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Unit – 3 ⇝ Statistics – I

Method 1 ⇝ Frequency Distributions

Introduction

→ Statistics is the branch of science where we plan, gather and analyze information about a particular collection of objects under investigation.

→ Statistics techniques are used in every other field of science, engineering and humanity, ranging from computer science to industrial engineering to sociology and psychology.

→ For any statistical problem the initial information collection from the sample may look messy, and hence confusing. This initial information needs to be organized first before we make any sense out of it.

Univariate Analysis

→ Univariate analysis involves the examination across cases of one variable at a time.

→ There are few major characteristics of a single variable that we tend to look at:

(1) Frequency distribution

(2) Central tendency

(3) Dispersion

(4) Moment

(5) Skewness

(6) Kurtosis

Frequency Distribution

→ The distribution is a summary of the frequency of individual values or ranges of values for a variable.

Types of Frequency Distribution

→ There are two types of frequency distribution which is

(1) Frequency distribution of **ungrouped data**

(2) Frequency distribution of **grouped data**

Frequency Distribution of Ungrouped Data

→ The ungrouped frequency distribution is a type of frequency distribution that displays the frequency of each **individual** data value.

→ For Example:

Marks of 10 students are 10, 25, 26, 35, 03, 08, 19, 29, 30, 18.

Frequency Distribution of Grouped Data

(1) **Discrete Frequency Distribution**

- A discrete frequency distribution is a type of frequency distribution that shows each number and the number of times it appears in a list.
- For Example:

  Data of students using library during exam time.

No. reading hours (x_i)	1	2	3	4	5	6
No. of students (f_i)	4	7	8	9	10	2

(2) **Continuous Frequency Distribution**

- A continuous frequency distribution is a series in which the data are classified into different **class intervals**.
- For Example:

  Data of students using library during exam time.

No. reading hours (x_i)	$0-2$	$3-5$	$6-8$	$9-11$
No. of students (f_i)	11	7	8	0

Exclusive Class

→ If classes of frequency distributions are $0-2, 2-4, 4-6, \ldots$ such classes are known as exclusive classes.

Inclusive Class

→ If classes of frequency distributions are $0-2, 3-5, 6-8, \ldots$ such classes are known as inclusive classes.

Lower Boundary & Upper Boundary

→ For the class $x_i - x_{i+1}$,

lower boundary is $\mathbf{x_i}$ and upper boundary is $\mathbf{x_{i+1}}$.

→ For Example:

For the class $3 - 5$, Lower boundary is 3 and upper boundary is 5.

Class Length

→ A class length for class $x_i - x_{i+1}$ is denoted by **c** and defined as

$c = \text{Upper Boundary} - \text{Lower Boundary} = x_{i+1} - x_i$

→ For Example:

For the class $3 - 5$, class length is $5 - 3 = 2$.

Mid-Point of Class

→ Mid point of class is used to find x_i in case of continuous frequency distribution.

→ It is defined as follow:

$$\text{Mid point of class} = x_i = \frac{\text{Lower Boundary} + \text{Upper Boundary}}{2}$$

→ For Example:

For the class $3 - 5$, mid point is $\frac{3+5}{2} = 4$.

Method 2 ⇝ Measure of Central Tendency

Central Tendency

→ The central tendency of a distribution is an estimate of the **center** of a distribution of values.

→ There are three measures to estimate central tendency which is

(1) Mean($\bar{x}$)

(2) Median(M)

(3) Mode(Z)

2.1 Mean

→ The mean means **average**.

→ Mean is denoted by " $\bar{x}$ " and read as x bar.

→ Table of different formulae of mean.

Method	Ungrouped Data	Discrete Grouped Data	Continuous Grouped Data
Direct Method	$\frac{\sum x_i}{n}$	$\frac{\sum f_i x_i}{\sum f_i}$	
Assumed Mean Method	$\frac{\sum d_i}{n}$	$A + \frac{\sum f_i d_i}{\sum f_i}$	
Step Deviation Method	-----	-----	$A + \frac{\sum f_i u_i}{\sum f_i} \times c$

→ n = total number of observations

→ In case of **continuous frequency distribution**,

x_i = mid value of the respective class.

→ In case of **assumed mean method**, A can be any value from x_i.

→ Use below formula to calculate d_i & u_i

$$d_i = x_i - A \quad ; \quad u_i = \frac{x_i - A}{c}$$

Example of Method-2.1: Examples of Mean

<table>
<tr><td>C</td><td>1</td><td>Find the mean of data 10.2, 9.5, 8.3, 9.7, 9.5, 11.1, 7.8, 8.8, 9.5, 10.

