Nonlinear and Adaptive Control: An Abbreviated Status Report*

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Abstract

We briefly review selected results in nonlinear and adaptive control, focusing on constructive concepts and design procedures: control Lyapunov functions, feedback passivation, robust and adaptive backstepping. For a more extensive discussion see Kokotović and Arcak (2001).

1 Control Lyapunov Functions

Early nonlinear concepts have been descriptive. Their 'feedback activation' began only recently, when some local properties were replaced with new concepts applicable to large regions of the state space. The main effort of activation is to make new concepts dependent on, and transformable by feedback control. A prominent example is the concept of *control Lyapunov function* (CLF) whose derivative depends on the control and can be made negative by feedback. This seemingly obvious concept, introduced by Artstein (1983) and Sontag (1983), made a tremendous impact on stabilization theory, which, at the end of the 1970's was stagnant. It converted stability descriptions into tools for solving stabilization tasks.

One way to stabilize a nonlinear system is to *select* a Lyapunov function $V(x)$ and then *try to find* a feedback control $u(x)$ that renders $\dot{V}(x,u(x))$ negative definite. With an arbitrary choice of $V(x)$ this attempt may fail, but if $V(x)$ is a CLF, we can find a stabilizing control law $u(x)$. For the nonlinear system

$$\dot{x} = f(x) + g(x)u\,, \tag{1}$$

$V(x)$ is a CLF if, for all $x \neq 0$,

$$L_gV(x) = 0 \quad \Rightarrow \quad L_fV(x) < 0\,, \tag{2}$$

where L_gV denotes $\frac{\partial V}{\partial x}g(x)$. By standard converse theorems, if (1) is stabilizable, a CLF exists. From (2), we see that the set where $L_gV(x) = 0$ is significant, because in this set the uncontrolled

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system has the property $L_fV(x) < 0$. However, if $L_fV(x) > 0$ when $L_gV(x) = 0$, then $V(x)$ is not a CLF and cannot be used for a feedback stabilization design (an observation that helps eliminate bad CLF candidates).

When $V(x)$ is a CLF, there are many control laws that render $\dot{V}(x, u(x))$ negative definite, one of which is given by a formula due to Sontag (1989). The construction of a CLF is a hard problem, which has been solved for special classes of systems. For example, when the system is feedback linearizable we can construct for it a quadratic CLF in the coordinates in which the system is forced to become linear by a feedback transformation that cancels all the nonlinearities. Once such a CLF is constructed, it can be used to design a control law $u(x)$ that avoids cancelation of useful nonlinearities. For a larger class of systems CLF's can be constructed by *backstepping*, as discussed in Section 4.

The CLF concept was extended by Freeman and Kokotović (1996a,b) to systems

$$\dot{x} = f(x, w) + g(x, w)u \,, \tag{3}$$

where w is a disturbance known to be bounded by $|w| \leq \Delta$, where Δ may depend on x. $V(x)$ is a *robust* CLF, if for all $|x| > c$, a control law $u(x)$ can be found to render $\dot{V}$ negative for any w such that $|w| \leq \Delta$. The value of c depends on Δ and on the chosen $u(x)$. For systems jointly affine in u and w,

$$\dot{x} = f(x) + g(x)u + p(x)w \,, \tag{4}$$

Krstić *et al.* (1995) considered a CLF $V(x)$ for which a class-$\mathcal{K}_\infty$ function $\rho(\cdot)$ exists such that

$$|x| > \rho(|w|) \quad \Rightarrow \quad \exists u : \; L_fV(x) + L_pV(x)w + L_gV(x)u < 0 \,. \tag{5}$$

Again, the set $L_gV(x) = 0$ is critical because in it we require that

$$L_fV(x) + |L_pV(x)|\rho^{-1}(|x|) < 0 \,, \tag{6}$$

which means that $L_fV(x)$ must be negative enough to overcome the effect of disturbances bounded by $|w| < \rho^{-1}(|x|)$. For systems with stochastic disturbances, Krstić and Deng (1998) introduced a notion of 'noise-to-state stability' and the corresponding CLF convenient for this type of stabilization.

