Calc II Notes

SuperExago and Tepoys

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Abstract

This will primarily consist of formulas necessary for calculations

1 Derivatives

Instantaneous rate of change at a point

1.1 Limit Definition of Derivatives

$f\prime(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
$f\prime(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

1.2 Basic Derivatives

Name	Derivative	Result
Constant	$\frac{d}{dx}[C]$	0
Power	$\frac{d}{dx}[u^n]$	$n * u^{n-1} * u\prime$
Exponential	$\frac{d}{dx}[a^u]$	$a^u * u\prime * ln(a)$
Constant Multiply	$\frac{d}{dx}[C * f(x)]$	$C * \frac{d}{dx}[f(x)]$
Multiply Functions	$\frac{d}{dx}[f(x)g(x)]$	$f\prime(x)g(x) + g\prime(x)f(x)$
Divide Functions	$\frac{d}{dx}[\frac{f(x)}{g(x)}]$	$\frac{f\prime(x)g(x)-g\prime(x)f(x)}{g(x)^2}$
Chain Rule	$\frac{d}{dx}[f(g(u))]$	$f\prime(g(u)) * g\prime(u) * u\prime$
Logarithm	$\frac{d}{dx}[log_a u]$	$\frac{u\prime}{uln(a)}$

1.3 Trigonometric Derivatives

Name	Derivative	Result
Sine	$\frac{d}{dx}[sin(u)]$	$cos(u)u\prime$
Cosine	$\frac{d}{dx}[cos(u)]$	$-sin(u)u\prime$
Tangent	$\frac{d}{dx}[tan(u)]$	$sec^2(u)u\prime$
Cotangent	$\frac{d}{dx}[cot(u)]$	$-csc^2(u)u\prime$
Secant	$\frac{d}{dx}[sec(u)]$	$sec(u)tan(u)u\prime$
Cosecant	$\frac{d}{dx}[csc(u)]$	$-csc(u)cot(u)u\prime$

1.4 Inverse Trigonometric Derivatives

For any “co-” version of the trig function multiply the result by -1

Name	Derivative	Result
Inverse Sine	$\frac{d}{dx}[sin^{-1}u]$	$\frac{u\prime}{\sqrt{1-u^2}}$
Inverse Tangent	$\frac{d}{dx}[tan^{-1}u]$	$\frac{u\prime}{1+u^2}$
Inverse Secant	$\frac{d}{dx}[sec^{-1}u]$	$\frac{u\prime}{u\sqrt{u^2-1}}$

1.5 Separable Differential Equation

If $\frac{dy}{dx} = f(y) * g(x)$ than $\frac{1}{f(y)}dy = g(x)dx$
If $\frac{dy}{dt} = ky$ where k is a constant than $|y| = Ce^{kt}$
If $f(x) = x + y$ then $f\prime(x) = 1 + \frac{dy}{dx}$

2 Integrals

Area under the curve (also known as anti-derivative)

2.1 Basic Integrals

Note that $F\prime(x) = f(x)$

Name	Integral	Result
Fundamental theorem of Calculus	$\int_a^b f(x)$	$F(b) - F(a)$
Simple Function	$\int f(x)$	$F(x) + C$
Power Rule	$\int [x^n] dx$	$\frac{x^{n+1}}{n+1} + C$
Reciprocal Function	$\int [\frac{1}{x}] dx$	$ln\|x\| + C$
Exponential Function	$\int [a^u] dx$	$\frac{a^u}{ln(a)u\prime}$

2.2 Trigometric Integrals

Name	Integral	Result
Sine	$\int[sinx]dx$	$-cosx + C$
Cosine	$\int[cosx]dx$	$sinx + C$
Secant	$\int[sec^2x]dx$	$tanx + C$
Cosecant	$\int[csc^2x]dx$	$-cotx + C$
Sectan	$\int[secxtanx]dx$	$secx + C$
Coseccotan	$\int[cscxcotx]dx$	$-cscx + C$

3 Applications of Integrals

Applying integrals to solve problems

3.1 Volumes of Revolution

Rotating a function(s) around an axis to find the volume of the 3d resulting object

Name	Formula
Total Volume	$\int_a^b A(x)$
Disk Method	$\int_a^b \pi f(x)^2 dx$s
Shell method	$\int_a^b 2\pi x f(x) dx$
Shell between	$\int_a^b 2\pi x(f(x) - g(x)) dx$
Washer method	$\int_a^b \pi(f(x)^2 - g(x)^2) dx$

3.2 Work

F is force which equals mass times acceleration, a and b are points along an axis with b being further than a, and W is work

Name	Formula	Notes
Constant Force Work	$F(b-a)$	Requires Force to be Constant
Varied Force Work	$\int_a^b F(x)dx$	If direction of force and motion are same than $F(x) >= 0$ else $F(x) <= 0$
Hooke's Law	$F(x) = kx$	k is the spring constant and x is the displacement
Pump Liquid	$\int_a^b d * h(y)A(y)dy$	d is density, $h(y)$ is distance the layer is being lifted, $A(y)$ is the area of the layer. 62.5 is water density

3.3 Moment and Center of Mass

Centroid is the center of mass. m represents mass.

Name	Formula	Notes
Moment	$m * distance$	m is mass
x Moment (M_x)	$\int_a^b \frac{1}{2}(f(x)^2 - g(x)^2)dx)$	x moment if infinite number of particles in the area between a, b, f(x), and g(x)
y Moment (M_y)	$\int_a^b x(f(x) - g(x))dx)$	y moment if infinite number of particles in the area between a, b, f(x), and g(x)
$\bar{x}$	$\frac{M_y}{A}$	Which is the x value of the centriod and A being area
$\bar{y}$	$\frac{M_x}{A}$	Which is the y value of the centriod and A being area

4 Parameterized Curves

Curve where any point on the curve is a function of t on [a,b]: $(x = f(t), y = g(t))$ for t in $[a, b]$

Name	Formula	Notes
Parameterized Curves	$x = f(t), y = g(t)$	Over t in [a,b], to find the equation solve to relate y and x removing t
Length of Curve	$\int_a^b \sqrt{f\prime(t)^2 + g\prime(t)^2} dy$	Divide curve into small subsections, estimate with line, sum over line, take the limit.
Length of Curve if $x = x$ and $y = h(x)$	$\int_a^b \sqrt{1 + h\prime(x)^2} dx$	A special case that comes from the general formula.