Appendices and Supporting Materials

To accompany the article *Speculative Bubbles and Control Theory - An Endogenous Approach*

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A.1 Model Availability

Accompanying the main article and this Appendix are the full models for the Thoroughbred Horse market, the eToys stock price system, and the GameStop stock price system (available as .mdl files) along with the necessary support files to performing the optimizations described in the main article, along with datafiles necessary to reproduce the figures in the main article.

https://github.com/jpain3/Endogenized-PID-Controllers-and-Speculative-Bubbles

More specifics on each file type and the layout of these datafiles is detailed below.

The .mdl files can be open and run using Vensim software, developed by Ventana Systems, Inc (*Vensim*, 2022). A free version of the Vensim software for personal use, along with a standalone model viewer, is available from Ventana Systems, Inc. These model files accompany this Appendix.

Ventana Systems, Inc provides detailed documentation on the Vensim software, including how to manipulate and examine specific formulations. However, the reader may quickly explore the influence of parameter choices on the model via the SyntheSim mode on the main Dashboard view of the model. Note that in more recent versions of the software, this mode has been renamed and may appear as a button labeled ‘Run simulation on each slider change.’

In either case, this interactive mode can be accessed by pressing the corresponding button in the top toolbar of the software as seen for either interface type in Figure A1.

Figure A1. Screenshots of Vensim Interface Buttons Relevant to Dynamic Model Exploration

Furthermore, the .mdl file provided may be opened in any program that is able to read UTF-8 encoding and the formulations directly viewed in plaintext. This plaintext viewable option, combined with a free dynamic viewer, is one of the primary reasons this specific software package was chosen for presenting this work.

Examples of programs that can open the .mdl file for direct viewing in plaintext include Notepad in the Windows operating system and Textpad in the Macintosh operating system. An example of this view of the model file is seen in Figure A2.