2 Nonlinear Relative Degree and Zero Dynamics

The development of nonlinear geometric methods was a remarkable achievement of the 1980's, presented in the books by Isidori (1995), Nijmeijer and van der Schaft (1990), Marino and Tomei (1995) and in the numerous papers referenced therein. Geometric concepts permeate our current thinking about nonlinear systems. Two of them need to be made explicit here: nonlinear *relative degree* and *zero dynamics*. These indispensable tools bring into focus the common input-output structure of linear and nonlinear systems.

For a scalar transfer function, the relative degree is the difference between the number of poles and zeros. This is also the number of times the output $y(t)$ needs to be differentiated for the input $u(t)$ to appear. For a state-space realization (A, b, c, d), the relative degree is zero if $d \neq 0$, it is one if $d = 0$ and $cb \neq 0$, it is two if $d = 0$, $cb = 0$ and $cAb \neq 0$, etc. For the nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) + j(x)u \,, \quad x \in I\!\!R^n \,, u, y \in I\!\!R \,, \end{aligned} \tag{7}$$

the relative degree at a point $x^\star$ is zero if $j(x^\star) \neq 0$, it is one if $j(x^\star)$ is identically zero on a neighborhood of $x^\star$ and $L_g h \neq 0$ at $x^\star$. This is so because

$$\dot{y} = \frac{\partial h}{\partial x}\dot{x} = L_f h + L_g h\, u\,, \tag{8}$$

so that, if $L_g h$ is nonzero, then the input $u(t)$ appears in the expression for the first derivative $\dot{y}(t)$ of the output $y(t)$. If $L_g h$ is zero, we can differentiate $\dot{y}$ once more and check whether u appears in the expression for $\ddot{y}(t)$, etc. In contrast to linear systems, the relative degree of nonlinear systems may not be defined.

When the system (7) has relative degree one, its *input-output linearization* is performed with the feedback transformation

$$u = (L_g h)^{-1}(v - L_f h) \quad \Rightarrow \quad \dot{y} = v\,, \tag{9}$$

which cancels the nonlinearities in the $\dot{y}$-equation and converts it into $\dot{y} = v$. Selecting new state coordinates in which y is one of the states, the remaining $n-1$ equations with $y(t) \equiv 0$ and $v(t) \equiv 0$ constitute the *zero dynamics*, that is, the dynamics which remain when the output is kept at zero. If the relative degree is two, then the linear part of the system is $\ddot{y} = v$, the chain of two integrators. In this case the zero dynamics are described by the remaining $n-2$ equations $y(t) = \dot{y}(t) \equiv 0$ and $v(t) \equiv 0$.

The relative degree and the zero dynamics cannot be altered by feedback. For this reason, systems with unstable zero dynamics, *nonminimum phase systems*, are much harder to control than *minimum phase systems* in which the zero dynamics are asymptotically stable. In *weakly minimum phase systems* the zero dynamics are stable, but not asymptotically stable.

Two *caveats* need to be made about input-output linearization (9) as a design tool. First, there may be nonlinearities that should not be canceled because they help the design task, like $-x^3$ which helps us to stabilize $\dot{x} = x - x^3 + u$. Second, in the presence of modeling errors, the concepts of relative degree and zero dynamics may be nonrobust. Sastry *et al.* (1989) showed that regular perturbations in a system may lead to singularly perturbed unstable zero dynamics. It is therefore important that geometric concepts be applied jointly with the analytical tools needed to guarantee robustness.

3 Feedback Passivation

Achieving strict passivity (SPR) with feedback was, in the 70's, a common tool for adaptive control of linear systems. A result of Fradkov (1976), made more accessible by Fradkov and Hill (1998), is that (A, B, C) can be rendered SPR with feedback if and only if it is minimum phase and relative degree one. In nonlinear control, the use of passivation was motivated by a difficulty encountered in feedback stabilization of linear-nonlinear cascade systems

$$\begin{aligned} \dot{x} &= f(x, \xi) \\ \dot{\xi} &= A\xi + Bu \end{aligned} \tag{10}$$

resulting from input-output linearization. The difficulty was that the global asymptotic stability (GAS) property of the subsystem $\dot{x} = f(x, 0)$ is not sufficient to achieve GAS of the whole cascade with ξ-feedback $u = K\xi$, as illustrated by

$$\begin{aligned} \dot{x} &= -x + x^2\xi \qquad (11) \\ \dot{\xi} &= u\,. \end{aligned}$$

