TSINGHUA UNIVERSITY

应用QMC与VRT于期权定价

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方法介绍

◎ 背景：风险中性世界中模拟Random Walk

◎ 目的：估计Black Scholes模型下期权的价格，并使其方差降低

◎ 方法：

- MC
- QMC—Sobol Sequence
- QMC —Sobol Sequence + Antithetic Variates
- QMC—Sobol Sequence + Control Variates
- QMC—Sobol Sequence + Importance Sampling

◎ 期权：

- European Option
- Asian Option—Arithmetic Average
- Asian Option—Geometric Average

方法介绍

MC

- $x_i \sim U(0,1)^d \rightarrow x_i' \sim N(0,1)^d \rightarrow Brownian\ Motion$

QMC—Sobol Sequence

比均匀分布更加均匀，可提高精度

- $x_i \sim Sobol\ Sequence^d \rightarrow random\ shift \rightarrow Brownian\ Motion$

$$P = \frac{1}{M}\sum_{i=1}^{M} C_T^i$$

方法介绍

Antithetic Variates

- $\frac{1}{M}\sum_{i=1}^{M} 0.5(f(x_i) + f(1 - x_i)), \forall x_i \sim U(0,1)^d$
- $\rightarrow \frac{1}{M}\sum_{i=1}^{M} 0.5(f(x_i') + f(-x_i')), \forall x_i' \sim N(0,1)^d$

Control Variates

- $\frac{1}{M}\sum_{i=1}^{M} f(x_i) + b\big(g(x_i) - Ig(x_i)\big), \forall\ x_i \sim U(0,1)^d, b = \frac{Cov(f,g)}{Var(g)}$

Importance Sampling

- $\frac{1}{M}\sum_{i=1}^{M} f(x_i)\frac{p(x_i)}{q(x_i)}, \forall x_i \sim q(x_i)$
- $\rightarrow E\big[I_{\{Z>b\}}\big] \approx \frac{1}{M}\sum_{i=1}^{M} I_{\{z_i>b\}} \approx \frac{1}{M}\sum_{i=1}^{M} I_{\{y_i>b\}}\frac{p(y_i)}{q(y_i)}$
- $\forall z_i \sim N(0,1), y_i \sim N(b,1)$

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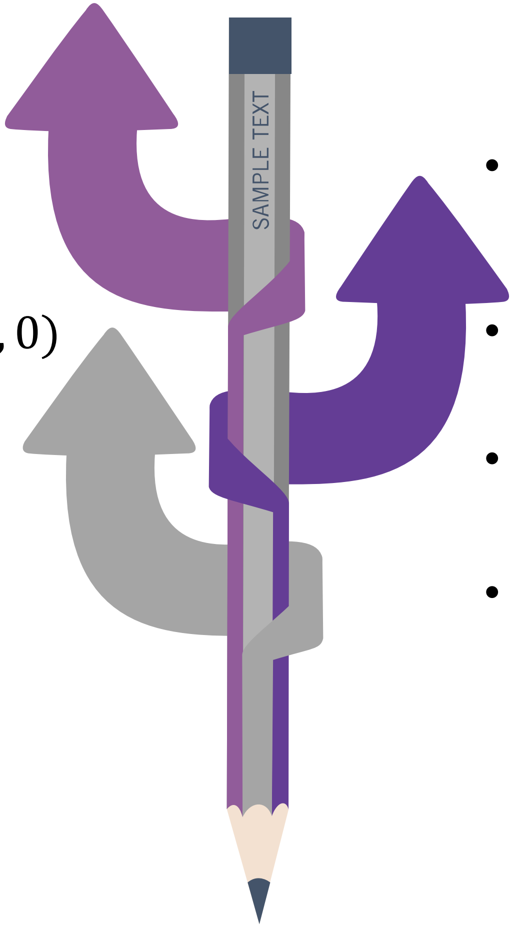
02 Part two 应用—欧式期权

应用—欧式期权

◎ Call：

- $C_{QMC} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(S_t^i(e) - K, 0)$
- $C_{QMCAV} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(\frac{\left(s_t^i(e)+s_t^i(-e)\right)}{2} - K, 0)$
- $C_{QMCCV} = C_{QMC} + b(S_{t-QMC} - e^{rt}S_0)$
- $C_{QMCIS} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt} \max\left(S_t^i(e')\frac{f(e')}{g(e')} - K, 0\right), \forall\, e' \sim N\frac{ln\frac{K}{S_0} - \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}, 1)$



◎ Put：

- $P_{QMC} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(K - S_t^i(e), 0)$
- $P_{QMCAV} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(K - \frac{\left(s_t^i(e)+s_t^i(-e)\right)}{2}, 0)$
- $P_{QMCCV} = P_{QMC} + b(S_{t-QMC} - e^{rt}S_0)$
- $P_{QMCIS} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt} \max\left(K - S_t^i(e')\frac{f(e')}{g(e')}, 0\right), \forall\, e' \sim N\frac{ln\frac{K}{S_0} - \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}, 1)$

