ECON5020 – Macroeconomics

Week 26 - Growth Theory II

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Solow Growth Model

- Consider an economy well represented by the Solow growth model whose production function is:

$$Y = AK^{\alpha}L^{1-\alpha}$$

  population growth rate $(\frac{\dot{L}}{L})$ is n and there is no TFP growth rate.

- **Analytical expression:**
  - Intensive form in the steady state:

$$y = Y/L = \frac{AK^{\alpha}L^{1-\alpha}}{L} = \frac{AK^{\alpha}L^{1-\alpha}}{L^{\alpha}L^{1-\alpha}} = Ak^{\alpha}$$

  - In the steady state

    $\Delta k = 0 \implies sy - (n+\delta)k = 0 \iff sAk^{\alpha} - (n+\delta)k = 0 \implies k = \left[\frac{sA}{n+\delta}\right]^{\frac{1}{1-\alpha}}$
  - Income per capita is then:

$$y = Ak^{\alpha} = A\left[\frac{sA}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$$

- **Steady state growth rate:**
  - y in the steady state doesn't change, given that there is no TFP growth rate: $\frac{\dot{y}}{y} = 0$

Solow Growth Model

- **Relative income pre capita**

$$\frac{y_{DRC}}{y_{US}} = \frac{\cancel{A}\left[\frac{s_{DRC}\cancel{A}}{\cancel{n+\delta}}\right]^{\frac{\alpha}{1-\alpha}}}{\cancel{A}\left[\frac{s_{US}\cancel{A}}{\cancel{n+\delta}}\right]^{\frac{\alpha}{1-\alpha}}} = \left[\frac{s_{DRC}}{s_{US}}\right]^{\frac{\alpha}{1-\alpha}} = \left[\frac{.154}{.202}\right]^{\frac{.3}{.7}} = .89$$

  - The model doesn't predict well the .055 value observed in the data.
  - In order to observe the value given by the data, α would be:

$$\frac{y_{DRC}}{y_{US}} = \left[\frac{s_{DRC}}{s_{US}}\right]^{\frac{\alpha}{1-\alpha}} \implies \log\left(\frac{y_{DRC}}{y_{US}}\right) = \frac{\alpha}{1-\alpha}\log\left(\frac{s_{DRC}}{s_{US}}\right) \implies$$

$$\alpha = \frac{\log\left(\frac{y_{DRC}}{y_{US}}\right)}{\log\left(\frac{y_{DRC}}{y_{US}}\right) + \log\left(\frac{s_{DRC}}{s_{US}}\right)} = .91$$

  - A value not really likely
  - So other things must be explaining the income per capita difference, possibly technology (A) .

Golden Rule

- From the previous exercise, we know that

$$k = \left[\frac{sA}{n+\delta}\right]^{\frac{1}{1-\alpha}} = \left[\frac{.5}{0+.1}\right]^{\frac{1}{1-.3}} \approx 9.96$$

- Now to answer about dynamic efficiency, we need to check if the economy is maximizing consumption per capita in the steady state:

$$c = y(k) - sy(k)$$

and remember that in steady state

$$sy(k) = (n+\delta)k$$

- Choosing capital to maximise consumption:

$$\frac{dc}{dk} = \underbrace{y'(k)}_{MPK} - (n+\delta) = 0 \implies MPK = (n+\delta)$$

if $MPK = \alpha k^{\alpha-1}$ then the choice of k that maximises consumption is

$$k_{gr} = \left[\frac{n+\delta}{\alpha}\right]^{\frac{1}{\alpha-1}}$$

Growth Accounting

- If we take the formula
$$Y = AK^{\alpha}L^{1-\alpha}$$
and apply logs to it, we have:
$$log(Y) = log(A) + \alpha log(K) + (1-\alpha)log(L)$$
- If we calculate the derivative of $log(x)$ with respect to time, where x is a random variable, we get $\frac{d\ log(x)}{d\ t} = \frac{1}{x}\frac{d\ x}{d\ t} = \frac{\dot{x}}{x}$, where $\dot{x} \equiv \Delta x$ when the time passed is really short.
- We can then write
$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}$$
- Plugging the values we have from the exercise, we have
$$.85 = \frac{\dot{A}}{A} + .3 \times .65 + .7 \times .5 \implies \frac{\dot{A}}{A} = .305 \text{ or } 30.5\%$$
- Which means that, out of the 85% of income increase observed, 30.5% came from TFP growth, and 85-30.5 = 54.5% came from factor accumulation. Roughly a 36/64 split.