With feedback $u = k\xi$, for every finite $k < 0$, there exist initial conditions from which $x(t)$ escapes to infinity. Thus, feedback is required from both ξ and x, that is,

$$u = K\xi + v(x, \xi)\,. \tag{12}$$

Such a control law was designed by Byrnes and Isidori (1989) for the special case of (10) with $\dot{\xi} = Bu$, where B is a square nonsingular matrix. Kokotović and Sussmann (1989) extended this design to *feedback passivation* where the cascade (10) is represented as a linear block H_1 in feedback with a nonlinear block H_2. The final result is arrived at in several steps. First, an output η of the linear block H_1 is selected to be the input of the nonlinear block H_2, that is, the x-subsystem of (10) is rewritten as

$$\dot{x} = f(x, 0) + g(x, \xi)\eta\,, \tag{13}$$

where several choices of $\eta = C\xi$ may be available. An output y is then chosen to render (13) passive from η to y. If a Lyapunov function $V(x)$ is known for $\dot{x} = f(x, 0)$ so that $L_f V \leq 0$, then $y = L_g V^T$ renders (13) passive because

$$\dot{V} = L_f V + L_g V\eta \leq L_g V\eta = y^T\eta\,. \tag{14}$$

Finally, if the linear block H_1 is rendered passive by feedback $K\xi$, the passivity theorem will be satisfied by closing the loop with $-y = -L_g V^T$.

For the existence of K in the global stabilization of the linear-nonlinear cascade (10) with (12), Kokotović and Sussmann (1989), and Saberi *et al.* (1990) showed that the weak minimum phase property of (A, B, C) is necessary unless some other restriction is imposed on the nonlinear part. Upon an extension by Ortega (1989), Byrnes *et al.* (1991) proceeded to prove that at $x = 0$, the nonlinear system (7) with $j(x) \equiv 0$ is *feedback passive* with a positive definite storage function $S(x)$ if and only if it is *relative degree one* and *weakly minimum phase.*

4 Backstepping

For nonlinear control the 1990's started with a breakthrough: *backstepping*, a recursive design for systems with nonlinearities not constrained by linear bounds. Although the idea of integrator backstepping may be implicit in some earlier works, its use as a design tool was initiated by Tsinias (1989*b*, 1991), Byrnes and Isidori (1989), Sontag and Sussmann (1988), Kokotović and Sussmann (1989), and Saberi *et al.* (1990). However, the true potential of backstepping was discovered only when this approach was developed for nonlinear systems with structured uncertainty. With *adaptive backstepping*, Kanellakopoulos *et al.* (1991*a*,*b*) achieved global stabilization in the presence of unknown parameters, and with *robust backstepping*, Freeman and Kokotović (1992, 1993), and Marino and Tomei (1993*b*) achieved it in the presence of disturbances. The emergence of adaptive, robust and observer-based backstepping was described in the 1991 Bode Lecture, Kokotović (1992).

The ease with which backstepping incorporated uncertainties and unknown parameters contributed to its instant popularity and rapid acceptance. At the same time, its limitation to a class of pure feedback (lower triangular) systems stimulated the development of other recursive procedures, such as *forwarding* by Teel (1992), Mazenc and Praly (1996), and Janković *et al.* (1996), applicable to feedforward systems. Interlacing the steps of these procedures, it is often possible to design other types of systems. The rapidly growing literature on recursive nonlinear designs includes the books by Krstić *et al.* (1995), Marino and Tomei (1995), Freeman and Kokotović (1996*b*), Sepulchre *et al.* (1997), Krstić and Deng (1998), Dawson *et al.* (1998), and Isidori (1999).