Answer: 9.44</td></tr>
<tr><td>H</td><td>2</td><td>Find mean of following data:
(a) 2, 8, 4, 6, 10, 12, 4, 8, 14, 16
(b) 10, 9, 21, 16, 14, 18, 20, 18, 14, 18, 23, 16, 18, 4

Answer: (a) 8.4, (b) 15.6429</td></tr>
<tr><td>C</td><td>3</td><td>Find the mean for following data:

<table>
<tr><td>Marks obtained</td><td>20</td><td>9</td><td>25</td><td>50</td><td>40</td><td>80</td></tr>
<tr><td>Number of students</td><td>6</td><td>4</td><td>16</td><td>7</td><td>8</td><td>2</td></tr>
</table>
Answer: 32.23</td></tr>
<tr><td>H</td><td>4</td><td>Find the mean for following data:

<table>
<tr><td>Weight of students</td><td>18</td><td>22</td><td>30</td><td>35</td><td>39</td><td>42</td><td>45</td><td>47</td></tr>
<tr><td>Number of students</td><td>4</td><td>5</td><td>8</td><td>8</td><td>16</td><td>4</td><td>2</td><td>3</td></tr>
</table>
Answer: 34.5</td></tr>
<tr><td>H</td><td>5</td><td>Find the mean for following data:

<table>
<tr><td>x</td><td>10</td><td>20</td><td>36</td><td>40</td><td>50</td><td>56</td><td>60</td><td>70</td><td>72</td><td>80</td><td>88</td><td>92</td><td>95</td></tr>
<tr><td>f</td><td>1</td><td>1</td><td>3</td><td>4</td><td>3</td><td>2</td><td>4</td><td>4</td><td>1</td><td>1</td><td>2</td><td>3</td><td>1</td></tr>
</table>
Answer: 59.3</td></tr>
<tr><td>C</td><td>6</td><td>Find the mean using direct method, assumed mean method and step deviation method:

<table>
<tr><td>Marks</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr>
<tr><td>No. of students</td><td>5</td><td>10</td><td>40</td><td>20</td><td>25</td></tr>
</table>
Answer: 30</td></tr>
</table>

H	7	Find the mean if survey regarding the weights (kg) of 45 students of class X of a school was conducted and the following data was obtained: x: 20 – 25 | 25 – 30 | 30 – 35 | 35 – 40 | 40 – 45 | 45 – 50 | 50 – 55 f: 2 | 5 | 8 | 10 | 7 | 10 | 3 **Answer: 38.83**
H	8	The following data represents the no. of foreign visitors in a multinational company in every 10 days during last 2 months. Use the data to find the mean. Class: 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 No. of visitors: 12 | 18 | 27 | 20 | 17 | 06 **Answer: 28**
T	9	Find the missing frequency from the following data if mean is 19.92. Class: 4 – 8 | 8 – 12 | 12 – 16 | 16 – 20 | 20 – 24 f: 11 | 13 | 16 | 14 | ? 24 – 28 | 28 – 32 | 32 – 36 | 36 – 40 9 | 17 | 6 | 4 **Answer: 10**
C	10	Find the missing frequency f_1 and f_2 in the table given below, it is being given that the mean of the given frequency distribution is 50. Class: 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 | Total f: 17 | f_1 | 32 | f_2 | 19 | 100 **Answer: $f_1 = 18$, $f_2 = 14$**
T	11	A co-operative bank has two branches employing 50 and 70 workers respectively. The average salaries paid by two respective branches are 360 and 390 rupees per month. Calculate the mean of the salaries of all the employees. **Answer: 377.5**

T	12	Find the mean of the following frequency distribution:

Mid value	15	20	25	30	35	40	45	50	55
Frequency	2	22	19	14	3	4	6	1	1
Cumulative	2	24	43	57	60	64	70	71	72

Answer: 27.8472

2.2 Median

→ The median is the value found at the **exact middle** of the set of values.

→ Median is denoted by capital letter "**M**".

→ To compute the median, list all observations in ascending order and then locate the value in the center of the sample.

→ Table of formula of median.

Data	Formula
Ungrouped Data	If n is **odd**, then $M = \left(\frac{n+1}{2}\right)^{th}$ observation
Discrete Grouped Data	If n is **even**, then $M = \frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$
Continuous Grouped Data	$M = L + \left(\frac{\frac{n}{2} - F}{f}\right) \times c$

→ Where,

Median class = Class whose cumulative frequency with property $\min\left\{ cf \mid cf \geq \frac{n}{2} \right\}$

L = Lower boundary point of the median class

n = Total number of observation (sum of the frequencies)

F = Cumulative frequency of the class preceding the median class

f = The frequency of the median class

Example of Method-2.2: Median

<table>
<tr><td>C</td><td>1</td><td>Find the median of following data:
20, 25, 30, 15, 17, 35, 26, 18, 40, 45, 50.

Answer: 26</td></tr>
<tr><td>H</td><td>2</td><td>Find the median of following data:
(a) 6, 20, 43, 50, 19, 53, 0, 37, 78, 1, 15.
(b) 10, 34, 27, 24, 12, 27, 20, 18, 15, 30.
(c) 110, 115, 108, 112, 120, 116, 140, 135, 128, 132.

