CS 536 HW 2

Types, Expressions, States, Quantified Predicates

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Answers: -

Question 1: -

a. Illegal
  - Because for execution the conditional statement we need the same types of output, but here one is Boolean and another is integer, so it is not possible.

b. Legal
  - match functions takes two arrays (b1, b2) and one integer 'n' as arguments.
    value of the function will be Boolean
    T = if matched
    F = if not matched

c. Illegal
  - Here b3 is two dimensional array in the right side, and for the addition of the two array we need the left side array value must be the two dimension, so for that reasons, it gives the error.

Question 2: -

a. Well-formed
  - X Is an array.
  - Value of an array is a function from index values to store values.

b. Well-formed
  - u is an array.
  - v is a syntactic symbol with semantic value 0.
  - w=4 because we can write u as τ = { u[0] = 3, u[1] = 4} (w is syntactic symbol while 4 is semantic)

c. Ill-formed
  - Here, for (r = one and s = four), r and s are syntactic while in t=r + s, they are semantic. It can't be both.

Question 3: -

a. σ = {x = 2, b = β} where β = (7, 12, 3, 0)
   - Now set ordered pair looks like,
   - β = {(0,7),(1,12),(2,3),(4,0)}
   - Then final function will be,
   - σ = {x = 2, b = β} = {(x,2),(b, β)}, where β = {(0,7),(1,12),(2,3),(4,0)}

b. Separate binding
   - b[0]=7, b[1]=12, b[2]=3, b[4]=0
   - σ = {x = 2, b[0]=7, b[1]=12, b[2]=3, b[3]=0}

Question 4: -

a. b=(1,3,4)
   z=5
   y=20
   x=100

   Given z=5, so y=20.
   B[1] and b[2] should be either 3 or 4.
   Lets take b[2] =4 b[1]=3 and so b[0]=1 as z = b[0] + b[2] (5=1+4)
   By taking this values , z = b[0] + b[2] ∧ 2 < b[1] < b[2] < 5 satisfies.
   And x=y*z=20*5=100

Question 5: -

a. Well-formed and proper. *Can ignore binding of c.
   We can write it as σ = {b[0] = 3, b[1] = 6, b[2] = 1, b[3] = 4, k = 0, c[0] = 2}
   Result: - 0*b[b[0]]
   - 0*b[3]
   - 0*4
   - 0
   It will terminate correctly.

b. Well-formed and proper. It is an empty state.

c. Well-formed but improper. Value of ‘b’ cannot be integer.

d. Well-formed and proper.
   We can write σ = {b[0] = 3, k = 0}
   Result: - 0*b[b[0]]
   - 0*b[3]
   - error
   It will cause runtime error as size of b is only one.

Question 6: -

a. There is no difference

- Here we can see their, there is no binding of z in σ_0
- So $\sigma_0[z \mapsto 1]$ is $\sigma_0 \cup \{(z, 1)\}$ which means it in σ_0 but added z=1

b. Here x has binding state σ_0, so that the value of x will be changed from x=2 to x=4

c. $\sigma_1 = \{x = \beta+3, y = 2\beta, b = (\beta, 0, 2\beta, \beta)\}$
   - putting $\beta = 2$
   - $\sigma_1 = \{ x=5, y=4, b=(2,0,4,2)\}$
   - Now, $\sigma_1[\ b[0] \mapsto \sigma_1(b[2])\ ]$
   - $\sigma_1[\ b[0] \mapsto 4]$
   - $\sigma_1 = \{x=5, y = 4 , b = (4, 0,4,2)\}$ -------(1)

d. Now taking values from the equation
   - $\tau[\ b[1] \mapsto \sigma_1(b[1]) + 8\ ]$
   - $\tau[\ b[1] \mapsto 0 + 8\ ]$
   - $\tau[\ b[1] \mapsto 8\ ]$
   - $\tau=\{x=5,y=4,b=(4,8,4,2)\}$

Question 7: -

a. No, Here $\sigma = \{ x = 4, y = 7, b[0] = 5, b[1] = 4, b[2] = 8\}$
   If we take any value of m between 0 to 2, it does not satisfies $b[m] < x < y$
   $4 \not< 4 < 7$
   $5 \not< 4 < 7$
   $8 \not< 4 <7$

b. Yes
   - since b[0], b[1], b[2] are positive and greater than x = 1.
   - So if value of k falls between 0 to 3 than b[k] are positive and greater than 1 which makes left side (let's say p) of implication true. Right side (let's say q) of implication is always true as b[k] is greater than 1.
   - Now in other possibility p will be false if value of k is outside 0 to 3. So if p is false, the whole implication will be true regardless of the value of q.

c. No,
   - It will be true if value of k falls between 0 to 3 as $0<k<3$ (call it as p) is true and $x < b[k]$ (call It as q) is also true.
   - But if value of k is outside range 0 to 3 than p will be false. If any proposition is false in $\wedge$, it will make whole expression false. Hence it will not satisfy.

Question 8: -

a. $\nvDash (\forall\ x \in V\ .\ (\exists\ y \in U\ .\ P(x, y)) \wedge (\forall\ z \in U\ .\ Q(x, z)))$
   - This holds true iff for this σ , for some $\alpha \in$ V, if for every $\beta \in$ U , $\sigma[x \mapsto \alpha][y \mapsto \beta] \nvDash P(x,y)$, or for some $\delta \in U$, $\sigma[x \mapsto \alpha][[z \mapsto \delta] \nvDash Q(x,z)$ (By applying DeMorgan's Law)
   - It holds when anyone of p or q is false.

b. $\not\models \forall\, y \in V\, .\, ((\exists\, x \in W.\ P(x, y)) \rightarrow (\exists\, y \in U\, .\, Q(x, y)))$

- Satisfaction occurs when $(\exists\, y \in U\, .\, Q(x, y))$ is false.
- This hold true iff for this σ , for some $\alpha \in V$, if for some $\beta \in W$, $\sigma[x \mapsto \alpha][y \mapsto \beta] \models P(x,y)$, then for every $\delta \in W$, $\sigma[x \mapsto \alpha][[y \mapsto \delta] \not\models Q(x,y)$

c. $\sigma \not\models (\exists\, x \in W\, .\, (\forall\, y \in U\, .\, P(x, y)))$

- This holds satisfaction when for some state σ , for every $\alpha \in W$, and for some $\beta \in U$, we have $\sigma[x \mapsto \alpha][y \mapsto \beta] \not\models P$