# 2020270026 应用统计硕士 王姿文

# 1 Dimension Reduction, PCA

## Problem 1

### 1 different components

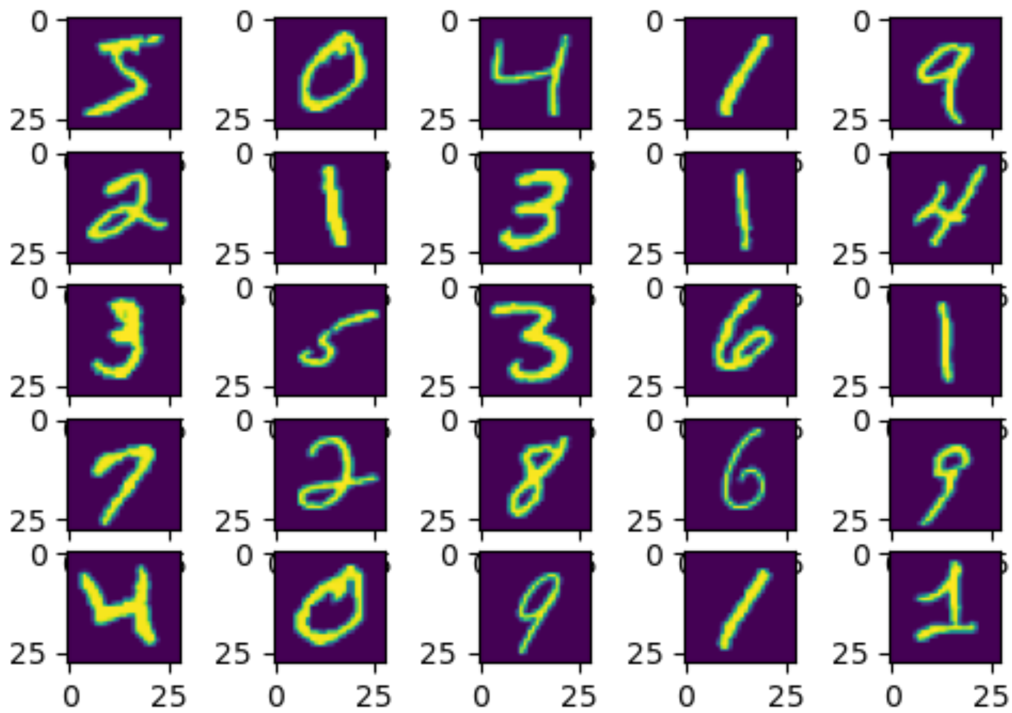
首先读取数据并来查看维度，接着将3d数据的60000笔中前25笔画出来，明显是手写数字图片（**由于此题的题目有表明需要附上code在旁边，因此附上**）：

In [14]:
```
import gzip
import numpy as np
import os
import matplotlib.pyplot as plt
import pandas as pd
import matplotlib as mpl
import warnings
warnings.filterwarnings('ignore')
mpl.rcParams['figure.dpi'] = 100
```

In [8]:
```
def load_data(data_folder):

    files = [
          'train-labels-idx1-ubyte.gz', 'train-images-idx3-ubyte.gz'
    ]
    paths = []
    for fname in files:
        paths.append(os.path.join(data_folder,fname))

    with gzip.open(paths[0], 'rb') as lbpath:
        y_train = np.frombuffer(lbpath.read(), np.uint8, offset=8)

    with gzip.open(paths[1], 'rb') as imgpath:
        x_train = np.frombuffer(
            imgpath.read(), np.uint8, offset=16).reshape(len(y_train),
28, 28)
    return (x_train, y_train)

#load
train_images, train_labels = load_data('/Users/wangziwen/Documents/Grad
uation/1st/Statistic Learning/HW/HW3/')

#dim
print('dim:',train_images.shape)

#reshape data
train_data=train_images.reshape(60000,784)
```

```
dim: (60000, 28, 28)
```

In [6]:
```
f, ax = plt.subplots(5, 5)
a = [i for i in range(25)]
a = np.array(a).reshape(5,5)
for i in range(5):
    for j in range(5):
        ax[i][j].imshow(train_data[a[i][j]].reshape(28,28))

plt.show()
```



已知此笔数据是60000x28x28，可以转换为2d的60000x784，此时便是60000笔数据涵盖784个特征
可以应用PCA以及PCA without centered于此笔数据，下面可以见到两种方法若要降维至累计解释变异高达99%，则会分别降为333个主成分和332个主成分

In [9]:
```
from sklearn.decomposition import PCA
from sklearn.decomposition import TruncatedSVD # without center
```

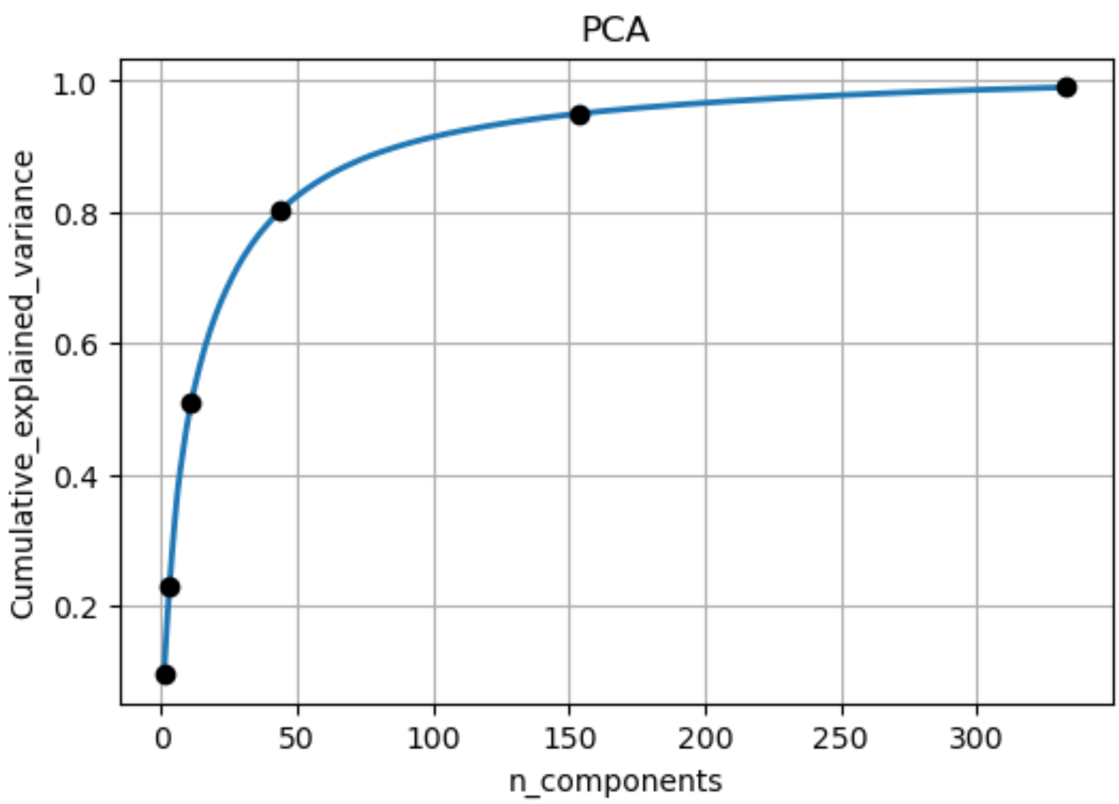
In [10]:
```
pca = PCA(n_components = 333) #np.argmax(np.cumsum(pca.explained_variance_ratio_)>=0.99)+1
train_data_pca = pca.fit_transform(train_data)
tsvd = TruncatedSVD(n_components = 332) #np.argmax(np.cumsum(pca.explained_variance_ratio_)>=0.99)+1
train_data_tsvd = tsvd.fit_transform(train_data)
```

In [14]:
```
print('PCA dim:',train_data_pca.shape,'-> reduce from 784 to 333') #784 -> 333
print('PCA without centered dim:',train_data_tsvd.shape,'-> reduce from 784 to 332')
```

