EC502 Macroeconomics
Week 17 Seminar: Consumption Theory

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1. Explain...

a) **Intertemporal Marginal Rate of Substitution**

- Trade-off between consuming more today or saving more today and consuming more tomorrow
- IMRS: Rate at which a consumer can give up some amount of one good in exchange for another good while maintaining the same level of utility.
- Gain in utility from $\uparrow C_1$ compared to the loss of utility of consuming less tomorrow $\downarrow C_2$
- Measures willingness to give up future consumption to consume more today

$$IMRS \equiv \frac{U'(C_1)}{U'(C_2)}$$

b) **Borrowing constraints**

- Agents cannot borrow as much as they want - only up to a certain amount
- Extreme case: $\cancel{C_1 > Y_1 - T_1} \quad \longleftrightarrow \quad C_1 \leq Y_1 - T_1$
- It may lead to a sub-optimal consumption bundle: lower utility
- Example: exercise 3 (graph)

1. Explain...

d) Effect of a fall in the real interest rate r in the case of a net lender

- A net lender is an agent that saves today's income to consume more tomorrow
- Optimal consumption C_1^*, C_2^*: where the slope of the indifference curve is equal to the slope of budget constraint: $IMRS = 1 + r$

$$C_1 < Y_1 - T_1$$

- It implies a 'flatter' budget constraint
- It implies lower utility via a lower indifference curve
- Income effect: lower future interest earnings imply lower lifetime wealth: $\downarrow C_1, \downarrow C_2$
- Substitution effect: future consumption is more expensive, or the rewards from saving are lower: $\uparrow C_1, \downarrow C_2$
- In this example: **INCOME** effect dominates: $\downarrow C_1$
- It implies lower utility via a lower indifference curve
- Income effect: lower future interest earnings imply lower lifetime wealth: $\downarrow C_1, \downarrow C_2$
- Substitution effect: future consumption is more expensive, or the rewards from saving are lower: $\uparrow C_1, \downarrow C_2$
- In this example: **SUBSTITUTION** effect dominates: $\uparrow C_1$

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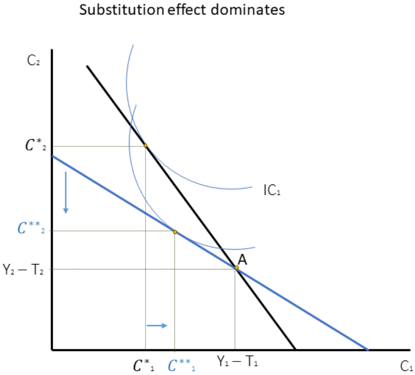
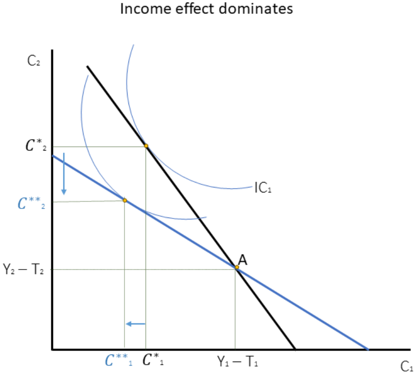


Substitution effect dominates
C₂
C*₂
C**₂
Y₂ – T₂
IC₁
A
C*₁
C**₁
Y₁ – T₁
C₁



Income effect dominates
C₂
C*₂
C**₂
Y₂ – T₂
IC₁
A
C**₁
C*₁
Y₁ – T₁
C₁

2. Find optimal consumption

a) Find the household's total wealth[1]

In terms of today's consumption:

$$W = Y_1 + \frac{Y_2}{1+r} = 5,000 + \frac{10,000}{1.05} = 14,523.81$$

In terms of future's consumption:

$$W = Y_1(1+r) + Y_2 + = 5,000(1.05) + 10,000 = 15,250$$

b) Find the optimal level of consumption today and tomorrow

$$\max U(C_1, C_2) = \sqrt{C_1 C_2}$$

$$\text{s.t.} \quad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

[1] Assume no taxes, i.e. $T_i = 0 \quad \forall i$

2. Find optimal consumption

$$\max U(C_1, C_2) = \sqrt{C_1 C_2} \quad \text{s.t.} \quad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