应用—欧式期权

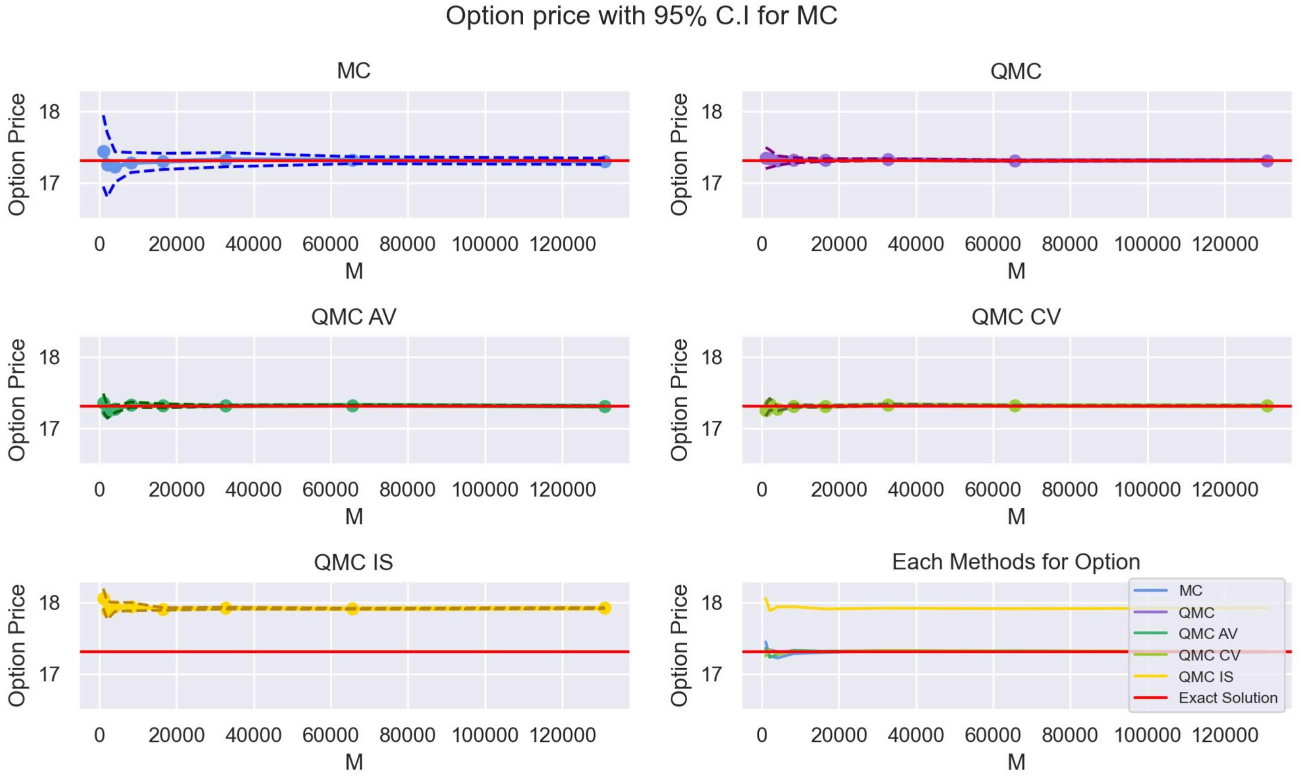
◎ Call Price：17.3114

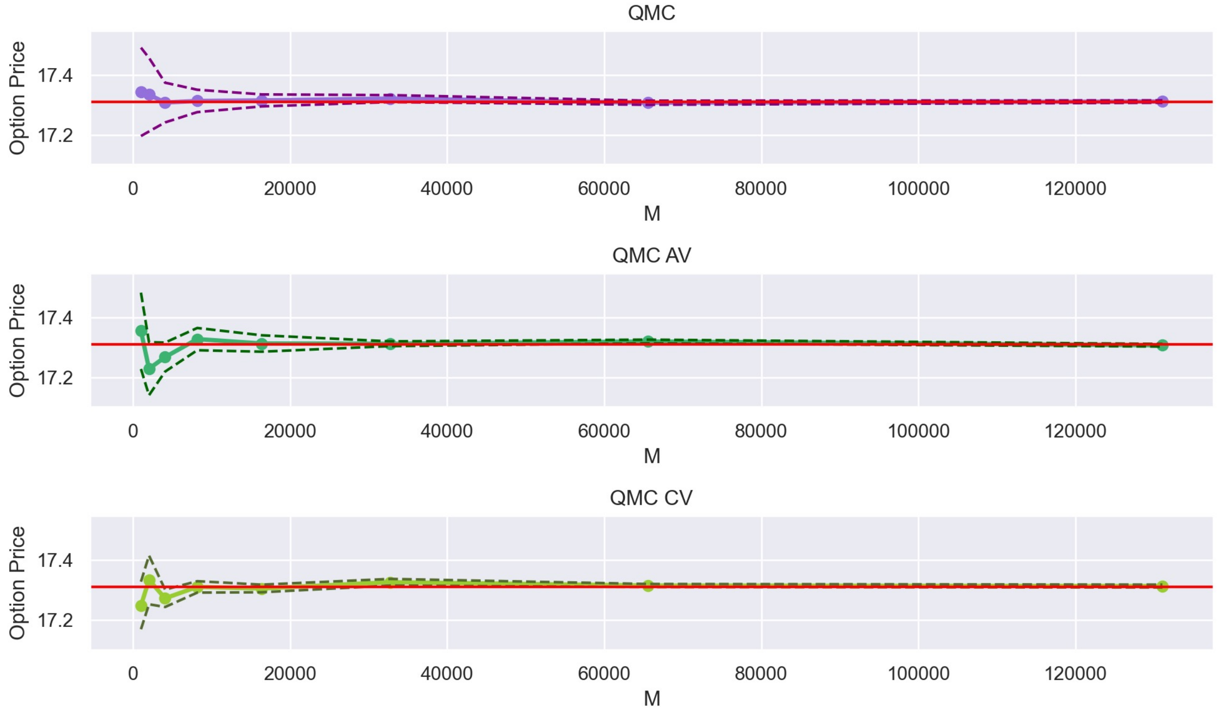
◎ Put Price：7.7952

- S0 = 100 # 资产价格
- K = 100 # 履约价
- r = 0.01 # 无风险利率
- Tt = 10 # 时间
- vol = 0.1 # 波动度
- N = 20 # 期数
- dt = Tt/N # 一期时间长度
- M # 模拟次数

