CS536 Homework 4

Sequential Nondeterminism, Hoare Triples 1 & 2

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Answers: -

Question 1: -

a. DO ≡ do $x \neq 0 \rightarrow x := x-1;\ y := y+1 \ \square\ x \neq 0 \rightarrow x := x-1;\ y := y+2$ od
   If $\sigma = \{x = \beta, y = \delta\}$ and $\beta \geq 2$ then there exist two possible paths to execute for single iteration
   $<DO, \sigma> \rightarrow <S1;DO, \sigma> = <x := x-1;\ y := y+1;\ DO, \sigma> \rightarrow <DO, \sigma[x \mapsto \beta -1, y \mapsto \delta +1]>$
   $<DO, \sigma> \rightarrow <S2;DO, \sigma> = <x := x-1;\ y := y+2;\ DO, \sigma> \rightarrow <DO, \sigma[x \mapsto \beta -1, y \mapsto \delta +2]>$
   So after one iteration possible τ are $\{\tau[x \mapsto \beta -1, y \mapsto \delta +1], \tau[x \mapsto \beta -1, y \mapsto \delta +2]\}$

b. We know from (a) that $\sigma_1 \in \{\tau[x \mapsto \beta -1, y \mapsto \delta +1], \tau[x \mapsto \beta -1, y \mapsto \delta +2]\}$. To get all possible σ_2 lets say σ_1 is $\tau[x \mapsto \beta -1, y \mapsto \delta +1]$, then one iteration will result in $\{\tau[x \mapsto (\beta-1) -1, y \mapsto (\delta+1) +1], \tau[x \mapsto (\beta-1) -1, y \mapsto (\delta+1) +2]\} = \{\tau[x \mapsto \beta -2, y \mapsto \delta +2], \tau[x \mapsto \beta -2, y \mapsto \delta +3]\}$

   if σ_1 is $\tau[x \mapsto \beta -1, y \mapsto \delta +2]$, then one iteration will result in $\{\tau[x \mapsto (\beta-1) -1, y \mapsto (\delta+2) +1], \tau[x \mapsto (\beta-1) -1, y \mapsto (\delta+2) +2]\} = \{\tau[x \mapsto \beta -2, y \mapsto \delta +3], \tau[x \mapsto \beta -2, y \mapsto \delta +4]\}$

   Combining both τ we get
   $\sigma_2 \in \{ \tau[x \mapsto \beta -2, y \mapsto \delta +2], \tau[x \mapsto \beta -2, y \mapsto \delta +3], \tau[x \mapsto \beta -2, y \mapsto \delta +4]\}$
   (two τ are same, so wrote them as one)

c. We can see from σ_1 and σ_2 that x gets updated to β-k, decrementing x in each iteration. For y we observed during each iteration that y is incremented 2k-n times where n is $0 \leq n \leq k$. So in general the set of all possible states for k iterations is
   $\Sigma' = \{ \sigma_0[x \mapsto \beta\text{-}k,\ y \mapsto \delta +2k\text{-}n \mid 0 \leq n \leq k\}$

Question 2: -

a. If $\sigma \vDash \{p\}\ S\ \{q\}$ and $\sigma \nvDash p$, then $\perp \in M(S, \sigma)$ **may or may not** occur.
   (Partial correctness in state σ describes what happens if $\sigma \vDash p$, but it doesn't say $\sigma \nvDash p$.)
b. If $\sigma \vDash \{p\}\ S\ \{q\}$ and $\sigma \nvDash p$, then $M(S, \sigma) - \{\perp\} \vDash q$ **may or may not** occur.
   (Partial correctness in state σ does not describes what happens if $\sigma \nvDash$ the precondition.)
c. If $\sigma \vDash \{p\}\ S\ \{q\}$ and $\sigma \vDash p$, then $\perp \in M(S, \sigma)$ **may or may not** occur.
   (Partial correctness in σ says that $\perp \in M(S, \sigma)$ or $M(S, \sigma) - \{\perp\} \vDash q$.)
d. If $\sigma \vDash \{p\}\ S\ \{q\}$ and $\sigma \vDash p$, then $M(S, \sigma) - \{\perp\} \vDash q$ **may or may not** occur.
   (Same reason as in 2(c))
e. If $\vDash_{tot} \{p\}\ S\ \{q\}$ then $\vDash_{tot} \{p\}\ S\ \{T\}$ **must** occur.
   (If S terminates satisfying q then it terminates satisfying true.)

f. If $\models_{tot}$ {p} S {T} then $\models_{tot}$ {p} S {q} **may or may not** occur.
(We can't say S terminates successfully satisfying q)

g. If σ ⊭ {p} S {q} and S is deterministic, then σ ⊨ p, ⊥ ∉ M(S, σ), and M(S, σ) ⊨ ¬q **must** all occur simultaneously.
(If ⊥ ∉ M(S, σ) and M(S, σ) ⊭ q, then the state ∈ M(S, σ) must satisfy ¬q.)