The purpose of backstepping is the construction of various types of CLF's: robust, adaptive etc. Backstepping constructions of robust CLF's by Freeman and Kokotović (1992), and Marino and Tomei (1993*b*) are illustrated on the system

$$\begin{aligned} \dot{x}_1 &= x_2 + w_1(x,t) \\ \dot{x}_2 &= u + w_2(x,t)\,, \end{aligned} \tag{15}$$

where the uncertainties w_1 and w_2 are bounded by known functions

$$\begin{aligned} |w_1(x,t)| &\leq \Delta_1(x_1) \\ |w_2(x,t)| &\leq \Delta_2(x_1,x_2)\,, \end{aligned} \tag{16}$$

which are allowed to grow faster than linear, like $\Delta_1(x_1) = x_1^2$. The crucial restriction of backstepping is imposed on the structure of bounding functions Δ_1, Δ_2 in (16), allowing Δ_i to depend only on $x_1, \cdots, x_i$. For the ease of presentation it will be assumed that $\Delta_1(0) = 0$, $\Delta_2(0,0) = 0$, and that the derivative of $\Delta_1(x_1)$ exists and is zero at $x_1 = 0$. When this is not the case, a slightly modified procedure achieves boundedness and convergence to a compact set around $x = 0$.

Backstepping starts with a part of the system for which the construction of a robust CLF is easy, as in the case when the uncertainty is *matched*. Lyapunov minmax designs for matched uncertainties were developed around 1980 by Gutman (1979), Corless and Leitmann (1981) and others, presented in (Khalil, 1996*b*, Section 13.1).

In the first equation of (15) the uncertainty w_1 is *matched* with x_2. This means that if x_2 were our control, it would be able to counteract the worst case of w_1 by $x_2 = \mu_1(x_1)$. To design such a *virtual* control law $\mu_1(x_1)$ for the x_1-equation we can use $V_1 = x_1^2$ as our robust CLF. Then to render $\dot{V}_1$ negative we seek $\mu_1(x_1)$ which, for $x_1 \neq 0$ and all $w_1(x,t)$ bounded by (16), satisfies

$$x_1[\mu_1(x_1) + w_1(x,t)] \;\leq\; x_1\mu_1(x_1) + |x_1|\Delta_1(x_1) < 0\,. \tag{17}$$

A possible choice is

$$\mu_1(x_1) = -x_1 - \mathrm{sgn}(x_1)\Delta_1(x_1)\,, \tag{18}$$

where $\mu_1'(x_1) := d\mu_1/dx_1$ exists because of the assumptions on Δ_1.

It is consistent with the idea of x_2 being a *virtual control* that we think of $x_2 - \mu_1(x_1)$ as an *error* to be regulated to zero by the actual control u. This suggests that we examine

$$V_2(x) = V_1(x_1) + [x_2 - \mu_1(x_1)]^2 \tag{19}$$

as a candidate robust CLF for the whole system (15). Our task is then to achieve, with some $u = \mu_2(x)$,

$$\dot{V}_2 = 2x_1[x_2 + w_1] + 2[x_2 - \mu_1(x_1)][u + w_2 - \mu_1'(x_1)(x_2 + w_1)] < 0 \tag{20}$$

for all $x \neq 0$, and all admissible $w_1(x,t)$ and $w_2(x,t)$. The choice of $\mu_1(x_1)$ in (18) to satisfy (17) has made this task easy, because it has reduced (20) to

$$\dot{V}_2 \leq -2x_1^2 + 2\big[x_2 - \mu_1(x_1)\big]\,\big[x_1 + u + w_2 - \mu_1'(x_1)(x_2 + w_1)\big] < 0\,, \tag{21}$$

where u matches the composite uncertainty

$$w_c(x,t) := w_2(x,t) - \mu_1'(x_1)w_1(x,t)\,, \tag{22}$$

with the bound $|w_c(x,t)| < \Delta_c(x)$ computed from Δ_1, Δ_2 and μ_1'. We first let

$$u = \mu_2(x) = -[x_2 - \mu_1(x)] - x_1 + \mu_1'(x_1)x_2 + u_r(x)\,. \tag{23}$$

Then, the inequality to be satisfied by $u_r(x)$ is of the same form as the inequality (17) and, hence,

$$u_r(x) = -\mathrm{sgn}[x_2 - \mu_1(x_1)]\Delta_c(x)\,. \tag{24}$$

The so designed $\mu_2(x)$ yields

$$\dot{V}_2 \leq -2x_1^2 - 2[x_2 - \mu_1(x_1)]^2\,, \tag{25}$$

which means that GAS is achieved.

