control 3.0.0

Computer-Aided Control System Design (CACSD) Tools for GNU Octave

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Preface

The GNU Octave control package from version 2 onwards was developed by Lukas F. Reichlin with major contributions by Thomas Vasileiou and is based on the proven open-source library SLICOT. This new package is intended as a replacement for control-1.0.11 by A. Scottedward Hodel and his students. Its main features are:

- Reliable solvers for Lyapunov, Sylvester and algebraic Riccati equations.
- Pole placement techniques as well as H_2 and H_∞ synthesis methods.
- Frequency-weighted model and controller reduction.
- System identification by subspace methods.
- Overloaded operators due to the use of classes introduced with Octave 3.2.
- Support for descriptor state-space models and non-proper transfer functions.
- Support for multiple systems in time- or frequency-domain plots.
- Improved MATLAB compatibility.

Acknowledgment

The author is indebted to several people and institutions who helped him to achieve his goals. I am particularly grateful to Luca Favatella who introduced me to Octave development as well as discussed and revised my early draft code with great patience. The continued support from the FHNW University of Applied Sciences Northwestern Switzerland has also been important. Namely, sincere thanks are given to my advisors, professors Hans Buchmann and Jürg Peter Keller. Furthermore, I thank the SLICOT authors Peter Benner, Vasile Sima and Andras Varga for their advice. Finally, I appreciate the feedback, bug reports and patches I have received from various people. The names of all contributors should be listed in the NEWS file.

Using the help function

Some functions of the control package are listed with the somewhat cryptic prefixes `@lti/` or `@iddata/`. These prefixes are only needed to view the help text of the function, e.g. `help norm` shows the built-in function while `help @lti/norm` shows the overloaded function for LTI systems. Note that there are LTI functions like `pole` that have no built-in equivalent. The same is true for IDDATA functions like `nkshift`.

When just using the function, the leading `@lti/` must **not** be typed. Octave selects the right function automatically. So one can type `norm (sys, inf)` and `norm (matrix, inf)` regardless of the class of the argument.

Bugs!

To err is human, and software is written by humans. Therefore, any larger piece of software is likely to contain bugs. If you find a bug in the control package, please take the time to report your findings! Feedback of any kind is highly appreciated by the author and vital for further enhancement of the software. Bug reports are to be sent to the Octave bug tracker, the mailing lists or directly to the author's e-mail: `lukas.reichlin@gmail.com`

Distribution

The GNU Octave control package is *free* software. Free software is a matter of the users' freedom to run, copy, distribute, study, change and improve the software. This means that everyone is free to use it and free to redistribute it on certain conditions. The GNU Octave control package is not, however, in the public domain. It is copyrighted and there are restrictions on its distribution, but the restrictions are designed to ensure that others will have the same freedom to use and redistribute Octave that you have. The precise conditions can be found in the GNU General Public License that comes with the GNU Octave control package and that also appears in Appendix A [Copying], page 104.

To download a copy of control, please visit `http://octave.sourceforge.net/control/`.

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1 Examples

1.1 MDSSystem

Robust control of a mass-damper-spring system. Type `which MDSSystem` to locate, `edit MDSSystem` to open and simply `MDSSystem` to run the example file.

1.2 optiPID

Numerical optimization of a PID controller using an objective function. The objective function is located in the file `optiPIDfun`. Type `which optiPID` to locate, `edit optiPID` to open and simply `optiPID` to run the example file. In this example called `optiPID`, loosely based on [1], it is assumed that the plant

$$P(s) = \frac{1}{(s^2 + s + 1)\ (s + 1)^4}$$

is controlled by a PID controller with second-order roll-off

$$C(s) = k_p\ (1 + \frac{1}{T_i\ s} + T_d\ s)\ \frac{1}{(\tau\ s + 1)^2}$$

in the usual negative feedback structure

$$T(s) = \frac{L(s)}{1 + L(s)} = \frac{P(s)\ C(s)}{1 + P(s)\ C(s)}$$

The plant P(s) is of higher order but benign. The initial values for the controller parameters k_p, T_i and T_d are obtained by applying the Astroem and Haegglund rules [2]. These values are to be improved using a numerical optimization as shown below. As with all numerical methods, this approach can never guarantee that a proposed solution is a global minimum. Therefore, good initial guesses for the parameters to be optimized are very important. The Octave function `fminsearch` minimizes the objective function J, which is chosen to be

$$J(k_p, T_i, T_d) = \mu_1 \cdot \int_0^\infty t\ |e(t)|\ dt\ +\ \mu_2 \cdot (||y(t)||_\infty - 1)\ +\ \mu_3 \cdot ||S(jw)||_\infty$$

This particular objective function penalizes the integral of time-weighted absolute error

$$ITAE = \int_0^\infty t\ |e(t)|\ dt$$

and the maximum overshoot

$$y_{max} - 1 = ||y(t)||_\infty - 1$$

to a unity reference step $r(t) = \varepsilon(t)$ in the time domain. In the frequency domain, the sensitivity

$$M_s = ||S(jw)||_\infty$$

is minimized for good robustness, where S(jw) denotes the *sensitivity* transfer function

$$S(s) = \frac{1}{1 + L(s)} = \frac{1}{1 + P(s)\ C(s)}$$

The constants μ_1, μ_2 and μ_3 are *relative weighting factors* or «tuning knobs» which reflect the importance of the different design goals. Varying these factors corresponds to changing the emphasis from, say, high performance to good robustness. The main advantage of this approach is the possibility to explore the tradeoffs of the design problem in a systematic way. In a first approach, all three design objectives are weigthed equally. In subsequent iterations, the parameters $\mu_1 = 1$, $\mu_2 = 10$ and $\mu_3 = 20$ are found to yield satisfactory closed-loop performance. This controller results in a system with virtually no overshoot and a phase margin of 64 degrees.

References

[1] Guzzella, L. *Analysis and Design of SISO Control Systems*, VDF Hochschulverlag, ETH Zurich, 2007

[2] Astroem, K. and Haegglund, T. *PID Controllers: Theory, Design and Tuning*, Second Edition, Instrument Society of America, 1995

1.3 Anderson

Frequency-weighted coprime factorization controller reduction.

1.4 Madievski

Demonstration of frequency-weighted controller reduction. The system considered in this example has been studied by Madievski and Anderson [1] and comprises four spinning disks. The disks are connected by a flexible rod, a motor applies torque to the third disk, and the angular displacement of the first disk is the variable of interest. The state-space model of eighth order is non-minimumphase and unstable. The continuous-time LQG controller used in [1] is open-loop stable and of eighth order like the plant. This eighth-order controller shall be reduced by frequency-weighted singular perturbation approximation (SPA). The major aim of this reduction is the preservation of the closed-loop transfer function. This means that the error in approximation of the controller K by the reduced-order controller Kr is minimized by

$$K_r \text{min} \; ||W \; (K - K_r) \; V||_\infty$$

where weights W and V are dictated by the requirement to preserve (as far as possible) the closed-loop transfer function. In minimizing the error, they cause the approximation process for K to be more accurate at certain frequencies. Suggested by [1] is the use of the following stability and performance enforcing weights:

$$W = (I - GK)^{-1}G, \qquad V = (I - GK)^{-1}$$

This example script reduces the eighth-order controller to orders four and two by the function call `Kr = spaconred (G, K, nr, 'feedback', '-')` where argument *nr* denotes the desired order (4 or 2). The key-value pair `'feedback', '-'` allows the reduction of negative feedback controllers while the default setting expects positive feedback controllers. The frequency responses of the original and reduced-order controllers are depicted in figure 1, the step responses of the closed loop in figure 2. There is no visible difference between the step responses of the closed-loop systems with original (blue) and fourth order (green) controllers. The second order controller (red) causes ripples in the step response, but otherwise the behavior of the system is unaltered. This leads to the conclusion that function `spaconred` is well suited to reduce the order of controllers considerably, while stability and performance are retained.

Reference

[1] Madievski, A.G. and Anderson, B.D.O. *Sampled-Data Controller Reduction Procedure*, IEEE Transactions of Automatic Control, Vol. 40, No. 11, November 1995

1.5 VLFamp

`VLFamp` [Function File]

`result` `= VLFamp (`*`verbose`*`)` [Function File]

Calculations on a two stage preamp for a multi-turn, air-core solenoid loop antenna for the reception of signals below 30kHz.

The Octave Control Package functions are used extensively to approximate the behavior of operational amplifiers and passive electrical circuit elements.

This example presents several 'screen' pages of documentation of the calculations and some reasoning about why. Plots of the results are presented in most cases.

The process is to display a 'screen' page of text followed by the calculation and a 'Press return to continue' message. To proceed in the example, press return. ^C to exit.

At one point in the calculations, the process may seem to hang, but, this is because of extensive calculations.

The returned transfer function is more than 100 characters long so will wrap in screens that are narrow and appear jumbled.

2 Linear Time Invariant Models

2.1 dss

sys = dss (*sys*) [Function File]
sys = dss (*d*, ...) [Function File]
sys = dss (*a*, *b*, *c*, *d*, *e*, ...) [Function File]
sys = dss (*a*, *b*, *c*, *d*, *e*, *tsam*, ...) [Function File]

Create or convert to descriptor state-space model.

Inputs

sys LTI model to be converted to state-space.

a State matrix (n-by-n).

b Input matrix (n-by-m).

c Output matrix (p-by-n).

d Feedthrough matrix (p-by-m).

e Descriptor matrix (n-by-n).

tsam Sampling time in seconds. If *tsam* is not specified, a continuous-time model is assumed.

... Optional pairs of properties and values. Type `set (dss)` for more information.

Outputs

sys Descriptor state-space model.

Option Keys and Values

'a', 'b', 'c', 'd', 'e'
State-space matrices. See 'Inputs' for details.

'stname' The name of the states in *sys*. Cell vector containing strings for each state. Default names are `{'x1', 'x2', ...}`

'scaled' Logical. If set to true, no automatic scaling is used, e.g. for frequency response plots.

'tsam' Sampling time. See 'Inputs' for details.

'inname' The name of the input channels in *sys*. Cell vector of length m containing strings. Default names are `{'u1', 'u2', ...}`

'outname' The name of the output channels in *sys*. Cell vector of length p containing strings. Default names are `{'y1', 'y2', ...}`

'ingroup' Struct with input group names as field names and vectors of input indices as field values. Default is an empty struct.

'outgroup' Struct with output group names as field names and vectors of output indices as field values. Default is an empty struct.

'name' String containing the name of the model.

'notes' String or cell of string containing comments.

'userdata' Any data type.

Equations

```
      .
    E x = A x + B u
      y = C x + D u
```

See also: ss, tf.

2.2 filt

`sys = filt (num, den, ...)` [Function File]
`sys = filt (num, den, tsam, ...)` [Function File]

Create discrete-time transfer function model from data in DSP format.

Inputs

num Numerator or cell of numerators. Each numerator must be a row vector containing the coefficients of the polynomial in ascending powers of z^-1. num{i,j} contains the numerator polynomial from input j to output i. In the SISO case, a single vector is accepted as well.

den Denominator or cell of denominators. Each denominator must be a row vector containing the coefficients of the polynomial in ascending powers of z^-1. den{i,j} contains the denominator polynomial from input j to output i. In the SISO case, a single vector is accepted as well.

tsam Sampling time in seconds. If *tsam* is not specified, default value -1 (unspecified) is taken.

... Optional pairs of properties and values. Type `set (filt)` for more information.

Outputs

sys Discrete-time transfer function model.

Option Keys and Values

'num' Numerator. See 'Inputs' for details.

'den' Denominator. See 'Inputs' for details.

'tfvar' String containing the transfer function variable.

'inv' Logical. True for negative powers of the transfer function variable.

'tsam' Sampling time. See 'Inputs' for details.

'inname' The name of the input channels in *sys*. Cell vector of length m containing strings. Default names are `{'u1', 'u2', ...}`

'outname' The name of the output channels in *sys*. Cell vector of length p containing strings. Default names are `{'y1', 'y2', ...}`

'ingroup' Struct with input group names as field names and vectors of input indices as field values. Default is an empty struct.

'outgroup' Struct with output group names as field names and vectors of output indices as field values. Default is an empty struct.

'name' String containing the name of the model.

'notes' String or cell of string containing comments.

'userdata' Any data type.

Example

```
                    3 z^-1
H(z^-1) = -------------------
          1 + 4 z^-1 + 2 z^-2

octave:1> H = filt ([0, 3], [1, 4, 2])

Transfer function 'H' from input 'u1' to output ...

              3 z^-1
 y1:  -------------------
      1 + 4 z^-1 + 2 z^-2

Sampling time: unspecified
Discrete-time model.
```

=

See also: tf.

2.3 frd

sys = frd (*sys*) [Function File]
sys = frd (*sys*, *w*) [Function File]
sys = frd (*H*, *w*, ...) [Function File]
sys = frd (*H*, *w*, *tsam*, ...) [Function File]

Create or convert to frequency response data.

Inputs

sys LTI model to be converted to frequency response data. If second argument *w* is omitted, the interesting frequency range is calculated by the zeros and poles of *sys*.

H Frequency response array (p-by-m-by-lw). H(i,j,k) contains the response from input j to output i at frequency k. In the SISO case, a vector (lw-by-1) or (1-by-lw) is accepted as well.

w Frequency vector (lw-by-1) in radian per second [rad/s]. Frequencies must be in ascending order.

tsam Sampling time in seconds. If *tsam* is not specified, a continuous-time model is assumed.

... Optional pairs of properties and values. Type `set (frd)` for more information.

Outputs

sys Frequency response data object.

Option Keys and Values

'H' Frequency response array. See 'Inputs' for details.

'w' Frequency vector. See 'Inputs' for details.

'tsam' Sampling time. See 'Inputs' for details.

'inname' The name of the input channels in *sys*. Cell vector of length m containing strings. Default names are `{'u1', 'u2', ...}`

'outname' The name of the output channels in *sys*. Cell vector of length p containing strings. Default names are `{'y1', 'y2', ...}`

'ingroup' Struct with input group names as field names and vectors of input indices as field values. Default is an empty struct.

'outgroup' Struct with output group names as field names and vectors of output indices as field values. Default is an empty struct.

'name' String containing the name of the model.

'notes' String or cell of string containing comments.

'userdata' Any data type.

See also: dss, ss, tf.

2.4 ss

sys = ss (*sys*) [Function File]
sys = ss (*d*, ...) [Function File]
sys = ss (*a*, *b*, ...) [Function File]
sys = ss (*a*, *b*, *c*, ...) [Function File]
sys = ss (*a*, *b*, *c*, *d*, ...) [Function File]
sys = ss (*a*, *b*, *c*, *d*, *tsam*, ...) [Function File]

Create or convert to state-space model.

Inputs

sys LTI model to be converted to state-space.

a State matrix (n-by-n).

b Input matrix (n-by-m).

c Output matrix (p-by-n). If *c* is empty `[]` or not specified, an identity matrix is assumed.

d Feedthrough matrix (p-by-m). If *d* is empty `[]` or not specified, a zero matrix is assumed.

tsam Sampling time in seconds. If *tsam* is not specified, a continuous-time model is assumed.

... Optional pairs of properties and values. Type `set (ss)` for more information.

Outputs

sys State-space model.

Option Keys and Values

'a', 'b', 'c', 'd', 'e'
State-space matrices. See 'Inputs' for details.

'stname' The name of the states in *sys*. Cell vector containing strings for each state. Default names are `{'x1', 'x2', ...}`

'scaled' Logical. If set to true, no automatic scaling is used, e.g. for frequency response plots.

'tsam' Sampling time. See 'Inputs' for details.

'inname' The name of the input channels in *sys*. Cell vector of length m containing strings. Default names are `{'u1', 'u2', ...}`

'outname' The name of the output channels in *sys*. Cell vector of length p containing strings. Default names are `{'y1', 'y2', ...}`

'ingroup' Struct with input group names as field names and vectors of input indices as field values. Default is an empty struct.

'outgroup' Struct with output group names as field names and vectors of output indices as field values. Default is an empty struct.

'name' String containing the name of the model.

'notes' String or cell of string containing comments.

'userdata' Any data type.

Equations

```
.
x = A x + B u
y = C x + D u
```

Example

```
octave:1> a = [1 2 3; 4 5 6; 7 8 9];
octave:2> b = [10; 11; 12];
octave:3> stname = {'V', 'A', 'kJ'};
octave:4> sys = ss (a, b, 'stname', stname)

sys.a =
         V   A  kJ
   V     1   2   3
   A     4   5   6
   kJ    7   8   9

sys.b =
        u1
   V    10
   A    11
   kJ   12

sys.c =
         V   A  kJ
   y1    1   0   0
   y2    0   1   0
   y3    0   0   1

sys.d =
        u1
   y1    0
   y2    0
   y3    0

Continuous-time model.
octave:5>
```

See also: tf, dss.

2.5 tf

`s = tf ('s')` [Function File]
`z = tf ('z', tsam)` [Function File]
`sys = tf (sys)` [Function File]
`sys = tf (mat, ...)` [Function File]
`sys = tf (num, den, ...)` [Function File]
`sys = tf (num, den, tsam, ...)` [Function File]

Create or convert to transfer function model.

Inputs

sys LTI model to be converted to transfer function.

mat Gain matrix to be converted to static transfer function.

num Numerator or cell of numerators. Each numerator must be a row vector containing the coefficients of the polynomial in descending powers of the transfer function variable. num{i,j} contains the numerator polynomial from input j to output i. In the SISO case, a single vector is accepted as well.

den Denominator or cell of denominators. Each denominator must be a row vector containing the coefficients of the polynomial in descending powers of the transfer function variable. den{i,j} contains the denominator polynomial from input j to output i. In the SISO case, a single vector is accepted as well.

tsam Sampling time in seconds. If *tsam* is not specified, a continuous-time model is assumed.

... Optional pairs of properties and values. Type `set (tf)` for more information.

Outputs

sys Transfer function model.

Option Keys and Values

'num' Numerator. See 'Inputs' for details.

'den' Denominator. See 'Inputs' for details.

'tfvar' String containing the transfer function variable.

'inv' Logical. True for negative powers of the transfer function variable.

'tsam' Sampling time. See 'Inputs' for details.

'inname' The name of the input channels in *sys*. Cell vector of length m containing strings. Default names are `{'u1', 'u2', ...}`

'outname' The name of the output channels in *sys*. Cell vector of length p containing strings. Default names are `{'y1', 'y2', ...}`

'ingroup' Struct with input group names as field names and vectors of input indices as field values. Default is an empty struct.

'outgroup' Struct with output group names as field names and vectors of output indices as field values. Default is an empty struct.

'name' String containing the name of the model.

'notes' String or cell of string containing comments.

'userdata' Any data type.

Example

```
octave:1> s = tf ('s');                                        =
octave:2> G = 1/(s+1)

Transfer function 'G' from input 'u1' to output ...

         1
 y1:  -----
      s + 1

Continuous-time model.
```

```
octave:3> z = tf ('z', 0.2);                                   =
octave:4> H = 0.095/(z-0.9)

Transfer function 'H' from input 'u1' to output ...

       0.095
 y1:  -------
      z - 0.9

Sampling time: 0.2 s
Discrete-time model.
```

```
octave:5> num = {[1, 5, 7], [1]; [1, 7], [1, 5, 5]};          =
octave:6> den = {[1, 5, 6], [1, 2]; [1, 8, 6], [1, 3, 2]};
octave:7> sys = tf (num, den)
```

```
Transfer function 'sys' from input 'u1' to output ...          =

      s^2 + 5 s + 7
 y1:  -------------
      s^2 + 5 s + 6

          s + 7
 y2:  -------------
      s^2 + 8 s + 6
```

```
Transfer function 'sys' from input 'u2' to output ...                =

        1
 y1:  -----
      s + 2

      s^2 + 5 s + 5
 y2:  -------------
      s^2 + 3 s + 2

Continuous-time model.
octave:8>
```

See also: filt, ss, dss.

2.6 zpk

s = zpk (*'s'*) [Function File]
z = zpk (*'z'*, *tsam*) [Function File]
sys = zpk (*sys*) [Function File]
sys = zpk (*k*, . . .) [Function File]
sys = zpk (*z*, *p*, *k*, . . .) [Function File]
sys = zpk (*z*, *p*, *k*, *tsam*, . . .) [Function File]
sys = zpk (*z*, *p*, *k*, *tsam*, . . .) [Function File]

Create transfer function model from zero-pole-gain data. This is just a stop-gap compatibility wrapper since zpk models are not yet implemented.

Inputs

sys LTI model to be converted to transfer function.

z Cell of vectors containing the zeros for each channel. z{i,j} contains the zeros from input j to output i. In the SISO case, a single vector is accepted as well.

p Cell of vectors containing the poles for each channel. p{i,j} contains the poles from input j to output i. In the SISO case, a single vector is accepted as well.

k Matrix containing the gains for each channel. k(i,j) contains the gain from input j to output i.

tsam Sampling time in seconds. If *tsam* is not specified, a continuous-time model is assumed.

. . . Optional pairs of properties and values. Type `set (tf)` for more information.

Outputs

sys Transfer function model.

See also: tf, ss, dss, frd.

3 Model Data Access

3.1 @lti/dssdata

`[a, b, c, d, e, tsam] = dssdata (sys)` [Function File]
`[a, b, c, d, e, tsam] = dssdata (sys, [])` [Function File]

Access descriptor state-space model data. Argument *sys* is not limited to descriptor state-space models. If *sys* is not a descriptor state-space model, it is converted automatically.

Inputs

sys Any type of LTI model.

[] In case *sys* is not a dss model (descriptor matrix *e* empty), `dssdata (sys, [])` returns the empty element `e = []` whereas `dssdata (sys)` returns the identity matrix `e = eye (size (a))`.

Outputs

a State matrix (n-by-n).

b Input matrix (n-by-m).

c Measurement matrix (p-by-n).

d Feedthrough matrix (p-by-m).

e Descriptor matrix (n-by-n).

tsam Sampling time in seconds. If *sys* is a continuous-time model, a zero is returned.

3.2 @lti/filtdata

`[num, den, tsam] = filtdata (sys)` [Function File]
`[num, den, tsam] = filtdata (sys, "vector")` [Function File]

Access discrete-time transfer function data in DSP format. Argument *sys* is not limited to transfer function models. If *sys* is not a transfer function, it is converted automatically.

Inputs

sys Any type of discrete-time LTI model.

"*v*", "*vector*"
For SISO models, return *num* and *den* directly as column vectors instead of cells containing a single column vector.

Outputs

num Cell of numerator(s). Each numerator is a row vector containing the coefficients of the polynomial in ascending powers of z^-1. num{i,j} contains the numerator polynomial from input j to output i. In the SISO case, a single vector is possible as well.

den Cell of denominator(s). Each denominator is a row vector containing the coefficients of the polynomial in ascending powers of z^-1. den{i,j} contains the denominator polynomial from input j to output i. In the SISO case, a single vector is possible as well.

tsam Sampling time in seconds. If *tsam* is not specified, -1 is returned.

3.3 @lti/frdata

`[H, w, tsam] = frdata (sys)` [Function File]
`[H, w, tsam] = frdata (sys, "vector")` [Function File]

Access frequency response data. Argument *sys* is not limited to frequency response data objects. If *sys* is not a frd object, it is converted automatically.