Answer: (a) 20, (b) 22, (c) 118</td></tr>
<tr><td>H</td><td>3</td><td>If the median of the data is 2, find the value of a: −9, −4, a, 5, 8, 11.

Answer: − 1</td></tr>
<tr><td>C</td><td>4</td><td>The given observations have been arranged in ascending order. If the median of the data is 63, find the value of x for the following data:
29, 32, 48, 50, x, x + 2, 72, 78, 84, 95.

Answer: x = 62</td></tr>
<tr><td>H</td><td>5</td><td>Obtain the median size of shoes sold from the following data:

<table>
<tr><td>Size</td><td>5</td><td>5.5</td><td>6</td><td>6.5</td><td>7</td><td>7.5</td><td>8</td></tr>
<tr><td>Pair</td><td>30</td><td>40</td><td>50</td><td>150</td><td>300</td><td>600</td><td>950</td></tr>
</table>
Answer: 7.5</td></tr>
<tr><td>C</td><td>6</td><td>Calculate the median for the following data:

<table>
<tr><td>Marks</td><td>20</td><td>9</td><td>25</td><td>50</td><td>40</td><td>80</td></tr>
<tr><td>No. of students</td><td>6</td><td>4</td><td>16</td><td>7</td><td>8</td><td>2</td></tr>
</table>
Answer: 25</td></tr>
</table>

<table>
<tr><td>C</td><td>7</td><td>The following table gives marks obtained by 50 students in statistics. Find the median.
<table>
<tr><td>Marks</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr>
<tr><td>No. of students</td><td>16</td><td>12</td><td>18</td><td>3</td><td>1</td></tr>
</table>
Answer: 17.5</td></tr>
<tr><td>H</td><td>8</td><td>Calculate the missing frequency from the following distribution, it is being given that the median of the distribution is 24.
<table>
<tr><td>Marks</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr>
<tr><td>No. of students</td><td>5</td><td>25</td><td>x</td><td>18</td><td>7</td></tr>
</table>
Answer: x = 25</td></tr>
<tr><td>C</td><td>9</td><td>The median of 60 observations (following data) is 28.5. Find x and y.
<table>
<tr><td>Marks</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td><td>50 – 60</td></tr>
<tr><td>No. of students</td><td>5</td><td>x</td><td>20</td><td>15</td><td>y</td><td>5</td></tr>
</table>
Answer: x = 8, y = 7</td></tr>
<tr><td>H</td><td>10</td><td>The following table gives the marks obtained by 50 students in mathematics. Find the median.
<table>
<tr><td>x</td><td>10–14</td><td>15–19</td><td>20–24</td><td>25–29</td><td>30–34</td><td>35–39</td><td>40–44</td><td>45–49</td></tr>
<tr><td>f</td><td>4</td><td>6</td><td>10</td><td>5</td><td>7</td><td>3</td><td>9</td><td>6</td></tr>
</table>
Answer: 29.5</td></tr>
</table>

2.3 Mode

→ The mode is the **most frequently** occurring value in the set.

→ Mode is denoted by capital letter "**Z**".

→ The mode is not necessarily unique, like mean and median. we can have data with two modes (bi-modal) or more than two modes (multi-modal).

→ Table of formula of mode.

Data	Formula
Ungrouped Data	Most repeated observation among given data
Discrete Grouped Data	Highest frequency among given data
Continuous Grouped Data	$Z = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c$

→ Where,

Modal class = A class with highest frequency

L = Lower boundary of modal class

c = Class length

f_1 = Frequency of the modal class

f_0 = Frequency of the class before the modal class

f_2 = Frequency of the class after the modal class

Relation Between Mean, Median and Mode

→ $Z = 3M - 2\bar{x}$; where $\bar{x}$ = Mean, M = Median, Z = Mode

Example of Method-2.3: Mode

<table>
<tr><td>C</td><td>1</td><td>If mean is 16 and median is 20. Calculate the mode.

Answer: 28</td></tr>
<tr><td>C</td><td>2</td><td>Find the mode of following data:
(a) 2, 4, 2, 5, 7, 2, 8, 9.
(b) 2, 8, 4, 6, 10, 12, 4, 8, 14, 16.