Two ways to solve it[2]

1. Substitution method
2. Optimality condition

[2] Use the one you feel easier

2. Find optimal consumption

$$\max U(C_1, C_2) = \sqrt{C_1 C_2} \quad \text{s.t.} \quad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

Two ways to solve it[2]

1. <u>Substitution method</u>

From IBS, given that $C_1 = Y_1 + \frac{Y_2 - C_2}{1+r}$, substitute into utility function:

$$\begin{aligned} \max U(C_2) &= \left[\left(Y_1 + \frac{Y_2 - C_2}{1+r}\right) \cdot C_2\right]^{1/2} \\ &= \left[Y_1 + \frac{Y_2 - C_2}{1+r}\right]^{1/2} \cdot C_2^{1/2} \end{aligned}$$

Which is an unconstrained maximization problem

[2] Use the one you feel easier

2. Find optimal consumption

$$\max U(C_2) = \left[Y_1 + \frac{Y_2 - C_2}{1+r} \right]^{1/2} \cdot C_2^{1/2}$$

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2. Find optimal consumption

$$\max U(C_2) = \left[Y_1 + \frac{Y_2 - C_2}{1+r}\right]^{1/2} \cdot C_2^{1/2}$$

- Calculate the first-order condition (FOC): $\partial U / \partial C_2 = 0$
- Use the chain rule

$$\frac{\partial U}{\partial C_2} = \frac{C_2^{-1/2}}{2} \cdot \left[Y_1 + \frac{Y_2 - C_2}{1+r}\right]^{1/2} + C_2^{1/2} \cdot \frac{1}{2} \cdot \frac{-\left(Y_1 + \frac{Y_2 - C_2}{1+r}\right)^{-1/2}}{1+r} = 0$$

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2. Find optimal consumption

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$$\frac{C_2^{-1/2}}{2} \cdot \left[Y_1 + \frac{Y_2 - C_2}{1+r}\right]^{1/2} = C_2^{1/2} \cdot \frac{1}{2} \cdot \frac{\left(Y_1 + \frac{Y_2 - C_2}{1+r}\right)^{-1/2}}{1+r}$$

$$\left[Y_1 + \frac{Y_2 - C_2}{1+r}\right]^{1/2} = C_2 \frac{\left(Y_1 + \frac{Y_2 - C_2}{1+r}\right)^{-1/2}}{1+r} \quad \longleftrightarrow$$ [3]

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2b. Find optimal consumption (continued)

$$\longleftrightarrow \quad \left[Y_1 + \frac{Y_2 - C_2}{1+r}\right]^{1/2} = C_2 \frac{\left(Y_1 + \frac{Y_2 - C_2}{1+r}\right)^{-1/2}}{1+r}$$

$$Y_1 + \frac{Y_2 - C_2}{1+r} = \frac{C_2}{1+r}$$

$$Y_1 + \frac{Y_2}{1+r} = \frac{2C_2}{1+r}$$

$$C_2^* = \frac{Y_1 \cdot (1+r) + Y_2}{2}$$

- Then substitute C_2^* into the budget constraint (IBC)

2b. Find optimal consumption (continued)

- Then substitute C_2^* into the budget constraint (IBC)

$$C_1 + \frac{\left[\frac{Y_1 \cdot (1+r) + Y_2}{2}\right]}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

$$C_1 + \left[\frac{Y_1 \cdot (1+r) + Y_2}{2(1+r)}\right] = Y_1 + \frac{Y_2}{1+r}$$

$$C_1 = Y_1 + \frac{Y_2}{1+r} - \frac{Y_1 \cdot (1+r)}{2(1+r)} - \frac{Y_2}{2(1+r)}$$

$$C_1 = \frac{Y_1}{2} - \frac{Y_2}{2(1+r)}$$

$$C_1^* = \frac{Y_1(1+r) - Y_2}{2(1+r)}$$

This implies the agent will consume half of its wealth in each period as preferences are homothetic