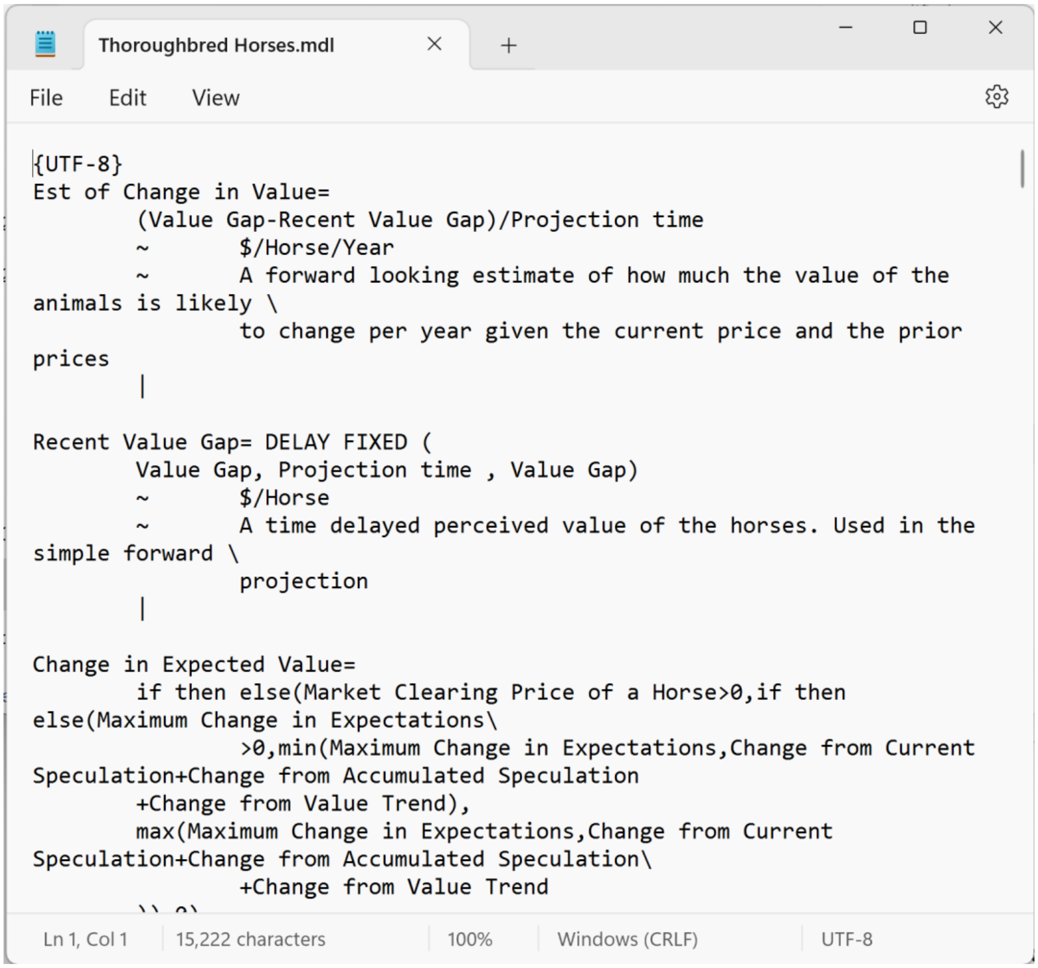

Figure A2. Example of Viewing the Supporting .mdl File in Notepad on Windows

The models developed for this article are also fully documented utilizing the SDM-Doc tool described in (Martinez-Moyano, 2012). The output from this documentation tool is available alongside the .mdl files.

The folder structure of the files included with each example model are illustrated in Figure A3 below. A description of each file and its purpose follows:

SDM Doc Output.zip

PID Speculation Model - eToys Stock Price.mdl

eToysHistoricPricesDaily.vdfx

eToys Daily.vdfx

EndoPIDController_eToys_Payoff_Daily.vpd

EndoPIDController_eToys_Daily.voc

Figure A3. Sample Folder Structure of the eToys Stock Price Model Files

- SDM Doc Output.zip
    - This is the output of the SDM tool. It contains within it a copy of the core model (the mdl file) that was used to generate the output to ensure transparency, along with an html file that gives a report of the content of the model. This includes detailed reports on the relationships between variables, full model structural details, along with flags for common errors or documentation hygiene issues. Care has been taken to minimize or outright eliminate any of these errors or warnings in these sample models.
- Files ending in *.mdl
    - These are the core Vensim model files. As discussed above these can be opened in any plain text editor and read directly. Additionally, they can be opened and run graphically using the free Vensim modeling tools described above.
- Files ending in *.vdfx
    - These are data sets used to generate traces of behavior seen in the figures in the main article and generated as outputs from running the *.mdl models in Vensim. Note that for these examples, there are typically two such files.

One contains a trace of historic prices, used for calibration and figure generation. The other is the trace of the model output itself.

- Files ending in *.vpd
  - These files define the payoff function for the calibration used to fit these models to historic data. These files can again be opened in any plain text editor and examined. Within Vensim, the optimization engine uses these files to define the payoff function in calibration. Note that the free versions of Vensim do not typically allow for direct manipulation of the optimization routines, though they can be edited indirectly by modifying these files in a text editor.
- Files ending in *.voc
  - Alongside the *.vpd files described above, these files are needed for the optimization and calibration routines in Vensim. This file serves to define the independent variables and their bounds available for the optimization and calibration routines. It also defines the routines themselves, with options ranging from hill-climbing Powell methods to stochastic Monte Carlo approaches. For all the examples developed in this specific paper, the Powell hill-climbing method with random restarts was used.

A.2 Numerical Methods for the Endogenized PID Controller

Consider first the block diagram of the traditional PID controller described in the main manuscript, and repeated below:

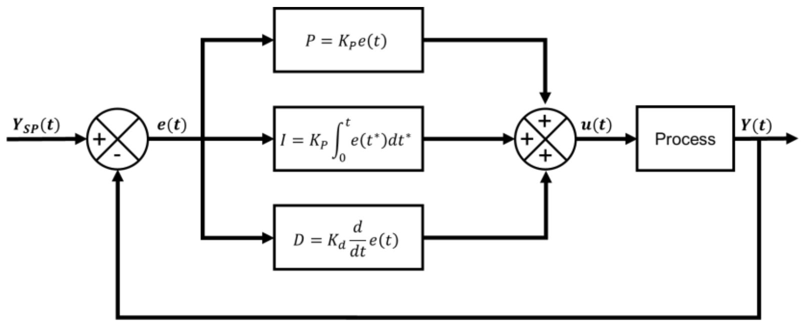

Figure A4. Block diagram of a Parallel PID Controller

In real systems, and discussed in more practice oriented control theory texts, the signal received and monitored is often not continuous and thus the derivative in the PID controller design cannot be computed directly, but rather inferred via numerical methods (Seborg et al., 2016). This lends itself towards translating Figure A4 into a compartmental model that utilizes the visual language of System Dynamics (Sterman, 2000), and in software which is able to readily perform the numerical integration needed for the *I* in the PID controller scheme, but with appropriate estimation simplifications for the differential *D* portion.