Inputs

sys Any type of LTI model.

"*v*", "*vector*"
In case *sys* is a SISO model, this option returns the frequency response as a column vector (lw-by-1) instead of an array (p-by-m-by-lw).

Outputs

H Frequency response array (p-by-m-by-lw). H(i,j,k) contains the response from input j to output i at frequency k. In the SISO case, a vector (lw-by-1) is possible as well.

w Frequency vector (lw-by-1) in radian per second [rad/s]. Frequencies are in ascending order.

tsam Sampling time in seconds. If *sys* is a continuous-time model, a zero is returned.

3.4 @lti/get

`get (sys)` [Function File]
`value = get (sys, "key")` [Function File]
`[val1, val2, ...] = get (sys, "key1", "key2", ...)` [Function File]

Access key values of LTI objects.

3.5 @lti/set

`set (sys)` [Function File]
`set (sys, "key", value, ...)` [Function File]
`retsys = set (sys, "key", value, ...)` [Function File]

Set or modify properties of LTI objects. If no return argument *retsys* is specified, the modified LTI object is stored in input argument *sys*. `set` can handle multiple properties in one call: `set (sys, 'key1', val1, 'key2', val2, 'key3', val3)`. `set (sys)` prints a list of the object's key names.

3.6 @lti/ssdata

`[a, b, c, d, tsam] = ssdata (sys)` [Function File]

Access state-space model data. Argument *sys* is not limited to state-space models. If *sys* is not a state-space model, it is converted automatically.

Inputs

sys Any type of LTI model.

Outputs

a State matrix (n-by-n).

b Input matrix (n-by-m).

c Measurement matrix (p-by-n).

d Feedthrough matrix (p-by-m).

tsam Sampling time in seconds. If *sys* is a continuous-time model, a zero is returned.

3.7 @lti/tfdata

`[num, den, tsam] = tfdata (sys)` [Function File]
`[num, den, tsam] = tfdata (sys, "vector")` [Function File]
`[num, den, tsam] = tfdata (sys, "tfpoly")` [Function File]

Access transfer function data. Argument *sys* is not limited to transfer function models. If *sys* is not a transfer function, it is converted automatically.

Inputs

sys Any type of LTI model.

"*v*", "*vector*"
For SISO models, return *num* and *den* directly as column vectors instead of cells containing a single column vector.

Outputs

num Cell of numerator(s). Each numerator is a row vector containing the coefficients of the polynomial in descending powers of the transfer function variable. num{i,j} contains the numerator polynomial from input j to output i. In the SISO case, a single vector is possible as well.

den Cell of denominator(s). Each denominator is a row vector containing the coefficients of the polynomial in descending powers of the transfer function variable. den{i,j} contains the denominator polynomial from input j to output i. In the SISO case, a single vector is possible as well.

tsam Sampling time in seconds. If *sys* is a continuous-time model, a zero is returned.

3.8 @lti/zpkdata

`[z, p, k, tsam] = zpkdata (sys)` [Function File]
`[z, p, k, tsam] = zpkdata (sys, "v")` [Function File]

Access zero-pole-gain data.

Inputs

sys Any type of LTI model.

"*v*", "*vector*"
For SISO models, return *z* and *p* directly as column vectors instead of cells containing a single column vector.

Outputs

z Cell of column vectors containing the zeros for each channel. z{i,j} contains the zeros from input j to output i.

p Cell of column vectors containing the poles for each channel. p{i,j} contains the poles from input j to output i.

k Matrix containing the gains for each channel. k(i,j) contains the gain from input j to output i.

tsam Sampling time in seconds. If *sys* is a continuous-time model, a zero is returned.

4 Model Conversions

4.1 @lti/c2d

sys = c2d (*sys*, *tsam*) [Function File]
sys = c2d (*sys*, *tsam*, *method*) [Function File]
sys = c2d (*sys*, *tsam*, *'prewarp'*, *w0*) [Function File]

Convert the continuous LTI model into its discrete-time equivalent.

Inputs

sys Continuous-time LTI model.

tsam Sampling time in seconds.

method Optional conversion method. If not specified, default method "*zoh*" is taken.

- *'zoh'* Zero-order hold or matrix exponential.
- *'tustin'*, *'bilin'* Bilinear transformation or Tustin approximation.
- *'prewarp'* Bilinear transformation with pre-warping at frequency *w0*.
- *'matched'* Matched pole/zero method.

Outputs

sys Discrete-time LTI model.

4.2 @lti/d2c

sys = d2c (*sys*) [Function File]
sys = d2c (*sys*, *method*) [Function File]
sys = d2c (*sys*, *'prewarp'*, *w0*) [Function File]

Convert the discrete LTI model into its continuous-time equivalent.

Inputs

sys Discrete-time LTI model.

method Optional conversion method. If not specified, default method "*zoh*" is taken.

- *'zoh'* Zero-order hold or matrix logarithm.
- *'tustin'*, *'bilin'* Bilinear transformation or Tustin approximation.
- *'prewarp'* Bilinear transformation with pre-warping at frequency *w0*.
- *'matched'* Matched pole/zero method.

Outputs

sys Continuous-time LTI model.

4.3 @lti/d2d

sys = d2d (*sys*, *tsam*) [Function File]
sys = d2d (*sys*, *tsam*, *method*) [Function File]
sys = d2d (*sys*, *tsam*, *'prewarp'*, *w0*) [Function File]

Resample discrete-time LTI model to sampling time *tsam*.

Inputs

sys Discrete-time LTI model.

tsam Desired sampling time in seconds.

method Optional conversion method. If not specified, default method "*zoh*" is taken.

- *'zoh'* Zero-order hold or matrix logarithm.
- *'tustin'*, *'bilin'* Bilinear transformation or Tustin approximation.
- *'prewarp'* Bilinear transformation with pre-warping at frequency *w0*.
- *'matched'* Matched pole/zero method.

Outputs

sys Resampled discrete-time LTI model with sampling time *tsam*.

4.4 @lti/prescale

[*scaledsys*, *info*] = prescale (*sys*) [Function File]

Scale state-space model. The scaled model *scaledsys* is equivalent to *sys*, but the state vector is scaled by diagonal transformation matrices in order to increase the accuracy of subsequent numerical computations. Frequency response commands perform automatic scaling unless model property *scaled* is set to *true*.

Inputs

sys LTI model.

Outputs

scaledsys Scaled state-space model.

info Structure containing additional information.

info.SL Left scaling factors. `Tl = diag (info.SL)`.

info.SR Right scaling factors. `Tr = diag (info.SR)`.

Equations

```
Es = Tl * E * Tr
As = Tl * A * Tr
Bs = Tl * B
Cs =      C * Tr
Ds =      D
```

=

For proper state-space models, *Tl* and *Tr* are inverse of each other.

Algorithm
Uses SLICOT TB01ID and TG01AD by courtesy of NICONET e.V. (`http://www.slicot.org`).

4.5 @lti/xperm

`retsys = xperm (sys, idx)` [Function File]

Reorder states in state-space models.

Inputs

sys State-space model.

idx Vector containing the state indices in the desired order. Alternatively, a cell vector containing the state names is possible as well. See `sys.stname`. State names only work if they were assigned explicitly before, i.e. `sys.stname` contains no empty strings. Note that if certain state indices of *sys* are missing or appear multiple times in *idx*, these states will be pruned or duplicated accordingly in the resulting state-space model *retsys*.

Outputs

retsys Resulting state-space model with states reordered according to *idx*.

5 Model Interconnections

5.1 append

sys = append (*sys1*, *sys2*, ..., *sysN*) [Function File]

Group LTI models by appending their inputs and outputs.

5.2 @lti/blkdiag

sys = blkdiag (*sys1*, *sys2*, ..., *sysN*) [Function File]

Block-diagonal concatenation of LTI models.

5.3 @lti/connect

sys = connect (*sys1*, *sys2*, ..., *sysN*, *inputs*, *outputs*) [Function File]
sys = connect (*sys*, *cm*, *inputs*, *outputs*) [Function File]

Name-based or index-based interconnections between the inputs and outputs of LTI models.

Inputs

sys1, ..., sysN
: LTI models to be connected. The properties 'inname' and 'outname' of each model should be set according to the desired input-output connections.

inputs
: For name-based interconnections, string or cell of strings containing the names of the inputs to be kept. The names must be part of the properties 'ingroup' or 'inname'. For index-based interconnections, vector containing the indices of the inputs to be kept.

outputs
: For name-based interconnections, string or cell of strings containing the names of the outputs to be kept. The names must be part of the properties 'outgroup' or 'outname'. For index-based interconnections, vector containing the indices of the outputs to be kept.

cm
: Connection matrix (not name-based). Each row of the matrix represents a summing junction. The first column holds the indices of the inputs to be summed with outputs of the subsequent columns. The output indices can be negative, if the output is to be substracted, or zero. For example, the row

```
[2 0 3 -4 0]
```

or

```
[2 -4 3]
```

will sum input u(2) with outputs y(3) and y(4) as

```
u(2) + y(3) - y(4).
```

Outputs

sys
: Resulting interconnected system with outputs *outputs* and inputs *inputs*.

See also: sumblk.

5.4 @lti/feedback

sys = feedback (*sys1*) [Function File]
sys = feedback (*sys1*, "+") [Function File]
sys = feedback (*sys1*, *sys2*) [Function File]
sys = feedback (*sys1*, *sys2*, "+") [Function File]
sys = feedback (*sys1*, *sys2*, *feedin*, *feedout*) [Function File]
sys = feedback (*sys1*, *sys2*, *feedin*, *feedout*, "+") [Function File]

Feedback connection of two LTI models.

Inputs

sys1 LTI model of forward transmission. `[p1, m1] = size (sys1)`.

sys2 LTI model of backward transmission. If not specified, an identity matrix of appropriate size is taken.

feedin Vector containing indices of inputs to *sys1* which are involved in the feedback loop. The number of *feedin* indices and outputs of *sys2* must be equal. If not specified, `1:m1` is taken.

feedout Vector containing indices of outputs from *sys1* which are to be connected to *sys2*. The number of *feedout* indices and inputs of *sys2* must be equal. If not specified, `1:p1` is taken.

"+" Positive feedback sign. If not specified, "-" for a negative feedback interconnection is assumed. *+1* and *-1* are possible as well, but only from the third argument onward due to ambiguity.

Outputs

sys Resulting LTI model.

Block Diagram

```
 u    +         +---------+             y
------>(+)----->|  sys1   |-------+------->
       ^ -      +---------+       |
       |                          |
       |        +---------+       |
       +--------|  sys2   |<------+
                +---------+
```

5.5 @lti/lft

sys = lft (*sys1*, *sys2*) [Function File]
sys = lft (*sys1*, *sys2*, *nu*, *ny*) [Function File]

Linear fractional tranformation, also known as Redheffer star product.

Inputs

sys1 Upper LTI model.

sys2 Lower LTI model.

nu The last nu inputs of *sys1* are connected with the first nu outputs of *sys2*. If not specified, `min (m1, p2)` is taken.

ny The last ny outputs of *sys1* are connected with the first ny inputs of *sys2*. If not specified, `min (p1, m2)` is taken.

Outputs

sys Resulting LTI model.

Block Diagram

```
      .............sys..............                                   =
      :         +--------+         :
w1 ------------>|        |------------> z1
      :         |  sys1  |         :
      : u +---->|        |-----+ y :
      :   |     +--------+     |   :          Lower LFT
      :   |                    |   :
      :   |     +--------+     |   :          lft (sys1, sys2)
      :   +-----|  sys2  |<----+   :
      :         +--------+         :
      :............................:

      .............sys..............                                   =
      :         +--------+         :
      : u +---->|  sys1  |-----+ y :
      :   |     +--------+     |   :          Upper LFT
      :   |                    |   :
      :   |     +--------+     |   :          lft (sys1, sys2)
      :   +-----|        |<----+   :
      :         |  sys2  |         :
z2 <------------|        |<------------ w2
      :         +--------+         :
      :............................:

      .............sys..............                                   =
      :         +--------+         :
w1 ------------>|        |------------> z1
      :         |  sys1  |         :
      : u +---->|        |-----+ y :
      :   |     +--------+     |   :
      :   |                    |   :          lft (sys1, sys2, nu, ny)
      :   |     +--------+     |   :
      :   +-----|        |<----+   :
      :         |  sys2  |         :
z2 <------------|        |<------------ w2
      :         +--------+         :
      :............................:
```

5.6 @lti/mconnect

sys = `mconnect` (*sys*, *m*) [Function File]
sys = `mconnect` (*sys*, *m*, *inputs*, *outputs*) [Function File]

Arbitrary interconnections between the inputs and outputs of an LTI model.

Inputs

sys LTI system.

m Connection matrix. Each row belongs to an input and each column represents an output.

inputs Vector of indices of those inputs which are retained. If not specified, all inputs are kept.

outputs Vector of indices of those outputs which are retained. If not specified, all outputs are kept.

Outputs

sys Interconnected system.

Example

```
Solve the system equations of
y(t) = G e(t)
e(t) = u(t) + M y(t)
in order to build
y(t) = H u(t)
The matrix M for a (p-by-m) system G
has m rows and p columns (m-by-p).

Example for a 3x2 system:
u1 = -1*y1 + 5*y2 + 0*y3
u2 = pi*y1 + 0*y2 - 7*y3

     | -1      5      0 |
M =  | pi      0      7 |
```

5.7 @lti/parallel

sys = parallel (*sys1*, *sys2*) [Function File]

Parallel connection of two LTI systems.

Block Diagram

```
     ..........................
     :      +--------+        :
     :  +-->|  sys1  |---+    :
 u   :  |   +--------+   | +  :  y
-------+                 O--------->
     :  |   +--------+   | +  :
     :  +-->|  sys2  |---+    :
     :      +--------+        :
     :.........sys............:

sys = parallel (sys1, sys2)
```

5.8 @lti/series

sys = series (*sys1*, *sys2*) [Function File]

sys = series (*sys1*, *sys2*, *outputs1*, *inputs2*) [Function File]

Series connection of two LTI models.

Block Diagram

```
    ..................................
 u  :  +--------+ y1    u2  +--------+  :  y
------>|  sys1  |---------->|  sys2  |------->
    :  +--------+           +--------+  :
    :...............sys................

sys = series (sys1, sys2)
```

```
    ..................................
    :                   v2  +--------+  :
    :            ---------->|        |  :  y
    :  +--------+ y1    u2  |  sys2  |------->
 u  :  |        |---------->|        |  :
------>|  sys1  |       z1  +--------+  :
    :  |        |---------->            :
    :  +--------+                       :
    :...............sys................

outputs1 = [1]
inputs2 = [2]
sys = series (sys1, sys2, outputs1, inputs2)
```

5.9 sumblk

`S = sumblk (formula)` [Function File]
`S = sumblk (formula, n)` [Function File]

Create summing junction *S* from string *formula* for name-based interconnections.

Inputs

formula String containing the formula of the summing junction, e.g. `e = r - y + d`

n Signal size. Default value is 1.

Outputs

S State-space model of the summing junction.

Example

```
octave:1> S = sumblk ('e = r - y + d')                                    =

S.d =
       r   y   d
   e   1  -1   1

Static gain.
octave:2> S = sumblk ('e = r - y + d', 2)

S.d =
       r1  r2  y1  y2  d1  d2
   e1   1   0  -1   0   1   0
   e2   0   1   0  -1   0   1

Static gain.
```

See also: connect.

6 Model Characteristics

6.1 ctrb

co = `ctrb` (*sys*) [Function File]
co = `ctrb` (*a*, *b*) [Function File]

Return controllability matrix.

Inputs

sys LTI model.

a State matrix (n-by-n).

b Input matrix (n-by-m).

Outputs

co Controllability matrix.

Equation

$$C_o = [B \;\; AB \;\; A^2B \;\; \ldots \;\; A^{n-1}B]$$

6.2 ctrbf

[*sysbar*, *T*, *K*] = `ctrbf` (*sys*) [Function File]
[*sysbar*, *T*, *K*] = `ctrbf` (*sys*, *tol*) [Function File]
[*Abar*, *Bbar*, *Cbar*, *T*, *K*] = `ctrbf` (*A*, *B*, *C*) [Function File]
[*Abar*, *Bbar*, *Cbar*, *T*, *K*] = `ctrbf` (*A*, *B*, *C*, *TOL*) [Function File]

If Co=ctrb(A,B) has rank r <= n = SIZE(A,1), then there is a similarity transformation Tc such that Tc = [t1 t2] where t1 is the controllable subspace and t2 is orthogonal to t1

```
Abar = Tc \ A * Tc ,  Bbar = Tc \ B ,  Cbar = C * Tc            =
```

and the transformed system has the form

```
       | Ac    A12|           | Bc |                              =
Abar = |----------|,   Bbar = | ---|,  Cbar = [Cc | Cnc].
       | 0     Anc|           |  0 |
```

where (Ac,Bc) is controllable, and Cc(sI-Ac)^(-1)Bc = C(sI-A)^(-1)B. and the system is stabilizable if Anc has no eigenvalues in the right half plane. The last output K is a vector of length n containing the number of controllable states.

6.3 @lti/dcgain

k = `dcgain` (*sys*) [Function File]

DC gain of LTI model.

Inputs

sys LTI system.

Outputs

k DC gain matrice. For a system with m inputs and p outputs, the array *k* has dimensions [p, m].

See also: freqresp.

6.4 gram

W = gram (*sys*, *mode*) [Function File]
Wc = gram (*a*, *b*) [Function File]

`gram (sys, "c")` returns the controllability gramian of the (continuous- or discrete-time) system *sys*. `gram (sys, "o")` returns the observability gramian of the (continuous- or discrete-time) system *sys*. `gram (a, b)` returns the controllability gramian *Wc* of the continuous-time system $dx/dt = ax + bu$; i.e., *Wc* satisfies $aWc + mWc' + bb' = 0$.

6.5 hsvd

hsv = hsvd (*sys*) [Function File]
hsv = hsvd (*sys*, "*offset*", *offset*) [Function File]
hsv = hsvd (*sys*, "*alpha*", *alpha*) [Function File]

Hankel singular values of the stable part of an LTI model. If no output arguments are given, the Hankel singular values are displayed in a plot.

Algorithm

Uses SLICOT AB13AD by courtesy of NICONET e.V. (`http://www.slicot.org`)

6.6 @lti/isct

bool = isct (*sys*) [Function File]

Determine whether LTI model is a continuous-time system.

Inputs

sys LTI system.

Outputs

bool = 0 *sys* is a discrete-time system.

bool = 1 *sys* is a continuous-time system or a static gain.

6.7 isctrb

[*bool*, *ncon*] = isctrb (*sys*) [Function File]
[*bool*, *ncon*] = isctrb (*sys*, *tol*) [Function File]
[*bool*, *ncon*] = isctrb (*a*, *b*) [Function File]
[*bool*, *ncon*] = isctrb (*a*, *b*, *e*) [Function File]
[*bool*, *ncon*] = isctrb (*a*, *b*, [], *tol*) [Function File]
[*bool*, *ncon*] = isctrb (*a*, *b*, *e*, *tol*) [Function File]

Logical check for system controllability. For numerical reasons, `isctrb (sys)` should be used instead of `rank (ctrb (sys))`.

Inputs

sys LTI model. Descriptor state-space models are possible. If *sys* is not a state-space model, it is converted to a minimal state-space realization, so beware of pole-zero cancellations which may lead to wrong results!

a State matrix (n-by-n).

b Input matrix (n-by-m).

e Descriptor matrix (n-by-n). If *e* is empty `[]` or not specified, an identity matrix is assumed.

tol Optional roundoff parameter. Default value is 0.

Outputs

bool = 0 System is not controllable.

bool = 1 System is controllable.

ncon Number of controllable states.

Algorithm
Uses SLICOT AB01OD and TG01HD by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: isobsv.

6.8 isdetectable

bool = isdetectable (*sys*) [Function File]
bool = isdetectable (*sys*, *tol*) [Function File]
bool = isdetectable (*a*, *c*) [Function File]
bool = isdetectable (*a*, *c*, *e*) [Function File]
bool = isdetectable (*a*, *c*, [], *tol*) [Function File]
bool = isdetectable (*a*, *c*, *e*, *tol*) [Function File]
bool = isdetectable (*a*, *c*, [], [], *dflg*) [Function File]
bool = isdetectable (*a*, *c*, *e*, [], *dflg*) [Function File]
bool = isdetectable (*a*, *c*, [], *tol*, *dflg*) [Function File]
bool = isdetectable (*a*, *c*, *e*, *tol*, *dflg*) [Function File]

Logical test for system detectability. All unstable modes must be observable or all unobservable states must be stable.

Inputs

sys LTI system.

a State transition matrix.

c Measurement matrix.

e Descriptor matrix. If *e* is empty `[]` or not specified, an identity matrix is assumed.

tol Optional tolerance for stability. Default value is 0.

dflg = 0 Matrices (*a*, *c*) are part of a continuous-time system. Default Value.

dflg = 1 Matrices (*a*, *c*) are part of a discrete-time system.

Outputs

bool = 0 System is not detectable.

bool = 1 System is detectable.

Algorithm
Uses SLICOT AB01OD and TG01HD by courtesy of NICONET e.V. (`http://www.slicot.org`) See `isstabilizable` for description of computational method.

See also: isstabilizable, isstable, isctrb, isobsv.

6.9 @lti/isdt

`bool = isdt (sys)` [Function File]

Determine whether LTI model is a discrete-time system.

Inputs

sys LTI system.

Outputs

bool = 0 *sys* is a continuous-time system.

bool = 1 *sys* is a discrete-time system or a static gain.

6.10 @lti/isminimumphase

`bool = isminimumphase (sys)` [Function File]

`bool = isminimumphase (sys, tol)` [Function File]

Determine whether LTI system has asymptotically stable zero dynamics. According to the definition of Byrnes/Isidori [1], the zeros of a minimum-phase system must be strictly inside the left complex half-plane (continuous-time case) or inside the unit circle (discrete-time case). Note that the poles are not tested.

M. Zeitz [2] discusses the inconsistent definitions of the minimum-phase property in a German paper. The abstract in English states the following [2]:

Originally, the minimum phase property has been defined by H. W. Bode [3] in order to characterize the unique relationship between gain and phase of the frequency response. With regard to the design of digital filters, another definition of minimum phase is used and a filter is said to be minimum phase if both the filter and its inverse are asymptotically stable. Finally, systems with asymptotically stable zero dynamics are named as minimum phase by C. I. Byrnes and A. Isidori [1]. Due to the inconsistent definitions, avoiding the minimum phase property for control purposes is advocated and the well-established criteria of Hurwitz or Ljapunow to describe the stability of filters and zero dynamics are recommended.

Inputs

sys LTI system.

tol Optional tolerance. *tol* must be a real-valued, non-negative scalar. Default value is 0.

Outputs

bool True if the system is minimum-phase and false otherwise.

```
real (z) < -tol*(1 + abs (z))    continuous-time
abs (z) < 1 - tol                discrete-time
```

=

References

[1] Byrnes, C.I. and Isidori, A. *A Frequency Domain Philosophy for Nonlinear Systems*. IEEE Conf. Dec. Contr. 23, pp. 15691573, 1984.

