STOCHASTIC PROGRAMMING FOR TRANSPORTATION AND LOGISTICS PROBLEMS: FREIGHT FORWARDING PROBLEM

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We consider a planning horizon consisting of D days and a set of consolidation points W. Connecting those consolidation points are legs L. Leg $l \in L$ has an origin consolidation point, $a_l \in W$, and a destination consolidation point, $b_l \in W$. It also has a travel time , τ_l, quoted in days. Capacity on leg $l \in L$ on day $d \in D$ is bought in units of size u_l^d with a per-unit cost of f_l^d.

Capacity is purchased to transport customers modeled with the set C. Associated with customer c is an origin location, a_c, and destination location b_c. Transporting customer c from its origin location, a_c, to consolidation point $w \in W$ has a per-unit cost $v_{a_c w}$ and takes $\tau_{a_c w}$ days. Transporting customer c from consolidation point $w \in W$ to its destination location b_c has a per-unit cost of v_{wb_c} and takes τ_{wb_c} days. Also associated with customer c is a day, s_c, in which it is to be picked up at its origin location, a_c, and a number of days, t_c, to deliver it to its destination location, b_c.

As such, we can construct a set of paths, P_c, of the form $p = \{(a_c, w_1, w_2, b_c)\}$ on which customer c's shipment can be transported. Formally, we must have $(w_1, w_2) \in L$ and $\tau_{a_c w_1} + \tau_{w_1 w_2} + \tau_{w_2 b_c} \leq t_c$. Relatedly, we let $l_p \in L$ denote the leg used by path p and g_p^d the day that leg is taken if the path begins on day d. Because transportation to and from customer locations is paid on a per-unit basis we observe there is no need for a shipment to arrive early to a consolidation point. Relatedly, we let $D_c^p, p \in P_c, c \in C$ denote the days in which the shipment for customer c can depart on path p. Abusing notation, we let $v_p = v_{a_c w_1} + v_{w_2 b_c}, p \in P_c$ denote the variable cost of path p. Lastly, each unit of customer c that is transported earns a per-unit revenue r_c.

We presume the size, q_c, of the shipment associated with customer c is unknown when decisions regarding booking capacity are made. Instead, we presume a random variable ω drawn from distribution Ω that induces the quantities $q_c(\omega)$.

Given this setting, we let the integer decision variable $y_l^d, l \in L, d \in D$ represent the number of units of capacity purchased on leg $l \in L$ on day $d \in D$. We then seek to solve the following stochastic program.

$$\text{maximize } -\sum_{l \in L} \sum_{d \in D} f_l^d y_l^d + \mathbf{E}_\omega[Q(y, q(\omega))] \tag{1}$$

in which $Q(y, q(\omega))$ represents the revenue earned given the capacity prescribed by y and the shipment sizes induced by ω. As $Q(y, q(\omega))$ is itself an optimization problem, we let z_c denote the amount of customer c's shipment that is transported and x_c^{pd} the amount that is transported on path $p \in P_c$ that departs the customer origin location on day $d \in D_c^p$.

Thus, we have

$$Q(y,q(\omega)) = \text{maximize } \sum_{c\in C} r_c z_c - \sum_{p\in P_c}\sum_{d\in D_c^p} v_p x_c^{pd} \tag{2}$$

subject to

$$z_c \le q_c(\omega) \ \ \forall c \in C, \tag{3}$$

$$\sum_{p\in P_c}\sum_{d\in D_c^p} x_c^{pd} = z_c \quad \forall c \in C, \tag{4}$$

$$\sum_{c\in C}\sum_{p\in P_c}\sum_{d\in D_c^p : g_p^d = d'} x_c^{pd} \le u_l^{d'} y_l^{d'} \quad \forall l \in L, d' \in D, \tag{5}$$

$$z_c \in \Re_+ \ \ \forall c \in C, \tag{6}$$

$$x_c^{pd} \in \Re_+ \ \ \forall c \in Cp \in P_c, d \in D_c^p. \tag{7}$$

With customer commitment Now, suppose we must decide whether to commit to serving a customer, in entirety, before knowing the sizes of their shipments. Further, we presume along with the paths identified previously, we have additional third party paths, E_c, in which the leg between consolidation points is operated by dedicated cargo planes. Associated with such a path e is a variable cost $v_e = v_{a_c o_l} + v_l + v_{d_l b_c}$. We presume unlimited capacity on dedicated cargo legs and they dispatch every day. Regardless, we define the set $D_c^e, e \in E_c, c \in C$ similarly to that of D_c^p.

