

# Lines

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## Lines in $\mathbb{R}^n$

### The Equation of a Line

We all know intuitively that a line is represented by the equation  $y = mx + b$ , but a line can also be represented using a pair of two vectors

**Definition 1** (Equation of a line). let  $\mathbf{d}$  be the direction vector and  $\mathbf{p}$  be the position vector, then we can represent a line as the position vector plus a scalar multiple of the direction vector

$$l = \{\mathbf{p} + t\mathbf{d} | t \in \mathbb{R}\}$$

where  $l$  is a line that passes through the point  $\mathbf{p}$  and is parallel to the direction vector  $\mathbf{d}$ . Given the set-builder notation on the definition of a line, we should consider a line to be the **set of all position vectors  $\overrightarrow{OX}$  to points on the line**

Similarly, we can view a line as a function corresponding to the scalar multiple  $t$

$$\mathbf{x}(t) = \mathbf{p} + t\mathbf{d}$$

Note that this implies that, given two position vectors, we can conclude that the direction vector is the distance between those two

$$\mathbf{x}(t) = \mathbf{p}_1 + t(\mathbf{p}_1 - \mathbf{p}_1)$$

a corollary of this definition is that the same lines can be represented using different position and direction vectors. in this case,  $\mathbf{p}$  can take different points on the line  $l$ , or  $\mathbf{d}$  can take any scalar multiple of the original direction vector.

### Intersections of two lines

The intersection of two lines is given by the system of equations

$$x_1 = x_2$$

where  $x_1$  is the vector at the intersection point and  $x_2$  is also the vector at the intersection point

### General Equation of a line

The general equation of a line is given by

$$ax_1 + by_1 = c$$

where  $x_1$  and  $y_1$  are points on the