2. Find optimal consumption

$$\max U(C_1, C_2) = \sqrt{C_1 C_2} \quad \text{s.t.} \quad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

Two ways to solve it[4]

1. Substitution method
2. Optimality condition

[4] Use the one you feel easier

2. Find optimal consumption

$$\max U(C_1, C_2) = \sqrt{C_1 C_2} \quad \text{s.t.} \quad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

Two ways to solve it[4]

1. Substitution method
2. Optimality condition

Optimal allocation where slope of indifference curve = slope of budget constraint

$$IMRS \equiv \frac{U'(C_1)}{U'(C_2)} = 1 + r$$

$$\frac{\frac{1}{2} \cdot C_1^{-1/2} \cdot C_2^{1/2}}{\frac{1}{2} \cdot C_1^{1/2} \cdot C_2^{-1/2}} = 1 + r$$

$$\frac{C_2}{C_1} = 1 + r$$

[4] Use the one you feel easier

2. Find optimal consumption

$$\frac{C_2}{C_1} = 1 + r$$

- This is the **Euler Equation**: the rate at which consumer is willing to substitute tomorrow's consumption with today's consumption
- Substitute C_1 into budget constraint (IBC)[5]

$$\begin{aligned}
\frac{C_2}{1+r} + \frac{C_2}{1+r} &= Y_1 + \frac{Y_2}{1+r} \\
2 \cdot \frac{C_2}{1+r} &= Y_1 + \frac{Y_2}{1+r} \\
C_2^* &= \frac{Y_1(1+r) + Y_2}{2}
\end{aligned}$$

- Then substitute C_2^* into optimal Euler equation:

$$C_1^* = \frac{C_2^*}{1+r} = \frac{Y_1(1+r) + Y_2}{2(1+r)}$$

[5] You can substitute whichever you prefer

2. Find optimal consumption

- Substitute for the values of $Y_1,\ Y_2,\ r$

$$C_1^* = \frac{Y_1(1+r)+Y_2}{2(1+r)} = \frac{5,000(1+0.05)+10,000}{2(1+0.05)} = 7,261.9$$
$$C_2^* = \frac{Y_1(1+r)+Y_2}{2} = \frac{5,000(1+0.05)+10,000}{2} = 7,625$$

- What is the type of the consumer?

$$C_1 \lesseqgtr Y_1?$$

2. Find optimal consumption

- Substitute for the values of $Y_1, \ Y_2, \ r$

$$C_1^* = \frac{Y_1(1+r)+Y_2}{2(1+r)} = \frac{5,000(1+0.05)+10,000}{2(1+0.05)} = 7,261.9$$

$$C_2^* = \frac{Y_1(1+r)+Y_2}{2} = \frac{5,000(1+0.05)+10,000}{2} = 7,625$$

- What is the type of the consumer?

$$C_1 \lesseqgtr Y_1? \quad \longleftrightarrow \quad 7,261.9 > 5,000$$

- This is a **net borrower**: uses future income to consume more in the present

2. Find optimal consumption

c) What if $\uparrow Y_1{}' = 5,500$ and Y_2 constant?

$$C_1^{*'} = \frac{Y_1(1+r)+Y_2}{2(1+r)} = \frac{5,500(1+0.05)+10,000}{2(1+0.05)} = 7,511.9$$

$$C_2^{*'} = \frac{Y_1(1+r)+Y_2}{2} = \frac{5,500(1+0.05)+10,000}{2} = 7,887.5$$

- Compare with previous result:

$$C_1^{*'} = 7,511.9 > C_1^* = 7,261.9 \qquad C_2^{*'} = 7,887.5 > C_2^* = 7,625$$

2. Find optimal consumption

c) What if $\uparrow Y_1{}' = 5,500$ and Y_2 constant?

