

## Motivation

Segmentation **Foundation Models**, like Segment Anything (SAM) [2], can still fail if:

- Data is scarce
- Objects are occluded
- Examples are out-of-distribution

Also: The prior knowledge of geometric properties is rarely considered in image segmentation

But: **Provably** ensuring constraints is hard and costly

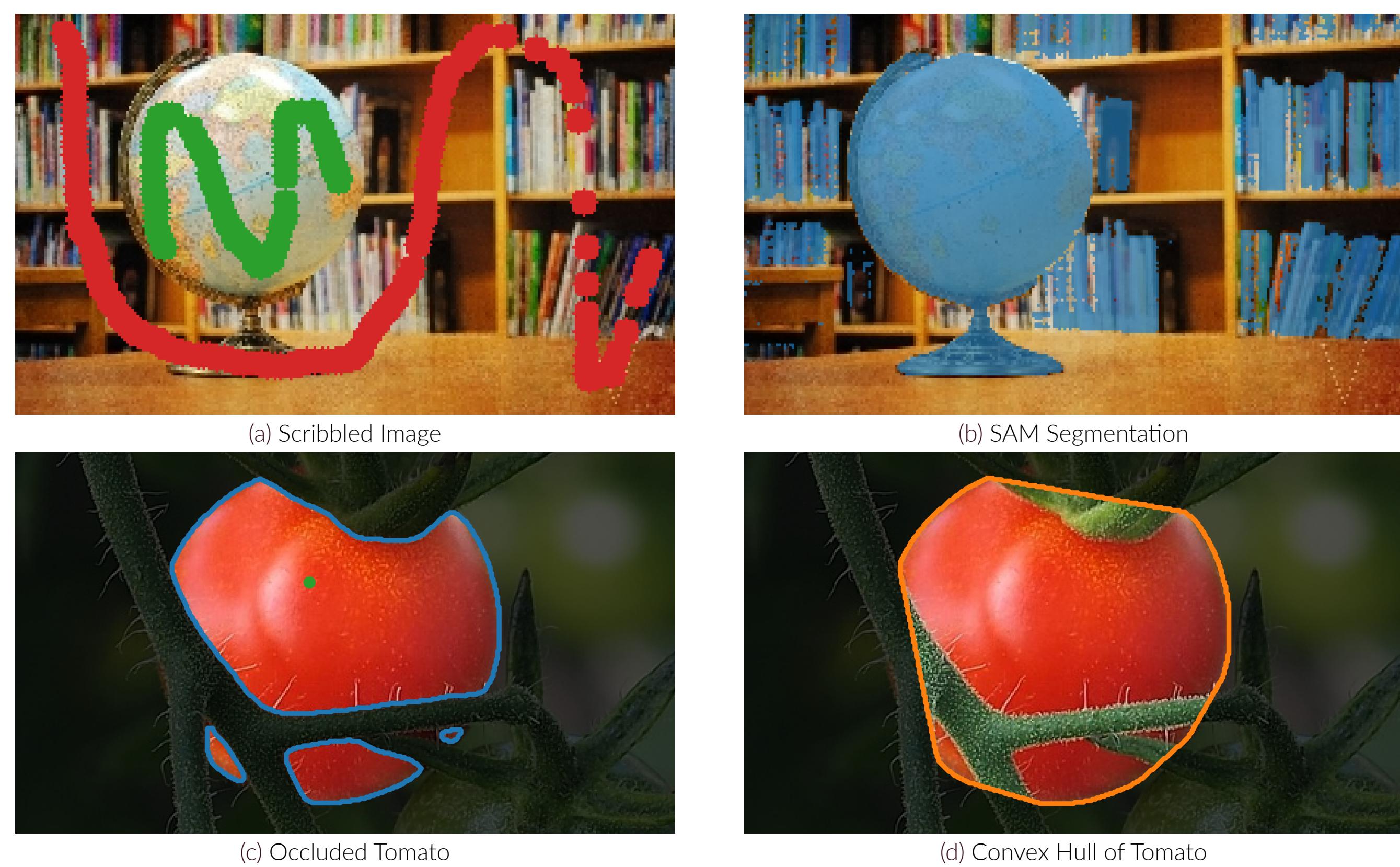


Figure 1. Segmenting a scribbled image (a), with SAM (b) leads to a scattered result. The partial occlusion of a tomato (c) leads to a disjointed segmentation in SAM, while for volume estimation, a volume-preserving segmentation (d) would be advantageous.

Can provable geometric constraints improve segmentation quality?