Answer: (a) 2, (b) 4 & 8</td></tr>
<tr><td>C</td><td>3</td><td>Find the mode of following data:
<table><tr><td>x</td><td>11</td><td>22</td><td>33</td><td>44</td></tr><tr><td>f</td><td>15</td><td>20</td><td>19</td><td>10</td></tr></table>
Answer: 22</td></tr>
<tr><td>H</td><td>4</td><td>Find the mode of following data:
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr><tr><td>f</td><td>8</td><td>10</td><td>11</td><td>16</td><td>20</td><td>25</td><td>15</td><td>9</td><td>6</td></tr></table>
Answer: 6</td></tr>
<tr><td>T</td><td>5</td><td>Find the mode from the following frequency distribution:
<table><tr><td>x</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td></tr><tr><td>f</td><td>5</td><td>6</td><td>8</td><td>7</td><td>9</td><td>8</td><td>9</td><td>6</td></tr></table>
Answer: 12 & 14</td></tr>
<tr><td>C</td><td>6</td><td>Find the mode of following data:
<table><tr><td>Class</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr><tr><td>f</td><td>3</td><td>5</td><td>7</td><td>10</td><td>12</td></tr><tr><td></td><td>50 – 60</td><td>60 – 70</td><td>70 – 80</td><td>80 – 90</td><td>90 – 100</td></tr><tr><td></td><td>15</td><td>12</td><td>6</td><td>2</td><td>8</td></tr></table>
Answer: 55</td></tr>
</table>

H | 7 | Find the mode of following data:

Class	200 – 220	220 – 240	240 – 260	260 – 280
f	7	15	21	19

280 – 300	300 – 320	320 – 340
6	4	2

Answer: 255

H | 8 | Find the mode of following data:

Class	400 – 500	500 – 600	600 – 700	700 – 800	800 – 900
f	8	16	20	17	3

Answer: 657.14

H | 9 | The mode of the following data is 67. Find the missing frequency x.

Amount	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Frequency	5	x	15	12	7

Answer: 8

T | 10 | An insurance company obtained the following data for accident claims (in thousand rupees) from a particular region. Find its mean, median and mode.

Amount	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency	6	47	75	46	18	8

Answer: $\bar{x} = 6.47$, $M = 6.2533$, $Z = 5.9825$

C | 11 | Obtain the mean, mode and median for the following information:

x	< 10	< 20	< 30	< 40	< 50	< 60
f	12	30	57	77	94	100

Answer: $\bar{x} = 28$, $M = 27.407$, $Z = 25.625$

T | 12 | Obtain the mean, median and mode for the following information:

Marks	0 <	10 <	20 <	30 <
Number of Students	50	38	20	5

Answer: $\bar{x} = 17.6$, $M = 17.2222$, $Z = 16.6667$

Method 3 ⇝ Dispersion

Dispersion

→ Dispersion refers to the **spread** of the values around the central tendency.

→ For Example:

$-5, 0, 5$ and $-50, 0, 50$ both have the same mean 0 but clearly the data given in the second case much more widely dispersed than those in the first case.

→ So, measures of central tendency are not sufficient for having some idea about dispersion.

→ Measures of dispersion gives the idea about the degree to which numerical data tend to spread about an average life.

→ There are certain measures of dispersion which is,

(1) Range

(2) Standard Deviation

(3) Mean Deviation

Range

→ Range is simply the highest value **minus** the lowest value of a set of data values.

→ For Example:

Range of $-5, 0, 5$ is 10

Reason: Range $=$ Highest value $-$ lowest value

$= 5 - (-5)$

$= 10$

Standard Deviation

→ Standard deviation is a measure that is used to quantify the amount of variation or dispersion of a set of data values.

→ It is denoted by "**σ**" and read as “sigma”.

→ Table of different formulae of standard deviation.

Method	Ungrouped Data	Discrete Grouped Data	Continuous Grouped Data
Direct Method	$\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$	$\sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$	
Assumed Mean Method	$\sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2}$	$\sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$	
Step Deviation Method	-----	-----	$\sqrt{\frac{\sum f_i u_i^2}{\sum f_i} - \left(\frac{\sum f_i u_i}{\sum f_i}\right)^2} \times c$

Variance

→ Variance is **expectation** of the squared deviation.

→ It informally measures how far a set of (random) numbers are spread out from their mean.

→ It is denoted by capital letter "**V**" and defined as $V = \sigma^2$.

Coefficient of Variation

→ The Coefficient of Variation is the **ratio** of the standard deviation to the mean and shows the extent of variability in relation to the mean of the population.

→ Coefficient of Variance is defined as

$$C.V. = \frac{\sigma}{\bar{x}} \times 100$$

→ If C.V. is high, then it is less consistent. Similarly, if C.V. is less, then it is more consistent.

→ The higher the Coefficient of Variation, the greater the dispersion.

Mean Deviation

→ The mean deviation is defined as a statistical measure that is used to calculate the average deviation from the mean value of the given data set.

→ In a simple word, the mean deviation is used to calculate how far the values fall from the middle of the data set.