This example highlights the key recursive feature of backstepping: the robust CLF for step $k+1$ is constructed as

$$V_{k+1} = V_k + [x_k - \mu_{k-1}(x_1, \cdots, x_{k-1})]^2, \tag{26}$$

where V_k is the k-th robust CLF and μ_{k-1} is the virtual control law which renders $\dot{V}_k < 0$ for $x_k = \mu_{k-1}(x_1, \cdots, x_{k-1})$.

Other backstepping constructions were developed by Praly and Jiang (1993), Jiang *et al.* (1994), Krstić *et al.* (1995). Marino *et al.* (1994), and Isidori (1996*b*,*a*) employed backstepping to solve an *almost disturbance decoupling* problem. For systems with stochastic disturbances backstepping designs were developed by Krstić and Deng (1998), and Pan and Başar (1999). Freeman and Praly (1998) extended backstepping to control inputs with magnitude and rate limits, and Jiang and Nijmeijer (1997) to nonholonomic systems. An undesirable property of backstepping is the growth of 'nonlinear gains', which Freeman and Kokotović (1993) counteracted by 'flattened' Lyapunov functions.

Several inverse optimal backstepping designs were proposed by Pan and Başar (1998), Krstić and Deng (1998) and Ezal *et al.* (2000). The design by Ezal *et al.* (2000) is particularly useful because it also achieves *local optimality*, that is, the linearization of the designed nonlinear feedback system is $\mathcal{H}_\infty$-optimal. In this way earlier optimal designs for linear systems are incorporated in nonlinear designs.

5 Adaptive Nonlinear Control

In the adaptive control problem the uncertainty is an unknown parameter vector θ and its estimate $\hat{\theta}(t)$ is used in the design of a control law. A *certainty equivalence* design, common in adaptive linear control, is not applicable to systems with strong nonlinearities like x^2. To see why, consider the system

$$\dot{x} = x + \theta x^2 + u\,, \tag{27}$$

and let its certainty equivalence control be $u = -2x - \hat{\theta}x^2$. It turns out that even with an exponentially convergent estimate $|\hat{\theta}(t) - \theta| \leq ce^{-at}$, some solutions of

$$\dot{x} = -x - (\hat{\theta} - \theta)x^2 \tag{28}$$

escape to infinity. For the matched case (27), the standard Lyapunov design furnishes a parameter update law which is faster than exponential. This design was extended by Kanellakopoulos *et al.* (1991*c*) to systems in which θ is separated from u by no more than one integrator, like $\dot{x}_1 = x_2 + \theta x_1^2;\ \dot{x}_2 = u$.

The real difficulties were encountered in the 'benchmark problem'

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta x_1^2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= u\,,\end{aligned} \tag{29}$$

presented by Kokotović and Kanellakopoulos (1990). Global stabilization of (29), and convergence of $x(t)$ were finally achieved with the first, overparametrized version of adaptive backstepping by Kanellakopoulos *et al.* (1991*a*,*b*), which also employed the *nonlinear damping* of Feuer and Morse (1978). Jiang and Praly (1991) reduced the overparametrization by one half, and the *tuning functions* method of Krstić *et al.* (1992) completely removed it.

The current form of adaptive backstepping, described in the book by Krstić *et al.* (1995), will now be explained with the help of the *adaptive* CLF. For the x-subsystem of the augmented system

$$\dot{x} = f(x) + F(x)\theta + g(x)\xi \tag{30}$$

$$\dot{\xi} = u\,, \qquad x \in I\!R^n;\ \xi, u \in I\!R,\ \theta \in I\!R^p, \tag{31}$$

with ξ as its virtual control, $V(x,\theta)$ is an adaptive CLF if there exists $\alpha_1(x,\theta)$ such that for all $x \neq 0$, and all θ,

$$\frac{\partial V_1}{\partial x_1}\left[f(x) + F(x)\left(\theta + \frac{\partial V_1}{\partial \theta}^T\right) + g(x)\alpha_1(x,\theta)\right] < -\sigma_1(x,\theta)\,, \tag{32}$$

where $\sigma_1(x,\theta) \geq 0$. Then, a virtual adaptive controller for the x-subsystem is

$$\begin{aligned}\xi &= \alpha_1(x,\hat{\theta}) \\ \dot{\hat{\theta}} &= \tau_1(x,\hat{\theta}) := F^T(x)\frac{\partial V_1}{\partial x_1}^T(x,\hat{\theta})\,,\end{aligned} \tag{33}$$

where τ_1 is the *first tuning function*. The stability properties of the feedback system (30),(33) are established with

$$\bar{V}_1(x,\hat{\theta}) = V_1(x,\hat{\theta}) + \frac{1}{2}|\hat{\theta} - \theta|^2\,. \tag{34}$$