## Proposal

### Implicit Segmentation Representations (ISR)

- Mapping spatial coordinates to fore- or background
- Foreground is **provably** assured to have one of the properties: Convex, Star-Shaped, Mirror & Rotational Symmetric or Path-Connected

### Regularization for Shapes

- Using a input convex neural network (ICNN) [3], different input coordinate systems or diffeomorphism + ICNN
- Goes with any prediction architecture: Constrained ISR is used as a regularizer
- Capable of joint optimization



Figure 2. Naïve segmentations (blue) and learned convex, star-shaped, mirror-symmetric, and path-connected ISRs (orange).

## Method

### Constraint ISR

- Represent segmentations as a function  $\mathcal{G}_\nu(x) : \mathbb{R}^2 \rightarrow \mathbb{R}$  implicitly via a neural network  
⇒  $\mathcal{G}_\nu(x)$  maps every pixel to its foreground likelihood
- Operating in the (image) coordinate system  $x = c(f), x \in \mathbb{R}^2$  for an image  $f \in \mathbb{R}^{n_y \times n_x \times 3}$
- Optional: coordinate transform function  $c : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^2$  mapping spatial image domain  $\Omega$
- For complete definitions and proofs of all ISRs we refer to our paper [4].

### Project or Regularize

Given:

- Segmentation predictor  $\mathcal{N}_\theta(f) \in [0, 1]^{n_y \times n_x}$ ; can be any neural network, or simple unaries
- One of our proposed constrained ISRs  $\mathcal{G}_\nu$

The output of  $\mathcal{N}_\theta$  can be projected on  $\mathcal{G}_\nu$  by:

$$\text{dist}(\mathcal{N}_\theta(f), S) = \min_\nu \|\sigma(\mathcal{N}_\theta(f)) - \sigma(\mathcal{G}_\nu(c(f)))\| \quad (1)$$

for  $S$  as the set of functions represented by  $\mathcal{G}_\nu$ , and  $\sigma$  being the sigmoid function.

Further: (1) can be used as a **regularizer** during training of  $\mathcal{N}_\theta \Rightarrow$  optimizing  $\theta$  and  $\nu$  jointly

## Numerical Experiments

- Convex ISR is investigated using a scribble-based convexity dataset and two simple architectures for  $\mathcal{N}_\theta$  [1]:
  1. Convolutional Neural Network (CNN)
  2. Fully Connected Network (FCN)
- Using RGB, spatial, and/or semantic input features per pixel

Table 1. Intersection over union (IoU) of foreground objects with predictors  $\mathcal{N}_\theta$  and our proposed convex ISR

	RGB+semantic CNN / convex	RGB+spatial+semantic FCN / convex	RGB+semantic CNN / convex	RGB+spatial+semantic FCN / convex
seq.	0.726 / <b>0.843</b>	0.714 / <b>0.851</b>	<b>0.778</b> / 0.766	0.736 / <b>0.746</b>
joint	0.818 / <b>0.899</b>	0.635 / <b>0.894</b>	0.805 / <b>0.809</b>	0.768 / <b>0.769</b>

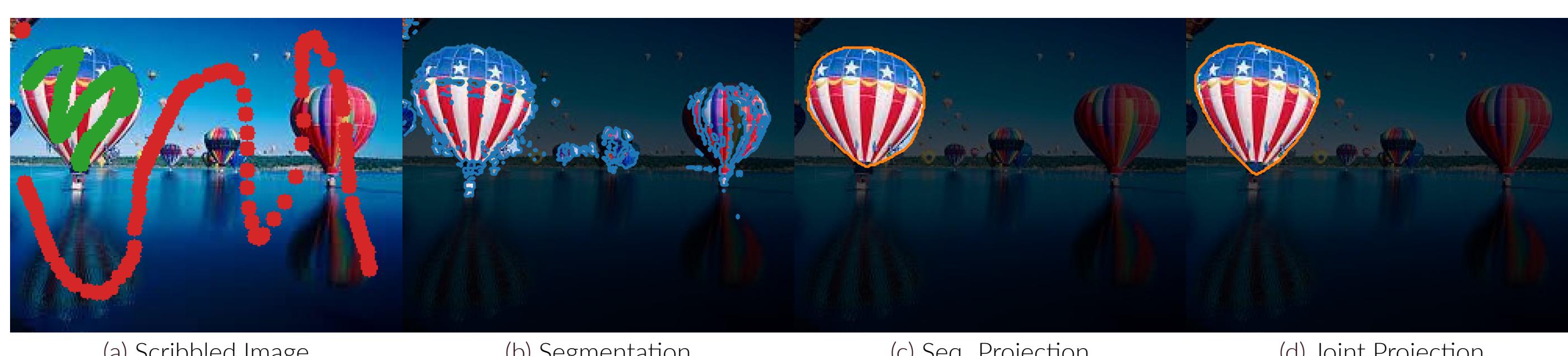


Figure 3. Results of the FCN architecture, trained on scribbles with RGB + semantic features.

