

Calc II Notes

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Abstract

This will primarily consist of formulas necessary for calculations

1 Derivatives

Instantaneous rate of change at a point

1.1 Limit Definition of Derivatives

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$
$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

1.2 Basic Derivatives

Name	Derivative	Result
Constant	$\frac{d}{dx}[C]$	0
Power	$\frac{d}{dx}[u^n]$	$n * u^{n-1} * u'$
Exponential	$\frac{d}{dx}[a^u]$	$a^u * u' * \ln(a)$
Constant Multiply	$\frac{d}{dx}[C * f(x)]$	$C * \frac{d}{dx}[f(x)]$
Multiply Functions	$\frac{d}{dx}[f(x)g(x)]$	$f'(x)g(x) + g'(x)f(x)$
Divide Functions	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$	$\frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$
Chain Rule	$\frac{d}{dx}[f(g(u))]$	$f'(g(u)) * g'(u) * u'$
Logarithm	$\frac{d}{dx}[\log_a u]$	$\frac{u'}{u \ln(a)}$

1.3 Trigonometric Derivatives

Name	Derivative	Result
Sine	$\frac{d}{dx}[\sin(u)]$	$\cos(u)u'$
Cosine	$\frac{d}{dx}[\cos(u)]$	$-\sin(u)u'$
Tangent	$\frac{d}{dx}[\tan(u)]$	$\sec^2(u)u'$
Cotangent	$\frac{d}{dx}[\cot(u)]$	$-\csc^2(u)u'$
Secant	$\frac{d}{dx}[\sec(u)]$	$\sec(u)\tan(u)u'$
Cosecant	$\frac{d}{dx}[\csc(u)]$	$-\csc(u)\cot(u)u'$

1.4 Inverse Trigonometric Derivatives

For any “co-” version of the trig function multiply the result by -1

Name	Derivative	Result
Inverse Sine	$\frac{d}{dx}[\sin^{-1}u]$	$\frac{u'}{\sqrt{1-u^2}}$
Inverse Tangent	$\frac{d}{dx}[\tan^{-1}u]$	$\frac{u'}{1+u^2}$
Inverse Secant	$\frac{d}{dx}[\sec^{-1}u]$	$\frac{u'}{u\sqrt{u^2-1}}$

1.5 Separable Differential Equation

If $\frac{dy}{dx} = f(y) * g(x)$ than $\frac{1}{f(y)}dy = g(x)dx$

If $\frac{dy}{dt} = ky$ where k is a constant than $|y| = Ce^{kt}$

If $f(x) = x + y$ then $f'(x) = 1 + \frac{dy}{dx}$

2 Integrals

Area under the curve

2.1 Basic Integrals

Note that $F'(x) = f(x)$

Name	Integral	Result
Fundamental theorem of Calculus	$\int_a^b f(x)$	$F(b) - F(a)$
Simple Function	$\int f(x)$	$F(x) + C$
Power Rule	$\int [x^n] dx$	$\frac{x^{n+1}}{n+1} + C$
Reciprocal Function	$\int [\frac{1}{x}] dx$	$\ln x + C$
Exponential Function	$\int [a^u] dx$	$\frac{a^u}{\ln(a)u'}$

2.2 Trigonometric Integrals

Name	Integral	Result
Sine	$\int [\sin x] dx$	$-\cos x + C$
Cosine	$\int [\cos x] dx$	$\sin x + C$
Secant	$\int [\sec^2 x] dx$	$\tan x + C$
Cosecant	$\int [\csc^2 x] dx$	$-\cot x + C$
Sectan	$\int [\sec x \tan x] dx$	$\sec x + C$
Coseccotan	$\int [\csc x \cot x] dx$	$-\csc x + C$

3 Applications of Integrals

Applying integrals to solve problems

3.1 Volumes of Revolution

Rotating a function(s) around an axis to find the volume of the 3d resulting object

Name	Formula
Total Volume	$\int_a^b A(x)$
Disk Method	$\int_a^b \pi f(x)^2 dx$
Shell method	$\int_a^b 2\pi x f(x) dx$
Shell between	$\int_a^b 2\pi x (f(x) - g(x)) dx$
Washer method	$\int_a^b \pi (f(x)^2 - g(x)^2) dx$

3.2 Work

F is force, a and b are points along an axis with b being further than a , and W is work

Name	Formula	Notes
Constant Force Work	$F(b - a)$	Requires Force to be Constant
Varied Force Work	$\int_a^b F(x)dx$	If direction of force and motion are same than $F(x) \geq 0$ else $F(x) \leq 0$
Hooke's Law	$F(x) = kx$	k is the spring constant and x is the spring constant
Pump Liquid	$\int_a^b d * h(y)A(y)dy$	d is density, $h(y)$ is distance the layer is being lifted, $A(y)$ is the area of the layer