应用—欧式期权—Call

实验次数：10

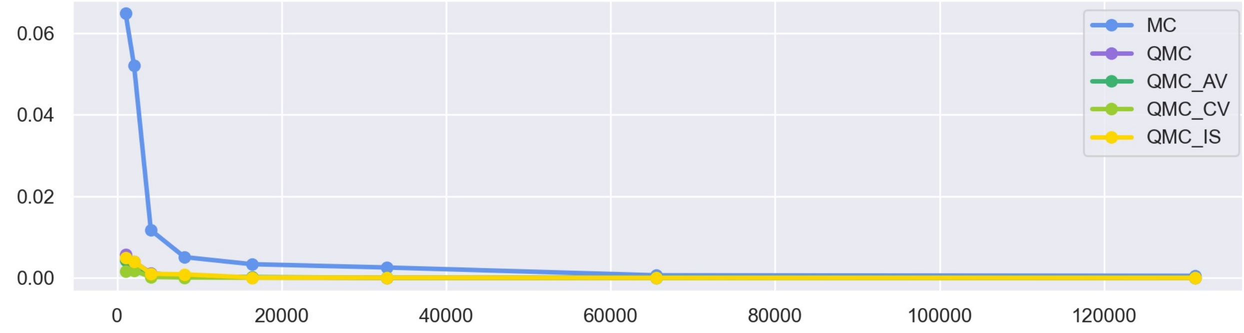




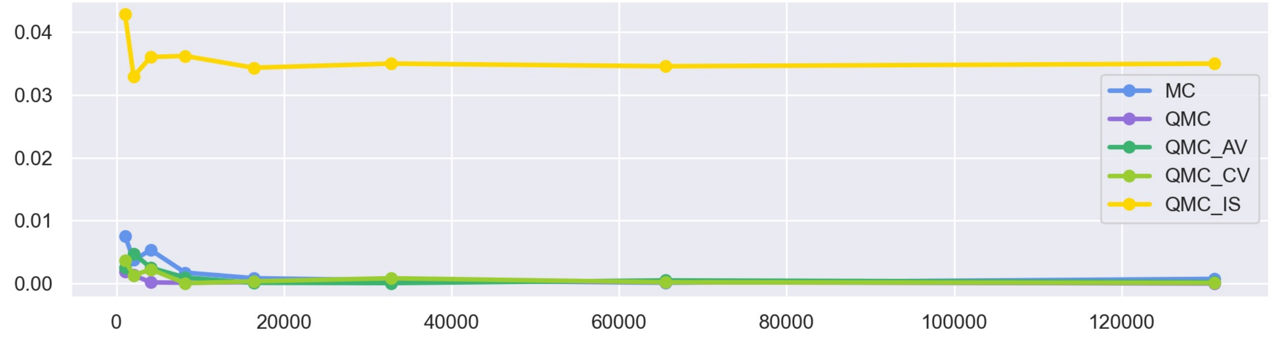
◎ 随着模拟路径次数的上升，C.I会降低

应用—欧式期权—Call

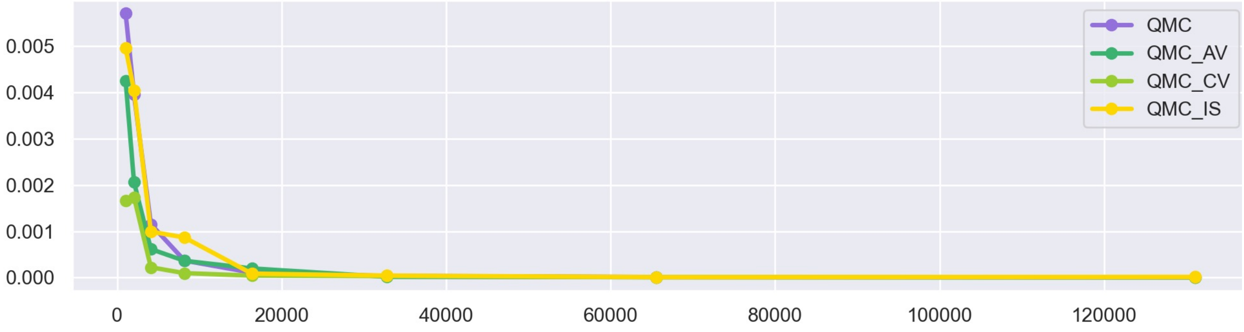
Var



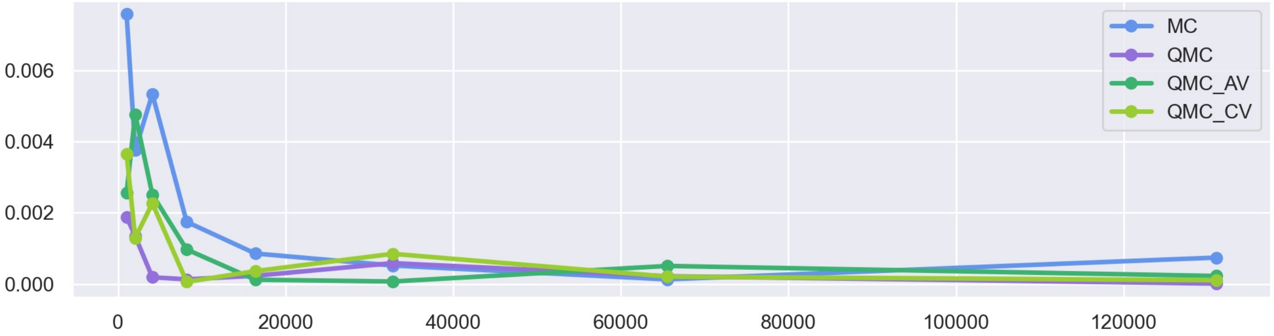
Error



Var without MC



Error without IS

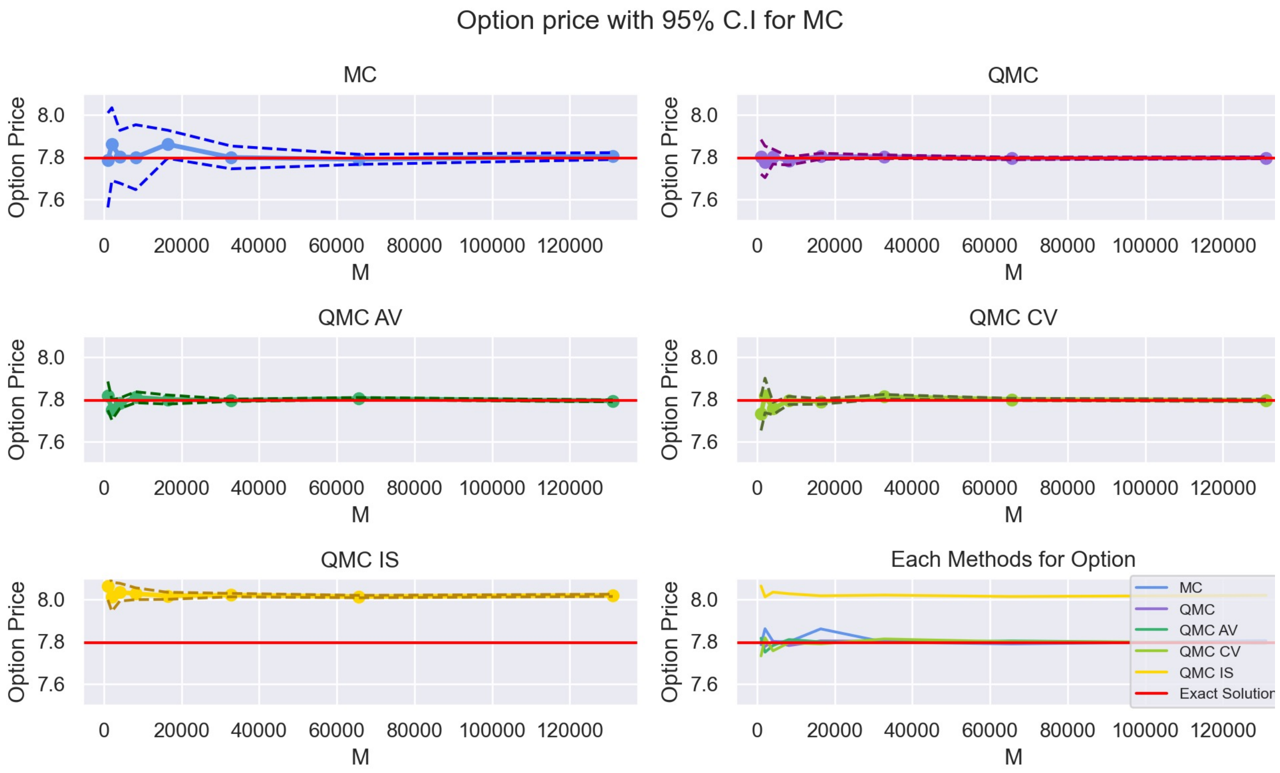


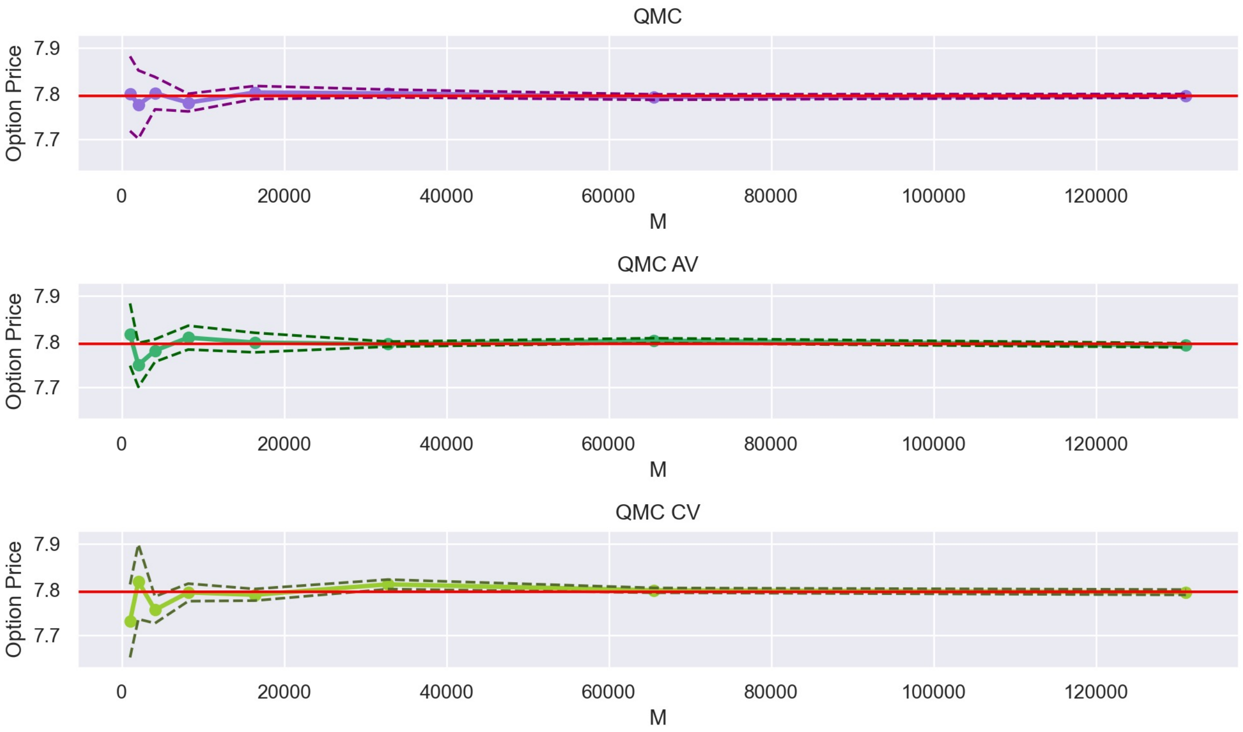
◎ Error： QMC_IS > MC > QMC_AV = QMC_CV = QMC

