Solutions to Workshop on Fiscal Policy

ECON5020 - Macroeconomics

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Question 1 – RBC Government Consumption Multiplier

0.1 Part 1

We'll follow the same steps that are in the lecture notes. We start by calculating the marginal utilities of consumption and labour:

$$U_C = \frac{1}{C+G} \tag{1}$$

$$U_L = -L \tag{2}$$

From the optimality condition $-U_L = U_c W$ we get:

$$L = \frac{W}{C+G} \tag{3}$$

Firm maximizes profit by choosing labour (normalise prices P to one):

$$\max_L \Pi = Y - WL = AL - WL \tag{4}$$

which implies $W = A$

We then solve the system of equation composed by

$$L = \frac{W}{C+G} \tag{5}$$

$$W = A \tag{6}$$

$$C = WL - T \tag{7}$$

$$C + G = Y \tag{8}$$

$$Y = AL \tag{9}$$

and the condition that $T = G$. Note that again (8) is redundant once we consider (6), (9) and the government balanced budget.

From (5) we have

$$C = \frac{W}{L} - G$$

which plugging into (7) we get

$$\frac{W}{L} - G = WL - T$$

using (6) and $T = G$ we get

$$\frac{A}{L} - \cancel{G} = AL - \cancel{G} \implies A = AL^2 \implies AL^2 - A = 0 \implies L = 1$$

.
We then get this result and get the expression of $Y = A$.

Part 2

Government spending multiplier:

$$\frac{\partial Y}{\partial G} = 0 \tag{10}$$

Part 3

How to explain this result? Note that private and public consumption are perfect substitute, so agents are indifferent whether their consumption comes from "their effort" or given to them by the government. So there's no need to increase work hours in order to achieve higher levels of indifference.

Question 2 – RBC goverment investment multiplier

The steps are similar to Question 1. Starting from $-U_L = U_c W$ where now $U_C = \frac{1}{C}$ we get

$$C = \frac{W}{L}.$$

The main difference is that now the optimality condition for the firm is

$$W = MPL \implies W = AG^{\gamma}.$$

Remember that the budget constrain from the agents is

$$\begin{aligned} C &= WL - T && \text{(Start from agent's balance budet constraint)} \\ \frac{W}{L} &= WL - G && \text{(Agents' optimality and gov. budget)} \\ \frac{AG^{\gamma}}{L} &= AG^{\gamma}L - G && \text{(firm's optimality)} \\ AG^{\gamma}L^2 - GL - AG^{\gamma} &= 0 && \text{(Rearrange and write in quadratic form)} \end{aligned}$$

Remember that $\gamma = \frac{1}{2}$, so we get

$$A\sqrt{G}L^2 - GL - A\sqrt{G} = 0 \tag{11}$$

Divigin everything by $\sqrt{G}$ we get

$$AL^2 - \sqrt{G}L - A = 0 \tag{12}$$

Note that it is in a similar format as the lecture notes, so we get L as

$$L = \frac{\sqrt{G} + \sqrt{G + 4A^2}}{2A} \tag{13}$$

Plug it back into the production function and we get

$$\begin{aligned}
Y &= AG^{\gamma}L \\
Y &= A\sqrt{G}L \\
Y &= \cancel{A}\sqrt{G}\frac{\sqrt{G} + \sqrt{G + 4A^2}}{2\cancel{A}} \\
Y &= \frac{1}{2}\left(G + \sqrt{G^2 + 4A^2G}\right) \qquad \text{(Production Funtion Solved)}
\end{aligned}$$

The government spending multiplier is

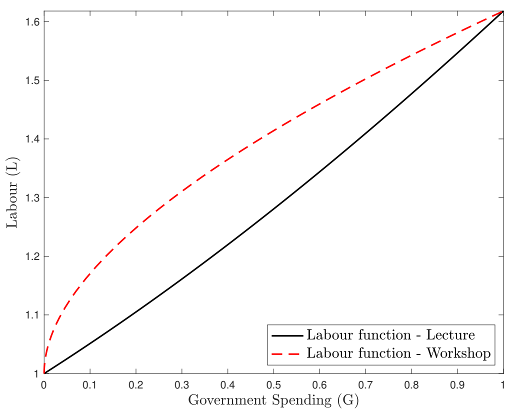
$$\begin{aligned}
\frac{\partial Y}{\partial G} &= \frac{1}{2}\left(1 + \frac{1}{2}\frac{2G + 4A^2}{\sqrt{G^2 + 4A^2G}}\right) && (14) \\
&= \frac{1}{2}\left(1 + \frac{G + 2A^2}{\sqrt{G^2 + 4A^2G}}\right) && (15) \\
& && (16)
\end{aligned}$$

Compare it to the solution in the lecture notes:

$$\begin{aligned}
\frac{\partial Y}{\partial G} &= \frac{1}{2}\left(1 + \frac{G + 2A^2}{\sqrt{G^2 + 4A^2G}}\right) && \text{(Our Solution)} \\
\frac{\partial Y}{\partial G} &= \frac{1}{2}\left(1 + \frac{G}{\sqrt{G^2 + 4A^2}}\right) && \text{(Lecture Solution)}
\end{aligned}$$

Figure 1: How the main variable are affected by government spending

(a) Labour



(b) Production