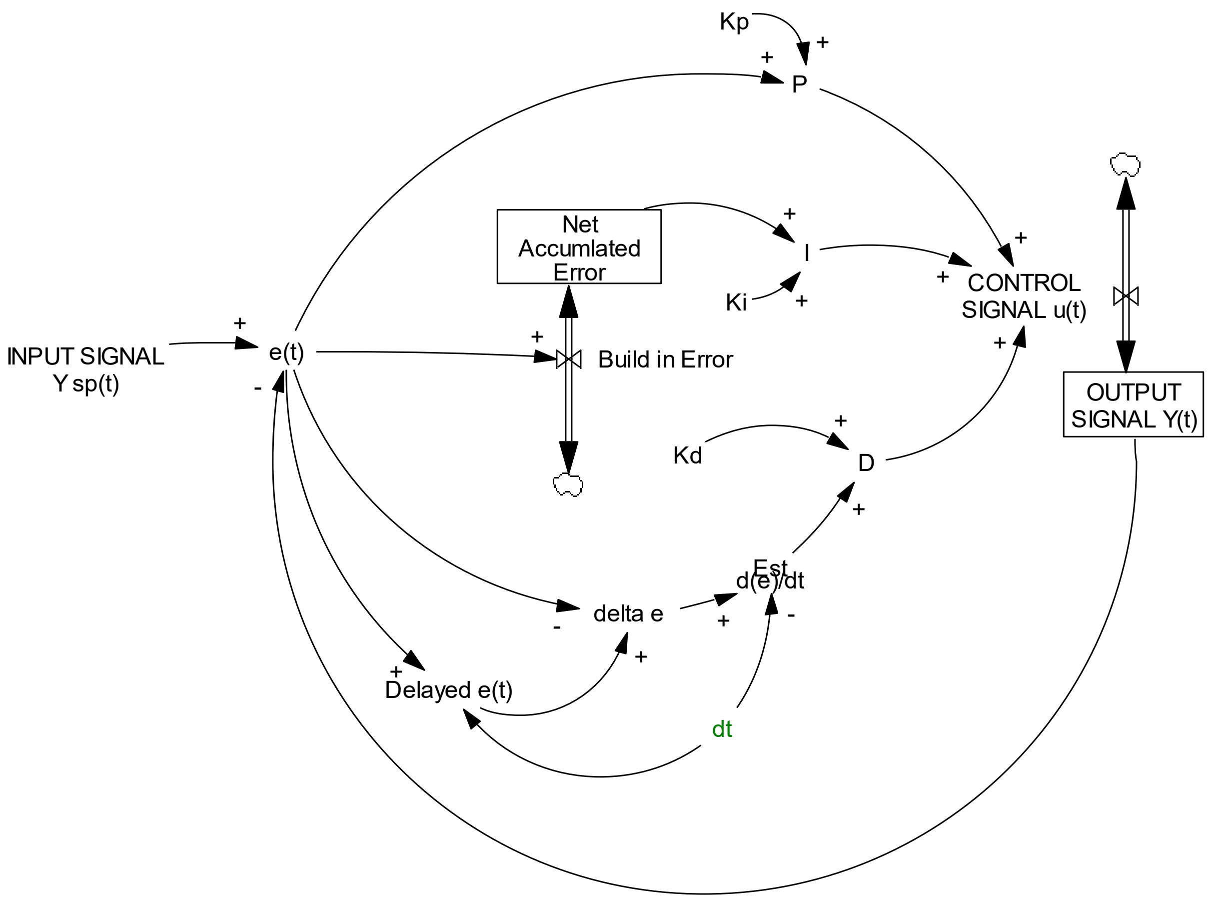

Figure A5. Compartmental Model of a Parallel PID Controller

The system described above and shown in Figure A4 and Figure A5 provides an endogenous formation of the system state, $Y(t)$, but relies on an exogenous input for the set point $Y_{SP}(t)$. While System Dynamics as a field has been strongly connected to control theory from its inception (Forrester, 1989), it derives its value relative to that original field by taking the mathematical frameworks given above and working to embed human-centric heuristic models and endogenizing meaningful feedbacks (Sterman, 2000).

In real processes that rely on human decision makers, the 'set point' describing the idealized state of the system is anchored towards the observed state of that system. Indeed the model shown in Figure A6 could be readily extended to consider anchoring-and-adjustment heuristics commonly invoked in behavioral sciences (Tversky & Kahneman, 1974), and often cited in Supply Chain management adjacent System Dynamics literature (Croson et al., 2014;

Paine, 2023; Sterman, 1989). By connecting the set point to the process output, we can more directly bridge the control theory roots of System Dynamics with the observations of human-centric decision making as illustrated in Figure A6.

