Skill Problem E: The Covering Problem

[9 points] The decision problem COVER is as follows. You are given a positive integer *k*, a set *S*, and a collection of *n* sets $\{S_1, S_2, ..., S_n\}$, where

$$S = \bigcup_{i=1}^{n} S_i .$$

The decision problem asks whether there is a subcollection of $\{S_1, S_2, ..., S_n\}$, consisting of at most *k* sets, whose union is also equal to *S*. In symbols, does there exist a subcollection $\{S_{i_1}, S_{i_2}, ..., S_{i_k}\}$ of the original collection so that

$$S = \bigcup_{j=1}^{k} S_{i_j}$$

Such a subcollection is called a *cover of size k.*

Example: Let $S = \{1,2,3,4\}$, $k = 2$, and suppose the given collection of sets is

$$\{\{1,2,3\}, \{2,3,4\}, \{1,2,4\}\}.$$

Notice that the subcollection $\{\{1,2,3\}, \{1,2,4\}\}$ is a cover of size 2, so the return value for this instance of the problem is "Yes".

For this problem, you must prove that VERTEXCOVER is polynomial reducible to COVER.

What to turn in:

Submit your document in which you give a detailed proof that VERTEXCOVER is polynomial reducible to COVER. You must provide as much detail as was given in class in the proof that HAMILTONIANCYCLE is polynomial reducible to TSP.