Solutions to Workshop on Demand Management

ECON5020 - Macroeconomics

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Question 1 – Conservative Central Banker

Your preferences

$$U = y - \frac{\theta}{2}\pi^2$$

Phillips Curve

$$\pi = \pi^e + by$$

Candidate preferences

$$\hat{U} = y - \frac{\hat{\theta}}{2}\pi^2$$

Your optimisation problem and candidate's yield similar results for inflation and output. Your optimal inflation and the candidate's will be

$$\pi^* = \frac{1}{b\theta} \qquad \hat{\pi}^* = \frac{1}{b\hat{\theta}}$$

respectively.

As Alfred said in the hints, you should try to set the Lagrangian and solve it, and only do what I do here if you don't have time. Again, let's assume that expectations are well anchored so $\pi^e = 0$. The Phillips curve under each inflation would be

$$y^* = \frac{1}{b^2\theta} \qquad \hat{y}^* = \frac{1}{b^2\hat{\theta}}$$

Which candidate would you choose?

Note in Figure 1 that if you choose a central bank that differs from your preferences, you'll be in an inferior indifference curve. So ideally, you'll choose one that has $\hat{\theta} = \theta$.

Figure 1: Inflation Bias - Conservative Central Banker and Well Anchored Expectations

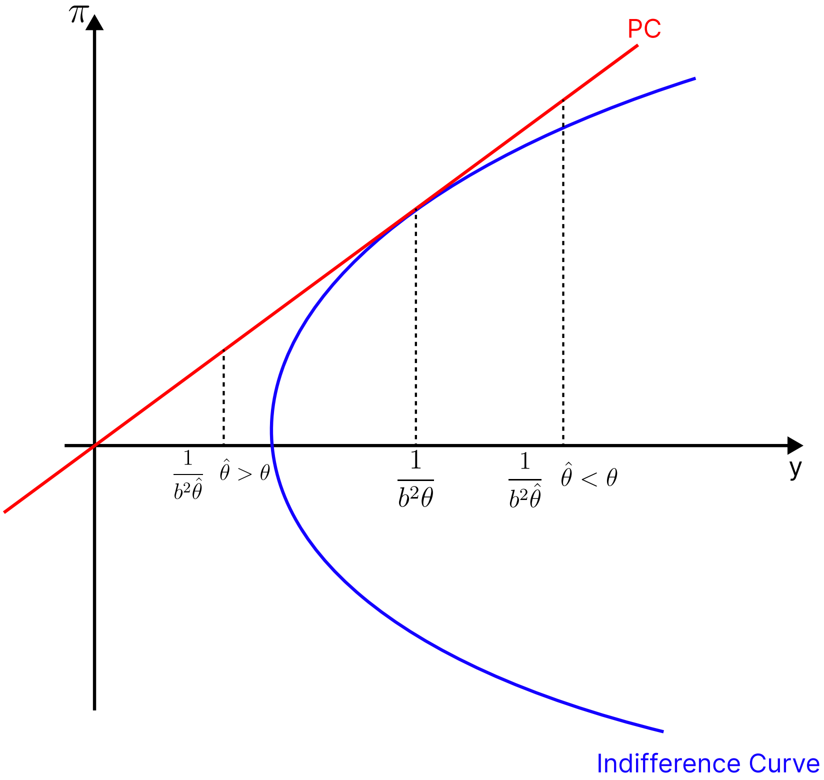
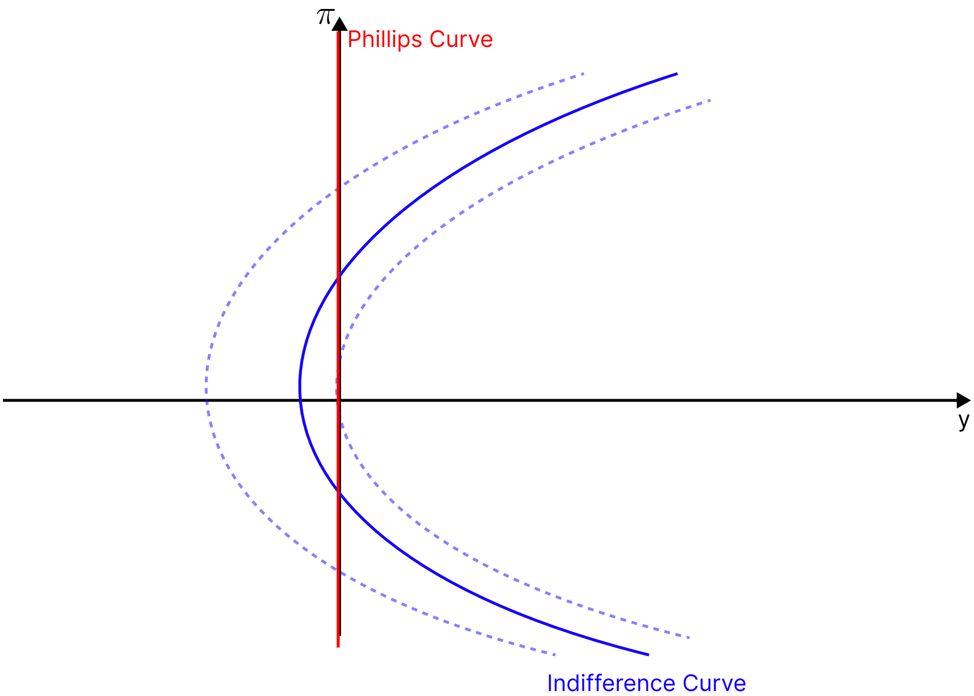