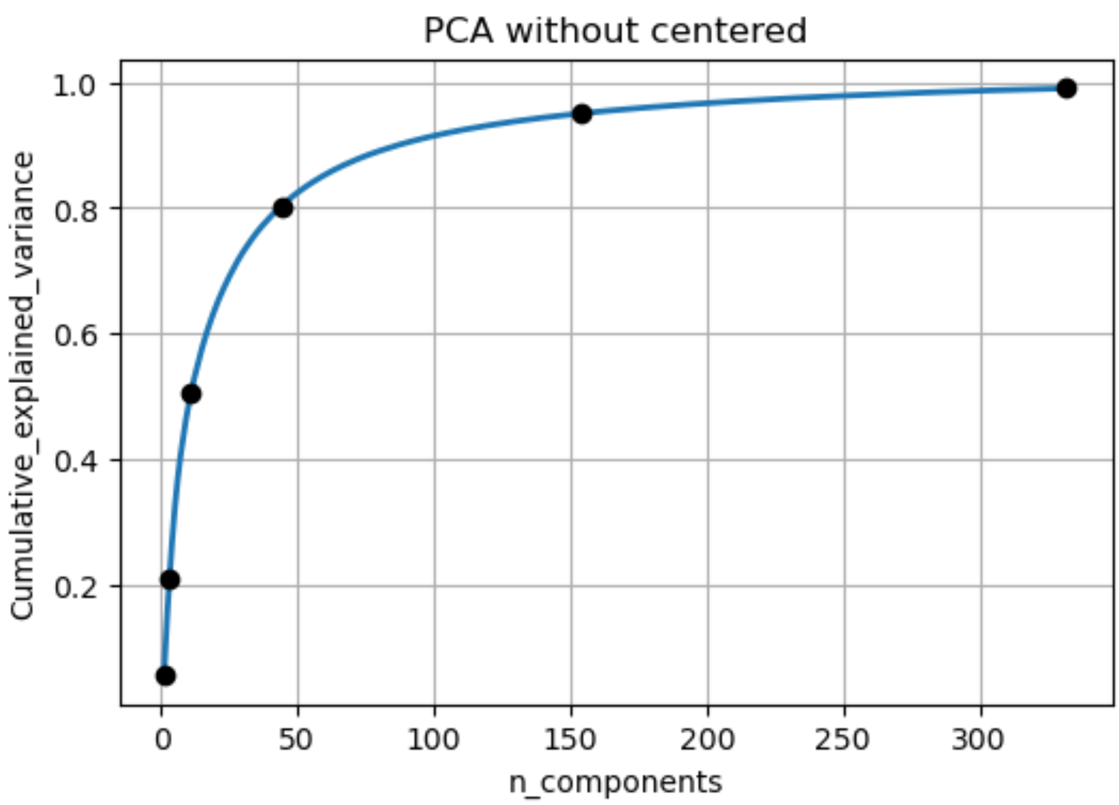
```
PCA dim: (60000, 333) -> reduce from 784 to 333
PCA without centered dim: (60000, 332) -> reduce from 784 to 332
```

接着来看两者的累计解释变异图，其图内的点分别为累计解释变异达至5%、20%、50%、80%、95%、99%时的主成分个数：

In [12]:
```
cumsum_pca = np.cumsum(pca.explained_variance_ratio_)
cumsum_tsvd = np.cumsum(tsvd.explained_variance_ratio_)

#1%, 5%, 20%, 50%, 80%, 95%, 99%
plt.figure(1, figsize=(6, 4))
plt.clf()
plt.plot([i+1 for i in range(333)],
         cumsum_pca,linewidth=2)
plt.plot(1, cumsum_pca[0], linestyle="None", marker="o",  color="black"
)
plt.plot(3, cumsum_pca[2], linestyle="None", marker="o",  color="black"
)
plt.plot(11, cumsum_pca[10], linestyle="None", marker="o",  color="blac
k")
plt.plot(44, cumsum_pca[43], linestyle="None", marker="o",  color="blac
k")
plt.plot(154, cumsum_pca[153], linestyle="None", marker="o",  color="bl
ack")
plt.plot(333, cumsum_pca[332], linestyle="None", marker="o",  color="bl
ack")
plt.axis('tight')
plt.grid()
plt.xlabel('n_components')
plt.ylabel('Cumulative_explained_variance')
plt.title('PCA')
plt.show()
```

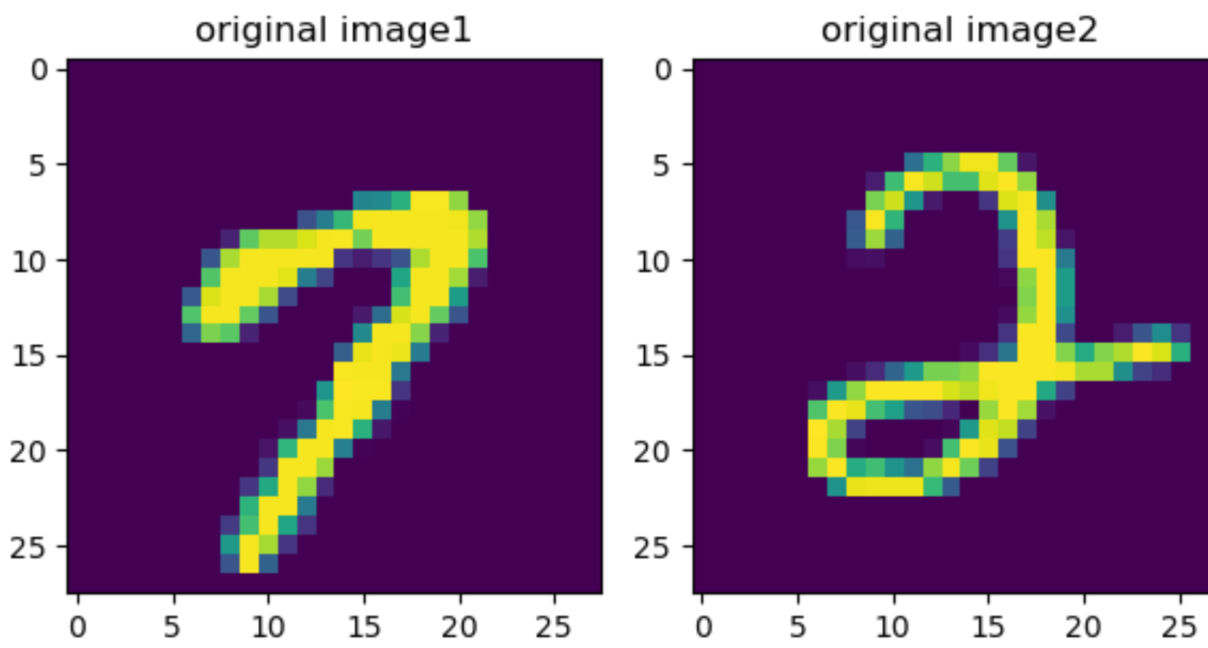


In [13]:
```
#1%, 5%, 20%, 50%, 80%, 95%, 99%
plt.figure(1, figsize=(6, 4))
plt.clf()
plt.plot([i+1 for i in range(332)],
         cumsum_tsvd,linewidth=2)
plt.plot(1, cumsum_tsvd[0], linestyle="None", marker="o",  color="black
")
plt.plot(3, cumsum_tsvd[2], linestyle="None", marker="o",  color="black
")
plt.plot(11, cumsum_tsvd[10], linestyle="None", marker="o",  color="bla
ck")
plt.plot(44, cumsum_tsvd[43], linestyle="None", marker="o",  color="bla
ck")
plt.plot(154, cumsum_tsvd[153], linestyle="None", marker="o",  color="b
lack")
plt.plot(332, cumsum_tsvd[331], linestyle="None", marker="o",  color="b
lack")
plt.axis('tight')
plt.grid()
plt.xlabel('n_components')
plt.ylabel('Cumulative_explained_variance')
plt.title('PCA without centered')
plt.show()
```




接着设定两张图片，下图为原始数据画出来的样子，希望能用两种降维方法后的数据来绘制此两张图：

In [16]:
```
f, ax = plt.subplots(1, 2)

ax[0].imshow(train_data[15,:].reshape(28,28))
ax[1].imshow(train_data[16,:].reshape(28,28))
ax[0].set_title('original image1')
ax[1].set_title('original image2')
plt.tight_layout()
plt.show()
```