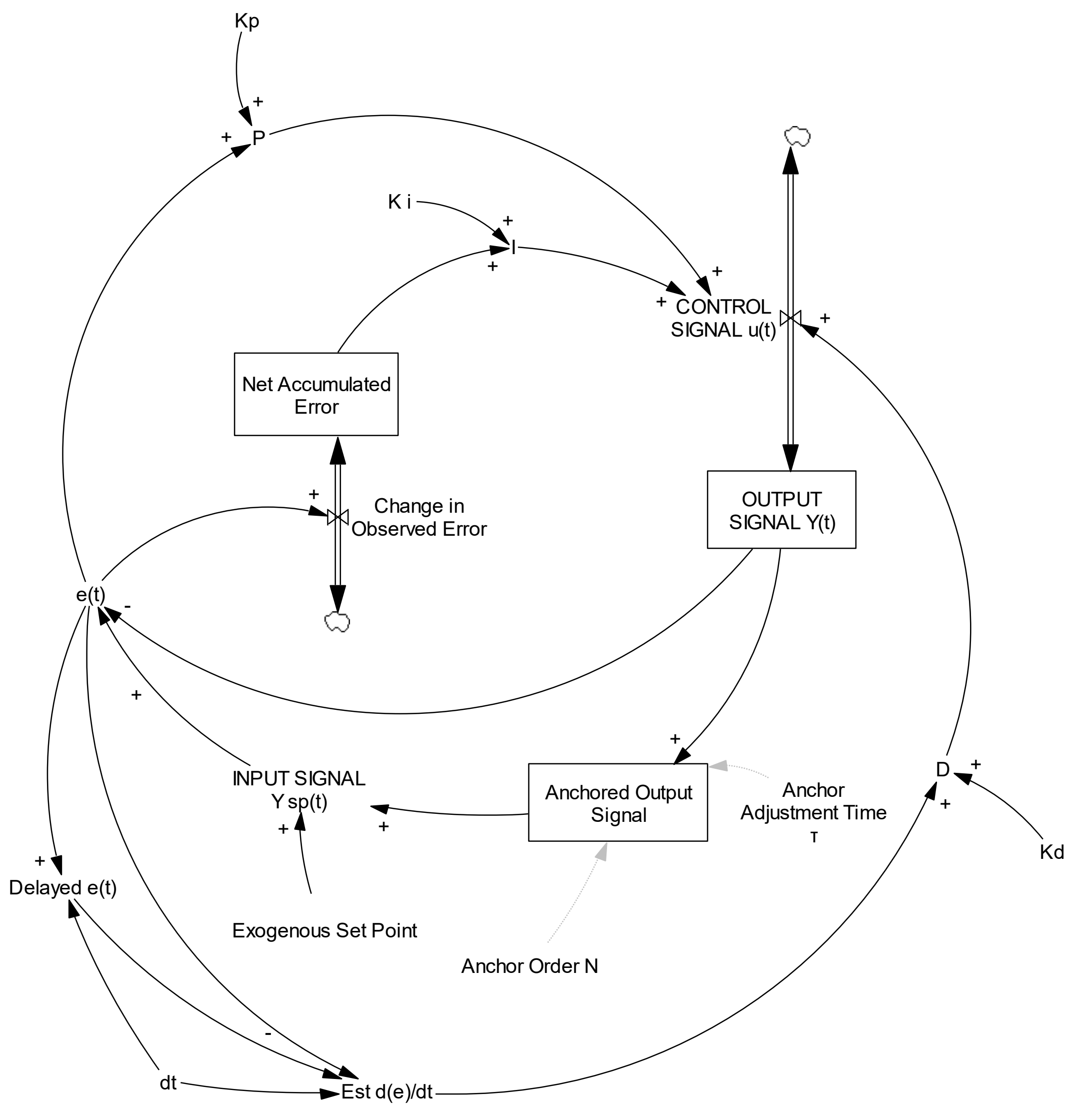

Figure A6. Compartmental Model of an Endogenized PID Controller

There are many methods of performing the numerical estimations for the derivative and integral terms seen in Figure A6, including popular numerical integration packages in coding languages like *R* or Python. Additionally, the block diagram models used in PID controller design lend themselves well to tools like Mathwork's Simulink software (*Matlab Simulink*, 2024). For the analyses in this manuscript, Ventana System's Vensim software package (*Vensim*, 2022) was the primary tool employed.

The expressions below provide a simple first-order approximation using the built-in fixed delay formulation and smooth formulation found in Vensim:

$$Delayed\ e(t) = FIXED\ DELAY[e(t), dt\ , e(t = 0)] \tag{A1}$$

$$Est\ d(e)/dt\ = \frac{e(t) - Delayed\ e(t)}{dt} \tag{A2}$$

$$Anchored\ Output\ Signal =\ SMOOTH\ N(Y(t), \tau\ , N) \tag{A3}$$

$$Y_{SP}(t) =\ Anchored\ Output\ Signal + Exogenous\ Set\ Point \tag{A4}$$

A.3 Endogenized PID Controller Model Recast for the Thoroughbred Horse Market

In the main manuscript, the endogenized PID controller design repeated here in Figure A6 is not explicitly shown recast specific language of the thoroughbred horse market stock pricing. This was done for compact presentation. Below Figure A7, provides this recast model.

