CS536 Homework 6

Syntactic Substitution, Forward Assignment, & sp

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Answers: -

Question 1

We will replace the x which is outside the quantified predicate (free variable).

Now replacing x in f(x*y) with y+z will make free x bounded after replacement. So will replace y with w first.

$p \equiv (x*y < f(a) \vee \exists x . x \geq a*y)[y+z/x] \rightarrow (\exists w . (f(x*y) > a-y+z)[w/y]) [y+z/x]$

$\equiv (y+z)*y < f(a) \vee \exists x . x \geq a*y \rightarrow (\exists w . f(x*w) > a-w+z) [y+z/x]$

$\equiv (y+z)*y < f(a) \vee \exists x . x \geq a*y \rightarrow \exists w . f((y+z)*w) > a-w+z$

Question 2

We will replace the free variable y which is not bound to any quantifier and the y included in ∃x as it is not bound. We will not replace y in ∃y as it is bounded.

$p \equiv x*y < f(a) \vee \exists x . x \geq a*y \rightarrow \exists y . f(x*y) > a-y+z [a-y/y]$

$\equiv x* (a-y) < f(a) \vee (\exists x . x \geq a*y \rightarrow \exists y . f(x*y) > a-y+z)[a-y/y]$

$\equiv x* (a-y) < f(a) \vee \exists x . x \geq a*(a-y) \rightarrow \exists y . f(x*y) > a-y+z$

Question 3

$p[a*y/a] \equiv (x*y < f(a) \vee \exists x . x \geq a*y \rightarrow \exists y . f(x*y) > a-y+z) [a*y/a]$

f(a) is free and not bounded so we can replace $[a*y/a]$

'a' in ∃x . x ≥ a*y is free and after replacement still it will be free.

Now a in ∃y is free but after replacing [a*y/a] it will get bounded. Hence to remove ambiguity of boundness first will replace y with u in ∃y . f(x*y) > a-y+z

$\equiv x*y < f(a*y) \vee \exists x . x \geq (a*y)*y \rightarrow (\exists u .(f(x*y) > a-y+z)[u/y]) [a*y/a]$

$\equiv x*y < f(a*y) \vee \exists x . x \geq (a*y)*y \rightarrow (\exists u .f(x*u)>a-u+z)[a*y/a]$

$\equiv x*y < f(a*y) \vee \exists x . x \geq (a*y)*y \rightarrow \exists u .f(x*u)>(a*y)-u+z$

Question 4

First will do $p_1 \equiv p[x \div y/a]$

$P1 \equiv (x*y < f(a) \vee \exists x . x \geq a*y \rightarrow \exists y . f(x*y) > a-y+z)[x \div y / a]$

Renaming x to u in ∃x and y to w in ∃y as replacing ‘a’ in both will make free ‘a’ to bounded.

We can replace a in f(a).

$\equiv x*y < f(x \div y) \vee ((\exists u .(x \geq a*y)[u/x]) \rightarrow (\exists w . (f(x*y) > a-y+z))[w/y]) [x \div y / a]$

$\equiv x*y < f(x \div y) \vee (\exists u . u \geq a*y \rightarrow \exists w . f(x*w) > a-w+z) [x \div y / a]$

$\equiv x*y < f(x \div y) \vee \exists u . u \geq (x \div y)*y \rightarrow \exists w . f(x*w) > (x \div y)-w+z$

Now will do p1 $[y-z / x]$ to get final p.

$P \equiv (x*y < f(x \div y) \vee \exists u . u \geq (x \div y)*y \rightarrow \exists w . f(x*w) > (x \div y)-w+z) [y-z/x]$

$\equiv (y-z)*y < f((y-z) \div y) \vee \exists u . u \geq ((y-z) \div y)*y \rightarrow \exists w . f((y-z)*w) > ((y-z) \div y)-w+z$

Question 5

Now, for checking the $\not\models_{tot}$ {T} S {sp(T, S)}, for that there are two possibilities available one is that it is invalid partial correctness and second is it does not terminate.

