

CS536 Homework 7

Proof Rules and Proofs

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Answers: -

Question 1

$$p_1 \equiv wp(k:=k+1, p) \equiv (x = 2^k \wedge k \leq n) [k / k+1] \equiv x = 2^{k+1} \wedge k+1 \leq n$$

$$\{p_2\} x := x * 2 \{p_1\}$$

$$p_1 \equiv x = 2^{k+1} \wedge k+1 \leq n$$

$$\text{Now, } p_2 \equiv x = 2^{k+1} \wedge k+1 \leq n [x * 2 / x]$$

$$\equiv x * 2 = 2^{k+1} \wedge k+1 \leq n$$

$$r_1 \equiv \text{precondition strengthen, 4, 3}$$

$$r_2 \equiv \text{loop, 5}$$

Question 2

$$p_1 \equiv p \wedge k < n \equiv x = 2^k \wedge k \leq n \wedge k < n$$

$$p_2 \equiv sp(p_1, x := x * 2) \equiv x = 2^k \wedge k \leq n \wedge k < n [x_0 / x] \wedge x = x * 2 [x_0 / x]$$

$$\equiv x_0 = 2^k \wedge k \leq n \wedge k < n \wedge x = x_0 * 2$$

$$p_3 \equiv sp(p_2, k := k+1) \equiv (x_0 = 2^k \wedge k \leq n \wedge k < n \wedge x = x_0 * 2) [k_0 / k] \wedge k = k+1 [k_0 / k]$$

$$\equiv x_0 = 2^{k_0} \wedge k_0 \leq n \wedge k_0 < n \wedge x = x_0 * 2 \wedge k = k_0 + 1$$

$$R_1 \equiv \text{post conditioning weakening 4,3}$$

$$R_2 \equiv \text{loop 5}$$

Question 3

$$q_1 \equiv wp(x := x/2, r = X * Y - x * y) \equiv r = X * Y - x * y [x / x/2] \equiv r = X * Y - (x/2) * y$$

$$R_1 \equiv \text{assignment(backward)}$$

$$q_2 \equiv wp(y := 2 * y, q_1) \equiv r = X * Y - (x/2) * y [y/2 * y] \equiv r = X * Y - (x/2) * (2 * y)$$

$$R_2 \equiv \text{assignment(backward)}$$

$$q3 \equiv (r = X * Y - x * y) \wedge (x \% 2 = 0)$$

$$q4 \equiv (r = X * Y - x * y) \wedge (x \% 2 \neq 0)$$

R3 \equiv assignment (forward)

$$q5 \equiv r0 = X * Y - x * y \wedge x \% 2 \neq 0 \wedge x = x0 \wedge r := r0 + y$$

$$q6 \equiv r0 = X * Y - x0 * y \wedge x0 \% 2 \neq 0 \wedge r := r0 + y \wedge x := x0 - 1$$

R4 \equiv assignment (forward)

R5 is conditional 5, 10