CS536 Homework 5

Weakest Preconditions 1 & 2; Domain Predicates

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Answers: -

Question 1:

IFN ≡ if B1 ➞ S1 ☐ B2 ➞ S2 fi

Then wp (IFN, q) ⇔ ((B1 ∧ W1) ∨ (B2 ∧ W2) ∨ (B1 ∧ B2 ∧ W1 ∧ W2)) But for

p' we need to conjunct B1 ∧ B2 ∧ W1 ∧ W2 Otherwise, wp (IFN, q) ➞ (B1 ∧

W1) ∨ (B2 ∧ W2) and

This cannot be true -wp (IFN, q) ⇐ (B1 ∧ W1) V (B2 ∧ W2)

Because B1 and B2 both hold true then we need W1 and W2 to hold the same.

And for the third conjunct it never holds true because it requires B ∧ -B.

Question 2:

Wp (S, p∨q) → wp (S, p) ∨ wp (S, q)

Here, if σ⊨ wp (S, p∨q), then ⊨p∨q but it does not imply U ⊨p and U ⊨ q

because τ ∈ U does not satisfy all p and q. So, this case is not valid. Wp (S, p) ∨ wp(S, q) → wp(S, p∨q)

If σ ⊨ wp (S, p) ∨ wp (S, q) then it implies that σ ⊨wp (S, p) or σ ⊨wp(S,q)

therefore τ ∈ U which implies that τ ⊨ p or τ ⊨q, so either way it implies τ ⊨p ∨ q that implies σ ⊨ wp (S, p ∨ q)

- wp (S, p∧ q) →and←wp(S, p) ∧wp(S,q)

In this case, for S in a state σ satisfies p ^ q from wp therefore τ ∈ U satisfies p ^ q which implies that τ ⊨ p and τ ⊨ q which implies that σ ⊨ wp (S, p) and σ ⊨ wp(S, q)

wp (S, p) ^ wp(S, q) → wp(S,p ^ q)

If σ ⊨ wp(S, p) ^ wp(S, q) then it implies that

σ ⊨ wp(S,p) and σ⊨ wp(S,q)

therefore τ ∈ U which implies that τ ⊨p and τ ⊨q, So it implies that τ ⊨ p^q that implies σ ⊨ wp(S, p ^ q

Question 3:

if S is deterministic, then ⊨tot {b} S {q}.

⊨tot{b}S{q}is true, because b is stronger that wpw (b⟶w)

- If S is nondeterministic, then there exists σ such that σ ⊨ {¬ c} S {¬ q}.

σ ⊨ {¬ c} S {¬ q) is true because σ ⊨ {¬w } S { ¬q} and w → c implies ¬ c → ¬ w that follows σ⊨{¬c}S{¬q)

- If S is nondeterministic, then there exist σ ⊨ ¬c and τ ∈ M (S, σ) such that τ ⊨ q.

True as per the definition of correctness. If τ ∈(S, σ) such that τ ⊨ q then σ is in the wp and wlp of S and q

Question 4

wlp(u := u*k; k := u, u > h(k))

≡ wlp(u := u*k, wlp(k := u, u > h(k)))

≡ wlp(u := u*k, u > h(u))

≡ u*k > h(u*k)

Question 5

wlp(if x < 0 then x := –x fi, $x^2 \geq x$)

≡ wlp(if x < 0 then x := –x else skip fi, $x^2 \geq x$)

≡ (x < 0 → wlp(x := -x, $x^2 \geq x$)) ∧ (x ≥ 0 → wlp(skip, $x^2 \geq x$))

≡ (x < 0 → $-x^2 \geq -x$) ∧ (x ≥ 0 → $x^2 \geq x$))

Question 6

wp(y := y/x, sqrt(y) < x)

- Let S ≡ y:=y/x, q ≡ sqrt(y) <x, and w ≡ wlp(S, q).
- We can expand w ≡ wlp(S, q) ≡ wlp(y:=y/x, sqrt(y) <x) ≡ sqrt(y/x) <x.
- We can calculate
  D(sqrt(y/x)<x
  ≡ D(y/x) ∧ y/x ≥ 0
  ≡ D(y) ∧ D(x) ∧ x ≠ 0 ∧ y/x ≥ 0 ≡ x ≠ 0 ∧ y/x ≥ 0 (D(y) and D(x) is T)
- wp(S, q) ≡ D(S) ∧ w ∧ D(w)
  ≡ D(y:=y/x) ∧ sqrt(y/x) <x ∧ D(sqrt(y/x) <x)

$\equiv (D(y) \wedge D(x) \wedge x \neq 0) \wedge sqrt(y/x) < x \wedge x \neq 0 \wedge y/x \geq 0$
$\equiv sqrt(y/x) < x \wedge x \neq 0 \wedge y/x \geq 0$

Question 7

Wp (if $y \geq 0$ then x: = y / x else x: = –x/y fi, $r < x \leq y$)

By definition,

wp (S, q) = $D(S) \wedge D(w) \wedge w$

where,

$q=r < x \leq y$

So, w = wlp (S, q)

= wlp (if $y \geq 0$ then x: = y/x else x: = -x/y, $q=r < x \leq y$)

$= (y \geq 0 \rightarrow wlp (x: = y/x, r < x \leq y)) \wedge (y < 0 \rightarrow wlp (x: = -x/y, q=r < x \leq y))$

$= (y \geq 0 \rightarrow r < (y/x) \leq y) \wedge (y < 0 \rightarrow r < (-x/y) \leq y)$

Now,

$D(w) = D ((y \geq 0 \rightarrow r < (y/x) \leq y) \wedge (y < 0 \rightarrow r < (-x/y) \leq y))$

$= D (y \geq 0 \rightarrow r < (y/x) \leq y) \wedge D (y < 0 \rightarrow r < (-x/y) \leq y)$

$= x \neq 0 \wedge y \neq 0$

Now,

D(S)=D (if $y \geq 0$ then x: = y/x else x: = -x/y)

$= D (y \geq 0) \wedge (y \geq 0 \rightarrow D (x: = y/x)) \wedge (z < 0 \rightarrow D (x: = -x/y))$

$= T \wedge (y \geq 0 \rightarrow x \neq 0) \wedge (z < 0 \rightarrow y \neq 0)$

$= (y \geq 0 \rightarrow x \neq 0) \wedge (z < 0 \rightarrow y \neq 0)$

So,

Wp (S, q) = $D(S) \wedge D(w) \wedge w$

$(y\geq 0 \rightarrow x \neq 0) \wedge (z<0 \rightarrow y\neq 0) \wedge x\neq 0 \wedge y\neq 0 \wedge (y\geq 0 \rightarrow r<(y/x) \leq y) \wedge (y<0 \rightarrow r<(-x/y) \leq y)$