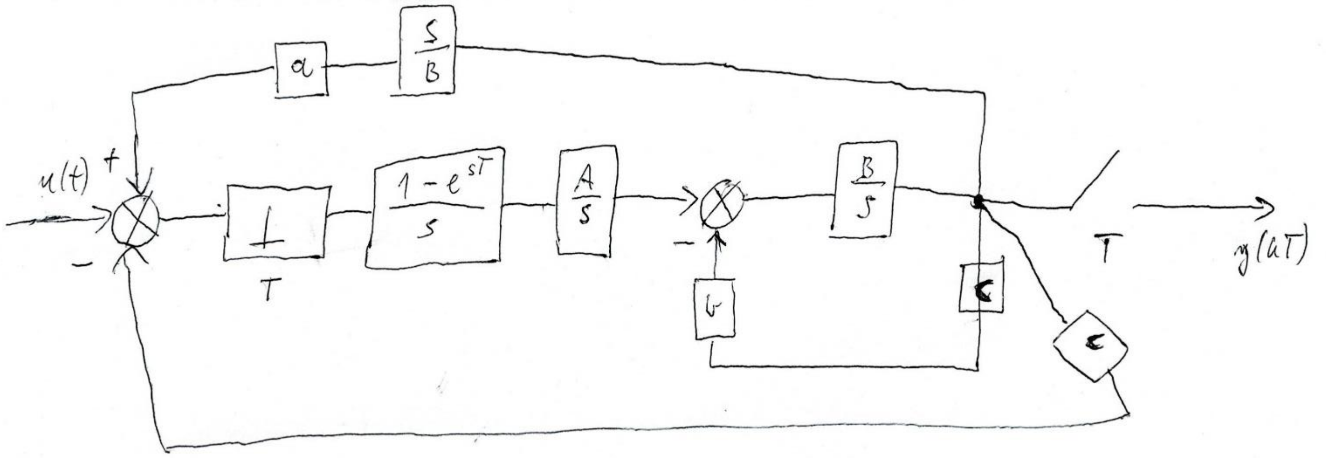
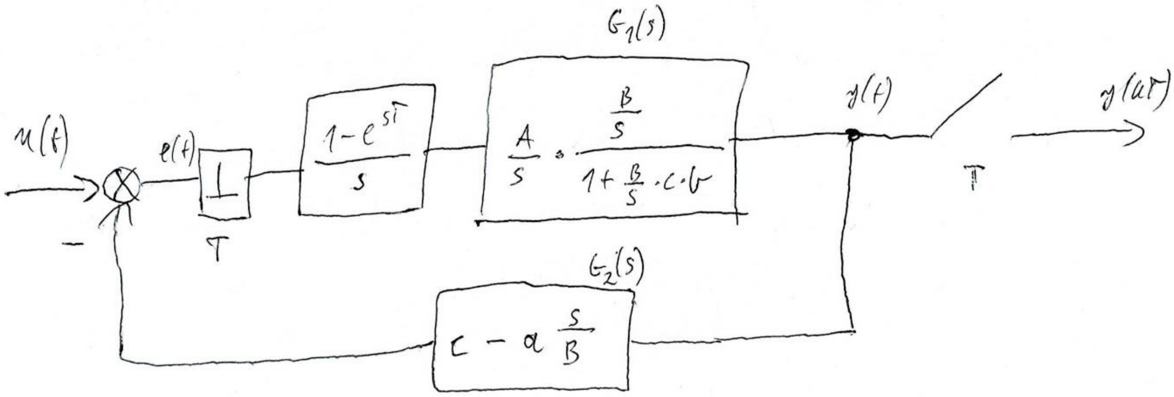
Zad. 1 – Wariant 1

1.





2.

$$G_1(s) = \frac{AB}{s^2 \cdot \left(1 + \frac{Bcb}{s}\right)} = \frac{2s}{s^3 + 3s^2}$$

$$G_2(s) = c - \frac{as}{B} = -2s + 1$$

3. Wzory z których policzyłem transmitancje:

3.