Now we are considering the case for the run time error, for this example,

$\{T\}\ a:=0;\ b:=2/a\ \{ a=0 \wedge b=2/a\}$

Question 6

$sp(x < y \wedge x+y \leq n,\ x := f(x+y);\ y := g(x*y))$

$\equiv sp(sp(x < y \wedge x+y \leq n,\ x := f(x+y)),\ y := g(x*y))$

$\equiv sp(x_0 < y \wedge x_0+y \leq n \wedge x = f(x_0+y),\ y := g(x*y))$

$\equiv x_0 < y_0 \wedge x_0+y_0 \leq n \wedge x = f(x_0+y_0) \wedge y = g(x*y_0)$

Question 7

$sp(x = 2^k, x := x/2)$

$\equiv x=2^k[x_0 /x] \wedge x = x/2[x_0 /x]$

$\equiv x_0=2^k \wedge x = x_0/2$

$\Rightarrow x= (2^k)/2$

$\Rightarrow x=2^k-1$

Question 8

$wp(x := x/2, x = 2^k)$

$\equiv x/2 = 2^k$

Question 9

S ≡ if even(x) then x := x+1 fi

a. wp(S, odd(x))
   ≡ wp(if even(x) then x := x+1 else skip fi, odd(x))
   ≡ (even(x) → wp(x:=x+1, odd(x))) ∧ (odd(x) → wp(skip, odd(x)))
   ≡ (even(x) → odd(x+1)) ∧ (odd(x) → odd(x))
b. sp(x = x_0, S)
   ≡ sp(x=x_0 , if even(x) then x := x+1 else skip fi)
   ≡ sp(x=x_0 ∧ even(x), x=x+1) ∨ sp(x=x_0 ∧ odd(x), skip)
   ≡ (x_0 = x_0 ∧ even(x_0) ∧ x=x_0 +1) ∨ (x=x_0 ∧ odd(x))
   ≡ (T ∧ even(x_0) ∧ x=x_0 +1) ∨ (x=x_0 ∧ odd(x))
   ≡ (even(x_0) ∧ x=x_0 +1) ∨ (x=x_0 ∧ odd(x))

Question 10

a. S ≡ if x < b[mid] then right := m else left := m fi
   p ≡ left< right–1 ∧ mid = (left + right)/2 ∧ b[left]≤ x < b[right].
   p′ ≡ left = $left_0$ ∧ right = $right_0$.

   sp(p ∧ p′, S)
   ≡ sp(left< right–1 ∧ mid = (left + right)/2 ∧ b[left]≤ x < b[right] ∧ left = $left_0$ ∧ right = $right_0$ ∧ x<b[mid], right:=m) ∨ sp(left< right–1 ∧ mid = (left + right)/2 ∧ b[left]≤ x < b[right] ∧ left = $left_0$ ∧ right = $right_0$ ∧ x ≥ b[mid], left:=m)

   ≡ (left<right-1 ∧ mid = (left + $right_0$)/2 ∧ b[left]≤ x < b[$right_0$] ∧ left = $left_0$ ∧ x<b[mid] ∧right=m) ∨ ($left_0$< right–1 ∧ mid = ($left_0$ + right)/2 ∧ b[$left_0$]≤ x < b[right] ∧ right = $right_0$ ∧ x ≥ b[mid] ∧ left=m)
b. Wp(S,p) ≡ (x < b[mid] – wp(right:= m, p)) <–> (x z b[mid] – wp(left :=mid p))
   ≡ (x < b[mid] -> p[mid]/right)) ∧ (x z b[mid] -> p[mid]/left))
   ≡ (x <b[mid]-L < m-1 / b[left] ≤ x < b[mid]) <-> (x ≥ b[mid] – m < right-1 b[mid] ≤ x < b[right]