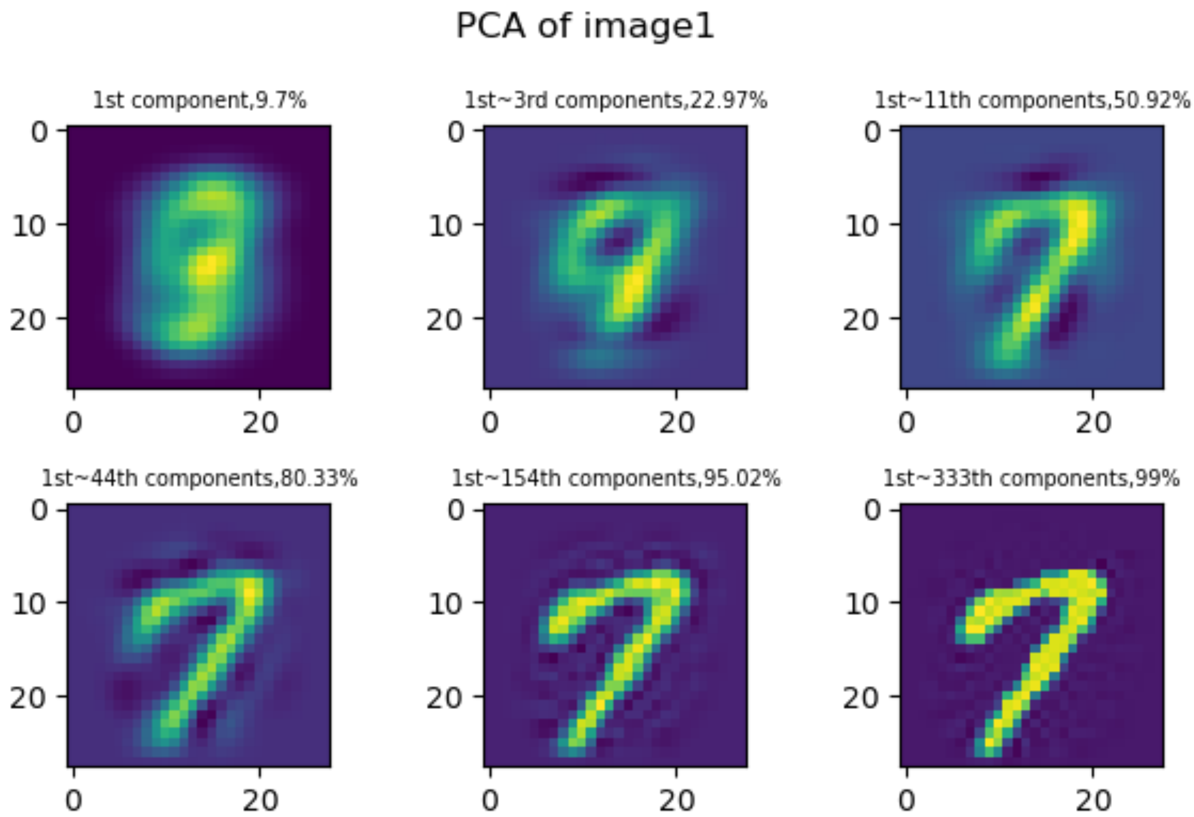

In [15]:
```
def train_pca(d):
    pca = PCA(n_components = d)
    train_data_pca = pca.fit_transform(train_data)
    return pca,train_data_pca

def tsvd_pca(d):
    tsvd = TruncatedSVD(n_components = d)
    train_data_tsvd = tsvd.fit_transform(train_data)
    return tsvd,train_data_tsvd

pca1,pca_1 = train_pca(1) #9.7%
pca2,pca_2 = train_pca(3) #22.97%
pca3,pca_3 = train_pca(11) #50.92%
pca4,pca_4 = train_pca(44) #80.33%
pca5,pca_5 = train_pca(154) #95.02%
pca6,pca_6 = train_pca(333) #99%

tsvd1,tsvd_1 = tsvd_pca(1) #5.8%
tsvd2,tsvd_2 = tsvd_pca(3) #21.15%
tsvd3,tsvd_3 = tsvd_pca(11) #50.77%
tsvd4,tsvd_4 = tsvd_pca(44) #80.31%
tsvd5,tsvd_5 = tsvd_pca(154) #95.02%
tsvd6,tsvd_6 = tsvd_pca(331) #99%
```

首先是第一张图'7'，可以从图的小标题看出主成分个数及其累计解释变异：

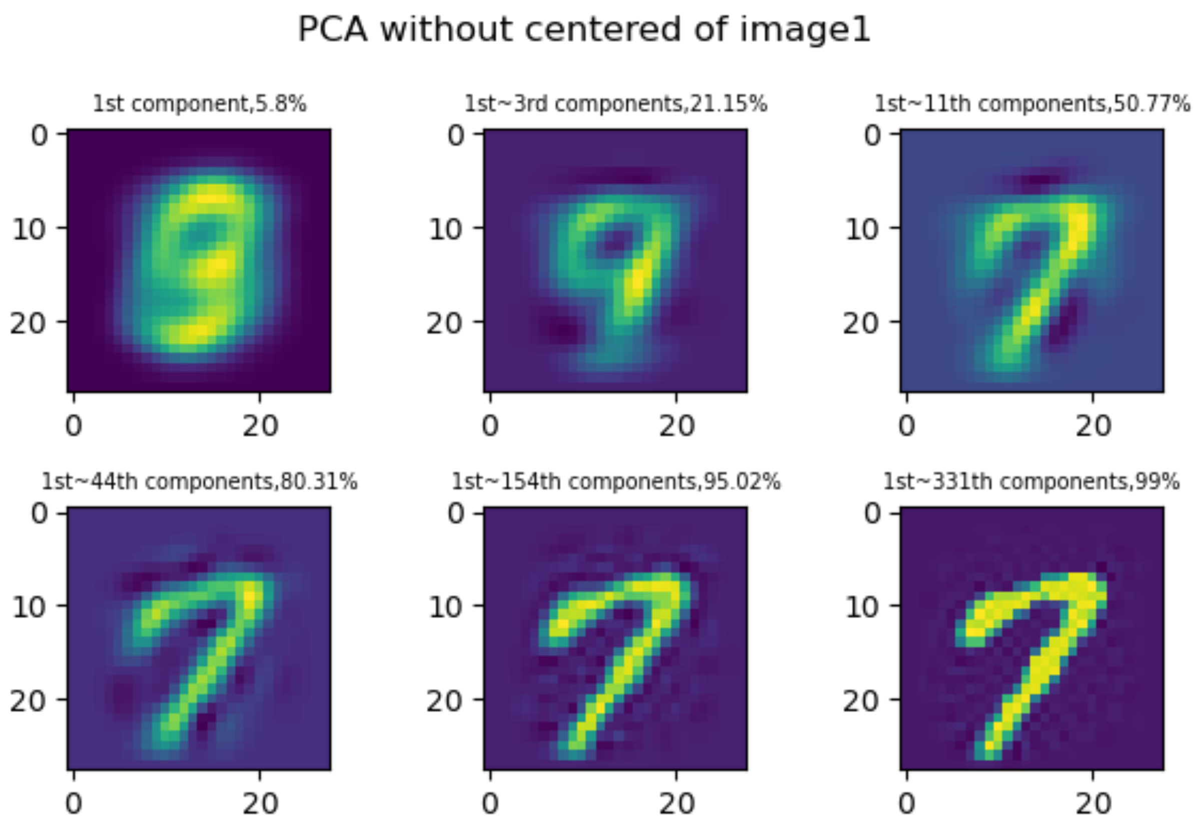
- PCA:
  在44个主成分的累计解释变异高达80.33%时，可以明显看出此图为'7'
- PCA without centered:
  在44个主成分的累计解释变异高达80.31%时，可以明显看出此图为'7'

这是非常棒的降维效果，因为原始是784个特征，只要降至44个主成分就能达到差不多的效果

In [17]:
```
f, ax = plt.subplots(2, 3)

f.suptitle("PCA of image1", fontsize=12)

ax[0,0].imshow(pca1.inverse_transform(pca_1[15, :]).reshape(28, 28))
ax[0,1].imshow(pca2.inverse_transform(pca_2[15, :]).reshape(28, 28))
ax[0,2].imshow(pca3.inverse_transform(pca_3[15, :]).reshape(28, 28))
ax[1,0].imshow(pca4.inverse_transform(pca_4[15, :]).reshape(28, 28))
ax[1,1].imshow(pca5.inverse_transform(pca_5[15, :]).reshape(28, 28))
ax[1,2].imshow(pca6.inverse_transform(pca_6[15, :]).reshape(28, 28))

ax[0,0].set_title('1st component,9.7%',fontsize=7)
ax[0,1].set_title('1st~3rd components,22.97%',fontsize=7)
ax[0,2].set_title('1st~11th components,50.92%',fontsize=7)
ax[1,0].set_title('1st~44th components,80.33%',fontsize=7)
ax[1,1].set_title('1st~154th components,95.02%',fontsize=7)
ax[1,2].set_title('1st~333th components,99%',fontsize=7)

plt.tight_layout()
plt.show()
```