→ Table of different formulae of mean deviation:

Method	Ungrouped Data	Grouped Data
M.D. about Mean	$\frac{\sum \lvert x_i - \bar{x} \rvert}{n}$	$\frac{\sum f_i \lvert x_i - \bar{x} \rvert}{\sum f_i}$
M.D. about Median	$\frac{\sum \lvert x_i - M \rvert}{n}$	$\frac{\sum f_i \lvert x_i - M \rvert}{\sum f_i}$
M.D. about Mode	$\frac{\sum \lvert x_i - Z \rvert}{n}$	$\frac{\sum f_i \lvert x_i - Z \rvert}{\sum f_i}$

Example of Method-3: Dispersion

C	1	Find the standard deviation for the following data: 6, 7, 10, 12, 13, 4, 8, 12. **Answer: 3.0414**
H	2	Find the standard deviation for the following distribution: x: 5, 15, 25, 30 f: 2, 1, 1, 3 **Answer: 10.6104**

C	3	Find the standard deviation and variance for the following distribution: <table><tr><td>x</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td><td>50 – 60</td><td>60 – 70</td></tr><tr><td>f</td><td>6</td><td>14</td><td>10</td><td>8</td><td>1</td><td>3</td><td>8</td></tr></table> **Answer: $\sigma = 19.6214$, $V = 384.9993$**
T	4	Find the standard deviation for the following distribution: <table><tr><td>Class</td><td>0 – 100</td><td>100 – 200</td><td>200 – 300</td><td>300 – 400</td></tr><tr><td>f</td><td>6</td><td>10</td><td>18</td><td>20</td></tr><tr><td></td><td>400 – 500</td><td>500 – 600</td><td>600 – 700</td><td>700 – 800</td></tr><tr><td></td><td>15</td><td>12</td><td>10</td><td>9</td></tr></table> **Answer: 196.21**
H	5	Find the standard deviation and variance of the mark distribution of 30 students at mathematics examination in a class as below: <table><tr><td>Class</td><td>10 – 25</td><td>25 – 40</td><td>40 – 55</td><td>55 – 70</td><td>70 – 85</td><td>85 – 100</td></tr><tr><td>f</td><td>2</td><td>3</td><td>0</td><td>14</td><td>8</td><td>3</td></tr></table> **Answer: $\sigma = 19.3391$, $V = 374.0008$**
C	6	The arithmetic means of runs scored by three batsmen A, B and C, in the same series of 10 innings, are 50, 48 and 12 respectively. The standard deviations of their runs are 15, 12 and 2 respectively. Who is the most consistent of the three? **Answer: Batsman C is more consistent.**
H	7	Runs scored by two batsmen A, B in 9 consecutive matches is given below: <table><tr><td>A</td><td>85</td><td>20</td><td>62</td><td>28</td><td>74</td><td>5</td><td>69</td><td>4</td><td>13</td></tr><tr><td>B</td><td>72</td><td>4</td><td>15</td><td>30</td><td>59</td><td>15</td><td>49</td><td>27</td><td>26</td></tr></table>Which of the batsman is more consistent? **Answer: Batsman B is more consistent.**

C	8	Two machines A, B are used to fill a mixture of cement concrete in a beam. Find the standard deviation of each machine & comment on the performances of two machines. A: 32, 28, 47, 63, 71, 39, 10, 60, 96, 14 B: 19, 31, 48, 53, 67, 90, 10, 62, 40, 80 **Answer:** $\sigma_A = 25.4950$, $\sigma_B = 24.4290$ **There is less variability in the performance of the machine B.**
H	9	Goals scored by two team A and B in a football season were as shown in the table. Find out which team is more consistent. Number of goals in a match: 0, 1, 2, 3, 4 Team A: 27, 9, 8, 5, 4 Team B: 17, 9, 6, 5, 3 **Answer: Team B is more consistent.**
T	10	An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results: Number of workers: Firm A 500, Firm B 600 Average daily wage: Firm A 186, Firm B 175 Variance of distribution of wages: Firm A 81, Firm B 100 (1) Which firm has a larger wage bill? (2) In which firm, is there greater variability in individual wages? (3) Calculate average daily wages of all the workers in the firms A & B taken together. **Answer: (1) Firm B, (2) Firm B, (3) 180**

H | 11 | Lives of two models of refrigerators turned in for new models in a recent survey are given in the adjoining table.

Life (in year)	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
Model A	5	16	13	7	5	4
Model B	2	7	12	19	9	1

(1) What is the average life of each model of these refrigerators?

(2) Which model shows more uniformity?

Answer: **(1) 5.12 & 6.16, (2) Model B**

H | 12 | Find the mean deviation about the mean and median for the following data:

2, 4, 7, 8, 9.

Answer: $\mathbf{MD(\bar{x}) = 2.4}$, $\mathbf{MD(M) = 2.2}$

T | 13 | Find the mean deviation about the mean, median and mode for the following data:

5, 10, 17, 20, 23, 20.