To formulate this problem, we treat the decision variable $z_c \in \{0,1\}, c \in C$ as a first stage decision variable as customer commitment decisions must be made in advance of knowing customer shipment sizes. Thus, we have the following stochastic program.

$$\text{maximize } -\sum_{l\in L}\sum_{d\in D} f_l^d y_l^d + \mathbf{E}_\omega[Q(z,y,q(\omega))] \tag{8}$$

where the subproblem includes the additional decision variable $o_c^{ed} \ge 0, c \in C, e \in E_c, d \in D_c^e$ that represents how much of customer c's shipment is transported on a path involving a dedicated cargo leg.

$$Q(z,y,q(\omega)) = \text{maximize } \sum_{c\in C} r_c q_c(\omega) z_c - \sum_{p\in P_c}\sum_{d\in D_c^p} v_p x_c^{pd} - \sum_{e\in E_c}\sum_{d\in D_c^e} v_e o_c^{ed} \tag{9}$$

subject to

$$\sum_{p\in P_c}\sum_{d\in D_c^p} x_c^{pd} + \sum_{e\in E_c}\sum_{d\in D_c^e} o_c^{ed} = z_c q_c(\omega) \quad \forall c \in C, \tag{10}$$

$$\sum_{c\in C}\sum_{p\in P_c}\sum_{d\in D_c^p : g_p^d = d'} x_c^{pd} \le u_l^{d'} y_l^{d'} \quad \forall l \in L, d' \in D, \tag{11}$$

$$x_c^{pd} \in \Re_+ \ \ \forall c \in Cp \in P_c, d \in D_c^p. \tag{12}$$

$$o_c^{ed} \in \Re_+ \ \ \forall c \in Ce \in E_c, d \in D_c^e. \tag{13}$$

We observe that as the term $\sum_{c \in C} r_c q_c(\omega) z_c$ is a constant in the subproblem, we can move its expectation to the objective of the first stage problem. Namely, we have

$$\text{maximize } -\sum_{l \in L} \sum_{d \in D} f_l^d y_l^d + \mathbf{E}_\omega[\sum_{c \in C} r_c q_c(\omega) z_c] - \mathbf{E}_\omega[Q'(y, z, q(\omega))] \tag{14}$$

Given the properties of expectations, we can rewrite as

$$\text{maximize } -\sum_{l \in L} \sum_{d \in D} f_l^d y_l^d + \sum_{c \in C} \mathbf{E}_\omega[r_c q_c(\omega)] z_c - \mathbf{E}_\omega[Q'(y, z, q(\omega))] \tag{15}$$

where $Q'(y, z, q(\omega)) = \text{minimize } \sum_{p \in P_c} \sum_{d \in D_c^p} v_p x_c^{pd} - \sum_{e \in E_c} \sum_{d \in D_c^e} v_e o_c^{ed}$.

With uncertain dedicated cargo leg pricing Now, we presume the per-unit costs, v_e, of dedicated cargo leg transportation is uncertain. Namely, we presume it is also a function of the random variable ω and thus we have $v_e(\omega)$. As such, we have the following subproblem.

$$\text{maximize } -\sum_{l \in L} \sum_{d \in D} f_l^d y_l^d + \mathbf{E}_\omega[Q(z, y, q(\omega))] \tag{16}$$

where the subproblem includes the additional decision variable $o_c^{ed} \geq 0, c \in C, e \in E_c, d \in D_c^e$ that represents how much of customer c's shipment is transported on a path involving a dedicated cargo leg.

$$Q(z, y, q(\omega), v(\omega)) = \text{maximize } \sum_{c \in C} r_c z_c - \sum_{p \in P_c} \sum_{d \in D_c^p} v_p x_c^{pd} - \sum_{e \in E_c} \sum_{d \in D_c^e} v_e(\omega) o_c^{ed} \tag{17}$$

subject to (10),(11),(12),(13).