In [18]:
```
f, ax = plt.subplots(2, 3)

f.suptitle("PCA without centered of image1", fontsize=12)

ax[0,0].imshow(tsvd1.inverse_transform(tsvd_1)[15, :].reshape(28, 28))
ax[0,1].imshow(tsvd2.inverse_transform(tsvd_2)[15, :].reshape(28, 28))
ax[0,2].imshow(tsvd3.inverse_transform(tsvd_3)[15, :].reshape(28, 28))
ax[1,0].imshow(tsvd4.inverse_transform(tsvd_4)[15, :].reshape(28, 28))
ax[1,1].imshow(tsvd5.inverse_transform(tsvd_5)[15, :].reshape(28, 28))
ax[1,2].imshow(tsvd6.inverse_transform(tsvd_6)[15, :].reshape(28, 28))

ax[0,0].set_title('1st component,5.8%',fontsize=7)
ax[0,1].set_title('1st~3rd components,21.15%',fontsize=7)
ax[0,2].set_title('1st~11th components,50.77%',fontsize=7)
ax[1,0].set_title('1st~44th components,80.31%',fontsize=7)
ax[1,1].set_title('1st~154th components,95.02%',fontsize=7)
ax[1,2].set_title('1st~331th components,99%',fontsize=7)

plt.tight_layout()
plt.show()
```

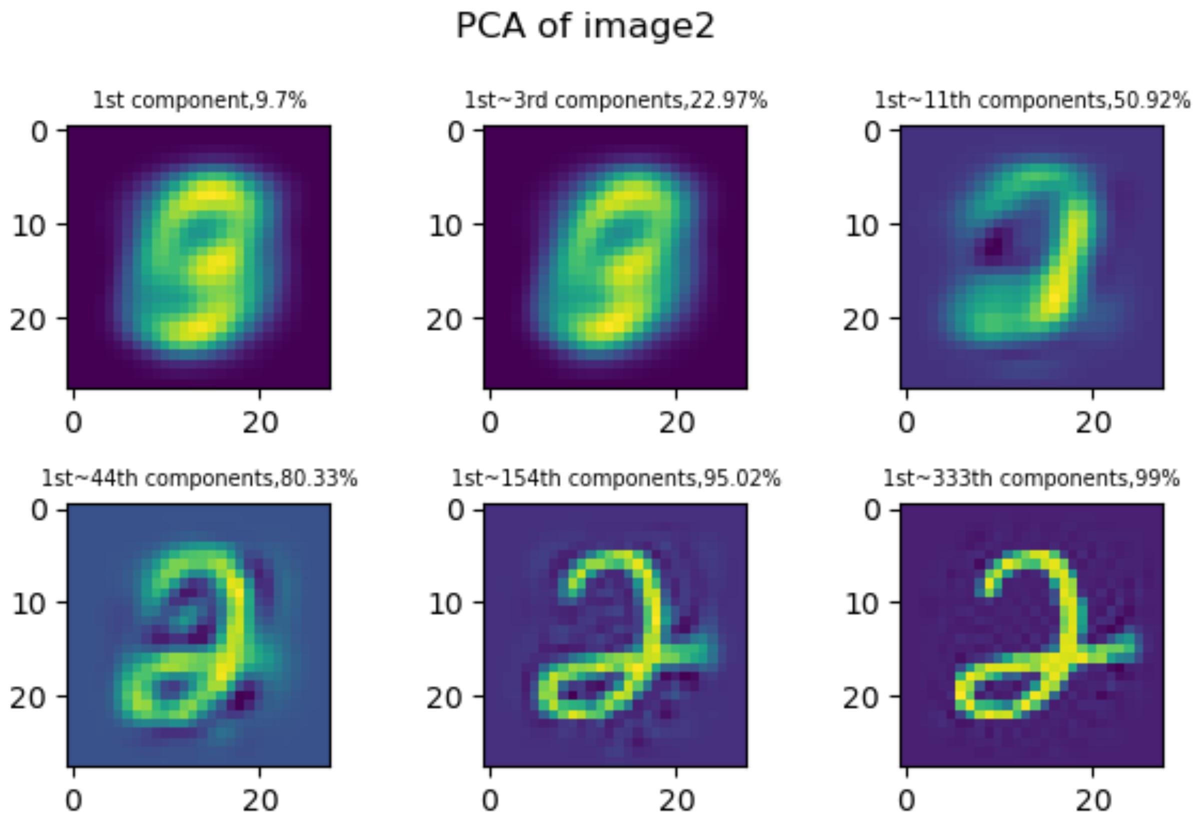



再来是第一张图'2'，可以从图的小标题看出主成分个数及其累计解释变异：

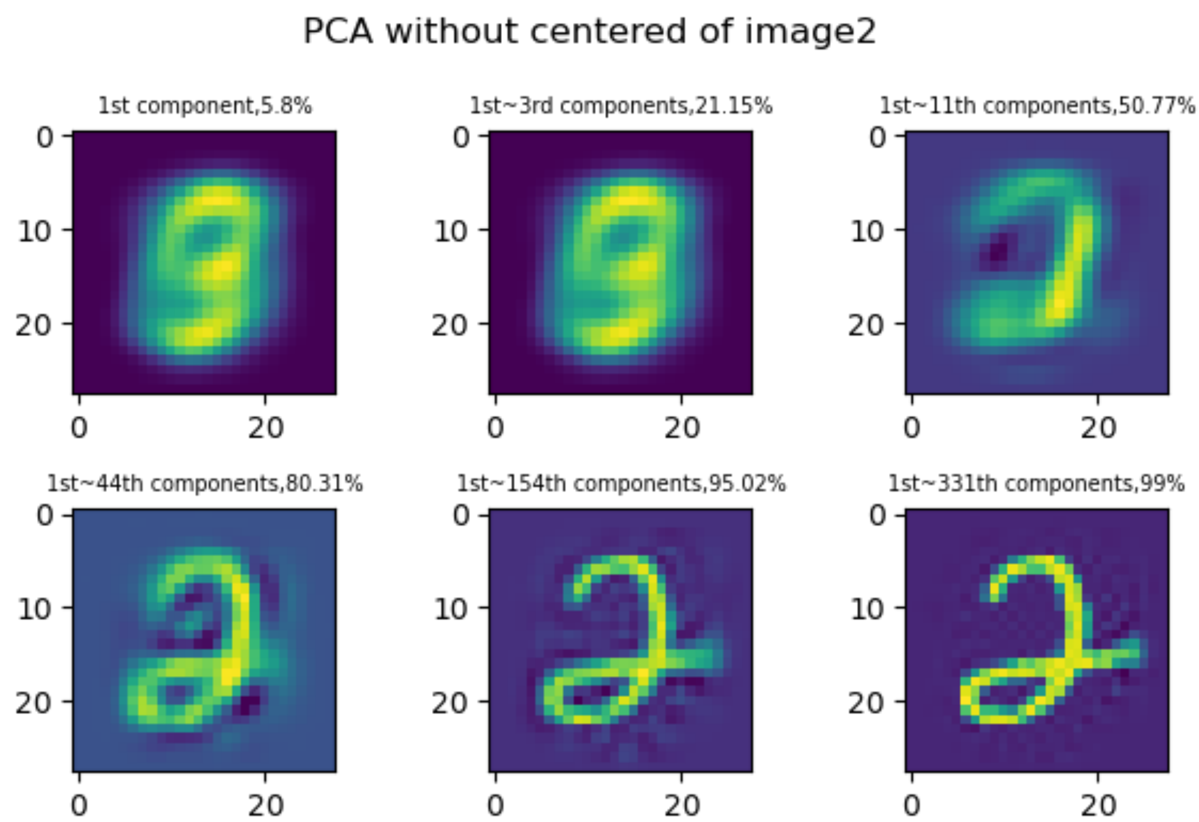
- PCA:
  在44个主成分的累计解释变异高达80.33%时，可以明显看出此图为'2'
- PCA without centered:
  在44个主成分的累计解释变异高达80.31%时，可以明显看出此图为'2'