Figure 2: Inflation Bias - Conservative Central Banker and Rational Expectations



Note that in Figure 2 you answer change, you'll tend to choose a more strict central banker.

Question 2 - Inflation targeting central bank

Note that now the utility of the central bank is different and no longer depends on output, just inflation

$$U = -\frac{\theta}{2}\pi^2$$

. You as a Chancellor set the desired inflation target π^* (which is...?), and agents set their expectation as $\pi^e = \pi^*$ Just as in the lecture notes, the Lagrangian is

$$\mathcal{L} = -\frac{\theta}{2}\pi^2 + \lambda(-by)$$

which will yield the know results of

$$\pi = 0 \quad \text{and} \quad y = 0$$

Note that now we don't have the time inconsistency problem, given that the central banker utility is independent of output levels.

Question 3 - Debt management

Lets think about how debt evolve from one period to another. Government in the next period will owe any outstanding debt increased by interest rate ($(1+r)B_t$), plus any new expenditure that they can't cover with tax income ($G_t - T_t$). We can then write:

$$B_{t+1} = (1+r)B_t + G_t - T_t \tag{1}$$

If we divide (1) by GDP (Y_t):

$$\frac{B_{t+1}}{Y_t} = (1+r)\frac{B_t}{Y_t} + \frac{G_t}{Y_t} - \frac{T_t}{Y_t}$$

Remember that $Y_{t+1} = (1+g^y)Y_t$ so we can write $Y_t = \frac{Y_{t+1}}{1+g^y}$. Which we can plug into the previous equation to write:

$$(1+g^y)\frac{B_{t+1}}{Y_{t+1}} = (1+r)\frac{B_t}{Y_t} + \frac{G_t}{Y_t} - \frac{T_t}{Y_t} \tag{2}$$

We can now rename the variables by substituting for lower case values, their equivalent "per GDP" values. Which means $x_t \equiv \frac{X_t}{Y_t}$ for any variable x_t. Equation (2) then becomes

$$(1+g^y)b_{t+1} = (1+r)b_t + g_t - t_t \tag{3}$$

Part 1

We have from the question that the primary deficit per GDP ($g_t - t_t$) is 3%, real interest rate(r) is 2%, the growth rate of the economy (g^y) is 3%, and the current debt per GDP equals to 100%. Plugging this values into (3) we have

$$b_{t+1} = \frac{(1+r)b_t + g_t - t_t}{(1+g^y)} = \frac{(1+2\%) \times 100\% + 3\%}{1+3\%} \approx 102\%$$

Part 2

Plugging the new value of debt to GDP into (3) we have

$$b_{t+1} = \frac{(1+r)b_t + g_t - t_t}{(1+g^y)} = \frac{(1+2\%) \times 200\% + 3\%}{1+3\%} \approx 201\%$$

Part 3 and 4

To find the steady state level of government debt we can use equation (3) again. Replace every variable by their steady state level:

$$(1+g^y)b_{ss} = (1+r)b_{ss} + g_{ss} - t_{ss}$$

Rearranging we get:

$$b_{ss} = \frac{g_{ss} - t_{ss}}{g^y - r} \tag{4}$$

If the primary deficit of government stays at 3%, from (4) forever we get

$$b_{ss} = \frac{g_{ss} - t_{ss}}{g^y - r} = \frac{3\%}{3\% - 2\%} = 300\%$$

At $r = 2.5\%$:

$$b_{ss} = \frac{g_{ss} - t_{ss}}{g^y - r} = \frac{3\%}{3\% - 2.5\%} = 600\%$$