h. If ⊥ ∉ M(S, σ), M(S, σ) ⊭ q, and S is deterministic, then M(S, σ) ⊨ ¬q **must** occur.
(Since S is deterministic, M(S, σ) = some {τ}; if τ ≠ ⊥ and ⊭ q, then it must ⊨ ¬q.)

i. If ⊥ ∉ M(S, σ), M(S, σ) ⊭ q, and S is nondeterministic, then M(S, σ) ⊨ ¬q **may or may not** occur.
(As S is nondeterministic, M(S, σ) ⊭ q says that one of the states doesn't satisfies q and not all)

j. If M(S, σ) ⊭ q, τ ∈ M(S, σ), and S is nondeterministic, then τ ⊨ q **may or may not** occur.

k. If S is deterministic and σ ⊨ {p} S {q}, then σ ⊨ {p} S {¬q} **may or may not** occur.

l. If σ $\not\models_{tot}$ {p} S {q} and S is deterministic, then σ ⊨ {p} S {¬q} **must** occur.

m. If σ $\not\models_{tot}$ {p} S {q} and S is nondeterministic, then σ ⊨ {p} S {¬q} **may or may not** occur.

n. If σ ⊭ {p} S {q} and S is deterministic, then σ $\models_{tot}$ {p} S {¬q} **must** occur.

o. If σ ⊭ {p} S {q} and S is non-deterministic, then σ $\models_{tot}$ {p} S {¬q} **may or may not** occur.

Question 3: -

- We know that for backward assignment {P(e)} v := e {P(v)}. So in our question we want this statement
b := b+b to terminate in a state that satisfies {b*c ≤ d–b}. Based on this we can say that we can start in a state which also satisfies {b*c ≤ d–b}, but every occurrence of 'b' is replaced with 'b+b'. So our precondition {P(e)} becomes
{(b+b) * c ≤ d – (b+b)}

Question 4: -

a. U ≡ {1 ≤ m*y ≤ n*m} (replacing 'x' with 'm' for backward assignment)

b. v ≡ {1 ≤ m*n ≤ n*m } = {1≤ m*n}

c. { v } y := n { u } ---- 1
{ u } x := m {1 ≤ x*y ≤ n*m} -----2
We need postcondition of 1 to match with precondition of 2. So from above we can see they are same so {w} in { w } y := n ; x := m {1 ≤ x*y ≤ n*m} will be {v} which is
{1 ≤ m*n} (or {1 ≤ m*n ≤ n*m })

Question 5: -

a. If σ ⊨ {p} S {q}
Given $p_0 \to p$, $p \to p_1$, $q_0 \to q$, and $q \to q_1$. Now moving towards left strengthens the precondition and moving towards right weakens the postcondition.
So by strengthen precondition we get $p_0 \to p$ and by Weakening postcondition we get $q \to q_1$.
Combining we get σ ⊨{p_0} S {q_1}

b. σ $\models_{tot}${p_0} S {q_1} for the same reason as (a)

Question 6: -

Let's say that S is deterministic and that running S on σ yields τ. If $\tau = \perp$, then we get partial correctness in all four cases (a) - (d), so assume $\tau \neq \perp$.

a. Yes: $\sigma \models \{p_1 \land p_2\}\ S\ \{q_1 \land q_2\}$. If $\sigma \models p_1 \land p_2$, then $\sigma \models p_1$ and $\sigma \models p_2$ both holds. Hence $\sigma \models p_1$ which means $\tau \models q_1$. Similarly, $\sigma \models p_2$, so $\tau \models q_2$.
b. Yes: $\sigma \models \{p_1 \lor p_2\}\ S\ \{q_1 \lor q_2\}$? $\sigma \models p_1 \lor p_2$ implies that at least one of $\sigma \models p_1$ and $\sigma \models p_2$ hold. If $\sigma \models p_1$ then $\tau \models q_1$, so $\tau \models q_1 \lor q_2$. Similarly, if $\sigma \models p_2$, then $\tau \models q_1 \lor q_2$.
c. Yes: $\sigma \models \{p_1 \land p_2\}\ S\ \{q_1 \lor q_2\}$? If $\sigma \models p_1 \land p_2$ then from (a), $\sigma \models q_1 \land q_2$, which implies $q_1 \lor q_2$. (Alternatively, $\sigma \models p_1 \land p_2$ implies $\sigma \models p_1$, which implies $\tau \models q_1$, which implies $\tau \models q_1 \lor q_2$.)
d. No, $\sigma \not\models \{p_1 \lor p_2\}\ S\ \{q_1 \land q_2\}$.
   This can be explained by giving an example:
   $\models \{x > 2\}\ y := x\ \{y > 2\}$ and $\models \{x \leq 2\}\ y := x\ \{y \leq 2\}$, but $\not\models \{x > 2 \lor x \leq 2\}\ y := x\ \{y > 2 \land y \leq 2\}$

This was assuming S was deterministic. If S is nondeterministic, then substitute M(S, σ) for τ above.