一样是非常棒的降维效果，因为原始是784个特征，只要降至44个主成分就能达到差不多的效果

In [19]:
```
f, ax = plt.subplots(2, 3)

f.suptitle("PCA of image2", fontsize=12)

ax[0,0].imshow(pca1.inverse_transform(pca_1[16, :]).reshape(28, 28))
ax[0,1].imshow(pca2.inverse_transform(pca_2[16, :]).reshape(28, 28))
ax[0,2].imshow(pca3.inverse_transform(pca_3[16, :]).reshape(28, 28))
ax[1,0].imshow(pca4.inverse_transform(pca_4[16, :]).reshape(28, 28))
ax[1,1].imshow(pca5.inverse_transform(pca_5[16, :]).reshape(28, 28))
ax[1,2].imshow(pca6.inverse_transform(pca_6[16, :]).reshape(28, 28))

ax[0,0].set_title('1st component,9.7%',fontsize=7)
ax[0,1].set_title('1st~3rd components,22.97%',fontsize=7)
ax[0,2].set_title('1st~11th components,50.92%',fontsize=7)
ax[1,0].set_title('1st~44th components,80.33%',fontsize=7)
ax[1,1].set_title('1st~154th components,95.02%',fontsize=7)
ax[1,2].set_title('1st~333th components,99%',fontsize=7)

plt.tight_layout()
plt.show()
```



In [20]:
```
f, ax = plt.subplots(2, 3)

f.suptitle("PCA without centered of image2", fontsize=12)

ax[0,0].imshow(tsvd1.inverse_transform(tsvd_1)[16, :].reshape(28, 28))
ax[0,1].imshow(tsvd2.inverse_transform(tsvd_2)[16, :].reshape(28, 28))
ax[0,2].imshow(tsvd3.inverse_transform(tsvd_3)[16, :].reshape(28, 28))
ax[1,0].imshow(tsvd4.inverse_transform(tsvd_4)[16, :].reshape(28, 28))
ax[1,1].imshow(tsvd5.inverse_transform(tsvd_5)[16, :].reshape(28, 28))
ax[1,2].imshow(tsvd6.inverse_transform(tsvd_6)[16, :].reshape(28, 28))

ax[0,0].set_title('1st component,5.8%',fontsize=7)
ax[0,1].set_title('1st~3rd components,21.15%',fontsize=7)
ax[0,2].set_title('1st~11th components,50.77%',fontsize=7)
ax[1,0].set_title('1st~44th components,80.31%',fontsize=7)
ax[1,1].set_title('1st~154th components,95.02%',fontsize=7)
ax[1,2].set_title('1st~331th components,99%',fontsize=7)

plt.tight_layout()
plt.show()
```




## 2 eigenvectors

PCA方法试图捕获最大方差的最优方向(特征向量)，下面描绘了PCA方法为MNIST生成的前100个最优方向或主成分轴(此时可解释变异为91.46%)。
可以明显看出图片越趋复杂，当把第1个特征向量和第100个特征向量进行比较时，很明显，在寻找最大方差中生成了更复杂的方向或成分，从而使新特征子空间的方差最大化。
此外，也能看出未中心化以及中心化的PCA的特征向量长相不同。

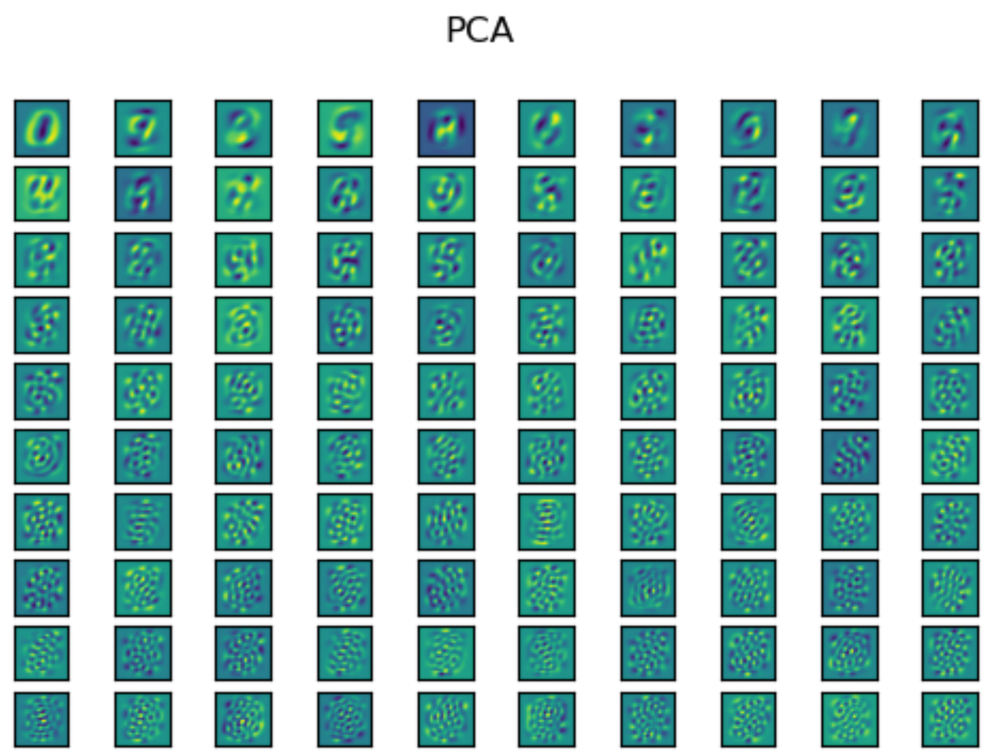
In [328]:
```
print('PCA eigenvector dim:',pca.components_.shape)
print('PCA without centered eigenvector dim:',tsvd.components_.shape)
```

```
PCA eigenvector dim: (333, 784)
PCA without centered eigenvector dim: (332, 784)
```

In [343]:
```
print('PCA 100th eigenvectors - cumulative explained variance ratio:', cumsum_pca[99]*100)
print('PCA without centered 100th eigenvectors - cumulative explained variance ratio:', cumsum_tsvd[99]*100)
```

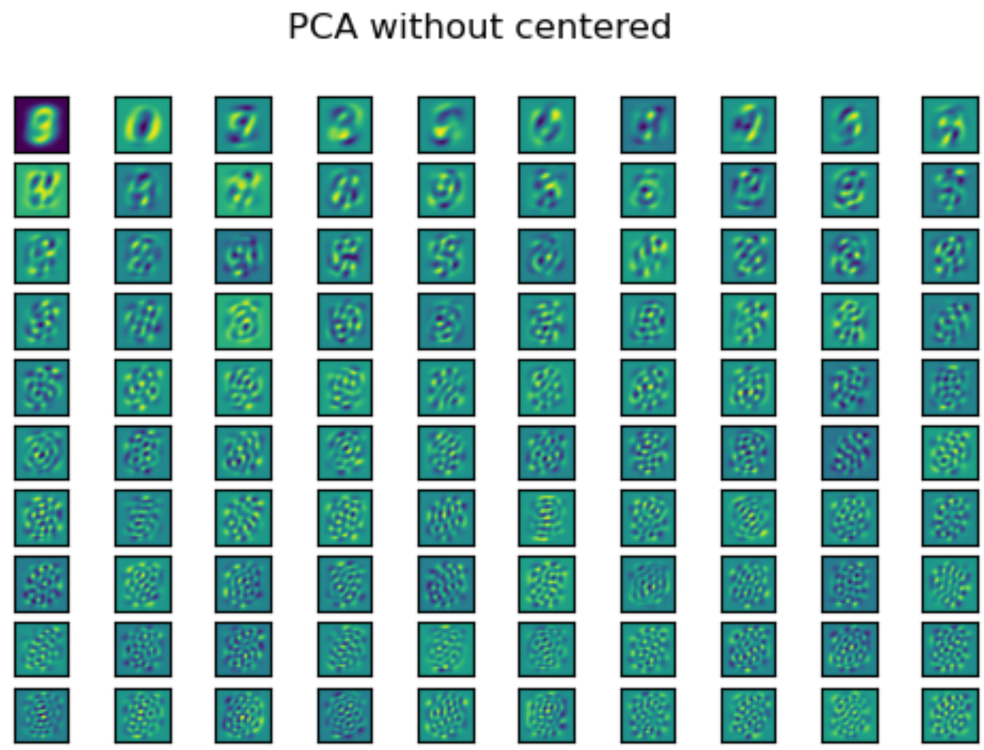
```
PCA 100th eigenvectors - cumulative explained variance ratio: 91.46285724330632
PCA without centered 100th eigenvectors - cumulative explained variance ratio: 91.45978111497294
```

