Lines

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Lines in $\mathbb{R}^n$

The Equation of a Line

We all know intuitively that a line is represented by the equation $y = mx + b$, but a line can also be represented using a pair of two vectors

Definition 1 (Equation of a line)**.** let $\mathbf{d}$ be the direction vector abd $\mathbf{p}$ be the position vector, then we can represent a line as the position vector plus a scalar multiple of the direction vector

$$l = \{\mathbf{p} + t\mathbf{d} | t \in \mathbb{R}\}$$

where l is a line that passes through the point $\mathbf{p}$ and is parallel to the direction vector $\mathbf{d}$ Given the set-builder notation on the definition of a line, we should consider a line to be the **set of all position vectors** $\overrightarrow{OX}$ **to points on the line**
Similarly, we can view a line as a function corresponding to the scalar multiple t

$$\mathbf{x(t)} = \mathbf{p} + t\mathbf{d}$$

Note that this implies that, given two position vectors, we can conclude that the direction vector is the distance between those two

$$\mathbf{x(t)} = \mathbf{p_1} + t(\mathbf{p_1} - \mathbf{p_1})$$

a corollary of this definition is that the same lines can be represented using different position and direction vectors. in this case, $\mathbf{p}$ can take different points on the line l, or $\mathbf{d}$ can take any scalar multiple of the original direction vecctor.

Intersections of two lines

The intersection of two lines is given by the system of equations

$$x_1 = x_2$$

where x_1 is the vector at the intersection point and x_2 is also the vector at the intersection point

General Equation of a line

The general equation of a line is given by

$$ax_1 + by_1 = c$$

where x_1 and y_1 correspond to some arbitrary point on the line and the vector $[a\ b]$ is a vector orthogonal to the line. This implies that

$$[a\ b] \cdot [x_2 - x_1\ y_2 - y_1] = 0$$

Explicitly, the equation of a line in normal form is represented by

$$\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$$