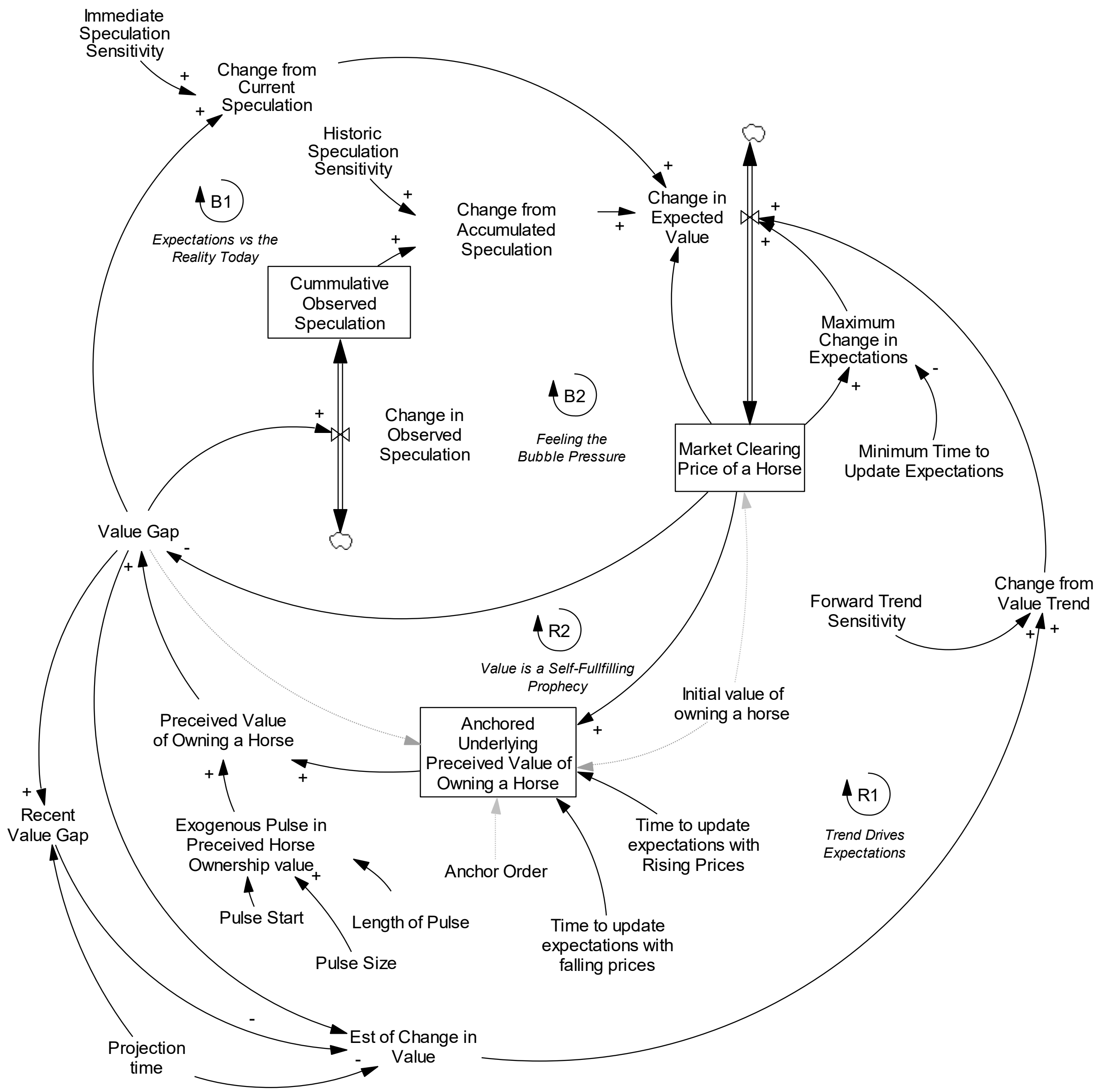

Figure A7. Compartmental Model of an Endogenized PID Controller Applied to the Thoroughbred Horse Example

First, the output and input described in the endogenized PID controller system above are now, respectively, the *Market Clearing Price of a Horse,* and *Perceived Value of Owning a Horse*. This model abstracts away from the full price formation and market clearing mechanisms of this market and assumes that, with some delay, the perceived value of owning a horse eventually translates into the market clearing price. In other words, arbitrage, if present, is insignificant.

Next, the *Anchored Output Signal* from the above model is more clearly recast as the *Anchored Underlying Perceived Value of Owning a Horse* and allowed to take on two different time constants, τ, depending on if market prices exceed or are below the perceived value of the actors in the market.

Finally, this specific speculative bubble requires a physical constraint on the output signal. Specially, it can be safe to assume that the *Market Clearing Price of a Horse* must be non-negative. The more generic structure developed above for an endogenized PID controller does not require this assumption, nor is it appropriate in many different real-world scenarios. In fact, in 2020 U.S. crude oil prices turned negative in a market that features speculation (Walker, 2020). However, for this specific scenario of forming market clearing prices for thoroughbred horses over 1965 to 1990, the assumption of non-negative prices can be incorporated by introducing a first order control on the *Market Clearing Price of a Horse* via introduction of a *Minimum Time to Update Expectations* time constant.

The final model of speculative price formation with endogenous updating of the underlying perceived value, with incorporates more realistic market elements from this specific scenario into the endogenized PID controller structure developed above.

This recast model can now also be considered in a more directly behavioral framework, rather than the more esoteric language of PID controllers. The reinforcing loop R1 clearly shows the classic forward-looking projection scheme described in many prior speculative bubble literature, and also the focus of prior System Dynamics work that highlights how the derivative term in PID controllers can be stabilizing when the input signal is truly exogenous versus destabilizing when even moderate endogeneity exists such in this case here (Forrester, 1961;

Saeed, 2009). The reinforcing loop R2 acts in a similar manner in that realized market value further drive actual market value, but this acts directly on the perceived underlying value of the asset, and as discussed below is an essential feature of this endogenized PID structure and is responsible for embedding some of the signal generated by the speculative bubble itself into the underlying value of the asset. Finally, the balancing loops act in much the same manner as the original PID controller, attempting to offset the error between the actual observed market clearing price of the horse and the underlying perceived value of the horse. Here that can be conceptualized as actors in this market being sensitive to either the instantaneous difference between these prices, B1, or the building pressure from prior speculative activity, B2.