In [329]:
```
f, ax = plt.subplots(10, 10)

f.suptitle("PCA", fontsize=12)

a = [i for i in range(100)]
a = np.array(a).reshape(10,10)
for i in range(10):
    for j in range(10):
        ax[i][j].imshow(pca.components_[a[i][j]].reshape(28,28))
        ax[i][j].set_xticks([])
        ax[i][j].set_yticks([])
plt.show()
```

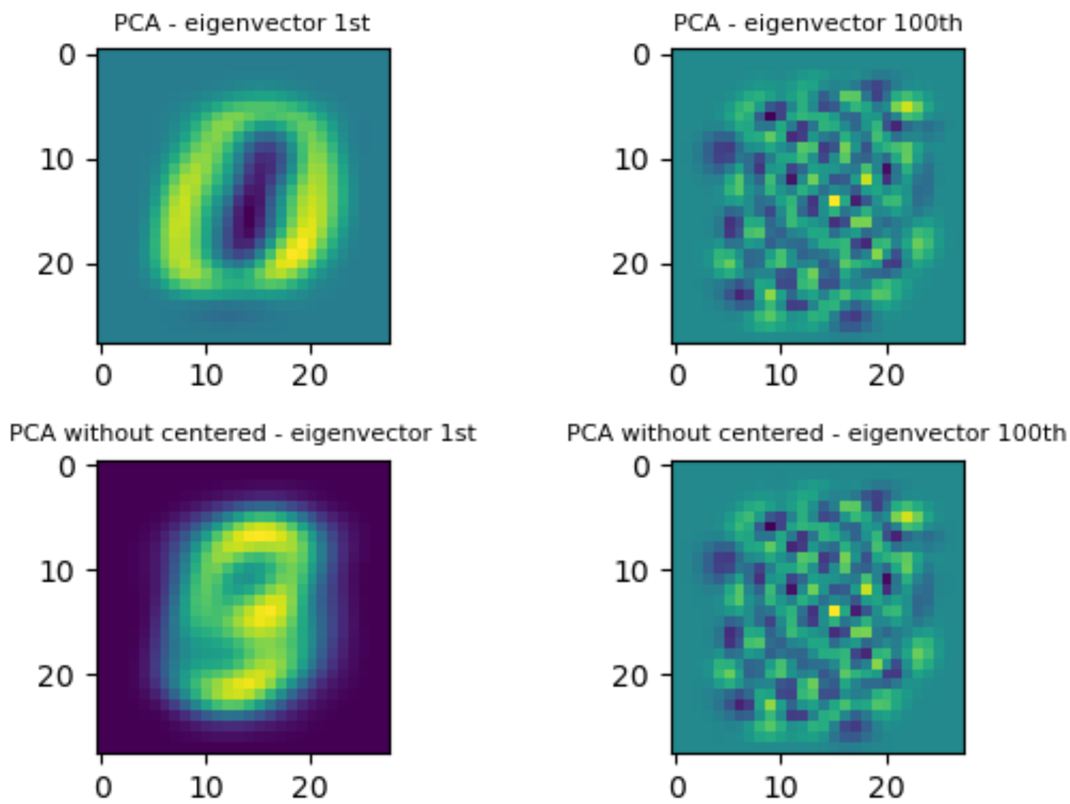


In [330]:

```
f, ax = plt.subplots(10, 10)

f.suptitle("PCA without centered", fontsize=12)

a = [i for i in range(100)]
a = np.array(a).reshape(10,10)
for i in range(10):
    for j in range(10):
        ax[i][j].imshow(tsvd.components_[a[i][j]].reshape(28,28))
        ax[i][j].set_xticks([])
        ax[i][j].set_yticks([])
plt.show()
```



In [333]:
```
f, ax = plt.subplots(2, 2)

ax[0,0].imshow(pca.components_[0,:].reshape(28,28))
ax[0,1].imshow(pca.components_[100,:].reshape(28,28))
ax[1,0].imshow(tsvd.components_[0,:].reshape(28,28))
ax[1,1].imshow(tsvd.components_[100,:].reshape(28,28))
ax[0,0].set_title('PCA - eigenvector 1st',fontsize = 8)
ax[0,1].set_title('PCA - eigenvector 100th',fontsize = 8)
ax[1,0].set_title('PCA without centered - eigenvector 1st',fontsize = 8
)
ax[1,1].set_title('PCA without centered - eigenvector 100th',fontsize =
8)
plt.tight_layout()
plt.show()
```




## 3 conclusion

此数据其实比较适合PCA without centering the dataset，尽管可能是否中心化在此题并无太明显的对比，然而要注意到的是此数据实际上是稀疏矩阵，举例来说，列出其中一个图片来看，可以明显看出其稀疏性，而做中心化会破坏矩阵的稀疏性
既然如此，为何PCA总是要做中心化呢？原因是因为PCA是为了寻找最大方差来获得主成分，以达到降维的目的，因此并不能说PCA是习惯做中心化，而该说去中心化才能求变异已推出PCA的结果
SVD则是和PCA不同的一个概念，有趣的是，若在做SVD前先对数据做中心化的预处理，其结果会和PCA等价，因此使用SVD来求解PCA便成为另一种大众所习惯的方法。正如最一开始提到的，若做中心化则稀疏性会被破坏，因此若遇到稀疏矩阵，可以移除PCA过程中对SVD所做的中心化预处理，以保留稀疏性来做后续的降维。最后也能看出两个方法从eigenvector到累计解释变异均不同，其中到非常高的累积解释变异时，才会显现出PCA without centering the dataset需要的主成分较少，但差距十分些微不明显

In [321]:
```
train_images[15,:] #稀疏矩阵
```

Out[321]:
```
array([[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
          0,   0],
       [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
          0,   0],
```

```
        [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0],
        [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0],
        [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0],
        [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0],
        [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0],
        [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
0,
           0,   0, 115, 121, 162, 253, 253, 213,   0,   0,   0,   0,
0,
           0,   0],
        [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,  6
3,
         107, 170, 251, 252, 252, 252, 252, 250, 214,   0,   0,   0,
0,
           0,   0],
        [  0,   0,   0,   0,   0,   0,   0,   0,  25, 192, 226, 226, 24
1,
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0,
          0,   0]], dtype=uint8)
```

