Solutions to Workshop on Monetary Policy

ECON5020 - Macroeconomics

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2023/24

Question 1 - 3 equation model

Part 1

The 3 equation model is expressed as follows:

$$y_t = -a(i_t - \pi_t^e) + u_t \tag{IS Curve}$$
$$\pi_t = \pi_t^e + by_t \tag{Phillips Curve}$$
$$i_t = c\pi_t + dy_t \tag{Taylor Rule}$$

Following the hints, we start by replacing Taylor Rule into IS Curve:

$$\begin{aligned} y_t &= -a(c\pi_t + dy_t - \pi_t^e) + u_t \\ (1+ad)y_t &= -ac\pi_t + a\pi_t^e + u_t \\ y_t &= -\frac{ac}{(1+ad)}\pi_t + \frac{a}{(1+ad)}\pi_t^e + \frac{1}{(1+ad)}u_t \end{aligned} \tag{1}$$

we then plug (1) into Phillips Curve and solve for π_t as function of π_t^e and u_t.

$$\begin{aligned} \pi_t &= \pi_t^e + by_t \\ \pi_t &= \pi_t^e + b\left[-\frac{ac}{(1+ad)}\pi_t + \frac{a}{(1+ad)}\pi_t^e + \frac{1}{(1+ad)}u_t\right] \\ \pi_t + \frac{abc}{(1+ad)}\pi_t &= \pi_t^e + \frac{ab}{(1+ad)}\pi_t^e + \frac{b}{(1+ad)}u_t \\ \frac{1+ad+abc}{\cancel{(1+ad)}}\pi_t &= \frac{1+ad+ab}{\cancel{(1+ad)}}\pi_t^e + \frac{b}{\cancel{(1+ad)}}u_t \\ \pi_t &= \frac{1+a(d+b)}{1+a(d+bc)}\pi_t^e + \frac{b}{1+a(d+bc)}u_t \end{aligned} \tag{2}$$

Equation (2) tells us how inflation behaves to expectations and shocks to the IS curve. If we use the assumption that expectations are adaptative ($\pi_t^e = \pi_{t-1}$) then it becomes:

$$\pi_t = \frac{1+a(d+b)}{1+a(d+bc)}\pi_{t-1} + \frac{b}{1+a(d+bc)}u_t \tag{3}$$

Now that we have solved the equation, we can apply it to any period t including the steady state. Lets call the inflation in the steady state as π_0, so $\pi_t = \pi_{t-1} = \pi_0$. Also, note that there is no shock in the steady state, so $u_0 = 0$. We have then:

$$\pi_0 = \frac{1+a(d+b)}{1+a(d+bc)}\pi_0 \implies \pi_0 = 0 \tag{4}$$

Which implies that once the shock happens in the first period we have

$$\pi_1 = \frac{1+a(d+b)}{1+a(d+bc)}\cancel{\pi_0} + \frac{b}{1+a(d+bc)}u_1 \implies \pi_1 = \frac{b}{1+a(d+bc)}u_1 \tag{5}$$

Plugging the result in (5) into (1) we have:

$$\begin{aligned}
y_1 &= -\frac{ac}{(1+ad)}\pi_1 + \frac{a}{(1+ad)}\cancelto{0}{\pi_0} + \frac{1}{(1+ad)}u_1 \\
y_1 &= -\frac{ac}{(1+ad)}\frac{b}{1+a(d+bc)}u_1 + \frac{1}{(1+ad)}u_1 \\
y_1 &= \frac{1}{(1+ad)}\left[1 - \frac{abc}{1+a(d+bc)}\right]u_1 \\
y_1 &= \frac{1}{(1+ad)}\left[\frac{1+ad+\cancel{abc}-\cancel{abc}}{1+a(d+bc)}\right]u_1 \\
y_1 &= \frac{1}{1+a(d+bc)}u_1
\end{aligned} \tag{6}$$

Finally, plugging (5) and (6) into Taylor Rule we have

$$\begin{aligned}
i_1 &= c\pi_1 + dy_1 \\
i_1 &= c\frac{b}{1+a(d+bc)}u_1 + d\frac{1}{1+a(d+bc)}u_1 \\
i_1 &= \frac{bc+d}{1+a(d+bc)}u_1
\end{aligned} \tag{7}$$

Part 2

In this part, we can use our general equation (3) substituting the value for π_1 we calculated in Part 1 and use $u_2 = 0$:

$$\begin{aligned}
\pi_2 &= \frac{1+a(d+b)}{1+a(d+bc)}\pi_1 + \frac{b}{1+a(d+bc)}\cancelto{0}{u_2} \\
\pi_2 &= \frac{1+a(d+b)}{1+a(d+bc)}\frac{b}{1+a(d+bc)}u_1 \implies \pi_2 = \frac{b\,(1+a(d+b))}{(1+a(d+bc))^2}u_1
\end{aligned} \tag{8}$$

Part 3

Note that because we solved the model in the most general format first to find the equation for π_t we can keep using (3). Note that the solution for π_1 doesn't change ($\pi_0 = 0$ and $u_1 \neq 0$). The solution to π_2 however changes to $\pi_2 = 0$.