▪ Path-connected ISR was evaluated on the FBMS-59 dataset

- Primary predictor  $\mathcal{N}_\theta$  is a UNet with training scheme proposed by [5]
- Sequential fit yields 1 %, joint projection 2%, IoU increase over baseline across all image sequences
- Path-Connected ISR is particularly useful when  $\mathcal{N}_\theta$  segments multiple objects. In joint training, these disappear completely, see Fig. 4.



Figure 4. Left: Sequential prediction (blue) and projection (orange), Right: Both, after joint training. ISR enforces  $\mathcal{N}_\theta$  to focus on the right car and accuracy increases significantly.

## Numerical Experiments Cont.

- Constraint ISRs are not limited to spatial image dimensions
- The input  $x$  of  $\mathcal{G}_\nu$  can be configured to include further features like depth or time

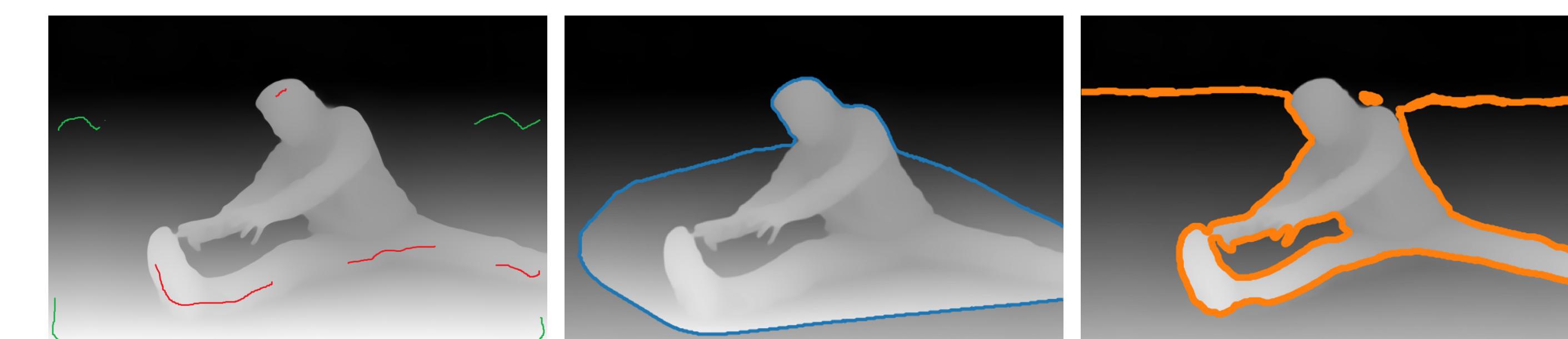


Figure 5. Illustrating of a naïve segmentation (blue) and convex ISR (orange) in  $(x, y, \text{depth})$ -space. For the ISR, we segment the floor "plane", which is convex in  $(x, y, \text{depth})$ .

