

Aprendizaje no supervisado

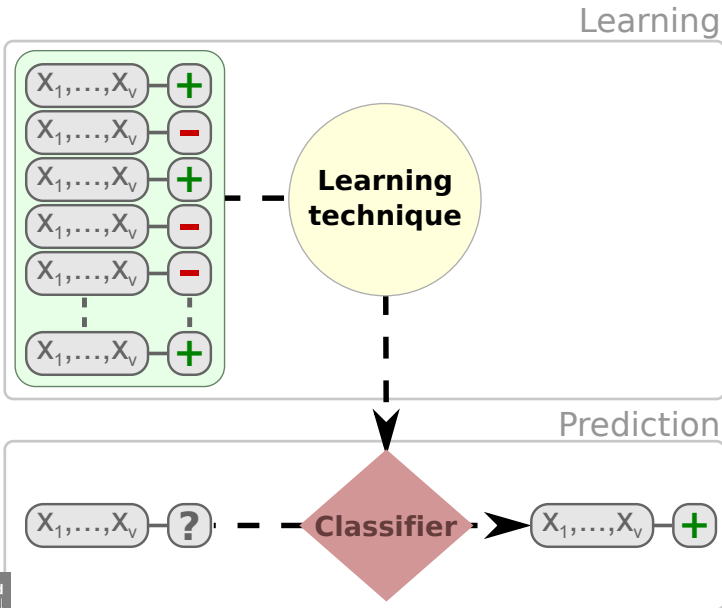
VC08: Naive Bayes

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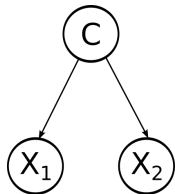
Clasificación Supervisada



Naive Bayes

$$p(X_1, X_2, C) = p(C|X_1, X_2) \times p(X_1, X_2)$$

$$p(C|X_1, X_2) = \frac{p(X_1, X_2, C)}{p(X_1, X_2)}$$



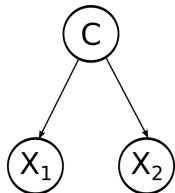
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$$p(C|X_1, X_2) = \frac{p(X_1, X_2, C)}{p(X_1, X_2)}$$

Regla de la cadena

$$p(X_1, X_2, C) = p(X_1|X_2, C) \times p(X_2, C)$$

$$p(X_1, X_2, C) = p(X_1|X_2, C) \times p(X_2|C) \times p(C)$$



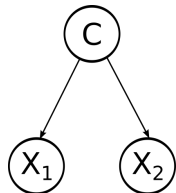
$$p(X_1, X_2, C) = p(C|X_1, X_2) \times p(X_1, X_2)$$

$$p(C|X_1, X_2) = \frac{p(X_1, X_2, C)}{p(X_1, X_2)}$$

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$$p(X_1, X_2, C) = p(X_1|X_2, C) \times p(X_2|C) \times p(C)$$



Asunción de independencia X's—C del Naive

Bayes $p(X_1, X_2, C) = p(X_1|C) \times p(X_2|C) \times p(C)$

$$p(C|X_1, X_2) = \frac{p(X_1|C) \times p(X_2|C) \times p(C)}{p(X_1, X_2)}$$

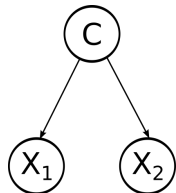
$$p(X_1, X_2, C) = p(C|X_1, X_2) \times p(X_1, X_2)$$

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Regla de la cadena

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$$p(X_1, X_2, C) = p(X_1|X_2, C) \times p(X_2|C) \times p(C)$$



Asunción de independencia X's—C del Naive

Bayes $p(X_1, X_2, C) = p(X_1|C) \times p(X_2|C) \times p(C)$

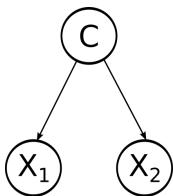
$$p(C|X_1, X_2) = \frac{p(X_1|C) \times p(X_2|C) \times p(C)}{p(X_1, X_2)}$$