a) Transmitancja Toru Głównego

$$G_{TG}(s) = G_1(s) \Rightarrow H_{TG}(s) = \frac{G_1(s)}{s} \Rightarrow H_{TG}(z) = \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ H_{TG}(s) \right\} \Big|_{t=kT} \right\}$$

$$G_{TG}(z) = \frac{z-1}{z} H_{TG}(z)$$

b) Transmitancja Układu Otwartego

$$G_0(s) = G_1(s) G_2(s) \Rightarrow H_0(s) = \frac{G_1(s) G_2(s)}{s} \Rightarrow H_0(z) = \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ H_0(s) \right\} \Big|_{t=kT} \right\}$$

$$G_0(z) = \frac{z-1}{z} H_0(z)$$

c) Transmitancja Układu Zamkniętego

$$G(z) = \frac{G_{TG}(z)}{1 + G_0(z)}$$

3. cd Transmitancje obliczone przy pomocy Matlaba:

```
G_ToruGlownego =

  0.0090707 (z+0.9049)
  --------------------
    (z-1) (z-0.7408)


G0 =

  -0.3365 (z-1.051)
  -----------------
  (z-1) (z-0.7408)


G =

   0.0090707 (z+0.9049)
  ----------------------
  (z^2 - 2.077z + 1.095)
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4. Czy układ $G(z)$ jest stabilny?

Mianownik $G(z)$:

$$M(z) = z^2 - 2{,}077z + 1{,}095$$

$$\big(z-(1{,}039 + i\,0{,}126)\big)\big(z-(1{,}039 - i\,0{,}126)\big) = 0$$

$$\Downarrow$$

$|z_{1,2}| = 1{,}046 > 1$, czyli układ niestabilny

5.

$$h_{ust} = \lim_{z \to 1} \frac{z-1}{z} G(z) \frac{z}{z-1} = G(1) = 0.0090707 \cdot \frac{1 + 0.9049}{1^2 - 2.077 + 0.1095} = 1$$

6.

$$e(t) = u(t) - y(t)$$

$$e_{ust} = 1 - h_{ust} = 1 - 1 = 0$$

7. Z Metody Bezpośredniej:

$$A = \begin{bmatrix} 0 & 1 \\ -1{,}095 & 2{,}077 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0{,}008 & 0{,}009 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

$$x\left((k+1)T\right) = \begin{bmatrix} 0 & 1 \\ -1{,}095 & 2{,}077 \end{bmatrix} x(kT) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(kT)$$

$$y(kT) = \begin{bmatrix} 0{,}008 & 0{,}009 \end{bmatrix} x(kT)$$

Zad. 2 – Wariant 1

a. (poprawione)

$$y(kT) = \{L^{-1}\frac{2(1-e^{-sT})}{s(0.4s+1)}U(s)\}\Big|_{t=kT}$$

$$y(kT) = \{L^{-1}\frac{2(1-e^{-sT})}{s^2(0.4s+1)}\}\Big|_{t=kT} = \{L^{-1}\frac{2}{s^2(0.4s+1)}\}\Big|_{t=kT} - \{L^{-1}\frac{2(1-e^{-sT})}{s^2(0.4s+1)}\}\Big|_{t=kT}$$

$$y(kT) = ((-0.8+2t+0.8e^{-2.5t}\cdot 1(t)) - (-0.8+2(t-T)+0.8e^{-2.5(t-T)})\cdot 1(t-T))\Big|_{t=kT} =$$

$$= (-0.8+2kT+0.8e^{-2.5kT}\cdot 1(kT)) - (-0.8+2(kT-T)+0.8e^{-2.5(kT-T)})\cdot 1(kT-T) =$$

$$= (-0.8+1.29k+0.8\cdot 0.2^{k}\cdot 1(kT)) - (-0.8+1.29(kT-T)+0.8\cdot 0.2^{k-1})\cdot 1(kT-T))$$

6.

$$G(s) = \frac{2}{0{,}4s+1} \Rightarrow H(s) = \frac{2}{s(0{,}4s+1)} \qquad u(z) = \frac{z}{z-1}$$

$$y(z) = \frac{z-1}{z} \mathcal{Z}\left\{ L^{-1}\{H(s)\}\Big|_{t=kT} \right\} \cdot \frac{z}{z-1} = \left\{ \tau = 0.4 \right\} =$$

$$= \frac{2z}{z-1} - \frac{2z}{z-e^{-\frac{T}{\tau}}} = \left\{ e^{-\frac{T}{\tau}} = 0.2 \right\} = \frac{2z}{z-1} - \frac{2z}{z-0.2}$$

$$y(kT) = 2 - 2\cdot(0{,}2)^k$$

Odpowiedź:

$$y(kT) = \left(2 - 2\cdot 0.2^k\right)\cdot u(kT)$$