

Normal Equation

Note: [8:00 to 8:44 - The design matrix X (in the bottom right side of the slide) given in the example should have elements x with subscript 1 and superscripts varying from 1 to m because for all m training sets there are only 2 features x_0 and x_1 . 12:56 - The X matrix is $(n+1)$ by $(n+1)$ and NOT n by n .]

Gradient descent gives one way of minimizing J . Let's discuss a second way of doing so, this time performing the minimization explicitly and without resorting to an iterative algorithm. In the "Normal Equation" method, we will minimize J by explicitly taking its derivatives with respect to the θ_j 's, and setting them to zero. This allows us to find the optimum theta without iteration. The normal equation formula is given below:

$$\theta = (X^T X)^{-1} X^T y$$

Examples: $m = 4$.

| | Size (feet ²) | Number of bedrooms | Number of floors | Age of home (years) | Price (\$1000) |
|-------|---------------------------|--------------------|------------------|---------------------|----------------|
| x_0 | x_1 | x_2 | x_3 | x_4 | y |
| 1 | 2104 | 5 | 1 | 45 | 460 |
| 1 | 1416 | 3 | 2 | 40 | 232 |
| 1 | 1534 | 3 | 2 | 30 | 315 |
| 1 | 852 | 2 | 1 | 36 | 178 |

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix}$$

$m \times (n+1)$

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

m -dimensional vector

$$\theta = (X^T X)^{-1} X^T y$$

There is **no need** to do feature scaling with the normal equation.

The following is a comparison of gradient descent and the normal equation:

| Gradient Descent | Normal Equation |
|------------------------------|---|
| Need to choose alpha | No need to choose alpha |
| Needs many iterations | No need to iterate |
| $O(kn^2)$ | $O(n^3)$, need to calculate inverse of $X^T X$ |
| Works well when n is large | Slow if n is very large |

With the normal equation, computing the inversion has complexity $\mathcal{O}(n^3)$. So if we have a very large number of features, the normal equation will be slow. In practice, when n exceeds 10,000 it might be a good time to go from a normal solution to an iterative process.

✓ Complete

