

## Assignment: NP-Completeness and Heuristic Algorithms

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*Note: You will discuss Question 1 as part of the Group Assignment. (Check this week's Group Assignment on Canvas for details).*

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- 1. NP-Completeness:** Consider the Travelling Salesperson (TSP) problem that was covered in the exploration.

Problem: Given a graph  $G$  with  $V$  vertices and  $E$  edges, determine if the graph has a TSP solution with a cost of at most  $k$ .

Prove that the above stated problem is NP-Complete.

- 2. Implement Heuristic Algorithm:**

- a. Below matrix represents the distance of 5 cities from each other. Represent it in the form of a graph

|   | A  | B  | C  | D  | E  |
|---|----|----|----|----|----|
| A | 0  | 2  | 3  | 20 | 1  |
| B | 2  | 0  | 15 | 2  | 20 |
| C | 3  | 15 | 0  | 20 | 13 |
| D | 20 | 2  | 20 | 0  | 9  |
| E | 1  | 20 | 13 | 9  | 0  |

- b. Apply Nearest-neighbor heuristic to this matrix and find the approximate solution for this matrix if it were for TSP problem.
- c. What is the approximation ratio of your approximate solution?
- d. Implement Travelling Salesman Problem using the nearest-neighbor heuristic.

**Input:** The input Graph is provided in the form of a 2-D matrix (adjacency matrix). Consider the first node as the starting point.

Sample input:

```
G = [
    [0, 2, 3, 20, 1],
    [2, 0, 15, 2, 20],
    [3, 15, 0, 20, 13],
    [20, 2, 20, 0, 9],
    [1, 20, 13, 9, 0],
]
```

**Output:** A list of indices indicating the path taken. You must return the sequence of nodes, the path taken starting from node 0. In this example,  $G$  is  $5 \times 5$ , indicating there are 5 nodes in this graph: 0-4. You will always begin with node 0, and your

path should include every node exactly once, and only go between nodes with a nonzero edge between them. Your path will end at the starting node.

Sample output (For above graph G):

```
[0, 4, 3, 1, 2, 0]
```

Note: Not all graphs are fully connected: some rows in G may have more than one 0. These indicate absence of an edge.

Name your function **solve\_tsp(G)**. Name your file **TSP.py**.