若中心化则破坏了稀疏性：

In [323]:
```
train_images[15,:] - np.mean(train_images[15,:])
```

Out[323]:
```
array([[-32.26530612, -32.26530612, -32.26530612, -32.26530612,
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```

---

# 2 Learning Theory

## Problem 2

### General setting of learning

已知$Z = data\ space,$ 其中$l$可用来衡量$h \in H$在$z \in Z$的表现
$H = hypothesis\ space,$
$l = H \times Z \to [0, \infty) = loss\ function$

现有一回归模型以及模型的$square\ error$，下面展示此回归模型的$General\ setting\ of\ learning$：

$Let\ X = input\ space,\ Y \subseteq \mathbb{R} = outputspace$
$\Rightarrow Z = X \times Y = (X, Y)$
$H = \{f : X \to w^T X + b = \hat{Y}\}$
$l = H \times Z = l(h, (x, y)) = l(h(x), y) = l(\hat{y}, y) = (\hat{y} - y)^2,$
$\forall h \in H,\ (x, y) \in (X, Y)$

---

## Problem 3

### Generalization error with random labels

已知$Z = X \times \{0, 1\},$
$H = \{f : f\ is\ a\ mapping\ from\ X\ to\ \{0, 1\}\},$
$l(h, z) = l(h, (x, y)) = I_{h(x) \neq y},$
$D = distribution\ of\ Z\ with\ equal\ probability \Rightarrow X \sim Ber(0.5),$
$R(h, D) \cong E_{z \sim D}[l(h, z)]$

下证$generalization\ error\ R(h, D)$永远是$\frac{1}{2}$:

已知$D : X \sim Ber(0.5) \Rightarrow p_{z \sim D}(x, 0) = p_{z \sim D}(x, 1) = 0.5,$
$R(h, D)$
$\cong E_{z \sim D}[l(h, z)]$
$= E_{z \sim D}[I_{h(x) \neq y}]$
$= \sum_{x, y \in X, Y} I_{h(x) \neq y} p_{z \sim D}(x, y)$

且知$I_{h(x) \neq y} = \begin{cases} 1,\ if h(x) \neq y \\ 0,\ if h(x) = y \end{cases}, \forall\ (h(x), y) = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

故$R(h, D) = \sum_{x, y \in X, Y} I_{h(x) \neq y} p_{z \sim D}(x, y)$
$= I_{h(x)=0, y=1} p_{z \sim D}(x, 1) + I_{h(x)=1, y=0} p_{z \sim D}(x, 0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$R(h, D)$永远是$\frac{1}{2}$得证

---

## Problem 4

### Bound the generalization error of ERM

#### 1

已知$Empirical\ Error = \hat{R}(h,S) \cong \frac{1}{m}\sum_{i=1} ml(h,z_i),\ \forall\ S=(z_1,z_2,\ldots,z_m) \in Z^m,$

$R^* \cong inf_{h\in H} R(h,D) = R(h^*,D)\ \forall\ Z \sim D,$

$empirical\ error\ minimization(ERM) = \hat{R}(h_s^{ERM},S),\ \forall\ h_s^{ERM} \in argmin_{h\in H} l(h,S)$

下证$R(h_s^{ERM},D) - R^* \le 2sup_{h\in H} \mid R(h,D) - \hat{R}(h,S)|$:

$R(h_s^{ERM},D) - R^*$

$= R(h_s^{ERM},D) - R(h^*,D)$

$= [R(h_s^{ERM},D) - \hat{R}(h_s^{ERM},D)] + [\hat{R}(h_s^{ERM},D) - R(h^*,D)]$

$\le [R(h_s^{ERM},D) - \hat{R}(h_s^{ERM},D)] + [\hat{R}(h^*,D) - R(h^*,D)]\ \ (\because \hat{R}(h^*,D) > \hat{R}(h_s^{ERM},D))$

$\le \mid R(h_s^{ERM},D) - \hat{R}(h_s^{ERM},D) \mid + \mid R(h^*,D) - \hat{R}(h^*,D) \mid$

$\le 2sup_{h\in H} \mid R(h,D) - \hat{R}(h,S)|$

$R(h_s^{ERM},D) - R^* \le 2sup_{h\in H} \mid R(h,D) - \hat{R}(h,S)|$得证

**2**

已知$H$是有限集合，且$l \leq M$

## 2.1 欲证$\forall \epsilon > 0,\ p_{s\sim D^m}(R(h_s^{ERM}, D) - R^* > \epsilon) \leq 2 \mid H \mid exp(-\frac{m\epsilon^2}{2M^2})$

已知$R(h_s^{ERM}, D) - R^* \leq 2sup_{h\in H} \mid R(h, D) - \hat{R}(h, S)|$

$\Rightarrow p_{s\sim D^m}(R(h_s^{ERM}, D) - R^* > \epsilon) \leq p_{s\sim D^m}(2sup_{h\in H} \mid R(h, D) - \hat{R}(h, S) \mid > \epsilon)$

$\Rightarrow$ 欲证$p_{s\sim D^m}(2sup_{h\in H} \mid R(h, D) - \hat{R}(h, S) \mid > \epsilon) \leq 2 \mid H \mid exp(-\frac{m\epsilon^2}{2M^2})$

$p_{s\sim D^m}(2sup_{h\in H} \mid R(h, D) - \hat{R}(h, S) \mid > \epsilon)$

$= p_{s\sim D^m}(2 \mid R(h_1, D) - \hat{R}(h_1, S) \mid > \epsilon \ \vee \ \ldots \ \vee \ 2 \mid R(h_{|H|}, D) - \hat{R}(h_{|H|}, S) \mid > \epsilon)$

$\leq \sum_{h\in H} p_{s\sim D^m}(2 \mid R(h, D) - \hat{R}(h, S) \mid > \epsilon) - (1)$

$By\ Hoeffding's\ inequality, \begin{cases} p(\mu - \frac{1}{m}\sum_{i=1}^m X_i > \epsilon) \leq exp(-\frac{2m\epsilon^2}{(b-a)^2}),\ \forall\ X_i \in [a, b],\ a < b \\ p(\frac{1}{m}\sum_{i=1}^m X_i - \mu > \epsilon) \leq exp(-\frac{2m\epsilon^2}{(b-a)^2}),\ \forall\ X_i \in [a, b],\ a < b \end{cases}$

$\Rightarrow p_{s\sim D^m}(2 \mid R(h, D) - \hat{R}(h, S) \mid > \epsilon)$

$= p_{s\sim D^m}(2R(h, D) - 2\hat{R}(h, S) > \epsilon) + p_{s\sim D^m}(2\hat{R}(h, S) - 2R(h, D) > \epsilon)$

$= p_{s\sim D^m}(2\mu_l - 2\frac{1}{m}\sum_{i=1}^m l(h, z_i) > \epsilon) + p_{s\sim D^m}(2\frac{1}{m}\sum_{i=1}^m l(h, z_i) - 2\mu_l > \epsilon)$

$= p_{s\sim D^m}(\mu_{2l} - \frac{1}{m}\sum_{i=1}^m 2l(h, z_i) > \epsilon) + p_{s\sim D^m}(\frac{1}{m}\sum_{i=1}^m 2l(h, z_i) - \mu_{2l} > \epsilon)$

$\because 0 < 2l \leq 2M \therefore (b-a)^2 = 4M^2$

$\Rightarrow p_{s\sim D^m}(\mu_{2l} - \frac{1}{m}\sum_{i=1}^m 2l(h, z_i) > \epsilon) + p_{s\sim D^m}(\frac{1}{m}\sum_{i=1}^m 2l(h, z_i) - \mu_{2l} > \epsilon)$

$\leq exp(-\frac{2m\epsilon^2}{4M^2}) + exp(-\frac{2m\epsilon^2}{4M^2})$

$= 2exp(-\frac{m\epsilon^2}{2M^2}) - (2)$

$By$ (1) & (2),

$p_{s\sim D^m}(2sup_{h\in H} \mid R(h, D) - \hat{R}(h, S) \mid > \epsilon)$

$\leq \sum_{h\in H} p_{s\sim D^m}(2 \mid R(h, D) - \hat{R}(h, S) \mid > \epsilon)$

$= 2 \mid H \mid exp(-\frac{m\epsilon^2}{2M^2})$

$\Rightarrow p_{s\sim D^m}(R(h_s^{ERM}, D) - R^* > \epsilon) \leq p_{s\sim D^m}(2sup_{h\in H} \mid R(h, D) - \hat{R}(h, S) \mid > \epsilon) \leq 2 \mid H \mid exp(-\frac{m\epsilon^2}{2M^2})$

$\Rightarrow\ p_{s\sim D^m}(R(h_s^{ERM}, D) - R^* > \epsilon) \leq 2 \mid H \mid exp(-\frac{m\epsilon^2}{2M^2})$得证

**2.2 欲证**

$$\forall\delta \in (0,1),\ p_{s\sim D^m}(R(h_s^{ERM}, D) \le R^* + \sqrt{\frac{2M^2(\ln 2|H|+\ln \delta^{-1})}{m}}) \ge 1-\delta$$

已知$p_{s\sim D^m}(R(h_s^{ERM}, D) - R^* > \epsilon) \le 2 \mid H \mid exp(-\frac{m\epsilon^2}{2M^2})$，且$p_{s\sim D^m}(R(h_s^{ERM}, D) - R^* > \epsilon) \in (0,1)$
可令$2 \mid H \mid exp(-\frac{m\epsilon^2}{2M^2}) = \delta \in (0,1)$

$\Rightarrow p_{s\sim D^m}(R(h_s^{ERM}, D) - R^* > \epsilon) \le \delta$
$\Rightarrow p_{s\sim D^m}(R(h_s^{ERM}, D) - R^* \le \epsilon) \ge 1-\delta$
$\Rightarrow p_{s\sim D^m}(R(h_s^{ERM}, D) \le R^* + \epsilon) \ge 1-\delta - (1)$

接着求$\epsilon$，$\because 2 \mid H \mid exp(-\frac{m\epsilon^2}{2M^2}) = \delta \in (0,1)$

$\Rightarrow \frac{2|H|}{exp(\frac{m\epsilon^2}{2M^2})} = \delta$

$\Rightarrow \delta exp(\frac{m\epsilon^2}{2M^2}) = 2 \mid H \mid$

$\Rightarrow exp(\frac{m\epsilon^2}{2M^2}) = 2 \mid H \mid \frac{1}{\delta}$

$\Rightarrow \frac{m\epsilon^2}{2M^2} = \ln 2 \mid H \mid + \ln\frac{1}{\delta}$

$\Rightarrow \epsilon^2 = \frac{2M^2}{m}[\ln 2 \mid H \mid + \ln\frac{1}{\delta}]$

$\Rightarrow \epsilon = \sqrt{\frac{2M^2(\ln 2|H|+\ln\frac{1}{\delta})}{m}} - (2)$

把(2)代入(1)，$p_{s\sim D^m}(R(h_s^{ERM}, D) \le R^* + \epsilon) \ge 1-\delta$

$\Rightarrow p_{s\sim D^m}(R(h_s^{ERM}, D) \le R^* + \sqrt{\frac{2M^2(\ln 2|H|+\ln \delta^{-1})}{m}}) \ge 1-\delta$得证

---

# 3 Deep Generative Models

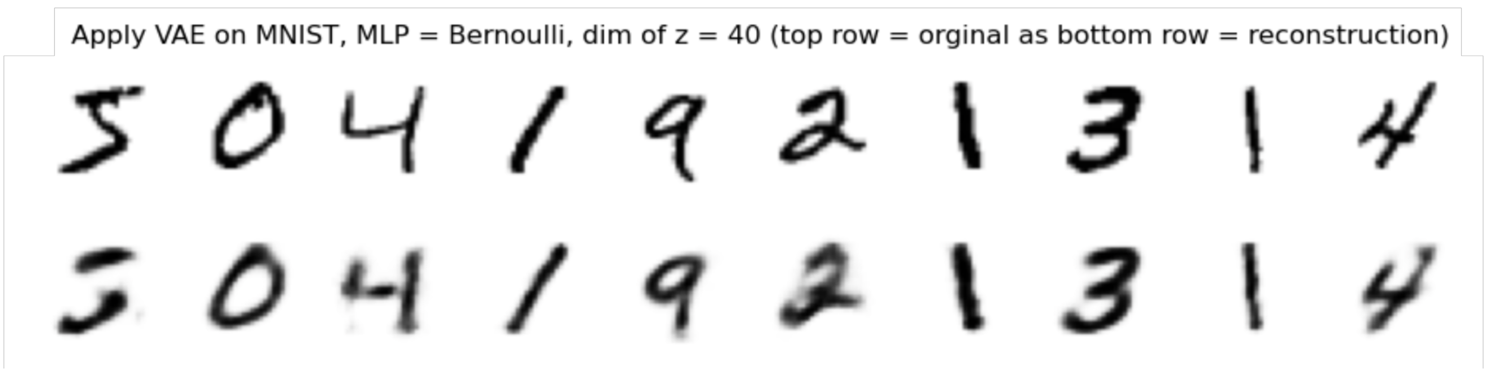
## Problem 5

### Gaussian VAE vs Bernoulli VAE

### 1

这张图是设定$dim\ of\ z = 40$，$MLP = Bernoulli$，并画出前十个MNIST里面的图，来看下重建的效果，其中可以从$Epoch$，$Batch\ Loss$，$Average\ Loss$的数值了解到随着$Epoch$的提升，$Loss\ function$的值会下降，所以若希望重建出来的效果更好，可以提升$Epoch$