[2] Zeitz, M. *Minimum phase no relevant property of automatic control!*. at Automatisierungstechnik. Volume 62, Issue 1, pp. 310, 2014.

[3] Bode, H.W. *Network Analysis and Feedback Amplifier Design*. D. Van Nostrand Company, pp. 312-318, 1945. pp. 341-351, 1992.

6.11 isobsv

`[bool, nobs] = isobsv (sys)` [Function File]
`[bool, nobs] = isobsv (sys, tol)` [Function File]
`[bool, nobs] = isobsv (a, c)` [Function File]
`[bool, nobs] = isobsv (a, c, e)` [Function File]
`[bool, nobs] = isobsv (a, c, [], tol)` [Function File]
`[bool, nobs] = isobsv (a, c, e, tol)` [Function File]

Logical check for system observability. For numerical reasons, `isobsv (sys)` should be used instead of `rank (obsv (sys))`.

Inputs

sys LTI model. Descriptor state-space models are possible.

a State matrix (n-by-n).

c Measurement matrix (p-by-n).

e Descriptor matrix (n-by-n). If *e* is empty `[]` or not specified, an identity matrix is assumed.

tol Optional roundoff parameter. Default value is 0.

Outputs

bool = 0 System is not observable.

bool = 1 System is observable.

nobs Number of observable states.

Algorithm

Uses SLICOT AB01OD and TG01HD by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: isctrb.

6.12 @lti/issiso

`bool = issiso (sys)` [Function File]

Determine whether LTI model is single-input/single-output (SISO).

6.13 isstabilizable

`bool = isstabilizable (sys)` [Function File]
`bool = isstabilizable (sys, tol)` [Function File]
`bool = isstabilizable (a, b)` [Function File]
`bool = isstabilizable (a, b, e)` [Function File]
`bool = isstabilizable (a, b, [], tol)` [Function File]
`bool = isstabilizable (a, b, e, tol)` [Function File]
`bool = isstabilizable (a, b, [], [], dflg)` [Function File]
`bool = isstabilizable (a, b, e, [], dflg)` [Function File]
`bool = isstabilizable (a, b, [], tol, dflg)` [Function File]
`bool = isstabilizable (a, b, e, tol, dflg)` [Function File]

Logical check for system stabilizability. All unstable modes must be controllable or all uncontrollable states must be stable.

Inputs

sys LTI system. If *sys* is not a state-space system, it is converted to a minimal state-space realization, so beware of pole-zero cancellations which may lead to wrong results!

a State transition matrix.

b Input matrix.

e Descriptor matrix. If *e* is empty `[]` or not specified, an identity matrix is assumed.

tol Optional tolerance for stability. Default value is 0.

dflg = 0 Matrices (*a*, *b*) are part of a continuous-time system. Default Value.

dflg = 1 Matrices (*a*, *b*) are part of a discrete-time system.

Outputs

bool = 0 System is not stabilizable.

bool = 1 System is stabilizable.

Algorithm
Uses SLICOT AB01OD and TG01HD by courtesy of NICONET e.V. (`http://www.slicot.org`)

```
* Calculate staircase form (SLICOT AB01OD)
* Extract unobservable part of state transition matrix
* Calculate eigenvalues of unobservable part
* Check whether
  real (ev) < -tol*(1 + abs (ev))   continuous-time
  abs (ev) < 1 - tol                discrete-time
```

See also: isdetectable, isstable, isctrb, isobsv.

6.14 @lti/isstable

bool = `isstable` (*sys*) [Function File]
bool = `isstable` (*sys*, *tol*) [Function File]

Determine whether LTI system is stable.

Inputs

sys LTI system.

tol Optional tolerance for stability. *tol* must be a real-valued, non-negative scalar. Default value is 0.

Outputs

bool True if the system is stable and false otherwise.

```
real (p) < -tol*(1 + abs (p))    continuous-time
abs (p) < 1 - tol                discrete-time
```

6.15 @lti/norm

gain = norm (*sys*, *2*) [Function File]
[*gain*, *wpeak*] = norm (*sys*, *inf*) [Function File]
[*gain*, *wpeak*] = norm (*sys*, *inf*, *tol*) [Function File]

Return H-2 or L-inf norm of LTI model.

Algorithm
Uses SLICOT AB13BD and AB13DD by courtesy of NICONET e.V. (http://www.slicot.org)

6.16 obsv

ob = obsv (*sys*) [Function File]
ob = obsv (*a*, *c*) [Function File]

Return observability matrix.

Inputs

sys LTI model.

a State matrix (n-by-n).

c Measurement matrix (p-by-n).

Outputs

ob Observability matrix.

Equation

$$O_b = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

6.17 obsvf

[*sysbar*, *T*, *K*] = obsvf (*sys*) [Function File]
[*sysbar*, *T*, *K*] = obsvf (*sys*, *tol*) [Function File]
[*Abar*, *Bbar*, *Cbar*, *T*, *K*] = obsvf (*A*, *B*, *C*) [Function File]
[*Abar*, *Bbar*, *Cbar*, *T*, *K*] = obsvf (*A*, *B*, *C*, *TOL*) [Function File]

If Ob=obsv(A,C) has rank r <= n = SIZE(A,1), then there is a similarity transformation Tc such that To = [t1;t2] where t1 is c and t2 is orthogonal to t1

```
Abar = To \ A * To ,  Bbar = To \ B ,  Cbar = C * To                =
```

and the transformed system has the form

```
       | Ao      0 |            | Bo  |                              =
Abar = |-----------|,   Bbar =  | --- |,  Cbar = [Co | 0 ].
       | A21   Ano|             | Bno |
```

where (Ao,Bo) is observable, and Co(sI-Ao)^(-1)Bo = C(sI-A)^(-1)B. And system is detectable if Ano has no eigenvalues in the right half plane. The last output K is a vector of length n containing the number of observable states.

6.18 @lti/pole

`p = pole (sys)` [Function File]

Compute poles of LTI system.

Inputs

sys LTI model.

Outputs

p Poles of *sys*.

Algorithm

For (descriptor) state-space models, `pole` relies on Octave's `eig`. For SISO transfer functions, `pole` uses Octave's `roots`. MIMO transfer functions are converted to a *minimal* state-space representation for the computation of the poles.

6.19 pzmap

`pzmap (sys)` [Function File]
`pzmap (sys1, sys2, ..., sysN)` [Function File]
`pzmap (sys1, 'style1', ..., sysN, 'styleN')` [Function File]
`[p, z] = pzmap (sys)` [Function File]

Plot the poles and zeros of an LTI system in the complex plane. If no output arguments are given, the result is plotted on the screen. Otherwise, the poles and zeros are computed and returned.

Inputs

sys LTI model.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

p Poles of *sys*.

z Invariant zeros of *sys*.

6.20 @lti/size

`nvec = size (sys)` [Function File]
`n = size (sys, dim)` [Function File]
`[p, m] = size (sys)` [Function File]

LTI model size, i.e. number of outputs and inputs.

Inputs

sys LTI system.

dim If given a second argument, `size` will return the size of the corresponding dimension.

Outputs

nvec Row vector. The first element is the number of outputs (rows) and the second element the number of inputs (columns).

n Scalar value. The size of the dimension *dim*.

p Number of outputs.

m Number of inputs.

6.21 @lti/zero

`z = zero (sys)` [Function File]
`z = zero (sys, type)` [Function File]
`[z, k, info] = zero (sys)` [Function File]

Compute zeros and gain of LTI model. By default, `zero` computes the invariant zeros, also known as Smith zeros. Alternatively, when called with a second input argument, `zero` can also compute the system zeros, transmission zeros, input decoupling zeros and output decoupling zeros. See paper [1] for an explanation of the various zero flavors as well as for further details.

Inputs

sys LTI model.

type String specifying the type of zeros:

'system', 's'
Compute the system zeros. The system zeros include in all cases (square, non-square, degenerate or non-degenerate system) all transmission and decoupling zeros.

'invariant', 'inv'
Compute invariant zeros. Default selection.

'transmission', 't'
Compute transmission zeros. Transmission zeros are a subset of the invariant zeros. The transmission zeros are the zeros of the Smith-McMillan form of the transfer function matrix.

'input', 'inp', 'id'
Compute input decoupling zeros. The input decoupling zeros are also known as the uncontrollable eigenvalues of the pair (A,B).

'output', 'o', 'od'
Compute output decoupling zeros. The output decoupling zeros are also known as the unobservable eigenvalues of the pair (A,C).

Outputs

z Depending on argument *type*, *z* contains the invariant (default), system, transmission, input decoupling or output decoupling zeros of *sys* as defined in [1].

k Gain of SISO system *sys*. For MIMO systems, an empty matrix `[]` is returned.

info Struct containing additional information. For details, see the documentation of SLICOT routines AB08ND and AG08BD.

info.rank The normal rank of the transfer function matrix (regular state-space models) or of the system pencil (descriptor state-space models).

info.infz Contains information on the infinite elementary divisors as follows: the system has info.infz(i) infinite elementary divisors of degree i, where i=1,2,...,length(info.infz).

info.kronr Right Kronecker (column) indices.

info.kronl Left Kronecker (row) indices.

Examples

```
[z, k, info] = zero (sys)        # invariant zeros
z = zero (sys, 'system')         # system zeros
z = zero (sys, 'invariant')      # invariant zeros
z = zero (sys, 'transmission')   # transmission zeros
z = zero (sys, 'output')         # output decoupling zeros
z = zero (sys, 'input')          # input decoupling zeros
```

Algorithm

For (descriptor) state-space models, `zero` relies on SLICOT AB08ND and AG08BD by courtesy of NICONET e.V. (`http://www.slicot.org`) For SISO transfer functions, `zero` uses Octave's `roots`. MIMO transfer functions are converted to a *minimal* state-space representation for the computation of the zeros.

References

[1] MacFarlane, A. and Karcanias, N. *Poles and zeros of linear multivariable systems: a survey of the algebraic, geometric and complex-variable theory*. Int. J. Control, vol. 24, pp. 33-74, 1976.

[2] Rosenbrock, H.H. *Correction to 'The zeros of a system'*. Int. J. Control, vol. 20, no. 3, pp. 525-527, 1974.

[3] Svaricek, F. *Computation of the structural invariants of linear multivariable systems with an extended version of the program ZEROS*. Systems & Control Letters, vol. 6, pp. 261-266, 1985.

[4] Emami-Naeini, A. and Van Dooren, P. *Computation of zeros of linear multivariable systems*. Automatica, vol. 26, pp. 415-430, 1982.

7 Model Simplification

7.1 @lti/minreal

sys = `minreal` (*sys*) [Function File]
sys = `minreal` (*sys*, *tol*) [Function File]

Minimal realization or zero-pole cancellation of LTI models.

7.2 @lti/sminreal

sys = `sminreal` (*sys*) [Function File]
sys = `sminreal` (*sys*, *tol*) [Function File]

Perform state-space model reduction based on structure. Remove states which have no influence on the input-output behaviour. The physical meaning of the states is retained.

Inputs

sys State-space model.

tol Optional tolerance for controllability and observability. Entries of the state-space matrices whose moduli are less or equal to *tol* are assumed to be zero. Default value is 0.

Outputs

sys Reduced state-space model.

See also: minreal.

8 Time Domain Analysis

8.1 covar

`[p, q] = covar (sys, w)` [Function File]

Return the steady-state covariance.

Inputs

sys LTI model.

w Intensity of Gaussian white noise inputs which drive *sys*.

Outputs

p Output covariance.

q State covariance.

See also: lyap, dlyap.

8.2 gensig

`[u, t] = gensig (sigtype, tau)` [Function File]
`[u, t] = gensig (sigtype, tau, tfinal)` [Function File]
`[u, t] = gensig (sigtype, tau, tfinal, tsam)` [Function File]

Generate periodic signal. Useful in combination with lsim.

Inputs

sigtype = "*sin*"
Sine wave.

sigtype = "*cos*"
Cosine wave.

sigtype = "*square*"
Square wave.

sigtype = "*pulse*"
Periodic pulse.

tau Duration of one period in seconds.

tfinal Optional duration of the signal in seconds. Default duration is 5 periods.

tsam Optional sampling time in seconds. Default spacing is tau/64.

Outputs

u Vector of signal values.

t Time vector of the signal.

See also: lsim.

8.3 impulse

`impulse (sys)` [Function File]
`impulse (sys1, sys2, ..., sysN)` [Function File]
`impulse (sys1, 'style1', ..., sysN, 'styleN')` [Function File]
`impulse (sys1, ..., t)` [Function File]
`impulse (sys1, ..., tfinal)` [Function File]
`impulse (sys1, ..., tfinal, dt)` [Function File]
`[y, t, x] = impulse (sys)` [Function File]
`[y, t, x] = impulse (sys, t)` [Function File]
`[y, t, x] = impulse (sys, tfinal)` [Function File]
`[y, t, x] = impulse (sys, tfinal, dt)` [Function File]

Impulse response of LTI system. If no output arguments are given, the response is printed on the screen.

Inputs

sys LTI model.

t Time vector. Should be evenly spaced. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

tfinal Optional simulation horizon. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

dt Optional sampling time. Be sure to choose it small enough to capture transient phenomena. If not specified, it is calculated by the poles of the system.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

y Output response array. Has as many rows as time samples (length of t) and as many columns as outputs.

t Time row vector.

x State trajectories array. Has `length (t)` rows and as many columns as states.

See also: initial, lsim, step.

8.4 initial

`initial (sys, x0)` [Function File]
`initial (sys1, sys2, ..., sysN, x0)` [Function File]
`initial (sys1, 'style1', ..., sysN, 'styleN', x0)` [Function File]
`initial (sys1, ..., x0, t)` [Function File]
`initial (sys1, ..., x0, tfinal)` [Function File]
`initial (sys1, ..., x0, tfinal, dt)` [Function File]
`[y, t, x] = initial (sys, x0)` [Function File]
`[y, t, x] = initial (sys, x0, t)` [Function File]
`[y, t, x] = initial (sys, x0, tfinal)` [Function File]
`[y, t, x] = initial (sys, x0, tfinal, dt)` [Function File]

Initial condition response of state-space model. If no output arguments are given, the response is printed on the screen.

Inputs

sys State-space model.

x0 Vector of initial conditions for each state.

t Optional time vector. Should be evenly spaced. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

tfinal Optional simulation horizon. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

dt Optional sampling time. Be sure to choose it small enough to capture transient phenomena. If not specified, it is calculated by the poles of the system.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

y Output response array. Has as many rows as time samples (length of t) and as many columns as outputs.

t Time row vector.

x State trajectories array. Has `length (t)` rows and as many columns as states.

Example

```
                       .
Continuous Time:    x = A x ,    y = C x ,    x(0) = x0

Discrete Time:   x[k+1] = A x[k] ,   y[k] = C x[k] ,   x[0] = x0
```

See also: impulse, lsim, step.

8.5 lsim

`lsim` (*sys*, *u*) [Function File]
`lsim` (*sys1*, *sys2*, ..., *sysN*, *u*) [Function File]
`lsim` (*sys1*, *'style1'*, ..., *sysN*, *'styleN'*, *u*) [Function File]
`lsim` (*sys1*, ..., *u*, *t*) [Function File]
`lsim` (*sys1*, ..., *u*, *t*, *x0*) [Function File]
`[y, t, x] = lsim` (*sys*, *u*) [Function File]
`[y, t, x] = lsim` (*sys*, *u*, *t*) [Function File]
`[y, t, x] = lsim` (*sys*, *u*, *t*, *x0*) [Function File]

Simulate LTI model response to arbitrary inputs. If no output arguments are given, the system response is plotted on the screen.

Inputs

sys LTI model. System must be proper, i.e. it must not have more zeros than poles.

u Vector or array of input signal. Needs `length(t)` rows and as many columns as there are inputs. If *sys* is a single-input system, row vectors *u* of length `length(t)` are accepted as well.

t Time vector. Should be evenly spaced. If *sys* is a continuous-time system and *t* is a real scalar, *sys* is discretized with sampling time `tsam = t/(rows(u)-1)`. If *sys* is a discrete-time system and *t* is not specified, vector *t* is assumed to be `0 : tsam : tsam*(rows(u)-1)`.

x0 Vector of initial conditions for each state. If not specified, a zero vector is assumed.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

y Output response array. Has as many rows as time samples (length of t) and as many columns as outputs.

t Time row vector. It is always evenly spaced.

x State trajectories array. Has `length (t)` rows and as many columns as states.

See also: impulse, initial, step.

8.6 ramp

`ramp (`*sys*`)` [Function File]
`ramp (`*sys1*, *sys2*, ..., *sysN*`)` [Function File]
`ramp (`*sys1*, *'style1'*, ..., *sysN*, *'styleN'*`)` [Function File]
`ramp (`*sys1*, ..., *t*`)` [Function File]
`ramp (`*sys1*, ..., *tfinal*`)` [Function File]
`ramp (`*sys1*, ..., *tfinal*, *dt*`)` [Function File]
`[y, t, x] = ramp (`*sys*`)` [Function File]
`[y, t, x] = ramp (`*sys*, *t*`)` [Function File]
`[y, t, x] = ramp (`*sys*, *tfinal*`)` [Function File]
`[y, t, x] = ramp (`*sys*, *tfinal*, *dt*`)` [Function File]

Ramp response of LTI system. If no output arguments are given, the response is printed on the screen.

$$r(t) = t \cdot h(t)$$

Inputs

sys LTI model.

t Time vector. Should be evenly spaced. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

tfinal Optional simulation horizon. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

dt Optional sampling time. Be sure to choose it small enough to capture transient phenomena. If not specified, it is calculated by the poles of the system.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

y Output response array. Has as many rows as time samples (length of t) and as many columns as outputs.

t Time row vector.

x State trajectories array. Has `length (t)` rows and as many columns as states.

See also: impulse, initial, lsim, step.

8.7 step

`step (`*sys*`)` [Function File]
`step (`*sys1*, *sys2*, ..., *sysN*`)` [Function File]
`step (`*sys1*, *'style1'*, ..., *sysN*, *'styleN'*`)` [Function File]
`step (`*sys1*, ..., *t*`)` [Function File]
`step (`*sys1*, ..., *tfinal*`)` [Function File]
`step (`*sys1*, ..., *tfinal*, *dt*`)` [Function File]
`[`*y*`, `*t*`, `*x*`] = step (`*sys*`)` [Function File]
`[`*y*`, `*t*`, `*x*`] = step (`*sys*, *t*`)` [Function File]
`[`*y*`, `*t*`, `*x*`] = step (`*sys*, *tfinal*`)` [Function File]
`[`*y*`, `*t*`, `*x*`] = step (`*sys*, *tfinal*, *dt*`)` [Function File]

Step response of LTI system. If no output arguments are given, the response is printed on the screen.

Inputs

sys LTI model.

t Time vector. Should be evenly spaced. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

tfinal Optional simulation horizon. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

dt Optional sampling time. Be sure to choose it small enough to capture transient phenomena. If not specified, it is calculated by the poles of the system.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

y Output response array. Has as many rows as time samples (length of t) and as many columns as outputs.

t Time row vector.

x State trajectories array. Has `length (t)` rows and as many columns as states.

See also: impulse, initial, lsim.

9 Frequency Domain Analysis

9.1 bode

`bode (`*sys*`)` [Function File]
`bode (`*sys1*, *sys2*, ..., *sysN*`)` [Function File]
`bode (`*sys1*, *sys2*, ..., *sysN*, *w*`)` [Function File]
`bode (`*sys1*, *'style1'*, ..., *sysN*, *'styleN'*`)` [Function File]
`[`*mag*, *pha*, *w*`] = bode (`*sys*`)` [Function File]
`[`*mag*, *pha*, *w*`] = bode (`*sys*, *w*`)` [Function File]

Bode diagram of frequency response. If no output arguments are given, the response is printed on the screen.

Inputs

sys LTI system. Must be a single-input and single-output (SISO) system.

w Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system. Alternatively, the cell `{wmin, wmax}` specifies a frequency range, where *wmin* and *wmax* denote minimum and maximum frequencies in rad/s.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

mag Vector of magnitude. Has length of frequency vector *w*.

pha Vector of phase. Has length of frequency vector *w*.

w Vector of frequency values used.

See also: nichols, nyquist, sigma.

9.2 bodemag

`bodemag (`*sys*`)` [Function File]
`bodemag (`*sys1*, *sys2*, ..., *sysN*`)` [Function File]
`bodemag (`*sys1*, *sys2*, ..., *sysN*, *w*`)` [Function File]
`bodemag (`*sys1*, *'style1'*, ..., *sysN*, *'styleN'*`)` [Function File]
`[`*mag*, *w*`] = bodemag (`*sys*`)` [Function File]
`[`*mag*, *w*`] = bodemag (`*sys*, *w*`)` [Function File]

Bode magnitude diagram of frequency response. If no output arguments are given, the response is printed on the screen.

Inputs

sys LTI system. Must be a single-input and single-output (SISO) system.

w Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system. Alternatively, the cell `{wmin, wmax}` specifies a frequency range, where *wmin* and *wmax* denote minimum and maximum frequencies in rad/s.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

mag Vector of magnitude. Has length of frequency vector *w*.

w Vector of frequency values used.

See also: bode, nichols, nyquist, sigma.

9.3 @lti/freqresp

H = `freqresp` (*sys*, *w*) [Function File]

Evaluate frequency response at given frequencies.

Inputs

sys LTI system.

w Vector of frequency values.

Outputs

H Array of frequency response. For a system with m inputs and p outputs, the array *H* has dimensions [p, m, length (w)]. The frequency response at the frequency w(k) is given by H(:,:,k).

See also: dcgain.

9.4 margin

[*gamma*, *phi*, *w_gamma*, *w_phi*] = `margin` (*sys*) [Function File]

[*gamma*, *phi*, *w_gamma*, *w_phi*] = `margin` (*sys*, *tol*) [Function File]

Gain and phase margin of a system. If no output arguments are given, both gain and phase margin are plotted on a bode diagram. Otherwise, the margins and their corresponding frequencies are computed and returned. A more robust criterion to assess the stability of a feedback system is the sensitivity Ms computed by function `sensitivity`.

Inputs

sys LTI model. Must be a single-input and single-output (SISO) system.

tol Imaginary parts below *tol* are assumed to be zero. If not specified, default value `sqrt (eps)` is taken.

Outputs

gamma Gain margin (as gain, not dBs).

phi Phase margin (in degrees).

w_gamma Frequency for the gain margin (in rad/s).

w_phi Frequency for the phase margin (in rad/s).

Algorithm

Uses function `roots` to calculate the frequencies *w_gamma*, *w_phi* from special polynomials created from the transfer function of *sys* as listed below in section «Equations».

