CS 536 HW 1

Logic Review (HW 1 : Lectures 1-2)

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Answers:-

Question 1

a. p is sufficient for q :- p →q
b. p only if q :- p →q
c. p if q :- q →p
d. p is necessary for q :- q →p

Question 2

a. As we studied in Lecture 1 that ≡→= so its contrapositive is also true which is ≠→≢.
   E.g., 3+3 ≠7, so 3+3 ≡ 7.
b. Sematic equality does not imply syntactic equality.
   E.g., 1) c+0 ≢ c , 2) p ∧ q ≢ q ∧ p .

Question 3

a. Ill – formed because it is missing binding for z.
b. Well-formed. It is proper but will give domain error at runtime.
c. Well-formed and proper. It will give runtime error that it cannot divide by 0.

Question 4

$p \wedge \neg(q \wedge r) \rightarrow q \wedge r \rightarrow \neg p$	
$p \wedge \neg(q \wedge r) \rightarrow (\neg(q \wedge r) \vee \neg p)$	Definition of →
$\neg(p \wedge \neg(q \wedge r)) \vee (\neg(p \wedge r) \vee \neg p)$	Definition of →
$\neg p \vee (q \wedge r) \vee \neg(q \wedge r) \vee \neg p$	DeMorgan's Law
$\neg p \ \vee p \ \wedge r \ \vee \neg q \ \vee \neg r \ \vee \neg p$	DeMorgan's Law
$\neg p \ \vee q \ \wedge r \ \vee \neg r \ \vee \neg q \ \vee \neg p$	Commutativity of ∨
$\neg p \ \vee q \ \wedge \mathrm{T} \vee \neg q \ \vee \neg p$	Excluded middle
$\neg p \ \vee q \ \vee \neg q \ \vee \neg p$	Identity
$\neg p \ \vee \mathrm{T} \vee \neg p$	Excluded middle
$\neg p \ \vee \mathrm{T}$	Idempotency
T	Domination

Question 5

$\neg (\forall x. (\exists y. x \le y) \vee \exists z. x \ge z)$

>>> Now we apply the de Morgan's laws,

$(\exists x. (\forall y. x > y) \wedge \forall z. x < z)$

>>> By negation of comparison

$\neg (\forall x. (\exists y. x \le y) \vee \exists z. x \ge z)$

$(\exists x. (\forall y. x > y) \wedge \forall z. x < z)$

Question 6

1) $p \wedge \neg r \wedge s \rightarrow \neg q \vee r \rightarrow \neg p \leftrightarrow \neg s \rightarrow t$

   $((((p \wedge (\neg r)) \wedge s) \rightarrow (((\neg q) \vee r) \rightarrow (\neg p))) \leftrightarrow ((\neg s) \rightarrow t))$

2) $\exists m. 0 \le m < n \wedge \forall j. 0 \le j < m \rightarrow b[0] \le b[j] \le b[m]$ *

   $(\exists m. ((0 \le m < n) \wedge (\forall j . ((0 \le j < m) \rightarrow ((b[0] \le b[j]) \wedge (b[j] \le b[m]))))))$

Question 7

1) $((\neg(p \vee q) \vee r) \rightarrow (((\neg q) \vee r) \rightarrow ((p \vee (\neg r)) \vee (q \wedge s))))$

   $(_1\ (_2\ \neg(_3\ p \vee q\ )_3\ \vee\ r\ )_2\ \rightarrow\ (_4\ (_5\ (_6\ \neg q\ )_6\ \vee\ r\ )_5\ \rightarrow (_7\ (_8\ p \vee (_9\ \neg r\ _9)\ )_8 \vee\ (_{10}\ q \wedge s\ _{10})\ _7)\ _4)\ _1) \equiv (\neg(p \vee q) \vee r) \rightarrow (\neg q \vee r) \rightarrow (p \vee \neg r \vee q \wedge s)$

2) $(\exists i . (((0 \le i) \wedge (i < m)) \wedge (\forall j . (((m \le j) \wedge (j < n)) \rightarrow (b[i] = b[j])))))$

   $\exists i . 0 \le i \wedge i < m \wedge \forall j . m \le j \wedge j < n \rightarrow b[i] = b[j]$

3) $\forall x.((\exists y.(p \rightarrow q)) \rightarrow (\forall z.(q \vee (r \wedge s)))))$

   $\forall x. \exists y. p \rightarrow q \rightarrow \forall z. q \vee (r \wedge s)$

Question 8

a. $p \wedge q \vee \neg r \rightarrow \neg p \rightarrow q \equiv ((p \wedge q) \vee ((\neg r \rightarrow ((\neg p) \rightarrow q))))$

P	q	r	p ∧ q	¬r	(p ∧ q) ∨ ¬r	¬p	¬p → q	(p ∧ q) ∨ ¬r → (¬p → q)	(¬r → (¬p → q))	(p ∧ q) ∨ (¬r → (¬p → q))
T	T	T	T	F	T	F	T	T	T	T
T	T	F	T	T	T	F	T	T	T	T
T	F	T	F	F	F	F	T	T	T	T
T	F	F	F	T	T	F	T	T	T	T
F	T	T	F	F	F	T	T	T	T	T
F	T	F	F	T	T	T	T	T	T	T
F	F	T	F	F	F	T	F	T	T	T
F	F	F	F	T	T	T	T	T	T	T

From the above the above truth table we can conclude that predicates are logically equivalence.

b. They are not syntactically equal. The reason is the perenthesis done on predicate on right side are not correct. As implication is right associativity, the parenthesis should be from right side.

c.

d. They are syntactically equal. The reason is the parenthesis done on right predicate do not effect due to associativity of ∨.

Question 9

a.

	p	q	R	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$(p \rightarrow q) \rightarrow r$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$
1	F	F	F	T	T	T	F	F
2	F	F	T	T	T	T	T	T
3	F	T	F	T	F	T	F	F
4	F	T	T	T	T	T	T	T
5	T	F	F	F	T	T	T	T
6	T	F	T	F	T	T	T	T
7	T	T	F	T	F	F	F	T
8	T	T	T	T	T	T	T	T

As from truth table we can observe contingency at 1) where p=q=r if False and 3) where p=r=False and q=True

b.

Question 10

- GT(b, x, m, k) that yields true iff x > b[m], … b[m+k-1]
  - To describe we can say each element in the list b[m], b[m+1], …, b[m+k-2], b[m+k-1] is < x.
  - Further expanding we can write b[m] < x, b[m+1] < x, …, b[m+k-1] < x.
  - We can generalize this to b[i] < x for i=m, m+1, m+2, …, m+k-1.
  - To get a formal predicate we need ∀ over i:
    ∀ i. m ≤ i<m+k-1 → b[i] < x