```
Epoch 1
[400/469] batch loss: 128.764
Average loss: 146.916995
Epoch 2
[400/469] batch loss: 115.003
Average loss: 119.899064
Epoch 3
[400/469] batch loss: 118.936
Average loss: 116.860961
Epoch 4
[400/469] batch loss: 118.478
Average loss: 115.866277
Epoch 5
[400/469] batch loss: 114.039
Average loss: 115.405274
```

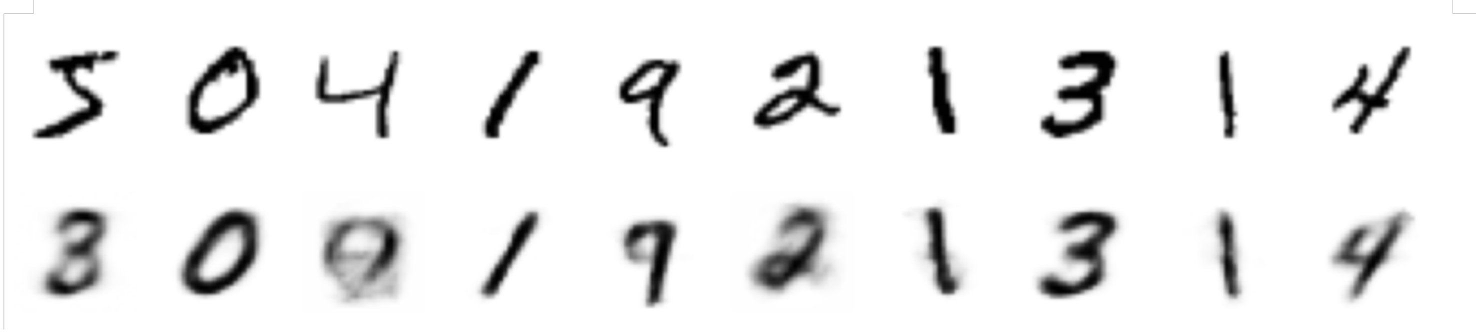



**2**

接着一样设定$dim\ of\ z = 40,\ MLP = Gaussian$，并画出前十个MNIST里面的图，来看下重建的效果

```
Epoch 1
[400/469] batch loss: 40.186
Average loss: 48.582390
Epoch 2
[400/469] batch loss: 40.315
Average loss: 40.026094
Epoch 3
[400/469] batch loss: 37.849
Average loss: 38.916014
Epoch 4
[400/469] batch loss: 37.444
Average loss: 38.522016
Epoch 5
[400/469] batch loss: 36.645
Average loss: 38.250036
```

Apply VAE on MNIST, MLP = Gaussian, dim of z = 40 (top row = orginal as bottom row = reconstruction)

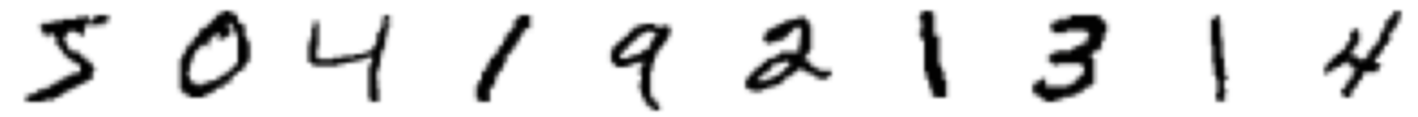


### 3

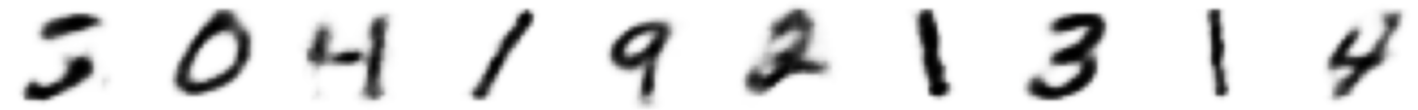
关于此题两个$VAE$，设定一样的$dim\ of\ z$、$Epoch$

- $Loss\ function$
  $MLP = Gaussian$比$MLP = Bernoulli$来得低
- 图片重建效果 $MLP = Gaussian$比$MLP = Bernoulli$来得不清晰

根据这个结果，我的想法是通常图片的值是二元的，我们看到的图片是以二进位制形成，虽然不完全像是Bernoulli一样只有0或1两个label，但仍然可以说接近binary的data形式（相较其他例如股票预测data来说），因此即便$MLP = Gaussian$的$Loss\ function$比$MLP = Bernoulli$来得低，对于为图片数据集的MNIST来说，考量图片结构会导致$MLP = Gaussian$的重建效果比$MLP = Bernoulli$来得不佳，因此我个人会倾向选择$MLP = Bernoulli$于MNIST的应用



Apply VAE on MNIST, MLP = Bernoulli



Apply VAE on MNIST, MLP = Gaussian