Equations

```
CONTINUOUS-TIME SYSTEMS                                                         =
Gain Margin
        _                 _
L(jw) = L(jw)      BTW: L(jw) = L(-jw) = conj (L(jw))

num(jw)   num(-jw)
------- = --------
den(jw)   den(-jw)

num(jw) den(-jw) = num(-jw) den(jw)

imag (num(jw) den(-jw)) = 0
imag (num(-jw) den(jw)) = 0



Phase Margin                                                                    =
          |num(jw)|
|L(jw)| = |-------| = 1
          |den(jw)|
  _     2      2
z z = Re z + Im z

num(jw)   num(-jw)
------- * -------- = 1
den(jw)   den(-jw)

num(jw) num(-jw) - den(jw) den(-jw) = 0

real (num(jw) num(-jw) - den(jw) den(-jw)) = 0



DISCRETE-TIME SYSTEMS                                                           =
Gain Margin
                                jwT         log z
L(z) = L(1/z)      BTW: z = e    --> w = -----
                                             j T
num(z)   num(1/z)
------ = --------
den(z)   den(1/z)

num(z) den(1/z) - num(1/z) den(z) = 0



Phase Margin                                                                    =
         |num(z)|
|L(z)| = |------| = 1
         |den(z)|
```

```
L(z) L(1/z) = 1                                                          =

num(z)   num(1/z)
------ * -------- = 1
den(z)   den(1/z)

num(z) num(1/z) - den(z) den(1/z) = 0


PS: How to get L(1/z)                                                    =
          4       3       2
p(z) = a z  +  b z  +  c z  +  d z  +  e

            -4      -3      -2      -1
p(1/z) = a z  +  b z  +  c z  +  d z  +  e

          -4                   2       3       4
       = z   ( a  +  b z  +  c z  +  d z  +  e z  )

              4       3       2                   4
       = ( e z  +  d z  +  c z  +  b z  +  a ) / ( z  )
```

See also: sensitivity, roots.

9.5 nichols

`nichols (`*sys*`)` [Function File]
`nichols (`*sys1*, *sys2*, . . ., *sysN*`)` [Function File]
`nichols (`*sys1*, *sys2*, . . ., *sysN*, *w*`)` [Function File]
`nichols (`*sys1*, *'style1'*, . . ., *sysN*, *'styleN'*`)` [Function File]
`[`*mag*`, `*pha*`, `*w*`] = nichols (`*sys*`)` [Function File]
`[`*mag*`, `*pha*`, `*w*`] = nichols (`*sys*, *w*`)` [Function File]

Nichols chart of frequency response. If no output arguments are given, the response is printed on the screen.

Inputs

sys LTI system. Must be a single-input and single-output (SISO) system.

w Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system. Alternatively, the cell `{wmin, wmax}` specifies a frequency range, where *wmin* and *wmax* denote minimum and maximum frequencies in rad/s.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

mag Vector of magnitude. Has length of frequency vector *w*.

pha Vector of phase. Has length of frequency vector *w*.

w Vector of frequency values used.

See also: bode, nyquist, sigma.

9.6 nyquist

`nyquist (`*sys*`)` [Function File]
`nyquist (`*sys1*, *sys2*, ..., *sysN*`)` [Function File]
`nyquist (`*sys1*, *sys2*, ..., *sysN*, *w*`)` [Function File]
`nyquist (`*sys1*, *'style1'*, ..., *sysN*, *'styleN'*`)` [Function File]
`[`*re*, *im*, *w*`] = nyquist (`*sys*`)` [Function File]
`[`*re*, *im*, *w*`] = nyquist (`*sys*, *w*`)` [Function File]

Nyquist diagram of frequency response. If no output arguments are given, the response is printed on the screen.

Inputs

sys LTI system. Must be a single-input and single-output (SISO) system.

w Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system. Alternatively, the cell `{wmin, wmax}` specifies a frequency range, where *wmin* and *wmax* denote minimum and maximum frequencies in rad/s.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

re Vector of real parts. Has length of frequency vector *w*.

im Vector of imaginary parts. Has length of frequency vector *w*.

w Vector of frequency values used.

See also: bode, nichols, sigma.

9.7 sensitivity

`[`*Ms*, *ws*`] = sensitivity (`*L*`)` [Function File]
`[`*Ms*, *ws*`] = sensitivity (`*P*, *C*`)` [Function File]
`[`*Ms*, *ws*`] = sensitivity (`*P*, *C1*, *C2*, ...`)` [Function File]

Return sensitivity margin *Ms*. The quantity *Ms* is simply the inverse of the shortest distance from the Nyquist curve to the critical point -1. Reasonable values of *Ms* are in the range from 1.3 to 2.

$$M_s = ||S(j\omega)||_\infty$$

If no output arguments are given, the critical distance 1/Ms is plotted on a Nyquist diagram. In contrast to gain and phase margin as computed by function `margin`, the sensitivity *Ms* is a more robust criterion to assess the stability of a feedback system.

Inputs

L Open loop transfer function. *L* can be any type of LTI system, but it must be square.

P Plant model. Any type of LTI system.

C Controller model. Any type of LTI system.

C1, *C2*, ...
If several controllers are specified, function `sensitivity` computes the sensitivity *Ms* for each of them in combination with plant *P*.

Outputs

Ms Sensitivity margin *Ms* as defined in [1]. Scalar value. If several controllers are specified, *Ms* becomes a row vector with as many entries as controllers.

ws The frequency [rad/s] corresponding to the sensitivity peak. Scalar value. If several controllers are specified, *ws* becomes a row vector with as many entries as controllers.

Algorithm

Uses SLICOT AB13DD by courtesy of NICONET e.V. (`http://www.slicot.org`) to calculate the infinity norm of the sensitivity function.

References

[1] Aström, K. and Hägglund, T. (1995) PID Controllers: Theory, Design and Tuning, Second Edition. Instrument Society of America.

9.8 sigma

`sigma (`*sys*`)` [Function File]
`sigma (`*sys1*, *sys2*, ..., *sysN*`)` [Function File]
`sigma (`*sys1*, *sys2*, ..., *sysN*, *w*`)` [Function File]
`sigma (`*sys1*, *'style1'*, ..., *sysN*, *'styleN'*`)` [Function File]
`[`*sv*`, `*w*`] = sigma (`*sys*`)` [Function File]
`[`*sv*`, `*w*`] = sigma (`*sys*, *w*`)` [Function File]

Singular values of frequency response. If no output arguments are given, the singular value plot is printed on the screen.

Inputs

sys LTI system. Multiple inputs and/or outputs (MIMO systems) make practical sense.

w Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system. Alternatively, the cell `{wmin, wmax}` specifies a frequency range, where *wmin* and *wmax* denote minimum and maximum frequencies in rad/s.

'style' Line style and color, e.g. 'r' for a solid red line or '-.k' for a dash-dotted black line. See `help plot` for details.

Outputs

sv Array of singular values. For a system with m inputs and p outputs, the array sv has `min (m, p)` rows and as many columns as frequency points `length (w)`. The singular values at the frequency `w(k)` are given by `sv(:,k)`.

w Vector of frequency values used.

See also: bodemag, svd.

10 Pole Placement

10.1 place

`f = place (sys, p)` [Function File]
`f = place (a, b, p)` [Function File]
`[f, info] = place (sys, p, alpha)` [Function File]
`[f, info] = place (a, b, p, alpha)` [Function File]

Pole assignment for a given matrix pair (A,B) such that `p = eig (A-B*F)`. If parameter *alpha* is specified, poles with real parts (continuous-time) or moduli (discrete-time) below *alpha* are left untouched.

Inputs

sys Continuous- or discrete-time LTI system.

a State matrix (n-by-n) of a continuous-time system.

b Input matrix (n-by-m) of a continuous-time system.

p Desired eigenvalues of the closed-loop system state-matrix A-B*F. `length (p) <= rows (A)`.

alpha Specifies the maximum admissible value, either for real parts or for moduli, of the eigenvalues of A which will not be modified by the eigenvalue assignment algorithm. `alpha >= 0` for discrete-time systems.

Outputs

f State feedback gain matrix.

info Structure containing additional information.

info.nfp The number of fixed poles, i.e. eigenvalues of A having real parts less than *alpha*, or moduli less than *alpha*. These eigenvalues are not modified by `place`.

info.nap The number of assigned eigenvalues. `nap = n-nfp-nup`.

info.nup The number of uncontrollable eigenvalues detected by the eigenvalue assignment algorithm.

info.z The orthogonal matrix z reduces the closed-loop system state matrix `A + B*F` to upper real Schur form. Note the positive sign in `A + B*F`.

Note

```
Place is also suitable to design estimator gains:
        L = place (A.', C.', p).'
        L = place (sys.', p).'   # useful for discrete-time systems
```

Algorithm

Uses SLICOT SB01BD by courtesy of NICONET e.V. (`http://www.slicot.org`)

10.2 rlocus

`rlocus (`*sys*`)` [Function File]
`[`*rldata*`,` *k*`] = rlocus (`*sys*`,` *increment*`,` *min_k*`,` *max_k*`)` [Function File]

Display root locus plot of the specified SISO system.

Inputs

sys LTI model. Must be a single-input and single-output (SISO) system.

increment The increment used in computing gain values.

min_k Minimum value of k.

max_k Maximum value of k.

Outputs

rldata Data points plotted: in column 1 real values, in column 2 the imaginary values.

k Gains for real axis break points.

Block Diagram

```
 u    +         +---+      +------+             y
------>(+)----->| k |----->| SISO |-------+------->
       ^ -      +---+      +------+       |
       |                                  |
       +----------------------------------+
```

=

11 Optimal Control

11.1 augstate

augsys = augstate (*sys*) [Function File]

Append state vector x of system *sys* to output vector y.

```
.                    .
x = A x + B u        x = A x + B u
y = C x + D u   =>   y = C x + D u
                     x = I x + O u
```

11.2 dlqe

[*m*, *p*, *z*, *e*] = dlqe (*a*, *g*, *c*, *q*, *r*) [Function File]
[*m*, *p*, *z*, *e*] = dlqe (*a*, *g*, *c*, *q*, *r*, *s*) [Function File]
[*m*, *p*, *z*, *e*] = dlqe (*a*, [], *c*, *q*, *r*) [Function File]
[*m*, *p*, *z*, *e*] = dlqe (*a*, [], *c*, *q*, *r*, *s*) [Function File]

Kalman filter for discrete-time systems.

```
x[k] = Ax[k] + Bu[k] + Gw[k]   (State equation)
y[k] = Cx[k] + Du[k] + v[k]    (Measurement Equation)
E(w) = 0, E(v) = 0, cov(w) = Q, cov(v) = R, cov(w,v) = S
```

Inputs

a State transition matrix of discrete-time system (n-by-n).

g Process noise matrix of discrete-time system (n-by-g). If *g* is empty `[]`, an identity matrix is assumed.

c Measurement matrix of discrete-time system (p-by-n).

q Process noise covariance matrix (g-by-g).

r Measurement noise covariance matrix (p-by-p).

s Optional cross term covariance matrix (g-by-p), s = cov(w,v). If *s* is empty `[]` or not specified, a zero matrix is assumed.

Outputs

m Kalman filter gain matrix (n-by-p).

p Unique stabilizing solution of the discrete-time Riccati equation (n-by-n). Symmetric matrix.

z Error covariance (n-by-n), cov(x(k|k)-x)

e Closed-loop poles (n-by-1).

Equations

```
x[k|k] = x[k|k-1] + M(y[k] - Cx[k|k-1] - Du[k])                          =

x[k+1|k] = Ax[k|k] + Bu[k] for S=0

x[k+1|k] = Ax[k|k] + Bu[k] + G*S*(C*P*C' + R)^-1*(y[k] - C*x[k|k-1]) for no


E = eig(A - A*M*C) for S=0

E = eig(A - A*M*C - G*S*(C*P*C' + Rv)^-1*C) for non-zero S
```

See also: dare, care, dlqr, lqr, lqe.

11.3 dlqr

`[g, x, l] = dlqr (sys, q, r)` [Function File]
`[g, x, l] = dlqr (sys, q, r, s)` [Function File]
`[g, x, l] = dlqr (a, b, q, r)` [Function File]
`[g, x, l] = dlqr (a, b, q, r, s)` [Function File]
`[g, x, l] = dlqr (a, b, q, r, [], e)` [Function File]
`[g, x, l] = dlqr (a, b, q, r, s, e)` [Function File]

Linear-quadratic regulator for discrete-time systems.

Inputs

sys Continuous or discrete-time LTI model (p-by-m, n states).

a State transition matrix of discrete-time system (n-by-n).

b Input matrix of discrete-time system (n-by-m).

q State weighting matrix (n-by-n).

r Input weighting matrix (m-by-m).

s Optional cross term matrix (n-by-m). If *s* is not specified, a zero matrix is assumed.

e Optional descriptor matrix (n-by-n). If *e* is not specified, an identity matrix is assumed.

Outputs

g State feedback matrix (m-by-n).

x Unique stabilizing solution of the discrete-time Riccati equation (n-by-n).

l Closed-loop poles (n-by-1).

Equations

```
x[k+1] = A x[k] + B u[k],   x[0] = x0                                    =

        inf
J(x0) = SUM (x' Q x  +  u' R u  +  2 x' S u)
        k=0

L = eig (A - B*G)
```

See also: dare, care, lqr.

11.4 estim

est = estim (*sys*, *l*) [Function File]
est = estim (*sys*, *l*, *sensors*, *known*) [Function File]

Return state estimator for a given estimator gain.

Inputs

sys LTI model.

l State feedback matrix.

sensors Indices of measured output signals y from *sys*. If omitted, all outputs are measured.

known Indices of known input signals u (deterministic) to *sys*. All other inputs to *sys* are assumed stochastic (w). If argument *known* is omitted, no inputs u are known.

Outputs

est State-space model of estimator.

Block Diagram

```
                                  u  +-------+         ^
      +----------------------------->|       |-------> y
      |    +-------+      +       y  |  est  |         ^
u ----+--->|       |----->(+)------->|       |-------> x
           |  sys  |       ^ +       +-------+
w -------->|       |       |
           +-------+       | v
```

See also: kalman, lqe, place.

11.5 kalman

[*est*, *g*, *x*] = kalman (*sys*, *q*, *r*) [Function File]
[*est*, *g*, *x*] = kalman (*sys*, *q*, *r*, *s*) [Function File]
[*est*, *g*, *x*] = kalman (*sys*, *q*, *r*, [], *sensors*, *known*) [Function File]
[*est*, *g*, *x*] = kalman (*sys*, *q*, *r*, *s*, *sensors*, *known*) [Function File]

Design Kalman estimator for LTI systems.

Inputs

sys Nominal plant model.

q Covariance of white process noise.

r Covariance of white measurement noise.

s Optional cross term covariance. Default value is 0.

sensors Indices of measured output signals y from *sys*. If omitted, all outputs are measured.

known Indices of known input signals u (deterministic) to *sys*. All other inputs to *sys* are assumed stochastic. If argument *known* is omitted, no inputs u are known.

Outputs

est State-space model of the Kalman estimator.

g Estimator gain.

x Solution of the Riccati equation.

Block Diagram

```
                                     u  +-------+         ^
      +---------------------------->|       |-------> y
      |    +-------+     +       y  |  est  |         ^
u ----+--->|       |----->(+)------>|       |-------> x
           |  sys  |       ^ +      +-------+
w -------->|       |       |
           +-------+       | v

Q = cov (w, w')     R = cov (v, v')     S = cov (w, v')
```

See also: care, dare, estim, lqr.

11.6 lqe

[*l*, *p*, *e*] = lqe (*sys*, *q*, *r*) [Function File]
[*l*, *p*, *e*] = lqe (*sys*, *q*, *r*, *s*) [Function File]
[*l*, *p*, *e*] = lqe (*a*, *g*, *c*, *q*, *r*) [Function File]
[*l*, *p*, *e*] = lqe (*a*, *g*, *c*, *q*, *r*, *s*) [Function File]
[*l*, *p*, *e*] = lqe (*a*, [], *c*, *q*, *r*) [Function File]
[*l*, *p*, *e*] = lqe (*a*, [], *c*, *q*, *r*, *s*) [Function File]

Kalman filter for continuous-time systems.

```
.
x = Ax + Bu + Gw   (State equation)
y = Cx + Du + v    (Measurement Equation)
E(w) = 0, E(v) = 0, cov(w) = Q, cov(v) = R, cov(w,v) = S
```

Inputs

sys Continuous or discrete-time LTI model (p-by-m, n states).

a State matrix of continuous-time system (n-by-n).

g Process noise matrix of continuous-time system (n-by-g). If *g* is empty `[]`, an identity matrix is assumed.

c Measurement matrix of continuous-time system (p-by-n).

q Process noise covariance matrix (g-by-g).

r Measurement noise covariance matrix (p-by-p).

s Optional cross term covariance matrix (g-by-p), s = cov(w,v). If *s* is empty `[]` or not specified, a zero matrix is assumed.

Outputs

l Kalman filter gain matrix (n-by-p).

p Unique stabilizing solution of the continuous-time Riccati equation (n-by-n). Symmetric matrix. If *sys* is a discrete-time model, the solution of the corresponding discrete-time Riccati equation is returned.

e Closed-loop poles (n-by-1).

Equations

```
.
x = Ax + Bu + L(y - Cx -Du)

E = eig(A - L*C)
```

See also: dare, care, dlqr, lqr, dlqe.

11.7 lqr

`[g, x, l] = lqr (`*sys*, *q*, *r*`)` [Function File]
`[g, x, l] = lqr (`*sys*, *q*, *r*, *s*`)` [Function File]
`[g, x, l] = lqr (`*a*, *b*, *q*, *r*`)` [Function File]
`[g, x, l] = lqr (`*a*, *b*, *q*, *r*, *s*`)` [Function File]
`[g, x, l] = lqr (`*a*, *b*, *q*, *r*, [], *e*`)` [Function File]
`[g, x, l] = lqr (`*a*, *b*, *q*, *r*, *s*, *e*`)` [Function File]

Linear-quadratic regulator.

Inputs

sys Continuous or discrete-time LTI model (p-by-m, n states).

a State matrix of continuous-time system (n-by-n).

b Input matrix of continuous-time system (n-by-m).

q State weighting matrix (n-by-n).

r Input weighting matrix (m-by-m).

s Optional cross term matrix (n-by-m). If *s* is not specified, a zero matrix is assumed.

e Optional descriptor matrix (n-by-n). If *e* is not specified, an identity matrix is assumed.

Outputs

g State feedback matrix (m-by-n).

x Unique stabilizing solution of the continuous-time Riccati equation (n-by-n).

l Closed-loop poles (n-by-1).

Equations

```
.
x = A x + B u,   x(0) = x0

        inf
J(x0) = INT (x' Q x  +  u' R u  +  2 x' S u)  dt
         0

L = eig (A - B*G)
```

See also: care, dare, dlqr.

12 Robust Control

12.1 augw

P = augw (*G*, *W1*, *W2*, *W3*) [Function File]

Extend plant for stacked S/KS/T problem. Subsequently, the robust control problem can be solved by h2syn or hinfsyn.

Inputs

G LTI model of plant.

W1 LTI model of performance weight. Bounds the largest singular values of sensitivity *S*. Model must be empty `[]`, SISO or of appropriate size.

W2 LTI model to penalize large control inputs. Bounds the largest singular values of *KS*. Model must be empty `[]`, SISO or of appropriate size.

W3 LTI model of robustness and noise sensitivity weight. Bounds the largest singular values of complementary sensitivity *T*. Model must be empty `[]`, SISO or of appropriate size.

All inputs must be proper/realizable. Scalars, vectors and matrices are possible instead of LTI models.

Outputs

P State-space model of augmented plant.

Block Diagram

```
                                                                       =
    | W1 | -W1*G |     z1 = W1 r  -  W1 G u
    | 0  |  W2   |     z2 =          W2   u
P = | 0  |  W3*G |     z3 =          W3 G u
    |----+-------|
    | I  |    -G |     e  =    r  -    G u


                                                           +------+  z1     =
            +-------------------------------------------->|  W1  |----->
            |                                              +------+
            |                                              +------+  z2
            |                 +--------------------------->|  W2  |----->
            |                 |                            +------+
 r   +    e |   +--------+  u |   +--------+  y            +------+  z3
----->(+)---+-->|  K(s)  |----+-->|  G(s)  |----+--------->|  W3  |----->
       ^ -      +--------+        +--------+    |          +------+
       |                                        |
       +----------------------------------------+


                +--------+                                          =
                |        |-----> z1 (p1x1)          z1 = W1 e
 r (px1) ----->|  P(s)  |-----> z2 (p2x1)          z2 = W2 u
                |        |-----> z3 (p3x1)          z3 = W3 y
 u (mx1) ----->|        |-----> e (px1)            e = r - y
                +--------+
```

```
             +--------+
  r ----->|        |-----> z
             |  P(s)  |
  u +---->|        |-----+ e
    |        +--------+     |
    |                       |
    |        +--------+     |
    +-----|  K(s)  |<----+
             +--------+
```

=

References

[1] Skogestad, S. and Postlethwaite I. (2005) *Multivariable Feedback Control: Analysis and Design: Second Edition*. Wiley.

See also: h2syn, hinfsyn, mixsyn.

12.2 fitfrd

`[sys, n] = fitfrd (dat, n)` [Function File]
`[sys, n] = fitfrd (dat, n, flag)` [Function File]

Fit frequency response data with a state-space system. If requested, the returned system is stable and minimum-phase.

Inputs

dat LTI model containing frequency response data of a SISO system.

n The desired order of the system to be fitted. `n <= length(dat.w)`.

flag The flag controls whether the returned system is stable and minimum-phase.

- *0* The system zeros and poles are not constrained. Default value.
- *1* The system zeros and poles will have negative real parts in the continuous-time case, or moduli less than 1 in the discrete-time case.

Outputs

sys State-space model of order *n*, fitted to frequency response data *dat*.

n The order of the obtained system. The value of *n* could only be modified if inputs `n > 0` and `flag = 1`.

Algorithm

Uses SLICOT SB10YD by courtesy of NICONET e.V. (`http://www.slicot.org`)

12.3 h2syn

`[K, N, gamma, info] = h2syn (P, nmeas, ncon)` [Function File]
`[K, N, gamma, info] = h2syn (P)` [Function File]

H-2 control synthesis for LTI plant.

Inputs

P Generalized plant. Must be a proper/realizable LTI model. If *P* is constructed with `mktito` or `augw`, arguments *nmeas* and *ncon* can be omitted.

nmeas Number of measured outputs v. The last *nmeas* outputs of *P* are connected to the inputs of controller *K*. The remaining outputs z (indices 1 to p-nmeas) are used to calculate the H-2 norm.

ncon Number of controlled inputs u. The last *ncon* inputs of *P* are connected to the outputs of controller *K*. The remaining inputs w (indices 1 to m-ncon) are excited by a harmonic test signal.

Outputs

K State-space model of the H-2 optimal controller.

N State-space model of the lower LFT of *P* and *K*.

info Structure containing additional information.

info.gamma
H-2 norm of *N*.

info.rcond Vector *rcond* contains estimates of the reciprocal condition numbers of the matrices which are to be inverted and estimates of the reciprocal condition numbers of the Riccati equations which have to be solved during the computation of the controller *K*. For details, see the description of the corresponding SLICOT routine.

Block Diagram

```
gamma = min||N(K)||             N = lft (P, K)
         K        2

               +--------+
       w ----->|        |-----> z
               |  P(s)  |
       u +---->|        |-----+ v
         |     +--------+     |
         |                    |
         |     +--------+     |
         +-----|  K(s)  |<----+
               +--------+

               +--------+
       w ----->|  N(s)  |-----> z
               +--------+
```

Algorithm
Uses SLICOT SB10HD and SB10ED by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: augw, lqr, dlqr, kalman.

