Calc II Notes

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Abstract

This will primarily consist of formulas necessary for calculations

1 Derivatives

Instantaneous rate of change at a point

1.1 Limit Definition of Derivatives

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

1.2 Basic Derivatives

| Name | Derivative | Result |
|--------------------|---|--|
| Constant | $\frac{d}{dx}[C]$ | 0 |
| Power | $\frac{d}{dx}[u^n]$ | $n * u^{n-1} * u\prime$ |
| Exponential | $\frac{d}{dx}[a^u]$ | $a^u * u \prime * ln(a)$ |
| Constant Multiply | $\frac{d}{dx}[C*f(x)]$ | $C * \frac{d}{dx}[f(x)]$ |
| Multiply Functions | $\frac{d}{dx}[f(x)g(x)]$ | f'(x)g(x) + g'(x)f(x) |
| Divide Functions | $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right]$ | $\frac{f\prime(x)g(x)-g\prime(x)f(x)}{g(x)^2}$ |
| Chain Rule | $\frac{d}{dx}[f(g(u))]$ | f'(g(u)) * g'(u) * u' |
| Logarithm | $rac{d}{dx}[log_a u]$ | $\frac{u\prime}{uln(a)}$ |

1.3 Trigonometric Derivatives

| Name | Derivative | Result |
|-----------|------------------------|-----------------------|
| Sine | $\frac{d}{dx}[sin(u)]$ | $cos(u)u\prime$ |
| Cosine | $\frac{d}{dx}[cos(u)]$ | $-sin(u)u\prime$ |
| Tangent | $\frac{d}{dx}[tan(u)]$ | $sec^2(u)u\prime$ |
| Cotangent | $\frac{d}{dx}[cot(u)]$ | $-csc^2(u)u\prime$ |
| Secant | $\frac{d}{dx}[sec(u)]$ | $sec(u)tan(u)u\prime$ |
| Cosecant | $\frac{d}{dx}[csc(u)]$ | -csc(u)cot(u)u |

1.4 Inverse Trigonometric Derivatives

For any "co-" version of the trig function multiply the result by -1

| Name | Derivative | Result |
|-----------------|---------------------------|-------------------------------------|
| Inverse Sine | $\frac{d}{dx}[sin^{-1}u]$ | $\frac{u'}{\sqrt{1-u^2}}$ |
| Inverse Tangent | $\frac{d}{dx}[tan^{-1}u]$ | $\frac{u\prime}{1+u^2}$ |
| Inverse Secant | $\frac{d}{dx}[sec^{-1}u]$ | $\frac{u\prime}{\&u\&\sqrt{u^2-1}}$ |

1.5 Separable Differential Equation

If
$$\frac{dy}{dx} = f(y) * g(x)$$
 than $\frac{1}{f(y)} dy = g(x) dx$
If $\frac{dy}{dt} = ky$ where k is a constant than $|y| = Ce^{kt}$
If $f(x) = x + y$ then $f'(x) = 1 + \frac{dy}{dx}$

2 Integrals

Area under the curve

2.1 Basic Integrals

Note that $F\prime(x)=f(x)$

| Name | Integral | Result |
|---------------------------------|------------------------------------|----------------------------|
| Fundamental theorem of Calculus | $\int_a^b f(x)$ | F(b) - F(a) |
| Simple Function | $\int f(x)$ | F(x) + C |
| Power Rule | $\int [x^n]dx$ | $\frac{x^{n+1}}{n+1} + C$ |
| Reciprocal Function | $\int \left[\frac{1}{x}\right] dx$ | ln x + C |
| Exponential Function | $\int [a^u]dx$ | $\frac{a^u}{ln(a)u\prime}$ |

2.2 Trigometric Integrals

| Name | Integral | Result |
|------------|---------------------|-----------|
| Sine | $\int [sinx]dx$ | -cosx + C |
| Cosine | $\int [cosx]dx$ | sinx + C |
| Secant | $\int [sec^2x]dx$ | tanx + C |
| Cosecant | $\int [csc^2x]dx$ | -cotx + C |
| Sectan | $\int [secxtanx]dx$ | secx + C |
| Coseccotan | $\int [cscxcotx]dx$ | -cscx + C |

2.3 Volumes of Revolution

Rotating a function(s) around an axis to find the volume of the 3d resulting object

| Name | Formula |
|---------------|--|
| Total Volume | $\int_a^b A(x)$ |
| Disk Method | $\int_a^b \pi f(x)^2 dx$ s |
| Shell method | $\int_{a}^{b} 2\pi x f(x) dx$ |
| Shell between | $\int_{a}^{b} 2\pi x (f(x) - g(x)) dx$ |
| Washer method | $\int_a^b \pi(f(x)^2 - g(x)^2) dx$ |