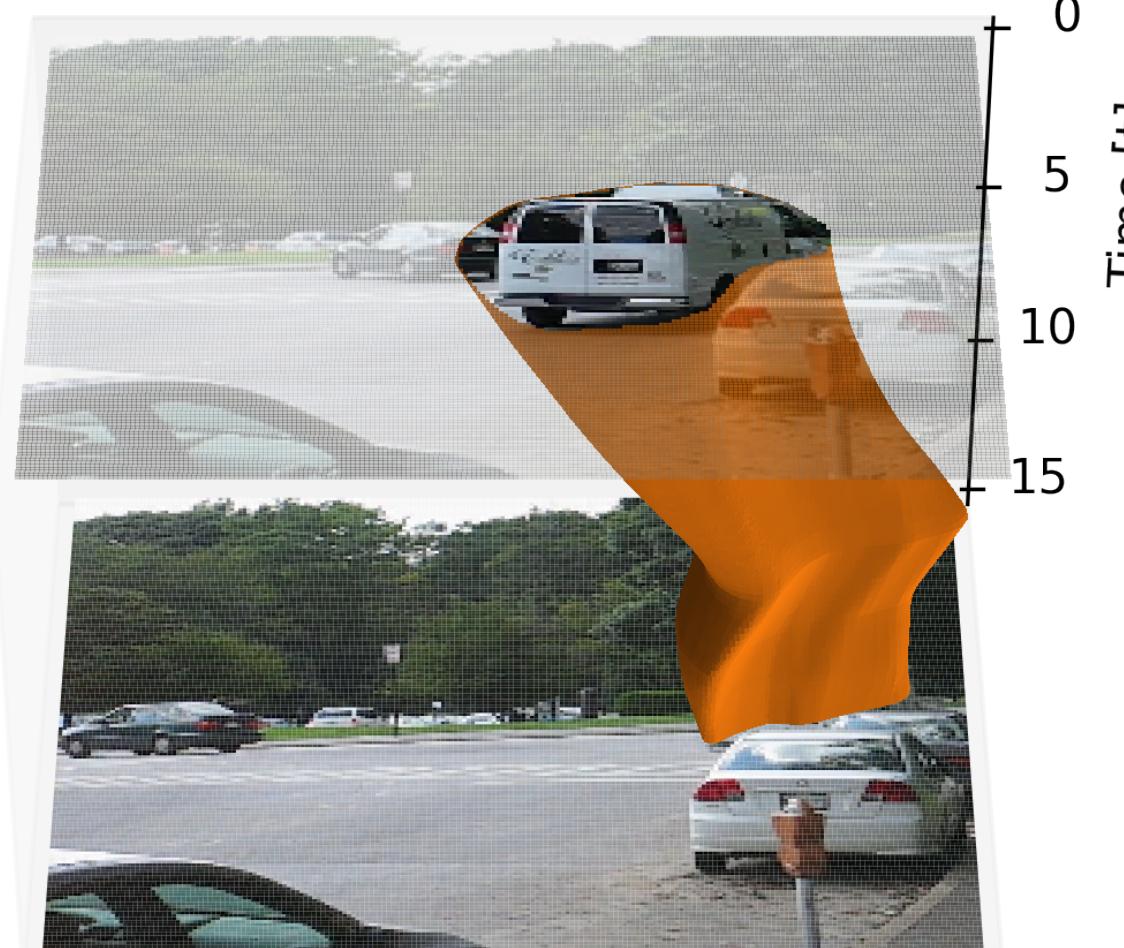


Figure 6. Spatio-temporal path-connected ISR along an image sequence. ISR was 6x super-resolved in time.

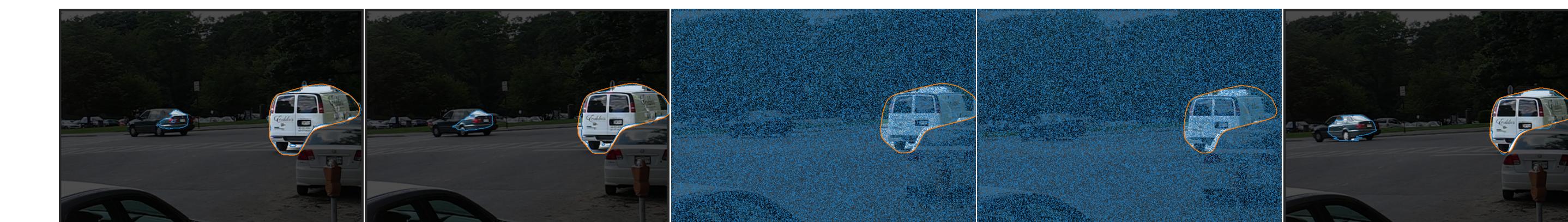


Figure 7. Spatio-temporal path-connected ISR, when trained on unaries which were partially (20 %) replaced by pure noise. The frames shown are index 10 to 14 of the cars3 sequence.

## Conclusion

- Proposed a variety of **provably** geometric constraints using *Implicit Representations* for image segmentation
- The ISRs with **implicitly enforced constraints** can improve the results: Especially if the missing condition is the main point of failure
- The **joint fit is superior** to a sequential one
- ISRs can be used in a variety of ways and further extended beyond segmentations in the spatial image domain

## References

- [1] Hannah Dröge and Michael Moeller, Learning or modelling? an analysis of single image segmentation based on scribble information. In *International Conference on Image Processing (ICIP)*, 2021.
  - [2] Alexander Kirillov et al. Segment anything. *CoRR*, 2023.
  - [3] Brandon Amos et al. Input convex neural networks. In *International Conference on Machine Learning*, 2017.
  - [4] Jan Philipp Schneider et al. Implicit Representations for Constrained Image Segmentation. In *International Conference on Machine Learning*, 2024.
  - [5] Amirhossein Kardoust and Margret Keuper. Uncertainty in minimum cost multicut for image and motion segmentation. In *Uncertainty in Artificial Intelligence*, 2021.
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For the code and link to our paper, scan the QR-Code or visit: <https://github.com/jp-schneider/awesome>