12.4 hinfsyn

`[K, N, gamma, info] = hinfsyn (P, nmeas, ncon)` [Function File]
`[K, N, gamma, info] = hinfsyn (P, nmeas, ncon, ...)` [Function File]
`[K, N, gamma, info] = hinfsyn (P, nmeas, ncon, opt, ...)` [Function File]
`[K, N, gamma, info] = hinfsyn (P, ...)` [Function File]
`[K, N, gamma, info] = hinfsyn (P, opt, ...)` [Function File]

H-infinity control synthesis for LTI plant.

Inputs

P Generalized plant. Must be a proper/realizable LTI model. If *P* is constructed with `mktito` or `augw`, arguments *nmeas* and *ncon* can be omitted.

nmeas Number of measured outputs v. The last *nmeas* outputs of *P* are connected to the inputs of controller *K*. The remaining outputs z (indices 1 to p-nmeas) are used to calculate the H-infinity norm.

ncon Number of controlled inputs u. The last *ncon* inputs of *P* are connected to the outputs of controller *K*. The remaining inputs w (indices 1 to m-ncon) are excited by a harmonic test signal.

... Optional pairs of keys and values. `'key1', value1, 'key2', value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

K State-space model of the H-infinity (sub-)optimal controller.

N State-space model of the lower LFT of *P* and *K*.

info Structure containing additional information.

info.gamma
L-infinity norm of *N*.

info.rcond Vector *rcond* contains estimates of the reciprocal condition numbers of the matrices which are to be inverted and estimates of the reciprocal condition numbers of the Riccati equations which have to be solved during the computation of the controller *K*. For details, see the description of the corresponding SLICOT routine.

Option Keys and Values

'method' String specifying the desired kind of controller:

'optimal', 'opt', 'o'
Compute optimal controller using gamma iteration. Default selection for compatibility reasons.

'suboptimal', 'sub', 's'
Compute (sub-)optimal controller. For stability reasons, suboptimal controllers are to be preferred over optimal ones.

'gmax' The maximum value of the H-infinity norm of *N*. It is assumed that *gmax* is sufficiently large so that the controller is admissible. Default value is 1e15.

'gmin' Initial lower bound for gamma iteration. Default value is 0. *gmin* is only meaningful for optimal discrete-time controllers.

'tolgam' Tolerance used for controlling the accuracy of *gamma* and its distance to the estimated minimal possible value of *gamma*. Default value is 0.01. If *tolgam* = 0, then a default value equal to `sqrt(eps)` is used, where *eps* is the relative machine precision. For suboptimal controllers, *tolgam* is ignored.

'actol' Upper bound for the poles of the closed-loop system *N* used for determining if it is stable. *actol* >= 0 for stable systems. For suboptimal controllers, *actol* is ignored.

Block Diagram

```
gamma = min||N(K)||             N = lft (P, K)
         K          inf

                 +--------+
         w ----->|        |-----> z
                 |  P(s)  |
         u +---->|        |-----+ v
           |     +--------+     |
           |                    |
           |     +--------+     |
           +-----|  K(s)  |<----+
                 +--------+

                 +--------+
         w ----->|  N(s)  |-----> z
                 +--------+
```

Algorithm
Uses SLICOT SB10FD, SB10DD and SB10AD by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: augw, mixsyn.

12.5 mixsyn

[*K*, *N*, *gamma*, *info*] = mixsyn (*G*, *W1*, *W2*, *W3*, ...) [Function File]

Solve stacked S/KS/T H-infinity problem. Mixed-sensitivity is the name given to transfer function shaping problems in which the sensitivity function $S = (I + GK)^{-1}$ is shaped along with one or more other closed-loop transfer functions such as $K\,S$ or the complementary sensitivity function $T = I - S = (I + GK)^{-1}GK$ in a typical one degree-of-freedom configuration, where G denotes the plant and K the (sub-)optimal controller to be found. The shaping of multivariable transfer functions is based on the idea that a satisfactory definition of gain (range of gain) for a matrix transfer function is given by the singular values σ of the transfer function. Hence the classical loop-shaping ideas of feedback design can be generalized to multivariable systems. In addition to the requirement that K stabilizes G, the closed-loop objectives are as follows [1]:

1. For *disturbance rejection* make $\overline{\sigma}(S)$ small.
2. For *noise attenuation* make $\overline{\sigma}(T)$ small.
3. For *reference tracking* make $\overline{\sigma}(T) \approx \underline{\sigma}(T) \approx 1$.
4. For *input usage (control energy) reduction* make $\overline{\sigma}(KS)$ small.
5. For *robust stability* in the presence of an additive perturbation $G_p = G + \Delta$, make $\overline{\sigma}(KS)$ small.
6. For *robust stability* in the presence of a multiplicative output perturbation $G_p = (I + \Delta)G$, make $\overline{\sigma}(T)$ small.

In order to find a robust controller for the so-called stacked $S/KS/T$ H_∞ problem, the user function `mixsyn` minimizes the following criterion

$$K\min||N(K)||_\infty, \quad N = |W_1S;\ W_2KS;\ W_3T|$$

`[K, N] = mixsyn (G, W1, W2, W3)`. The user-defined weighting functions *W1*, *W2* and *W3* bound the largest singular values of the closed-loop transfer functions S (for performance),

K S (to penalize large inputs) and T (for robustness and to avoid sensitivity to noise), respectively [1]. A few points are to be considered when choosing the weights. The weigths *Wi* must all be proper and stable. Therefore if one wishes, for example, to minimize S at low frequencies by a weighting *W1* including integral action, $\frac{1}{s}$ needs to be approximated by $\frac{1}{s+\epsilon}$, where $\epsilon \ll 1$. Similarly one might be interested in weighting K S with a non-proper weight *W2* to ensure that K is small outside the system bandwidth. The trick here is to replace a non-proper term such as $1 + \tau_1 s$ by $\frac{1+\tau_1 s}{1+\tau_2 s}$, where $\tau_2 \ll \tau_1$ [1, 2].

Inputs

G LTI model of plant.

W1 LTI model of performance weight. Bounds the largest singular values of sensitivity *S*. Model must be empty `[]`, SISO or of appropriate size.

W2 LTI model to penalize large control inputs. Bounds the largest singular values of *KS*. Model must be empty `[]`, SISO or of appropriate size.

W3 LTI model of robustness and noise sensitivity weight. Bounds the largest singular values of complementary sensitivity *T*. Model must be empty `[]`, SISO or of appropriate size.

... Optional arguments of `hinfsyn`. Type `help hinfsyn` for more information.

All inputs must be proper/realizable. Scalars, vectors and matrices are possible instead of LTI models.

Outputs

K State-space model of the H-infinity (sub-)optimal controller.

N State-space model of the lower LFT of *P* and *K*.

info Structure containing additional information.

info.gamma
L-infinity norm of *N*.

info.rcond Vector *rcond* contains estimates of the reciprocal condition numbers of the matrices which are to be inverted and estimates of the reciprocal condition numbers of the Riccati equations which have to be solved during the computation of the controller *K*. For details, see the description of the corresponding SLICOT routine.

Block Diagram

=

```
                                            | W1 S   |
     gamma = min||N(K)||             N  =   | W2 K S | = lft (P, K)
              K        inf                  | W3 T   |
```

```
                                                             +------+  z1     =
                  +-------------------------------------->|  W1  |----->
                  |                                          +------+
                  |                                          +------+  z2
                  |                 +---------------------->|  W2  |----->
                  |                 |                        +------+
 r   +    e |    +--------+  u |    +--------+  y            +------+  z3
----->(+)---+-->|  K(s)  |----+-->|  G(s)  |----+---->|  W3  |----->
       ^ -       +--------+         +--------+    |          +------+
       |                                            |
       +--------------------------------------------+
```

```
               +--------+                                                 =
               |        |-----> z1 (p1x1)          z1 = W1 e
 r (px1) ----->|  P(s)  |-----> z2 (p2x1)          z2 = W2 u
               |        |-----> z3 (p3x1)          z3 = W3 y
 u (mx1) ----->|        |-----> e (px1)            e = r - y
               +--------+
```

```
               +--------+                                                 =
       r ----->|        |-----> z
               |  P(s)  |
       u +---->|        |-----+ e
         |     +--------+     |
         |                    |
         |     +--------+     |
         +-----|  K(s)  |<----+
               +--------+
```

```
               +--------+                                                 =
       r ----->|  N(s)  |-----> z
               +--------+
```

```
Extended Plant:  P = augw (G, W1, W2, W3)                                 =
Controller:      K = mixsyn (G, W1, W2, W3)
Entire System:   N = lft (P, K)
Open Loop:       L = G * K
Closed Loop:     T = feedback (L)
```

Algorithm
Relies on functions `augw` and `hinfsyn`, which use SLICOT SB10FD, SB10DD and SB10AD by courtesy of NICONET e.V. (`http://www.slicot.org`)

References
[1] Skogestad, S. and Postlethwaite I. (2005) *Multivariable Feedback Control: Analysis and Design: Second Edition*. Wiley, Chichester, England.
[2] Meinsma, G. (1995) *Unstable and nonproper weights in H-infinity control* Automatica, Vol. 31, No. 11, pp. 1655-1658

See also: hinfsyn, augw.

12.6 mktito

P = `mktito` (*P*, *nmeas*, *ncon*) [Function File]

Partition LTI plant *P* for robust controller synthesis. If a plant is partitioned this way, one can omit the inputs *nmeas* and *ncon* when calling the functions `hinfsyn` and `h2syn`.

Inputs

P Generalized plant.

nmeas Number of measured outputs v. The last *nmeas* outputs of *P* are connected to the inputs of controller *K*. The remaining outputs z (indices 1 to p-nmeas) are used to calculate the H-2/H-infinity norm.

ncon Number of controlled inputs u. The last *ncon* inputs of *P* are connected to the outputs of controller *K*. The remaining inputs w (indices 1 to m-ncon) are excited by a harmonic test signal.

Outputs

P Partitioned plant. The input/output groups and names are overwritten with designations according to [1].

Block Diagram

```
min||N(K)||               N = lft (P, K)
 K         norm

                +--------+
        w ----->|        |-----> z
                |  P(s)  |
        u +---->|        |-----+ v
          |     +--------+     |
          |                    |
          |     +--------+     |
          +-----|  K(s)  |<----+
                +--------+

                +--------+
        w ----->|  N(s)  |-----> z
                +--------+
```

Reference

[1] Skogestad, S. and Postlethwaite, I. (2005) *Multivariable Feedback Control: Analysis and Design: Second Edition*. Wiley, Chichester, England.

12.7 ncfsyn

[*K*, *N*, *gamma*, *info*] = `ncfsyn` (*G*, *W1*, *W2*, *factor*) [Function File]

Loop shaping H-infinity synthesis. Compute positive feedback controller using the McFarlane/Glover loop shaping design procedure [1]. Using a precompensator *W1* and/or a post-compensator *W2*, the singular values of the nominal plant *G* are shaped to give a desired open-loop shape. The nominal plant *G* and shaping functions *W1*, *W2* are combined to form the shaped plant, *Gs* where `Gs = W2 G W1`. We assume that *W1* and *W2* are such that *Gs* contains no hidden modes. It is relatively easy to approximate the closed-loop requirements by the following open-loop objectives [2]:

1. For *disturbance rejection* make $\underline{\sigma}(W_2GW_1)$ large; valid for frequencies at which $\underline{\sigma}(G_S) \gg 1$.
2. For *noise attenuation* make $\overline{\sigma}(W_2GW_1)$ small; valid for frequencies at which $\overline{\sigma}(G_S) \ll 1$.
3. For *reference tracking* make $\underline{\sigma}(W_2GW_1)$ large; valid for frequencies at which $\underline{\sigma}(G_S) \gg 1$.
4. For *robust stability* to a multiplicative output perturbation $G_p = (I + \Delta)G$, make $\overline{\sigma}(W_2GW_1)$ small; valid for frequencies at which $\overline{\sigma}(G_S) \ll 1$.

Then a stabilizing controller *Ks* is synthesized for shaped plant *Gs*. The final positive feedback controller K is then constructed by combining the H_∞ controller *Ks* with the shaping functions *W1* and *W2* such that `K = W1 Ks W2`. In [1] is stated further that the given robust stabilization objective can be interpreted as a H_∞ problem formulation of minimizing the H_∞ norm of the frequency weighted gain from disturbances on the plant input and output to the controller input and output as follows:

$$K \min ||N(K)||_\infty,$$

$$N = |W_1^{-1}; W_2G|\ (I - KG)^{-1}\ |W_1,\ GW_2^{-1}|$$

`[K, N] = ncfsyn (G, W1, W2, f)` The function `ncfsyn` - the somewhat cryptic name stands for *normalized coprime factorization synthesis* - allows the specification of an additional argument, factor *f*. Default value `f = 1` implies that an optimal controller is required, whereas `f > 1` implies that a suboptimal controller is required, achieving a performance that is f times less than optimal.

Inputs

G LTI model of plant.

W1 LTI model of precompensator. Model must be SISO or of appropriate size. An identity matrix is taken if *W1* is not specified or if an empty model `[]` is passed.

W2 LTI model of postcompensator. Model must be SISO or of appropriate size. An identity matrix is taken if *W2* is not specified or if an empty model `[]` is passed.

factor `factor = 1` implies that an optimal controller is required. `factor > 1` implies that a suboptimal controller is required, achieving a performance that is *factor* times less than optimal. Default value is 1.

Outputs

K State-space model of the H-infinity loop-shaping controller. Note that K is a *positive* feedback controller.

N State-space model of the closed loop depicted below.

info Structure containing additional information.

info.gamma
L-infinity norm of *N*. `gamma = norm (N, inf)`.

info.emax Nugap robustness. `emax = inv (gamma)`.

info.Gs Shaped plant. `Gs = W2 * G * W1`.

info.Ks Controller for shaped plant. `Ks = ncfsyn (Gs)`.

info.rcond Estimates of the reciprocal condition numbers of the Riccati equations and a few other things. For details, see the description of the corresponding SLICOT routine.

Block Diagram of N

=

```
            ^ z1               ^ z2
            |                  |
 w1  +      |   +---------+    |            +---------+
----->(+)---+-->|   Ks    |----+--->(+)---->|   Gs    |----+
       ^ +      +---------+          ^      +---------+    |
       |                         w2  |                     |
       |                                                   |
       +---------------------------------------------------+
```

Algorithm

Uses SLICOT SB10ID, SB10KD and SB10ZD by courtesy of NICONET e.V. (`http://www.slicot.org`)

References

[1] D. McFarlane and K. Glover, *A Loop Shaping Design Procedure Using H-infinity Synthesis*, IEEE Transactions on Automatic Control, Vol. 37, No. 6, June 1992.

[2] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design: Second Edition*. Wiley, Chichester, England, 2005.

13 Matrix Equation Solvers

13.1 care

[*x*, *l*, *g*] = care (*a*, *b*, *q*, *r*) [Function File]
[*x*, *l*, *g*] = care (*a*, *b*, *q*, *r*, *s*) [Function File]
[*x*, *l*, *g*] = care (*a*, *b*, *q*, *r*, [], *e*) [Function File]
[*x*, *l*, *g*] = care (*a*, *b*, *q*, *r*, *s*, *e*) [Function File]

Solve continuous-time algebraic Riccati equation (ARE).

Inputs

a Real matrix (n-by-n).

b Real matrix (n-by-m).

q Real matrix (n-by-n).

r Real matrix (m-by-m).

s Optional real matrix (n-by-m). If *s* is not specified, a zero matrix is assumed.

e Optional descriptor matrix (n-by-n). If *e* is not specified, an identity matrix is assumed.

Outputs

x Unique stabilizing solution of the continuous-time Riccati equation (n-by-n).

l Closed-loop poles (n-by-1).

g Corresponding gain matrix (m-by-n).

Equations

$$A'X + XA - XB\,R^{-1}\,B'X + Q = 0$$

$$A'X + XA - (XB + S)\,R^{-1}\,(B'X + S') + Q = 0$$

$$G = R^{-1}\,B'X$$

$$G = R^{-1}\,(B'X + S')$$

$$L = \mathrm{eig}\,(A - B*G)$$

```
                          -1                                              =
A'XE + E'XA - E'XB R   B'XE + Q = 0

                                -1
A'XE + E'XA - (E'XB + S) R   (B'XE + S') + Q = 0

       -1
G = R  B'XE

       -1
G = R  (B'XE + S)

L = eig (A - B*G, E)
```

Algorithm

Uses SLICOT SB02OD and SG02AD by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: dare, lqr, dlqr, kalman.

13.2 dare

[*x*, *l*, *g*] = **dare** (*a*, *b*, *q*, *r*) [Function File]
[*x*, *l*, *g*] = **dare** (*a*, *b*, *q*, *r*, *s*) [Function File]
[*x*, *l*, *g*] = **dare** (*a*, *b*, *q*, *r*, [], *e*) [Function File]
[*x*, *l*, *g*] = **dare** (*a*, *b*, *q*, *r*, *s*, *e*) [Function File]

Solve discrete-time algebraic Riccati equation (ARE).

Inputs

a Real matrix (n-by-n).

b Real matrix (n-by-m).

q Real matrix (n-by-n).

r Real matrix (m-by-m).

s Optional real matrix (n-by-m). If *s* is not specified, a zero matrix is assumed.

e Optional descriptor matrix (n-by-n). If *e* is not specified, an identity matrix is assumed.

Outputs

x Unique stabilizing solution of the discrete-time Riccati equation (n-by-n).

l Closed-loop poles (n-by-1).

g Corresponding gain matrix (m-by-n).

Equations

```
                           -1
A'XA - X - A'XB (B'XB + R)   B'XA + Q = 0

                                 -1
A'XA - X - (A'XB + S) (B'XB + R)   (B'XA + S') + Q = 0

                 -1
G = (B'XB + R)   B'XA

                 -1
G = (B'XB + R)   (B'XA + S')

L = eig (A - B*G)
```

```
                              -1
A'XA - E'XE - A'XB (B'XB + R)   B'XA + Q = 0

                                    -1
A'XA - E'XE - (A'XB + S) (B'XB + R)   (B'XA + S') + Q = 0

                 -1
G = (B'XB + R)   B'XA

                 -1
G = (B'XB + R)   (B'XA + S')

L = eig (A - B*G, E)
```

Algorithm
Uses SLICOT SB02OD and SG02AD by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: care, lqr, dlqr, kalman.

13.3 dlyap

x = `dlyap` (*a*, *b*) [Function File]
x = `dlyap` (*a*, *b*, *c*) [Function File]
x = `dlyap` (*a*, *b*, [], *e*) [Function File]

Solve discrete-time Lyapunov or Sylvester equations.

Equations

```
AXA' - X + B = 0      (Lyapunov Equation)

AXB' - X + C = 0      (Sylvester Equation)

AXA' - EXE' + B = 0   (Generalized Lyapunov Equation)
```

Algorithm
Uses SLICOT SB03MD, SB04QD and SG03AD by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: dlyapchol, lyap, lyapchol.

13.4 dlyapchol

u = dlyapchol (*a*, *b*) [Function File]
u = dlyapchol (*a*, *b*, *e*) [Function File]

Compute Cholesky factor of discrete-time Lyapunov equations.

Equations

```
A U' U A'  -  U' U  +  B B'  =  0          (Lyapunov Equation)        =

A U' U A'  -  E U' U E'  +  B B'  =  0      (Generalized Lyapunov Equation)
```

Algorithm
Uses SLICOT SB03OD and SG03BD by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: dlyap, lyap, lyapchol.

13.5 lyap

x = lyap (*a*, *b*) [Function File]
x = lyap (*a*, *b*, *c*) [Function File]
x = lyap (*a*, *b*, [], *e*) [Function File]

Solve continuous-time Lyapunov or Sylvester equations.

Equations

```
AX + XA' + B = 0      (Lyapunov Equation)                             =

AX + XB + C = 0       (Sylvester Equation)

AXE' + EXA' + B = 0   (Generalized Lyapunov Equation)
```

Algorithm
Uses SLICOT SB03MD, SB04MD and SG03AD by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: lyapchol, dlyap, dlyapchol.

13.6 lyapchol

u = lyapchol (*a*, *b*) [Function File]
u = lyapchol (*a*, *b*, *e*) [Function File]

Compute Cholesky factor of continuous-time Lyapunov equations.

Equations

```
A U' U  +  U' U A'  +  B B'  =  0          (Lyapunov Equation)        =

A U' U E'  +  E U' U A'  +  B B'  =  0      (Generalized Lyapunov Equation)
```

Algorithm
Uses SLICOT SB03OD and SG03BD by courtesy of NICONET e.V. (`http://www.slicot.org`)

See also: lyap, dlyap, dlyapchol.

14 Model Reduction

14.1 bstmodred

[*Gr*, *info*] = bstmodred (*G*, ...) [Function File]
[*Gr*, *info*] = bstmodred (*G*, *nr*, ...) [Function File]
[*Gr*, *info*] = bstmodred (*G*, *opt*, ...) [Function File]
[*Gr*, *info*] = bstmodred (*G*, *nr*, *opt*, ...) [Function File]

Model order reduction by Balanced Stochastic Truncation (BST) method. The aim of model reduction is to find an LTI system *Gr* of order *nr* (nr < n) such that the input-output behaviour of *Gr* approximates the one from original system *G*.

BST is a relative error method which tries to minimize

$$||G^{-1}(G - G_r)||_\infty = \min$$

Inputs

G LTI model to be reduced.

nr The desired order of the resulting reduced order system *Gr*. If not specified, *nr* is chosen automatically according to the description of key *'order'*.

... Optional pairs of keys and values. `"key1", value1, "key2", value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

Gr Reduced order state-space model.

info Struct containing additional information.

- *info.n* The order of the original system *G*.
- *info.ns* The order of the *alpha*-stable subsystem of the original system *G*.
- *info.hsv* The Hankel singular values of the phase system corresponding to the *alpha*-stable part of the original system *G*. The *ns* Hankel singular values are ordered decreasingly.
- *info.nu* The order of the *alpha*-unstable subsystem of both the original system *G* and the reduced-order system *Gr*.
- *info.nr* The order of the obtained reduced order system *Gr*.

Option Keys and Values

'order', 'nr'
The desired order of the resulting reduced order system *Gr*. If not specified, *nr* is the sum of NU and the number of Hankel singular values greater than `MAX(TOL1,NS*EPS)`; *nr* can be further reduced to ensure that `HSV(NR-NU) > HSV(NR+1-NU)`.

'method' Approximation method for the H-infinity norm. Valid values corresponding to this key are:

'sr-bta', 'b'
Use the square-root Balance & Truncate method.

'bfsr-bta', 'f'
: Use the balancing-free square-root Balance & Truncate method. Default method.

'sr-spa', 's'
: Use the square-root Singular Perturbation Approximation method.

'bfsr-spa', 'p'
: Use the balancing-free square-root Singular Perturbation Approximation method.

'alpha' Specifies the ALPHA-stability boundary for the eigenvalues of the state dynamics matrix $G.A$. For a continuous-time system, ALPHA <= 0 is the boundary value for the real parts of eigenvalues, while for a discrete-time system, 0 <= ALPHA <= 1 represents the boundary value for the moduli of eigenvalues. The ALPHA-stability domain does not include the boundary. Default value is 0 for continuous-time systems and 1 for discrete-time systems.