Answer: $\mathbf{MD(\bar{x}) = 5.5556}$, $\mathbf{MD(M) = 5.1667}$, $\mathbf{MD(Z) = 5}$

H | 14 | Find mean deviation about the mean, median and mode for the following data:

x	2	5	6	8	10	12
f	2	8	10	7	8	5

Answer: $\mathbf{MD(\bar{x}) = MD(M) = MD(Z) = 2.3}$

C | 15 | Find mean deviation about the mean, median and mode for the following data:

x	5	10	15	20	25
f	7	4	6	3	5

Answer: $\mathbf{MD(\bar{x}) = 6.32}$, $\mathbf{MD(M) = 6.2}$, $\mathbf{MD(Z) = 9}$

C	16	Find mean deviation about the mean, median and mode for the following data: Class: 5 – 25 | 25 – 45 | 45 – 65 | 65 – 85 | 85 – 105 f: 12 | 8 | 14 | 20 | 6 **Answer: $MD(\bar{x}) = 21.33$, $MD(M) = 21.904$, $MD(Z) = 23.466$**
T	17	Find mean deviation about the mean, median and mode for the following data: Class: 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 f: 5 | 10 | 20 | 9 | 6 **Answer: $MD(\bar{x}) = 8.28$, $MD(M) = 8.20$, $MD(Z) = 8.30$**

Method 4 ⇝ Moments

Moments

→ Moment is the arithmetic **mean** of the various powers of the deviations of items from their assumed mean or actual mean.

→ There are three types of moments which is

(1) Moment about mean

(2) Moment about assumed mean

(3) Moment about zero

Moment bout mean

→ If deviations of data are taken from mean, then it is known as moment about mean.

→ It is also known as central moment.

→ It is denoted by " μ_r ".

Moment about assumed mean

→ If deviations of data are taken about any assumed value(a) from x_i, then it is known as moment about assumed mean.

→ It is denoted by " μ'_r ".

Moment about zero

→ If deviations of data are taken about any zero, it is known as moment about zero.

→ It is denoted by "v_r".

→ Table of different formulae of moment.

Moment about	Ungrouped Data	Grouped Data
Mean (μ_r)	$\frac{\sum(x_i - \bar{x})^r}{n}$	$\frac{\sum f_i (x_i - \bar{x})^r}{\sum f_i}$
Assumed mean (μ'_r)	$\frac{\sum(x_i - a)^r}{n}$	$\frac{\Sigma f_i (x_i - a)^r}{\sum f_i}$
Zero (v_r)	$\frac{\sum {x_i}^r}{n}$	$\frac{\Sigma f_i {x_i}^r}{\sum f_i}$

Where, $r = 1, 2, 3, 4, \ldots$

Example of Method-4: Moments

C	1	Find the first four moments about assumed mean 5, actual mean and zero for the data 1, 3, 7, 9, 10. **Answer:** $\mu = 0, \quad 12, \quad -12, \quad 208.8$ $\mu' = 1, \quad 13, \quad 25, \quad 233.8$ $v = 6, \quad 48, \quad 420, \quad 3808.8$
H	2	Find the first four moments about assumed mean 14, actual mean and zero for the data 11, 12, 14, 16, 20. **Answer:** $\mu = 0, \quad 10.24, \quad 19.162, \quad 213.5872$ $\mu' = 0.6, \quad 10.6, \quad 37.8, \quad 281.8$ $v = 14.6, \quad 223.4, \quad 3579.8, \quad 57225.8$
C	3	Calculate the four moments about assumed mean 5, actual mean and zero for following distribution. **Answer:** $\mu = 0, \quad 0.9333, \quad 0, \quad 2.5333$ $\mu' = -1, \quad 1.9333, \quad -3.8, \quad 9.1333$ $v = 4, \quad 16.9333, \quad 75.2, \quad 348.1333$
T	4	Calculate the four moments about assumed mean 15, actual mean and zero for following distribution. **Answer:** $\mu = 0, \quad 34, \quad 40.5, \quad 2707$ $\mu' = -1, \quad 35, \quad -62.5, \quad 2750$ $v = 14, \quad 230, \quad 4212.5, \quad 83375$

x	2	3	4	5	6
f	1	3	7	3	1

x	5	10	15	20	25
f	6	10	14	6	4

H	5	Calculate the moments about assumed mean 3, actual mean and zero for following distribution: x: 1, 2, 3, 4, 5, 6 f: 5, 4, 3, 7, 1, 1 **Answer:** $\mu = 0,\ 2.0862,\ 0.5017,\ 9.0299$ $\mu' = -0.0952,\ 2.0952,\ -0.0952,\ 8.9524$ $v = 2.9048,\ 10.5238,\ 43.1905,\ 191.6667$
C	6	Calculate the moments about assumed mean 25, actual mean and zero for following distribution: Class: 0 – 10, 10 – 20, 20 – 30, 30 – 40 f: 1, 3, 4, 2 **Answer:** $\mu = 0,\ 81,\ -144,\ 14817$ $\mu' = -3,\ 90,\ -900,\ 21000$ $v = 22,\ 565,\ 15850,\ 471625$
H	7	Calculate the moments about assumed mean 35, actual mean and zero for following distribution: x: 0 – 10, 10 – 20, 20 – 30, 30 – 40, 40 – 50, 50 – 60, 60 – 70 f: 8, 12, 20, 30, 15, 10, 5 **Answer:** $\mu = 0,\ 236.76,\ 264.336,\ 141290.0876$ $\mu' = -1.8,\ 240,\ -1020,\ 144000$ $v = 33.2,\ 1339,\ 60440,\ 2957125$

