ECON5020 Workshop Solutions

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3 equation model

$$y_t = -a(i_t - \pi_t^e) + u_t \tag{IS Curve}$$
$$\pi_t = \pi_t^e + by_t \tag{Phillips Curve}$$
$$i_t = c\pi_t + dy_t \tag{Taylor Rule}$$

0.1 Part A

Substitute (i) from Taylor Rule in the IS eq.

$$y_t = -a(c\pi_t + dy_t - \pi_t^e) + u_t$$

$$y_t + ady_t = (1+ad)y_t = -ac\pi_t + a\pi_t^e + u_t$$

$$y_t = \frac{-ac\pi_t + a\pi_t^e + u_t}{(1+ad)} \tag{1}$$

Sub y_t from eq (1) in PC

$$\pi_t = \pi_t^e + b\left(\frac{-ac\pi_t + a\pi_t^e + u_t}{(1+ad)}\right)$$

$$\pi_t + \frac{bac}{(1+ad)}\pi_t = \pi_t^e + \frac{ba}{(1+ad)}\pi_t^e + \frac{b}{(1+ad)}u_t$$

Simplify to solve for π_t

$$\frac{(1+ad+bac)\pi_t}{(1+ad)} = \frac{(1+ad+ba)\pi_t^e + bu_t}{(1+ad)}$$

$$\pi_t = \frac{1+ad+ba}{1+ad+bac}\pi_t^e + \frac{b}{1+ad+bac}u_t \tag{2}$$

Eq (2) expresses inflation as the function of expected inflation and shock. Assuming adaptive expectations ($\pi_t^e = \pi_{t-1}$), we can write eq (2) for any time t as follows:

$$\pi_t = \frac{1+ad+ba}{1+ad+bac}\pi_{t-1} + \frac{b}{1+ad+bac}u_t \tag{3}$$

(PS. We can use eq (2) as well)

Eq (3) is the generalised form for any period t. Suppose that we are starting at some initial point, such that $\pi_t = \pi_{t-1} = \pi_0 = 0$ and with no shock i.e., $u_0 = 0$, then following eq (3), π_1 can be written as

$$\pi_1 = \frac{1+ad+ba}{1+ad+bac}\pi_0 + \frac{b}{1+ad+bac}u_1 \tag{4}$$

$$\pi_1 = \frac{b}{1+ad+bac}u_1 \tag{5}$$

Substitute eq (5) in the AD eq (1) to solve for y_1

$$y_1 = \frac{-ac\pi_1 + a\pi_0 + u_1}{(1+ad)}$$

(note: $\pi_0 = 0$)

$$y_1 = \frac{-ac}{(1+ad)}\frac{b}{1+ad+bac}u_1 + \frac{u_1}{(1+ad)}$$

$$y_1 = \frac{u_1}{(1+ad)}\left(\frac{1+ad+bac-acb}{1+ad+bac}\right) = \frac{1}{1+ad+bac}u_1$$

$$y_1 = \frac{1}{1+ad+bac}u_1 \tag{6}$$

Now, substitute eq (6) in Taylor Rule to solve for i_1

$$i_1 = c\pi_1 + dy_1 \tag{Taylor Rule}$$

$$i_1 = c\frac{b}{1+ad+bac}u_1 + d\frac{1}{1+ad+bac}u_1 = \frac{bc+d}{1+ad+bac}u_1$$

$$i_1 = \frac{bc+d}{1+ad+bac}u_1 \tag{7}$$

0.2 Part B

When $u_2 = 0$, using eq (3), solving for π_2.

$$\pi_2 = \frac{1+ad+ba}{1+ad+bac}\pi_1 + \frac{b}{1+ad+bac}u_2$$

Now, substitute π_1

$$\pi_2 = \left(\frac{1+ad+ba}{1+ad+bac}\right)\left(\frac{b}{1+ad+bac}\right)u_1$$

$$\pi_2 = \frac{b(1+ad+ba)}{(1+ad+bac)^2}u_1 \tag{8}$$

0.3 Part C

Part A remains unchanged.
Part B, $\pi_2 = 0$

$$\pi_2 = \frac{1 + ad + ba}{1 + ad + bac}\pi_1 + \frac{b}{1 + ad + bac}u_2$$

where $\pi_2^e = 0, \pi_2^e = \pi_1$; we know that $u_2 = 0$

$$\pi_2 = 0$$