'beta' Use `[G, beta*I]` as new system G to combine absolute and relative error methods. BETA > 0 specifies the absolute/relative error weighting parameter. A large positive value of BETA favours the minimization of the absolute approximation error, while a small value of BETA is appropriate for the minimization of the relative error. BETA = 0 means a pure relative error method and can be used only if rank(G.D) = rows(G.D) which means that the feedthrough matrice must not be rank-deficient. Default value is 0.

'tol1' If *'order'* is not specified, *tol1* contains the tolerance for determining the order of reduced system. For model reduction, the recommended value of *tol1* lies in the interval [0.00001, 0.001]. *tol1* < 1. If *tol1* <= 0 on entry, the used default value is *tol1* = NS*EPS, where NS is the number of ALPHA-stable eigenvalues of A and EPS is the machine precision. If *'order'* is specified, the value of *tol1* is ignored.

'tol2' The tolerance for determining the order of a minimal realization of the phase system (see METHOD) corresponding to the ALPHA-stable part of the given system. The recommended value is TOL2 = NS*EPS. TOL2 <= TOL1 < 1. This value is used by default if *'tol2'* is not specified or if TOL2 <= 0 on entry.

'equil', 'scale'
: Boolean indicating whether equilibration (scaling) should be performed on system G prior to order reduction. Default value is true if `G.scaled == false` and false if `G.scaled == true`. Note that for MIMO models, proper scaling of both inputs and outputs is of utmost importance. The input and output scaling can **not** be done by the equilibration option or the `prescale` function because these functions perform state transformations only. Furthermore, signals should not be scaled simply to a certain range. For all inputs (or outputs), a certain change should be of the same importance for the model.

BST is often suitable to perform model reduction in order to obtain low order design models for controller synthesis.

Approximation Properties:

- Guaranteed stability of reduced models
- Approximates simultaneously gain and phase
- Preserves non-minimum phase zeros
- Guaranteed a priori error bound

$$||G^{-1}(G - G_r)||_\infty \leq 2 \sum_{j=r+1}^{n} \frac{1 + \sigma_j}{1 - \sigma_j} - 1$$

Algorithm
Uses SLICOT AB09HD by courtesy of NICONET e.V. (`http://www.slicot.org`)

14.2 btamodred

[*Gr*, *info*] = btamodred (*G*, ...) [Function File]
[*Gr*, *info*] = btamodred (*G*, *nr*, ...) [Function File]
[*Gr*, *info*] = btamodred (*G*, *opt*, ...) [Function File]
[*Gr*, *info*] = btamodred (*G*, *nr*, *opt*, ...) [Function File]

Model order reduction by frequency weighted Balanced Truncation Approximation (BTA) method. The aim of model reduction is to find an LTI system *Gr* of order *nr* (nr < n) such that the input-output behaviour of *Gr* approximates the one from original system *G*.

BTA is an absolute error method which tries to minimize

$$||G - G_r||_\infty = \min$$

$$||V\ (G - G_r)\ W||_\infty = \min$$

where *V* and *W* denote output and input weightings.

Inputs

G LTI model to be reduced.

nr The desired order of the resulting reduced order system *Gr*. If not specified, *nr* is chosen automatically according to the description of key *'order'*.

... Optional pairs of keys and values. `"key1", value1, "key2", value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

Gr Reduced order state-space model.

info Struct containing additional information.

- *info.n* The order of the original system *G*.
- *info.ns* The order of the *alpha*-stable subsystem of the original system *G*.
- *info.hsv* The Hankel singular values of the *alpha*-stable part of the original system *G*, ordered decreasingly.
- *info.nu* The order of the *alpha*-unstable subsystem of both the original system *G* and the reduced-order system *Gr*.
- *info.nr* The order of the obtained reduced order system *Gr*.

Option Keys and Values

'order', *'nr'*
The desired order of the resulting reduced order system *Gr*. If not specified, *nr* is chosen automatically such that states with Hankel singular values *info.hsv* > *tol1* are retained.

'left', *'output'*
LTI model of the left/output frequency weighting *V*. Default value is an identity matrix.

'right', 'input'
: LTI model of the right/input frequency weighting W. Default value is an identity matrix.

'method'
: Approximation method for the L-infinity norm to be used as follows:

  'sr', 'b'
  : Use the square-root Balance & Truncate method.

  'bfsr', 'f'
  : Use the balancing-free square-root Balance & Truncate method. Default method.

'alpha'
: Specifies the ALPHA-stability boundary for the eigenvalues of the state dynamics matrix $G.A$. For a continuous-time system, ALPHA <= 0 is the boundary value for the real parts of eigenvalues, while for a discrete-time system, 0 <= ALPHA <= 1 represents the boundary value for the moduli of eigenvalues. The ALPHA-stability domain does not include the boundary. Default value is 0 for continuous-time systems and 1 for discrete-time systems.

'tol1'
: If *'order'* is not specified, *tol1* contains the tolerance for determining the order of the reduced model. For model reduction, the recommended value of *tol1* is c*info.hsv(1), where c lies in the interval [0.00001, 0.001]. Default value is info.ns*eps*info.hsv(1). If *'order'* is specified, the value of *tol1* is ignored.

'tol2'
: The tolerance for determining the order of a minimal realization of the ALPHA-stable part of the given model. TOL2 <= TOL1. If not specified, ns*eps*info.hsv(1) is chosen.

'gram-ctrb'
: Specifies the choice of frequency-weighted controllability Grammian as follows:

  'standard'
  : Choice corresponding to a combination method [4] of the approaches of Enns [1] and Lin-Chiu [2,3]. Default method.

  'enhanced'
  : Choice corresponding to the stability enhanced modified combination method of [4].

'gram-obsv'
: Specifies the choice of frequency-weighted observability Grammian as follows:

  'standard'
  : Choice corresponding to a combination method [4] of the approaches of Enns [1] and Lin-Chiu [2,3]. Default method.

  'enhanced'
  : Choice corresponding to the stability enhanced modified combination method of [4].

'alpha-ctrb'
: Combination method parameter for defining the frequency-weighted controllability Grammian. abs(alphac) <= 1. If alphac = 0, the choice of Grammian corresponds to the method of Enns [1], while if alphac = 1, the choice of Grammian corresponds to the method of Lin and Chiu [2,3]. Default value is 0.

'alpha-obsv'
: Combination method parameter for defining the frequency-weighted observability Grammian. abs(alphao) <= 1. If alphao = 0, the choice of Grammian corresponds to the method of Enns [1], while if alphao = 1, the choice of Grammian corresponds to the method of Lin and Chiu [2,3]. Default value is 0.

'equil', 'scale'
: Boolean indicating whether equilibration (scaling) should be performed on system G prior to order reduction. This is done by state transformations. Default value is true if `G.scaled == false` and false if `G.scaled == true`. Note that for MIMO models, proper scaling of both inputs and outputs is of utmost importance. The input and output scaling can **not** be done by the equilibration option or the `prescale` function because these functions perform state transformations only. Furthermore, signals should not be scaled simply to a certain range. For all inputs (or outputs), a certain change should be of the same importance for the model.

Approximation Properties:

- Guaranteed stability of reduced models
- Lower guaranteed error bound
- Guaranteed a priori error bound

$$\sigma_{r+1} \le ||(G - G_r)||_\infty \le 2 \sum_{j=r+1}^{n} \sigma_j$$

References

[1] Enns, D. *Model reduction with balanced realizations: An error bound and a frequency weighted generalization.* Proc. 23-th CDC, Las Vegas, pp. 127-132, 1984.

[2] Lin, C.-A. and Chiu, T.-Y. *Model reduction via frequency-weighted balanced realization.* Control Theory and Advanced Technology, vol. 8, pp. 341-351, 1992.

[3] Sreeram, V., Anderson, B.D.O and Madievski, A.G. *New results on frequency weighted balanced reduction technique.* Proc. ACC, Seattle, Washington, pp. 4004-4009, 1995.

[4] Varga, A. and Anderson, B.D.O. *Square-root balancing-free methods for the frequency-weighted balancing related model reduction.* (report in preparation)

Algorithm

Uses SLICOT AB09ID by courtesy of NICONET e.V. (`http://www.slicot.org`)

14.3 hnamodred

`[Gr, info] = hnamodred (G, ...)` [Function File]
`[Gr, info] = hnamodred (G, nr, ...)` [Function File]
`[Gr, info] = hnamodred (G, opt, ...)` [Function File]
`[Gr, info] = hnamodred (G, nr, opt, ...)` [Function File]

Model order reduction by frequency weighted optimal Hankel-norm (HNA) method. The aim of model reduction is to find an LTI system *Gr* of order *nr* (nr < n) such that the input-output behaviour of *Gr* approximates the one from original system *G*.

HNA is an absolute error method which tries to minimize

$$||G - G_r||_H = \min$$

$$||V\ (G - G_r)\ W||_H = \min$$

where V and W denote output and input weightings.

Inputs

G
: LTI model to be reduced.

nr
: The desired order of the resulting reduced order system *Gr*. If not specified, *nr* is chosen automatically according to the description of key `"`*order*`"`.

... Optional pairs of keys and values. `"key1", value1, "key2", value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

Gr Reduced order state-space model.

info Struct containing additional information.

info.n The order of the original system *G*.

info.ns The order of the *alpha*-stable subsystem of the original system *G*.

info.hsv The Hankel singular values corresponding to the projection `op(V)*G1*op(W)`, where G1 denotes the *alpha*-stable part of the original system *G*. The *ns* Hankel singular values are ordered decreasingly.

info.nu The order of the *alpha*-unstable subsystem of both the original system *G* and the reduced-order system *Gr*.

info.nr The order of the obtained reduced order system *Gr*.

Option Keys and Values

'order', 'nr'
The desired order of the resulting reduced order system *Gr*. If not specified, *nr* is the sum of *info.nu* and the number of Hankel singular values greater than `max(tol1, ns*eps*info.hsv(1);`

'method' Specifies the computational approach to be used. Valid values corresponding to this key are:

'descriptor'
Use the inverse free descriptor system approach.

'standard' Use the inversion based standard approach.

'auto' Switch automatically to the inverse free descriptor approach in case of badly conditioned feedthrough matrices in V or W. Default method.

'left', 'v' LTI model of the left/output frequency weighting. The weighting must be antistable. $||V\ (G - G_r) \ldots ||_H = \min$

'right', 'w' LTI model of the right/input frequency weighting. The weighting must be antistable. $|| \ldots (G - G_r)\ W||_H = \min$

'left-inv', 'inv-v'
LTI model of the left/output frequency weighting. The weighting must have only antistable zeros. $||inv(V)\ (G - G_r) \ldots ||_H = \min$

'right-inv', 'inv-w'
LTI model of the right/input frequency weighting. The weighting must have only antistable zeros. $|| \ldots (G - G_r)\ inv(W)||_H = \min$

'left-conj', 'conj-v'
LTI model of the left/output frequency weighting. The weighting must be stable. $||conj(V)\ (G - G_r) \ldots ||_H = \min$

'right-conj', 'conj-w'
LTI model of the right/input frequency weighting. The weighting must be stable. $|| \ldots (G - G_r)\ conj(W)||_H = \min$

'left-conj-inv', *'conj-inv-v'*
LTI model of the left/output frequency weighting. The weighting must be minimum-phase. $||conj(inv(V))\ (G - G_r) \ldots ||_H = \min$

'right-conj-inv', *'conj-inv-w'*
LTI model of the right/input frequency weighting. The weighting must be minimum-phase. $|| \ldots (G - G_r)\ conj(inv(W))||_H = \min$

'alpha' Specifies the ALPHA-stability boundary for the eigenvalues of the state dynamics matrix $G.A$. For a continuous-time system, ALPHA <= 0 is the boundary value for the real parts of eigenvalues, while for a discrete-time system, 0 <= ALPHA <= 1 represents the boundary value for the moduli of eigenvalues. The ALPHA-stability domain does not include the boundary. Default value is 0 for continuous-time systems and 1 for discrete-time systems.

'tol1' If *'order'* is not specified, *tol1* contains the tolerance for determining the order of the reduced model. For model reduction, the recommended value of *tol1* is c*info.hsv(1), where c lies in the interval [0.00001, 0.001]. *tol1* < 1. If *'order'* is specified, the value of *tol1* is ignored.

'tol2' The tolerance for determining the order of a minimal realization of the ALPHA-stable part of the given model. *tol2* <= *tol1* < 1. If not specified, ns*eps*info.hsv(1) is chosen.

'equil', *'scale'*
Boolean indicating whether equilibration (scaling) should be performed on system G prior to order reduction. Default value is true if `G.scaled == false` and false if `G.scaled == true`. Note that for MIMO models, proper scaling of both inputs and outputs is of utmost importance. The input and output scaling can **not** be done by the equilibration option or the `prescale` function because these functions perform state transformations only. Furthermore, signals should not be scaled simply to a certain range. For all inputs (or outputs), a certain change should be of the same importance for the model.

Approximation Properties:

- Guaranteed stability of reduced models
- Lower guaranteed error bound
- Guaranteed a priori error bound

$$\sigma_{r+1} \leq ||(G - G_r)||_\infty \leq 2 \sum_{j=r+1}^{n} \sigma_j$$

Algorithm
Uses SLICOT AB09JD by courtesy of NICONET e.V. (`http://www.slicot.org`)

14.4 spamodred

[*Gr*, *info*] = spamodred (*G*, ...) [Function File]
[*Gr*, *info*] = spamodred (*G*, *nr*, ...) [Function File]
[*Gr*, *info*] = spamodred (*G*, *opt*, ...) [Function File]
[*Gr*, *info*] = spamodred (*G*, *nr*, *opt*, ...) [Function File]

Model order reduction by frequency weighted Singular Perturbation Approximation (SPA). The aim of model reduction is to find an LTI system *Gr* of order *nr* (nr < n) such that the input-output behaviour of *Gr* approximates the one from original system *G*.

SPA is an absolute error method which tries to minimize

$$||G - G_r||_\infty = \min$$

$$||V\ (G - G_r)\ W||_\infty = \min$$

where V and W denote output and input weightings.

Inputs

G LTI model to be reduced.

nr The desired order of the resulting reduced order system *Gr*. If not specified, *nr* is chosen automatically according to the description of key *'order'*.

... Optional pairs of keys and values. `"key1", value1, "key2", value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

Gr Reduced order state-space model.

info Struct containing additional information.

info.n The order of the original system *G*.

info.ns The order of the *alpha*-stable subsystem of the original system *G*.

info.hsv The Hankel singular values of the *alpha*-stable part of the original system *G*, ordered decreasingly.

info.nu The order of the *alpha*-unstable subsystem of both the original system *G* and the reduced-order system *Gr*.

info.nr The order of the obtained reduced order system *Gr*.

Option Keys and Values

'order', 'nr'
The desired order of the resulting reduced order system *Gr*. If not specified, *nr* is chosen automatically such that states with Hankel singular values *info.hsv* `>` *tol1* are retained.

'left', 'output'
LTI model of the left/output frequency weighting *V*. Default value is an identity matrix.

'right', 'input'
LTI model of the right/input frequency weighting *W*. Default value is an identity matrix.

'method' Approximation method for the L-infinity norm to be used as follows:

'sr', 's' Use the square-root Singular Perturbation Approximation method.

'bfsr', 'p' Use the balancing-free square-root Singular Perturbation Approximation method. Default method.

'alpha' Specifies the ALPHA-stability boundary for the eigenvalues of the state dynamics matrix *G.A*. For a continuous-time system, ALPHA `<=` 0 is the boundary value for the real parts of eigenvalues, while for a discrete-time system, 0 `<=` ALPHA `<=` 1 represents the boundary value for the moduli of eigenvalues. The ALPHA-stability domain does not include the boundary. Default value is 0 for continuous-time systems and 1 for discrete-time systems.

'tol1' If *'order'* is not specified, *tol1* contains the tolerance for determining the order of the reduced model. For model reduction, the recommended value of *tol1* is c*info.hsv(1), where c lies in the interval [0.00001, 0.001]. Default value is info.ns*eps*info.hsv(1). If *'order'* is specified, the value of *tol1* is ignored.

'tol2' The tolerance for determining the order of a minimal realization of the ALPHA-stable part of the given model. TOL2 <= TOL1. If not specified, ns*eps*info.hsv(1) is chosen.

'gram-ctrb'
Specifies the choice of frequency-weighted controllability Grammian as follows:

'standard' Choice corresponding to a combination method [4] of the approaches of Enns [1] and Lin-Chiu [2,3]. Default method.

'enhanced'
Choice corresponding to the stability enhanced modified combination method of [4].

'gram-obsv'
Specifies the choice of frequency-weighted observability Grammian as follows:

'standard' Choice corresponding to a combination method [4] of the approaches of Enns [1] and Lin-Chiu [2,3]. Default method.

'enhanced'
Choice corresponding to the stability enhanced modified combination method of [4].

'alpha-ctrb'
Combination method parameter for defining the frequency-weighted controllability Grammian. abs(alphac) <= 1. If alphac = 0, the choice of Grammian corresponds to the method of Enns [1], while if alphac = 1, the choice of Grammian corresponds to the method of Lin and Chiu [2,3]. Default value is 0.

'alpha-obsv'
Combination method parameter for defining the frequency-weighted observability Grammian. abs(alphao) <= 1. If alphao = 0, the choice of Grammian corresponds to the method of Enns [1], while if alphao = 1, the choice of Grammian corresponds to the method of Lin and Chiu [2,3]. Default value is 0.

'equil', 'scale'
Boolean indicating whether equilibration (scaling) should be performed on system G prior to order reduction. Default value is true if `G.scaled == false` and false if `G.scaled == true`. Note that for MIMO models, proper scaling of both inputs and outputs is of utmost importance. The input and output scaling can **not** be done by the equilibration option or the `prescale` function because these functions perform state transformations only. Furthermore, signals should not be scaled simply to a certain range. For all inputs (or outputs), a certain change should be of the same importance for the model.

References

[1] Enns, D. *Model reduction with balanced realizations: An error bound and a frequency weighted generalization*. Proc. 23-th CDC, Las Vegas, pp. 127-132, 1984.

[2] Lin, C.-A. and Chiu, T.-Y. *Model reduction via frequency-weighted balanced realization*. Control Theory and Advanced Technology, vol. 8, pp. 341-351, 1992.

[3] Sreeram, V., Anderson, B.D.O and Madievski, A.G. *New results on frequency weighted balanced reduction technique*. Proc. ACC, Seattle, Washington, pp. 4004-4009, 1995.

[4] Varga, A. and Anderson, B.D.O. *Square-root balancing-free methods for the frequency-weighted balancing related model reduction.* (report in preparation)

Algorithm

Uses SLICOT AB09ID by courtesy of NICONET e.V. (`http://www.slicot.org`)

15 Controller Reduction

15.1 btaconred

`[Kr, info] = btaconred (G, K, ...)` [Function File]
`[Kr, info] = btaconred (G, K, ncr, ...)` [Function File]
`[Kr, info] = btaconred (G, K, opt, ...)` [Function File]
`[Kr, info] = btaconred (G, K, ncr, opt, ...)` [Function File]

Controller reduction by frequency-weighted Balanced Truncation Approximation (BTA). Given a plant *G* and a stabilizing controller *K*, determine a reduced order controller *Kr* such that the closed-loop system is stable and closed-loop performance is retained.

The algorithm tries to minimize the frequency-weighted error

$$||V \ (K - K_r) \ W||_{\infty} = \min$$

where *V* and *W* denote output and input weightings.

Inputs

G LTI model of the plant. It has m inputs, p outputs and n states.

K LTI model of the controller. It has p inputs, m outputs and nc states.

ncr The desired order of the resulting reduced order controller *Kr*. If not specified, *ncr* is chosen automatically according to the description of key *'order'*.

... Optional pairs of keys and values. `"key1", value1, "key2", value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

Kr State-space model of reduced order controller.

info Struct containing additional information.

info.ncr The order of the obtained reduced order controller *Kr*.

info.ncs The order of the alpha-stable part of original controller *K*.

info.hsvc The Hankel singular values of the alpha-stable part of *K*. The *ncs* Hankel singular values are ordered decreasingly.

Option Keys and Values

'order', 'ncr'
The desired order of the resulting reduced order controller *Kr*. If not specified, *ncr* is chosen automatically such that states with Hankel singular values *info.hsvc* `>` *tol1* are retained.

'method' Order reduction approach to be used as follows:

'sr', 'b' Use the square-root Balance & Truncate method.

'bfsr', 'f' Use the balancing-free square-root Balance & Truncate method. Default method.

'weight' Specifies the type of frequency-weighting as follows:

'none' No weightings are used (V = I, W = I).

'left', 'output'
Use stability enforcing left (output) weighting

$$V = (I - GK)^{-1}G, \qquad W = I$$

'right', 'input'
Use stability enforcing right (input) weighting

$$V = I, \qquad W = (I - GK)^{-1}G$$

'both', 'performance'
Use stability and performance enforcing weightings

$$V = (I - GK)^{-1}G, \qquad W = (I - GK)^{-1}$$

Default value.

'feedback' Specifies whether K is a positive or negative feedback controller:

'+' Use positive feedback controller. Default value.

'-' Use negative feedback controller.

'alpha' Specifies the ALPHA-stability boundary for the eigenvalues of the state dynamics matrix $K.A$. For a continuous-time controller, ALPHA <= 0 is the boundary value for the real parts of eigenvalues, while for a discrete-time controller, 0 <= ALPHA <= 1 represents the boundary value for the moduli of eigenvalues. The ALPHA-stability domain does not include the boundary. Default value is 0 for continuous-time controllers and 1 for discrete-time controllers.

'tol1' If *'order'* is not specified, *tol1* contains the tolerance for determining the order of the reduced controller. For model reduction, the recommended value of *tol1* is c*info.hsvc(1), where c lies in the interval [0.00001, 0.001]. Default value is info.ncs*eps*info.hsvc(1). If *'order'* is specified, the value of *tol1* is ignored.

'tol2' The tolerance for determining the order of a minimal realization of the ALPHA-stable part of the given controller. TOL2 <= TOL1. If not specified, ncs*eps*info.hsvc(1) is chosen.

'gram-ctrb'
Specifies the choice of frequency-weighted controllability Grammian as follows:

'standard' Choice corresponding to standard Enns' method [1]. Default method.

'enhanced'
Choice corresponding to the stability enhanced modified Enns' method of [2].

'gram-obsv'
Specifies the choice of frequency-weighted observability Grammian as follows:

'standard' Choice corresponding to standard Enns' method [1]. Default method.

'enhanced'
Choice corresponding to the stability enhanced modified Enns' method of [2].

'equil', 'scale'
: Boolean indicating whether equilibration (scaling) should be performed on *G* and *K* prior to order reduction. Default value is false if both `G.scaled == true, K.scaled == true` and true otherwise. Note that for MIMO models, proper scaling of both inputs and outputs is of utmost importance. The input and output scaling can **not** be done by the equilibration option or the `prescale` function because these functions perform state transformations only. Furthermore, signals should not be scaled simply to a certain range. For all inputs (or outputs), a certain change should be of the same importance for the model.