A.4 Further Details on the Model fit to the Thoroughbred Horse Example

The tables below provide the point-outputs (in Table A1) and goodness-of-fit statistics (in Table A2) for the fitted model for this thoroughbred horse pricing data.

Table A1. Historic Data vs Model Fitted Values for the Thoroughbred Horse Market Model

Year	Yearling Price (Thousands of 1965$)	Model Prediction
1965	$ 17.23	$ 27.07
1966	$ 17.23	$ 27.07
1967	$ 19.53	$ 27.07
1968	$ 23.74	$ 27.07
1969	$ 19.14	$ 27.07
1970	$ 22.97	$ 27.07
1971	$ 23.74	$ 27.07
1972	$ 25.27	$ 27.07
1973	$ 36.37	$ 27.07
1974	$ 30.25	$ 27.07
1975	$ 27.18	$ 27.07
1976	$ 31.39	$ 28.33
1977	$ 39.05	$ 34.85
1978	$ 52.83	$ 42.88
1979	$ 57.43	$ 52.76
1980	$ 64.32	$ 64.91
1981	$ 76.57	$ 79.86
1982	$ 89.97	$ 98.25
1983	$ 124.81	$ 120.87
1984	$ 140.51	$ 136.41
1985	$ 122.89	$ 118.49
1986	$ 84.99	$ 92.47
1987	$ 84.23	$ 79.79
1988	$ 72.36	$ 77.97
1989	$ 78.87	$ 79.10
Observations		25
Residual Sum of Squares SS_{res}		839
Total Sum of Squares SS_{tot}		33,010
Coefficient of Determination R^2		**0.975**

Table A2. Summary Fit Statistics for the eToys Daily Closing Price Model

Observations	448
Residual Sum of Squares SS_{res}	24,067
Total Sum of Squares SS_{tot}	231,158
Coefficient of Determination R^2	**0.896**

A.5 Endogenized PID Controller Model Recast for eToys and GameStop

Similar to the above, the endogenize controller design was shown in only the general form in the main manuscript. Figure A8 below provides the recast model for the context of the eToys and GameStop examples. Note that these models are all structurally identical. The only changes are to the names of variables and structures (the references to 'horses' replaces with 'stocks') and the time scale (the time units of 'years' replaced with 'days').