◎ Var： MC > QMC > QMC_IS > QMC_AV > QMC_CV

应用—欧式期权—Put

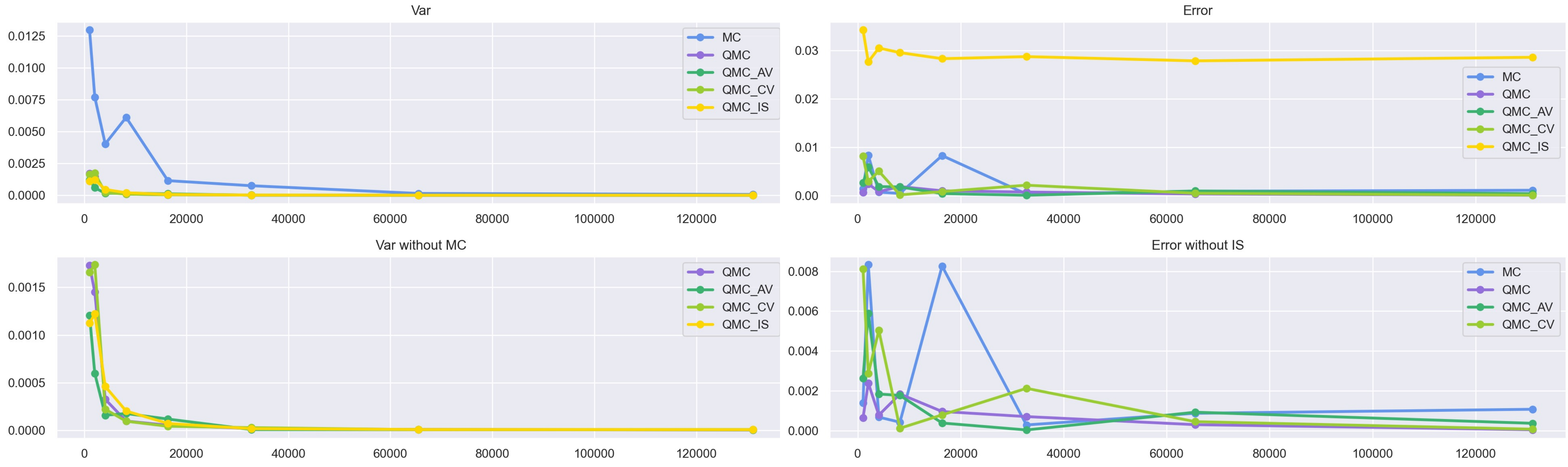
实验次数：10





◎ 随着模拟路径次数的上升，C.I会降低

应用—欧式期权—Put



◎ Error： QMC_IS > MC > QMC_CV = QMC = QMC_AV

◎ Var： MC > QMC = QMC_CV > QMC_IS = QMC_AV

应用—欧式期权

◎ Error：虽然QMC_IS的Error比较高，但若实验次数增加，
可以得出QMC_IS的Var相对MC和QMC小

◎ Var：QMC、QMC_AV、QMC_CV的Error和Var很接近，也相对小

◎ 随着模拟路径次数的上升，Var和Error会降低

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03

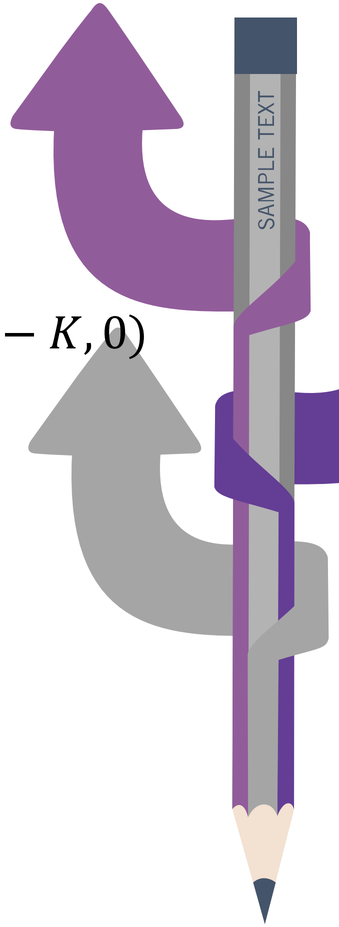
Part three

应用—亚式几何平均期权

应用—亚式几何平均期权

◎ Call：

- $CG_{QMC} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(SG_t^i(e) - K, 0)$
- $CG_{QMCAV} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(\frac{\left(SG_t^i(e)+SG_t^i(-e)\right)}{2} - K, 0)$
- $CG_{QMCCV} = CG_{QMC} + b(SG_{t-QMC} - e^{rt}S_0)$
- $CG_{QMCIS} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max\left(SG_t^i(e')\frac{f(e')}{g(e')} - K, 0\right), \forall\, e' \sim N\left(\frac{ln\frac{K}{S_0}-\left(r-\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}, 1\right)$



◎ Put：

- $PG_{QMC} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(K - SG_t^i(e), 0)$
- $PG_{QMCAV} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(K - \frac{\left(SG_t^i(e)+SG_t^i(-e)\right)}{2}, 0)$
- $PG_{QMCCV} = PG_{QMC} + b(SG_{t-QMC} - e^{rt}S_0)$
- $PG_{QMCIS} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max\left(K - SG_t^i(e')\frac{f(e')}{g(e')}, 0\right), \forall\, e' \sim N\left(\frac{ln\frac{K}{S_0}-\left(r-\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}, 1\right)$

应用—亞式几何平均期权

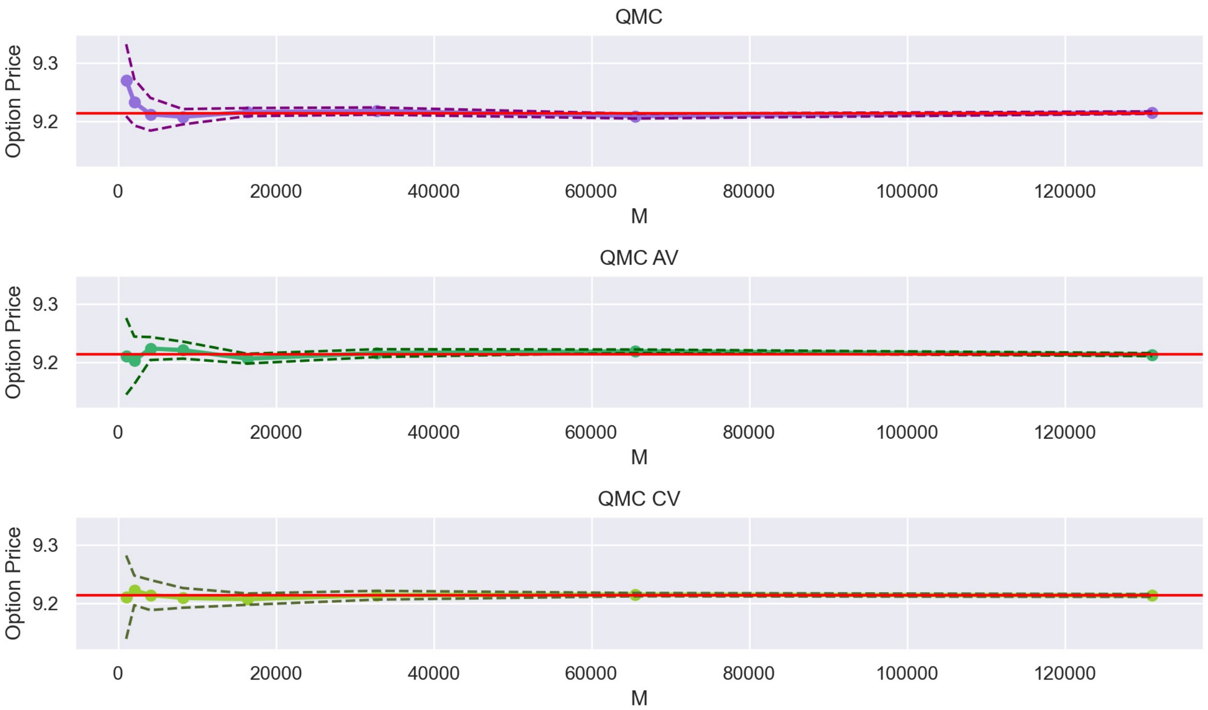
◎ Call Price：9.2137

◎ Put Price：5.1258

- S0 = 100　　# 资产价格
- K = 100　　# 履约价
- r = 0.01　　# 无风险利率
- Tt = 10　　# 时间
- vol = 0.1　　# 波动度
- N = 20　　# 期数
- dt = Tt/N　　# 一期时间长度
- M　# 模拟次数