Algorithm

Uses SLICOT SB16AD by courtesy of NICONET e.V. (`http://www.slicot.org`)

15.2 cfconred

`[Kr, info] = cfconred (G, F, L, ...)` [Function File]
`[Kr, info] = cfconred (G, F, L, ncr, ...)` [Function File]
`[Kr, info] = cfconred (G, F, L, opt, ...)` [Function File]
`[Kr, info] = cfconred (G, F, L, ncr, opt, ...)` [Function File]

Reduction of state-feedback-observer based controller by coprime factorization (CF). Given a plant *G*, state feedback gain *F* and full observer gain *L*, determine a reduced order controller *Kr*.

Inputs

G LTI model of the open-loop plant (A,B,C,D). It has m inputs, p outputs and n states.

F Stabilizing state feedback matrix (m-by-n).

L Stabilizing observer gain matrix (n-by-p).

ncr The desired order of the resulting reduced order controller *Kr*. If not specified, *ncr* is chosen automatically according to the description of key *'order'*.

... Optional pairs of keys and values. `"key1", value1, "key2", value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

Kr State-space model of reduced order controller.

info Struct containing additional information.

info.hsv The Hankel singular values of the extended system?!?. The *n* Hankel singular values are ordered decreasingly.

info.ncr The order of the obtained reduced order controller *Kr*.

Option Keys and Values

'order', 'ncr'
: The desired order of the resulting reduced order controller *Kr*. If not specified, *ncr* is chosen automatically such that states with Hankel singular values *info.hsv* > *tol1* are retained.

'method' Order reduction approach to be used as follows:

'sr-bta', 'b'
: Use the square-root Balance & Truncate method.

'bfsr-bta', 'f'
Use the balancing-free square-root Balance & Truncate method. Default method.

'sr-spa', 's'
Use the square-root Singular Perturbation Approximation method.

'bfsr-spa', 'p'
Use the balancing-free square-root Singular Perturbation Approximation method.

'cf' Specifies whether left or right coprime factorization is to be used as follows:

'left', 'l' Use left coprime factorization. Default method.

'right', 'r' Use right coprime factorization.

'feedback' Specifies whether F and L are fed back positively or negatively:

'+' A+BK and A+LC are both Hurwitz matrices.

'-' A-BK and A-LC are both Hurwitz matrices. Default value.

'tol1' If *'order'* is not specified, *tol1* contains the tolerance for determining the order of the reduced system. For model reduction, the recommended value of *tol1* is c*info.hsv(1), where c lies in the interval [0.00001, 0.001]. Default value is n*eps*info.hsv(1). If *'order'* is specified, the value of *tol1* is ignored.

'tol2' The tolerance for determining the order of a minimal realization of the coprime factorization controller. TOL2 <= TOL1. If not specified, n*eps*info.hsv(1) is chosen.

'equil', 'scale'
Boolean indicating whether equilibration (scaling) should be performed on system G prior to order reduction. Default value is true if `G.scaled == false` and false if `G.scaled == true`. Note that for MIMO models, proper scaling of both inputs and outputs is of utmost importance. The input and output scaling can **not** be done by the equilibration option or the `prescale` function because these functions perform state transformations only. Furthermore, signals should not be scaled simply to a certain range. For all inputs (or outputs), a certain change should be of the same importance for the model.

Algorithm
Uses SLICOT SB16BD by courtesy of NICONET e.V. (`http://www.slicot.org`)

15.3 fwcfconred

`[Kr, info] = fwcfconred (G, F, L, ...)` [Function File]
`[Kr, info] = fwcfconred (G, F, L, ncr, ...)` [Function File]
`[Kr, info] = fwcfconred (G, F, L, opt, ...)` [Function File]
`[Kr, info] = fwcfconred (G, F, L, ncr, opt, ...)` [Function File]

Reduction of state-feedback-observer based controller by frequency-weighted coprime factorization (FW CF). Given a plant G, state feedback gain F and full observer gain L, determine a reduced order controller Kr by using stability enforcing frequency weights.

Inputs

G LTI model of the open-loop plant (A,B,C,D). It has m inputs, p outputs and n states.

F Stabilizing state feedback matrix (m-by-n).

L Stabilizing observer gain matrix (n-by-p).

ncr The desired order of the resulting reduced order controller *Kr*. If not specified, *ncr* is chosen automatically according to the description of key *'order'*.

... Optional pairs of keys and values. `"key1", value1, "key2", value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

Kr State-space model of reduced order controller.

info Struct containing additional information.

info.hsv The Hankel singular values of the extended system?!?. The *n* Hankel singular values are ordered decreasingly.

info.ncr The order of the obtained reduced order controller *Kr*.

Option Keys and Values

'order', 'ncr'
The desired order of the resulting reduced order controller *Kr*. If not specified, *ncr* is chosen automatically such that states with Hankel singular values *info.hsv* > *tol1* are retained.

'method' Order reduction approach to be used as follows:

'sr', 'b' Use the square-root Balance & Truncate method.

'bfsr', 'f' Use the balancing-free square-root Balance & Truncate method. Default method.

'cf' Specifies whether left or right coprime factorization is to be used as follows:

'left', 'l' Use left coprime factorization.

'right', 'r' Use right coprime factorization. Default method.

'feedback' Specifies whether *F* and *L* are fed back positively or negatively:

'+' A+BK and A+LC are both Hurwitz matrices.

'-' A-BK and A-LC are both Hurwitz matrices. Default value.

'tol1' If *'order'* is not specified, *tol1* contains the tolerance for determining the order of the reduced system. For model reduction, the recommended value of *tol1* is c*info.hsv(1), where c lies in the interval [0.00001, 0.001]. Default value is n*eps*info.hsv(1). If *'order'* is specified, the value of *tol1* is ignored.

Algorithm
Uses SLICOT SB16CD by courtesy of NICONET e.V. (`http://www.slicot.org`)

15.4 spaconred

`[Kr, info] = spaconred (G, K, ...)` [Function File]
`[Kr, info] = spaconred (G, K, ncr, ...)` [Function File]
`[Kr, info] = spaconred (G, K, opt, ...)` [Function File]
`[Kr, info] = spaconred (G, K, ncr, opt, ...)` [Function File]

Controller reduction by frequency-weighted Singular Perturbation Approximation (SPA). Given a plant *G* and a stabilizing controller *K*, determine a reduced order controller *Kr* such that the closed-loop system is stable and closed-loop performance is retained.

The algorithm tries to minimize the frequency-weighted error

$$||V\ (K - K_r)\ W||_\infty = \min$$

where V and W denote output and input weightings.

Inputs

G LTI model of the plant. It has m inputs, p outputs and n states.

K LTI model of the controller. It has p inputs, m outputs and nc states.

ncr The desired order of the resulting reduced order controller *Kr*. If not specified, *ncr* is chosen automatically according to the description of key *'order'*.

... Optional pairs of keys and values. `"key1", value1, "key2", value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

Kr State-space model of reduced order controller.

info Struct containing additional information.

info.ncr The order of the obtained reduced order controller *Kr*.

info.ncs The order of the alpha-stable part of original controller *K*.

info.hsvc The Hankel singular values of the alpha-stable part of *K*. The *ncs* Hankel singular values are ordered decreasingly.

Option Keys and Values

'order', 'ncr'
The desired order of the resulting reduced order controller *Kr*. If not specified, *ncr* is chosen automatically such that states with Hankel singular values *info.hsvc* > *tol1* are retained.

'method' Order reduction approach to be used as follows:

'sr', 's' Use the square-root Singular Perturbation Approximation method.

'bfsr', 'p' Use the balancing-free square-root Singular Perturbation Approximation method. Default method.

'weight' Specifies the type of frequency-weighting as follows:

'none' No weightings are used (V = I, W = I).

'left', 'output'
Use stability enforcing left (output) weighting

$$V = (I - GK)^{-1}G, \qquad W = I$$

'right', 'input'
Use stability enforcing right (input) weighting

$$V = I, \qquad W = (I - GK)^{-1}G$$

'both', 'performance'
Use stability and performance enforcing weightings

$$V = (I - GK)^{-1}G, \qquad W = (I - GK)^{-1}$$

Default value.

'feedback' Specifies whether K is a positive or negative feedback controller:

- '+' Use positive feedback controller. Default value.
- '-' Use negative feedback controller.

'alpha' Specifies the ALPHA-stability boundary for the eigenvalues of the state dynamics matrix $K.A$. For a continuous-time controller, ALPHA <= 0 is the boundary value for the real parts of eigenvalues, while for a discrete-time controller, 0 <= ALPHA <= 1 represents the boundary value for the moduli of eigenvalues. The ALPHA-stability domain does not include the boundary. Default value is 0 for continuous-time controllers and 1 for discrete-time controllers.

'tol1' If *'order'* is not specified, *tol1* contains the tolerance for determining the order of the reduced controller. For model reduction, the recommended value of *tol1* is c*info.hsvc(1), where c lies in the interval [0.00001, 0.001]. Default value is info.ncs*eps*info.hsvc(1). If *'order'* is specified, the value of *tol1* is ignored.

'tol2' The tolerance for determining the order of a minimal realization of the ALPHA-stable part of the given controller. TOL2 <= TOL1. If not specified, ncs*eps*info.hsvc(1) is chosen.

'gram-ctrb'
Specifies the choice of frequency-weighted controllability Grammian as follows:

- *'standard'* Choice corresponding to standard Enns' method [1]. Default method.
- *'enhanced'* Choice corresponding to the stability enhanced modified Enns' method of [2].

'gram-obsv'
Specifies the choice of frequency-weighted observability Grammian as follows:

- *'standard'* Choice corresponding to standard Enns' method [1]. Default method.
- *'enhanced'* Choice corresponding to the stability enhanced modified Enns' method of [2].

'equil', 'scale'
Boolean indicating whether equilibration (scaling) should be performed on G and K prior to order reduction. Default value is false if both `G.scaled == true, K.scaled == true` and true otherwise. Note that for MIMO models, proper scaling of both inputs and outputs is of utmost importance. The input and output scaling can **not** be done by the equilibration option or the `prescale` function because these functions perform state transformations only. Furthermore, signals should not be scaled simply to a certain range. For all inputs (or outputs), a certain change should be of the same importance for the model.

Algorithm
Uses SLICOT SB16AD by courtesy of NICONET e.V. (`http://www.slicot.org`)

16 Experimental Data Handling

16.1 iddata

dat = iddata (*y*) [Function File]
dat = iddata (*y*, *u*) [Function File]
dat = iddata (*y*, *u*, *tsam*, ...) [Function File]
dat = iddata (*y*, *u*, [], ...) [Function File]

Create identification dataset of output and input signals.

Inputs

y Real matrix containing the output signal in time-domain. For a system with *p* outputs and *n* samples, *y* is a n-by-p matrix. For data from multiple experiments, *y* becomes a e-by-1 or 1-by-e cell vector of n(i)-by-p matrices, where *e* denotes the number of experiments and n(i) the individual number of samples for each experiment.

u Real matrix containing the input signal in time-domain. For a system with *m* inputs and *n* samples, *u* is a n-by-m matrix. For data from multiple experiments, *u* becomes a e-by-1 or 1-by-e cell vector of n(i)-by-m matrices, where *e* denotes the number of experiments and n(i) the individual number of samples for each experiment. If *u* is not specified or an empty element `[]` is passed, *dat* becomes a time series dataset.

tsam Sampling time. If not specified, default value -1 (unspecified) is taken. For multi-experiment data, *tsam* becomes a e-by-1 or 1-by-e cell vector containing individual sampling times for each experiment. If a scalar *tsam* is provided, then all experiments have the same sampling time.

... Optional pairs of properties and values.

Outputs

dat iddata identification dataset.

Option Keys and Values

'expname' The name of the experiments in *dat*. Cell vector of length e containing strings. Default names are `{'exp1', 'exp2', ...}`

'y' Output signals. See 'Inputs' for details.

'outname' The name of the output channels in *dat*. Cell vector of length p containing strings. Default names are `{'y1', 'y2', ...}`

'outunit' The units of the output channels in *dat*. Cell vector of length p containing strings.

'u' Input signals. See 'Inputs' for details.

'inname' The name of the input channels in *dat*. Cell vector of length m containing strings. Default names are `{'u1', 'u2', ...}`

'inunit' The units of the input channels in *dat*. Cell vector of length m containing strings.

'tsam' Sampling time. See 'Inputs' for details.

'timeunit' The units of the sampling times in *dat*. Cell vector of length e containing strings.

'name' String containing the name of the dataset.

'notes' String or cell of string containing comments.

'userdata' Any data type.

16.2 @iddata/cat

dat = `cat` (*dim*, *dat1*, *dat2*, ...) [Function File]

Concatenate iddata sets along dimension *dim*.

Inputs

dim Dimension along which the concatenation takes place.

- *1* Concatenate samples. The samples are concatenated in the following way: `dat.y{e} = [dat1.y{e}; dat2.y{e}; ...] dat.u{e} = [dat1.u{e}; dat2.u{e}; ...]` where *e* denotes the experiment. The number of experiments, outputs and inputs must be equal for all datasets. Equivalent to `vertcat`.
- *2* Concatenate inputs and outputs. The outputs and inputs are concatenated in the following way: `dat.y{e} = [dat1.y{e}, dat2.y{e}, ...]` `dat.u{e} = [dat1.u{e}, dat2.u{e}, ...]` where *e* denotes the experiment. The number of experiments and samples must be equal for all datasets. Equivalent to `horzcat`.
- *3* Concatenate experiments. The experiments are concatenated in the following way: `dat.y = [dat1.y; dat2.y; ...] dat.u = [dat1.u; dat2.u; ...]` The number of outputs and inputs must be equal for all datasets. Equivalent to `merge`.

dat1, dat2, ...
iddata sets to be concatenated.

Outputs

dat Concatenated iddata set.

See also: horzcat, merge, vertcat.

16.3 @iddata/detrend

dat = `detrend` (*dat*) [Function File]
dat = `detrend` (*dat*, *ord*) [Function File]

Detrend outputs and inputs of dataset *dat* by removing the best fit of a polynomial of order *ord*. If *ord* is not specified, default value 0 is taken. This corresponds to removing a constant.

16.4 @iddata/diff

dat = `diff` (*dat*) [Function File]
dat = `diff` (*dat*, *k*) [Function File]

Return *k*-th difference of outputs and inputs of dataset *dat*. If *k* is not specified, default value 1 is taken.

16.5 @iddata/fft

dat = `fft` (*dat*) [Function File]
dat = `fft` (*dat*, *n*) [Function File]

Transform iddata objects from time to frequency domain using a Fast Fourier Transform (FFT) algorithm.

Inputs

dat iddata set containing signals in time-domain.

n Length of the FFT transformations. If *n* does not match the signal length, the signals in *dat* are shortened or padded with zeros. *n* is a vector with as many elements as there are experiments in *dat* or a scalar with a common length for all experiments. If not specified, the signal lengths are taken as default values.

Outputs

dat iddata identification dataset in frequency-domain. In order to preserve signal power and noise level, the FFTs are normalized by dividing each transform by the square root of the signal length. The frequency values are distributed equally from 0 to the Nyquist frequency. The Nyquist frequency is only included for even signal lengths.

16.6 @iddata/filter

`dat = filter (dat, sys)` [Function File]
`dat = filter (dat, b, a)` [Function File]

Filter output and input signals of dataset *dat*. The filter is specified either by LTI system *sys* or by transfer function polynomials *b* and *a* as described in the help text of Octave's built-in filter function. Type `help filter` for more information.

Inputs

dat iddata identification dataset containing signals in time-domain.

sys LTI object containing the discrete-time filter.

b Numerator polynomial of the discrete-time filter. Must be a row vector containing the coefficients of the polynomial in ascending powers of z^-1.

a Denominator polynomial of the discrete-time filter. Must be a row vector containing the coefficients of the polynomial in ascending powers of z^-1.

Outputs

dat iddata identification dataset with filtered output and input signals.

16.7 @iddata/get

`get (dat)` [Function File]
`value = get (dat, 'key')` [Function File]
`[val1, val2, ...] = get (dat, 'key1', 'key2', ...)` [Function File]

Access key values of iddata objects. Type `get(dat)` to display a list of available keys.

16.8 @iddata/ifft

`dat = ifft (dat)` [Function File]

Transform iddata objects from frequency to time domain.

Inputs

dat iddata set containing signals in frequency domain. The frequency values must be distributed equally from 0 to the Nyquist frequency. The Nyquist frequency is only included for even signal lengths.

Outputs

dat iddata identification dataset in time domain. In order to preserve signal power and noise level, the FFTs are normalized by multiplying each transform by the square root of the signal length.

16.9 @iddata/merge

dat = merge (*dat1*, *dat2*, ...) [Function File]

Concatenate experiments of iddata datasets. The experiments are concatenated in the following way: `dat.y = [dat1.y; dat2.y; ...]` `dat.u = [dat1.u; dat2.u; ...]` The number of outputs and inputs must be equal for all datasets.

16.10 @iddata/nkshift

dat = nkshift (*dat*, *nk*) [Function File]

dat = nkshift (*dat*, *nk*, *'append'*) [Function File]

Shift input channels of dataset *dat* according to integer *nk*. A positive value of *nk* means that the input channels are delayed *nk* samples. By default, both input and output signals are shortened by *nk* samples. If a third argument *'append'* is passed, the output signals are left untouched while *nk* zeros are appended to the (shortened) input signals such that the number of samples in *dat* remains constant.

16.11 @iddata/plot

plot (*dat*) [Function File]

plot (*dat*, *exp*) [Function File]

Plot signals of iddata identification datasets on the screen. The signals are plotted experiment-wise, either in time- or frequency-domain. For multi-experiment datasets, press any key to switch to the next experiment. If the plot of a single experiment should be saved by the `print` command, use `plot(dat,exp)`, where *exp* denotes the desired experiment.

16.12 @iddata/resample

dat = resample (*dat*, *p*, *q*) [Function File]

dat = resample (*dat*, *p*, *q*, *n*) [Function File]

dat = resample (*dat*, *p*, *q*, *h*) [Function File]

Change the sample rate of the output and input signals in dataset *dat* by a factor of `p/q`. This is performed using a polyphase algorithm. The anti-aliasing FIR filter can be specified as follows: Either by order *n* (scalar) with default value 0. The band edges are then chosen automatically. Or by impulse response *h* (vector). Requires the signal package to be installed.

Algorithm

Uses functions `fir1` and `resample` from the signal package.

References

[1] J. G. Proakis and D. G. Manolakis, Digital Signal Processing: Principles, Algorithms, and Applications, 4th ed., Prentice Hall, 2007. Chap. 6

[2] A. V. Oppenheim, R. W. Schafer and J. R. Buck, Discrete-time signal processing, Signal processing series, Prentice-Hall, 1999

16.13 @iddata/set

set (*dat*) [Function File]

set (*dat*, *'key'*, *value*, ...) [Function File]

dat = set (*dat*, *'key'*, *value*, ...) [Function File]

Set or modify keys of iddata objects. If no return argument *dat* is specified, the modified IDDATA object is stored in input argument *dat*. `set` can handle multiple keys in one call: `set (dat, 'key1', val1, 'key2', val2, 'key3', val3)`. `set (dat)` prints a list of the object's key names.

16.14 @iddata/size

nvec = size (*dat*) [Function File]
ndim = size (*dat*, *dim*) [Function File]
[*n*, *p*, *m*, *e*] = size (*dat*) [Function File]

Return dimensions of iddata set *dat*.

Inputs

dat iddata set.

dim If given a second argument, `size` will return the size of the corresponding dimension.

Outputs

nvec Row vector. The first element is the total number of samples (rows of dat.y and dat.u). The second element is the number of outputs (columns of dat.y) and the third element the number of inputs (columns of dat.u). The fourth element is the number of experiments.

ndim Scalar value. The size of the dimension *dim*.

n Row vector containing the number of samples of each experiment.

p Number of outputs.

m Number of inputs.

e Number of experiments.

17 System Identification

17.1 arx

[*sys*, *x0*] = arx (*dat*, *n*, ...) [Function File]
[*sys*, *x0*] = arx (*dat*, *n*, *opt*, ...) [Function File]
[*sys*, *x0*] = arx (*dat*, *opt*, ...) [Function File]
[*sys*, *x0*] = arx (*dat*, 'na', *na*, 'nb', *nb*) [Function File]

Estimate ARX model using QR factorization.

$$A(q)\,y(t) = B(q)\,u(t) \,+\, e(t)$$

Inputs

dat iddata identification dataset containing the measurements, i.e. time-domain signals.

n The desired order of the resulting model *sys*.

... Optional pairs of keys and values. `'key1', value1, 'key2', value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

sys Discrete-time transfer function model. If the second output argument *x0* is returned, *sys* becomes a state-space model.

x0 Initial state vector. If *dat* is a multi-experiment dataset, *x0* becomes a cell vector containing an initial state vector for each experiment.

Option Keys and Values

'na' Order of the polynomial A(q) and number of poles.

'nb' Order of the polynomial B(q)+1 and number of zeros+1. *nb* <= *na*.

'nk' Input-output delay specified as number of sampling instants. Scalar positive integer. This corresponds to a call to function `nkshift`, followed by padding the B polynomial with *nk* leading zeros.

Algorithm

Uses the formulae given in [1] on pages 318-319, 'Solving for the LS Estimate by QR Factorization'. For the initial conditions, SLICOT IB01CD is used by courtesy of NICONET e.V. (`http://www.slicot.org`)

References

[1] Ljung, L. (1999) *System Identification: Theory for the User: Second Edition*. Prentice Hall, New Jersey, USA.

17.2 moen4

[*sys*, *x0*, *info*] = moen4 (*dat*, ...) [Function File]
[*sys*, *x0*, *info*] = moen4 (*dat*, *n*, ...) [Function File]
[*sys*, *x0*, *info*] = moen4 (*dat*, *opt*, ...) [Function File]
[*sys*, *x0*, *info*] = moen4 (*dat*, *n*, *opt*, ...) [Function File]

Estimate state-space model using combined subspace method: MOESP algorithm for finding the matrices A and C, and N4SID algorithm for finding the matrices B and D. If no output arguments are given, the singular values are plotted on the screen in order to estimate the system order.

Inputs

dat iddata set containing the measurements, i.e. time-domain signals.

n The desired order of the resulting state-space system *sys*. If not specified, *n* is chosen automatically according to the singular values and tolerances.

... Optional pairs of keys and values. `'key1', value1, 'key2', value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

sys Discrete-time state-space model.

x0 Initial state vector. If *dat* is a multi-experiment dataset, *x0* becomes a cell vector containing an initial state vector for each experiment.

info Struct containing additional information.

- *info.K* Kalman gain matrix.
- *info.Q* State covariance matrix.
- *info.Ry* Output covariance matrix.
- *info.S* State-output cross-covariance matrix.
- *info.L* Noise variance matrix factor. LL'=Ry.

Option Keys and Values

'n' The desired order of the resulting state-space system *sys*. $s > n > 0$.

's' The number of block rows *s* in the input and output block Hankel matrices to be processed. $s > 0$. In the MOESP theory, *s* should be larger than *n*, the estimated dimension of state vector.

'alg', 'algorithm'
Specifies the algorithm for computing the triangular factor R, as follows:

- *'C'* Cholesky algorithm applied to the correlation matrix of the input-output data. Default method.
- *'F'* Fast QR algorithm.
- *'Q'* QR algorithm applied to the concatenated block Hankel matrices.