$$\operatorname{argm\acute{a}x}_c p(C|X_1, X_2)$$

$$p(C|X_1, X_2) \propto p(X_1|C) \times p(X_2|C) \times p(C)$$

$$\operatorname{argm\acute{a}x}_c p(X_1|C) \times p(X_2|C) \times p(C)$$

Naive Bayes



$$\theta_C = p(C = 1)$$

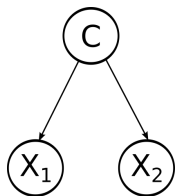
$$\theta_{X_1|C=0} = p(X_1 = 1|C = 0)$$

$$\theta_{X_1|C=1} = p(X_1 = 1|C = 1)$$

$$\theta_{X_2|C=0} = p(X_2 = 1|C = 0)$$

$$\theta_{X_2|C=1} = p(X_2 = 1|C = 1)$$

Naive Bayes



$$\theta_C = p(C = 1)$$

$$\theta_{X_1|C=0} = p(X_1 = 1|C = 0)$$

$$\theta_{X_1|C=1} = p(X_1 = 1|C = 1)$$

$$\theta_{X_2|C=0} = p(X_2 = 1|C = 0)$$

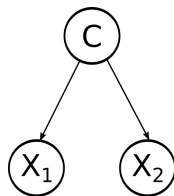
$$\theta_{X_2|C=1} = p(X_2 = 1|C = 1)$$

C	$p(C)$
0	0,4
1	0,6

X_1	C	$p(X_1 C)$
0	0	0,50
1	0	0,50
0	1	0,33
1	1	0,66

X_2	C	$p(X_2 C)$
0	0	0,25
1	0	0,75
0	1	0,66
1	1	0,33

Naive Bayes



$$\theta_C = p(C = 1)$$

$$\theta_{X_1|C=0} = p(X_1 = 1|C = 0)$$

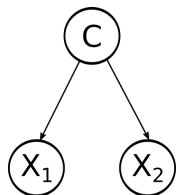
$$\theta_{X_1|C=1} = p(X_1 = 1|C = 1)$$

$$\theta_{X_2|C=0} = p(X_2 = 1|C = 0)$$

$$\theta_{X_2|C=1} = p(X_2 = 1|C = 1)$$

C		X_1		$p(X_1 C)$	X_2		$p(X_2 C)$
$p(C)$		0	1		0	1	
0	0,4	0	0	0,50	0	0	0,25
1	0,6 = θ_C	1	0	0,50 = $\theta_{X_1 C=0}$	1	0	0,75 = $\theta_{X_2 C=0}$
		0	1	0,33	0	1	0,66
		1	1	0,66 = $\theta_{X_1 C=1}$	1	1	0,33 = $\theta_{X_2 C=1}$

Naive Bayes



C	$p(C)$
0	0,4
1	0,6

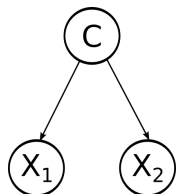
X_1	C	$p(X_1 C)$
0	0	0,50
1	0	0,50
0	1	0,33
1	1	0,66

X_2	C	$p(X_2 C)$
0	0	0,25
1	0	0,75
0	1	0,66
1	1	0,33

$$\hat{c} = f(\mathbf{x}) = \operatorname{argmax}_c p(c) \prod_{i=1}^2 p(x_i|c)$$

X_1	X_2	C
0	0	?
1	0	?

Naive Bayes



C	$p(C)$
0	0,4
1	0,6

X_1	C	$p(X_1 C)$
0	0	0,50
1	0	0,50
0	1	0,33
1	1	0,66

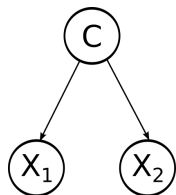
X_2	C	$p(X_2 C)$
0	0	0,25
1	0	0,75
0	1	0,66
1	1	0,33

$$\hat{c} = f(\mathbf{x}) = \underset{c}{\operatorname{argmax}} p(c) \prod_{i=1}^2 p(x_i|c)$$

X_1	X_2	C
0	0	?
1	0	?

$$\begin{aligned} C=0 & (1 - \theta_C) \times (1 - \theta_{X_1|C=0}) \times (1 - \theta_{X_2|C=0}) \\ C=1 & \theta_C \times (1 - \theta_{X_1|C=1}) \times (1 - \theta_{X_2|C=1}) \end{aligned}$$

Naive Bayes



C	$p(C)$
0	0,4
1	0,6

X_1	C	$p(X_1 C)$
0	0	0,50
1	0	0,50
0	1	0,33
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X_2	C	$p(X_2 C)$
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0	1	0,66
1	1	0,33

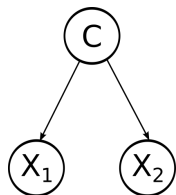
$$\hat{c} = f(\mathbf{x}) = \operatorname{argmax}_c p(c) \prod_{i=1}^2 p(x_i|c)$$

X_1	X_2	C
0	0	?
1	0	?

$$C=0 \quad 0,4 \times 0,5 \times 0,25$$

$$C=1 \quad 0,6 \times 0,33 \times 0,66$$

Naive Bayes



C	$p(C)$
0	0,4
1	0,6

X_1	C	$p(X_1 C)$
0	0	0,50
1	0	0,50
0	1	0,33
1	1	0,66

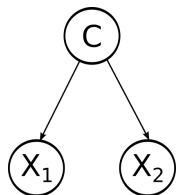
X_2	C	$p(X_2 C)$
0	0	0,25
1	0	0,75
0	1	0,66
1	1	0,33

$$\hat{c} = f(\mathbf{x}) = \underset{c}{\operatorname{argmax}} p(c) \prod_{i=1}^2 p(x_i|c)$$