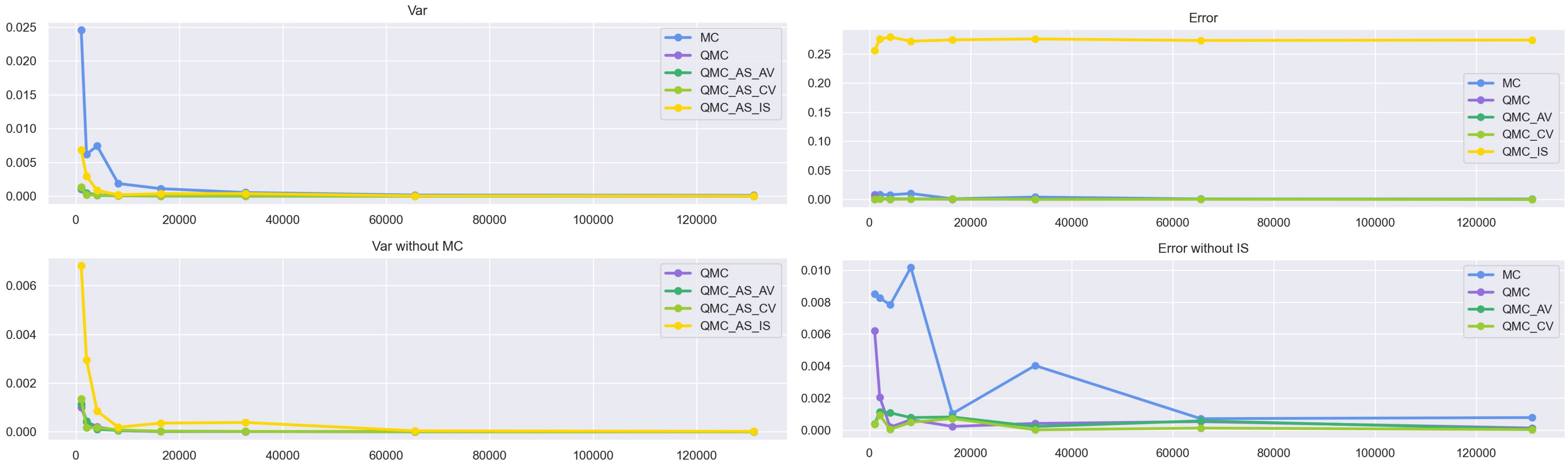
应用—亞式几何平均期权—Call

实验次数：10



◎ 随着模拟路径次数的上升，C.I会降低

应用—亚式几何平均期权—Call

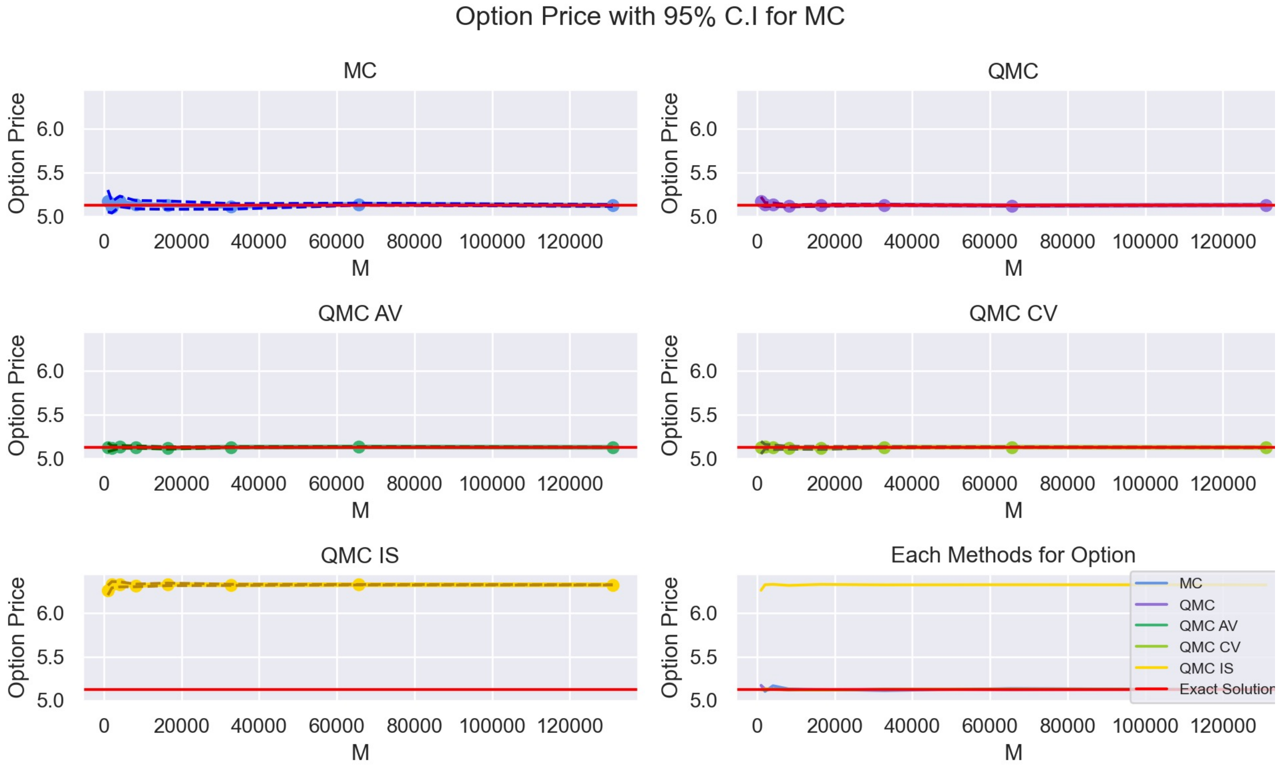


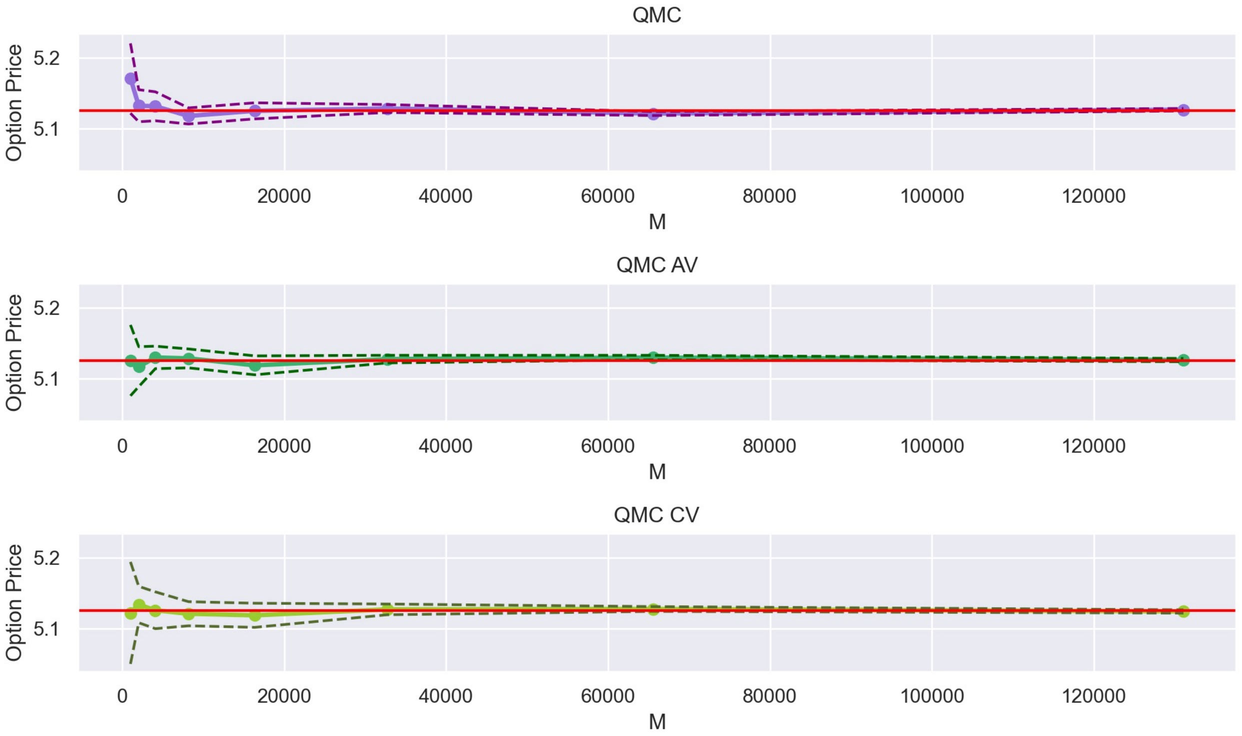
◎ Error： QMC_IS > MC > QMC > QMC_AV = QMC_CV

