

Planes

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December 14, 2022

Planes in \mathbb{R}^3

Planes are a flat surface that exist in the vector space \mathbb{R}^3 .

The equation of a plane

For $n \geq 2 \in \mathbb{R}^n$, the plane equation is as follows:

Definition 1 (Plane).

$$\{\mathbf{p} + s\mathbf{v} + t\mathbf{w} \mid s, t \in \mathbb{R}\}$$

At a certain point \mathbf{x} , we have that the equation of the plane becomes

$$\mathbf{x} = \mathbf{p} + s\mathbf{v} + t\mathbf{w}$$

Given three position vectors, we can deduce that the direction vectors \mathbf{v} and \mathbf{w} become the differences $\mathbf{p}_2 - \mathbf{p}_1$, $\mathbf{p}_3 - \mathbf{p}_1$. Since the difference of two vectors creates a separate 'distance' vector that points from head to head.

Parametric equations

The equation for a specific point in a plane $x = [x_1 \ x_2 \ x_3]$ for a plane in \mathbb{R}^n , $n \geq 3$ is given by the equation

$$\mathbf{x} = \mathbf{p} + s\mathbf{v} + t\mathbf{w}$$

which satisfies

$$\mathbf{x}_1 = \mathbf{p}_1 + s\mathbf{v}_1 + t\mathbf{w}_1, \mathbf{x}_2 = \mathbf{p}_2 + s\mathbf{v}_2 + t\mathbf{w}_2, \mathbf{x}_3 = \mathbf{p}_3 + s\mathbf{v}_3 + t\mathbf{w}_3$$

The general equation of a plane

Similar to lines, the general equation of a plane is given by

$$ax + by + cz = d$$

where the non-zero normal vector $\mathbf{n} = [a \ b \ c]$ is orthogonal to the plane - implying that it is orthogonal to the direction vectors $\mathbf{v} = \mathbf{p}_2 - \mathbf{p}_1$ and $\mathbf{w} = \mathbf{p}_3 - \mathbf{p}_1$. Also similar to lines, the general equation for the plane is equivalent to the equation

$$\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$$

Which is called the **normal form of the equation of the plane**. since it is equivalent to $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$. In vector notation, this equation is represented as

$$[a \ b \ c] \cdot [x \ y \ z] = [a \ b \ c] \cdot [p_1 \ p_2 \ p_3]$$