

# Planes

Julian Pagcaliwagan

December 14, 2022

## Planes in $\mathbb{R}^3$

Planes are a flat surface that exist in the vector space  $\mathbb{R}^3$ .

### The equation of a plane

For  $n \geq 2 \in \mathbb{R}^n$ , the plane equation is as follows:

**Definition 1** (Plane).

$$\{\mathbf{p} + s\mathbf{v} + t\mathbf{w} \mid s, t \in \mathbb{R}\}$$

At a certain point  $\mathbf{x}$ , we have that the equation of the plane becomes

$$\mathbf{x} = \mathbf{p} + s\mathbf{v} + t\mathbf{w}$$

Given three position vectors, we can deduce that the direction vectors  $\mathbf{v}$  and  $\mathbf{w}$  become the differences  $\mathbf{p}_2 - \mathbf{p}_1$ ,  $\mathbf{p}_3 - \mathbf{p}_1$ . Since the difference of two vectors creates a separate 'distance' vector that points from head to head.

### Parametric equations

The equation for a specific point in a plane  $x = [x_1 \ x_2 \ x_3]$  for a plane in  $\mathbb{R}^n$ ,  $n \geq 3$  is given by the equation

$$\mathbf{x} = \mathbf{p} + s\mathbf{v} + t\mathbf{w}$$

which satisfies

$$\mathbf{x}_1 = \mathbf{p}_1 + s\mathbf{v}_1 + t\mathbf{w}_1, \mathbf{x}_2 = \mathbf{p}_2 + s\mathbf{v}_2 + t\mathbf{w}_2, \mathbf{x}_3 = \mathbf{p}_3 + s\mathbf{v}_3 + t\mathbf{w}_3$$

### The general equation of a plane

Similar to lines, the general equation of a plane is given by

$$ax + by + cz = d$$

where the non-zero normal vector  $\mathbf{n} = [a \ b \ c]$  is orthogonal to the plane - implying that it is orthogonal to the direction vectors  $\mathbf{v} = \mathbf{p}_2 - \mathbf{p}_1$  and  $\mathbf{w} = \mathbf{p}_3 - \mathbf{p}_1$ . Also similar to lines, the general equation for the plane is equivalent to the equation

$$\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$$

Which is called the **normal form of the equation of the plane**. since it is equivalent to  $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$ . In vector notation, this equation is represented as

$$[a \ b \ c] \cdot [x \ y \ z] = [a \ b \ c] \cdot [p_1 \ p_2 \ p_3]$$

a