As always, the purpose of backstepping is to construct an adaptive CLF for the augmented system (30),(31). Again, a candidate is

$$V_2(x,\xi,\theta) = V_1(x,\theta) + \frac{1}{2}(\xi - \alpha_1(x,\theta))^2\,. \tag{35}$$

This candidate wins, because there exists $\alpha_2(x,\xi,\theta)$ and $\sigma_2(x,\xi,\theta) \geq 0$ such that

$$\frac{\partial V_2}{\partial(x,\xi)}\begin{bmatrix} f(x) + F(x)\left(\theta + \frac{\partial V_2}{\partial \theta}^T\right) + g(x)\xi \\ \alpha_2(x,\xi,\theta)\end{bmatrix} < -\sigma_2(x,\xi,\theta)\,, \tag{36}$$

for all $x \neq 0$, $\xi \neq 0$, where expressions for $\alpha_2(x,\xi,\theta)$ and $\sigma_2(x,\xi,\theta)$ can be obtained by a short calculation. With $V_2(x,\xi,\theta)$ as an adaptive CLF, an adaptive controller for (30), (31) is

$$u = \alpha_2(x,\xi,\hat{\theta})\,, \qquad \dot{\hat{\theta}} = \tau_2(x,\xi,\hat{\theta})\,, \tag{37}$$

where the update law is the *second tuning function*

$$\tau_2(x,\xi,\hat{\theta}) = \tau_1(x,\hat{\theta}) - \left(\frac{\partial \alpha_1}{\partial x}\right)^T(\xi - \alpha_1)\,. \tag{38}$$

The boundedness of $x(t)$, $\xi(t)$, $\hat{\theta}(t)$ and the convergence $x(t) \to 0$, $\xi(t) \to 0$ are easy to prove with

$$\bar{V}_2 = V_1(x, \xi, \hat{\theta}) + \frac{1}{2}|\hat{\theta} - \theta|^2 \,. \tag{39}$$

The recursive formula for V_i is as in (35) and for τ_i is as in (38). A similar recursive formula is available for α_i.

An alternative estimation-based approach to adaptive nonlinear control was motivated by adaptive designs for linear systems. The status of this line of research in 1990 was described by Praly *et al.* (1991). For an estimation-based design to succeed in nonlinear systems, the traditional *certainty equivalence* control law had to be replaced by a stronger control law, developed by Krstić and Kokotović (1995, 1996). This control law can be used in conjunction with most standard adaptive estimators.

Because the newly developed adaptive nonlinear controllers had no counterparts in adaptive linear control, it was of interest to specialize them to linear systems and compare them with traditional adaptive controllers. Krstić *et al.* (1994) showed that the new designs far outperformed their predecessors.

Extensions of adaptive backstepping to a wider class of systems were made by Seto *et al.* (1994). Asymptotic properties, transient performance, robustness and dynamic extensions of the new adaptive controllers were further investigated by Zhang *et al.* (1996), Ikhouane and Krstić (1998), Lin and Kanellakopoulos (1998), Sira-Ramírez *et al.* (1997), Jiang and Praly (1998) and several other authors. Systems containing both unknown parameters θ and bounded disturbances $w(x,t)$ can be handled by a combination of adaptive and robust backstepping as described by Freeman *et al.* (1998). The difficult problem of nonlinear parameterizations has recently been addressed by Bošković (1998), Annaswamy *et al.* (1998), and Kojić *et al.* (1998).

6 Output Feedback Designs

Progress in nonlinear output feedback design has been slower. First, nonlinear observers are available only for very restrictive classes of systems. Second, even when a nonlinear observer is available, it may not be applicable for output feedback design because the *separation principle* does not hold.