T	8	Calculate the moments about assumed mean 65, actual mean and zero for following distribution:

Class	60 – 62	63 – 65	66 – 68	69 – 71	72 – 74
f	5	18	42	27	8

Answer: $\mu = 0, \quad 8.5275, \quad -2.6933, \quad 199.3759$

$\mu' = 2.45, \quad 14.53, \quad 74.69, \quad 516.13$

$v = 67.45, \quad 4558.03, \quad 308586.79, \quad 20930221.03$

Method 5 ⇝ Skewness

Skewness

→ In symmetrical distribution, the mean, median, and mode is equal.

→ When a frequency distribution is not symmetrical, it is known as asymmetrical or skewed.

→ Skewness means **lake** of symmetry.

Measures of Skewness

→ Measure of skewness give us an idea about the extent of lopsidedness in a data.

→ The various measures of skewness are

(1) Karl Pearson's method

(2) Method of moments

Karl Pearson's Method

→ Karl Pearson's coefficient of skewness is defined as

$$\text{Skewness} = \frac{\text{Mean} - \text{Mode}}{\text{Standard Deviation}}$$

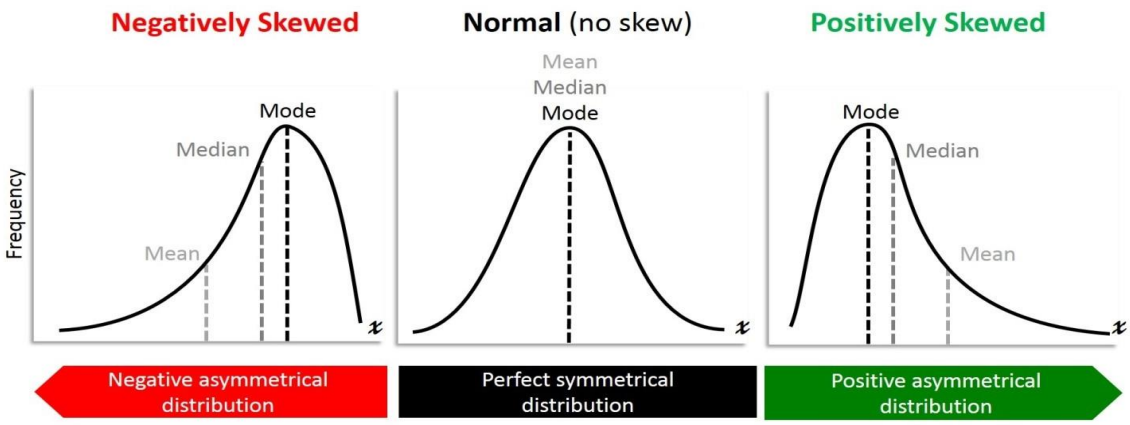
Method of Moments

→ Measure of skewness is obtained from moment about mean and defined as

$$\text{Skewness} = \beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3}$$

→ Note:

- The skewness value can be either positive or negative or zero or even undefined.
- If skewness value is zero, then the distribution is known as symmetric.
- If standard deviation is zero, then the skewness is not defined.
- **Positive skewness:** The right tail is longer; the mass of the distribution is concentrated on the left.
- **Negative skewness:** The left tail is longer; the mass of the distribution is concentrated on the right.



Example of Method-5: Skewness

C	1	Compute the Karl Pearson's coefficient of skewness for data: 25, 15, 23, 40, 27, 25, 23, 25, 20 **Answer: -0.03**
H	2	Find skewness by the method of moments for data: 38.2, 40.9, 39.5, 44, 39.6, 40.5, 39.5. **Answer: 1.3035**
C	3	Karl Pearson's coefficient of skewness of a distribution is 0.32, its standard deviation is 6.5 and mean is 29.6. Find the mode for the distribution. **Answer: 27.52**
H	4	Karl Pearson's coefficient of skewness of a distribution is 0.3, its variance is 8 and mean is 200. Find the mode and median for the distribution. **Answer: 195.2, 198.4**
C	5	For a group of 10 items, $\sum x = 452$, $\sum x^2 = 24270$ and mode is 43.7 then, find Karl Pearson's coefficient of skewness. **Answer: 0.077**

<table>
<tr><td>C</td><td>6</td><td>From the marks scored by 100 students in section A and 100 students in section B of a class, the following measures were obtained
<table>
<tr><td>Section A</td><td>$\mu_A = 55$</td><td>$\sigma_A = 15.4$</td><td>Mode = 58.72</td></tr>
<tr><td>Section A</td><td>$\mu_B = 53$</td><td>$\sigma_B = 15.4$</td><td>Mode = 48.83</td></tr>
</table>
Determine which distribution of marks is more skewed.