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- Compare with previous result:

$$C_1^{*'} = 7,511.9 > C_1^* = 7,261.9 \qquad C_2^{*'} = 7,887.5 > C_2^* = 7,625$$

- C_1 increased **LESS** than the increase in income Y_1: $\Delta C_1 = 250$
- Agent smoothes consumption across both periods
- Note still $C^{*'} > Y_1$, so net borrower. The net borrower position is reduced

2. Find optimal consumption

d) What if $\uparrow Y_1{}' = 5,500$ AND $Y_2 = 10,500$, i.e., same increase?

$$C_1^{*''} = \frac{Y_1(1+r)+Y_2}{2(1+r)} = \frac{5,500(1+0.05)+10,500}{2(1+0.05)} = 7750$$

$$C_2^{*''} = \frac{Y_1(1+r)+Y_2}{2} = \frac{5,500(1+0.05)+10,500}{2} = 8137.5$$

- Compare with previous result:

$$C_1^{*''} = 7750 > C_1^* = 7,261.9 \qquad C_2^{*''} = 8137.5 > C_2^* = 7,625$$

2. Find optimal consumption

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- Compare with previous result:

$$C_1^{*''} = 7750 > C_1^* = 7,261.9 \qquad C_2^{*''} = 8137.5 > C_2^* = 7,625$$

- BOTH consumptions increase proportionally

$$\Delta C_1 = 488.1 = \Delta C_2 = 512.5/1.05$$

- Agent smoothes consumption across both periods
- Note still $C^{*''} > Y_1$, so net borrower.

2. Find optimal consumption

e) What if $\uparrow r' = 0.1$?

$$C_1^{*e} = \frac{Y_1(1+r)+Y_2}{2(1+r)} = \frac{5,000(1+0.1)+10,000}{2(1+0.1)} = 7045.5$$

$$C_2^{*e} = \frac{Y_1(1+r)+Y_2}{2} = \frac{5,000(1+0.1)+10,000}{1+0.1} = 7750$$

- Because agent is net borrower we would expect

2. Find optimal consumption

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- Because agent is net borrower we would expect
  - $\downarrow C_1$, $\downarrow C_2$: higher r implies she faces higher future payments on the loan $\rightarrow$ Income effect
  - $\uparrow C_2$: higher r implies higher willingness to exchange C_2 and C_1 (Euler) $\rightarrow$ Substitution effect

2. Find optimal consumption

e) What if $\uparrow r' = 0.1$?

$$C_1^{*e} = \frac{Y_1(1+r)+Y_2}{2(1+r)} = \frac{5,000(1+0.1)+10,000}{2(1+0.1)} = 7045.5$$

$$C_2^{*e} = \frac{Y_1(1+r)+Y_2}{2} = \frac{5,000(1+0.1)+10,000}{1+0.1} = 7750$$

- Compare with previous result:

$$C_1^{*e} = 7045.5 < C_1^* = 7,261.9 \qquad C_2^{*e} = 7750 > C_2^* = 7,625$$

- Big decrease in C_1, small increase in C_2

$$\Delta C_1 = -216.5 < 0 \quad \Delta C_2 = 125 > 0$$

2. Find optimal consumption

e) What if $\uparrow r' = 0.1$?

$$C_1^{*e} = \frac{Y_1(1+r)+Y_2}{2(1+r)} = \frac{5,000(1+0.1)+10,000}{2(1+0.1)} = 7045.5$$

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- Compare with previous result:

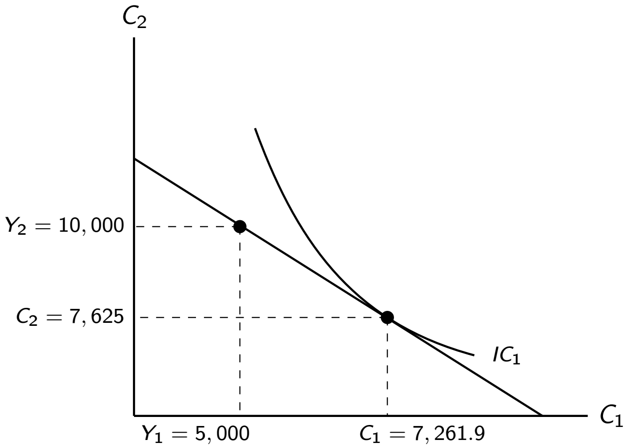
$$C_1^{*e} = 7045.5 < C_1^* = 7,261.9 \qquad C_2^{*e} = 7750 > C_2^* = 7,625$$

- Big decrease in C_1, small increase in C_2

$$\Delta C_1 = -216.5 < 0 \qquad \underbrace{\Delta C_2 = 125 > 0}_{\text{substitution effect dominates}}$$

2. Find optimal consumption

e) What if $\uparrow r' = 0.1$?



2. Find optimal consumption

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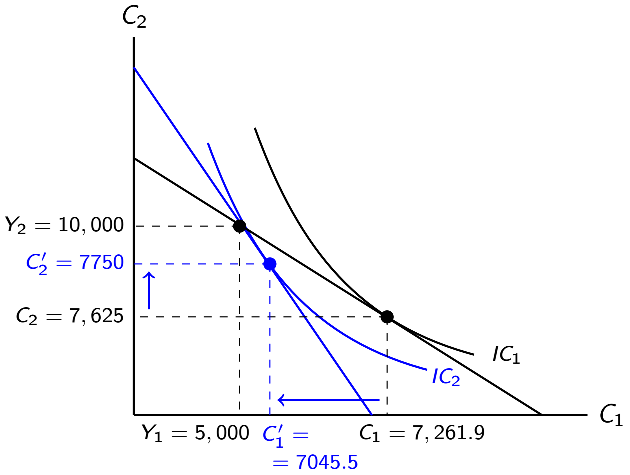


2. Find optimal consumption

f) What if lifetime utility is $U = \log(C_1) + \beta \log(C_2)$?

- Follow the same steps as before:

$$IMRS \equiv \frac{U'(C_1)}{U'(C_2)} = \frac{1/C_1}{\beta/C_2} = 1 + r$$
$$\frac{C_2}{C_1} = (1+r)\beta$$
$$\frac{C_2}{C_1} = (1+0.05) \cdot 0.952 \approx 1$$

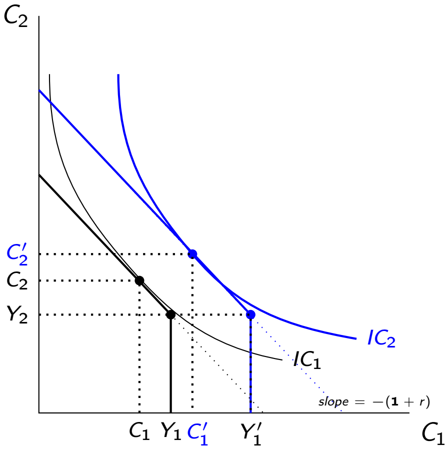
- The optimal condition implies that $C_1 = C_2$: consumption smoothing
- Note if $\downarrow \beta$, then $C_2 << C_1$: bias towards today's consumption
- Substitute into budget constraint (IBC)

$$C_1 + \frac{C_1}{1+r} = Y_1 + \frac{Y_2}{1+r} \quad \longleftrightarrow$$
$$\frac{(1+r+1)C_1}{1+r} = Y_1 + \frac{Y_2}{1+r} \quad \longleftrightarrow \quad C_1^* = C_2^* = \frac{Y_1(1+r) + Y_2}{2+r}$$

3. Borrowing constraints

d) Effect of an increase in current income $\uparrow Y_1$

- Overall wealth increase $\rightarrow$ shift IBC
- If net lender, does not need borrowing $\rightarrow \uparrow$ consumption both periods



