Planes

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Planes in $\mathbb{R}^3$

Planes are a flat surface that exist in the vector space $\mathbb{R}^3$.

The equation of a plane

For $n \geq 2 \in \mathbb{R}^n$, the plane equation is as follows:

Definition 1 (Plane)**.**

$$\{\mathbf{p} + s\mathbf{v} + t\mathbf{w} \mid s, t \in \mathbb{R}\}$$

At a certain point $\mathbf{x}$, we have that the equation of the plane becomes

$$\mathbf{x} = \mathbf{p} + s\mathbf{v} + t\mathbf{w}$$

Given three position vectors, we can deduce that the direction vectors $\mathbf{v}$ and $\mathbf{w}$ become the differences $\mathbf{p_2} - \mathbf{p_1},\ \mathbf{p_3} - \mathbf{p_1}$. Since the difference of two vectors creates a separate 'distance' vector that points from head to head.

Parametric equations

The equation for a specific point in a plane $x = [x_1\ x_2\ x_3]$ for a plane in $\mathbb{R}^n,\ n \geq 3$ is given by the equation

$$\mathbf{x} = \mathbf{p} + s\mathbf{v} + t\mathbf{w}$$

which satisfies

$$\mathbf{x_1} = \mathbf{p_1} + s\mathbf{v_1} + t\mathbf{w_2},\ \mathbf{x_2} = \mathbf{p_2} + s\mathbf{v_2} + t\mathbf{w_2},\ \mathbf{x_3} = \mathbf{p_3} + s\mathbf{v_3} + t\mathbf{w_3}$$

The general equation of a plane

Similar to lines, the general equation of a plane is given by

$$ax + by + cz = d$$

where the non-zero normal vector $\mathbf{n} = [a\ b\ c]$ is orthogonal to the plane - implying that it is orthogonal to the direction vectors $\mathbf{v} = \mathbf{p_2} - \mathbf{p_1}$ and $\mathbf{w} = \mathbf{p_3} - \mathbf{p_1}$. Also similar to lines, the general equation for the plane is equivalent to the equation

$$\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$$

Which is called the **normal form of the equation of the plane.** since it is equivalent to $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$. In vector notation, this equation is represented as

$$[a\ b\ c] \cdot [x\ y\ z] = [a\ b\ c] \cdot [p_1\ p_2\ p_3]$$