Answer: Section B is more skewed</td></tr>
<tr><td>T</td><td>7</td><td>From the marks scored by 120 students in section A and 120 students in section B of a class, the following measures were obtained
<table>
<tr><td>Section A</td><td>$\mu_A = 46.83$</td><td>$\sigma_A = 14.8$</td><td>Mode = 51.67</td></tr>
<tr><td>Section A</td><td>$\mu_B = 47.83$</td><td>$\sigma_B = 14.8$</td><td>Mode = 47.07</td></tr>
</table>
Determine which distribution of marks is more skewed.

Answer: Section A is more skewed</td></tr>
<tr><td>H</td><td>8</td><td>Find Karl Pearson’s coefficient of skewness and skewness based on the method of moments for the following data:
<table>
<tr><td>Class</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr>
<tr><td>f</td><td>13</td><td>20</td><td>30</td><td>25</td><td>12</td></tr>
</table>
Answer: -0.1135, 0.0085</td></tr>
<tr><td>T</td><td>9</td><td>Find skewness of the following data using method of moment.
<table>
<tr><td>x</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td><td>30</td><td>35</td></tr>
<tr><td>f</td><td>8</td><td>15</td><td>20</td><td>32</td><td>23</td><td>17</td><td>5</td></tr>
</table>
Answer: 0.01141</td></tr>
<tr><td>H</td><td>10</td><td>Prove that the skewness of the following data is 0.0390 using method of moment.
<table>
<tr><td>Class</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td></tr>
<tr><td>Frequency</td><td>1</td><td>3</td><td>4</td><td>2</td></tr>
</table></td></tr>
</table>

Method 6 ⇝ Kurtosis

Kurtosis

→ Measures of central tendency, dispersion and skewness of a random variable cannot give a complete idea about the probability distribution.

→ So, another characteristic, Kurtosis is required.

→ Kurtosis measures the flatness/peakedness of a distribution.

→ It is denoted by "β_2" defined as

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2}$$

→ Note:

- The greater the value of β_2, the more peaked is the distribution.
- When the value of $\beta_2 = 3$, the curve is normal curve and the distribution is known as **mesokurtic**.
- When the value of $\beta_2 > 3$, the curve is more peaked than normal curve and the distribution is known as **leptokurtic**.
- When the value of $\beta_2 < 3$, the curve is less peaked than normal curve and the distribution is known as **platykurtic**.

Example of Method-6: Kurtosis

<table>
<tr><td>H</td><td>1</td><td>Find the Kurtosis for the data 1, 3, 7, 9, 10. Also comment on type of distribution.

Answer: Kurtosis = 1.45, Distribution is platykurtic</td></tr>
<tr><td>C</td><td>2</td><td>Find the Kurtosis for the following data. Also comment on type of distribution.
<table>
<tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>f</td><td>1</td><td>3</td><td>7</td><td>3</td><td>1</td></tr>
</table>
Answer: Kurtosis = 2.9082, Distribution is platykurtic</td></tr>
</table>

<table>
<tr><td>H</td><td>3</td><td>Find the kurtosis for the following data. Also, Comment on type of distribution.
<table>
<tr><td>Class</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td></tr>
<tr><td>f</td><td>1</td><td>3</td><td>4</td><td>2</td></tr>
</table>
Answer: Kurtosis = 2.2583, Distribution is platykurtic</td></tr>
<tr><td>C</td><td>4</td><td>Find out the kurtosis of the following data:
<table>
<tr><td>Class</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr>
<tr><td>f</td><td>10</td><td>20</td><td>40</td><td>20</td><td>10</td></tr>
</table>
Answer: 2.5</td></tr>
<tr><td>H</td><td>5</td><td>Find out the kurtosis of the following data:
<table>
<tr><td>Class</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td></tr>
<tr><td>f</td><td>1</td><td>4</td><td>3</td><td>2</td></tr>
</table>
Answer: 2.102</td></tr>
<tr><td>T</td><td>6</td><td>Find the coefficient of variation, β_1 and β_2 for the following data:
<table>
<tr><td>x</td><td>170 – 180</td><td>180 – 190</td><td>190 – 200</td><td>200 – 210</td><td>210 – 220</td><td>220 – 230</td><td>230 – 240</td><td>240 – 250</td></tr>
<tr><td>f</td><td>52</td><td>68</td><td>85</td><td>92</td><td>100</td><td>95</td><td>70</td><td>28</td></tr>
</table>
Answer: C.V. = 9.4, $\beta_1 = 0.0034$, $\beta_2 = 2.0340$</td></tr>
</table>

*** * * * * End of the Unit * * * * ***