'tol' Absolute tolerance used for determining an estimate of the system order. If *tol* >= 0, the estimate is indicated by the index of the last singular value greater than or equal to *tol*. (Singular values less than *tol* are considered as zero.) When *tol* = 0, an internally computed default value, *tol* = *s***eps**SV(1), is used, where SV(1) is the maximal singular value, and *eps* is the relative machine precision. When *tol* < 0, the estimate is indicated by the index of the singular value that has the largest logarithmic gap to its successor. Default value is 0.

'rcond' The tolerance to be used for estimating the rank of matrices. If the user sets *rcond* > 0, the given value of *rcond* is used as a lower bound for the reciprocal condition number; an m-by-n matrix whose estimated condition number is less than 1/*rcond* is considered to be of full rank. If the user sets *rcond* <= 0, then an implicitly computed, default tolerance, defined by *rcond* = m*n**eps*, is used instead, where *eps* is the relative machine precision. Default value is 0.

'confirm' Specifies whether or not the user's confirmation of the system order estimate is desired, as follows:

true User's confirmation.

false No confirmation. Default value.

'noiseinput'

The desired type of noise input channels.

'n' No error inputs. Default value.

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k + Du_k$$

'e' Return *sys* as a (p-by-m+p) state-space model with both measured input channels u and noise channels e with covariance matrix *Ry*.

$$x_{k+1} = Ax_k + Bu_k + Ke_k$$

$$y_k = Cx_k + Du_k + e_k$$

'v' Return *sys* as a (p-by-m+p) state-space model with both measured input channels u and white noise channels v with identity covariance matrix.

$$x_{k+1} = Ax_k + Bu_k + KLv_k$$

$$y_k = Cx_k + Du_k + Lv_k$$

$$e = Lv, \; LL^T = R_y$$

'k' Return *sys* as a Kalman predictor for simulation.

$$\widehat{x}_{k+1} = A\widehat{x}_k + Bu_k + K(y_k - \widehat{y}_k)$$

$$\widehat{y}_k = C\widehat{x}_k + Du_k$$

$$\widehat{x}_{k+1} = (A - KC)\widehat{x}_k + (B - KD)u_k + Ky_k$$

$$\widehat{y}_k = C\widehat{x}_k + Du_k + 0y_k$$

Algorithm

Uses SLICOT IB01AD, IB01BD and IB01CD by courtesy of NICONET e.V. (`http://www.slicot.org`)

17.3 moesp

`[sys, x0, info] = moesp (dat, ...)` [Function File]
`[sys, x0, info] = moesp (dat, n, ...)` [Function File]
`[sys, x0, info] = moesp (dat, opt, ...)` [Function File]
`[sys, x0, info] = moesp (dat, n, opt, ...)` [Function File]

Estimate state-space model using MOESP algorithm. MOESP: Multivariable Output Error State sPace. If no output arguments are given, the singular values are plotted on the screen in order to estimate the system order.

Inputs

dat iddata set containing the measurements, i.e. time-domain signals.

n The desired order of the resulting state-space system *sys*. If not specified, *n* is chosen automatically according to the singular values and tolerances.

... Optional pairs of keys and values. `'key1', value1, 'key2', value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

sys Discrete-time state-space model.

x0 Initial state vector. If *dat* is a multi-experiment dataset, *x0* becomes a cell vector containing an initial state vector for each experiment.

info Struct containing additional information.

- *info.K* Kalman gain matrix.
- *info.Q* State covariance matrix.
- *info.Ry* Output covariance matrix.
- *info.S* State-output cross-covariance matrix.
- *info.L* Noise variance matrix factor. LL'=Ry.

Option Keys and Values

'n' The desired order of the resulting state-space system *sys*. $s > n > 0$.

's' The number of block rows *s* in the input and output block Hankel matrices to be processed. $s > 0$. In the MOESP theory, *s* should be larger than *n*, the estimated dimension of state vector.

'alg', 'algorithm'
Specifies the algorithm for computing the triangular factor R, as follows:

- *'C'* Cholesky algorithm applied to the correlation matrix of the input-output data. Default method.
- *'F'* Fast QR algorithm.
- *'Q'* QR algorithm applied to the concatenated block Hankel matrices.

'tol' Absolute tolerance used for determining an estimate of the system order. If *tol* >= 0, the estimate is indicated by the index of the last singular value greater than or equal to *tol*. (Singular values less than *tol* are considered as zero.) When *tol* = 0, an internally computed default value, *tol* = *s***eps**SV(1), is used, where SV(1) is the maximal singular value, and *eps* is the relative machine precision. When *tol* < 0, the estimate is indicated by the index of the singular value that has the largest logarithmic gap to its successor. Default value is 0.

'rcond' The tolerance to be used for estimating the rank of matrices. If the user sets *rcond* > 0, the given value of *rcond* is used as a lower bound for the reciprocal condition number; an m-by-n matrix whose estimated condition number is less than 1/*rcond* is considered to be of full rank. If the user sets *rcond* <= 0, then an implicitly computed, default tolerance, defined by *rcond* = m*n**eps*, is used instead, where *eps* is the relative machine precision. Default value is 0.

'confirm' Specifies whether or not the user's confirmation of the system order estimate is desired, as follows:

true User's confirmation.

false No confirmation. Default value.

'noiseinput'

The desired type of noise input channels.

'n' No error inputs. Default value.

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k + Du_k$$

'e' Return *sys* as a (p-by-m+p) state-space model with both measured input channels u and noise channels e with covariance matrix *Ry*.

$$x_{k+1} = Ax_k + Bu_k + Ke_k$$

$$y_k = Cx_k + Du_k + e_k$$

'v' Return *sys* as a (p-by-m+p) state-space model with both measured input channels u and white noise channels v with identity covariance matrix.

$$x_{k+1} = Ax_k + Bu_k + KLv_k$$

$$y_k = Cx_k + Du_k + Lv_k$$

$$e = Lv, \ LL^T = R_y$$

'k' Return *sys* as a Kalman predictor for simulation.

$$\widehat{x}_{k+1} = A\widehat{x}_k + Bu_k + K(y_k - \widehat{y}_k)$$

$$\widehat{y}_k = C\widehat{x}_k + Du_k$$

$$\widehat{x}_{k+1} = (A - KC)\widehat{x}_k + (B - KD)u_k + Ky_k$$

$$\widehat{y}_k = C\widehat{x}_k + Du_k + 0y_k$$

Algorithm

Uses SLICOT IB01AD, IB01BD and IB01CD by courtesy of NICONET e.V. (`http://www.slicot.org`)

17.4 n4sid

`[sys, x0, info] = n4sid (dat, ...)` [Function File]
`[sys, x0, info] = n4sid (dat, n, ...)` [Function File]
`[sys, x0, info] = n4sid (dat, opt, ...)` [Function File]
`[sys, x0, info] = n4sid (dat, n, opt, ...)` [Function File]

Estimate state-space model using N4SID algorithm. N4SID: Numerical algorithm for Subspace State Space System IDentification. If no output arguments are given, the singular values are plotted on the screen in order to estimate the system order.

Inputs

dat iddata set containing the measurements, i.e. time-domain signals.

n The desired order of the resulting state-space system *sys*. If not specified, *n* is chosen automatically according to the singular values and tolerances.

... Optional pairs of keys and values. `'key1', value1, 'key2', value2`.

opt Optional struct with keys as field names. Struct *opt* can be created directly or by function `options`. `opt.key1 = value1, opt.key2 = value2`.

Outputs

sys Discrete-time state-space model.

x0 Initial state vector. If *dat* is a multi-experiment dataset, *x0* becomes a cell vector containing an initial state vector for each experiment.

info Struct containing additional information.

info.K Kalman gain matrix.

info.Q State covariance matrix.

info.Ry Output covariance matrix.

info.S State-output cross-covariance matrix.

info.L Noise variance matrix factor. LL'=Ry.

Option Keys and Values

'n' The desired order of the resulting state-space system *sys*. $s > n > 0$.

's' The number of block rows *s* in the input and output block Hankel matrices to be processed. $s > 0$. In the MOESP theory, *s* should be larger than *n*, the estimated dimension of state vector.

'alg', 'algorithm'
Specifies the algorithm for computing the triangular factor R, as follows:

'C' Cholesky algorithm applied to the correlation matrix of the input-output data. Default method.

'F' Fast QR algorithm.

'Q' QR algorithm applied to the concatenated block Hankel matrices.

'tol' Absolute tolerance used for determining an estimate of the system order. If *tol* >= 0, the estimate is indicated by the index of the last singular value greater than or equal to *tol*. (Singular values less than *tol* are considered as zero.) When *tol* = 0, an internally computed default value, *tol* = *s***eps**SV(1), is used, where SV(1) is the maximal singular value, and *eps* is the relative machine precision. When *tol* < 0, the estimate is indicated by the index of the singular value that has the largest logarithmic gap to its successor. Default value is 0.

'rcond' The tolerance to be used for estimating the rank of matrices. If the user sets *rcond* > 0, the given value of *rcond* is used as a lower bound for the reciprocal condition number; an m-by-n matrix whose estimated condition number is less than 1/*rcond* is considered to be of full rank. If the user sets *rcond* <= 0, then an implicitly computed, default tolerance, defined by *rcond* = m*n**eps*, is used instead, where *eps* is the relative machine precision. Default value is 0.

'confirm' Specifies whether or not the user's confirmation of the system order estimate is desired, as follows:

true User's confirmation.

false No confirmation. Default value.

'noiseinput'
The desired type of noise input channels.

'n' No error inputs. Default value.

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k + Du_k$$

'e' Return *sys* as a (p-by-m+p) state-space model with both measured input channels u and noise channels e with covariance matrix Ry.

$$x_{k+1} = Ax_k + Bu_k + Ke_k$$

$$y_k = Cx_k + Du_k + e_k$$

'v' Return *sys* as a (p-by-m+p) state-space model with both measured input channels u and white noise channels v with identity covariance matrix.

$$x_{k+1} = Ax_k + Bu_k + KLv_k$$

$$y_k = Cx_k + Du_k + Lv_k$$

$$e = Lv, \ LL^T = R_y$$

'k' Return *sys* as a Kalman predictor for simulation.

$$\widehat{x}_{k+1} = A\widehat{x}_k + Bu_k + K(y_k - \widehat{y}_k)$$

$$\widehat{y}_k = C\widehat{x}_k + Du_k$$

$$\widehat{x}_{k+1} = (A - KC)\widehat{x}_k + (B - KD)u_k + Ky_k$$

$$\widehat{y}_k = C\widehat{x}_k + Du_k + 0y_k$$

Algorithm
Uses SLICOT IB01AD, IB01BD and IB01CD by courtesy of NICONET e.V. (`http://www.slicot.org`)

18 Overloaded LTI Operators

18.1 @lti/ctranspose

Conjugate transpose or pertransposition of LTI objects. Used by Octave for "sys'". For a transfer-function matrix G, G' denotes the conjugate of G given by G.'(-s) for a continuous-time system or G.'(1/z) for a discrete-time system. The frequency response of the pertransposition of G is the Hermitian (conjugate) transpose of G(jw), i.e. freqresp (G', w) = freqresp (G, w)'. **WARNING:** Do **NOT** use this for dual problems, use the transpose "sys.'" (note the dot) instead.

18.2 @lti/end

End indexing for LTI objects. Used by Octave for "sys(1:end, end-1)".

18.3 @lti/horzcat

Horizontal concatenation of LTI objects. If necessary, object conversion is done by sys_group. Used by Octave for "[sys1, sys2]".

18.4 @lti/inv

Inversion of LTI objects.

18.5 @lti/minus

Binary subtraction of LTI objects. If necessary, object conversion is done by sys_group. Used by Octave for "sys1 - sys2".

18.6 @lti/mldivide

Matrix left division of LTI objects. If necessary, object conversion is done by sys_group in mtimes. Used by Octave for "sys1 \ sys2".

18.7 @lti/mpower

Matrix power of LTI objects. The exponent must be an integer. Used by Octave for "sys^int".

18.8 @lti/mrdivide

Matrix right division of LTI objects. If necessary, object conversion is done by sys_group in mtimes. Used by Octave for "sys1 / sys2".

18.9 @lti/mtimes

Matrix multiplication of LTI objects. If necessary, object conversion is done by sys_group. Used by Octave for "sys1 * sys2".

18.10 @lti/plus

Binary addition of LTI objects. If necessary, object conversion is done by sys_group. Used by Octave for "sys1 + sys2". Operation is also known as "parallel connection".

18.11 @lti/repmat

`rsys = repmat (sys, m, n)` [Function File]
`rsys = repmat (sys, [m, n])` [Function File]
`rsys = repmat (sys, m)` [Function File]

Form a block transfer matrix of *sys* with *m* copies vertically and *n* copies horizontally. If *n* is not specified, it is set to *m*. `repmat (sys, 2, 3)` is equivalent to `[sys, sys, sys; sys, sys, sys]`.

18.12 @lti/subsasgn

Subscripted assignment for LTI objects. Used by Octave for "sys.property = value".

18.13 @lti/subsref

Subscripted reference for LTI objects. Used by Octave for "sys = sys(2:4, :)" or "val = sys.prop".

18.14 @lti/times

Hadamard/Schur product of transfer function matrices. Also known as element-wise multiplication. Used by Octave for "sys1 .* sys2".

Example

```
# Compute Relative-Gain Array
G = tf (Boeing707)
RGA = G .* inv (G).'
# Gain at 0 rad/s
RGA(0)
```

18.15 @lti/transpose

Transpose of LTI objects. Used by Octave for "sys.'". Useful for dual problems, i.e. controllability and observability or designing estimator gains with `lqr` and `place`.

18.16 @lti/uminus

Unary minus of LTI object. Used by Octave for "-sys".

18.17 @lti/uplus

Unary plus of LTI object. Used by Octave for "+sys".

18.18 @lti/vertcat

Vertical concatenation of LTI objects. If necessary, object conversion is done by sys_group. Used by Octave for "[sys1; sys2]".

19 Overloaded IDDATA Operators

19.1 @iddata/end

End indexing for IDDATA objects. Used by Octave for "dat(1:end)".

19.2 @iddata/horzcat

dat = `horzcat` (*dat1*, *dat2*, ...) [Function File]

Horizontal concatenation of iddata datasets. The outputs and inputs are concatenated in the following way: `dat.y{e} = [dat1.y{e}, dat2.y{e}, ...] dat.u{e} = [dat1.u{e}, dat2.u{e}, ...]` where *e* denotes the experiment. The number of experiments and samples must be equal for all datasets.

19.3 @iddata/subsasgn

Subscripted assignment for iddata objects. Used by Octave for "dat.property = value".

19.4 @iddata/subsref

Subscripted reference for iddata objects. Used by Octave for "dat = dat(2:4, :)" or "val = dat.prop".

19.5 @iddata/vertcat

dat = `vertcat` (*dat1*, *dat2*, ...) [Function File]

Vertical concatenation of iddata datasets. The samples are concatenated in the following way: `dat.y{e} = [dat1.y{e}; dat2.y{e}; ...] dat.u{e} = [dat1.u{e}; dat2.u{e}; ...]` where *e* denotes the experiment. The number of experiments, outputs and inputs must be equal for all datasets.

20 Miscellaneous

20.1 db2mag

mag = db2mag (*db*) [Function File]

Convert Decibels (dB) to Magnitude.

Inputs

db Decibel (dB) value(s). Both real-valued scalars and matrices are accepted.

Outputs

mag Magnitude value(s).

See also: mag2db.

20.2 mag2db

db = mag2db (*mag*) [Function File]

Convert Magnitude to Decibels (dB).

Inputs

mag Magnitude value(s). Both real-valued scalars and matrices are accepted.

Outputs

db Decibel (dB) value(s).

See also: db2mag.

20.3 options

opt = options (*'key1'*, *value1*, *'key2'*, *value2*, ...) [Function File]

Create options struct *opt* from a number of key and value pairs. For use with order reduction and system identification functions. Option structs are a way to avoid typing the same key and value pairs over and over again.

Inputs

key, property
The name of the property.

value The value of the property.

Outputs

opt Struct with fields for each key.

Example

```
octave:1> opt = options ("method", "spa", "tol", 1e-6)                    =
opt =

  scalar structure containing the fields:

    method = spa
    tol =  1.0000e-06
```

```
octave:2> save filename opt
octave:3> # save the struct 'opt' to file 'filename' for later use
octave:4> load filename
octave:5> # load struct 'opt' from file 'filename'
```

20.4 pid

C = pid (*Kp*) [Function File]
C = pid (*Kp*, *Ki*) [Function File]
C = pid (*Kp*, *Ki*, *Kd*) [Function File]
C = pid (*Kp*, *Ki*, *Kd*, *Tf*) [Function File]

Return the transfer function *C* of the PID controller in parallel form with first-order roll-off.

```
             Ki      Kd s
C(s) = Kp + ---- + --------
             s     Tf s + 1
```

20.5 pidstd

C = pidstd (*Kp*) [Function File]
C = pidstd (*Kp*, *Ti*) [Function File]
C = pidstd (*Kp*, *Ti*, *Td*) [Function File]
C = pidstd (*Kp*, *Ti*, *Td*, *N*) [Function File]

Return the transfer function *C* of the PID controller in standard form with first-order roll-off.

```
                 1        Td s
C(s) = Kp (1 + ---- + ----------)
               Ti s   Td/N s + 1
```

20.6 repsys

rsys = repsys (*sys*, *m*, *n*) [Function File]
rsys = repsys (*sys*, [*m*, *n*]) [Function File]
rsys = repsys (*sys*, *m*) [Function File]

Form a block transfer matrix of *sys* with *m* copies vertically and *n* copies horizontally. If *n* is not specified, it is set to *m*. `repsys (sys, 2, 3)` is equivalent to `[sys, sys, sys; sys, sys, sys]`.

20.7 strseq

strvec = strseq (*str*, *idx*) [Function File]

Return a cell vector of indexed strings by appending the indices *idx* to the string *str*.

```
strseq ("x", 1:3) = {"x1"; "x2"; "x3"}
strseq ("u", [1, 2, 5]) = {"u1"; "u2"; "u5"}
```

20.8 test_control

test_control [Script File]

Execute all available tests at once. The Octave control package is based on the SLICOT (`http://www.slicot.org`) library. SLICOT needs BLAS and LAPACK libraries which are also prerequisites for Octave itself. In case of failing tests, it is highly recommended to use Netlib's reference BLAS (`http://www.netlib.org/blas/`) and LAPACK (`http://www.netlib.org/lapack/`) for building Octave. Using ATLAS may lead to sign changes in some entries of the state-space matrices. In general, these sign changes are not 'wrong' and can be regarded as the result of state transformations. Such state transformations (but not input/output transformations) have no influence on the input-output behaviour of the system. For better numerics, the control package uses such transformations by default when calculating the frequency responses and a few other things. However, arguments like the Hankel singular Values (HSV) must not change. Differing HSVs and failing algorithms are known for using Framework Accelerate from Mac OS X 10.7.

20.9 thiran

`sys = thiran (tau, tsam)` [Function File]

Approximation of continuous-time delay using a discrete-time allpass Thiran filter.

Thiran filters can approximate continuous-time delays that are non-integer multiples of the sampling time (fractional delays). This approximation gives a better matching of the phase shift between the continuous- and the discrete-time system. If there is no fractional part in the delay, then the standard discrete-time delay representation is used.

Inputs

tau A continuous-time delay, given in time units (seconds).

tsam The sampling time of the resulting Thiran filter.

Outputs

sys Transfer function model of the resulting filter. The order of the filter is determined automatically.

Example

```
octave:1> sys = thiran (1.33, 0.5)                                    =

Transfer function 'sys' from input 'u1' to output ...

      0.003859 z^3 - 0.03947 z^2 + 0.2787 z + 1
 y1:  -----------------------------------------
       z^3 + 0.2787 z^2 - 0.03947 z + 0.003859

Sampling time: 0.5 s
Discrete-time model.

octave:2> sys = thiran (1, 0.5)                                       =

Transfer function 'sys' from input 'u1' to output ...

       1
 y1:  ---
      z^2

Sampling time: 0.5 s
Discrete-time model.
```

See also: absorbdelay, pade.

20.10 BMWengine

sys = BMWengine () [Function File]
sys = BMWengine ("*scaled*") [Function File]
sys = BMWengine ("*unscaled*") [Function File]

Model of the BMW 4-cylinder engine at ETH Zurich's control laboratory.

```
OPERATING POINT                                                    =
Drosselklappenstellung     alpha_DK = 10.3 Grad
Saugrohrdruck              p_s = 0.48 bar
Motordrehzahl              n = 860 U/min
Lambda-Messwert            lambda = 1.000
Relativer Wandfilminhalt   nu = 1


INPUTS                                                             =
U_1 Sollsignal Drosselklappenstellung   [Grad]
U_2 Relative Einspritzmenge             [-]
U_3 Zuendzeitpunkt                      [Grad KW]
M_L Lastdrehmoment                      [Nm]


STATES                                                             =
X_1 Drosselklappenstellung     [Grad]
X_2 Saugrohrdruck              [bar]
X_3 Motordrehzahl              [U/min]
X_4 Messwert Lamba-Sonde       [-]
X_5 Relativer Wandfilminhalt   [-]


OUTPUTS                                                            =
Y_1 Motordrehzahl              [U/min]
Y_2 Messwert Lambda-Sonde      [-]


SCALING                                                            =
U_1N, X_1N   1 Grad
U_2N, X_4N, X_5N, Y_2N   0.05
U_3N   1.6 Grad KW
X_2N   0.05 bar
X_3N, Y_1N   200 U/min
```

20.11 Boeing707

sys = Boeing707 () [Function File]

Creates a linearized state-space model of a Boeing 707-321 aircraft at v=80 m/s ($M = 0.26$, $G_{a0} = -3°$, $\alpha_0 = 4°$, $\kappa = 50°$).

System inputs: (1) thrust and (2) elevator angle.

System outputs: (1) airspeed and (2) pitch angle.

Reference: R. Brockhaus: *Flugregelung* (Flight Control), Springer, 1994.

20.12 WestlandLynx

sys = WestlandLynx () [Function File]

Model of the Westland Lynx Helicopter about hover.

```
INPUTS                                                              =
main rotor collective
longitudinal cyclic
lateral cyclic
tail rotor collective

STATES                                                              =
pitch attitude           theta       [rad]
roll attitude            phi         [rad]
roll rate (body-axis)    p           [rad/s]
pitch rate (body-axis)   q           [rad/s]
yaw rate                 xi          [rad/s]
forward velocity         v_x         [ft/s]
lateral velocity         v_y         [ft/s]
vertical velocity        v_z         [ft/s]

OUTPUTS                                                             =
heave velocity           H_dot       [ft/s]
pitch attitude           theta       [rad]
roll attitude            phi         [rad]
heading rate             psi_dot     [rad/s]
roll rate                p           [rad/s]
pitch rate               q           [rad/s]
```

References

[1] Skogestad, S. and Postlethwaite I. (2005) *Multivariable Feedback Control: Analysis and Design: Second Edition.* Wiley. http://www.nt.ntnu.no/users/skoge/book/2nd_edition/matlab_m/matfiles.html

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