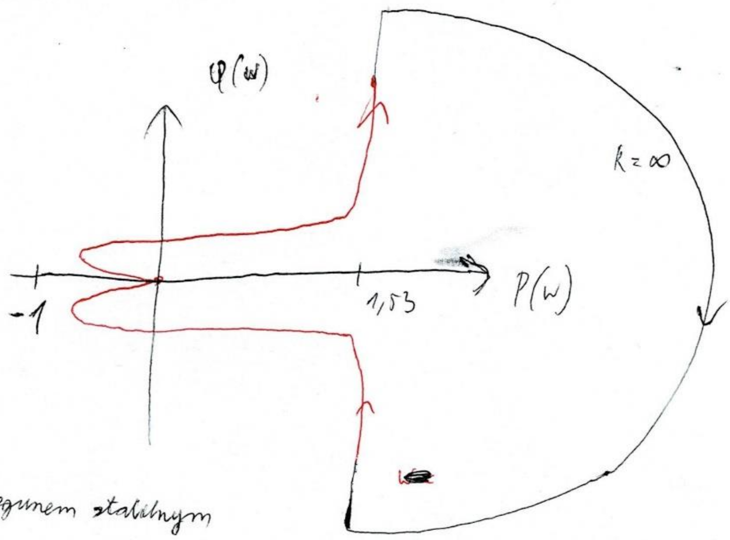
3.

$$G_0(j\omega) = P(\omega) + jQ(\omega)$$

$$P(\omega) = \frac{-22162(\omega-2{,}63)(\omega+2{,}63)}{(\omega^2+10)(\omega^2+100)^2}$$

$$Q(\omega) = \frac{1000(\omega+11{,}93)(\omega-11{,}93)(\omega^2+2{,}22)}{\omega(\omega^2+10)(\omega^2+100)^2}$$

ω	0^+	...	2,627	...	∞
$P(\omega)$	1,53	−	0	+	0
$Q(\omega)$	$-\infty$	−	−2,43	−	0



założyłem, że $s=0$ jest biegunem stabilnym

Układ jest stabilny, ponieważ punkt $(-1, 0j)$ nie leży wewnątrz charakterystyki

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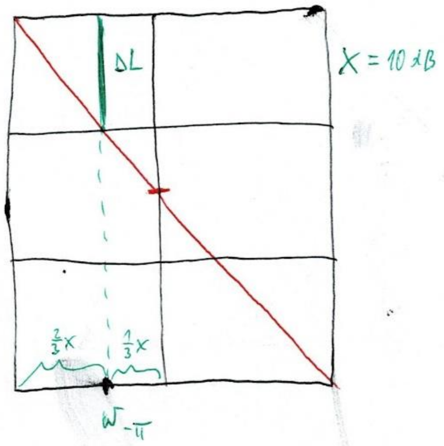
z charakterystyk asymptotycznych:

- $\omega_0 = 10 \frac{rad}{s}$
- $\Delta\varphi = \frac{1}{8}\pi = 22{,}5^{\circ}$
- $\omega_{-\pi} = 10^{1+\frac{2}{3}\cdot\frac{1}{4}} = 14{,}68 \frac{rad}{s}$
- $\Delta L =$



$= \Delta L = 10\, dB$

$$\tau < \tau_{kr} = \frac{\Delta\varphi}{\omega_0} = \frac{\frac{1}{8}\pi}{10} = 0{,}04\, s$$

$$0 < k < 10^{\frac{\Delta L}{20}} = 10^{\frac{10}{20}} = \sqrt{10}$$

Wartości dokładne:

$$G_0(j\omega) = A(\omega)\, e^{j\varphi(\omega)}$$

$$A(\omega) = \frac{\sqrt{10}\,\sqrt{\omega^2+1}}{\omega\sqrt{\frac{\omega^2}{10}+1}\cdot\left(\frac{\omega^2}{100}+1\right)}$$

$$\varphi(\omega) = \operatorname{arctg}\omega - \frac{\pi}{2} - \operatorname{arctg}\frac{\sqrt{10}}{10}\omega - 2\operatorname{arctg}\frac{\omega}{10}$$

$\omega_{-\pi}$:
$$\arg\{G_0(j\omega_{-\pi})\} = -\pi$$
$$\omega_{-\pi} = 11{,}93\ \frac{rad}{s}$$

ω_0:
$$A(\omega_0) = 1$$
$$\omega_0 = 6{,}43\ \frac{rad}{s}$$

$$\Delta L = 20\lg\frac{1}{A(\omega_{-\pi})} = 9{,}49\,dB$$

$$\Delta\varphi = \pi + \arg\left(G_0(j\omega_0)\right) = 0{,}73\ rad = 41{,}83^\circ$$

$$k < 10^{\frac{\Delta L}{20}} = 2{,}98$$

$$T_0 < \frac{\Delta\varphi}{\omega_0} = 0{,}11\ s$$