ECON5020 – Macroeconomics

Week 29 - Consumption theory

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Address the following questions

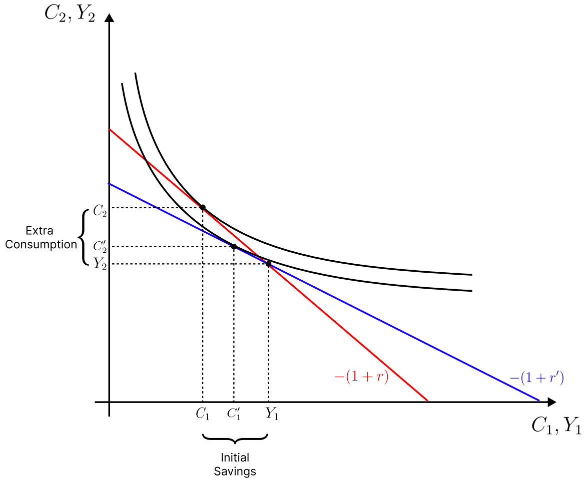
- **What is the Intertemporal Marginal Rate of Substitution?**
  - Amount of today consumption an agent is willing to exchange for future consumption in order to maintain the same level of utility. If there's only 2 periods:

$$dU = dC_1 \times \mathrm{MgU}_1 + dC_2 \times \mathrm{MgU}_2 = 0 \implies \frac{dC_1}{dC_2} = \frac{\mathrm{MgU}_2}{\mathrm{MgU}_1}$$

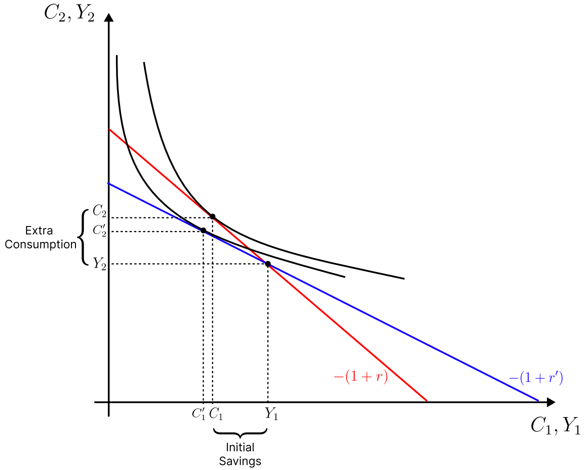
- **What is a borrowing constraint?**
  - When there is some sort of limitation, which prevents agent to borrowing as much as they want.
  - One example is the one seen in lecture, where agent can save as much as they want, but can't borrow.
- **Explain the effect of a fall in the real interest rate r in the case of a net lender**
  - The IBC will turn counter-clockwise.
  - We know from lecture that $\frac{\beta \mathrm{MgU}_2}{\mathrm{MgU}_1} = \frac{1}{1+r\downarrow} \uparrow$
  - We know then that C_1 should increase and/or C_2 should decrease.
  - There are 2 effects and depending on who dominates, the agent can keep being net lender or become net borrower:
    - ◇ **Income:** Today income is higher, now that interest rate is lower $(Y_1 + \frac{Y_2}{(1+r)})$
    - ◇ **Substitution:** Future consumption is more expensive $(C_1 + \frac{C_2}{(1+r)})$.

Address the following questions

- If substitution effect dominates, consumption in the future decreases and consumption today increases



- If income effect dominates, consumption in the future decreases but less and consumption today decreases.



Household Problem

- £5,000 of income today, £10,000 of income tomorrow and 5% real interest rate
- Total wealth in:
  - Today's consumption: $\pounds 5,000 + \frac{\pounds 10,000}{1+5\%} \approx \pounds 14,524$
  - Tomorrow's consumption: $\pounds 5,000 \times (1+5\%) + \pounds 10,000 \approx \pounds 15,250$
- Optimal consumption:
  - Trick: Whenever we have Cobb-Douglas as utility function $(C_1^{\alpha} C_2^{\beta})$:

$$C_1 = \frac{\alpha}{\alpha+\beta}\frac{W}{P_1} \qquad C_2 = \frac{\beta}{\alpha+\beta}\frac{W}{P_2}$$

  - In this case: $W = Y_1 + \frac{Y_2}{1+5\%} = \pounds 14,524$, $P_1 = 1$ and $P_2 = \frac{1}{1+5\%}$
  - Plugging the values we have that:

$$C_1 = \frac{1}{2} \times \pounds 14,524 = \pounds 7,261.9$$

$$C_2 = \frac{1}{2} \times (1+5\%) \times \pounds 14,524 = \pounds 7,625$$

  - Since $C1 > Y1$, agent is net borrower

Household Problem

- Increase in current income to £5,500: $5,500 + \frac{10{,}000}{1+5\%} = £15,024$
  - Apply the trick from last slide:

$$C_1 = \frac{1}{2} \times £15,024 = £7,511.9$$

$$C_2 = \frac{1}{2} \times (1+5\%) \times £15,024 = £7,887.5$$

- Increase in current income to £5,500 and future income to £10,500:$5,500 + \frac{10{,}500}{1+5\%} = £15,500$
  - Apply the trick from last slide:

$$C_1 = \frac{1}{2} \times £15,500 = £7,750$$

$$C_2 = \frac{1}{2} \times (1+5\%) \times £15,500 = £8,137.5$$

Household Problem

- Back to original income, but $r = 10\%$: $5,000 + \frac{10,000}{1+10\%} = £14,091$
  - Apply the trick from last slide:

$$C_1 = \frac{1}{2} \times £14,091 = £7,045$$

$$C_2 = \frac{1}{2} \times (1 + 5\%) \times £14,091 = £7,750$$

- $U = \log(C_1) + \beta \log(C_2)$:
  - Using the condition: $\text{MRS} = 1 + r$:

$$\frac{\text{MgU}_1}{\text{MgU}_2} = 1 + r \implies \frac{C_2}{\beta C_1} = 1 + r \implies C_2 = \beta(1 + r)C_1$$

  - Note that $\beta \times (1 + r) \approx 1$, so $C1 = C2$, which solves to:

$$C_1 = C_2 \approx £7,440$$

Borrowing Constraint

- If there're borrowing constraints, then no borrowing would be possible.

- If agent were to be net lender under no borrowing constraints, not effected
- If agent were to be net borrower under no contraints, then they can achieve (unconstrained) optimal. Consequence: consumes all income in first period.
  - Can't smooth consumption across time and is susceptible to income shocks $\implies$ higher consumption volatility
  - There might be a point of income that shifts to net saver.