◎ Var： MC > QMC_IS > QMC = QMC_AV = QMC_CV

应用—亚式几何平均期权—Put

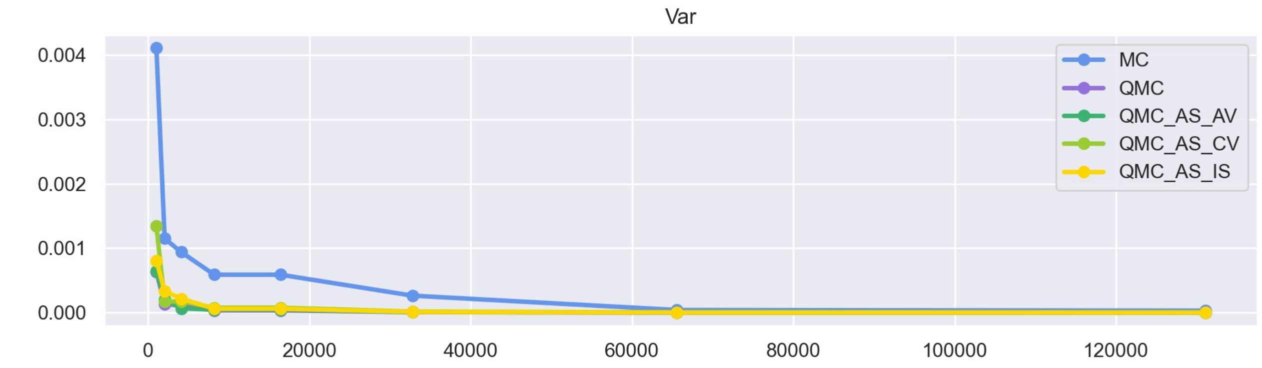
实验次数：10

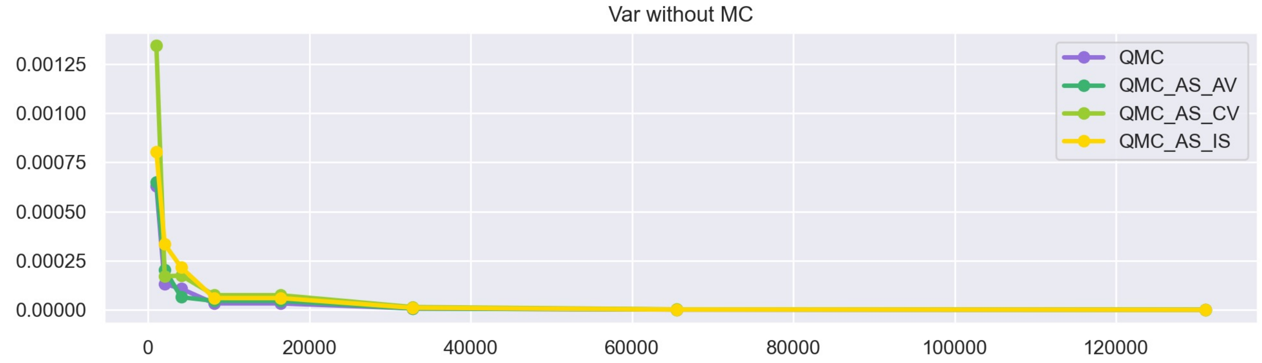


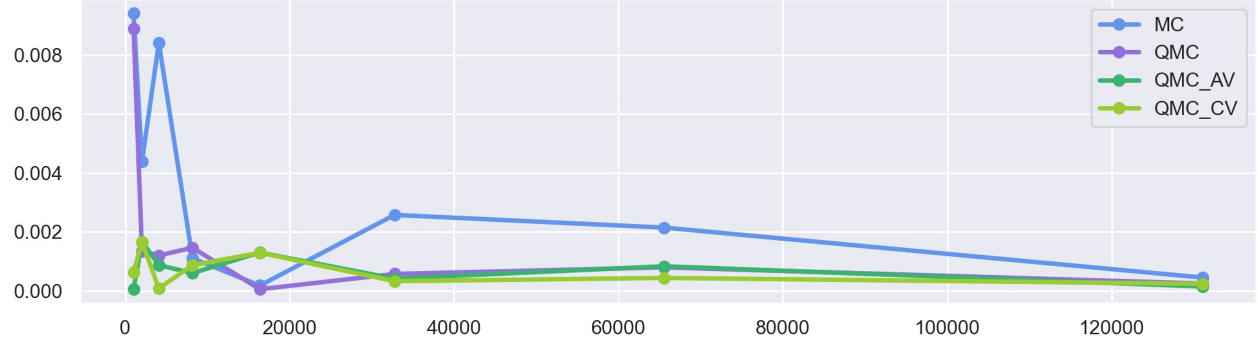


◎ 随着模拟路径次数的上升，C.I会降低

应用—亞式几何平均期权—Put







◎ Error： QMC_IS > MC > QMC > QMC_AV = QMC_CV

◎ Var： MC > QMC_CV > QMC_IS = QMC_AV = QMC

应用—亞式几何平均期权

◎ Error：QMC_IS的Var在亚式几何平均期权比欧式几何平均期权来得高

◎ Var：QMC_AV的Error和Var很接近，也相对小

◎ 随着模拟路径次数的上升，Var和Error会降低

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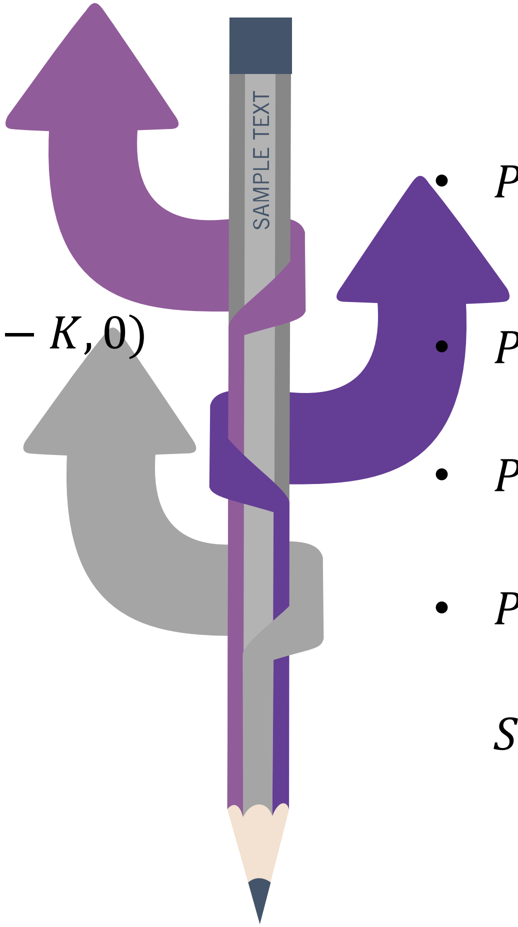
04 Part four

应用—亚式算数平均期权

应用—亚式算数平均期权

◎ Call：

- $CA_{QMC} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(SA_t^i(e) - K, 0)$
- $CA_{QMCAV} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(\frac{\left(SA_t^i(e)+SA_t^i(-e)\right)}{2} - K, 0)$
- $CA_{QMCCV} = CA_{QMC} + b(SA_{t-QMC} - e^{rt}S_0)$
- $CA_{QMCIS} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max\left(SA_t^i(e')\frac{f(e')}{g(e')} - K, 0\right), \forall\, e' \sim N\left(\frac{\ln\frac{K}{S_0}-\left(r-\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}, 1\right)$



◎ Put：

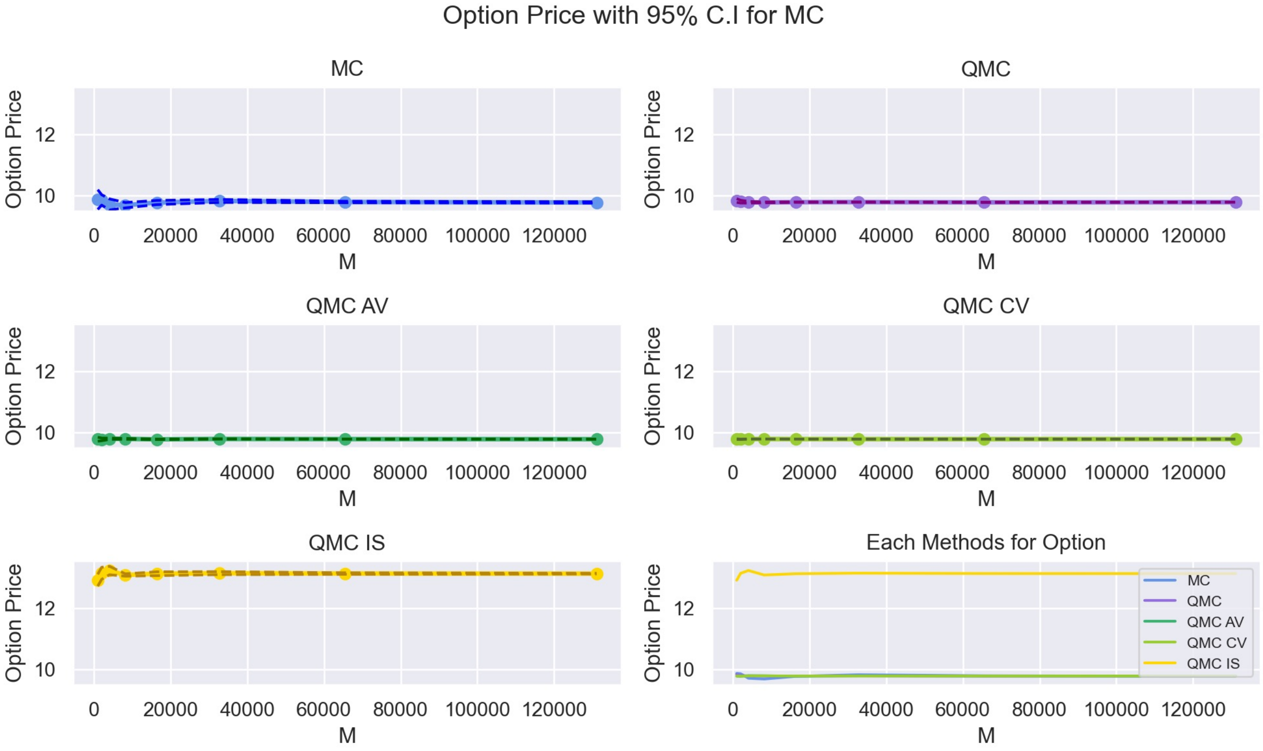
- $PA_{QMC} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(K - SA_t^i(e), 0)$
- $PA_{QMCAV} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max(K - \frac{\left(SA_t^i(e)+SA_t^i(-e)\right)}{2}, 0)$
- $PA_{QMCCV} = PG_{QMC} + b(SA_{t-QMC} - e^{rt}S_0)$
- $PA_{QMCIS} = \frac{1}{M}\sum_{i=1}^{M} e^{-rt}\max\left(K - SA_t^i(e')\frac{f(e')}{g(e')}, 0\right), \forall\, e' \sim N\left(\frac{\ln\frac{K}{S_0}-\left(r-\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}, 1\right)$