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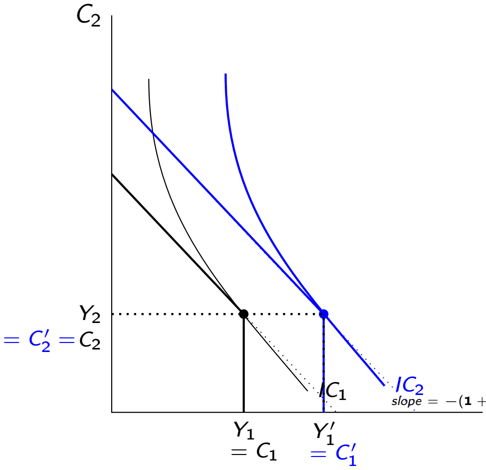
d) Effect of an increase in current income $\uparrow Y_1$

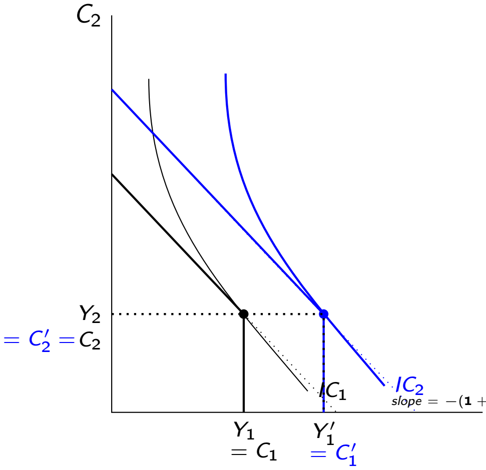
- Overall wealth increase $\rightarrow$ shift IBC
- If net lender, does not need borrowing $\rightarrow \uparrow$ consumption both periods
- It net borrower, $C_1 > Y_1$. Facing borrowing constraints agent can only consume up to their income, $Y_1 = C_1$
- $\uparrow Y_1$ results in only $\uparrow C_1$ as cannot smooth consumption by borrowing



3. Borrowing constraints

d) Effect of an increase in current income $\uparrow Y_1$

- Overall wealth increase $\rightarrow$ shift IBC
- If net lender, does not need borrowing $\rightarrow \uparrow$ consumption both periods
- It net borrower, $C_1 > Y_1$. Facing borrowing constraints agent can only consume up to their income, $Y_1 = C_1$
- $\uparrow Y_1$ results in only $\uparrow C_1$ as cannot smooth consumption by borrowing
- Optimal consumption theory predicts $C_1 = C_2$ (from previous exercise). Borrowing constraints might explain why $C_1 \neq C_2$ (**volatility**)



Exercise 2: $U = \log(C_1) + \beta \log(C_2)$

- Optimal condition implies that $C_1 = C_2$: consumption smoothing

$$IMRS \equiv \frac{U'(C_1)}{U'(C_2)} = \frac{1/C_1}{\beta/C_2} = 1 + r$$
$$\frac{C_2}{C_1} = (1+r)\beta = (1+0.05) \cdot 0.952 \approx 1$$

- Substitute into budget constraint (IBC)

$$C_1 + \frac{C_1}{1+r} = Y_1 + \frac{Y_2}{1+r} \quad \longleftrightarrow \frac{(1+r+1)C_1}{1+r} = Y_1 + \frac{Y_2}{1+r} \quad \longleftrightarrow$$
$$C_1^* = C_2^* = \frac{Y_1(1+r) + Y_2}{2+r} = 7,439.02$$

- If $\uparrow Y_1 = 500$, then $C_1 = C_2 = 7,595.12$
- If $\uparrow Y_1 = \uparrow Y_2 = 500$, then $C_1 = C_2 = 7,939.02$
- If $\uparrow r = 0.1$, then $C_1 = C_2 = 7,380.95$

INCOME effect dominates: $\Delta C_1 = \Delta C_2 = -58.07 < 0$