CS 536 HW3

Language Syntax, Semantics, Runtime Errors

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Question 1

(a)
```
x:=0;
m: = x;
y:=1;
while (m<n)
do
m: =m+1;
x: =x+1;
y: =y*x
od;
m: = m+1
```

(b)
```
m: =n;
z: =1;
x: =z;
m= m-1;
while (m<n)
do
x: =x+1;
x: =x+2
m = m-1
od
```

(c)
```
m: =n;
x: =1;
v: =x;
m: =m-1;
while (m<n)
do
x: = x+1;
v: = v+x;
m: = m-1
od
```

(d) m:= n;
y: =1;
p: =y;
m: = m-1;
while (m<n)
do
p: =p+1;
p: = p+y;
p: =p+1;
p: p+z;
m: = m-1;
od

QUESTION 2

(a) < IF {x=3, y=5,z=9} >

↓

< x: =x*2, {x=3, y=5, z=9} > (σ(x>0) = T)

↓

<E, σ [x ↦ α] >

Where α = σ(x*z)

= σ (x ↦3*9)

= σ (x ↦ 21)

(b) < IF, {x=3, y=4, z=3} >

↓

< y: ≠y*z, {x=3, y=4, z=3y} >

↓

<E, σ [y ↦α] >

Where α= σ(y*z)

= σ (y ↦4*3)

= σ(y↦12)

(c) <IF, {x=5, y=-2, z=-2} >

↓

< IF y>0 then y: =y*2 else skip f: σ)

↓

<skip, σ>

↓

<E, σ>

Where σ= {x=-5, y=-2, z=-2)

QUESTION 3

Since S is going to execute multiple times, so its execution of S in an arbitrary state $\tau [m \mapsto \alpha] [x \mapsto \beta]$ is

$< S, \tau [m \mapsto \alpha] [x \mapsto \beta] >$

$<m: = m+1; x: =x+m *m, \tau [m \mapsto \alpha] [x \mapsto \beta] >$

$< x: = x+m*m, \tau [m \mapsto \alpha+1] [x \mapsto \beta] >$

$< E, \tau [m \mapsto \alpha+1] [x \mapsto \beta +(\alpha+1) *(\alpha+1)>$

$<W, \tau [m \mapsto \alpha+1] [x \mapsto \beta + (\alpha+1) *(\alpha+1)] >$

Now lets evaluate

$<W, \sigma_0>$

$\rightarrow^2 <W, \sigma 1[m \mapsto] [x \mapsto 2]>$ (After loop initialization)

$\rightarrow^3 < W, \sigma 2[m \mapsto 2] [x \mapsto 6]>$ (one iteration)

$\rightarrow^3 <W, \sigma 3 [m \mapsto 3] [x \mapsto 15] >$ (two iteration)

$\rightarrow^3 <W, \sigma 4 [m \mapsto 4] [x \mapsto 31] >$ (three iteration)

$< E, \sigma 4 [m \mapsto 4] [x \mapsto 31]>$ (now we stop as $m \neq n$ is false)

QUESTION 4:

W= while m≠n do s od
Where s=m: =m+1; x: = x+m≠m
σ_0 = {m=0, x=1, n=4}
(a) <w, σ0> = M (x: =x≠2, {x=3, y=5, z=9}) M (s, *σ)*
=M (s, σ)
=M(σ(x) ↦ σ(x) σ(z))
=M(σ(x) ↦3*9)
=M(σ(x) ↦27)
So, the value of x in σ state will be 27.
(b) M (y: =y*z, σ)
M(σ(y) ↦ σ(y) * σ(z))
M(σ(y) ↦ (4) *(3))
M(σ(y) ↦ 12)
(c) M < s, {x=-5, y=-2, z=-2y>
<skip, *σ*>
<E, *σ*>

QUESTION 5:

S= m: = m+1;X: = X+m*m

W= while m≠n do s od

(a) Now we considering the τ for s then
M (s, τ) = M (m: = m+1;X: = X+m*m, τ [m ↦β] [x↦α])
={τ[m↦β+1] [x↦α+(β+1)*(β+1)]}

(b) σ0= {m=0, x=0}
Here we are looking for the value in which the β ε z that make M (w, σ0[n↦β]) =⊥d

So, from the entire loop condition, if the value of m and n is equal then loop is stops and in the given value of, is zero and it is increasing by 1 for the n<0 there is now option available for m equal to n.

(c) $\sigma = \{m = \beta, x = \delta, n = \beta\}$

From previous problem we know $M(s, \tau) = \{\tau[m \mapsto \beta+1]\ [x \mapsto \alpha+(\beta+1)*(\beta+1)]\}$

so the sequence of states

$M(S, \sigma_0[n \mapsto \beta]) = \{m=1, x=0+(1*1), n=\beta\}$

$M(S, \sigma_1[n \mapsto \beta]) = \{m=2, x=(0+(1*1)) + 2*2, n=\beta\}$

$M(S, \sigma_2[n \mapsto \beta]) = \{m=3, x=((0+(1*1))+2*2) + 3*3, n=\beta\}$ and so on.

when $m=\beta$ than $x=0+1^2 + 2^2 + 3^2 + \ldots + \beta^2$

so by given statement in question we can say

$\delta=0+1^2 + 2^2 + 3^2 + \ldots + \beta^2$

which is sum of squares of all integers from 0 to β.

QUESTION 6

According to the given statement, three cases can be considered for ⊥e

1. $\sigma(b[m+1]) = \perp e$
   this diverges when array index out of bounds so we can write
   $\sigma\ (m+1)<0$ or $\sigma\ (m+1) \geq \sigma\ (size(b))$
   iff $\alpha < -1$ or $\alpha \geq \delta - 1$ ($\sigma\ (m) = \alpha$, size of array = δ)

2. $\sigma(sqrt(k)) = \perp e$
   if k is negative , square root of negative number gives error.
   so iff $\sigma(k)<0$
   iff $\beta < 0$

3. $\sigma(k) = \perp e$
   if k is 0, its square root will be 0. This will lead to division by zero error.
   iff $\beta = 0$

   so from above cases we can say runtime errors occur when
   $\alpha < -1$ or $\alpha \geq \delta - 1$ or $\beta \leq 0$