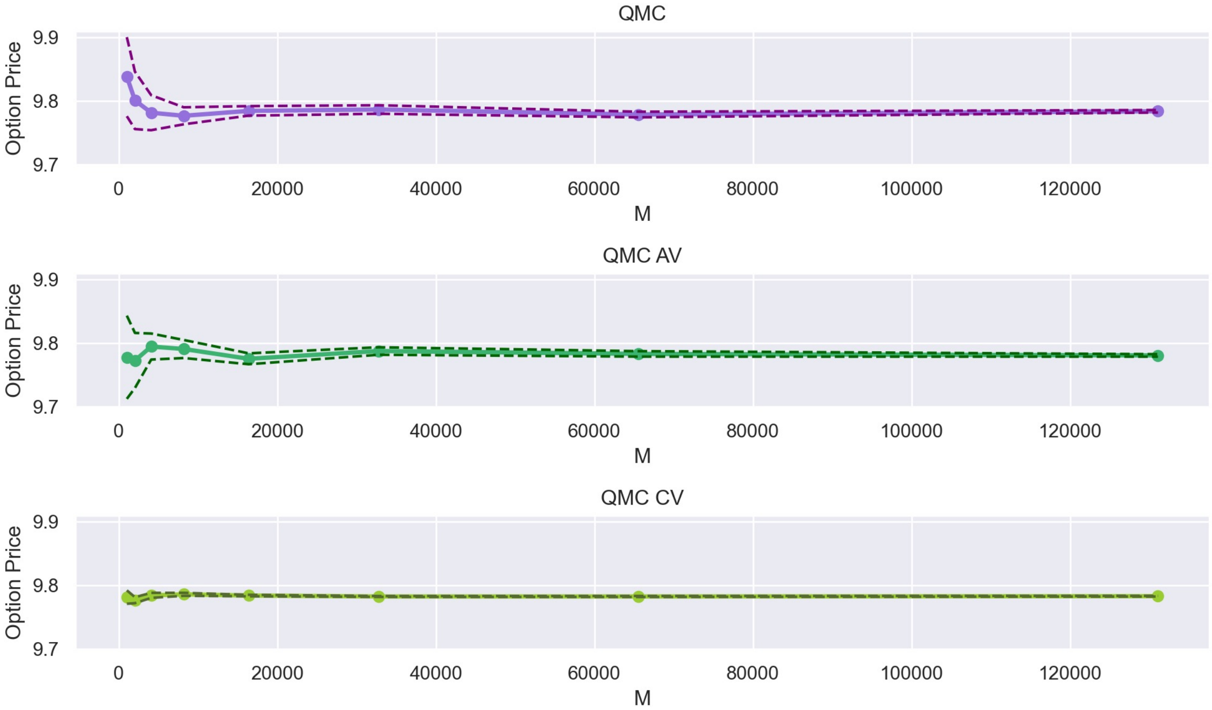
应用—亚式算数平均期权

- S0 = 100   # 资产价格
- K = 100   # 履约价
- r = 0.01   # 无风险利率
- Tt = 10   # 时间
- vol = 0.1   # 波动度
- N = 20   # 期数
- dt = Tt/N   # 一期时间长度
- M # 模拟次数

应用—亚式算数平均期权—Call

实验次数：10





◎ 随着模拟路径次数的上升，C.I会降低

应用—亚式算数平均期权—Call

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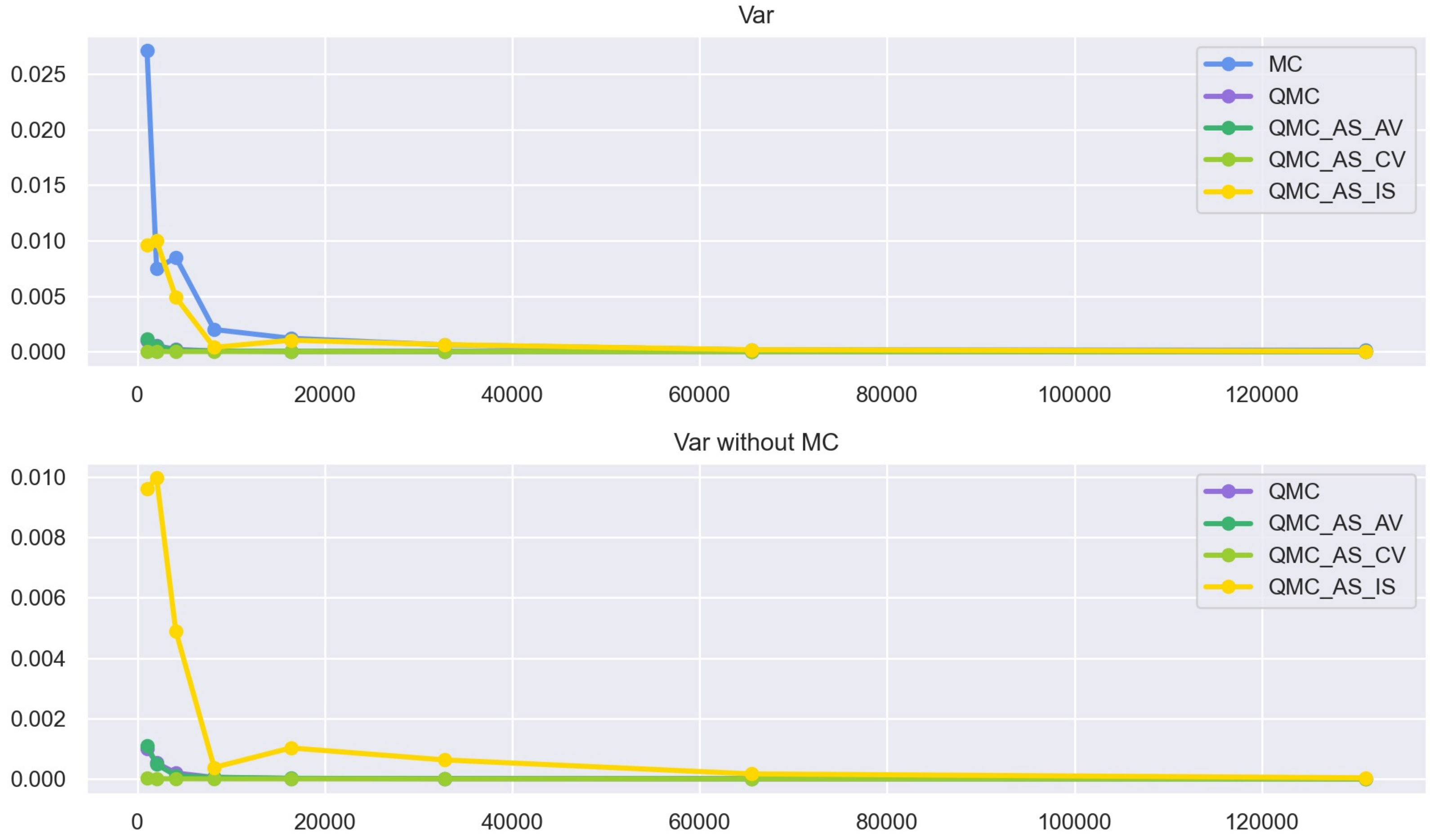
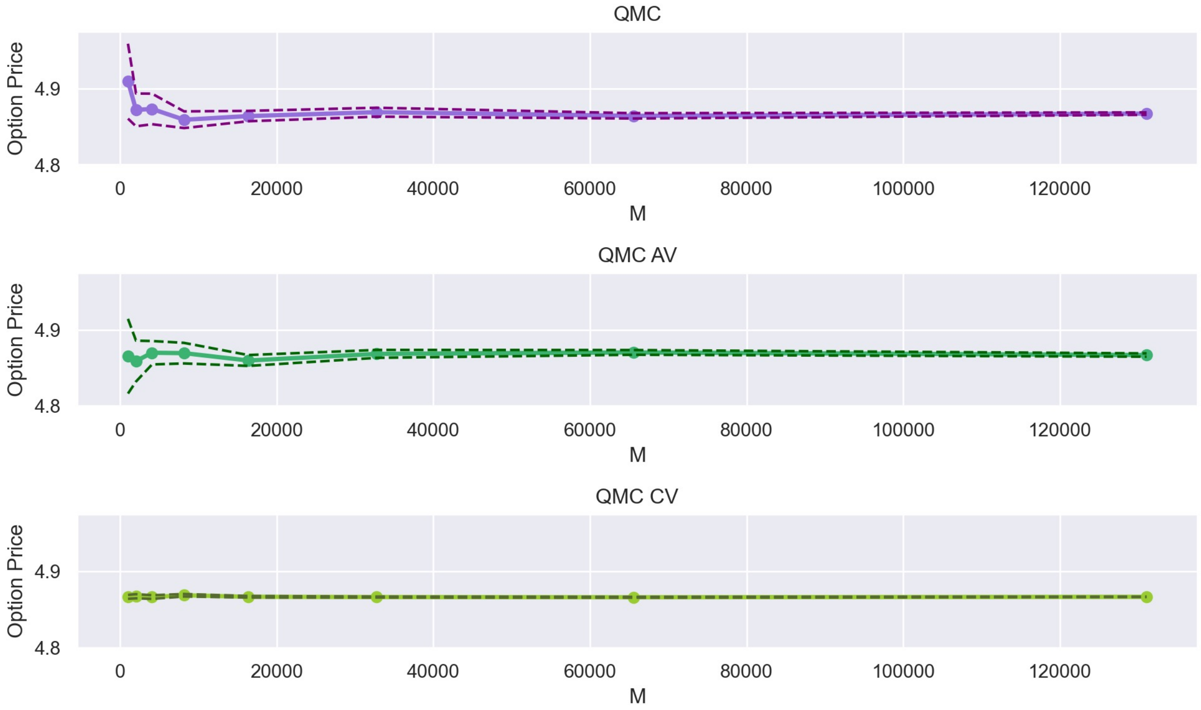