X_1	X_2	C
0	0	?
1	0	?

$$\begin{aligned} C=0 & \quad (1 - \theta_C) \times \theta_{X_1|C=0} \times (1 - \theta_{X_2|C=0}) \\ C=1 & \quad \theta_C \times \theta_{X_1|C=1} \times (1 - \theta_{X_2|C=1}) \end{aligned}$$

Naive Bayes



C	$p(C)$
0	0,4
1	0,6

X_1	C	$p(X_1 C)$
0	0	0,50
1	0	0,50
0	1	0,33
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X_2	C	$p(X_2 C)$
0	0	0,25
1	0	0,75
0	1	0,66
1	1	0,33

$$\hat{c} = f(\mathbf{x}) = \underset{c}{\operatorname{argmax}} p(c) \prod_{i=1}^2 p(x_i|c)$$

X_1	X_2	C
0	0	?
1	0	?

$$C=0 \quad 0,4 \times 0,5 \times 0,25$$

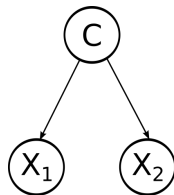
$$C=1 \quad 0,6 \times 0,66 \times 0,66$$

Naive Bayes

Aprendizaje del modelo

Estimando los parámetros (tablas de probabilidades):

X_1	X_2	C
1	0	0
0	0	1
0	1	0
0	1	0
1	0	1
0	0	1
1	1	0
1	1	1
0	1	0
1	1	0



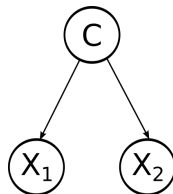
$$p(C)$$
$$p(X_1|C)$$
$$p(X_2|C)$$

Naive Bayes

Aprendizaje del modelo

Estimando los parámetros (tablas de probabilidades):

X_1	X_2	C
1	0	0
0	0	1
0	1	0
0	1	0
1	0	1
0	0	1
1	1	0
1	1	1
0	1	0
1	1	0



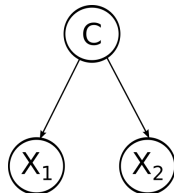
C	$p(C)$
0	
1	

Naive Bayes

Aprendizaje del modelo

Estimando los parámetros (tablas de probabilidades):

X_1	X_2	C
1	0	0
0	0	1
0	1	0
0	1	0
1	0	1
0	0	1
1	1	0
1	1	1
0	1	0
1	1	0



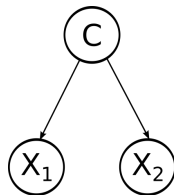
C	$p(C)$
0	$6/10 = 0,6$
1	$4/10 = 0,4$

Naive Bayes

Aprendizaje del modelo

Estimando los parámetros (tablas de probabilidades):

X_1	X_2	C
1	0	0
0	0	1
0	1	0
0	1	0
1	0	1
0	0	1
1	1	0
1	1	1
0	1	0
1	1	0



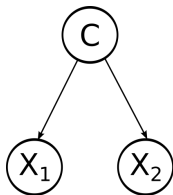
X_1	C	$p(X_1 C)$
0	0	
1	0	
0	1	
1	1	

Naive Bayes

Aprendizaje del modelo

Estimando los parámetros (tablas de probabilidades):

X_1	X_2	C
1	0	0
0	0	1
0	1	0
0	1	0
1	0	1
0	0	1
1	1	0
1	1	1
0	1	0
1	1	0



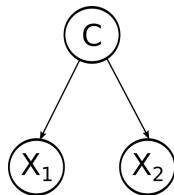
X_1	C	$p(X_1 C)$
0	0	$3/6 = 0,50$
1	0	$3/6 = 0,50$
0	1	$2/4 = 0,50$
1	1	$2/4 = 0,50$

Naive Bayes

Aprendizaje del modelo

Estimando los parámetros (tablas de probabilidades):

X_1	X_2	C
1	0	0
0	0	1
0	1	0
0	1	0
1	0	1
0	0	1
1	1	0
1	1	1
0	1	0
1	1	0



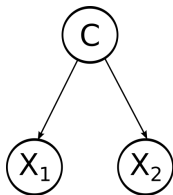
X_2	C	$p(X_2 C)$
0	0	
1	0	
0	1	
1	1	

Naive Bayes

Aprendizaje del modelo

Estimando los parámetros (tablas de probabilidades):

X_1	X_2	C
1	0	0
0	0	1
0	1	0
0	1	0
1	0	1
0	0	1
1	1	0
1	1	1
0	1	0
1	1	0



X_2	C	$p(X_2 C)$
0	0	$1/6 = 0,17$
1	0	$5/6 = 0,83$
0	1	$3/4 = 0,75$
1	1	$1/4 = 0,25$

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