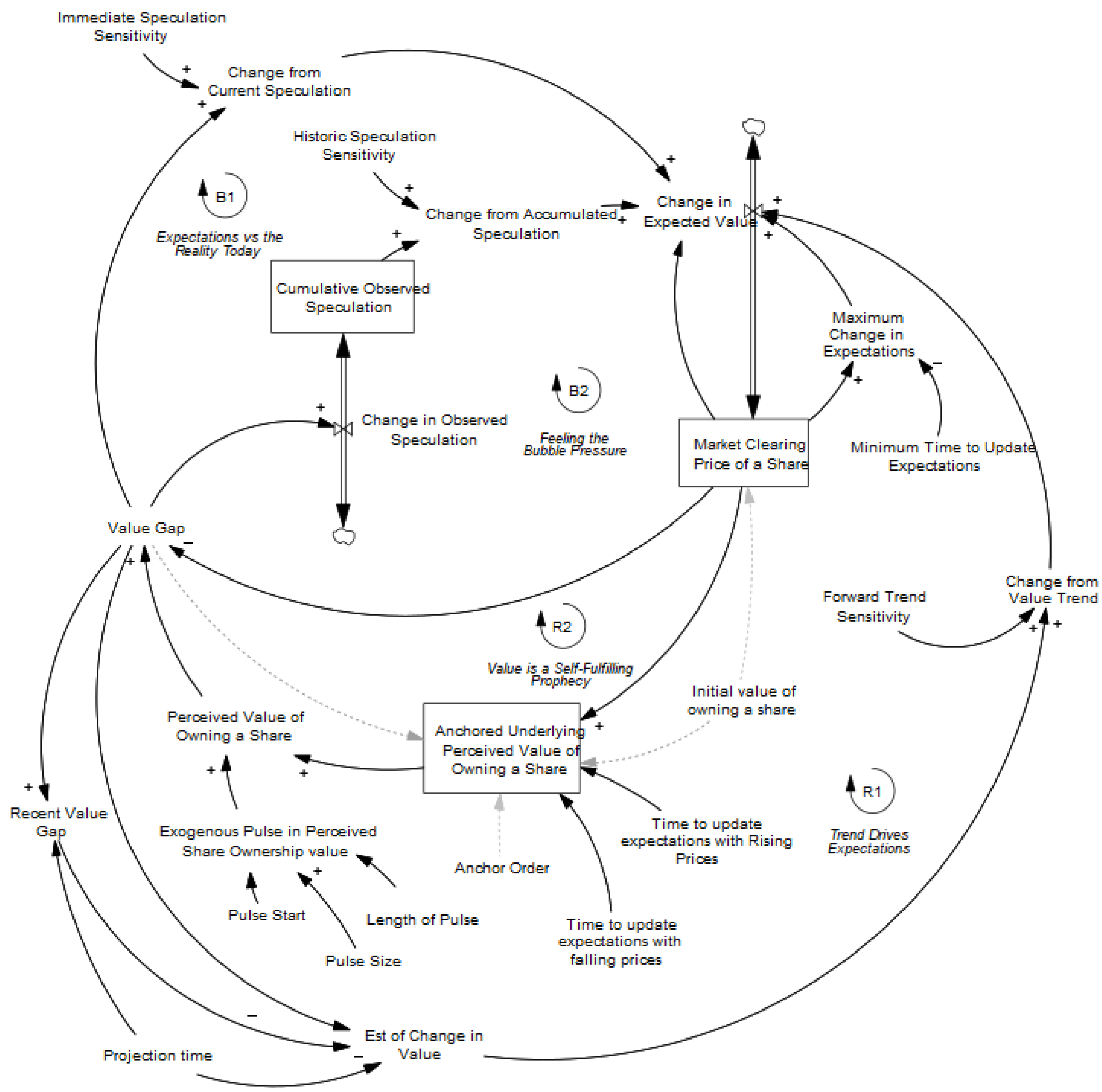

Figure A8. Compartmental Model of an Endogenized PID Controller Applied to the eToys and GameStop Stock Price Example

As stated in the main document, the specific daily closing values for the eToys stock was obtained via a licensed dataset from the *Center for Research in Security Prices, LLC* and hosted by *Wharton Research Data Services* (*eToys (ETYS) Price History May 1999 through*

February 2001, 2024). As a condition of using that dataset, the specific values are not able to be reported here. Table A3 shows the fitted model when recast for the eToys data.

Table A3. Fitted Values of the eToys Model

Parameter	Fitted Value	Units
Initial value of owning a share	20	$/share
Time to update expectations with falling prices	0.321	Days
Time to update expectations with Rising Prices	44.592	Days
Immediate Speculation Sensitivity	1.553	dmnl/Day
Historic Speculation Sensitivity	0.2956	dmnl/Day/Day
Forward Trend Sensitivity	72.4852	dmnl
Projection time	146.656	Day
Minimum Time to Update Expectations	0.8	Day
Pulse Size	100	$/share
Length of Pulse	1	Day

Applying this same method to the GameStop timeseries data yields the calibrated values shown in Table A4:

Table A4. Fitted Values of the GameStop Model

Parameter	Fitted Value	Units
Time to update expectations with falling prices	0.6805	Days
Time to update expectations with Rising Prices	6.4584	Days
Immediate Speculation Sensitivity	0.5807	dmnl/Day
Historic Speculation Sensitivity	0.0113	dmnl/Day/Day
Forward Trend Sensitivity	92.7935	dmnl
Projection time	317.808	Day
Minimum Time to Update Expectations	0.2110	Day

A.6 References to the Appendix

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