◎ Var：MC > QMC_IS > QMC = QMC_AV > QMC_CV

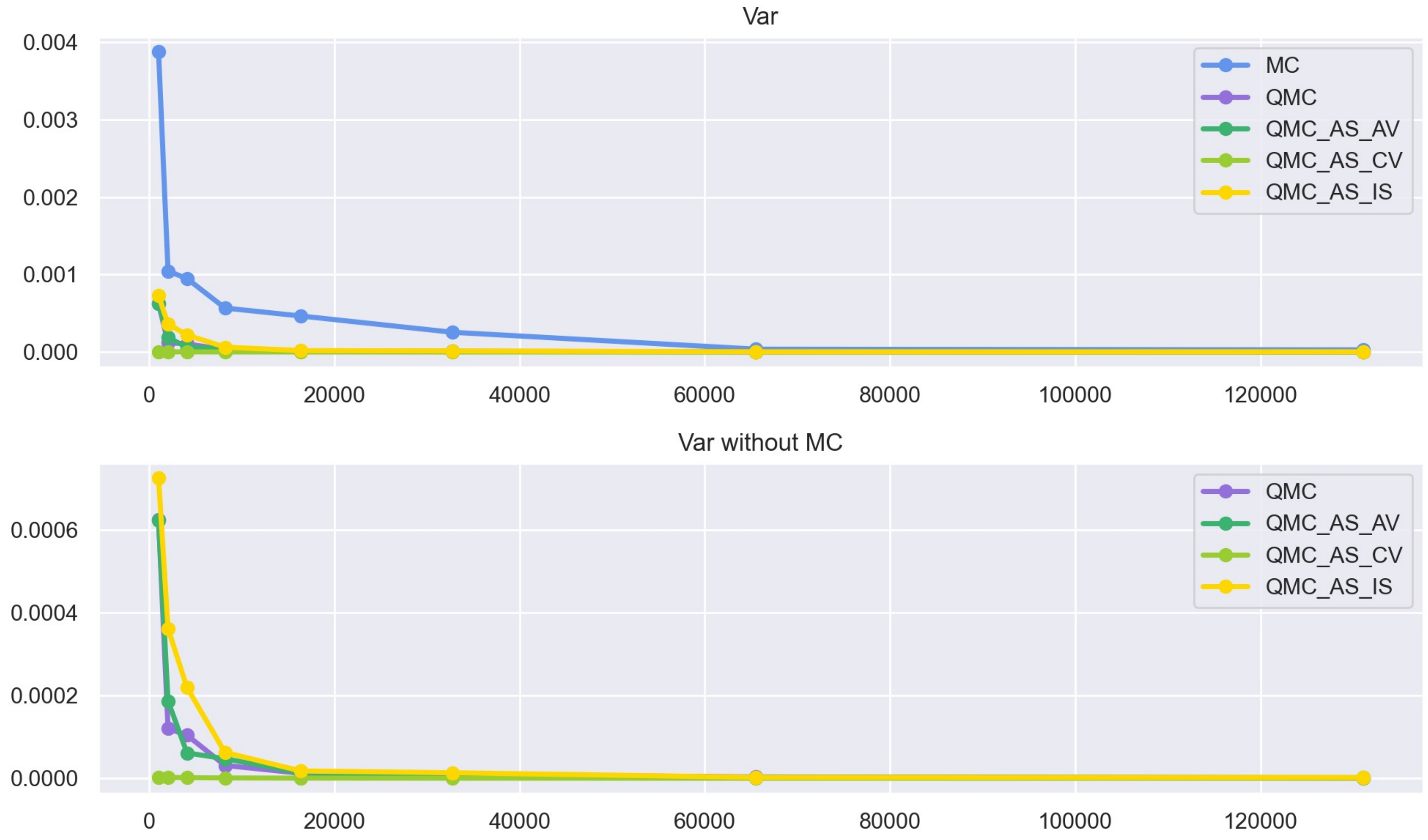
应用—亚式算数平均期权—Put

实验次数：10



◎ 随着模拟路径次数的上升，C.I会降低

应用—亚式算数平均期权—Put



◎ Var： MC > QMC_IS > QMC = QMC_AV > QMC_CV

应用—亚式算数平均期权

◎ Var：因为没有精确解，单看Var的话，是QMC_CV最好

◎ QMC_IS的定价计算结果相对高

◎ 随着模拟路径次数的上升，Var会降低

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05 Part five

结论

结论

实验次数：10	Error					Var				
	MC	QMC	QMC–AV	QMC–CV	QMC–IS	MC	QMC	QMC–AV	QMC–CV	QMC–IS
European Call	2	3	3	3	1	1	2	4	5	3
European Put	2	3	3	3	1	1	2	4	3	4
Asian Geometric Call	2	3	4	4	1	1	3	4	4	2
Asian Geometric Put	2	3	4	4	1	1	3	3	2	3
Asian Arithmetic Call						1	3	3	4	2
Asian Arithmetic Put						1	3	3	4	2

◎ Error： QMC_IS > MC > QMC > QMC_AV = QMC_CV

◎ Var： MC > QMC = QMC_IS > QMC_AV > QMC_CV

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Thank you

谢谢倾听