For systems in which the nonlinearities appear as functions of the measured output, the nonlinearity is canceled by an 'output injection' term. This class of systems has been characterized by Krener and Isidori (1983), Bestle and Zeitz (1983), Besançon (1999), among others. *Output injection observers* have been incorporated in observer-based control designs by Kanellakopoulos *et al.* (1992), Praly and Jiang (1993), Marino and Tomei (1993*a*), and, for stochastic nonlinear systems, by Deng and Krstić (1999).

A class of nonlinear observers by Thau (1973), Kou *et al.* (1975), Banks (1981), Tsinias (1989*a*), Yaz (1993), (Boyd *et al.*, 1994, Section 7.6), Raghavan and Hedrick (1994), and Rajamani (1998) require that the state-dependent nonlinearities be globally Lipschitz, so that quadratic Lyapunov functions can be used for observer design.

A broader class of systems is characterized by linear dependence on unmeasured states. For this class, dynamic output feedback designs have been proposed by Praly (1992), Pomet *et al.* (1993), Marino and Tomei (1995), and Freeman and Kokotović (1996*c*).

For feedback linearizable systems Esfandiari and Khalil (1992), Khalil and Esfandiari (1993), Atassi and Khalil (1999) developed an output feedback design which achieves semiglobal stabilization and approximately recovers the performance of the underlying full state feedback. The key idea is to use a high-gain observer, but to pass the state estimates through saturation elements, thus avoiding the destabilizing effects of observer transients with large magnitudes. The high-gain observer has been employed in semiglobal output feedback designs by Teel and Praly (1995), Lin and Saberi (1995), Praly and Jiang (1998), and Isidori *et al.* (1999). Janković (1996) and Khalil (1996*a*) used the same approach in adaptive control.

Khalil's high-gain observer with saturation, along with the notion of *complete uniform observability* of Gauthier and Bornard (1981), led to the conceptually appealing 'separation theorem' by Teel and Praly (1994): If the equilibrium $x^\star$ is globally stabilizable by state feedback and the system is completely uniformly observable, then $x^\star$ is semiglobally stabilizable by dynamic output feedback. Extensions and interpretations of this result have been presented by Atassi and Khalil (1999), and (Isidori, 1999, Section 12.3).

To achieve global convergence of high-gain observers, Gauthier *et al.* (1992) resorted to a global Lipschitz condition - a common restriction in most global designs. In the absence of such a restriction, global stabilization by output feedback may not be possible, as shown by the counterexamples of Mazenc *et al.* (1994).

Arcak and Kokotović (1999) designed observers for systems with monotonic nonlinearities such as x^3, $exp(x)$, *etc.* Their approach is to represent the observer error system as the feedback interconnection of a linear system and a time-varying sector nonlinearity. The convergence of the observer error to zero is then achieved by rendering the linear system SPR with the help of LMI computations.

Isidori and Byrnes (1990) developed a nonlinear counterpart of the linear servomechanism design of Davison, Francis and Wonham, which incorporates an *internal model* of the disturbance. The internal model makes it possible to create and locally stabilize an *invariant manifold* on which the tracking error is zero. The local property restricts the disturbances and the initial conditions to be small. Huang and Rugh (1992) allowed large disturbances by restricting the exosystem to be slow. Khalil (1994), Mahmoud and Khalil (1996) and Khalil (1998) used a high-gain observer to solve the nonlinear servomechanism problem with arbitrarily large initial conditions. Developments in this area are treated in the book by Byrnes *et al.* (1997), and the survey by Byrnes and Isidori (1998).

7 Conclusions

In this abbreviated progress report, we focused on constructive nonlinear and adaptive control. We believe that this constructive trend will continue, with further development of structure-specific procedures applicable to broader classes of systems. This has already happened for structures induced by physical laws for electromechanical systems, with new challenges at micro- and nanoscales.

Constructive procedures have been developed for only a few output feedback problems. This is an area where discoveries of new structures may lead to significant breakthroughs.

Physically motivated characterizations of nonlinear uncertainties, that is, unmodeled dynamics, deterministic and stochastic disturbances, are needed to help robustify the constructive procedures, without undue conservativeness. To reduce complexity of feedback designs, attention must be paid to structuring and simplification of models.

Nonlinear control designs are increasingly important in a wide range of technologies. With a solid knowledge of nonlinear control, new generations of engineers will be better equipped for new creative tasks.

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