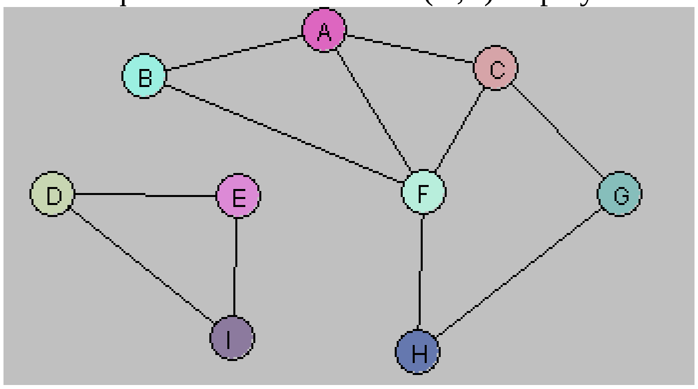
Lab 11 Solutions

1. Answer questions about the G = (V,E) displayed below.



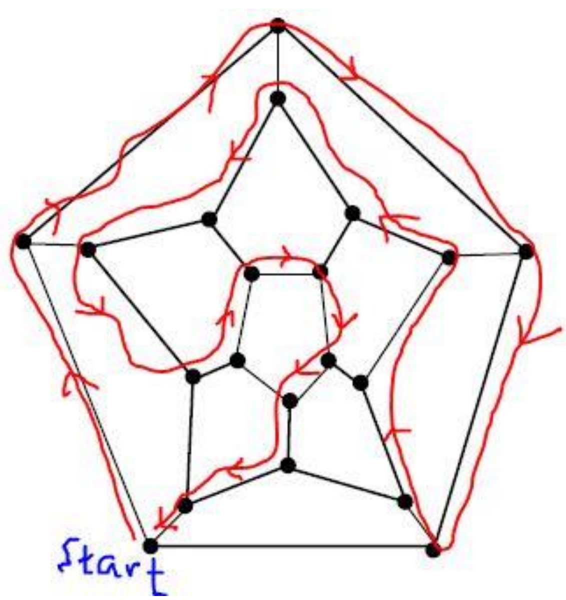
   A. Is the graph G connected? If not, what are the connected components for G?
   Solution: G is not connected. It has two connected components…

   B. Draw a spanning tree/forest for G.
   Solution: T = {DE, EI, FB, FA, FC, FH, GH}

   C. Is G a Hamiltonian graph?
   Solution: No, it has no Hamiltonian Cycle.

   D. Is there a Vertex Cover of size less than or equal to 5 for G? If so, what is the Vertex Cover?
   Solution: Yes. C={D, E, F, A, G}

2. *Hamiltonian Graphs.* The following graph has a Hamiltonian cycle. Find it.



3. Express in pseudo-code an algorithm which accepts as input a graph G and which outputs a vertex cover for G of smallest possible size. You may make use of the PowerSet algorithm without showing any pseudo-code details indicating how it works. Also, you may assume that your algorithm can make use of these operations freely:

   `computeEndpoints(e)` //returns the two endpoints of the edge e

   `belongsTo(x, U)` // returns true if vertex x belongs to set U; false otherwise

   Follow the rules for the pseudo-code language as completely as possible.

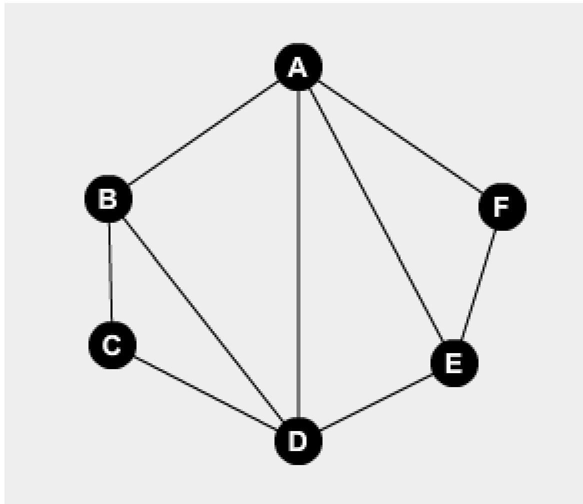
Solution:

```
Algorithm: SmallestVertexCover
   Input: A graph G whose set of vertices is denoted V and set
      of edges is denoted E
   Output: Smallest size of a vertex cover U for G

   pow ← PowerSet(V)
   minCover ←  V
   minVal ← |V|
   for each U in pow do
      isCover ← true
      //verify U is a vertex cover
      for each e in E do
         (u,v)  ← computeEndpoints(e)
         if( !(belongsTo(u,U) and !belongsTo(v,U))
            isCover ← false

      if(isCover and U.size() < minCover.size()) then
            minCover ←  U
            minVal ←  |U|
   return minVal
```

4. Compute two spanning trees for the graphs below using algorithms we discuss in class. (You can start with vertex A) Are the two spanning trees same?



Solution:
Using DFS starting with A: AB, BC, CD, DE, EF
Using BFS starting with A: AB, AD, AE, AF, BC
The two spanning tree are not same.

5. Write the pesudo-code for compute connected components algorithm discussed in class. Your algorithm can be built on top of DFS discussed in the slides.

Solution:
Make a ConnectedComponentSearch subclass of DFS

Initialize ArrayList<List<Vertex>> componentMap;
Initialize HashMap<Vertex,Integer> vertexComponentMap;
CurrentComponentNumber $\leftarrow$ 0

Algorithm: additionalProcessing
    currentComponentNumber++

Algorithm: processVertex(v)
    vertexComponentMap.put(v, currentComponentNumber)
    componentMap.get(currentComponentNumber).add(v)

```
Algorithm: computeConnectedComponents
    //start DFS
    start()
    Graph[] components ← new Graph[currentComponentNumber];
    for i ← 0 to currentComponentNumber do
        // For each component i, we get a list of vertices for that component
        List<Vertex> vlist ← componentMap.get(i)
        List<Edge> elist ← new ArrayList<Edge>()
        foreach Vertex v in vlist
            List<Vertex> adjList ← adjacencyList.get(v);
                foreach Vertex u in adjList
                    Edge e ← new Edge(v, u))
                    if e not yet in elist then
                        elist.add(e)
        components[i] ← new Graph(elist)
    return components
```

6. Write the pesudo-code for the algorithm, discussed in class, that computes the shortest path length between two vertices in a graph. You can assume that:
    a. The graph is connected.
    b. A version of BFS is provided that accepts a specified starting vertex.

Solution:

Make ShortestPath a subclass of BFS

```
Initialize HashMap<Vertex, Integer> levelsMap
Initialize HashMap<Vertex, Vertex> parentMap

Algorithm processEdge(Vertex v, Vertex w)
    //v is the parent, and w is the child
    parentMap.put(w, v);

Algorithm processVertex(Vertex v)
    parentVertex ← parentMap.get(v)
    if parentVertex is null then    //v is the starting vertex
            parentMap.put(v,  null)
            levelsMap.put(v, 0)
    else
            plevel ← levelsMap.get(parentVertex)
            levelsMap.put(v, plevel +1)

Algorithm computeShortestPathLength(Vertex s, Vertex v)
    //start BFS with starting vertex s
    start(s)
    //now levels and parents have been computed
    return levelsMap.get(v)
```